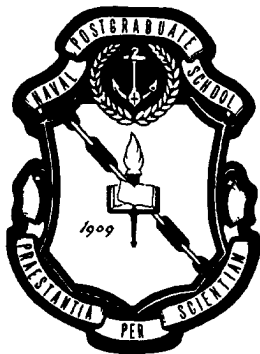


Naval Postgraduate School
Monterey, California 93943-5138



SUMMARY OF RESEARCH 1997

Department of Mathematics

**W. Max Woods
Chair**

**Beny Neta
Associate Chair for Research**

Approved for public release; distribution is unlimited.

Prepared for: Naval Postgraduate School¹
Monterey, CA 93943-5000

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral R.C. Chaplain
Superintendent

R. Elster
Provost

This report was prepared for the Naval Postgraduate School, Monterey, CA.

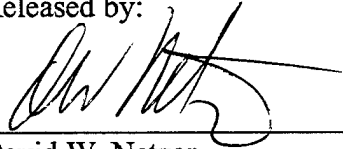
Reproduction of all or part of this report is authorized.

Reviewed by:



Danielle A. Kuska
Director, Research Administration

Released by:



David W. Netzer
Associate Provost and Dean of Research

REPORT DOCUMENTATION PAGE

Form approved

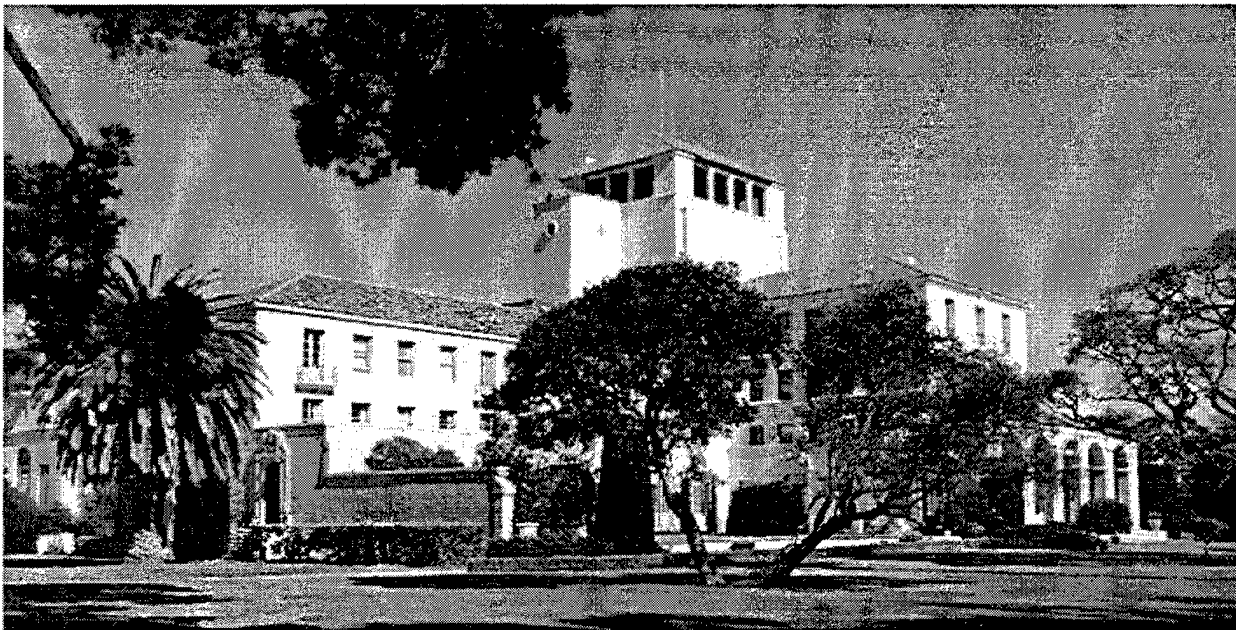
OMB No 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE January 1999	3. REPORT TYPE AND DATES COVERED Summary Report, 1 January 1997 - 31 December 1997	
4. TITLE AND SUBTITLE Summary of Research 1997, Department of Mathematics			5. FUNDING	
6. AUTHOR(S) Faculty of the Department of Mathematics, Naval Postgraduate School				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			8. PERFORMING ORGANIZATION REPORT NUMBER NPS-09-98-014	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES The views expressed in this report are those of the authors and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE A	
13. ABSTRACT (Maximum 200 words.) This report contains summaries of research projects in the Department of Mathematics. A list of recent publications is also included which consists of conference presentations and publications, books, contributions to books, published journal papers, technical reports, and thesis abstracts.				
14. SUBJECT TERMS			15. NUMBER OF PAGES 56 -	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	

THE NAVAL POSTGRADUATE SCHOOL MISSION

The mission of the Naval Postgraduate School is to increase the combat effectiveness of U.S. and Allied armed forces and enhance the security of the USA through advanced education and research programs focused on the technical, analytical, and managerial tools needed to confront defense-related challenges



CONTENTS

Preface	7
Introduction	9
Faculty Listing	11
Department Summary	13
Project Summaries	15
Algebraic Multigrid for Large-Scale Simulations on Unstructured Grids: Phase II	24
Algorithms for Segmented Data	21
Applications of Bifurcation Control by State Feedback	24
Application of Periodic Optimal Control to Space Maneuvers	18
Bi-orthogonality Relations Applied to Scattering in Waveguides	30
Close Combat Anti-Armor Weapon System (CCAWS) Technology Analysis	26
Comanche Program Review	17
Compression of Images in Support of Tactical Operations	15
Covariance Functions for 3-dimensional Data Assimilation I, II	21
De Bruijn Sequences for Cryptographic Applications	22
Evaluation of Completion-Based Heuristics for Graph Coloring	28
Fast, Stable, Computational Algorithms for Spectral Decomposition of Stationary Time Series	23
Finite Differences Versus Finite Elements	27
Integrated Assessment of Ship Missile Defense Effectiveness	31
Least Squares Multiquadric Approximation	20
Line and Circle Tracking for Nonholonomic Autonomous Vehicles	19
Linear Elastic Behavior of Orthogonally Stiffened Plate Panels	16
Modeling Amphibious Landings Involving Mine Warfare	15
Numerical Simulation of Thermocapillary Convection in Welding	16
Numerical Study of Exponential Stability of Coupled Fluid/Structure Systems	19
Optimal Design of Damping and Control Mechanisms for Distributed Parameter Systems	18
p-Competition Graphs: Chromatic Properties and Characterizations	29
Parallel Version of Special Perturbation	26
Reliability Modeling of Concurrent Software Modules	24
Removal of the Assumption of Cellular Targets in the Computation of Damage Aggregation to an Area Target From a Salvo of N Weapons	30
Targeting Sub-State Political Groups	28
Publications and Presentations	33
Thesis Abstracts	37
Initial Distribution List	47

PREFACE

Research at the Naval Postgraduate School is carried out by faculty in the School's eleven academic departments, four interdisciplinary groups, and the School of Aviation Safety. This volume contains research summaries for the projects undertaken by faculty in the Department of Mathematics during 1997. Also included is an overview of the department, faculty listing, a compilation of publications/presentations, and abstracts from theses directed by the department faculty.

Questions about particular projects may be directed to the faculty Principal Investigator listed, the Department Chair, or the Department Associate Chair for Research. Questions may also be directed to the Office of the Associate Provost and Dean of Research. General questions about the NPS Research Program should be directed to the Office of the Associate Provost and Dean of Research at (831) 656-2098 (voice) or research@nps.navy.mil (e-mail). Additional information is also available at the RESEARCH AT NPS website, <http://web.nps.navy.mil~code09/>.

INTRODUCTION

The research program at the Naval Postgraduate School exists to support the graduate education of our students. It does so by providing militarily relevant thesis topics that address issues from the current needs of the Fleet and Joint Forces to the science and technology that is required to sustain the long-term superiority of the Navy/DoD. It keeps our faculty current on Navy/DoD issues, permitting them to maintain the content of the upper division courses at the cutting edge of their disciplines. At the same time, the students and faculty together provide a very unique capability within the DoD for addressing warfighting problems. This capability is especially important at the present time when technology in general, and information operations in particular, are changing rapidly. Our officers must be able to think innovatively and have the knowledge and skills that will let them apply technologies that are being rapidly developed in both the commercial and military sectors. Their unique knowledge of the operational Navy, when combined with a challenging thesis project that requires them to apply their focussed graduate education, is one of the most effective methods for both solving Fleet problems and instilling the life-long capability for applying basic principles to the creative solution of complex problems.

The research program at NPS consists of both reimbursable (sponsored) and institutionally funded research. The research varies from very fundamental to very applied, from unclassified to all levels of classification.

- **Reimbursable (Sponsored) Program:** This program includes those projects externally funded on the basis of proposals submitted to outside sponsors by the School's faculty. These funds allow the faculty to interact closely with RDT&E program managers and high-level policymakers throughout the Navy, DoD, and other government agencies as well as with the private sector in defense-related technologies. The sponsored program utilizes Cooperative Research and Development Agreements (CRADAs) with private industry, participates in consortia with other government laboratories and universities, provides off-campus courses either on-site at the recipient command or by VTC, and provides short courses for technology updates.
- **NPS Institutionally Funded Research Program (NIFR):** The institutionally funded research program has several purposes: (1) to provide the initial support required for new faculty to establish a Navy/DoD relevant research area, (2) to provide support for major new initiatives that address near-term Fleet and OPNAV needs, (3) to enhance productive research that is reimbursable sponsored, (4) to contribute to the recapitalization of major scientific equipment, and (5) to cost-share the support of a strong post-doctoral program.
- **Institute for Joint Warfare Analysis (IJWA) Program:** The IJWA Program provides funding to stimulate innovative research ideas with a strong emphasis on joint, interdisciplinary areas. This funding ensures that joint relevance is a consideration of research faculty.

In 1997, the overall level of research effort at NPS was 151 faculty workyears and exceeded \$32 million. The Department of Mathematics' effort was 5.67 faculty workyears and exceeded \$500 thousand. The sponsored research program has grown steadily to provide the faculty and staff support that is required to sustain a strong and viable graduate school in times of reduced budgets. In FY97, over 87% percent of the NPS research program was externally supported. In the Department of Mathematics 67% was externally supported.

The department's research sponsorship in FY97 is provided in Figure 1.

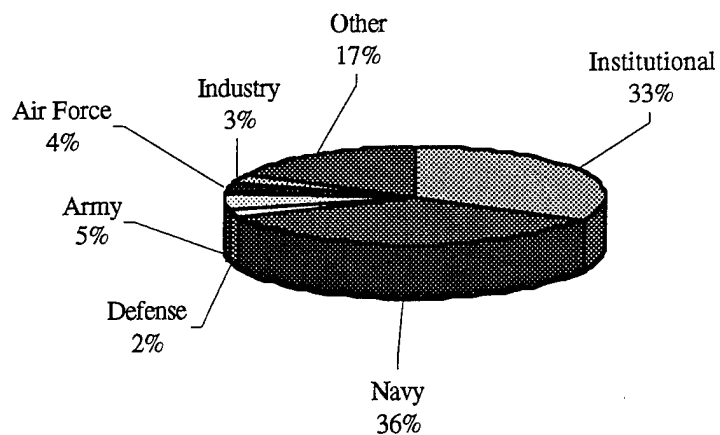


Figure 1. FY97 Sponsor Profile of the Department of Mathematics

These are both challenging and exciting times at NPS and the research program exists to help ensure that we remain unique in our ability to provide graduate education for the warfighter.

DAVID W. NETZER
Associate Provost and Dean of Research

January 1999

FACULTY LISTING

Woods, W. Max
Professor and Chair
MA/Wo
831-656-2206 (phone)
831-656-2355 (fax)
mwoods@wposmtp.nps.navy.mil

Neta, Beny
Professor and Associate Chair for Research
MA/Nd
656-2124
bneta@nps.navy.mil

Borges, Carlos
Associate Professor
MA/Bc
656-2124
borges@nps.navy.mil

Frenzen, Chris
Associate Professor
MA/Fr
656-2435
cfrenzen@math.nps.navy.mil

Owen, Guillermo
Professor
MA/On
656-2720
owen@nps.navy.mil

Canright, David
Associate Professor
MA/Ca
656-2782
dcanright@nps.navy.mil

Gragg, William B.
Professor
MA/Gr
656-2194
gragg@nps.navy.mil

Rasmussen, Craig W.
Associate Professor
MA/Ra
656-2763
ras@nps.navy.mil

Danielson, Don A.
Professor
MA/Dd
656-2622
dad@nps.navy.mil

Jayachandran, Toke
Professor
MA/Jy
656-2600
tj@nps.navy.mil

Russak, I. Bert
Associate Professor
MA/Ru
656-2293
ibrussak@nps.navy.mil

Fahroo, Fariba
Assistant Professor
MA/Ff
656-2664
fahroo@math.nps.navy.mil

Kang, Wei
Assistant Professor
MA/Kw
656-3337
wkang@nps.navy.mil

Scandrett, Clyde
Associate Professor
MA/Sd
656-2027
cscand@nps.navy.mil

Franke, Richard
Professor
MA/Fe
656-2758
rfranke@nps.navy.mil

Latta, G.E.
Professor Emeritus
MA/Lz
656-3540
gelatta@nps.navy.mil

Schoenstadt, Arthur
Professor
MA/Zh
656-2662
alschoen@nps.navy.mil

Fredricksen, Harold
Professor
MA/Fs
656-3249
half@nps.navy.mil

Mansager, Bard K.
Senior Lecturer
MA/Ma
656-2695
bardman@nps.navy.mil

Weir, Maurice
Professor
MA/Wc
656-2608/3059
mweir@nps.navy.mil

DEPARTMENT SUMMARY

The research program of the Department of Mathematics seeks to advance the state of knowledge in the areas important to the Department of the Navy and Department of Defense, such as scientific and parallel computing, fluid flow, orbital mechanics, graph theory, and simulation and modeling.

The specific areas of our faculty and their students are reported in detail later in this book. Output in the form of student theses, technical reports, conference presentations, and refereed journal articles is listed.

Professors Borges and Mansager continued their work on mine countermeasures. Part of this work was presented at the 65th MORS conference and has been nominated for the Barchi prize. Professor Borges also worked on tactical image compression with Professor Fredricksen. This project involved an investigation of some current DoN capabilities, in particular, the radiant TIN algorithm. As part of this work Professor Borges developed a number of improvements to the *histogram* method of estimating the parameters of a Gibbs distribution. These improvements include an MMSE estimator for the local interaction sums of an auto-binomial Markov Random Field, and a direct solution for the field parameters following the generation of the histogram.

Professor Canright continued his work on thermocapillary convection in welding, particularly trying to determine the scaling and structure of the "cold-corner singularity" in thermocapillary flow in weld pools. His current efforts involve extending the working simulation code through the development, implementation, and testing of two additional components: the moving phase-change boundary and the Eulerian-Lagrangian Method (ELM) for the nonlinear convective terms. The details of discretization and implementation for these two components were fully worked out, and most of the programming was completed and partly debugged.

Professor Danielson continued to investigate the mechanical behavior of stiffened plates, basic structural components of ships and submarines. He is also a collaborator on a research project to study vibration/structural dynamics of the RAH66 Comanche helicopter.

Professor Franke continued his research into the approximation of scattered data using radial basis function methods, in particular multiquadric functions in a least squares setting. Professor Franke also worked on developing methods for more rigorous modeling of the three-dimensional spatial covariance function for the error in numerical weather forecasts, with the goal of improving data assimilation methods. This effort also involved applying methods derived in the first part to real data. In addition, Professor Franke worked extensively on developing algorithms for segmented data, in particular, computationally simple algorithms for constructing a surface that separates the various classes of data and the volumes that contain only data of one of the various classes.

Professor Fahroo worked extensively on applying Optimal Periodic Control theory to determine the optimal reboosting strategy for a Low-Earth-orbiting or LEO spacecraft to achieve minimum consumption of fuel. This research, which is a joint effort with Professor I.M. Ross of the Department of Aeronautics and Astronautics, involved the development of both analytical and numerical solutions for this complex optimization problem. Professor Fahroo also worked on the optimal design of damping and control mechanisms for distributed parameter systems. This work focuses on examining different damping designs for achieving exponential stability of flexible structures. Professor Fahroo also continued her study of the exponential stability of several acoustic-structure models by numerical approximation of these models. The focus of the work was on establishing uniform exponential stability for the model using the multiplier technique that has been used successfully in establishing exponential decay rates for wave equations with boundary feedback damping.

Professor Fahroo was also involved in a joint research project with Professor Y. Kanayama of the Department of Computer Science whose focus was developing an algorithm for the movement of a vehicle under the nonholonomic constraint to track a given directed line without allowing any spinning motion. This effort led to a new principle of computing the derivative of path curvature as a linear combination of the current vehicle path curvature, vehicle orientation, and positional difference. Numerous simulation results as well as experimental results were obtained on the autonomous robot Yamabico at the Naval Postgraduate School that showed the effectiveness of this method.

Professors Frenzen and Scandrett worked jointly to investigate the behavior of Scholte/Rayleigh-Lamb surface wave propagation along the interface of a elastic/poro-elastic solid underlying a fluid layer. In this effort the bi-orthogonality relationships developed for a porous/elastic-fluid layered system were implemented to determine the interfacial wave scattering from a vertical discontinuity in the medium. The dispersion relationship was found and used to determine wavenumbers for the discrete eigenfunctions of the media.

Professor Gragg set out to implement, rigorously test, and prove numerical stability of, new algorithms for executing the second phase of Pisarenko's signal processing algorithm, now reformulated so as to solve a well-conditioned problem. The new algorithms, stabilized forms of the unitary Hessenberg QR (uhqr) algorithm, permit fast, $O(n^2)$, solution of

DEPARTMENT SUMMARY

problems of (essentially) arbitrarily high order. Together, these provide algorithms for fast adaptive recursive least squares modeling of stationary time series by trigonometric polynomials (spectral estimation).

Professors Russak and Jayachandran collaborated in an effort to improve the accuracy of damage calculations to an area target by removing certain non-real world assumptions used in its model. In particular, analysts who do computation of damage aggregation to an area target from a salvo of weapons sometimes use the simplifying assumption of a target consisting of cells. This often leads to the implicit assumption of weapons hits to the target always being at the center of a cell. The removal of this assumption provides a more accurate model with more accurate calculations of damage aggregation.

Professor Jayachandran also worked on developing probability models for the reliability of concurrent software modules used to build redundancy to increase reliability. Unlike hardware, failures of redundant software are correlated and, therefore, the determination of the improvement in reliability is difficult. An algorithm to compute the probability of failure for one of the models proposed in the literature was developed. Work to develop alternative models is in progress.

Professor Kang's work focused on developing a stabilization feedback design methodology for nonlinear control systems near bifurcation points. Two problems of immediate practical interest to the Department of the Navy are control of rotating stall and surge in gas engine compressors, and the design of feedbacks for depth control in the dive plane of submarines near the critical Froude number where pitchfork bifurcation occurs. This exceptional research effort led to four journal publications, several conference presentations, and one PhD thesis this year alone.

Professor Neta worked on a linear analysis of the shallow water equations in spherical coordinates for the Turkel-Zwas explicit large time-step scheme. The analysis suggests that the Turkel-Zwas scheme must be staggered in a certain way in order to get eigenvalues and eigenfunctions approaching those of the continuous case. This is joint work with Frank Giraldo of the Naval Research Laboratory. Professor Neta also investigated a control decomposition approach for parallelizing a numerical orbit propagator.

Professor Owen continued his collaborative work with Professor McCormick, Special Operations Curriculum Committee, targeting sub-state political groups. The goal of this research is to develop a formal framework for evaluating the dynamics of sub-state conflict and to employ this framework to improve our ability to target terrorist and other sub-state politico-military organizations.

Professor Rasmussen continued his efforts to develop and evaluate heuristics for finding approximate solutions to hard combinatorial optimization problems, such as graph coloring, that arise in diverse problems such as scheduling and frequency assignment. These problems are generally NP-complete, but typically are easily solved on certain families of highly structured problem instances. These families themselves possess more internal structure than was previously known, and the idea is to exploit this structure to obtain useful approximate solutions to the hard instances. Professor Rasmussen also continued his work on characterizing competition graphs and p-competition graphs of various highly structured families of graphs and digraphs.

PROJECT SUMMARIES

MODELING AMPHIBIOUS LANDINGS INVOLVING MINE WARFARE

C. F. Borges, Associate Professor
Department of Mathematics
Sponsor: Office of Naval Research

OBJECTIVE: To continue to build upon a model representation of special operations units and mine warfare components in a traditionally land-based combat model. The research will evaluate the ability of the model to capture the salient aspects of these warfare types. These additions, if practical, will make the current JANUS Joint Warfare scenario more robust in its ability to represent operations in a littoral environment. This work will extend our current scenarios of an amphibious landing to include the continuation of operations ashore.

SUMMARY: This research was integrated into the student theses of CPT Bob Lazzell (USA), Capt Ron Middlebrook (USMC), and CPT Kent Wineingar (USA). A fairly complex scenario for an amphibious assault on a mined beach in JANUS was built. This scenario was based on the KERNEL BLITZ 1995 exercise. An attempt was made to duplicate it in some detail. Actual information was used for staging this exercise to determine troop and equipment placement and routes, etc. Some aspects of this work were presented at the 65th MORS conference. Various aspects of this work have been briefed to visiting flag officers and others.

CONFERENCE PRESENTATION:

Middlebrook, E.E., Mansager, B, and Borges, C.F. "A Combat Simulation Analysis of Autonomous Legged Underwater Vehicles," 65th Military Operations Research Society Symposium, Quantico, VA, June 1997. (This presentation was voted best in working group and was nominated for the Barchi prize.)

DoD KEY TECHNOLOGY AREA: Modeling and Simulation

KEYWORDS: Mine Countermeasures, High-Resolution Combat Simulation

COMPRESSION OF IMAGES IN SUPPORT OF TACTICAL OPERATIONS

C. F. Borges, Associate Professor
Hal Fredricksen, Professor
Department of Mathematics
Sponsor: Navy Engineering Logistics Office

OBJECTIVE: To investigate image compression algorithms, especially the Radiant TIN algorithm and its use on tactical imagery.

SUMMARY: Several aspects of this problem were examined. The main initial thrust focused on trying to understand the workings of the Radiant TIN algorithm and to look for areas which might be improved. It was noted that the algorithm might benefit from the use of simple representations of texture that were not deterministic. The focused was primarily on investigating that possibility and developing a number of methods that might allow the use of Markov Random Fields in this capacity. One thesis student worked on this project, and one paper is in review containing results from this project.

PUBLICATION:

Borges, C.F., "On Estimating the Parameters of a Markov Random Field," submitted to *IEEE Transactions on Pattern Analysis and Machine Intelligence*.

PROJECT SUMMARIES

THESIS DIRECTED:

Korn, Chris, "Using Markov Random Fields for Texture Compression," Master's Thesis, Naval Postgraduate School, March 1997.

DoD KEY TECHNOLOGY AREA: Other (Image Processing)

KEYWORDS: Image Compression, Markov Random Fields

NUMERICAL SIMULATION OF THERMOCAPILLARY CONVECTION IN WELDING

D. R. Canright, Associate Professor
Department of Mathematics
Sponsor: Naval Postgraduate School

OBJECTIVE: The goal of this project is to determine the scaling and structure of the "cold-corner singularity" in thermocapillary flow in weld pools. This is a continuing project.

SUMMARY: Recent work in modeling thermocapillary convection in materials processing, for example in the pool of liquid metal formed during welding, shows a region of rapid flow and intense heat transfer, concentrated in the "cold corner" region. A theoretical understanding of this region, currently lacking, is essential for accurate numerical models. The objective of this study is to analyze the coupled thermal and flow fields in this important region. The results should be useful in developing more complete numerical models of the welding process, to understand how to make welds more reliable.

Work in 1997 extended the working simulation code through the development, implementation, and testing of two additional components: the moving phase-change boundary and the Eulerian-Lagrangian Method (ELM) for the nonlinear convective terms. The phase interface moves through the local fixed triangular grid (adapted to the smallest local length scales) as material melts or solidifies, as governed by the heat balance. Because of the variety of ways the interface can cross the grid elements, many different cases had to be considered. The nonlinear convective terms were initially implemented using Finite-Volume Elements (FVE) for the flow velocity as well as the convected quantity (heat or vorticity, depending), but this was found to be inaccurate for convectively-dominated regimes. Hence these terms were reformulated using ELM, which is known to be both accurate and stable (with appropriate restrictions on the local time step) for the full range of parameters. The details of discretization and implementation for these two components were fully worked out, and most of the programming was completed and partly debugged.

DoD KEY TECHNOLOGY AREA: Materials, Processes, and Structures

KEYWORDS: Thermocapillary, Solidification, Welding, Crystal Growth, Marangoni, Convection

LINEAR ELASTIC BEHAVIOR OF ORTHOGONALLY STIFFENED PLATE PANELS

D. A. Danielson, Professor
Department of Mathematics
Sponsor: Naval Surface Warfare Center-Carderock Division

OBJECTIVE: To improve structural design of ships.

SUMMARY: The subject of this work is the mechanical behavior of stiffened plates, basic structural components of ships and submarines. The buckling loads of grillages subjected to axial compression with and without lateral pressure are calculated using a finite element based eigenvalue analysis.

PROJECT SUMMARIES

PUBLICATION:

Danielson, D., "Buckling of Ship Grillages," Naval Postgraduate School Technical Report, NPS-MA-97-005, September 1997.

DoD KEY TECHNOLOGY AREAS: Computing and Software, Modeling and Simulation, Other (Structures, Ships)

KEYWORDS: Plates, Ships, Structures, Buckling

COMANCHE PROGRAM REVIEW

D. A. Danielson, Professor

Department of Mathematics

E.R. Wood, Professor

Department of Aeronautics and Astronautics

J. H. Gordis, Assistant Professor

Department of Mechanical Engineering

Sponsor: U.S. Army Comanche Program Office

OBJECTIVE: To study vibration/structural dynamics of the RAH66 Comanche helicopter.

SUMMARY: Nine structural modifications to the Comanche tailcone were developed conceptually, then analyzed using the NASTRAN/PATRAN finite element code. The addition of radar absorbing material (RAM) on the outer skin of the modified model costs only a 6 per cent reduction in torsional stiffness from baseline values, as compared to a 24 per cent reduction in tailcone stiffness for adding the same amount of RAM were these structural modifications not incorporated into the design. Additional suggested improvements include reversing the tail landing gear assembly, so that it would be anchored to the forward landing gear bay bulkhead and not to the aft landing gear bay bulkhead.

PUBLICATIONS:

Danielson, D. A., "Buckling of Ship Grillages," Naval Postgraduate School Technical Report, NPS-MA-97-005, September 1997.

Danielson, D. A., et al., "Research in the Structural Dynamic Response of the RAH-66 Comanche Helicopter," Naval Postgraduate School Report, December 1997.

THESES DIRECTED:

Shoop, B., "Structural Design Analysis of the Tail Landing Gear Bay and the Vertical/Horizontal Stabilizer of the RAH-66 Comanche Helicopter," Master's Thesis, Naval Postgraduate School, September 1997.

Tobin, V., "Analysis of Potential Structural Design Modification for the Tail Section of the RAH-66 Comanche Helicopter," Master's Thesis, Naval Postgraduate School, June 1997.

DoD KEY TECHNOLOGY AREAS: Computing and Software, Air Vehicles, Modeling and Simulation, Other (Structures)

KEYWORDS: Helicopters, Dynamics, Vibrations

PROJECT SUMMARIES

APPLICATION OF PERIODIC OPTIMAL CONTROL TO SPACE MANEUVERS

F. Fahroo, Assistant Professor
Department of Mathematics
Sponsor: Naval Postgraduate School

OBJECTIVE: To apply the theory of Optimal Periodic Control theory to the problem of orbit control of Low-Earth-orbiting or LEO spacecraft.

SUMMARY: In this research the theory of Optimal Periodic Control was applied to determine the optimal reboosting strategy for a Low-Earth-orbiting or LEO spacecraft to achieve minimum consumption of fuel. Analytical as well as numerical solutions were sought for this optimization problem. This project is a joint project with Professor I. Michael Ross, Department of Aeronautics and Astronautics.

PUBLICATIONS:

Jensen, K., Fahroo, F., and Ross, I.M., "Application of Optimal Periodic Control Theory to the Orbit Reboost Problem," *Proceedings of the AAS/AIAA Space Flight Mechanics Meeting*, February 1998, in the Advances in the Astronomical Sciences Series.

CONFERENCE PRESENTATION:

Fahroo, F. and Ross, I.M., "A Spectral Collocation Method for Solving Optimal Periodic Control Problems," to be presented in 1998 AIAA GNC Conference, Boston, MA, August 1998.

DoD KEY TECHNOLOGY AREA: Space Vehicles

KEYWORDS: Necessary Optimality Conditions, Periodic Optimal Control Theory, Low-Earth Orbiting Spacecraft, Minimum Fuel Consumption

OPTIMAL DESIGN OF DAMPING AND CONTROL MECHANISMS FOR DISTRIBUTED PARAMETER SYSTEMS

F. Fahroo, Assistant Professor
Department of Mathematics
Sponsor: Naval Postgraduate School

OBJECTIVE: To examine different damping designs for achieving exponential stability of flexible structures.

SUMMARY: This study addressed the question of "optimal" damping design for flexible structures in an abstract setting and precisely defined and analyzed various design criteria which are of importance in applications. In particular, damping designs as well as optimal parameter design of feedback controllers were considered to achieve not only exponential stability but moreover obtain better and faster rate of decay for the energy of the system. The results were illustrated in application to a damped wave equation and a flexible beam, and performed numerous numerical experiments for different damping designs for these examples.

PUBLICATIONS:

Fahroo, F. and Ito, K., "Variational Formulation of Optimum Damping Designs," *Optimization Methods in PDEs*, in the Contemporary Mathematics Series by the American Mathematical Society, Vol 209, pp. 95-115.

Fahroo, F. and Wang, Y., "Optimal Location of Piezoceramic Actuators for Vibration Suppression of a Flexible Structure," in the *Proceedings of IEEE Conference on Decision and Control*, San Diego, CA, December 1997.

PROJECT SUMMARIES

CONFERENCE PRESENTATION:

Fahroo, F. and Wang, Y., "Optimal Location of Piezoceramic Actuators for Vibration Suppression of a Flexible Structure," Conference on Decision and Control, San Diego, December 1997.

DoD KEY TECHNOLOGY AREA: Materials, Processes, and Structures

KEYWORDS: Distributed Parameter Systems, Damping Mechanism, Optimization

NUMERICAL STUDY OF EXPONENTIAL STABILITY OF COUPLED FLUID/STRUCTURE SYSTEMS

F. Fahroo, Assistant Professor

Department of Mathematics

Sponsor: Naval Postgraduate School

OBJECTIVE: To study the exponential stability of several acoustic-structure models by numerical approximation of these models. Convergence and stability of these numerical approximations are also studied.

SUMMARY: In this project a fluid-structure model was considered which consisted of a two dimensional air cavity and a vibrating flexible beam that formed a portion of the boundary of the cavity. A "porous" boundary condition was proposed for the beam equation which allowed the flow of air through the beam. The focus of the work was on establishing uniform exponential stability for the model, and to achieve this goal the multiplier technique was used which has already been used successfully in establishing exponential decay rates for wave equations with boundary feedback damping. After proving the desired stability result for the infinite-dimensional model, the effect of choosing different boundary conditions on the stability of the model was explored by performing numerous numerical simulations and different numerical schemes were also investigated that would preserve the exponential stability of the original model under approximation.

PUBLICATION:

Fahroo, F. and Wang, C., "Numerical Experiments on Approximated Acoustic-Structure Systems," *Journal of Mathematical Systems, Estimation, and Control*, Vol 8, No 2, to appear February 1998.

CONFERENCE PRESENTATION:

Fahroo, F. and Wang, C., "In the Infinite-Horizon LQR Problem for Acoustic-Structure System," the SIAM Annual Conference, Stanford, CA, July 1997.

DoD KEY TECHNOLOGY AREA: Other (Active Noise Control)

KEYWORDS: Exponential Stability, Acoustic-Structure Models, Numerical Approximations

LINE AND CIRCLE TRACKING FOR NONHOLONOMIC AUTONOMOUS VEHICLES

F. Fahroo, Assistant Professor

Department of Mathematics

Sponsor: Naval Postgraduate School

OBJECTIVE: To develop a new algorithm for nonholonomic vehicles for tracking a given line or a circle.

SUMMARY: In this joint research project with the Department of Computer Science, the problem of finding an algorithm for the movement of a vehicle under the nonholonomic constraint to track a given directed line without allowing any

PROJECT SUMMARIES

spinning motion was investigated. A new principle was proposed for computing the derivative of path curvature as a linear combination of the current vehicle path curvature, vehicle orientation, and positional difference. This function is called a steering function. By linearization an optimal selection of parameters for critically damped motions was found and a single parameter, smoothness, for tracking was obtained. Numerous simulation results as well as experimental results were obtained on the autonomous robot, Yamabico, at the Naval Postgraduate School which showed the effectiveness of this method.

This research project has resulted in submission of one journal paper and appearance of two conference papers.

PUBLICATIONS:

Kanayama, Y. and Fahroo, F., "A New Line Tracking Method for Nonholonomic Vehicles," *ICRA Conference Proceedings*, Albuquerque, NM, 1997, pp. 2908-2913.

Kanayama, Y. and Fahroo, F., "A Circle Tracking Method for Nonholonomic Vehicles," *Proceedings of the SYROCO '97 Conference on Robot Control*, Nantes, France, September 1997.

CONFERENCE PRESENTATIONS:

Kanayama, Y. and Fahroo, F., "A New Line Tracking Method for Nonholonomic Vehicles," the ICRA Conference, Albuquerque, NM, 1997.

Kanayama, Y. and Fahroo, F., "A Circle Tracking Method for Nonholonomic Vehicles, SYROCO 97 Conference on Robot Control, Nantes, France, September 1997.

DoD KEY TECHNOLOGY AREAS: Ground Vehicles, Other (Robotics)

KEYWORDS: Nonholonomic Vehicles, Path Tracking, Steering Function

LEAST SQUARES MULTIQUADRIC APPROXIMATION

Richard Franke, Professor
Department of Mathematics
Sponsor: Unfunded

OBJECTIVE: The objective of this project was to continue research in the approximation of scattered data using radial basis function methods, in particular multiquadric functions in a least squares setting.

SUMMARY: This work builds on previous work of the investigator. Here, the nonlinear optimization was over the location of the basis functions (centers or knots) and simultaneously for a transformation of the domain to another, over which the actual approximation is performed. An eleven parameter biquadratic Bezier function was used for the transformation. It was designed in such a way that the one-to-one transformed region is a convex quadrilateral. Examples were constructed showing that the additional eleven degrees of freedom were more valuable in the transformation than simply adding an equivalent number of additional terms to the multiquadric approximation. In addition, the qualitative behavior of the approximations tended to be improved by the method.

PUBLICATION:

Franke, R. and Hagen, H., "Least Squares Surface Approximation Using Multiquadrics and Parametric Domain Distortion," manuscript submitted to CAGD.

PROJECT SUMMARIES

DoD KEY TECHNOLOGY AREAS: Computing and Software, Modeling and Simulation

KEYWORDS: Scattered Data, Radial Basis Functions, Surface Approximation, Least Squares, Domain Distortion

COVARIANCE FUNCTIONS FOR 3-DIMENSIONAL DATA ASSIMILATION I, II

Richard Franke, Professor

Department of Mathematics

Sponsor: Naval Research Laboratory-Monterey

OBJECTIVE: The objective of the first part of the project was to investigate methods for more rigorous modeling of the three-dimensional spatial covariance function for the error in numerical weather forecasts, with the goal of improving data assimilation methods. One aim was to determine properties that will guarantee that the model has the requisite positive definiteness. The objective of the second phase of the research was to continue the investigation by applying methods derived in the first part to real data. Innovation data for a two-month period from the NOGAPS model was used in an investigation of the properties of the data and the generation of a consistent model of the forecast and observation errors.

SUMMARY: A survey of methods for modeling covariance 3-dimensional functions was conducted. A method incorporating a simultaneous transformation of the domain and a fit to the data was experimented with using data from the literature. A 3-dimensional model of the innovation data was constructed using a second order autoregressive (SOAR) function in the horizontal with parameters varying with pressure level. Fitting the innovation data with the model resulted in estimates of the prediction and observation error covariances as a function of pressure level, and also a parameter known as the correlation distance. The prediction and observation error covariances were then fit as functions of pressure level by simultaneous transformations to a domain and fit using a SOAR plus constant model in each case. The variations in the correlation distance at various pressure levels was fit using a cubic curve. The overall 3-d covariance model was then a partially separated model, the product of a horizontal covariance function (varying with pressure level) and a vertical correlation function. Extensive simulations found no evidence that the resulting model was not positive definite. As a by-product of the investigation it was determined that the predictive errors for NOGAPS tend to be smaller than radiosonde observation errors over the United States.

PUBLICATIONS:

Franke, R., "Three Dimensional Covariance Functions: Theory," NRL/MR/7531/97/7231, October 1997 (submitted to *Monthly Weather Review*).

Franke, R., "Three Dimensional Covariance Functions: Real Data," NRL/MR/7532/97/7232, October 1997 (submitted to *Monthly Weather Review*).

DoD KEY TECHNOLOGY AREAS: Computing and Software, Modeling and Simulation

KEYWORDS: Objective Analysis, Data Assimilation, Covariance Functions, Numerical Weather Prediction

ALGORITHMS FOR SEGMENTED DATA

Richard Franke, Professor

Department of Mathematics

Sponsor: Unfunded

OBJECTIVE: Segmented data is data in 3-space that has a classification associated with each point. The objective was to construct computationally simple algorithms for constructing a surface that separates the various classes of data and the volumes that contain only data of one of the various classes.

PROJECT SUMMARIES

SUMMARY: The approach taken was to first construct a Delaunay tetrahedrization of the (possibly scattered) data points. Then a marching cubes approach was taken in that each tetrahedron was processed nearly independently of the others. Since there are only four vertices in a tetrahedron, there are only four (nontrivial) cases to consider: three vertices of one class, one of another; two vertices of one class, two of another; two vertices of one class, one each of two other (different) classes; and all vertices of a different classes. Using these four cases it is then possible to write out a list of triangles composing the separating surface and a list of tetrahedra composing the volume for each class. In certain cases care must be taken to avoid the crack problem of marching cubes, but this is easily handled assuming an order on the input points.

PUBLICATIONS:

Nielson, G. and Franke, R., "Computing the Separating Surface for Segmented Data," *Proceedings of Visualization '97*, IEEE Press, October 1997, pp. 229-233.

Nielson, G. and Franke, R., "Computing Segmented Volumes," to appear in the *Proceedings of Dagstuhl Seminar on Scientific Visualization*.

Freedden, W., Schreiner, M. and Franke, R., "A Survey on Spherical Spline Approximation," *Surveys on Mathematics for Industry*, 7(1997)28-85.

CONFERENCE PRESENTATIONS:

Franke, R., "Computing the Separating Surface for Segmented Data," IEEE Visualization '97, Phoenix, AZ October 1997.

Franke, R., "Computing Segmented Volumes," Dagstuhl Seminar on Scientific Visualization, Dagstuhl Castle, Germany, June 1997.

DoD KEY TECHNOLOGY AREAS: Computing and Software, Modeling and Simulation

KEYWORDS: Scattered Data, Segmented Data, Marching Cubes, Separating Surface

DE BRUIJN SEQUENCES FOR CRYPTOGRAPHIC APPLICATIONS

H. Fredericksen, Professor
Department of Mathematics
Sponsor: Teledyne Industries

OBJECTIVE: The work under this agreement will be pursued in three phases as follows: the mathematical foundations of the generalized combs and nonlinear sequences associated with them will be developed. Classes of nonlinear sequence possessing good auto-correlation and cross-correlation properties will be identified. Permutations for the generation of the sequences will be determined. Cryptographic systems employing the new sequences and their permutations will be developed.

DoD KEY TECHNOLOGY AREA: Other (Cryptographic Applications)

KEYWORDS: Nonlinear Sequences, De Bruijn Sequences, Cryptographic

PROJECT SUMMARIES

FAST, STABLE, COMPUTATIONAL ALGORITHMS FOR SPECTRAL DECOMPOSITION OF STATIONARY TIME SERIES

W. B. Gragg, Professor
Department of Mathematics
Sponsor: Naval Postgraduate School

OBJECTIVE: To implement, rigorously test, and prove numerical stability of, new stable algorithms for executing the second phase of Pisarenko's signal processing algorithm, now reformulated so as to solve a well-conditioned problem. The new algorithms, stabilized forms of the unitary Hessenberg QR (uhqr) algorithm, permit fast, $O(n^2)$, solution of problems of (essentially) arbitrarily high order. To do the same for the related inverse algorithms (ihqr). Together, these algorithms provide algorithms for fast adaptive recursive least squares modeling of stationary time series by trigonometric polynomials (spectral estimation).

SUMMARY: The "stabilization" of the uhqr algorithm developed gives rather massive numerical evidence that it is effective. A generalization of the uhqr algorithm permits, in principle, the $O(n^2)$ computation of all zeros of n th degree polynomials. Thus, it solves a long-standing open problem of computational complex analysis. The details of implementation of a stable form of this algorithm will be decidedly nontrivial.

PUBLICATIONS:

Gates, K. and Gragg, W.B., "Notes on tqr Algorithms," *Journal of Computational. Applied Mathematics*, 86 (1997) 195-203.

Gragg, W.B., "Stabilization of the uhqr Algorithm?" *Proceedings of Guangzhou International Conference on Computational Mathematics*, Guangzhou, China, August 1997.

PRESENTATIONS:

Gragg, W.B., "Stabilization of the uhqr Algorithm," University of Kentucky, Lexington, KY, 3 March 1997.

Gragg, W.B., "Stabilization of the uhqr Algorithm," Royal Institute of Technology, Stockholm, Sweden, 18 March 1997.

Gragg, W.B., "Stabilization of the uhqr Algorithm," University of California-Berkeley, Berkeley, CA, 25 March 1997.

Gragg, W.B., "Stabilization of the uhqr Algorithm," Arizona State University, Tempe, AZ, 14 April 1997.

Gragg, W.B., "Stabilization of the uhqr Algorithm," Air Force Institute of Technology, Dayton, OH, 23 June 1997.

Gragg, W.B., "Stabilization of the uhqr Algorithm," Northern Illinois University, DeKalb, IL, 1 August 1997.

Gragg, W.B., "Stabilization of the uhqr Algorithm?" *Guangzhou International Conference on Computational Mathematics*, Guangzhou, China, 12 August 1997.

Gragg, W.B., "An $O(n^2)$ QR Algorithm (shqz) for Polynomial Zeros," *International Conference on Computational Methods and Function Theory*, Nicosia, Cyprus, 12 October 1997.

DoD KEY TECHNOLOGY AREA: Computing and Software

KEYWORDS: Pisarenko's Method, Signal Processing, Fast Algorithms, QR Algorithms, Unitary Hessenberg Matrices, Spectral Estimation, Stationary Time Series

PROJECT SUMMARIES

ALGEBRAIC MULTIGRID FOR LARGE-SCALE SIMULATIONS ON UNSTRUCTURED GRIDS: PHASE II

Emden van Henson, Assistant Professor

Department of Mathematics

Department of Energy-Lawrence Livermore National Laboratory

OBJECTIVE: Several applications important to the nation's science-based stockpile stewardship mission require the numerical solution of elliptic PDEs on extremely large grids whose gridpoints are irregularly space-unstructured grids. Algebraic multigrid (AMG) is a method developed for use with unstructured grids. In phase I of this project, it was determined that AMG can be applied to such problems in general, but that extensive algorithmic development will be required to solve the specific problems of interest. This proposal, for Phase II, is intended to design AMG-based algorithms that solve the specific problems of interest; and to develop a prototype serial AMG code with a design allowing later conversion to a parallel version.

DoD KEY TECHNOLOGY AREA: Computing and Software

KEYWORDS: Multigrid, Algebraic Multigrid, Parallel Processing, Stockpile Stewardship

RELIABILITY MODELING OF CONCURRENT SOFTWARE MODULES

T. Jayachandran, Professor

Department of Mathematics

Sponsor: Unfunded

OBJECTIVE: To develop probability models for the reliability of concurrent software modules used to build redundancy to increase reliability.

SUMMARY: This is an ongoing unfunded project. Unlike for hardware, failures of redundant software are correlated and, therefore, the determination of the improvement in reliability is difficult. An algorithm to compute the probability of failure of one of the models proposed in the literature has been developed. Work to develop alternative models is in progress.

PUBLICATION:

Jayachandran, T., "An Intensity Distribution for Concurrent Software Failures," submitted to the *IEEE Transactions on Software Engineering*.

DoD KEY TECHNOLOGY AREA: Computing and Software

KEYWORDS: Software Reliability, Redundancy, Intensity Distribution

APPLICATIONS OF BIFURCATION CONTROL BY STATE FEEDBACK

Wei Kang, Assistant Professor

Department of Mathematics

Sponsor: Naval Postgraduate School

OBJECTIVE: The objective of this project was to: (1) To develop a stabilization feedback design methodology for nonlinear control systems near bifurcation points; (2) To control the rotating stall and surge in gas engine compressors; (3) To design feedbacks for depth control in the dive plane of submarines near the critical Froude number where pitchfork bifurcation occurs; and (4). To enhance the research partnership of the Mathematics Department with engineering departments of NPS in the fields involving analysis, design, and scientific computation.

PROJECT SUMMARIES

SUMMARY: A control system, in general, has infinitely many equilibrium points. The stability properties at different equilibrium points are not always the same, especially when bifurcation occurs. The equilibrium sets of control systems and their bifurcations are classified based on normal forms and invariants of the system. The bifurcation analysis for control systems is applied to engineering problems such as engine compressor control and submersible vehicles.

PUBLICATIONS:

Barbot, J.P. and Kang, W., "Higher Order Approximations for Chained Form," *Proceedings of European Control Conference*, Brussels, Belgium, 4 July 1997.

Byrnes, C.I., Priscoli, F.S., Isidori, A., and Kang, W., "Structurally Stable Output Regulation of Nonlinear Systems," *Automatica*, 33 (1997), 369-385.

Kang, W. and Huang, J., "Calculation of the Minimal Dimension-order Robust Servo-regulator," *IEEE Transaction Automatic Control*, 42 (1997), 382-386.

Kang, W., "Bifurcation and Normal Form of Nonlinear Control Systems - Part I," *SIAM Journal of Control and Optimization*, to appear.

Kang, W., "Bifurcation and Normal Form of Nonlinear Control Systems - Part II," *SIAM Journal of Control and Optimization*, to appear.

Kang, W., Gu, G., Sparks, A., and Banda, S., "Surge Control and Test Functions for Axial Flow Compressors," *Proceedings of American Control Conference*, Albuquerque, NM, 6 June 1997, 3721-3725.

Kang, W. and Papoulias, F., "Bifurcation and Normal Forms of Dive Plane Reversal of Submersible Vehicles," *Proceedings of 7th International Offshore and Polar Engineering Conference*, Vol. II, pp. 62-69, Honolulu, HI, 25-30 May 1997.

Kang, W., "The Stability and Invariants of Control Systems with Pitchfork or Cusp Bifurcations," *Proceedings of IEEE Conference on Decision and Control*, San Diego, CA, 10-12 December 1997.

Kang, W., "Invariants and Stability of Control Systems with Transcritical and Saddle-node Bifurcations," *Proceedings of IEEE Conference on Decision and Control*, San Diego, CA, 10-12 December 1997.

CONFERENCE PRESENTATIONS:

Kang, W., "Bifurcation Control for Systems with a Single Uncontrollable Mode," PRET Workshop, AFOSR-PRET Center, UCSB, Santa Barbara, CA, 1 February 1997.

Kang, W., "Bifurcation Control via State Feedback," 45th SIAM Annual Meeting, Stanford, CA, 15 July 1997.

Ph.D. DISSERTATION DIRECTED:

Fitch, O., "The Control of Bifurcations with Engineering Applications," Ph.D. Dissertation, Naval Postgraduate School, September 1997.

DoD KEY TECHNOLOGY AREAS: Air Vehicles, Space Vehicles, Surface/Under Surface Vehicles – Ships and aircraft

KEYWORDS: Nonlinear Control Systems, Bifurcations, Invariants, Normal Forms

PROJECT SUMMARIES

CLOSE COMBAT ANTI-ARMOR WEAPON SYSTEM (CCAWS) TECHNOLOGY ANALYSIS

B. K. Mansager, Senior Lecturer

Department of Mathematics

Sponsor: CCAWS Project Office

OBJECTIVE: To provide PM CCAWS Information regarding the sensitivity of weapon parameters (system preparation time ability to fire on the move, and the use of "shoot and scoot" tactics) for three candidate systems (XJAV, XTOW, and CLOS) using the measures of effectiveness of survivability and engagement range.

DoD KEY TECHNOLOGY AREA: Other (Environmental Effects)

KEYWORDS: Antitank Missiles, Close Combat Antitank Systems, Tow Missile Improvements

PARALLEL VERSION OF SPECIAL PERTURBATION

B. Neta, Professor

Department of Mathematics

Sponsor: U.S. Air Force-Phillips Laboratory

OBJECTIVE: The objective of this work is to develop a parallel version of a special perturbation code using task rather than data decomposition. Code for the solution of systems of differential equations has been implemented

SUMMARY: Parallelization can be achieved by either control or domain decomposition. The latter was tried for analytic (by Neta et al.), semianalytic (by Wallace) and numerical propagators (by Neal and Coffey). The control decomposition idea is inefficient for analytic propagators (Neta et al.), because the computation time is too short. A control decomposition approach to parallelize a numerical orbit propagator which is more computationally intensive is discussed.

PUBLICATIONS:

Neta, B., "Parallelization of Satellite Motion Models," *SIAM News*, 30, November 1997.

Neta, B. and Vallado, D., "On Satellite Umbra/Penumbra Entry and Exit Positions," *Proceedings of Seventh AAS/AIAA Space Flight Mechanics Meeting*, Huntsville, AL, in 10-12 February 1997, Paper Number AAS 97-155.

Neta, B., "Parallel Version of Special Perturbations Orbit Propagator," *Proceedings of AAS/AIAA Astrodynamics Conference*, Sun Valley, ID, 4-7 August 1997, Paper Number 97-688.

CONFERENCE PRESENTATION:

Neta, B., "On Satellite Umbra/Penumbra Entry and Exit Positions," *Seventh AAS/AIAA Space Flight Mechanics Meeting*, Huntsville, AL, 10-12 February 1997.

Neta, B., "Parallel Version of Special Perturbations Orbit Propagator," *AAS/AIAA Astrodynamics Conference*, Sun Valley, ID, 4-7 August 1997.

DoD KEY TECHNOLOGY AREAS: Computing and Software, Space Vehicles, Modeling and Simulation

KEYWORDS: Satellites, Orbit prediction, Umbra/penumbra

PROJECT SUMMARIES

FINITE DIFFERENCES VERSUS FINITE ELEMENTS

B. Neta, Professor

Department of Mathematics

Sponsor: Unfunded

OBJECTIVE: The objective of this research is: (1) to investigate a linear analysis of the shallow water equations in spherical coordinates for the Turkel-Zwas explicit large time-step scheme, and (2) to analyze the stability of finite element approximation to the linearized two dimensional advection-diffusion equation.

SUMMARY: A linear analysis of the shallow water equations in spherical coordinates for the Turkel-Zwas explicit large time-step scheme is presented. This coordinate system is more realistic in meteorology and more complicated to analyze, since the coefficients are no longer constant. The analysis suggests that the Turkel-Zwas scheme must be staggered in a certain way in order to get eigenvalues and eigenfunctions approaching those of the continuous case. The importance of such an analysis is the fact that it is also valid for non-constant coefficients and thereby applicable to any numerical scheme.

Another paper analyzed the stability of the finite element approximation to the linearized two-dimensional advection-diffusion equation. Bilinear basis functions on rectangular elements are considered. Giraldo and Neta have numerically compared the Eulerian and semi-Lagrangian finite element approximation to the advection-diffusion equation. This paper analyzes the finite element schemes used there.

PUBLICATIONS:

Neta, B., Giraldo, F.X., and Navon, I.M., "Analysis of the Turkel-Zwas Scheme for the Two-Dimensional Shallow Water Equations in Spherical Coordinates," *Journal of Computational Physics*, 133, pp. 102-112, 1997.

Giraldo, F.X. and Neta, B., "A Comparison of a Family of Eulerian and Semi-Lagrangian Finite Element Methods for the Advection-Diffusion Equation," in *Computer Modeling of Seas and Coastal Regions III*, J. R. Acinas and C. A. Brebbia (eds), Computational Mechanics Publications, Southampton, U. K., pp. 217-229, 1997.

CONFERENCE PRESENTATION:

Giraldo, F.X. and Neta, B., "A Comparison of a Family of Eulerian and Semi-Lagrangian Finite Element Methods for the Advection-Diffusion Equation," *Computer Modeling of Seas and Coastal Regions*, La Coruna, Spain, 23-25 June 1997.

THESIS DIRECTED:

Hamrick, T.A., "Analysis of the Numerical Solution of the Shallow Water Equations," Master's Thesis, Naval Postgraduate School, September 1997.

DoD KEY TECHNOLOGY AREAS: Computing and Software, Modeling and Simulation

KEYWORDS: Finite Elements, Finite Differences, Shallow Water, Stability Analysis

PROJECT SUMMARIES

TARGETING SUB-STATE POLITICAL GROUPS

Guillermo Owen, Professor

Department of Mathematics

Gordon H. McCormick, Associate Professor

Special Operations Curriculum Committee

Sponsor: Assistant Secretary of Defense, Special Operations-Low Intensity Conflict (SOLIC)

OBJECTIVES: The objective of this research was to: (1) To develop a formal framework for evaluating the dynamics of sub-state conflict, and (2) To employ this framework to improve our ability to target terrorist and other sub-state politico-military organizations.

SUMMARY: The growing incidence of direct and indirect U.S. involvement in locally driven sub-state conflicts—coupled with the poor track record in this area—requires that the analytical aids used to evaluate, measure and respond to such engagement effectively be improved. This project employed formal modeling to isolate and examine the variables and relationships that define the dynamics of internal wars in an effort to improve the ability to control and diffuse such conflicts.

PUBLICATIONS:

Owen, G. and McCormick, G., "Violence, Factionalism and State-Terrorist Bargaining."

Owen, G. and McCormick, G., "Security and Coordination in a Clandestine Organization."

DoD KEY TECHNOLOGY AREA: Other (Sub-State Political Group)

KEYWORDS: Terrorist, Sub-State Politico-Military Organizations

EVALUATION OF COMPLETION-BASED HEURISTICS FOR GRAPH COLORING

Craig W. Rasmussen, Associate Professor

Department of Mathematics

Sponsor: Naval Postgraduate School

OBJECTIVE: To develop and evaluate heuristics for finding approximate solutions to hard combinatorial optimization problems, such as graph coloring, that arise in diverse problems such as scheduling and frequency assignment. These problems are generally NP-complete, but typically are easily solved on certain families of highly structured problem instances. These families themselves possess more internal structure than was previously known, and the idea is to exploit this structure to obtain useful approximate solutions to the hard instances.

SUMMARY: This summary covers the continuation of a project that was initiated in FY94 and which was partially funded by the Naval Postgraduate School in FY95 - FY97. The focus during the period covered by this summary was implementation of an algorithm for approximating the chromatic number of a graph. MATLAB codes were developed for the following: (1) Recognition of chordal graphs, which have good algorithmic properties; (2) Construction of completion sequences of chordal graphs; (3) Location of a maximal chordal subgraph of an arbitrary input graph; and (4) Constructively determining the chromatic number of a chordal graph.

Equipped with these codes, the idea is to accept as input a randomly generated graph G , locate a maximal chordal subgraph H of G , use H to initialize a completion algorithm that terminates when a chordal supergraph K of G is found. K is then assigned a coloring that is subsequently inherited by G .

PROJECT SUMMARIES

THESIS DIRECTED:

Eggen, L., "Approximating the Chromatic Number of an Arbitrary Graph Using a Supergraph Heuristic," Master's Thesis, Naval Postgraduate School, June 1997.

DoD KEY TECHNOLOGY AREA: Other (Applied Mathematics)

KEYWORDS: Chordal Graphs, Graph Coloring

p-COMPETITION GRAPHS: CHROMATIC PROPERTIES AND CHARACTERIZATIONS

Craig W. Rasmussen, Associate Professor

Department of Mathematics

Sponsor: Naval Postgraduate School

OBJECTIVE: Characterize competition graphs and p-competition graphs of various highly structured families of graphs and digraphs.

SUMMARY: This is ongoing work that is conducted jointly with colleagues at Colorado University-Denver, Kenyon College, and the University of the Pacific. The project is an outgrowth of a project that was supported during FY93 and FY94 by the Research Initiation Program (RIP) at NPS. A related area is that of upper-bound graphs of posets. Joint work in that area is with the Denver group and with a colleague at the University of Louisville.

PUBLICATIONS:

Langley, L., Lundgren, J.R., McKenna, P.A., Merz, S.K., and Rasmussen, C.W., "p-Competition Graphs of Strongly Connected and Hamiltonian Digraphs," *Ars Combinatoria* v47, December 1997.

Langley, L., Lundgren, J.R., Merz, S.K., and Rasmussen, C.W., "Posets with Interval or Chordal Strict Upper and Lower Bound Graphs," *Congressus Numerantium*, December 1997.

McMorris, F.R. and Rasmussen, C.W., "Phi-Tolerance Upper-Bound Graphs of Partially Ordered Sets," *Congress Numerantium*, December 1997.

CONFERENCE PRESENTATIONS:

Lundgren, J.R., Langley, L., Merz, S.K., and Rasmussen, C.W., "Digraphs with Interval or Chordal Competition and Resource Graphs," 28th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1997.

Lundgren, J.R., Langley, L., Merz, S.K., and Rasmussen, C.W., "Posets with Interval Upper and Lower Bound Graphs," 28th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1997.

McMorris, F.R. and Rasmussen, C.W., "Phi-Tolerance Upper-Bound Graphs of Partially Ordered Sets," 28th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1997.

DoD KEY TECHNOLOGY AREAS: Computing and Software, Other (Resource Allocation)

KEYWORDS: Ordered Sets, Chordal Graphs, Interval Graphs

PROJECT SUMMARIES

REMOVAL OF THE ASSUMPTION OF CELLULAR TARGETS IN THE COMPUTATION OF DAMAGE AGGREGATION TO AN AREA TARGET FROM A SALVO OF N WEAPONS

I. Bert Russak, Associate Professor

T. Jayachandran, Professor

Department of Mathematics

Sponsor: Naval Postgraduate School-Institute of Joint Warfare Analysis

OBJECTIVE: To improve the accuracy of damage calculations to an area target by removing certain non-real world assumptions used in its model.

SUMMARY: Analysts who do computation of damage aggregation to a area target from a salvo of weapons sometimes use the simplifying assumption of a target consisting of cells. This often has the implicit assumption of weapons hits to the target always being at the center of a cell, which is certainly not true. The removal of this assumption provides a more accurate model with more accurate calculations of damage aggregation.

PUBLICATION:

Russak, I.B. and Jayachandran, T. "Removal of the Assumption of Cellular Targets in the Computation of Damage Aggregation to an Area Target from a Salvo of N Weapons," NPS Technical Report, NPS-MA-98-001, December 1997.

DoD KEY TECHNOLOGY AREAS: Computing and Software, Modeling and Simulation

KEYWORDS: Damage Aggregation, Area Target, Cellular Target

BI-ORTHOGONALITY RELATIONS APPLIED TO SCATTERING IN WAVEGUIDES

Clyde Scandrett, Associate Professor

Chris Frenzen, Associate professor

Department of Mathematics

Sponsor: Naval Postgraduate School

OBJECTIVE: To investigate the behavior of Scholte/Rayleigh-Lamb surface wave propagation along the interface of an elastic/poro-elastic solid underlying a fluid layer.

SUMMARY: The bi-orthogonality relationships developed for a porous/elastic-fluid layered system were implemented to determine the interfacial wave scattering from a vertical discontinuity in the medium. The dispersion relationship was found and used to determine wavenumbers for the discrete eigenfunctions of the media. Non-uniqueness of the solution led the investigators to develop compatibility conditions in the form of infinite matrix identities. The compatibility condition was obtained analytically for the case of two fluids in lateral contact. Compatibility conditions for elastic and poro-elastic media were also discussed.

PUBLICATION:

Scandrett, C. and Frenzen, C. L., "Bi-orthogonality Relationships and Scattering from Material Discontinuities," to appear in the Proceedings of the Fourth International Conference on Mathematical and Numerical Aspects of Wave Propagation, Golden, CO, 1-5 June 1998.

CONFERENCE PRESENTATION:

Scandrett, C. and Frenzen, C.L., "Bi-orthogonality Relationships," to be given at the Fourth International Conference on Mathematical and Numerical Aspects of Wave Propagation, Golden, CO, 1-5 June 1998.

PROJECT SUMMARIES

DoD KEY TECHNOLOGY AREAS: Environmental Quality, Sensors

KEYWORDS: Wave Propagation, Porous Media, Bi-Orthogonality, Layered Media

INTEGRATED ASSESSMENT OF SHIP MISSILE DEFENSE EFFECTIVENESS

W. M. Woods, Professor

Department of Mathematics

Sponsor: Naval Warfare Assessment Center

OBJECTIVE: To provide technical support to NWAC (QA30) in integrated assessment of DD963 ship missile defense effectiveness for a specific threat.

DoD KEY TECHNOLOGY AREA: Conventional Weapons

KEYWORDS: Ship Missile Defense, System Effectiveness

PUBLICATIONS/PRESENTATIONS

JOURNAL PAPERS

- Byrnes, C.I., Priscoli, F.S., Isidori, A., and Kang, W., "Structurally Stable Output Regulation of Nonlinear Systems," *Automatica*, 33 (1997), 369-385.
- Fahroo, F. and Ito, K., "Variational Formulation of Optimum Damping Designs," *Optimization Methods in PDEs*, in the Contemporary Mathematics Series by the American Mathematical Society, Vol. 209, pp. 95-115.
- Freedden, W., Schreiner, M., and Franke, R., "A Survey on Spherical Spline Approximation," *Surveys on Mathematics for Industry* 7(1997)28-85.
- Gates, K. and Gragg, W.B., "Notes on tqr Algorithms," *Journal of Computational Applied Mathematics*, 86 (1997) 195-203.
- Kang, W. and Huang, J., "Calculation of the Minimal Dimension-order Robust Servo-regulator," *IEEE Transactions on Automatic Control*, 42 (1997), 382-386.
- Langley, L., Lundgren, J.R., McKenna, P.A., Merz, S.K., and Rasmussen, C.W., "p-Competition Graphs of Strongly Connected and Hamiltonian Digraphs," *Ars Combinatoria*, v47, December 1997.
- Langley, L., Lundgren, J.R., Merz, S.K., and Rasmussen, C.W., "Posets with Interval or Chordal Strict Upper and Lower Bound Graphs," *Congressus Numerantium*, December 1997.
- McMorris, F.R. and Rasmussen, C.W., "Phi-Tolerance Upper-Bound Graphs of Partially Ordered Sets," *Congress Numerantium*, December 1997.
- Neta, B., "Parallelization of Satellite Motion Models," *SIAM News*, 30, November 1997.
- Neta, B., Giraldo, F.X., and Navon, I.M., "Analysis of the Turkel-Zwas Scheme for the Two-Dimensional Shallow Water Equations in Spherical Coordinates," *Journal of Computational Physics*, 133, (1997), 102-112.
- Owen, G. and Carreras, F., "Automorphisms and Weighted Values of Games," *International Journal Game Theory*, pp. 1-10, 1997.
- Owen, G., Glazer, A., and Grofman, B.N., "A Neo-Downsian Model of Group-Oriented Voting and Racial Backlash," *Public Choice*, 1997.
- Owen, G. and Grofman, B.N. "Estimating the Likelihood of Fallacious Ecological Inference: Linear Ecological Regression in the Presence of Context Effects," *Political Geography*, 1997, 675-690.

CONFERENCE PAPERS

- Barbot, J.P. and Kang, W., "Higher Order Approximations for Chained Form," *Proceedings of European Control Conference*, Brussels, Belgium, 4 July 1997.
- Fahroo, F., and Wang, Y., "Optimal Location of Piezoceramic Actuators for Vibration Suppression of a Flexible Structure," *Proceedings of IEEE Conference on Decision and Control*, San Diego, CA, December 1997.
- Gragg, W.B., "Stabilization of the uhqr Algorithm?" *Proceedings of Guangzhou International Conference on Computational Mathematics*, Guangzhou, China, August 1997.

PUBLICATIONS/PRESENTATIONS

Kanayama, Y. and Fahroo, F., "A New Line Tracking Method for Nonholonomic Vehicles," *ICRA Conference Proceedings*, Albuquerque, NM, 1997, pp. 2908-2913.

Kanayama, Y. and Fahroo, F., "A Circle Tracking Method for Nonholonomic Vehicles," *Proceedings of the SYROCO 97 Conference on Robot Control*, Nantes, France, September 1997.

Kang, W., Gu, G., Sparks, A., and Banda, S., "Surge Control and Test Functions for Axial Flow Compressors," *Proceedings of American Control Conference*, Albuquerque, NM, 6 June 1997, 3721-3725.

Kang, W. and Papoulias, F., "Bifurcation and Normal Forms of Dive Plane Reversal of Submersible Vehicles," *Proceedings of 7th International Offshore and Polar Engineering Conference*, Vol. II, pp. 62-69, Honolulu, HI, 25-30 May 1997.

Kang, W., "The Stability and Invariants of Control Systems with Pitchfork or Cusp Bifurcations," *Proceedings of IEEE Conference on Decision and Control*, San Diego, CA, 10-12 December 1997.

Kang, W., "Invariants and Stability of Control Systems with Transcritical and Saddle-node Bifurcations," *Proceedings of IEEE Conference on Decision and Control*, San Diego, CA, 10-12 December 1997.

Neta, B. and Vallado, D., "On Satellite Umbra/Penumbra Entry and Exit Positions," *Proceedings of Seventh AAS/AIAA Space Flight Mechanics Meeting*, Huntsville, AL, in 10-12 February 1997, Paper Number AAS 97-155.

Neta, B., "Parallel Version of Special Perturbations Orbit Propagator," *Proceedings of AAS/AIAA Astrodynamics Conference*, Sun Valley, ID, 4-7 August 1997, Paper Number 97-688.

Nielson, G. and Franke, R., "Computing the Separating Surface for Segmented Data," *Proceedings of Visualization '97*, IEEE Press, October 1997, pp. 229-233.

CONFERENCE PRESENTATIONS

Fahroo, F. and Wang, Y., "Optimal Location of Piezoceramic Actuators for Vibration Suppression of a Flexible Structure," *Conference on Decision and Control*, San Diego, CA, December 1997.

Fahroo, F. and Wang, Y., "In the Infinite-Horizon LQR Problem for Acoustic-Structure System," *SIAM Annual Conference*, Stanford, CA, July 1997.

Fahroo, F. and Wang, Y., "Optimal Location of Piezoceramic Actuators for Vibration Suppression of a Flexible Structure," *Conference on Decision and Control*, San Diego, CA, December 1997.

Franke, R., "Computing Segmented Volumes," *Dagstuhl Seminar on Scientific Visualization*, Dagstuhl Castle, Germany, June 1997.

Franke, R., "Computing the Separating Surface for Segmented Data," *IEEE Visualization '97*, Phoenix, AZ, October 1997.

Giraldo, F.X. and Neta, B., "A Comparison of a Family of Eulerian and Semi-Lagrangian Finite Element Methods for the Advection-Diffusion Equation," *Computer Modeling of Seas and Coastal Regions*, La Coruna, Spain, 23-25 June 1997.

Gragg, W.B., "Stabilization of the uhqr Algorithm?" *Guangzhou International Conference on Computational Mathematics*, Guangzhou, China, 12 August 1997.

Gragg, W.B., "An $O(n^2)$ QR Algorithm (shqz) for Polynomial Zeros," *International Conference on Computational Methods and Function Theory*, Nicosia, Cyprus, 12 October 1997.

PUBLICATIONS/PRESENTATIONS

Kanayama, Y. and Fahroo, F., "A New Line Tracking Method for Nonholonomic Vehicles," ICRA Conference, Albuquerque, NM, 1997.

Kanayama, Y. and Fahroo, F., "A Circle Tracking Method for Nonholonomic Vehicles, SYROCO 97 Conference on Robot Control, Nantes, France, September 1997.

Kang, W., "Bifurcation Control for Systems with a Single Uncontrollable Mode," PRET Workshop, AFOSR-PRET Center, University of California-Santa Barbara, Santa Barbara, CA, 1 February 1997.

Kang, W., "Bifurcation Control via State Feedback," 45th SIAM Annual Meeting, Stanford, CA, 15 July 1997.

Lundgren, J.R., Langley, L., Merz, S.K., and Rasmussen, C.W., "Digraphs with Interval or Chordal Competition and Resource Graphs," 28th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1997.

Lundgren, J.R., Langley, L., Merz, S.K., and Rasmussen, C.W., "Posets with Interval Upper and Lower Bound Graphs," 28th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1997.

McMorris, F.R. and Rasmussen, C.W., "Phi-Tolerance Upper-Bound Graphs of Partially Ordered Sets," 28th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1997.

Middlebrook, E.E., Mansager, B. and Borges, C.F. "A Combat Simulation Analysis of Autonomous Legged Underwater Vehicles," 65th Military Operations Research Society Symposium, Quantico, VA, June 1997.

Neta, B., "On Satellite Umbra/Penumbra Entry and Exit Positions," Seventh AAS/AIAA Space Flight Mechanics Meeting, Huntsville, AL, 10-12 February 1997.

Neta, B., "Parallel Version of Special Perturbations Orbit Propagator," AAS/AIAA Astrodynamics Conference, Sun Valley, ID, 4-7 August 1997.

Owen, G., "Applications of Game Theory to Economic Equilibrium," joint meeting of German Society for Operations Research (DGOR) and Society for Mathematical Economics and Operations Research (GM(tm)OR), Jena, Germany, 3 September 1997.

Owen, G., "Bargaining between Heterogeneous Coalitions," Meeting on Cooperative Game Theory, Chicago, IL, 26 October 1997.

Weir, M.D., Finney, R.L., and Lepp, M., "Engineering Calculus in Two Semesters," 10th Annual International Conference on Technology in Collegiate Mathematics, Chicago, IL, November 1997.

TECHNICAL REPORTS

Danielson, D. A., "Buckling of Ship Grillages," Naval Postgraduate School Technical Report, NPS-MA-97-005, September 1997.

Danielson, D. A., et al., "Research in the Structural Dynamic Response of the RAH-66 Comanche Helicopter," Naval Postgraduate School Report, December 1997.

Franke, R., "Three Dimensional Covariance Functions: Theory," NRL/MR/7531/97/7231, October 1997.

PUBLICATIONS/PRESENTATIONS

Franke, R., "Three Dimensional Covariance Functions: Real Data," NRL/MR/7532/97/7232, October 1997.

Mansager, B.K., Pearman, G., Larimer, L., and Shenk, M., "JANUS Version 6.0 Tutorial," Naval Postgraduate School Technical Report, NPS-MA-97-007, December 1997.

Middlebrook, E.E., Mansager, B.K., and Borges, C.F., "A Combat Simulation Analysis of Autonomous Legged Underwater Vehicles," Naval Postgraduate School Technical, NPS-MA-97-006, December 1997.

Owen, G. and McCormick, G. "Violence, Factionalism and State-Terrorist Bargaining."

Owen, G. and McCormick, G. "Security and Coordination in a Clandestine Organization."

Russak, I.B. and Jayachandran, T. "Removal of the Assumption of Cellular Targets in the Computation of Damage Aggregation to an Area Target From a Salvo of N Weapons," NPS Technical Report, NPS-MA-98-001, December 1997.

BOOKS

Weir, M.D., Giordano, Frank R., and Fox, William P., *A First Course in Mathematical Modeling*, Second Edition, Brooks/Cole Publishing Company, Pacific Grove, CA, 1997, 526 pp.

Weir, M.D., Giordano, Frank R., and Finney, Ross L., *Calculus for Engineers and Scientists*, Preliminary Edition, Vol. 1, Addison-Wesley-Longman Publishing Company, Reading, MA, 1997, 372 pp.

CONTRIBUTION TO BOOKS

Giraldo, F.X. and Neta, B., "A Comparison of a Family of Eulerian and Semi-Lagrangian Finite Element Methods for the Advection-Diffusion Equation," in *Computer Modeling of Seas and Coastal Regions III*, J. R. Acinas and C. A. Brebbia (eds), Computational Mechanics Publications, Southampton, U. K., (1997), 217-229.

Owen, G., "Pari-Mutuel as a System of Aggregation of Information," *Game-Theoretical Applications to Operations Research and Economics*, ed. T. K. Parthasarathy et al., Dordrecht (Kluwer), 1997, 183-196

1997 THESIS ABSTRACTS

KNAPSACK CUTS AND EXPLICIT-CONSTRAINT BRANCHING FOR SOLVING INTEGER PROGRAMS

Jeffrey A. Appleget-Lieutenant Colonel, United States Army

B.S. United States Military Academy, 1979

M.S., Rensselaer Polytechnic Institute, 1989

Doctor of Philosophy in Operations Research-June 1997

Advisors: R. Kevin Wood, Department of Operations Research

Gerald G. Brown, Department of Operations Research

Alan R. Washburn, Department of Operations Research

Craig M. Rasmussen, Department of Mathematics

François Melese, Defense Resources Management Institute

Enhanced solution techniques are developed for solving integer programs (IPs) and mixed-integer programs (MIPs). Previously unsolvable problems can be solved with these new techniques. We develop knapsack cut-finding procedures for minimal cover cuts, and convert existing cut-strengthening theory into practical procedures that lift and tighten violated minimal cover valid inequalities to violated knapsack facets in polynomial time. We find a new class of knapsack cuts called "non-minimal cover cuts" and a method of lifting them called "deficit lifting." Deficit lifting enables all of these cuts to be lifted and tightened to facets as well. Extensions of these techniques enable us to find cuts for elastic knapsack constraints and cuts for non-standard knapsack constraints. We also develop the new technique of "explicit-constraint branching" (ECB). ECB enables the technique of constraint branching to be used on IPs and MIPs that do not have the structure required for known "implicit constraint branching" techniques. When these techniques are applied to 84 randomly generated generalized assignment problems, the combination of knapsack cuts and explicit-constraint branching were able to solve 100 percent of the problems in under 1000 CPU seconds. Explicit constraint branching alone solved 94 percent, and knapsack cuts solved 93 percent. Standard branch and bound alone solved only 38 percent. The benefits of these techniques are also demonstrated on some real world generalized assignment and set-partitioning problems.

A CASCADE APPROACH FOR STAIRCASE LINEAR PROGRAMS WITH AN APPLICATION TO AIR FORCE MOBILITY OPTIMIZATION

Steven F. Baker-Major, United States Air Force

B.S., United States Air Force Academy, 1981

M.S., Air Force Institute of Technology, 1991

Doctor of Philosophy in Operations Research-June 1997

Advisors: Richard E. Rosenthal, Department of Operations Research

Gerald G. Brown, Department of Operations Research

R. Kevin Wood, Department of Operations Research

Craig M. Rasmussen, Department of Mathematics

David P. Morton, University of Texas at Austin

We develop a method to approximately solve a large staircase linear program that optimizes decisions over time. Also developed is a method to bound that approximation's error. A feasible solution is derived by a *proximal cascade*, which sequentially considers overlapping subsets of the model's time periods, or other ordinally defined set. In turn, we bound the cascade's deviation from the optimal objective value by a *Lagrangian cascade* which penalizes unfeasibility by incorporating dual information provided by the proximal cascade solution. When tested on a large temporal LP developed for U.S. Air Force mobility planners, we often observe gaps between the approximation and bound of less than 10 percent, and save as much as 80 percent of the time required to solve the original problem. We also address methods to reduce the gap, including constraint extension of the *Lagrangian cascade*, as well as exploitation of dual multipliers within the proximal cascade.

1997 THESIS ABSTRACTS

DETERMINATION OF HUB FORCES AND MOMENTS OF THE RAH-46 COMANCHE HELICOPTER

William F. Beaver, Jr.-Lieutenant, United States Navy

B.A.E., Georgia Institute of Technology, 1986

Master of Science in Aeronautical Engineering-December 1996

Advisor: E. Roberts Wood, Department of Aeronautics and Astronautics

Second Reader: Donald A. Danielson, Department of Mathematics

Efforts to establish a better understanding of the performance of the RAH-66 *Comanche* helicopter were performed as part of an engineering internship with the Sikorsky Aircraft Comanche Dynamics group in Trumbull (Stratford), Connecticut. Test data from whirl stand testing and the *Comanche* Propulsion System Testbed (the ground test vehicle replacement) was evaluated. Fixed and rotating frame measurements were used to determine hub moments and forces generated by cyclic inputs. Flapping response phase to control input was also determined. Other mast loads were examined to determine the cause for greater than anticipated hub forces. Edgewise bending of the rotor blades was found to be a significant contributor to hub forces.

SIMULATION OF THE DYNAMIC BEHAVIOR OF EXPLOSION GAS BUBBLES IN A COMPRESSIBLE FLUID MEDIUM

James Earl Chisum-Lieutenant Commander, United States Navy

B.S., Southern Oregon State College, 1982

M.S., Naval Postgraduate School, 1992

Mech. Eng., Naval Postgraduate School, 1992

Doctor of Philosophy in Mechanical Engineering-December 1996

Dissertation Supervisor: Young S. Shin, Department of Mechanical Engineering

Doctoral Committee: Young S. Shin, Department of Mechanical Engineering

Anthony Healey, Department of Mechanical Engineering

Young W. Kwon, Department of Mechanical Engineering

Clyde Scandrett, Department of Mathematics

Steven R. Baker, Department of Physics

Data from one-dimensional (spherically symmetric) analyses was used to examine the effects of compressibility and gas energy on the dynamic behavior of an explosion gas bubble, by comparing the bubble's behavior with experimental results and with analytical results which neglect these factors. Results from two-dimensional (axially symmetric) analyses were used to investigate the behavior of a deep explosion gas bubble in the vicinity of plane rigid or constant pressure boundaries. Previous analytical research into explosion gas bubbles near such boundaries has primarily led to results of a qualitative nature, owing to a complete breakdown of the assumptions made in the analysis at the critical juncture. In the present investigation, it was found possible to characterize the effect of the boundary surface on both the change in the first oscillation period of the bubble and its location at the end of the first oscillation cycle. For a broad range of bubble-boundary standoff distances, these semi-empirical characterizations have a functional form particularly suitable for extension of the quantitative results of this investigation to other explosive charge types, weights, and depths, as has been done for the Willis formula for the free-field oscillation period of explosion gas bubbles.

1997 THESIS ABSTRACTS

APPROXIMATING THE CHROMATIC NUMBER OF AN ARBITRARY GRAPH USING A SUPERGRAPH HEURISTIC

Loren G. Eggen-Captain, United States Army

B.A., Saint Cloud State University, 1988

Master of Science in Applied Mathematics-June 1997

Advisor: Craig W. Rasmussen, Department of Mathematics

Second Reader: Harold M. Fredricksen, Department of Mathematics

We color the vertices of a graph G , so that no two adjacent vertices have the same color. We would like to do this as cheaply as possible. An efficient coloring would be very helpful in optimization models, with applications to bin packing, examination timetable construction, and resource allocations, among others. Graph coloring with the minimum number of colors is in general an NP-complete problem. However, there are several classes of graphs for which coloring is a polynomial-time problem. One such class is the chordal graphs. This thesis deals with an experimental algorithm to approximate the chromatic number of an input graph G . We first find a maximal edge-induced chordal subgraph H of G . We then use a completion procedure to add edges to H , so that the chordality is maintained, until the missing edges from G are restored to create a chordal supergraph S . The supergraph S can then be colored using the greedy approach in polynomial time. The graph G now inherits the coloring of the supergraph S .

MATHEMATICAL MODELING USING MICROSOFT EXCEL

Nelson L. Emmons, Jr.-Captain, United States Army

B.S., United States Military Academy, 1989

Master of Science in Applied Mathematics-June 1997

Advisor: Maurice D. Weir, Department of Mathematics

Second Reader: Bard K. Mansager, Department of Mathematics

The entry into higher mathematics begins with calculus. Rarely, however, does the calculus student recognize the full power and applications for the mathematical concepts and tools that are taught. Frank R. Giordano, Maurice D. Weir, and William P. Fox produced *A First Course in Mathematical Modeling*, a unique text designed to address this shortcoming and teach the student how to identify, formulate, and interpret the real world in mathematical terms. Mathematical modeling is the application of mathematics to explain or predict real-world behavior. Often real-world data are collected and used to verify or validate (and sometimes formulate) a hypothetical model or scenario. Inevitably, in such situations, it is desirable and necessary to have computational support available to analyze the large amounts of data. Certainly this eliminates the tedious and inefficient hand calculations necessary to validate and apply the model (assuming the calculations can even be reasonably done by hand).

The primary purpose of *Mathematical Modeling Using Microsoft Excel* is to provide instructions and examples for using the spreadsheet program Microsoft Excel to support a wide range of mathematical modeling applications. Microsoft Excel is a powerful spreadsheet program which allows the user to organize numerical data into an easy-to-follow on-screen grid of columns and rows. Our version of Excel is based on Microsoft Windows. In this text, it is not the intent to teach mathematical modeling, but rather to provide computer support for most of the modeling topics covered in *A First Course in Mathematical Modeling*. The examples given here support that text as well.

THE CONTROL OF BIFURCATIONS WITH ENGINEERING APPLICATIONS

Osa F. Fitch-Lieutenant Commander, United States Navy

M.S., Massachusetts Institute of Technology, 1982

Doctor of Philosophy in Aeronautics and Astronautics-September 1997

Dissertation Supervisor: Wei Kang, Department of Mathematics

Committee Chairman: Richard M. Howard, Department of Aeronautics and Astronautics

This dissertation develops a general method for the control of the class of local bifurcations of engineering interest, including saddle-node, transcritical, pitchfork, and Hopf bifurcations. The method is based on transforming a general affine

1997 THESIS ABSTRACTS

single-input control system into quadratic normal form through coordinate transformations and feedback. (The quadratic normal form includes the quadratic order Poincare normal form of the uncontrolled system as a natural subset.) Then, linear and quadratic state feedback control laws are developed which control the shape of the center manifold of the transformed system. It is shown that control of the center manifold allows the quadratic and cubic order terms of the center dynamics to be influenced to produce non-linear stability. Specific matrix operations necessary to transform a general affine single-input control system into quadratic normal form are provided. Specific control laws to stabilize a general system experiencing a linearly unstabilizable saddle-node, transcritical, pitchfork, or Hopf bifurcation are also provided.

A DYNAMIC TARGETING MODEL OF THE COCAINE TRADE FOR POLICY MAKERS AND INTELLIGENCE ANALYSTS (U)

Marc A. Flicker-Captain, United States Army

B.S., Pennsylvania State University, December 1986

Master of Arts in National Security Affairs-December 1996

and

Kirk H. Nilsson-Captain, United States Army

B.S., University of Idaho, May 1986

Master of Arts in National Security Affairs, December 1996

Advisors: Carlos Borges, Department of Mathematics

Gordon McCormick, Special Operations Low Intensity Conflict (SOLIC) Curriculum Committee

Our purpose in this thesis is to develop a model for use as a tool for national level policy makers. It will assist them to understand the cocaine trade, develop effective counter cocaine strategies and to make informed decisions on the allocation of resources. We created three models in the process of developing a dynamic targeting model of the cocaine trade. Our first model is a static description of the cocaine trade that explains the production, processing, and transportation of cocaine from source countries to destination countries. We used the values and spreadsheet organization derived from our static model to develop our second model, the corridor model. We use this model to generate two important values for our dynamic targeting model: Monthly estimates for how much cocaine should flow through the various corridors and from which countries this cocaine originates.

In our final model, we incorporated values from the static model and corridor model to generate a dynamic model of the cocaine system. This model allows us to evaluate various targeting scenarios over time. By evaluating various targeting scenarios, we will determine vulnerabilities and predict responses in the system over the short and long term.

COMPARING THE MAXIMUM LIKELIHOOD METHOD AND A MODIFIED MOMENT METHOD TO FIT A WEIBULL DISTRIBUTION TO AIRCRAFT ENGINE FAILURE TIME DATA

Fernando Guimil-Lieutenant Commander, Spanish Navy

B.S., Spanish Naval Academy, 1978

Licenciado Ciencias Economicas, Universidad Nacional de Educacion a Distancia, 1993

Master of Science in Operations Research-September 1997

Advisor: W. Max Woods, Department of Mathematics

Second Reader: Harold J. Larson, Department of Operations Research

This thesis provides a comparison of the accuracies of two methods for fitting a Weibull distribution to a set of aircraft engines time-between-failure data.

One method used is the Maximum Likelihood Method and assumes that these engine failure times are independent. The other method is a Modified Method of Moments procedure and uses the fact that if time to failure T has a Weibull distribution with scale parameter λ and shape parameter β , then T^β has an exponential distribution with scale parameter λ^β . The latter method makes no assumption about independent failure times.

A comparison is made from times that are randomly generated with a program. The program generates times in a manner that resembles the way in which engine failures occur in the real world for an engine with three subsystems. These

1997 THESIS ABSTRACTS

generated operating times between failures for the same engine are not statistically independent. This comparison was extended to real data.

Although the two methods gave good fits, the Maximum Likelihood Method produced a better fit than the Modified Method of Moments. Explanations for this fact are analyzed and presented in the conclusions.

ANALYSIS OF THE NUMERICAL SOLUTION OF THE SHALLOW WATER EQUATIONS

Thomas A. Hamrick-Lieutenant, United States Navy
B.S., Economics, North Carolina State University, Raleigh, 1987
Master of Science in Applied Mathematics-September 1997
Advisor: Beny Neta, Department of Mathematics
Second Reader: Clyde Scandrett, Department of Mathematics

This thesis is concerned with the analysis of various methods for the numerical solution of the shallow water equations along with the stability of these methods. Most of the thesis is concerned with the background and formulation of the shallow water equations. The derivation of the basic equations will be given, in the primitive variable and vorticity-divergence formulation. Also the shallow water equations will be written in spherical coordinates. Two main types of methods used in approximating differential equations of this nature will be discussed. The two schemes are finite difference method (FDM) and the finite element method (FEM). After presenting the shallow water equations in several formulations, some examples will be presented. The use of the Fourier transform to find the solution of a semidiscrete analog of the shallow water equations is also demonstrated.

MARKOV RANDOM FIELD TEXTURES AND APPLICATIONS IN IMAGE PROCESSING

Christopher A. Korn-Lieutenant, United States Navy
B.S., United States Naval Academy, 1988
Master of Science in Applied Mathematics-June 1997
Advisor: Harold M. Fredricksen, Department of Mathematics
Second Reader: Carlos Borges, Department of Mathematics

In the field of image compression, transmission and reproduction, the foremost objective is to reduce the amount of information which must be transmitted. Currently the methods used to limit the amount of data which must be transmitted are compression algorithms using either lossless or lossy compression. Both of these methods start with the entire initial image and compress it using different techniques. This paper will address the use of Markov Random Field Textures in image processing. If there is a texture region in the initial image, the concept is to identify that region and match it to a suitable texture which can then be represented by a Markov random field. Then the region boundaries and the identifying parameters for the Markov texture can be transmitted in place of the initial or compressed image for that region.

PREDICTION AND GEOMETRY OF CHAOTIC TIME SERIES

Mary L. Leonardi-Captain, United States Marine Corps
B.A., Northwestern University, 1991
Master of Science in Applied Mathematics-June 1997
Advisor: Christopher L. Frenzen, Department of Mathematics
Second Reader: Phil Beaver, Department of Mathematics

This thesis examines the topic of chaotic time series. An overview of chaos, dynamical systems, and traditional approaches to time series analysis is provided, followed by an examination of the method of state space reconstruction. State space reconstruction is a nonlinear, deterministic approach whose goal is to use the immediate past behavior of the time series to reconstruct the current state of the system. The choice of delay parameter and embedding dimension are crucial to this reconstruction. Once the state space has been properly reconstructed, one can address the issue of whether apparently random data has come from a low-dimensional chaotic (deterministic) source or from a "random" process. Specific tech-

1997 THESIS ABSTRACTS

niques for making this determination include attractor reconstruction, estimation of fractal dimension and Lyapunov exponents, and short-term prediction.

If the time series data appears to be from a low-dimensional chaotic source, then one can predict the "continuation" of the data in the short term, exploiting the fact that chaotic systems are fairly predictable in the short term. This is the "inverse problem" of dynamical systems. In this thesis, the technique of local fitting is used to accomplish the prediction. Finally the issue of noisy data is treated, with the purpose of highlighting where further research may be beneficial.

AN ANNULAR THERMOACOUSTIC PRIME MOVER

Tseng-Hsiao Lin-Major, Taiwan, Republic of China Army

B.S., Chung Cheng Institute of Technology in Taiwan, 1984

M.S., Naval Postgraduate School, 1989

Doctor of Philosophy in Engineering Acoustics-September 1997

Dissertation Supervisor: Anthony A. Atchley, Department of Physics

Advisors: Thomas J. Hofler, Department of Physics

Andrés Larraza, Department of Physics

Robert M. Keolian, Department of Physics

Christopher Frenzen, Department of Mathematics

James H. Miller, Professor of Ocean Engineering, University of Rhode Island

The dissertation constitutes the first detailed theoretical and experimental investigation of a thermoacoustic prime mover with periodic boundary conditions. There are five significant aspects to this research: (1) using DeltaE to analyze an annular prime mover, (2) developing an entirely new analysis program using MATLAB, (3) designing, building, and experimentally investigating a single stack, annular prime mover, (4) experimentally investigating a constricted, single stack prime mover, and (5) predicting the performance of a two stack annular prime mover. The major conclusions are: (1) A single stack annular prime mover will not reach onset because the eigenmodes of the system do not support thermoacoustic growth, (2) A constricted annular prime mover will reach onset because the constriction forces dominating boundary conditions that alter the eigenmodes, and (3) A two stack prime mover is predicted to reach onset because one of the eigenmodes of the symmetric system does support thermoacoustics.

INTERPOLATION WEIGHTS OF ALGEBRAIC MULTIGRID

Gerald N. Miranda, Jr.-Lieutenant, United States Navy

B.A., University of California, San Diego, 1990

Master of Science in Applied Mathematics-June 1997

Advisor: Van Emden Henson, Department of Mathematics

Second Reader: Christopher L. Frenzen, Department of Mathematics

Algebraic multigrid (AMG) is a numerical method used to solve particular algebraic systems, and interest in it has risen because of its multigrid-like efficiency. Variations in methodology during the interpolation phase result in differing convergence rates. We have found that regular interpolation weight definitions are inadequate for solving certain discretized systems so an iterative approach to determine the weights will prove useful. This iterative weight definition must balance the requirement of keeping the interpolatory set of points "small" in order to reduce solver complexity while maintaining accurate interpolation to correctly represent the coarse-grid function on the fine grid. Furthermore, the weight definition process must be efficient enough to reduce setup phase costs.

We present systems involving matrices where this iterative method significantly outperforms regular AMG weight definitions. Experimental results show that the iterative weight definition does not improve the convergence rate over standard AMG when applied to M-matrices; however, the improvement becomes significant when solving certain types of complicated, non-standard algebraic equations generated by irregular operators.

1997 THESIS ABSTRACTS

AN ANALYTICAL MODEL FOR EVALUATING AEGIS MISSILE DEFENSE EFFECTIVENESS

Curt A. Renshaw-Lieutenant, United States Navy

B.S., United States Naval Academy, 1990

Master of Science in Operations Research-September 1997

Advisor: W. Max Woods, Department of Mathematics

Second Reader: Wayne P. Hughes, Department of Operations Research

The purpose of this thesis is to model the effectiveness of the Aegis combat system against supersonic anti-ship cruise missiles. The model takes into consideration the overall integration of all weapons and sensor systems on board, the availability and reliability of the weapons systems, the threat range at detection, the proficiency of the crew in employing the weapons systems, the threat range at engagement, and the probability of kill for each weapons system. This model is used to compare the effectiveness of the Aegis system operated by crews of varied proficiency with the effectiveness of the system using automated engagement systems. Additionally, a number of new, more potent weapons systems are proposed as additions to the Aegis system. An analysis of the resulting improvement in air defense capability due to the addition of these weapons and also the required level of automation for these systems is examined. The results of this analysis indicate that for certain cruise missile threats improved reaction time is more important than improved lethality for the Aegis system. Recommendations for the tactical employment of the system are given.

MODELING AND ANALYSIS OF HELICOPTER GROUND RESONANCE UTILIZING SYMBOLIC PROCESSING AND DYNAMIC SIMULATION SOFTWARE

Christopher S. Robinson-Lieutenant, United States Navy

B.S., Rensselaer Polytechnic Institute, 1989

Aeronautical and Astronautical Engineer-March 1997

Advisor: E. Roberts Wood, Department of Aeronautics and Astronautics

Second Reader: Donald Danielson, Department of Mathematics

This thesis develops a technique for formulating the full nonlinear equations of motion for a coupled rotor-fuselage system utilizing the symbolic processing software MAPLE®. The symbolic software is further utilized to automatically convert the equations of motion into C, Fortran or MATLAB® source code formatted specifically for numerical integration. The compiled source code can be accessed and numerically integrated by the dynamic simulation software SIMULINK®. SIMULINK® is utilized to generate time history plots of blade and fuselage motion. These time traces can be used to explore the effects of damping nonlinearities, structural nonlinearities, active control, individual blade control, and damper failure on ground resonance. In addition, a MATLAB® program was developed to apply the Moving Block Technique for determining modal damping of the rotor-fuselage system from the time marching solutions.

STRUCTURAL DESIGN ANALYSIS OF THE TAIL LANDING GEAR BAY AND THE VERTICAL/HORIZONTAL STABILIZERS OF THE RAH-66 COMANCHE HELICOPTER

Brian Paul Shoop-Major, United States Army

B.S., United States Military Academy, 1986

Master of Science in Aeronautical Engineering-September 1997

Advisors: E. Roberts Wood, Department of Aeronautics and Astronautics

Donald A. Danielson, Department of Mathematics

Joshua H. Gordis, Department of Mechanical Engineering

The RAH-66 Comanche's stealth design requires the use of radar-absorbing material (RAM) on the outer skin of the aircraft. The reduced stiffness properties of RAM produce insufficient tail torsional stiffness, necessitating the use of non-radar-absorbing graphite on the outer skin of the prototype's tail section. This thesis investigates structural design modifications to increase the tail section's stiffness to allow the use of RAM on the outer skin and still meet all structural requirements. An original model represents the prototype aircraft at first flight. The goal is to create a model using RAM on the

1997 THESIS ABSTRACTS

outer skin that matches the structural stiffness of the original model. This thesis builds on earlier work conducted at the Naval Postgraduate School (NPS). Two new design modifications to the tailcone are developed. The best modification increases the torsional stiffness of a baseline model by six percent. Integrating earlier NPS modifications increases torsional stiffness by 12 percent. When RAM is applied to the outer skin of the modified model, torsional stiffness is reduced by only six percent from the baseline as compared to a 24 percent reduction with no modifications. Additional modifications to the vertical and horizontal stabilizers further increase structural stiffness and reduce weight.

THE UTILITY OF HIGH RESOLUTION MODELING IN ARMY SPECIAL OPERATIONS AVIATION MISSION PLANNING

Robin R. Smith-Captain, United States Army

B.A., University of Arizona, 1982

M.Ed., Arizona State University, 1986

Master of Arts in National Security Affairs-December 1996

Advisor: Bard Mansager, Department of Mathematics

Second Reader: Maurice D. Weir, Department of Mathematics

The purpose of this thesis is to explore the application of high resolution modeling in the Army Special Operations Aviation mission planning process. This thesis looks at the unique missions Special Operations Forces are expected to perform, often at very high levels of public scrutiny, and how the use of combat simulation can assist commanders, planners and staffs in simplifying the frictions encountered in the planning process. The main objective of this study is to define common practical uses for combat simulation in deliberate and time sensitive mission planning.

This investigation surveys the use of special operations to achieve key foreign policy objectives and the ability of combat simulation to provide answers to potential questions and to stimulate queries to subjects that operators may not have considered important. By applying combat simulation in the mission planning process, planners can streamline decision-making capabilities by constructing correct and visible paths to valid conclusions. An historical case study, the raid on the Son Tay prisoner of war camp in North Vietnam in 1970, serves as an instructive example to demonstrate basic applications of combat simulation in the mission planning process and investigating variables possibly cogent to the outcome of the mission.

Finally, a discussion on high resolution special operations models used at the United States Special Operations Command and their architecture for future mission planning modeling will assist in spanning the chasm from the Cold War paradigm to new and unexpected tactical scenarios.

AN ANALYSIS OF THE DETECTION RANGE FOR THE MARK V SPECIAL OPERATIONS CRAFT USING HIGH RESOLUTION COMPUTER MODELING. (U)

Todd L. Tinsley-Lieutenant, United States Navy

B.S., University of Nebraska, Lincoln, 1989

Master of Arts in National Security Affairs-June 1997

Master of Science in Applied Mathematics-June 1997

Advisor: Bard K. Mansager, Department of Mathematics

Second Reader: Carlos F. Borges, Department of Mathematics

Naval Special Warfare (NSW) forces are designed to be difficult to detect, and not to defeat the enemy in a head-to-head confrontation. It follows that detection is a primary concern for these elite units. The latest, most versatile, high performance combatant craft introduced into the NSW Special Boat Unit (SBU) inventory to improve maritime special operations capabilities is the MARK V Special Operations Craft (MK V SOC). The role of this craft is for medium range insertion and extraction of special operations forces (SOF) in a low to medium threat environment. This thesis uses a high resolution computer simulation model, Janus, to represent the characteristics and operating parameters of the MK V SOC along with three, electro-optical night vision devices (NVD). Through repeated computer simulation runs, detections of the MK V SOC by these sensors in three varying meteorological conditions is tested. The range to first detection is recorded for each

1997 THESIS ABSTRACTS

case and analyzed using graphical and statistical methods. Based on the sample data of detection ranges, statistical inferences are made about a sensor's performance under prescribed environmental conditions and its ability to detect an approaching MK V SOC.

ANALYSIS OF POTENTIAL STRUCTURAL DESIGN MODIFICATIONS FOR THE TAIL SECTION OF THE RAH-46 COMANCHE HELICOPTER

Vincent M. Tobin-Major, United States Army

B.S., Bucknell University, 1985

Master of Science in Aeronautical Engineering-June 1997

Advisors: E. Roberts Wood, Department of Aeronautics and Astronautics

Donald A. Danielson, Department of Mathematics

Joshua H. Gordis, Department of Mechanical Engineering

The Army RAH-66 Comanche Helicopter made its first flight in January of 1996. Its current structural configuration, however, does not meet the Army's requirements for radar signature. Structural configurations of the tailcone that meet radar cross-section requirements tend to lack sufficient structural stiffness due to the presence of Kevlar in place of graphite on the outer mold line. This thesis investigates potential structural design modifications to the Comanche tailcone that would move the design closer to meeting both its structural and radar signature requirements. Structural geometry modifications with baseline (current configuration) materials increased torsional stiffness by six percent. Geometry modifications using radar signature-compliant materials reduced torsional stiffness by 15 percent. The geometry changes analyzed produce structural performance improvements insufficient to allow the use of radar-compliant materials without further geometry changes.

HIGH RESOLUTION MODELING OF SOF MINE-COUNTERMEASURE OPERATIONS (U)

Robert C. Wilson-Lieutenant Commander, United States Navy

B.A., College of the Holy Cross, 1986

Master of Arts in National Security Affairs-March 1997

Advisor: Bard K. Mansager, Department of Mathematics

Second Reader: Maurice D. Weir, Department of Mathematics

This thesis explores ways in which stochastic high-resolution modeling may be utilized by maritime special operations forces (SOF) as a tool for tactics development and mission planning. Using SOF mine countermeasure (MCM) operations for illustrative purposes, the study focuses on testing and evaluation of the Janus high-resolution model (HRM). Model development includes terrain, amphibious minefields, enemy shore-based surveillance systems, SOF MCM units, and tactics pertinent to SOF mine reconnaissance operations. Model execution tests three SOF MCM search tactics in minefields laid according to enemy doctrine. Following multiple iterations, sensitivity analysis is conducted on search tactics and various surface support craft detection vulnerabilities.

Study findings demonstrate HRM utility for optimization of tactics and model-assisted mission planning. The model developed in this study may augment ongoing maritime craft detection vulnerability studies. Model development, testing, and analysis reveals shortfalls and limitations pertaining to the model and databases used. The study provides recommendations for the improvement of future high-resolution models that include maritime SOF operations. Recommendations may be applicable to Janus derivative models such as the Joint Tactical Simulation (JTS) and Joint Conflict Model (JCM).

INITIAL DISTRIBUTION LIST

- | | | |
|----|---|---|
| 1. | Defense Technical Information Center
8725 John J. Kingman Rd., STE 0944
Ft. Belvoir, VA 22060-6218 | 2 |
| 2. | Dudley Knox Library, Code 013
Naval Postgraduate School
411 Dyer Rd.
Monterey, CA 93943-5101 | 2 |
| 3. | Associate Provost and Dean of Research
Code 09
Naval Postgraduate School
Monterey, CA 93943-5138 | 2 |
| 4. | Chair
Department of Mathematics
Naval Postgraduate School
Monterey, CA 93943-5000 | 5 |
| 5. | Associate Chair for Research
Department of Mathematics
Naval Postgraduate School
Monterey, CA 93943-5000 | 1 |
| 6. | Dean, Division of Science and Engineering
Code 07
Naval Postgraduate School
Monterey, CA 93943-5000 | 1 |
| 7. | Provost and Academic Dean
Code 01
Naval Postgraduate School
Monterey, CA 93943-5000 | 1 |