

AD

REPORT 26-04-R-017

**TECHNICAL ADVICE IN SUPPORT OF  
THE WES CENTRIFUGE RESEARCH CENTER**

**(PHASE 4C)**

**FINAL TECHNICAL REPORT**

**by**

**A N Schofield and R S Steedman**

**1 February 1999**

**United States Army**

**EUROPEAN RESEARCH OFFICE OF THE U.S. ARMY**

**London England**

**CONTRACT NUMBER N68171-97-M-5510**

**ANDREW N SCHOFIELD & ASSOCIATES LTD  
9 LITTLE ST MARYS LANE,  
CAMBRIDGE CB2 1RR, U.K.**

**Approved for Public Release; distribution unlimited**

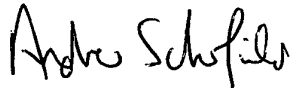
**REPORT DOCUMENTATION PAGE**

Contract No.: N68171-97-M-5510

Title of Proposal: Phase 4C: Technical Advice in support of the WES  
Centrifuge Research Center

Name of Institution: Andrew N Schofield & Associates Ltd.,  
9 Little St Marys Lane,  
Cambridge CB2 1RR, U.K.

Principal Investigator: Dr Andrew N Schofield



A N Schofield



R S Steedman

1 February 1999

## SUMMARY

The development of the WES Centrifuge Research Center was a response to a request for novel research facilities to support the Army Corps of Engineers and its work programmes. The new Army centrifuge, which was inaugurated on 20 November 1997, is the most powerful beam centrifuge currently operating in the world and is unique in its range of capabilities. This report describes the physics of centrifuge modeling and explains the basis by which these capabilities may be derived. There are an infinite number of ways of describing the capabilities of a research centrifuge in terms of the equivalent prototype or field condition, or in terms of its mechanical design. A fundamental appreciation of the physics of a centrifuge as well as an understanding of the behavior of materials under high stress is necessary for the safe and effective use of such a facility to generate world class research results.

**LIST OF KEYWORDS**

centrifuge  
test  
model  
capabilities  
research  
containment  
buildings  
equipment  
appurtenances  
instrumentation  
safety  
geotechnical  
soil mechanics  
civil engineering

## CONTENTS

	Page
Report Cover Sheet	i
Report Documentation Page	ii
Summary	iii
List of Keywords	iv
Contents	v
<b>1.0 COMMISSIONING OF THE ARMY CENTRIFUGE</b>	<b>1</b>
1.1 Research Background	1
1.2 Capabilities in Research	2
<b>2.0 CENTRIFUGE MODELING</b>	<b>3</b>
2.1 Mechanics and Theory	3
2.2 Coriolis and centrifugal acceleration	5
<b>3.0 SCALING RELATIONS</b>	<b>9</b>
<b>4.0 CAPABILITIES OF THE ARMY CENTRIFUGE</b>	<b>10</b>
<b>5.0 CONCLUSIONS</b>	<b>14</b>
<b>BIBLIOGRAPHY</b>	<b>15</b>
<b>APPENDIX A Table of Calculations for Coriolis effects</b>	<b>A-1</b>

## **1.0 COMMISSIONING OF THE ARMY CENTRIFUGE**

### **1.1 RESEARCH BACKGROUND**

This report is one of a series of reports prepared by Andrew N Schofield & Associates Ltd (ANS&A) addressing the development and commissioning of new capabilities for physical modeling research at the Waterways Experiment Station (WES), through the acquisition of the powerful Army centrifuge facility. The research described herein forms Phase 4C of the programme of work first proposed under ANS&A's response (of 17 April 1989) to the WES Broad Agency Announcement (BAA) of December 1988.

Phase 1 of this project, entitled "Safety Factor Analysis for Centrifuge Systems", addressed the specification, Quality Assurance (QA) procedures and safety of operations that would be required to successfully commission a new centrifuge center at WES. In the Final Technical Report under Phase 1 (Contract Number DAJA45-90-C-018), ANS&A (1992), it was recommended that WES should buy the Acutronic 684-1 centrifuge subject to the implementation of QA procedures designed to ensure the swift integration of the new facility into the research activities of WES, Schofield and Steedman (1991).

Phase 2 of this project (Contract number DAJA45-91-C-0012) entitled "Development of a WES Centrifuge" initiated the Quality Assurance process under which ANS&A worked with the Laboratories of the US Army Corps of Engineers through the Centrifuge Coordinating Committee to prepare specifications for appurtenances and data acquisition equipment that would be needed during the commissioning of capabilities. ANS&A's Phase 2 Final Technical Report made specific recommendations concerning the development of appurtenances for initial experiments which would be compatible with the design of the Acutronic 684-1 centrifuge, Schofield and Steedman (1992).

Phase 3A entitled "Centrifuge facility design and development of capabilities" (Contract number DAJA45-91-C-0025) and 3B "Report on Quality Assurance for the WES Centrifuge" (Contract number DAJA45-92-C-0021) addressed the continuing role of ANS&A in providing advice and guidance during the design phase of the WES Centrifuge by Acutronic France SA. ANS&A's Final Technical Report covering Phases 3A and 3B recommended acceptance of the detailed design of the Acutronic 684-1 centrifuge and that the operating envelope of the centrifuge be revised to maximise the potential capability of the facility in the mid-range of operating levels (150-350g), Schofield and Steedman (1993).

Phase 3C, entitled "Coordination of operations for centrifuge quality control" (Contract number DAJA45-93-C-0021), presented recommendations concerning the initial research experimentation on the WES centrifuge and addressed in detail the mechanical commissioning of the centrifuge following its arrival in Vicksburg. A key recommendation arising from Phase 3C was the separation of the initial mechanical commissioning (upto a level of around 250 gravities) from the final commissioning (to full capability) and the necessity for careful and close control of the commissioning operations, Schofield and Steedman (1995). This approach was considered essential because of the unique nature of the centrifuge and the uncertainty over the available Manufacturer's documentation concerning Quality Control.

Phase 3D, entitled "Integration" (Contract number N68171-94-C-9066), addressed the preliminary experiments being planned to demonstrate the range of novel capabilities achieved for the new facility. Recommendations were also made on staffing for the facility and a set of operating procedures were prepared for general use, Schofield and Steedman (1995).

Phase 3D was followed by Phases 4 (Contract Number N68171-95-C-9047) and 4A (Contract Number N68171-97-C-9013). In reviewing the history of the project leading upto the inauguration of the new research center it was concluded that three main factors could be identified which had contributed to the success of the project. These were a) the focus on the research product, which led to early investment in general purpose and specific test equipment and appurtenances so that a rapid start could be made on use of the facility, b) an emphasis on safety of components and safety of operations, and c) the continuity of staff working on the project, at WES, within ANS&A and with the centrifuge designers in France.

The subsequent contract Phase 4B (Contract Number N68171-97-M-5713) entitled "Commissioning", followed the process of commissioning of the WES centrifuge center in the months immediately before and after the inauguration of the centrifuge facility on 20 November 1997. Phase 4B addressed the geotechnical context of the centrifuge and its historical development, concluding that a key element in the marketing and development of the center must be to emphasise the link between the modeling of soil and theory of plastic design.

This Final Technical Report concludes the series of ANS&A reports on the WES Centrifuge Research Center development, entitled Phase 4C (Contract Number N68171-97-M-5510), and covering the development of capabilities during the latter half of 1998 and early months of 1999, some ten years after the submission of ANS&A's response on 1 April 1989 to the Broad Agency Announcement (BAA) of December 1988.

## 1.2 CAPABILITIES IN RESEARCH

ANS&A's proposal, in April 1989, was to develop a novel centrifuge research facility at the Waterways Experiment Station in Vicksburg, Mississippi which would quickly be recognised as world class and which would be unique in its range of capabilities. The early focus on the research product, rather than the mechanical equipment (and in particular the large beam centrifuge at the centre of the facility), was to ensure that the new research facility would be capable of generating outstanding data in a wide range of fields at an early date following commissioning. This focus on research was based on the experience of the principal investigators, Drs Schofield and Steedman, at Cambridge and other centrifuge centers around the world. The division of responsibility was further emphasised during the procurement process by the parallel and separate contract arrangements for the centrifuge and its supporting services by French designers (through their US operating division), the containment building, which was designed by an American A&E consultant, and the capabilities in research, for which ANS&A, based in Cambridge, England, was responsible. In this report, those capabilities are quantified and the fundamental basis by which centrifuge model behavior and field behavior may be compared is explained.

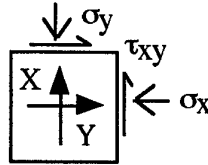
## 2.0 CENTRIFUGE MODELING

### 2.1 MECHANICS AND THEORY

Consider plane equilibrium of a prototype soil body such as a dam. The plane stress components on an element satisfy :

$$\delta\sigma_x/\delta x + \delta\tau_{xy}/\delta y + X = 0$$

$$\delta\tau_{xy}/\delta x + \delta\sigma_y/\delta y + Y = 0$$



where  $X$  and  $Y$  are inertial body forces. A sinusoidal base motion in a horizontal direction can be defined by :

$$x = a \sin \omega t$$

$$dx/dt = -a \omega \cos \omega t$$

$$d^2x/dt^2 = a \omega^2 \sin \omega t$$

The inertial body forces are then given by :

$$X = \rho a \omega^2 \sin \omega t$$

$$Y = -\rho g$$

In the corresponding centrifuge model subject to a steady centrifugal acceleration field defined as  $ng$ , lengths are reduced by the factor  $n$  and accelerations are increased by  $n$ . The base shaking motion, which for geometrical similarity must be subject to the same criterion, lengths being reduced by  $n$  and accelerations increased by  $n$ , requires to be modeled with time also reduced by a factor  $n$  (ie. frequencies increased by  $n$ ) :

$$x' = a/n \sin n\omega t$$

$$dx'/dt = -a \omega \cos n\omega t$$

$$d^2x'/dt^2 = na \omega^2 \sin n\omega t$$

Amplitudes of motion are reduced by  $n$ , velocities are identical and accelerations are increased by a factor  $n$  between model and prototype. Model stress components satisfy

$$\delta\sigma_x/\delta(x/n) + \delta\tau_{xy}/\delta(y/n) + nX = 0$$

$$\delta\tau_{xy}/\delta(x/n) + \delta\sigma_y/\delta(y/n) + nY = 0$$

and therefore identical stresses may be seen to act at homologous points in model and prototype.

Local velocities within a model held in equilibrium in steady flight in a centrifuge with a model velocity  $V$  (m/s) at a radius of  $r$  metres may be expressed using a transformation

of axes :

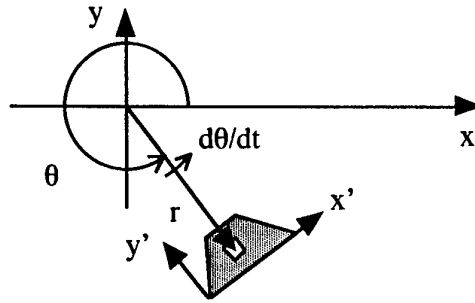
$$x = r \cos \theta$$

$$y = r \sin \theta$$

Then,

$$dx/dt = dr/dt \cos \theta - r d\theta/dt \sin \theta$$

$$dy/dt = dr/dt \sin \theta + r d\theta/dt \cos \theta$$



and local accelerations are given by :

$$d^2x/dt^2 = d^2r/dt^2 \cos \theta - 2(dr/dt)(d\theta/dt) \sin \theta - r d^2\theta/dt^2 \sin \theta - r (d\theta/dt)^2 \cos \theta$$

$$d^2y/dt^2 = d^2r/dt^2 \sin \theta + 2(dr/dt)(d\theta/dt) \cos \theta + r d^2\theta/dt^2 \cos \theta - r (d\theta/dt)^2 \sin \theta$$

Expressing the acceleration components in local axes  $x'$ ,  $y'$  requires a further transformation

$$x' = \text{const.} - x \sin \theta + y \cos \theta$$

$$y' = \text{const.} - x \cos \theta - y \sin \theta$$

giving local accelerations :

$$d^2x'/dt^2 = -d^2x/dt^2 \sin \theta + d^2y/dt^2 \cos \theta, \text{ or}$$

$$d^2x'/dt^2 = 2(dr/dt)(d\theta/dt) - r d^2\theta/dt^2$$

and

$$d^2y'/dt^2 = -d^2x/dt^2 \cos \theta + d^2y/dt^2 \sin \theta, \text{ or}$$

$$d^2y'/dt^2 = -d^2r/dt^2 + r (d\theta/dt)^2$$

These four terms provide the four components of acceleration which an element experiences. In the radial direction,  $r (d\theta/dt)^2$ , corresponds to the prototype acceleration  $g$ . It is simply the familiar 'centrifugal acceleration' term,  $r\omega^2$ . The second  $y'$  term,  $d^2r/dt^2$ , corresponds to vertical shaking of the model, not a common feature of model actuators to date, although all actuators experience some vertical motion, generally at high frequency.

In the  $x'$  direction, horizontal base shaking is described by the term  $r d^2\theta/dt^2$ . The second term is the Coriolis component  $2(dr/dt)(d\theta/dt)$ , described in more detail below. The Coriolis term can provide a significant error in certain classes of experiment.

## 2.2 CORIOLIS AND CENTRIFUGAL ACCELERATION

A centrifuge exerts a centripetal force on a payload (specimen), ie. a force towards the centre of rotation, and it is the reaction to that force, experienced by the specimen, which is labelled the centrifugal force.

As the magnitude of the force depends on the mass of the object in the rotating field it is often more convenient to talk of a centrifugal acceleration. Centrifugal acceleration, together with its partner Coriolis acceleration, are frequently described as 'fictitious', in the sense that they arise out of the kinematics of the problem and not from a physical interaction. They are inertial acceleration fields, whose effects are indistinguishable from the effects of a gravitational acceleration field.

Centrifugal force increases with distance, unlike forces due to interaction, such as gravitational attraction between bodies. As was seen in Section 2.1 above, these terms arise from the transformation between an inertial (or Newtonian) coordinate system to a rotational system. They are necessary for an observer on one reference frame to interpret the motion of an object on another reference frame rotating (and thus accelerating) relative to his own. For most engineering applications, the earth is considered a suitable inertial frame of reference. A rotating system such as a centrifuge, is a system accelerating relative to the earth. If the Coriolis and centrifugal terms are applied to each particle in the rotating system, then that rotating frame may be treated as a Newtonian frame. Expressed vectorially, the equation of motion for a particle, mass  $m$ , which in the inertial system is simply :

$$\underline{F} = m \underline{a}_S$$

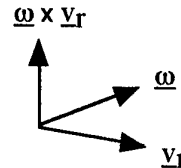
where  $\underline{a}_S$  is the acceleration relative to the fixed space system, becomes :

$$\underline{F} - 2m (\underline{\omega} \times \underline{v}_R) - m \underline{\omega} \times (\underline{\omega} \times \underline{r}) = m \underline{a}_R$$

where  $\underline{r}$ ,  $\underline{v}_R$ , and  $\underline{a}_R$  are the position, velocity and acceleration vectors of the particle relative to the rotating system and  $\underline{\omega}$  is the constant angular velocity. The Coriolis force is seen to be proportional to the cross-product of  $\underline{\omega}$  and  $\underline{v}_R$  and is of magnitude

$$|\underline{F}_{cor}| = 2m |\underline{\omega}| |\underline{v}_R| \sin \theta$$

Its direction follows a righthand rule and is perpendicular to both  $\underline{\omega}$  and  $\underline{v}_R$ .



Clearly if  $\underline{\omega}$  and  $\underline{v}_R$  lie in the same direction the Coriolis force is zero. Also if the particle remains at rest in the rotating frame  $|\underline{F}_{cor}| = 0$ .

An example illustrates the significance of Coriolis in centrifuge model tests. A particle dropped from a hopper mounted at a radius,  $r$  from the centre of rotation falls through a 'height'  $h$  to reach the ground surface in a centrifuge model.

Assuming the change in radius is small in relation to the centrifuge radius, then :

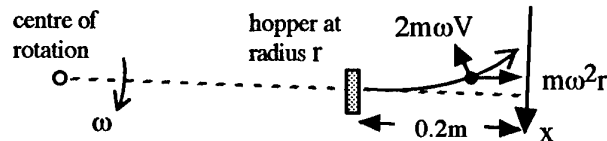
$$m d^2x/dt^2 = -2m\omega V$$

substituting  $V = \omega^2 r t$ , then

$$d^2x/dt^2 = -2\omega^3 r t$$

and integrating gives

$$x = -\omega^3 r^3 / 3$$



Note: observer on centrifuge model container

Considering the radial direction, the radial acceleration is simply  $d^2y/dt^2 = \omega^2 r$ , as seen from the centrifuge container. Integrating gives  $t = \sqrt{2h/\omega^2 r}$ , and hence  $x = 0.943 h^{3/2}/\sqrt{r}$ . For typical dimensions,  $h = 0.2\text{m}$  and  $r = 6\text{m}$  ( $h/r = 1/30$ ), then  $x \approx 34\text{ mm}$ , *behind*.

Illustrations are often shown in centrifuge texts of ejecta thrown *forward* from a crater formed in flight in a centrifuge model specimen, rather than being distributed uniformly around the perimeter in a symmetrical manner, and this is described as a Coriolis effect. This may be explained using the calculation above quite simply. In essence a particle which is ejected 'upwards' relative to the model surface in a centrifuge spinning in a horizontal plane is released with a high tangential velocity which it maintains as there is no longer any acceleration field acting on it (except for earth's gravity). The 'ground' beneath, in the model container, must travel around an arc at the same velocity. Particles which are ejected in the plane of rotation of the centrifuge will be affected to a maximum (as in the calculation above). Particles which are ejected in a direction normal to the plane of rotation will be unaffected. Directions in between will be affected to varying degrees, as a function of the vector cross product.

The Coriolis 'error' is related to the relative velocities of the particle and the centrifuge. Too fast and the movement of the centrifuge will be of no consequence. Too slow and there will be no distortion of the trajectory. Comparing the Coriolis acceleration,  $A^*$  to the steady centrifugal acceleration,  $A$

$$A^*/A = 2V\omega/v\omega = 2V/v$$

where  $v$  is the steady tangential velocity of the centrifuge,  $v = \omega r$ , and  $V$  is the velocity of the particle. Clearly if the particle velocity exceeds 5% of the tangential velocity of the centrifuge, then the error in neglecting Coriolis will exceed 10%. The Coriolis acceleration may be defined instantaneously in terms of a radius, comparable to the centrifuge radius. This Coriolis radius  $R = V/2\omega$ . For  $r = R$ ,  $v/\omega = V/2\omega$ , giving  $V = 2v$ . Therefore, for particle velocities in excess of twice the centrifuge velocity, the radius of curvature of its path relative to the centrifuge model will exceed the radius of

the centrifuge.

Such high velocities might be generated using a gas gun or other system to study projectile penetration. In this case, consider that the particle is ejected from the gun with an initial velocity,  $V_0$ . Then, as above,

$$m d^2x/dt^2 = -2m\omega V$$

substituting  $V = \omega^2 r t + V_0$ , then

$$d^2x/dt^2 = -2\omega^3 r t - 2\omega V_0$$

and integrating gives

$$dx/dt = -\omega^3 r t^2 - 2\omega V_0 t, \text{ and}$$

$$x = -\omega^3 r t^3/3 - \omega V_0 t^2$$

Considering the radial direction, the radial acceleration is simply  $d^2y/dt^2 = \omega^2 r$ , as seen from the centrifuge container. The velocity is  $dy/dt = \omega^2 r t + V_0$ . Integrating gives

$$\omega^2 r t^2 + 2V_0 t - 2h = 0$$

Three cases are considered,  $V_0 = v/2$ ,  $v$ , and  $2v$ , where  $v = \omega r$ . Typical dimensions are  $h = 0.15\text{m}$ ,  $r = 6\text{m}$ , giving  $h/r = 1/40$ . At impact,  $t = T$ . For reference, at 100g and  $r = 6\text{m}$ ,  $v = 76.7\text{ m/s}$ . Appendix A presents a table of the relevant calculations. Tables 1 and 2 present the results of the calculation for two drop heights, 1/40 and 1/20.

$V_0$	=	$v/2$	$v$	$2v$
$\omega T$	=	0.04772	0.02470	0.01246
$dx/dy$	=	0.09129	0.04880	0.02485
$x$	=	7.05	3.69	1.87 mm <i>behind</i> (for $r = 6\text{m}$ , 100g)

**Table 1** Coriolis distortion on a particle, initial velocity  $V_0$ , falling a height  $h/r = 1/40$

$V_0$	=	$v/2$	$v$	$2v$
$\omega T$	=	0.09161	0.04881	0.02485
$dx/dy$	=	0.16903	0.09535	0.04939
$x$	=	26.71	14.53	7.44 mm <i>behind</i> (for $r = 6\text{m}$ , 100g)

**Table 2** Coriolis distortion on a particle, initial velocity  $V_0$ , falling a height  $h/r = 1/20$

For a drop height of  $1/40$ , at  $V_o = v/2$ , the slope of the trajectory on impact is less than 10% off 'vertical'. As the velocity of the particle increases further, the error caused by the Coriolis effect drops by half with each doubling of the particle velocity. In the case of an initial velocity of  $2v$ , at which stage the instantaneous Coriolis 'radius' equals the radius of the centrifuge, the slope of the trajectory on impact is less than 2.5% off vertical for the same drop height. For the given criteria,  $r = 6m$  and  $100g$ , the particle will strike the surface just 1.9mm 'behind' a direct line projected from the line of the gun barrel.

It is important to note that the Coriolis and centrifugal accelerations shown in the sketch above as acting on the particle, are as seen from the centrifuge model reference frame. From outside the centrifuge 'chamber', the instant the particle leaves the hopper, or the ejecta leaves the surface of the specimen, relative to the earth its path is entirely predictable again without appeal to either of these 'fictitious forces'. In the case of the particle leaving the hopper, flying in a horizontal plane, the particle takes a tangential path with the tangential velocity of the hopper. It travels in a straight line (in the horizontal plane) until it reaches the surface of the container below. Its path, seen from above the centrifuge, would be entirely different to the path shown in the sketch.

### 3.0 SCALING RELATIONS

In presenting the capabilities of a civil engineering centrifuge, it is common practice to discuss these in terms of equivalent prototype units. Scaling relationships are used to compare the model and the field condition. In studying geotechnical phenomena, it is typical to adopt a linear scaling, where the increased acceleration in the centrifuge, expressed as a multiple of earth's gravity,  $n$ , is used as the scale factor for length. Thus dimensions are reduced by  $1/n$  in the model, areas by  $1/n^2$ , masses and volumes by  $1/n^3$ . Stress and strain are identical at homologous points in model and prototype. From these criteria, others follow, such as time for inertial events, which is reduced by  $n$  in the model, and frequencies of inertial events, which are correspondingly increased by  $n$ . As dimensions are reduced by  $n$ , but stresses are identical, then hydraulic gradients and seepage velocities are increased by  $n$  times, compared to the equivalent prototype. In many groundwater cases this is not significant, but in certain models which are investigating high velocities of flow, this may become important.

As seepage velocities are increased by  $n$ , but the distance fluid has to travel to reach a 'drain' is reduced by  $n$ , the time for consolidation is reduced by  $n^2$ , which has many advantages for the study of long term diffusion events. Similar arguments can be used for the scaling of heat flow and other similar phenomena. It was a feature of the design of the Army centrifuge that containers have been developed that control the thermal environment within a specimen, not simply that ambient temperatures may be maintained but also that conditions as varied as a polar environment or desert heat may be simulated.

Such arguments are simple but effective in presenting the capabilities of the Army centrifuge to non-expert users. Tables of scaling relations may be found in the literature, but without a physical interpretation, there is an implication that the centrifuge 'model' is simply a scale replica of some larger field event. To potential users who are unfamiliar with physical modeling techniques, such an approach suggests scale model 'toys', which may or may not bear close relation to their full scale counterparts. This attitude has been reinforced in the literature by the unfortunate habit of placing toys next to centrifuge models, in the misplaced belief that this gives an indication of 'scale'.

A specimen contained in a centrifuge strong box or chamber and subjected to high inertial accelerations is not a toy. The physics of the experiment can be fully explained, as outlined in the previous sections. The pressures and strains experienced by the specimen are not scaled (with the implication that they are reduced) compared to prototype conditions, but are identical. The engineering involved in the operation of large centrifuges is complex and very challenging, for the simple reason that most engineers have been trained to work on the planet earth, under conditions of one (earth) gravity, and their experience is non-existent of the true conditions on board the centrifuge in-flight, where robotic equipment and instrumentation is required to work safely in an environment which may be subject to inertial accelerations upto several hundred times earth's gravity.

The centrifuge specimen is a world in equilibrium relative to our own. It is a world which we will never experience. But using scaling relations, we may interpret the behavior of models in such an environment in terms of an equivalent field event on the planet earth, or on other planets, and this can bring considerable value.

#### 4.0 CAPABILITIES OF THE ARMY CENTRIFUGE

The capability of large centrifuges is commonly expressed in g-tonnes (or g-kg) because this is a convenient (albeit non standard) expression of force. It is convenient because these are the two parameters which people most easily relate to in the description of capabilities. For example, people can readily appreciate that a 100 g-tonne centrifuge can carry a 1 tonne payload (1000 kg) to an acceleration equivalent to 100 times earth's gravity (980.665 m/s<sup>2</sup>).

An alternative is to express the capability of the centrifuge in terms of the equivalent prototype which the payload represents, using the scaling relations described above. This has also many benefits, as comparisons may be made with field experiments, or laboratory equipment operating at full scale or near full scale. Thus a centrifuge experiment of a crater test conducted in the field may be carried out at 300g, using a 1 cubic metre specimen. At one gravity, in the field, the equivalent volume of soil required to conduct the same experiment would be 27 million cubic metres, a huge volume which could not possibly be constructed under controlled conditions to the same geometry.

The operating envelope of the Army centrifuge was selected to meet research demands, as perceived at an early stage. The centrifuge is limited in the absolute value of force to which its booms may be subjected, and by balance and motor considerations. Thus the design of the centrifuge was optimised based on a range of criteria which limit its capability in different fields of operation.

The comments that follow are an interpretation of the design from a user's perspective, and are intended solely to assist in explaining the capability of the Army centrifuge from this standpoint. They have not been discussed with the designer, for example.

The centrifuge cannot be used for experiments at less than 38.6 rpm. (The nominal radius of the centrifuge is 6m, being the radius to the centroid of a hypothetical specimen. The actual radius to the surface of the swinging platform in rotation is 6.5m.)

Between 38.6 rpm and 146 rpm, the stress in the arms (booms) and platform is not the critical factor. Instead the maximum size of payload that can be carried is limited by the size and position of the counterweights. This limits the payload at these operating speeds to 8000kg. This was a substantial increase over the original specification and arose because of the high quality of the steel booms, which permitted the designer to make use of the spare metal at the ends of the booms which would normally have been disposed of.

At 146 rpm and a payload of 8000 kg, the force exerted by the payload on the booms of the centrifuge is a maximum and it is this force which is used to describe the published 'capability' of the centrifuge. For convenience, the published capability is based on the mass of the payload alone, using the 'standard' swinging platform.

Based on a nominal radius to the centroid of a payload of 6m, the force exerted by the payload is simply calculated as

$$F = M r \omega^2$$

where M is the payload mass, r the radius and  $\omega$  the angular velocity, as in section 2 above. Therefore,

$F = 8000.6(2\pi 146/60)^2 = 11,220,300 \text{ kgm/s}^2$ , or Newtons (N). This may be commonly expressed as  $F = 11.2 \text{ MN}$ .

This force may also be expressed in terms of g-tonnes, giving

$F = 11,220,300 / 9.80665 = 1,144,000 \text{ g kg} = 1,144 \text{ g tonnes}$ .

Strictly, the mass of the swinging platform could be added to the mass of the payload, and used to describe the capability of the centrifuge, as the present platform was designed such that it could be removed and replaced with a special, purpose built capsule if required in the future. The mass of the swinging platform alone is around 5.5 tonnes (the hangers weigh around 1300 kg each; the platform 2000 kg, pins (6 No.) 91 kg, and hinges (2 No.) 125 kg). This would give a total force capacity at 143g (146 rpm) of 18.9 MN.

Between 146 and 228 rpm the capability of the centrifuge is limited by the force in the pins connecting the swinging arms of the platform to the booms, and the platform itself. Because the platform is 'dead weight' then the payload has to reduce in mass, so that the force in the pins connecting the platform to the booms is constant. The reduction in payload with increasing rpm gives the characteristic curved shape of the performance envelope.

At 228 rpm, the payload is reduced to 2000kg. Using this point on the envelope gives an apparently reduced 'capability' of 700 g tonnes, or 6.84 MN. For this reason, the capability is generally quoted at 143g. However, the design of the centrifuge includes the capability to remove the swinging 'flat' platform and to replace the entire assembly with a purpose designed swinging capsule, combining the swing and container. In an early version of the design, this was envisaged as the only option for high g testing. By eliminating the flat platform, and adopting a shape more reminiscent of a pressure vessel, perhaps with a hemi-spherical base, considerable mass could be saved in the containment, which would allow the mass of the specimen to be increased proportionately. The entire mass of the payload and swinging platform is around 7.5 tonnes at 350g. This would give a total force capacity at 350g (228 rpm) of 25.8 MN. This is the largest force computation that may be made for the Army centrifuge relating to its carrying capacity. It is a remarkable number, implying that the pin at the end of each boom is carrying a force of nearly 13 MN, or 1,300 tonnes, at full speed and with a full payload. Under these conditions, the centrifuge platform is travelling at a speed of over 320 mph.

The design of the centrifuge is limited to 228 rpm, which was the revised design requirement (350g at 6m). This limit cannot be exceeded, even with a smaller payload, because of the capacity of the motor drive. The bluff area of the centrifuge in rotation exerts a considerable aerodynamic drag, which was taken into account in the sizing of the motor set. During the commissioning of the centrifuge, the machine was taken to 228 rpm, with a payload of 2000kg of steel plate, without the aerodynamic shielding.

Ref.	Description	Model dimension	Field condition	Assumptions
i.	Maximum depth of sample (using standard existing sample chambers)	1.0 m	350 m	linear scaling; depth could be below a free surface, or below a horizon (see iii. below)
ii.	Maximum self-weight pressure at the base of a sample with a free surface	20 kPa	7 MPa	based on material unit weight of 20 kN/m <sup>3</sup> ; 7 MPa also corresponds to the design udl for the surface of the platform
iii.	Maximum pressure within a sample without a free surface	practically unlimited	practically unlimited	a sealed sample chamber can be backpressured
iv.	Maximum surface area of sample	1.131 m <sup>2</sup>	139000 m <sup>2</sup>	1.2 m diameter sample chamber
v.	Maximum width/length of sample	1.2 m	420 m	1.2 m square sample chamber
vi.	Maximum payload at 350g	2000 kg	85.75 million tonnes	total design payload distributed over circular footprint
vii.	Maximum mass of sample at 350g excluding sample chamber	1225 kg	52.5 million tonnes	based on existing 1.2 m dia chamber, filled to a depth of 0.54 m with saturated soil
viii.	Maximum volume of sample at 350g (inside existing 1.2m diameter sample chamber)	1.13 m <sup>3</sup>	48.5 million m <sup>3</sup>	based on existing chamber, filled with water
ix.	Maximum payload at 143g	8000 kg	23.4 million tonnes	total design payload, distributed over whole platform area
x.	Maximum explosive charge (proof tested to date)	15 grams	643 tonnes	based on proof test carried out by SL on the existing high pressure sample chamber
xi.	Maximum pressure on platform		6.67 MPa	design condition at 350 and 143g (see ii.)
xii.	Maximum centrifugal force (exerted on the 2 pins connecting the swing to the booms)		25.8 MN	based on entire swing and payload at 350g (eg using a special purpose container)
xiii.	Maximum centrifugal force (based on payload only)		11.2 MN	based on 8000 kg payload at 143g

**Table 3** Selected examples of the capability of the Army centrifuge expressed in terms of specimen and equivalent prototype dimensions and stresses

In its physical size, the WES centrifuge does not have either the biggest platform or the biggest radius of any centrifuge in the world. However, the WES centrifuge is the biggest beam centrifuge (to public knowledge) that exists anywhere in terms of the volume, mass and dimensions of the equivalent prototype (field) conditions that can be represented in its sample or specimen chambers (as discussed above in Section 3). This is because of its combination of high speed and large payload capacity.

There are an infinite number of ways of expressing the capability of the centrifuge in terms of the equivalent prototype conditions, as a specimen can be configured and tested in a multiplicity of ways. In Table 3, a series of examples are presented which each represent the capability of the Army centrifuge in a different way.

## 5.0 CONCLUSIONS

The Army centrifuge at the Waterways Experiment Station is unique in its range and depth of capabilities. This project, which arose out of a Broad Agency Announcement issued in December 1988, came to fruition at a formal inauguration held on 20 November 1997. ANS&A's role in advising the Corps of Engineers about the development of capabilities has been documented in a series of research reports over a period of ten years.

The physics underlying the operation of a centrifuge, and the corresponding forces and accelerations which are applied to specimens are well established and may be readily explained. The centrifuge model is not a scale replica of a field condition, but there is an equivalence between the model and field problem which may be interpreted using scaling relations.

The capabilities of the Army centrifuge may be expressed in a wide range of ways. The centrifuge operates within a performance envelope, determined by design parameters. Maximum payloads vary, depending on the speed of the centrifuge. These payloads may be expressed in terms of prototype or field conditions, such as self-weight pressure at the base of a specimen, surface area, mass or volume of material.

The Army centrifuge is the most powerful of any beam centrifuge ever constructed in the world and certainly the most powerful centrifuge operating today to public knowledge. The engineering of research experiments which utilise this facility requires a fundamental appreciation of the physics of a centrifuge as well as of material behavior under high stresses. It is a complex and challenging working environment which requires respect and rigorous attention to detail.

## BIBLIOGRAPHY

- Schofield, A.N. and Steedman, R.S. (1991) Safety Factor Analysis for Centrifuge Systems, Final Technical Report, ANS&A Report 26-01-R-001 for US Army European Research Office, London (Contract No. DAJA45-90-C-018).
- Schofield, A.N. and Steedman, R.S. (1992) Development of a WES Centrifuge, Final Technical Report, ANS&A Report 26-02-R-004 for US Army European Research Office, London (Contract No. DAJA45-91-C-0012).
- Schofield, A.N. and Steedman, R.S. (1993) Design of the WES Centrifuge (Phases 3A & 3B), Final Technical Report, ANS&A Report 26-03-R-006 for US Army European Research Office, London (Contract No. DAJA45-92-C-0021).
- Schofield, A.N. and Steedman, R.S. (1995) Coordination of operations for centrifuge quality control (Phase 3C), Final Technical Report, ANS&A Report 26-03-R-009 for US Army European Research Office, London (Contract No. DAJA45-93-C-0021).
- Schofield, A.N. and Steedman, R.S. (1995) Integration (Phase 3D), Final Technical Report, ANS&A Report 26-03-R-012 for US Army European Research Office, London (Contract No. N68171-94-C-9066).
- Schofield, A.N. and Steedman, R.S. (1997) Commissioning (Phase 4, 4A), Final Technical Report, ANS&A Report 26-04-R-010 for US Army European Research Office, London (Contract No. N68171-97-C-9013).
- Schofield, A.N. and Steedman, R.S. (1998) Commissioning (Phase 4B), Final Technical Report, ANS&A Report 26-04-R-014 for US Army European Research Office, London (Contract No. N68171-97-M-5713).

**APPENDIX A**  
**CORIOLIS EFFECTS**

**Table of calculations for Coriolis effects,  
with and without initial velocity, Vo**

$dx/dt = -w^3rt^2 - 2wVot$  , and

$x = -w^3rt^3/3 - wVot^2$

$x = -r(wt)^3/3 - B(wt)^2$

$dy/dt = w^2rt + Vo$

if the height of drop = h , reached at time t = T, then

$w^2rT^2 + 2VoT - 2h = 0$  or

$wT^2 + 2BT - 2h/wr = 0$

	100	981 m/s <sup>2</sup>
r		6 m
w		12.79 rad/s
or		122.10 rpm
v		76.72 m/s

notes :

<b>h/r</b>	<b>0.025</b>	<b>0.025</b>	<b>0.025</b> 0.15/6 or 1/40
<b>Vo = Bv, B =</b>	<b>0.5</b>	<b>1</b>	<b>2</b>
<b>-b</b>	<b>-1</b>	<b>-2</b>	<b>-4</b>
<b>b<sup>2</sup> - 4ac</b>	<b>1.2</b>	<b>4.2</b>	<b>16.2</b>
<b>(-b+sqrt()/2)</b>	<b>0.04772</b>	<b>0.02470</b>	<b>0.01246</b> wT
	<b>20.95</b>	<b>40.49</b>	<b>80.25</b> 1/wT
<b>x</b>	<b>-0.00705</b>	<b>-0.00369</b>	<b>-0.00187</b> mm
<b>dx/dy</b>	<b>-0.09129</b>	<b>-0.04880</b>	<b>-0.02485</b>

<b>h/r</b>	<b>0.05</b>	<b>0.05</b>	<b>0.05</b> 0.3/6 or 1/20
<b>Vo = Bv, B =</b>	<b>0.5</b>	<b>1</b>	<b>2</b>
<b>-b</b>	<b>-1</b>	<b>-2</b>	<b>-4</b>
<b>b<sup>2</sup> - 4ac</b>	<b>1.4</b>	<b>4.4</b>	<b>16.4</b>
<b>(-b+sqrt()/2)</b>	<b>0.09161</b>	<b>0.04881</b>	<b>0.02485</b> wT
	<b>10.92</b>	<b>20.49</b>	<b>40.25</b> 1/wT
<b>x</b>	<b>-0.02671</b>	<b>-0.01453</b>	<b>-0.00744</b> mm
<b>dx/dy</b>	<b>-0.16903</b>	<b>-0.09535</b>	<b>-0.04939</b>

<b>h/r</b>	<b>0.03333</b>	<b>0.03333</b>	<b>0.03333</b> 0.2/6 or 1/30
<b>Vo = Bv, B =</b>	<b>0</b>	<b>0.1</b>	<b>0.2</b>
<b>-b</b>	<b>0</b>	<b>-0.2</b>	<b>-0.4</b>
<b>b<sup>2</sup> - 4ac</b>	<b>0.26667</b>	<b>0.30667</b>	<b>0.42667</b>
<b>(-b+sqrt()/2)</b>	<b>0.25820</b>	<b>0.17689</b>	<b>0.12660</b> wT
	<b>3.87</b>	<b>5.65</b>	<b>7.90</b> 1/wT
<b>x</b>	<b>-0.03443</b>	<b>-0.02984</b>	<b>-0.02329</b> mm, quoted in text for Vo = 0
<b>dx/dy</b>	<b>-0.25820</b>	<b>-0.24077</b>	<b>-0.20412</b>