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A SHORT RANGE FORECAST OF THE ATMOSPHERIC PRESSURE FIELD FOR A THREE LAYER ATMOSPHERIC MODEL

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An atmospheric model, consisting of three levels -- 250, 500 and 750 mb, is used for a forecast of the atmospheric pressure field in 1-2 days, on the basis of the solution of the linearized system of hydro-thermodynamic equations.

The condition of a maximum vertical velocity at the level of the 500 mb surface is used when forming an initial closed system of differential equations. The solution is given in analytic form, on the basis of which a method of forecasting the maps of absolute topography of isobaric surfaces is proposed.

Considering that the flow in the free atmosphere is adiabatic and quasi-geostrophic, let us denote for the solution of the presented problem, the following system of equations of hydro-thermodynamics [1]:

$$\varsigma^2 \frac{\partial^2 \Phi}{\partial \varsigma \partial t} + \frac{c^2}{l}\tau + \frac{\varsigma^2}{l}\left(\Phi, \frac{\partial \Phi}{\partial \varsigma}\right) = 0$$

$$\Delta \frac{\partial \Phi}{\partial t} - \frac{l^2}{P}\frac{\partial \tau}{\partial \varsigma} + \frac{1}{l}(\Phi, \Delta\Phi) + \beta \frac{\partial \Phi}{\partial x} = 0 \quad (1.1)$$

Here Φ is the geopotential, $\varsigma = p/P$ is the reduced pressure, l is the Coriolis parameter, $\beta = dl/dy$, $c^2 = \alpha R T_1$, R is the gas constant, T_1 is the mean temperature of the troposphere, $\alpha = \frac{R(\gamma_a - \gamma)}{g}$, g is the force of gravity, γ_a is the adiabatic temperature gradient, γ is the vertical temperature gradient.

In the nature of boundary conditions let us take the following:

$$\text{at } \varsigma = 0 \quad \tau = 0 \quad , \text{at } \varsigma = 1 \quad \tau = 0$$

In part, let us examine the solution of the system of equations (1.1) numerically dividing the atmosphere into $2n - 1$ layers, equidistant from each other (along ς), and replacing derivatives along ς, i.e. $\partial\Phi/\partial\varsigma$ and $\partial\tau/\partial\varsigma$, with centered finite differences:

$$\left(\frac{\partial\Phi}{\partial\varsigma}\right)_K \approx \frac{\Phi_{K+1} - \Phi_{K-1}}{\delta\varsigma}, \quad \left(\frac{\partial\tau}{\partial\varsigma}\right)_K \approx \frac{\tau_{K+1} - \tau_{K-1}}{\delta\varsigma} \tag{1.2}$$

where $\delta\varsigma = 1/n$, while $\left(\frac{\partial\Phi}{\partial\varsigma}\right)_K$ and $\left(\frac{\partial\tau}{\partial\varsigma}\right)_K$ correspond to the quantities $\varsigma = \varsigma_K = K/2n$. In addition the parameter $K = 0$ will correspond to $\varsigma = 0$, and $K = 2N$ to $\varsigma = 1$. In this case the boundary conditions adapt the form:

$$\tau = \tau_{2n} = 0.$$

Now if we give the parameter n various values, we shall obtain various prognostic atmospheric models. In particular, for $n = 1$ we shall have the so called one level model, coinciding completely with the well known model of the mean level (or the barotropic model).

Let us put the expressions for $(\partial\Phi/\partial\varsigma)_K$ and $(\partial\tau/\partial\varsigma)_K$ into (1.1):

$$\begin{gathered}
\varsigma_K^2 \frac{\partial}{\partial t} \frac{\Phi_{K+1} - \Phi_{K-1}}{\delta\varsigma} + \frac{C_K^2}{P} \tau_K + \frac{\varsigma_K^2}{\ell}\left(\Phi_K, \frac{\Phi_{K+1} - \Phi_{K-1}}{\delta\varsigma}\right) = 0 \\
\Delta \frac{\partial\Phi_{K+1}}{\partial t} + \frac{\ell^2}{P} \frac{\tau_{K+2} - \tau_K}{\delta\varsigma} + \frac{1}{\ell}\left(\Phi_{K+1}, \Delta\Phi_{K+1}\right) + \beta \frac{\partial\Phi_{K+1}}{\partial x} = 0
\end{gathered} \tag{1.3}$$

Putting $n = 2$, let us write the first of equations (1.3) at $K = 2$, and the second at $K = 0$ and $K = 2$. Then eliminating the function τ from the obtained system of equations (for this condition use of the boundary conditions $\tau_0 = \tau_4 = 0$), we arrive at the following system of equations:

$$\Delta \frac{\partial \Phi_1}{\partial t} - \frac{\ell^2}{c_2^2}\left\{\frac{1}{\ell}(\Phi_3, \Phi_2) + \frac{1}{\ell}(\Phi_2, \Phi_1) + \frac{\partial \Phi_1}{\partial t} - \frac{\partial \Phi_3}{\partial t}\right\} + \frac{1}{\ell}(\Phi_1, \Delta\Phi_1) + \beta\frac{\partial \Phi_1}{\partial x} = 0$$

$$\Delta \frac{\partial \Phi_3}{\partial t} + \frac{\ell^2}{c_2^2}\left\{\frac{1}{\ell}(\Phi_3, \Phi_2) + \frac{1}{\ell}(\Phi_2, \Phi_1) + \frac{\partial \Phi_1}{\partial t} - \frac{\partial \Phi_3}{\partial t}\right\} + \frac{1}{\ell}(\Phi_3, \Delta\Phi_3) + \beta\frac{\partial \Phi_3}{\partial x} = 0 \qquad (1.4)$$

It is clear, that in the given case the index "1" corresponds to the geopotential of the 250 mb surface of absolute topography, the index "2" -- 500 mb, the index "3" -- 750 mb. The system of two equations (1.4) contains three unknown functions -- Φ_1, Φ_2 and Φ_3. In order to make (1.4) a closed system, it is necessary to introduce another condition, which connects the unknown functions. It is possible for example, to take

$$\Phi_2 \approx \frac{\Phi_1 + \Phi_3}{2} \qquad (1.5)$$

Then we shall have a model of the atmosphere consisting of two levels. Since Φ_2 appears as the geopotential of the 500mb surface, condition (1.5) is not completely precise, since Φ_2 satisfies the equation of the mean level:

$$\Delta \frac{\partial \Phi_2}{\partial t} + \frac{1}{\ell}(\Phi_2, \Delta\Phi_2) + \beta\frac{\partial \Phi_2}{\partial x} = 0 \qquad (1.6)$$

and could be found independently of Φ_1 and Φ_3. But then the system of equations (1.4) and (1.6) would be complete and could be taken for the determination of Φ_1, Φ_2 and Φ_3.

For the solution of the system of equations (1.4) and (1.6) let us look into their linearization, considering, that the unknown function Φ_i could be presented in the form $\Phi_i = \overline{\Phi}_i(y) + \Phi_i'(x, y, t)$, where $\overline{\Phi}_i(y)$ is the zonal distribution of Φ_i, related to the speed of the west-east flow V in the mean troposphere by the relation

$d\bar{\Phi}_i/dy = -\ell v_i$ while Φ_i' are small deviations, and whose products of derivatives may be neglected. At the same time let us introduce the non-dimensional quantities for the horizontal coordinates, time, and velocity: L is a characteristic length, T is a characteristic time, $V = L/T$ is a characteristic velocity. The system of equations (1.4) and (1.6) after linearization take the form:

$$\left(\frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x}\right)\Delta\Phi_1' + \bar{\beta}\frac{\partial \Phi_1'}{\partial x} = -\lambda^2\left[(V_3 - V_1)\frac{\partial \Phi_2'}{\partial x} + \left(\frac{\partial}{\partial t} + V_2\frac{\partial}{\partial x}\right)(\Phi_1' - \Phi_3')\right]$$

$$\left(\frac{\partial}{\partial t} + V_3 \frac{\partial}{\partial x}\right)\Delta\Phi_3' + \bar{\beta}\frac{\partial \Phi_3'}{\partial x} = \lambda^2\left[(V_3 - V_1)\frac{\partial \Phi_2'}{\partial x} + \left(\frac{\partial}{\partial t} + V_2\frac{\partial}{\partial x}\right)(\Phi_1' - \Phi_3')\right] \tag{1.7}$$

$$\left(\frac{\partial}{\partial t} + V_2 \frac{\partial}{\partial x}\right)\Delta\Phi_2' + \bar{\beta}\frac{\partial \Phi_2'}{\partial x} = 0$$

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where λ and $\bar{\beta}$ are non dimensional parameters:

$$\lambda = \frac{\ell^2 L^2}{C_2^2}, \quad \bar{\beta} = \beta L T$$

Let us call upon the Fourier method to find the solution of the linear system of equations (1.7). Let us assume, that

$$\Phi_i'(x,y,t) = \iint_{-\infty}^{+\infty} \Phi_i^{(mn)} e^{i\sigma t + imx + iny}\, dm\, dn \tag{1.8}$$

Considering, that at the initial moment of time we know the geopotential distribution, we must assume that at $t = 0$, $\Phi_{io}'(x, y, 0)$ is a known function. But then, it is clear,

$$\Phi_i^{(mn)} = \frac{1}{2\pi}\iint_{-\infty}^{+\infty} \Phi_{io}'(x', y', 0)\, e^{-imx' - iny'}\, dx'\, dy' \tag{1.9}$$

will also be a known function. Putting (1.8) into (1.7), we have:

$$
\begin{aligned}
&[\bar{\beta}-\rho^2(r+\delta)-(r+\delta_0)\lambda^2]\bar{\Phi}_1^{(mn)}-2\delta\lambda^2\bar{\Phi}_2^{(mn)}\\
&\quad+\lambda^2(r+\delta_0)\bar{\Phi}_3^{(mn)}=0\\
&\lambda^2(r+\delta_0)\bar{\Phi}_1^{(mn)}+2\delta\lambda^2\bar{\Phi}_2^{(mn)}+[\bar{\beta}-\rho^2(r-\delta)-\lambda^2(r+\delta_0)]\bar{\Phi}_3^{(mn)}=\\
&\qquad[\bar{\beta}-\rho^2(r+\delta_0)]\bar{\Phi}_2^{(mn)}=0
\end{aligned}
\tag{1.10}
$$

where

$$
r+\delta=\frac{\sigma}{m}+V_1,\quad r-\delta=\frac{\sigma}{m}+V_3,\quad r+\delta_0=\frac{\sigma}{m}+V_2
$$

$$
\delta=\frac{V_1-V_3}{2},\quad \delta_0=V_2-\frac{V_1+V_3}{2},\quad \rho^2=m^2+n^2
$$

Setting the determinant of the homogeneous linear algebraic system of equations (1.10) equal to zero, we obtain the equation for the determination of the frequency r . If we denote

$$
\begin{aligned}
&\bar{\beta}-\rho^2(r\pm\delta)-(r+\delta_0)\lambda^2=L(r,\pm\delta),\\
&\lambda^2(r+\delta_0)=Q(r),\quad 2\delta\lambda^2=N
\end{aligned}
\tag{1.11}
$$

Then the equation for the determination of r has the form:

$$
\begin{vmatrix}
L(r,\delta) & -N & Q(r)\\
Q(r) & N & L(r,-\delta)\\
0 & \bar{\beta}^2-\rho^2(r+\delta_0) & 0
\end{vmatrix}=0
$$

From this

$$
\begin{aligned}
&\bar{\beta}-\rho^2(r+\delta_0)=0\\
&L(r,\delta)L(r,-\delta)-Q^2(r)=0
\end{aligned}
\tag{1.12}
$$

The first equality from (1.12) gives one value for r

$$r_2 = \frac{\bar{\beta}}{\rho^2} - \delta_0 \tag{1.13}$$

The second condition yields a quadratic equation, ______________ which have the following values:

$$r_{1,3} = \frac{\bar{\beta}}{\rho^2} + \frac{-a \pm \sqrt{a^2+b}}{\rho^2(\rho^2+2\lambda^2)}, \quad a = \lambda^2(\bar{\beta}+\delta_0\rho^2), \quad b = \rho^6\sigma^2(\rho^2+2\lambda^2) \tag{1.14}$$

Since r has three different values, then correspondingly σ will have three values. Since equation (1.7) is linear, the solution for Φ_i' could now be presented in the form:

$$\Phi_i' = \iint_{-\infty}^{+\infty} \left(C_i^{(1)} e^{i\sigma_1 t} + C_i^{(2)} e^{i\sigma_2 t} + C_i^{(3)} e^{i\sigma_3 t} \right) e^{imx+iny}\, dm\, dn \tag{1.15}$$

A further problem will consist in finding the nine coefficients $C_i^{(j)}$ $(i, j = 1, 2, 3)$, using both equation (1.7) and the initial conditions. Let us note that Φ_2' satisfies the third equation of (1.7) when $\sigma = \sigma_2$, therefore $C_2^{(1)} = C_2^{(3)} \equiv 0$. For the determination of the remaining seven coefficients $C_i^{(j)}$, let us again put Φ_i' into (1.7) and, using (1.11) we shall have:

$$C_1^{(1)} L(r_1, \delta) + C_3^{(1)} Q(r_1) = 0, \quad C_3^{(3)} L(r_3, \delta) + C_3^{(3)} Q(r_3) = 0$$
$$C_1^{(2)} L(r_2, \delta) - C_2^{(2)} N + C_3^{(2)} Q(r_2) = 0 \tag{1.16}$$
$$C_1^{(2)} Q(r_2) + C_2^{(2)} N + C_3^{(2)} L(r_2, -\delta) = 0$$

It follows that we must supplement the initial conditions at $t = 0$ to the system (1.16):

$$C_1^{(1)} + C_1^{(2)} + C_1^{(3)} = \bar{\Phi}_{10}^{(mn)}, \quad C_2^{(2)} = \bar{\Phi}_{20}^{(mn)} \tag{1.17}$$

$$C_3^{(1)} + C_3^{(2)} + C_3^{(3)} = \bar{\Phi}_{30}^{(mn)}$$

Systems (1.16) and (1.17) together are now sufficient for the determination of the coefficients $C_i^{(j)}$. Not writing out in full the values of $C_i^{(j)}$ separately, let us directly introduce the expressions for the combinations of $C_i^{(j)}$, entering in (1.15):

$$C_1^{(1)} e^{i\sigma_1 t} + C_1^{(2)} e^{i\sigma_2 t} + C_1^{(3)} e^{i\sigma_3 t} = \bar{\Phi}_1^{(mn)} e^{i\sigma_3 t}$$

$$+ \frac{\varkappa_3}{\varkappa_3 - \varkappa_1} \left(e^{i\sigma_1 t} - e^{i\sigma_3 t}\right) \bar{\Phi}_1^{(mn)} + \frac{e^{i\sigma_1 t} - e^{i\sigma_3 t}}{\varkappa_3 - \varkappa_1} \bar{\Phi}_3^{(mn}$$

$$+ Y_1 \bar{\Phi}_2^{(mn)}$$

$$C_3^{(1)} e^{i\sigma_1 t} + C_3^{(2)} e^{i\sigma_2 t} + C_3^{(3)} e^{i\sigma_3 t} = \bar{\Phi}_3^{(mn)} e^{i\sigma_1 t} \tag{1.18}$$

$$- \frac{\varkappa_3}{\varkappa_3 - \varkappa_1} \left(e^{i\sigma_1 t} - e^{i\sigma_3 t}\right) \bar{\Phi}_3^{(mn)} + Y_2 \bar{\Phi}_2^{(mn)} - \frac{\varkappa_1 \varkappa_3}{\varkappa_3 - \varkappa_1} \left(e^{i\sigma_1 t} - e^{i\sigma_3 t}\right) \bar{\Phi}_1^{(mn)}$$

$$C_2^{(2)} e^{i\sigma_2 t} = e^{i\sigma_2 t} \bar{\Phi}_2^{(mn)}$$

where

$$Y_1 = \frac{\varkappa_5 \left[\varkappa_1 (1 + \varkappa_4) - (1 + \varkappa_2)\right]}{(1 - \varkappa_2 \varkappa_4)(\varkappa_3 - \varkappa_1)} \left(e^{i\sigma_1 t} - e^{i\sigma_3 t}\right)$$

$$+ \frac{\varkappa_5 (1 + \varkappa_4)}{1 - \varkappa_2 \varkappa_4} \left(e^{i\sigma_1 t} - e^{i\sigma_2 t}\right)$$

$$Y_2 = \frac{\mathcal{H}_3 \mathcal{H}_5 \left[\mathcal{H}_1 (1+\mathcal{H}_4) - (1-\mathcal{H}_2)\right]}{(1-\mathcal{H}_2 \mathcal{H}_4)(\mathcal{H}_3 - \mathcal{H}_1)} \left(e^{i\sigma_1 t} - e^{i\sigma_3 t}\right)$$

$$- \frac{\mathcal{H}_5 (1+\mathcal{H}_2)}{1-\mathcal{H}_2 \mathcal{H}_4} \left(e^{i\sigma_1 t} - e^{i\sigma_2 t}\right)$$

$$\mathcal{H}_{13} = \frac{L(r_{1,3}, \delta)}{Q(r_{1,3})}, \quad \mathcal{H}_2 = \frac{L(r_2, \delta)}{Q(r_2)}, \quad \mathcal{H}_4 = \frac{L(r_2 - \delta)}{Q(r_2)}$$

$$\mathcal{H}_5 = \frac{\mathcal{N}}{Q(r_2)}$$

Putting (1.18) into (1.15) and substituting $\Phi_i^{(mn)}$ from (1.9), we arrive at the final form of the solution for the unknown function Φ_i' $(i = 1, 2, 3)$:

$$\begin{aligned}
\Phi_1'(x,y,t) &= \Phi_{10}'(x - V_1 t, y, 0) \\
&+ \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Phi_{10}'(x',y') G_1^{(1)}(x,x',y,y',t)\, dx'dy' \\
&+ \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Phi_{20}'(x',y') G_2^{(1)}(x,x',y,y',t)\, dx'dy' \\
&+ \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Phi_{30}'(x',y') G_3^{(1)}(x,x',y,y',t)\, dx'dy'
\end{aligned} \tag{1.19}$$

$$\begin{aligned}
\Phi_3'(x,y,t) &= \Phi_{30}'(x - V_3 t, y, 0) \\
&+ \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Phi_{10}'(x',y') G_3^{(1)}(x,x',y,y',t)\, dx'dy' \\
&+ \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Phi_{20}'(x',y') G_3^{(2)}(x,x',y,y',t)\, dx'dy' \\
&+ \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Phi_{30}'(x',y') G_3^{(3)}(x,x',y,y',t)\, dx'dy'
\end{aligned}$$

$$\Phi_2'(x,y,t) = \Phi_{20}'(x - V_2 t, y, 0) + \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Phi_{20}'(x',y') G_2^{(2)}(x,x',y,y',t)\, dx'dy'$$

here $G_i^{(j)}$ are the Green's functions, having the following form:

$$G_1^{(1)} = \int_0^\infty \left[J_0(\rho A_3) - J_0(\rho B_3) + \frac{\mathcal{H}_3}{\mathcal{H}_3 - \mathcal{H}_1} \left(J_0(\rho A_1) - J_0(\rho A_3) \right) \right] \rho \, d\rho$$

$$G_2^{(1)} = \int_0^\infty \left[\frac{\mathcal{H}_5 [(1+\mathcal{H}_4) - (1+\mathcal{H}_2)]}{(1-\mathcal{H}_2\mathcal{H}_4)(\mathcal{H}_3 - \mathcal{H}_1)} \left(J_0(\rho A_1) - J_0(\rho A_3) \right) \right.$$

$$\left. + \frac{\mathcal{H}_5 (1+\mathcal{H}_4)}{1-\mathcal{H}_2\mathcal{H}_4} \left(J_0(\rho A_1) - J_0(\rho A_2) \right) \right] \rho \, d\rho$$

$$G_3^{(1)} = \int_0^\infty \frac{J_0(\rho A_1) - J_0(\rho A_3)}{\mathcal{H}_3 - \mathcal{H}_1} \rho \, d\rho \tag{2.20}$$

$$G_1^{(3)} = \int_0^\infty \mathcal{H}_1 \mathcal{H}_3 \frac{J_0(\rho A_3) - J_0(\rho A_1)}{\mathcal{H}_3 - \mathcal{H}_1} \rho \, d\rho$$

$$G_2^{(3)} = \int_0^\infty \left[\frac{\mathcal{H}_3 \mathcal{H}_5 [\mathcal{H}_1 (1+\mathcal{H}_4) - (\mathcal{H}_2 + 1)]}{(1-\mathcal{H}_2\mathcal{H}_4)(\mathcal{H}_3 - \mathcal{H}_1)} \left(J_0(\rho A_3) - J_0(\rho A_1) \right) \right.$$

$$\left. - \frac{\mathcal{H}_5 (1+\mathcal{H}_2)}{1-\mathcal{H}_2\mathcal{H}_4} \left(J_0(\rho A_1) - J_0(\rho A_2) \right) \right] \rho \, d\rho$$

$$G_3^{(3)} = \int_0^\infty \left[J_0(\rho A_1) - J_0(\rho B_3) - \frac{\mathcal{H}_3}{\mathcal{H}_3 - \mathcal{H}_1} \left(J_0(\rho A_1) - J_0(\rho A_3) \right) \right] \rho \, d\rho$$

$$G_2^{(2)} = \int_0^\infty \left[J_0(\rho A_2) - J_0(\rho B_2) \right] \rho \, d\rho$$

$$A_i = \sqrt{[(r_i - \delta_{mean})t + x - x']^2 + (y - y')^2}, \qquad i = 1, 2, 3$$

$$B_i = \sqrt{(x - v_i t - x')^2 + (y - y')^2} \qquad \delta_{mean} = \frac{v_1 + v_3}{2}$$

2. Before going into the use of the solution obtained for Φ_i' for prognostic purposes, let us stop briefly to analyze the stability of the solution. The frequencies σ_i are connected with r_i $(i = 1, 2, 3)$

on the basis of (1.10) by means of the equality $\sigma_i = (r_i - \delta_{mean})' m$.
But, through (1.13) and (1.14) r_i is a real number, therefore σ_i is also real, and the obtained solution, generally speaking, is stable. Nevertheless, as further analysis shows, there are possible cases, when the functions $G_2^{(1)}$ and $G_{(2)}^{(3)}$, will always contain a secular term of the order t [2], (and not of the order e^{at} $a > 0$, as this would have a place for complex values of σ_i) for some wave number ρ_i.

Let us examine the expression $D_1(r_2) = 1 - \mathcal{H}_2 \mathcal{H}_4$, which after substitution of the values $\mathcal{H}_2$ and $\mathcal{H}_4$ takes the form:

$$D_1(r_2) = 1 - \mathcal{H}_2 \mathcal{H}_4 = \frac{Q^2(r_2) - L(r_2, \delta) L(r_2, -\delta)}{Q^2(r_2)} \tag{21}$$

Equation $D_1(r_1) = D_1(r_3) = 0$ determines r_1 and r_3. Therefore if

$$D_1(r_2) = 0 \tag{2.2}$$

Then this means that for some $\rho = \rho_1$, $r_2 = r_1$ or $r_2 = r_3$. It is possible to show, that only the equality $r_2 = r_1$ is possible at $\rho = \rho_1$. But then the expression

$$\frac{J_0(\rho A_1) - J_0(\rho A_2)}{1 - \mathcal{H}_2 \mathcal{H}_4} \tag{2.3}$$

appearing in $G_2^{(1)}$ and $G_2^{(3)}$, will contain the indeterminate form $0/0$, since from $r_1 = r_2$ at $\rho = \rho_1$ it will follow that $A_1 = A_2$ by definition. The uncovered indeterminateness gives the following value for the rates (2.3):

$$\lim_{\substack{r_2 \to r_1 \\ \rho \to \rho_1}} \frac{J_0(\rho A_1) - J_0(\rho A_2)}{1 - \mathcal{H}_2 \mathcal{H}_4} = \rho\left[(r_1 - \delta_{mean})t + \varkappa - \varkappa'\right]t \left.\frac{J_1(\rho A_1)}{A_1}\right|_{\substack{at \\ \rho = \rho_1}}$$

This circumstance must be realized during numerical calculations of the functions $G_2^{(1)}$ and $G_2^{(3)}$. For the determination of ρ_1 let us put the quantities Q and L into (2.1):

$$D_1(r_2) = \frac{\rho^4}{\lambda^2 \bar{\beta}}\left[2\delta_0 + \frac{\rho^4(\delta^2 - \delta_0^2)}{\lambda^2 \bar{\beta}}\right] \tag{2.4}$$

It is easy to see, that $D_1(r_2) = 0$ when $\rho = \rho_1$ only in the case, when

$$1)\ \delta_0 < 0, \quad i.e. \quad V_2 < \frac{V_1 + V_3}{2} \tag{2.5}$$

$$2)\ \delta_0 > 0, \ but\ \delta < \delta_0, \ i.e. \quad V_2 > V_1 \tag{2.6}$$

Then ρ_1 has the following value:

$$\rho_1 = \sqrt[4]{\frac{2\lambda^2 \bar{\beta}\,|\delta_0|}{|\delta_0^2 - \delta^2|}} \tag{2.7}$$

In the first case the velocity of the zonal flow increases with height a little less than __________ by the linear law. In the second case there exists an extremum (a maximum) of the speed of the zonal flow in the layer, bounded by the 250 and 750 mb. surfaces. Let us note, that when using condition (1.5) together with equation (1.6) for the solution of the system of equations (1.4), we would have obtained the following values for r_1 and r_3:

$$r_{1,3} = \frac{\beta}{\rho^2(\rho^2+2\lambda^2)}\left[\rho^2+\lambda^2 \pm \lambda^2\sqrt{1+\frac{\rho^4\delta^2(\rho^4-4\lambda^4)}{\lambda^4\,\overline{\beta}^{(2)}}}\right] \tag{2.8}$$

The extremum of the term under the radical in (2.8) exists for $\rho = \lambda\sqrt{2}$. With a value of ρ so that the expression under the radical can appear as negative, then $r_{1,3}$ is complex.

The conditions for stability of the solution in this case take the form:

$$\delta < \frac{\overline{\beta}}{2\lambda^2} \quad \text{or} \quad V_1 - V_3 < \frac{\overline{\beta}}{\lambda^2} \tag{2.9}$$

It is easy to see, it does not depend on the value of the velocity of the zonal flow within the layer, bounded by the 250 and 750 mb surfaces.

3. In the nature of an example of the use of the obtained solution for Φ_L' for prognostic purposes, let us apply it to a forecast of the 700 mb map. Although Φ_3 is the geopotential of the 750 mb surface, in the future we shall identify it with the 700 mb surface, since the latter appears as a standard level, and the indicated substitution does not give rise to principle objections.

Since, generally speaking, it is possible to show [3], that

$$\iint_{-\infty}^{+\infty} G_i^{(j)}(x, x', y, y', t)\, dx' = 0 \tag{3.1}$$

Then, putting $\bar{\Phi}_i' = \Phi_i - \bar{\Phi}_i$ into (1.19) we deduce that

$$\iint_{-\infty}^{+\infty} \bar{\Phi}_i(y') G_i^{(j)} dx' dy' = \int_{-\infty}^{+\infty} \bar{\Phi}_i(y') dy' \int_{-\infty}^{+\infty} G_i^{(j)} dx' \equiv 0 \tag{3.2}$$

on the basis of (3.1). From here it follows, that in the obtained solution (1.19) it is possible to neglect шт/п ихи with Φ_i'.

In other words, the obtained solution describes the changes of its geopotential.

Therefore, for a forecast of the map of absolute topography of the 700 mb surface let us use the following formula:

$$\bar{\Phi}_3(x,y,t) = \bar{\Phi}_{30}(x - V_3 t, y, 0) + \frac{1}{2\pi}\iint_{-\infty}^{+\infty} \bar{\Phi}_{10} G_1^{(3)} dx' dy' \tag{3.3}$$

$$+ \frac{1}{2\pi}\iint_{-\infty}^{+\infty} \bar{\Phi}_{20} G_2^{(3)} dx' dy' + \frac{1}{2\pi}\iint_{-\infty}^{+\infty} \bar{\Phi}_{30} G_3^{(3)} dx' dy'$$

Calculations of the influence functions $G_1^{(3)}$, $G_2^{(3)}$ and $G_3^{(3)}$ should be carried out partly numerically and partly analytically if we use the asymptotic integrand function. Let us place the initial coordinate at the point $x = V_3 t$, $y = 0$. In this case the function $G_i^{(3)}$ $(i = 1, 2, 3)$ could be presented in the following convenient form for integration:

$$G_1^{(3)} = \frac{(\delta - \delta_0)}{2\delta} \int_0^{\infty} \frac{J_0(\rho a_3) - J_0(\rho a_1)}{\rho} d\rho + I_1(N)$$

$$G_2^{(3)} = \frac{2\delta\lambda^2}{\delta + \delta_0} \int_0^{\infty} \frac{J_0(\rho a_1) - J_0(\rho a_2)}{\rho} d\rho + I_2(N)$$

$$G_3^{(3)} = \int_0^{\infty} \left[J_0\left(\rho \sqrt{\left(\frac{\bar{\Phi} t}{\rho^2} - x'\right)^2 + y'^2}\right) - J_0\left(\rho\sqrt{x'^2 + y'^2}\right) \right] \rho\, d\rho + I_3(N)$$

$$I_1(N) = \int_0^N \left(\frac{\mathcal{H}_1 \mathcal{H}_3}{\mathcal{H}_3 - \mathcal{H}_1} - \frac{(\delta-\delta_0)\lambda^2}{2\delta\rho^2} \right)(J_0(\rho A_3) - J_0(\rho A_1))\rho d\rho \tag{3.4}$$

$$I_2(N) = \int_0^N \Big[\frac{\mathcal{H}_3 \mathcal{H}_5 [\mathcal{H}_1(1+\mathcal{H}_4) - (1-\mathcal{H}_2)]}{(1-\mathcal{H}_2\mathcal{H}_4)(\mathcal{H}_3 - \mathcal{H}_1)} \times$$

$$(J_0(\rho A_3) - J_0(\rho A_1)) - \left(\frac{\mathcal{H}_5(1+\mathcal{H}_2)}{1-\mathcal{H}_2\mathcal{H}_4} - \frac{2\delta\lambda^2}{(\delta+\delta_0)\rho^2} \right) \times$$

$$(J_0(\rho A_1) - J_0(\rho A_2)) \Big] \rho d\rho$$

$$I_3(N) = \int_0^N \Big[J_0(\rho A_1) - J_0\left(\rho\sqrt{\left(\frac{\beta t}{\rho^2} - x'\right)^2 + y'^2}\right)$$

$$- \frac{\mathcal{H}_3}{\mathcal{H}_3 - \mathcal{H}_1} \left(J_0(\rho A_1) - J_0(\rho A_3) \right) \Big] \rho d\rho$$

$$a_1 = \sqrt{x'^2 + y'^2}, \quad a_2 = \sqrt{[(\delta+\delta_0)t + x']^2 + y'^2}$$

$$a_3 = \sqrt{(2\delta t + x')^2 + y'^2}$$

since

$$\int_0^\infty \frac{f(px) - f(qx)}{x} dx = [f(0) - f(\infty)] \ln \frac{q}{p} \tag{3.5}$$

if $f(x)$ is a function, continuous as $x \to 0$ and if a finite limit $f(+\infty) = \lim_{x \to +\infty} f(x)$ [4] exists, then, obviously, using ~~[illegible]~~ (3.5) it is possible to cond ude, that

$$\int \frac{J_0(\rho a_3) - J_0(\rho a_1)}{\rho} d\rho = \ln \frac{a_1}{a_3} \quad and \quad \int_0^\infty \frac{J_0(\rho a_1) - J_0(\rho a_2)}{\rho} d\rho = \ln \frac{a_2}{a_1} \tag{3.6}$$

On the other hand, the definite integral which appears in $G_3^{(3)}$, completely coincides with the influence function $G_{\psi'}(x', y', t)$ for the forecast of the absolute topography of the mean level ([1], chapter V, Section 5,3). Now it is possible to write that

$$ {}^{(3)} = \frac{(\delta-\delta_0)\lambda^2}{2\delta} \ln \frac{a_1}{a_3} + I_1, \quad G_2^{(3)} = \frac{2\delta\lambda^2}{\delta+\delta_0} \ln \frac{a_2}{a_1} + I_2, \quad G_3^{(3)} = G_{\psi'} + I_3 \tag{3.7} $$

A graph of the function $G_1^{(3)}$ is presented in fig. 1, function $G_{(2)}^{(3)}$ in fig. 2 and function $G_3^{(3)}$ in fig. 3. During the calculations the following values of the parameters were used:

$$ L = 10^6 \text{ m}, \quad T = 1 \text{ day}, \quad \bar{\beta} = 1.1, \quad \lambda^2 = 1.6, \quad \delta = 0.5, $$
$$ \delta_0 = 0.4, \quad t = 1, \quad N = 3. $$

For the realization of a forecast, now calculation of the integrals appearing on the right side of (3.3) remains. It is possible to do this, since it was proposed, for example, in [2], i.e. a polar system of coordinates was introduced and an approximate integration was constructed over little rectangles, bounded by radii and circles. Using the approximate integration in (3.3), let us bring it into the given case with the following form: let us take the (x, y) plane in the equivalent "small" squares, in order that it could be possible to consider Φ_{i0} and $G_i^{(3)}$ constants within each of them, equal to their values in the center of the square. If a side of the square takes the value h, and the centers of the squares are denoted at the points by $(\pm mh, \pm nh)$ $m, n = 1, 2, \ldots\ldots\ldots$, then formula (3.3) can be presented in the following form:

$$ {}_3(V_3 t, 0, t) \approx \Phi_3(0,0,0) + \sum_{m=-M}^{M} \sum_{n=-S}^{S} \sum_{i=1,2,3} \Phi_{i0}(mh, nh) G_{mn}^{(i)}(t) \tag{3.8} $$

where

$$ G_{mn}^{(i)}(t) = \frac{h^2}{2\pi} G_i^{(3)}(mh, nh, t) \quad i = 1, 2, 3 $$

a 24 hr forecast we produce the tables of the values of $G_{mn}^{(i)}(t)$, uted for $h =$ 0.25 and $t =$ 1 (tables 1-3).

For a forecast, it follows that we must calculate Φ_3^T at every rsection of the given grid by formula (3.8), and then "advect" the obed field from west to east a distance equal to $V_3 t$.

Examples of forecasts of the AT_{700} map for 24 hours, on the s of the model of the atmosphere considered and with the obtained solution Φ_3^T , are illustrated in figs. 4-8. In figs. 4 and 5 the initial of AT_{700} for 1800 on the 11 and 12 of January 1954 are given, in . 6 and 8 the predictions are given, while in figs. 5 and 7 the correspond- actual AT_{700} maps for 1800 on the 12 and 13 of January 1954 are given. The fores were made by N. M. Kirev.

In conclusion, it should be mentioned that the given examples of prois still can not serve as an evaluation of the given method of forecasting. this, it follows that we should amass the necessary statistical material. pite of the simplicity of the idea of construction of the atmospheric model obtained solution of the linearized system of equations, by which the foremethod becomes quite simple, its realization, nevertheless, calls upon the iderable difficulty of numerical calculation of a series. It is easy to calte, that for every forecast it was necessary to make 94,770 operations of iplication and addition. For that reason tests of the indicated atmospheric l for prognostic purposes should only be carried out through the use of high l electronic-computing machines. In addition, it is necessary to note the es of the parameters δ and δ_0 , which play an important role ie calculation of the influence functions. For the realization of a 24 hr cast of the pressure field the values of these parameters must be close to : actual values at the initial moment of time.

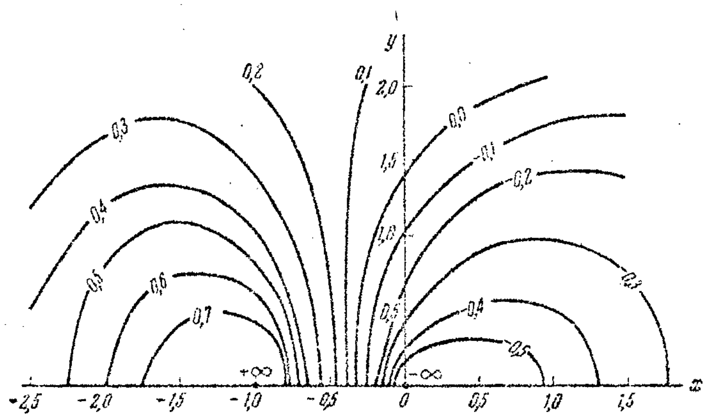

Fig. 1. Graph of the function $G_1^{(3)}$

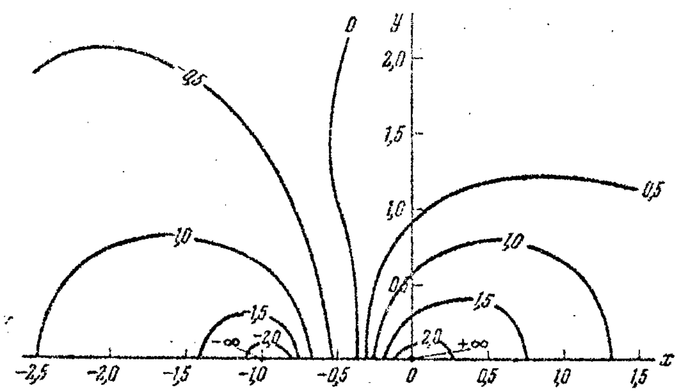

Fig. 2. Graph of the function $G_2^{(3)}$

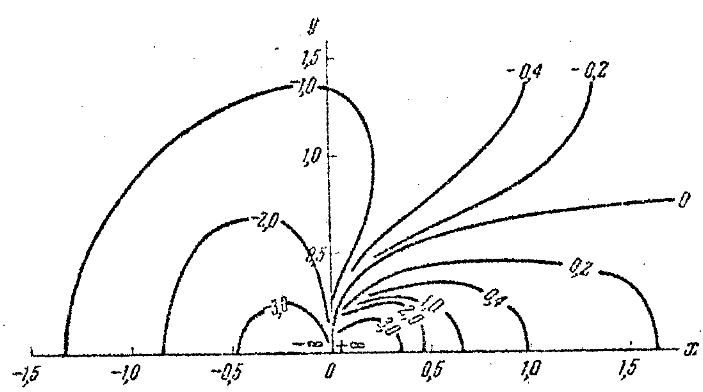

Fig. 3. Graph of the function $G_3^{(3)}$

Таблица 1

$100\ G^{(3)}_{mn}(1)$

m/n	−5	−4	−3	−2	−1	0	1	2	3	4	5
−5		−0,7	−0,9	−1,0	−1,1	−1,0	−0,9	−0,7	−0,5	−0,4	
−4	−0,7	−0,9	−1,2	−1,3	−1,4	−1,1	−0,8	−0,6	−0,4	−0,3	−0,2
−3	−0,8	−1,2	−1,5	−1,8	−1,5	−1,4	−0,7	−0,4	−0,2	−0,1	−0,1
−2	−0,9	−1,4	−1,8	−2,1	−1,9	−1,4	−0,2	0,1	0,1	0,1	0,1
−1	−1,1	−1,5	−2,1	−2,5	−2,8	−0,7	1,3	0,9	0,5	0,3	0,2
0	−1,1	−1,7	−2,2	−2,9	−4,6	80,0	3,7	1,4	0,7	0,4	0,3
1	−1,1	−1,5	−2,1	−2,5	−2,8	−0,7	1,3	0,9	0,5	0,3	0,2
2	−0,9	−1,4	−1,8	−2,1	−1,9	−1,1	−0,2	0,1	0,1	0,1	0,1
3	−0,8	−1,2	−1,5	−1,8	−1,5	−1,4	−0,7	−0,4	−0,2	−0,1	−0,1
4	−0,7	−0,9	−1,2	−1,3	−1,4	−1,1	−0,8	−0,6	−0,4	−0,3	−0,2
5		−0,7	−0,9	−1,0	−1,1	−1,0	−0,9	−0,7	−0,5	−0,4	

Таблица 2

$100\ G^{(2)}_{mn}(1)$

m/n	−7	−6	−5	−4	−3	−2	−1	0	1	2	3
−5		−0,7	−0,5	−0,4	−0,2	0	0,1	0,2	0,3	0,3	
−4	−0,9	−0,8	−0,7	−0,5	−0,2	0,1	0,3	0,4	0,8	0,8	0,8
−3	−1,1	−1,0	−0,9	−0,7	−0,4	0,1	0,5	0,8	1,0	1,0	1,0
−2	−1,2	−1,4	−1,2	−1,0	−0,6	0	0,6	1,1	1,3	1,2	1,2
−1	−1,3	−1,4	−1,4	−1,5	−0,9	−0,1	0,9	1,6	1,6	1,5	1,3
0	−1,3	−1,4	−1,6	−12,3	−1,4	−0,1	1,2	16,2	1,9	1,6	1,4
1	−1,3	−1,4	−1,4	−1,5	−0,9	−0,1	0,9	1,6	1,6	1,5	1,3
2	−1,2	−1,4	−1,2	−1,0	−0,6	0	0,6	1,1	1,3	1,2	1,2
3	−1,1	−1,0	−0,9	−0,7	−0,4	0,1	0,5	0,8	1,0	1,0	1,0
4	−0,9	−0,8	−0,7	−0,5	−0,2	0,1	0,3	0,4	0,8	0,8	0,8
5		−0,7	−0,5	−0,4	−0,2	0	0,1	0,2	0,3	0,3	

Таблица 3

$100\ G^{(1)}_{mn}(1)$

m/n	−7	−6	−5	−4	−3	−2	−1	0	1	2	3
−5		0,4	0,4	0,3	0,2	0,1	0	−0,1	−0,1	−0,2	
−4	0,5	0,5	0,4	0,4	0,3	0,1	0	−0,1	−0,2	−0,2	−0,3
−3	0,5	0,6	0,5	0,4	0,3	0,2	0	−0,2	−0,3	−0,3	−0,3
−2	0,6	0,6	0,6	0,5	0,4	0,2	−0,1	−0,2	−0,3	−0,4	−0,4
−1	0,6	0,7	0,7	0,6	0,4	0,2	−0,1	−0,3	−0,4	−0,4	−0,4
0	0,7	0,7	0,7	17,4	0,4	0,2	−0,1	−17,1	−0,5	−0,5	−0,5
1	0,6	0,7	0,7	0,6	0,4	0,2	−0,1	−0,3	−0,4	−0,4	−0,4
2	0,6	0,6	0,6	0,5	0,4	0,2	−0,1	−0,2	−0,3	−0,4	−0,4
3	0,5	0,6	0,5	0,4	0,3	0,2	0	−0,2	−0,3	−0,3	−0,3
4	0,5	0,5	0,4	0,4	0,3	0,1	0	−0,1	−0,2	−0,2	−0,3
5		0,4	0,4	0,3	0,2	0,1	0	−0,1	−0,1	−0,2	

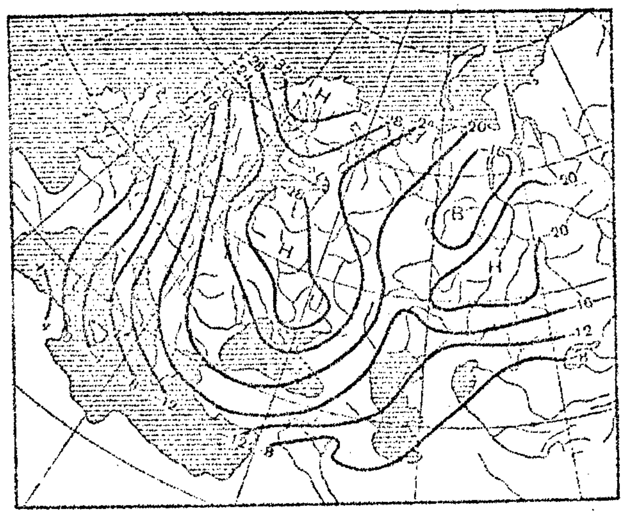

Fig. 4 Map AT_{700} for 18Z 11 January 1954

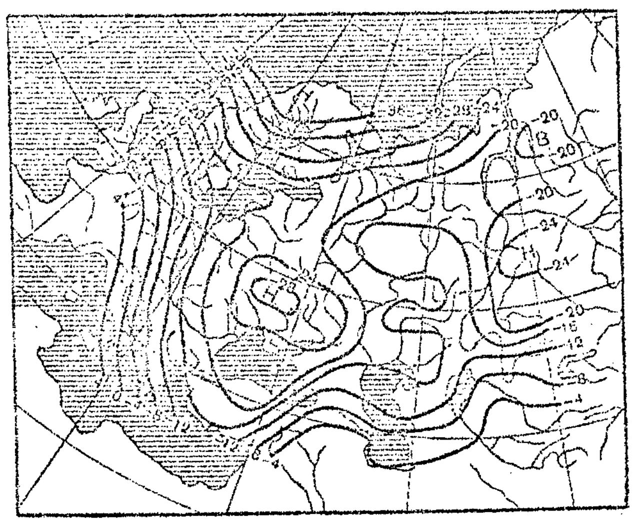

Fig. 5. Map AT_{700} for 18Z 12 January 1954

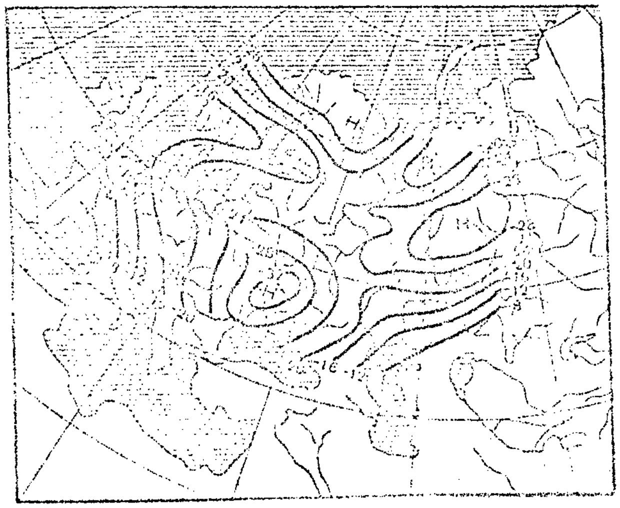

Fig. 6. Forecast Map AT 700 at 18Z 12 January 1954

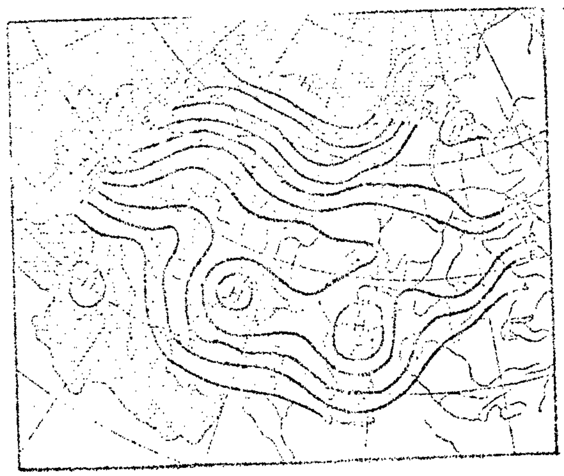

Fig. 7. Map AT_{700} for 18Z 13 January 1954

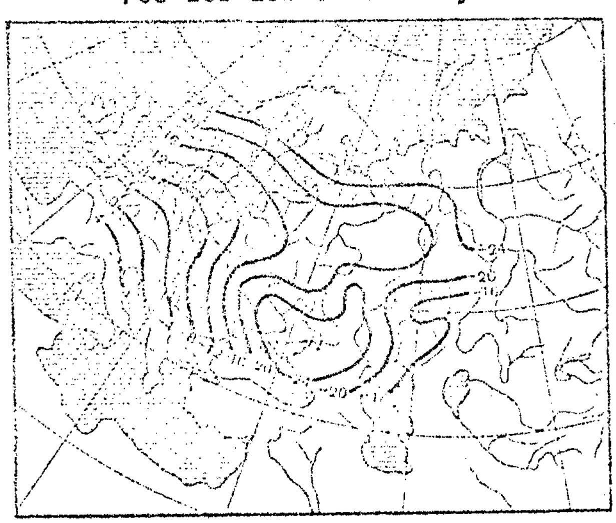

Fig. 8. Forecast Map AT_{700} at 18Z 13 January 1954

REFERENCES

Kibel, I.A., 1957: Introduction to hydrodynamic methods of short range weather forecasts.

Nemchinov, C.V., 1958: A short range forecast of the non zonal height deviations of the absolute topography for the lower and middle troposphere. Izvestiia, Ser. Geofiz., No. 6.

Nemchinov, C.V., 1955: Toward the prognosis of sea level pressure on the basis of the solution of the linearized system of dynamical equations of the atmosphere. Dissertation, Inst. of Atmos. Physics, Academy of Sciences, USSR.

Fichtengoltz, G.M., 1948: Course in differential and integral calculus.

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