OTS: 60-31,796

JPRS: 3984

27 September 1960

ON THE LOSS OF STABILITY OF A CYLINDRICAL SHELL UNDER DYNAMIC LOADING

- USSR -

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19990305 037

DTIC QUALITY INSPECTED 2

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U. S. DEPARTMENT OF COMMERCE
WASHINGTON 25, D. C.

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FOREWORD

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In contrast to statistical studies, the analysis of the
loss of stability of cylinders in dynamics has the characteristic
peculiarity that it is not the determination of the moment of the
beginning of stability loss but the study of the magnitude of the
flexure /progib/ in the process of loading which acquires decisive
significance.

In an engineering sense, the critical load is taken as that
in which the flexures of the loss of stability begin to increase
vigorously or reach dangerous proportions. The motion of a cylinder
is studied on the assumption that it has a beginning flexure which
coincides in form with the flexure of stability loss.

Such an approach to a determination of critical loads
applicable to rods was developed in the works of N. Khoff /1/.
It was first applied to shells by A. S. Vol'mir who studied the
stability of a cylindrical panel during longitudinal compression
/2/. A study by V. V. Bolotin and others /3/ was also devoted
to this problem. The loss of stability of a cylindrical shell during
loading on all sides which increased linearly over time was
examined in an article by V. L. Agamirn and A. S. Vol'mir /4/.

The effect of an equally-distributed transverse dynamic
load is studied below. Unlike study /4/, we consider not only
the inertness of the formation of indentations of the stability
loss but also the inertness of axially symmetric compression of
the shell. This permits the use of the system of equations of the
motion of the shell, which was obtained, for cases of more rapid
loading than is permitted by the system of equations presented in
study /4/.

Let us consider an infinitely long cylindrical shell, supported
by equi-distant lateral ribs, absolutely inflexible to bending but
not resisting compression, loaded with a transverse dynamic load.

In this case the loss of stability will take place only between the ribs. We shall not consider the mass of the ribs during the motion of the shell. We shall confine ourselves to the case where the distance between the ribs is such that during the stability loss on the perimeter of the shell, a large number of waves ($n > 5$) is formed. Then for the study we can use the non-linear theory of flattened /pologikh/ shells.

For an approximation of the flexure of a shell during the loss of stability, we shall take advantage of the expression:

$$w = f_0 + f_1 \sin \alpha x \sin \beta y + f_2 \sin^2 \alpha x \qquad \left(\alpha = \frac{\pi}{L}, \ \beta = \frac{n}{R} \right) \quad (1)$$

Here L -- the distance between the ribs; R -- radius of the shell; n -- the number of waves of stability loss; x, y -- the coordinates, read along the generatrix and the directrix.

Expression (1) has been used by many authors to study the statistical loss of stability of cylindrical shells. In article /4/ it is applied to a dynamic problem.

We shall assume that the shell has an initial flexure coinciding in form with the flexure during stability loss:

$$w_0 = f_{10} \sin \alpha x \sin \beta y + f_{20} \sin^2 \alpha x \qquad (2)$$

We shall avail ourselves of the LaGrange equation of the second order /roda/ to form the equations of motion:

$$\frac{d}{dt} \frac{\partial K}{\partial p_i} - \frac{\partial K}{\partial p_i} + \frac{\partial \Pi}{\partial p_i} = 0 \qquad (3)$$

Here K -- kinetic energy of the shell; Π -- full potential energy; p_i -- the generalized coordinate; t -- time.

We shall take the parameters of flexure f_1, f_2 and f_0 as generalized coordinates.

Since the tangential displacements are small in comparison with normal displacements for a large number of waves, we shall define the kinetic energy of the motion of the shell by the formula:

$$K = \frac{\rho h}{2} \int_0^L \int_0^{2\pi R} \left(\frac{\partial w}{\partial t} \right)^2 dx\, dy \qquad (4)$$

-- 2 --

Here ρ — the density of the material of the shell; h — the thickness of the shell.

The full potential energy consists of the energy of flexure of the shell, the energy of deformation of the middle surface and the work of external forces in a given cass of transverse pressure. We shall determine these magnitudes by the familiar formula of the theory of flattened shells $/\ 5\ /$.

The magnitude of the average annular stress is described by the parameters of flexure with the aid of the equation of the closed condition of the shell

$$\int_0^{2\pi R} \frac{\partial v}{\partial y}\, dy = 0 \tag{5}$$

(where v — the tangential displacement of points of the middle surface of the shell) and is eliminated from the expression for the potential energy of the shell.

The equations of the motion of a shell in dimensionless magnitudes have the form

$$\frac{d^2 \zeta_1}{d\tau^2} = a_1 \zeta_1^3 - a_2 \zeta_1 \zeta_2^2 + a_3 \zeta_1 \zeta_2 + a_4 \zeta_2^2 - a_5 \zeta_1 - a_6 \zeta_2 +$$

$$+ a_7 + \eta_1 \zeta_1 = 0 \tag{6}$$

$$\frac{d^2 \zeta_2}{d\tau^2} = - 2a_2 \zeta_1^2 \zeta_2 + a_8 \zeta_1^2 + 4a_4 \zeta_1 \zeta_2 - a_9 \zeta_1 - a_{10} \zeta_2 + a_{11}$$

$$\frac{d^2 \zeta_0}{d\tau^2} = a_2 \zeta_1^2 \zeta_2 + a_{12} \zeta_1^2 - 2a_4 \zeta_1 \zeta_2 + \tfrac{1}{2} a_9 \zeta_1 - a_{13} \zeta_2 +$$

$$+ a_{14} - \zeta_0 + q^* (\tau)$$

Here

$$\zeta_0 = \frac{f_0}{h} , \quad \zeta_1 = \frac{f_1 + f_{10}}{h} , \quad \zeta_2 = \frac{f_2 + f_{20}}{h} ,$$

$$\eta_1 = \frac{n^2 h}{R} , \quad q^*(\tau) = \frac{q(\tau) R^2}{Eh^2} , \quad \tau = \frac{tV}{R} \tag{7}$$

- 3)

in which $q\,(\tau)$ -- the transverse load; v -- speed of the propagation of sound in the material of the shell.

The coefficients of system (6) have the following values:

$$a_1 = \frac{1}{16}\,\pi^4\delta^2(1 + 3b_1), \qquad a_2 = \pi^4\delta^2(b_2 + b_3),$$

$$a_3 = \pi^2\lambda^2\delta\left(\frac{3}{4}b_1 + 2b_2\right), \qquad a_4 = \pi^4\delta^2\zeta_1(2b_2 + 3b_3)$$

$$a_5 = q_+^*\,\eta - a_1(\zeta_1^0)^2 + \frac{3}{4}\eta\,\delta_2^0,$$

$$a_6 = \pi^4\delta\zeta_1^0\zeta_2^0(b_2 + 2b_3) + \pi^2\lambda^2\delta\zeta_1^0 b_2,$$

$$a_7 = q_+^*\zeta_1^0 - \pi^2\lambda^2\delta\zeta_2^0\zeta_1^0 b_2$$

$$a_8 = \pi^2\lambda^2\delta\left(\frac{1}{4}b_1 + 2b_2\right)$$

$$a_9 = 2\pi^4\delta^2\zeta_1^0\zeta_2^0(b_2 + 2b_3) + 2\pi^2\lambda^2\delta\zeta_1^0 b_2,$$

$$a_{10} = 1 + \frac{1}{3}\left(\frac{4\pi^4\delta^2}{1 - \mu^2}\right) + 4\pi^4\delta^2(\zeta_1^0)^2(2b_2 + 3b_3),$$

$$a_{11} = \left[1 + \frac{4\pi^4\delta^2}{3(1 - \mu^2)}\right]\zeta_2^0 + 2\pi^4\delta^2(\zeta_1^0)^2\zeta_2^0(2b_2 + 3b_3) - \frac{1}{4}\pi^2\delta\lambda^3 b_1,$$

$$a_{12} = \frac{1}{8}\eta - \frac{1}{2}a_8$$

$$a_{13} = \frac{1}{2} - \frac{1}{2}a_{10}$$

$$a_{14} = \frac{1}{2}a_{11} + \frac{1}{8}\eta\,(\zeta_1^0)^2$$

Here

$$\zeta_1^0 = \frac{f_{10}}{h}, \quad \zeta_2^0 = \frac{f_{20}}{h}, \quad \delta = \frac{hR}{L^2}, \quad \lambda = \frac{\pi R}{nL}, \quad b_1 = \frac{1}{\lambda^4}$$

$$b_2 = \frac{1}{(1 + \lambda^2)^2}, \quad b_3 = \frac{1}{(1 + 9\lambda^2)^2}, \quad q_+^* = \frac{(1 + \lambda^2)^2\,\eta}{12(1 - \mu^2)} + \frac{\lambda^4}{(1 + \lambda^2)^2\,\eta}$$

q_+^* — upper static critical load.

The dynamic loss of stability of a shell is studied in the example of a sudden application of a load of constant magnitude. The calculations for system (6) were carried out on the MN-7 analog computer, with the inclusion of additional units of non-linearity. A shell characterized by the parameter $\pi^2 \xi = 1$ was selected as an example.

Figure 1 shows a typical graph of the dynamic flexure of a shell.

Let us take note of the following characteristic peculiarities of the motion of a shell.

1. The flexures of the loss of stability increase very slowly in the beginning of motion and basically axially symmetrical compression of the shell takes place. At the same time, the coefficient of dynamics /dinamichnost'/ is equal to 2.

2. Beginning at some moment of time, flexures of stability loss begin to increase vigorously and then change into nonlinear oscillations. It is characteristic that the static connection of parameters ξ_1 and ξ_2 is not observed in dynamics.

3. With great flexures of stability loss, axially symmetric compression of the shell reaches more than twice the magnitude of the static value.

4. At the moment that the magnitude of axially symmetric compression of the shell declines, a reconstruction of the wave surface takes place: the indentations change to bulges and conversely.

A brief and prolonged application of the load was investigated. In the first case, the load was removed at the moment in time equal to the period of natural axial symmetrical oscillations of the shell. By that time, the first maximum of the flexures of stability loss ξ_1 had time to form. After removing the load, the free oscillation of the shell occured with the amplitude of the bending flexures not exceeding the first maximum. Therefore, the first maximum of the bending flexures can be taken in a given case of loading as determining. Figure 2 shows the dependence of the magnitude of the first peak of the bending flexures ξ_1 on the initial deflections of the shell and the magnitude of the load.

We shall agree to call that load safe under which the summary stresses at any point of the shell do not exceed the flow limit. This hypothesis is introduced to explain the qualitative illustration of the phenomenon since reaching the stresses of the flow limit at different points is still not dangerous for the shell. We propose to specify the criteria of safe loads precisely below.

With transverse loads the greatest stress occurs in the longitudinal cross-sections in the middle of the span between the ribs on the crests or in the troughs of the waves of stability loss. They are equal to

$$\epsilon_y^* = \frac{\epsilon_y R}{Eh} = -\zeta_0 - (\zeta_2 - \zeta_2^0) + \frac{1}{4}\eta(\zeta_1^2 - \zeta_1^{02}) +$$

$$+ \left\{ \eta(\zeta_1\zeta_2 - \zeta_1^0\zeta_2^0)\left[\frac{1}{(1+\lambda^2)^2} - \frac{9}{(9+\lambda^2)^2}\right] - \right.$$

$$- \frac{\zeta_1 - \zeta_1^0}{(1+\lambda^2)^2}\sin\beta y \pm \frac{1}{12(1-\mu^2)}\left[(\zeta_1 - \zeta_1^0)\right.$$

$$(\gamma_1 + \mu\pi^2\delta)\sin\beta y + \mu\pi^2\delta(\zeta_2 - \zeta_2^0)\right]$$

(10)

Since the dependence of the flexure parameters on time is known from the solution to system (6), it is not difficult to find an illustration of the change of stresses in the process of stability loss.

We shall determine the value of safe dynamic loads, taking the parameter of permissable stress equal to $\sigma_x^* = 1$ (This takes place, for example, when $\sigma_T = 4{,}000$ kg/cm^2; $E = 2 \cdot 10^6$ kg/cm^2; $R/h = 500$).

The values of safe loads for $\lambda = 0.5$ are shown in Figure 2 from which it is clear that safe loads depend heavily on the initial flexure of the shell.

It is characteristic that when $\zeta_1^0 = 0.1$ and 0.01, the dangerous stresses occured at the moment when the maximum flexures of stability loss were formed. With $\zeta_1^0 = 0.001$, the stress from axially symmetric compression exceeded the permissable size before the flexures of stability loss had time to develop. Since the problem was solved in the limits of elasticity, the motion of the shell in this case is no longer described by system (6). The values of the safe loads for various n when $\zeta_1^0 = 0.01$ and $R/h = 100$ are shown in Figure 3.

It is interesting to note that the least safe load corresponds to a larger number of waves than during the loss of stability in statics.

As is known, such a tendency can be detected in an analysis of experimental data on dynamic stresses /4, 6/.

Thus, the least dangerous stress can be assumed to be calculated for the known order of magnitudes of the initial technological irregularities of the shell. In the example investigated, the safe load proved to be equal to 0.87 (critical in statics) and the stresses in the middle of the surface of the shell were significantly in excess of the critical.

The study of flexures of a shell during prolonged loading (the time of load activation was approximately equal to five periods of the natural axially symmetric oscillations) revealed the following characteristic phenomena (Figure 2): if the initial deviations are sufficiently great, the maximum size of the bending flexures practically coincide with the first peak; on the other hand, if the initial deviations are small, the curve of maximum flexures differs markedly from the curve of the first peaks and has some regions of rapid growth. In this case, it is natural to take as the safe load that which corresponds to the beginning of this region. It is necessary to note that during prolonged load activation, the magnitude of the maximum flexures of stability loss, beginning with some load, practically does not depend on the initial deviations of the shell.

Besides the sudden application of a constant load, the effect of a load increasing linearly over time on the shell was also investigated. In this regard, it was established that the influence of inertness of flexures ζ_0 can be ignored for low loading velocities when the stability loss occurs at a moment in time in a period of natural axially symmetric oscillations of the shell several times greater. Such loading velocities as these were also investigated in the examples in article $\underline{/4/}$ where the inertness of axially symmetric compression was not considered.

One must not ignore, however, the effect of inertness of axially symmetric compression during quite rapid loading. In Figure 4, the functions $\zeta_0(\tau)$, $\zeta_2(\varphi)$, $\zeta_2(\psi)$ were presented as curves J_0, J_1, J_2 respectively. As is clear, the function ζ_0 differs significantly from the linear, although it does not take inertness into account.

Submitted 20-1-60

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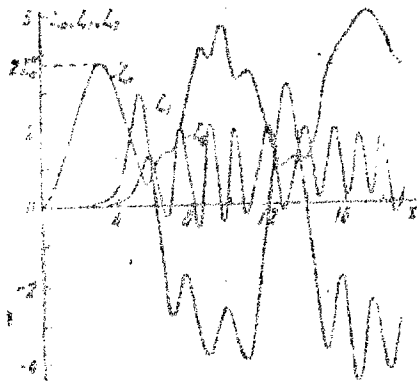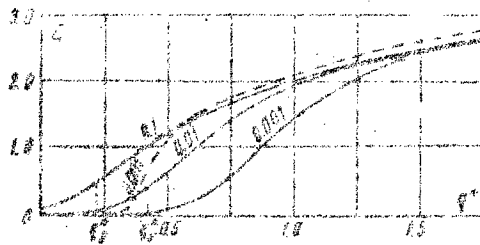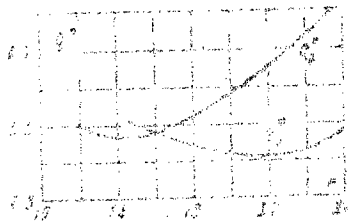
Figure 1.

Figure 2.

Figure 3.

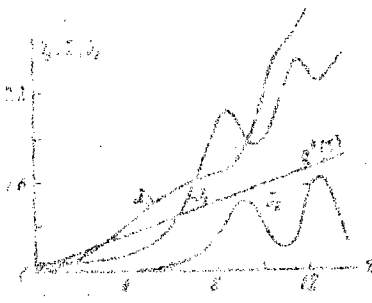

Figure 4.

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