*Aerospace Structures Information and Analysis Center*

# Nonlinear Internal Flow Analysis

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Space re-entry vehicles like ICBM's, manned space crafts, and space shuttles are subjected to excessive heat fluxes at the time of space re-entry. One method of preventing high internal temperatures is to let the outer surface ablate. While this solves the problem of overheating and protects the internal structure of these vehicles, it also changes the shape of these vehicles and makes them unsuitable from an aerodynamic viewpoint. This disadvantage can be overcome by using internal ablators protected by a fixed shape outer radiating structure. During periods of excessive heating internal ablation occurs, and the gaseous products transpire through the outer porous radiating structure.

In this report, a simple one dimensional analysis of the thermal ablation problem is presented. The model consists of a fixed shape outer structure whose temperature is limited to T*. An ablator is placed behind this outer structure. The ablator is assumed to act like a heat sink both in terms of mass supply and thermal energy supply so that it can be assumed to be at a constant temperature $T_b$. When the surface temperature of the ablator $T_s$ rises due to heat radiation and reaches a value of $T_a$, it ablates and gaseous products transpire through the fixed outer surface thereby reducing the heat transfer to the outer surface. The heat transfer mechanism between the outer structure and the ablation surface is by radiation across the gap. Of the heating rate $q_o(t)$ incident on the outer structure, a part $q_{r,o}(t)$ is radiated outward while the rest $q_{r,i}(t)$ is radiated inward to the ablation surface. When ablation occurs, the surface temperature of the ablator increases to a fixed value of $T_a$.

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# FOREWORD

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This technical report covers work accomplished from September 1995 through February 1997

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## NONLINEAR INTERNAL FLOW ANALYSIS

The aim of this report is to report the findings in nonlinear internal flow convection heat transfer analysis. One of the major types of loads which a high speed aerospace structure encounters is due to high thermal fluxes especially on the leading edges and the associated thermal structural interactions.. Often these type of fluxes are associated with nonlinear phenomena like compressible effects, viscous effects, turbulence, etc. and the only possible solution to these is through numerical methods. The finite element, finite difference, and finite volume methods, and to some extent boundary element method are among the most popular numerical methods which have found widespread use in computational mechanics. Each of these methods has its own advantages and disadvantages. The finite element methods is mathematically very elegant, can handle complex geometries easily, and can model very complex boundary conditions. The finite difference and finite volume methods on the other hand, are conceptually very simple and straight forward to implement on computers. The advent of powerful computers and work stations has resulted in these numerical methods becoming very powerful design tools. Despite all their advantages, the results of these methods are very heavily dependent on the uncertainty of the input data.

For, after all, numerical methods are approximate solutions to differential equations (both ordinary and partial), which in turn are often approximations to physical models. Hence the quality of the numerical solution depends strongly not only on how good a particular numerical method is, but also on how accurate the physical model is. A very obvious example of this is numerical solutions of turbulence equations. Because of their stiff nonlinearities, one has to often resort to several tricks, the most extreme, yet popular one being the adjustment of the values of turbulence constants. This is often required to get a converged solution, the accuracy of which is not known to the designer.

Space reentry vehicles like ICBM's, manned space crafts, and space shuttles are subjected to excessive heat fluxes at the time of space reentry. One method of preventing them from reaching hign temperatures would be to let the outer surfaces which are subjected to such extreme fluxes ablate. While this solves the problem of overheating and protects the internal structure of these vehicles, it also changes the shape of these vehicles and makes them unsuitable from an aerodynamic viewpoint. This disadvantage can be overcome by using internal ablators protected by a fixed shape outer radiating structure. During periods of excessive heating internal ablation occurs and the gaseous products transpire through the outer porous radiating structure.

In the following pages a simple one dimensional analysis of the thermal ablation problem is presented. This has the inherent disadvantage that the analysis is applicable at the point on the surface where heat flux is likely to be the highest, like the stagnation point. In fact it is this point on the surface where a one dimensional approximation is most suitable. A schematic of the stagnation point geometry is shown in Fig. 1. It consists of a fixed shape outer structurethe temperature of which is limited to $T^*$. An ablator is placed behind this outer structure. The ablator is assumed to act like a sink both in terms of mass supply and thermal energy supply so that it can be assumed to be at a constant temperature $T_b$. When the surface temperature of the ablator $T_s$ rises due to heat radiation and reaches a value of $T_a$, it ablates and gaseous products transpire through the fixed outer surface thereby reducing the heat transfer to the outer surface. The heat transfer mechanism between the outer structure and the ablation surface is by radiation across the gap. Of the heating rate $q_o(t)$ incident on the outer structure, a part $q_{r,o}(t)$ is radiated outward while the rest $q_{r,i}(t)$ is radiated inward to the ablation surface. When ablation occurs, the surface temperature of the ablator increases to a fixed value of $T_a$.

## GOVERNING EQUATIONS:

At the porous shield the following equation is obtained by a thermal energy balance:

$$q_o(t) = q_{r,o}(t) + q_{r,i}(t) \tag{1}$$

In the above equation $q_o(t)$ is the aerothermodynamic heating on the vehicle surface, while $q_{r,o}(t)$, $q_{r,i}(t)$ are the radiative exchanges between the shield and outer atmosphere and inner ablator surface respectively. Since the outside atmosphere can be assumed to be at absolute zero, $q_{r,o}(t)$ can be approximated as

$$q_{r,o}(t) = \varepsilon_w \sigma T_w^4(t) \tag{2}$$

$$q_{r,i}(t) = \varepsilon_w \sigma \frac{[T_w^4(t) - T_s^4(t)]}{[1 + \frac{\varepsilon_w}{\varepsilon_a} - \varepsilon_w]} \tag{3}$$

where it has been assumed in Eq. (3) that the heat transfer mechanism between the porous shield and ablator surface is due to radiation. During ablation as the gaseous products move through the porous surface they reduce the heat flux $q_o(t)$. By making a heat balance at the ablator surface the following equation is obtained.

$$q_{net}(t) = q_o(t) - \eta \dot{m} H_o(t) \tag{4}$$

where $H_o(t)$ is the total free stream enthalpy, and $\eta$ is an effiency for the vaporization process. During ablation at steady state, a thermal energy flux babalce equation at the shield wall is given by

$$q_o(t) - \eta \dot{m} H_o(t) = \varepsilon_w \sigma T_w^4(t) + \varepsilon_w \sigma \frac{[T_w^4(t) - T_s^4(t)]}{[1 + \frac{\varepsilon_w}{\varepsilon_a} - \varepsilon_w]} \tag{5}$$

While ablation takes place, $T_s(t) = T_a$. At the ablator surface an energy balance gives the following equation:

$$\varepsilon_w \sigma \frac{[T_w^4(t) - T_s^4(t)]}{[1 + \frac{\varepsilon_w}{\varepsilon_a} - \varepsilon_w]} = \dot{m} H_v + \varrho_b c_b \frac{d}{dt}[(T_s - T_b)\theta] \tag{6}$$

In the above equation $H_v$ consists of two parts, namely the ehthalpy of vaporization also called the enthalpy of ablation, and the enthalpy rise before vaporization. For a planar slab, integral thermal thickness can be used to approximate the thermal conduction into the interior as follows:

$$-\varkappa(\frac{\partial T}{\partial z}) = \varkappa[\frac{(T_s - T_b)}{\theta(t)}] = \varrho_b c_b \frac{d}{dt}[(T_s - T_b)\theta] \tag{7}$$

Hence Eq. (6) can be rewritten as

$$\varepsilon_w \sigma \frac{[T_w^4(t) - T_s^4(t)]}{[1 + \frac{\varepsilon_w}{\varepsilon_a} - \varepsilon_w]} = \dot{m} \Delta H_v + \varkappa_b[\frac{(T_s - T_b)}{\theta}] \tag{8}$$

Thus for a combination of geometry of the trajectory, sheild and ablator properties the behaviour of the system can be obtained by solving Eqs. (5), (6), and (8). When no ablation takes place, the mass ablation rate can be set to zero and the equations solved for the variables $T_w(t)$, $T_s(t)$, and $\theta(t)$. When ablation takes place, the surface temperature $T_s(t)$ is set equal to a constant $T_a$ and now the variables to be solved forare $T_w(t)$, $m(t)$, and $\theta(t)$. The next step in the process is to recast the equations in nondimensional form which is shown below.

Non dimensional variables cab be easily obtained by normalizing each variable against some reference state as follows:

$$
\begin{aligned}
&\overline{T}_w = \frac{T_w}{T^*} && \overline{T}_{w,o} < \overline{T}_w < \overline{T}_{w,max} \\
&\overline{T}_s = \frac{T_s}{T_a} && \frac{T_b}{T_a} < \overline{T}_s < 1 \\
&\delta = \frac{\theta}{\theta_g} && \delta < \delta_{\max} \\
&\mu = m\frac{H_v}{q_m t_m} && \mu < \mu_{\max} \\
&\tau = \frac{t}{t_m} && \tau < \tau_f \\
\text{and}\quad &F(\tau) = \frac{q_o(\tau)}{q_m} && F(\tau) < 1
\end{aligned}
\tag{9}
$$

In the above set of equations $q_m$ is the maximum input heat flus of the trajectory which occurs when $t = t_m$. Using these non dimensional quantities, Eqs (5), (6), and (8) can be reformulated in dimensionaless form. With no ablation, we can set the mass flow rate term to be zero and write equation (5) as

$$F(\tau) = R_w \overline{T}_w^4(\tau) - R_a \overline{T}_s^4(\tau) \tag{10}$$

where $R_w$ ans $R_a$ are the radiation parameters defined as follows:

$$R_w = \frac{q_r^*}{q_m}\left(1 + \frac{\varepsilon_a}{\varepsilon_a + \varepsilon_w - \varepsilon_a\varepsilon_w}\right) \tag{11a}$$

$$R_a = \frac{q_r^*}{q_m} \left(\frac{T_a}{T^*}\right)^4 \left(1 + \frac{\varepsilon_a}{\varepsilon_a + \varepsilon_w - \varepsilon_a\varepsilon_w}\right) \tag{11b}$$

with

$$q_r^* = \varepsilon_w \ \sigma \ T^{*4}$$

We can use Eq. (10) to solve for the temperature ratio to get

$$\overline{T}_w^4(\tau) = \frac{1}{R_w}[F(\tau) + R_a \overline{T}_s^4(\tau)] \tag{12}$$

Using the above in the non-dimensional form for Eqs. (6), and (8) we get

$$\varepsilon_{a,w} \ F(\tau) + (\varepsilon_{a,w} - 1) \ \ R_a \overline{T}_s^4(\tau) = \beta_a \frac{d}{d\tau}\overline{[(\overline{T}_s(\tau) - \overline{T}_b)\delta]} \tag{13}$$

$$\varepsilon_{a,w} \ F(\tau) + (\varepsilon_{a,w} - 1) \ \ R_a \overline{T}_s^4(\tau) = \frac{\gamma_a}{\delta(\tau)}[(\overline{T}_s(\tau) - \overline{T}_b)] \tag{14}$$

with

$$\beta_a = \frac{\varrho_b c_b T_a \theta_g}{q_m t_m}$$

$$\gamma_a = \frac{\varkappa_b T_a}{q_m \theta_g} \tag{15}$$

$$\overline{T}_b = \frac{T_b}{T_a}$$

$$\varepsilon_{a,w} = \frac{\varepsilon_a}{2\varepsilon_a + \varepsilon_w - \varepsilon_a\varepsilon_w} \tag{16}$$

Equations (12) - (14) are a set of three nonlinear coupled set of equations which need to be solved for $\overline{T}_s$, $\overline{T}_w$, and $\delta$. A further simplification can be made to the set of equations (10), (13), and (14) if a temperature function $\psi$ is introduced as follows:

$$\psi(\tau) = [\overline{T}_s(\tau) - \overline{T}_b]^2 \tag{17}$$

and an expression for the derivative for $\psi$ can be written as

$$\frac{d}{dt}\psi(\tau) = \frac{B^3[\psi] + A \;\; \varepsilon_{a,w} \;\; \psi F'}{AC[\psi]} \tag{18}$$

where $B[\psi]$, $C[\psi]$, and A are defined as

$$B[\psi] = \varepsilon_{a,w}\, F(\tau) + (\varepsilon_{a,w} - 1) \;\; R_a(\overline{T}_b + \sqrt{\psi})^4 \tag{19}$$

$$C[\psi] = \varepsilon_{a,w}\, F(\tau) + (\varepsilon_{a,w} - 1) \;\; R_a(\overline{T}_b + \sqrt{\psi})^3(\overline{T}_b - \sqrt{\psi}) \tag{20}$$

$$A = \beta_a\gamma_a \tag{21}$$

and $F' = dF/d\tau$. A numerical integration of Eq. (18) gives us the funciton $\psi(\tau)$. Once the temperature function $\psi(\tau)$ is known, we can easily get $\overline{T}_s(\tau)$, the nondimensional surface temperature from Eq. (17). Using $\overline{T}_s(\tau)$, we van get $\overline{T}_w$ from Eq. (12) and $\delta(\tau)$ from Eq. (14) respectively.

On the other hand when vaporization takes place as when the surface temperature of the ablator reaches the ablation temperature $T_a$, mass flux must be taken into account. Thus we have from Eq. (5) after nondimensionalizing,

$$\overline{T}_w^4(\tau) = \frac{1}{R_w}[F(\tau) - \eta\dot{\mu}\overline{H}_o(\tau) + R_a] \tag{22}$$

where

$$\overline{H}_o(\tau) = \frac{H_o(t)}{H_v}$$

Since the material is ablating, we have $\overline{T}_s = 1$. The dimensionless forms of eqs. (6) and (8) are

$$\varepsilon_{a,w}\ F(\tau) + (\varepsilon_{a,w} - 1)\ \ R_a = (1 + \varepsilon_{a,w}\eta\overline{H}_o)\frac{d\mu}{d\tau} + \beta_a(1 - \overline{T}_b)\frac{d\delta}{d\tau} \tag{23}$$

$$\varepsilon_{a,w}\ F(\tau) + (\varepsilon_{a,w} - 1)\ \ R_a = (\frac{\Delta h_v}{H_v} + \varepsilon_{a,w}\eta\overline{H}_o)\frac{d\mu}{d\tau} + \gamma_a\frac{(1 - \overline{T}_b)}{\delta(\tau)} \tag{24}$$

By rearranging Eq. (24) we get

$$\frac{d\mu}{d\tau} = \frac{\varepsilon_{a,w}\ F(\tau) + (\varepsilon_{a,w} - 1)\ \ R_a - \frac{\gamma_a}{\delta}(1 - \overline{T}_b)}{(\frac{\Delta h_v}{H_v} + \varepsilon_{a,w}\eta\overline{H}_o)} \tag{25}$$

A single differential equation for δ is obtained by combining Eqs. (23) and (24)

$$\frac{d\delta}{d\tau} = \frac{\varepsilon_{a,w}\ F(\tau) + (\varepsilon_{a,w} - 1)\ \ R_a(1 - \Pi)}{\beta_a(1 - \overline{T}_b)} + (\frac{\gamma_a}{\beta_a})\frac{\Pi}{\delta} \tag{26}$$

where parameter Π is defined as

$$\Pi = \frac{1 + \varepsilon_{a,w}\eta\overline{H}_o}{\frac{\Delta h_v}{H_v} + \varepsilon_{a,w}\eta\overline{H}_o} \tag{27}$$

It has been pointed out by Camberos (1989, 1996) that for typical ablation materials $\Pi \sim 1$. Equation (26) is solved numerically for $\delta$ and the information is used in Eq. (25) to solve for dimensionless mass flux. Presently calculations are underway to verify the one dimensional equations. For the case of two dimensional problem, the resulting set of equations would be integral equations.

## NOMENCLATURE:

| | | | |
|---|---|---|---|
| $A$ | = | dimensionless ablation parameter | |
| $c$ | = | specific heat at constant pressure, | J/kg-K |
| $F$ | = | dimensionless heating rate function | |
| $\Delta h_v$ | = | enthalpy of ablation | J/kg |
| $q$ | = | heating rate per unit area, | $W/m^2$ |
| $R$ | = | dimensionless radiation parameter | |
| $T$ | = | temperature | K |
| $t$ | = | time | s |
| $V$ | = | velocity | m/s |
| $x,y$ | = | coordinates | m |

Greek Symbols

| | | |
|---|---|---|
| $\beta$ | = | dimensionless parameter |
| $\gamma$ | = | dimensionless parameter |

$\delta$ = dimensionless thermal thickness

$\varepsilon$ = emissivity

$\varrho$ = density kg/m$^3$

$\varkappa$ = thermal conductivity W/m-K

$\mu$ = dimensionless mass flux

$\sigma$ = Stephan-Boltzmann constant = 5.67 x 10-8 W/m$^2$K$^4$

$\tau$ = dimensionless time

$\psi$ = dimensionless temperature function

SUBSCRIPTS:

a = ablator property

b = ablation material

o = stagnation point value

r = radiative

s = ablation surface

w = porous shield wall value

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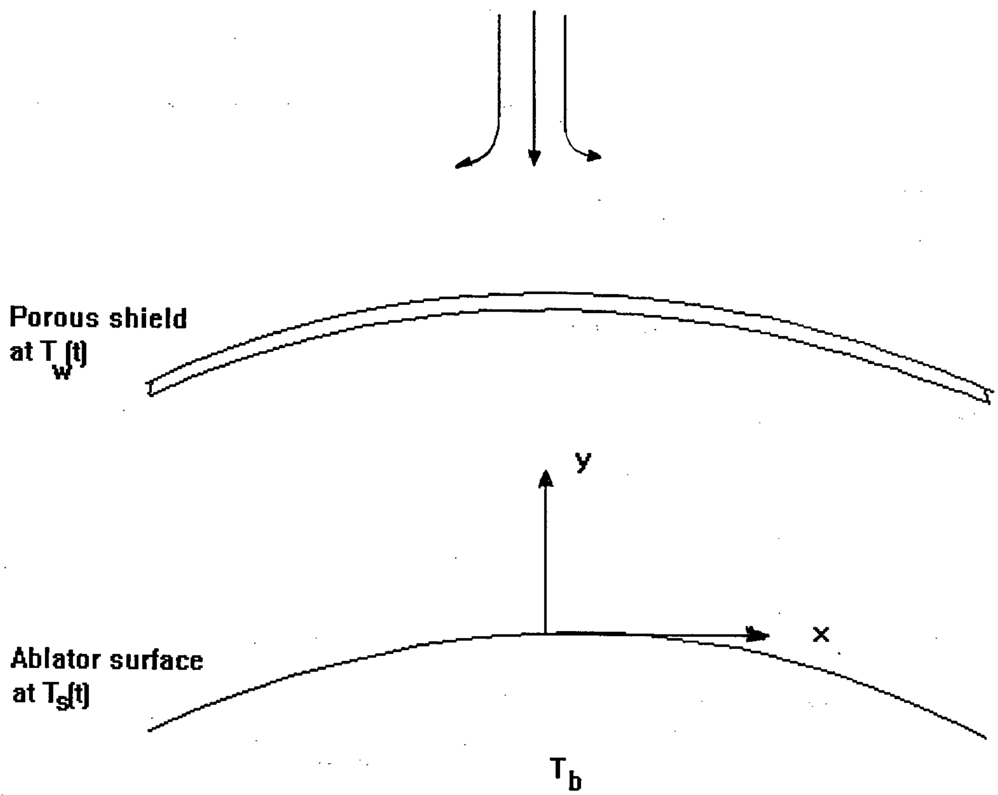


Fig. 1: One dimensional geometry near stagnation point of ablator surface