

RESEARCH AND DEVELOPMENT NEEDS  
IN FINITE ELEMENT ANALYSIS  
OF VISCOELASTICALLY DAMPED STRUCTURES

December 1980

Approved for public release;  
distribution unlimited

**AEROSPACE STRUCTURES  
INFORMATION AND ANALYSIS CENTER**

**OPERATED FOR THE AIRFORCE FLIGHT DYNAMICS LABORATORY  
BY ANAMET LABORATORIES, INC.**

19990518 019

copy IV

RESEARCH AND DEVELOPMENT NEEDS  
IN FINITE ELEMENT ANALYSIS  
OF VISCOELASTICALLY DAMPED STRUCTURES

December 1980

Approved for public release;  
distribution unlimited

*Aerospace Structures  
Information and Analysis Center*

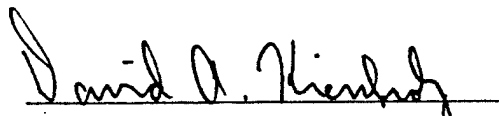
AEROSTRUCTURES IAC  
ANAMET LABORATORIES, INC.  
100 INDUSTRIAL WAY  
SAN CARLOS, CALIFORNIA 94070  
TEL. (415) 593-2125

AEROSTRUCTURES IAC  
AFFDL/FBR  
WRIGHT-PATTERSON AFB,  
OHIO 45433  
TEL. (513) 255-6688

*Aerospace Structures  
Information and Analysis Center*

This document describes a number of current problem areas in finite element analysis of structures with integral or add-on viscoelastic damping. The intent is to identify areas where development effort could yield near term benefits by improving design methods. Both theoretical and practical questions are considered.

This report was prepared by the Aerospace Structures Information and Analysis Center, which is operated for the Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories by Anamet Laboratories, Inc. under Contract No. F33615-77-C-3046. The work was performed in support of the Advanced Metallic Structures ADP (FIBAA) under ASIAC Problem No. 120, Finite Element Analysis of Damped Laminated Structures.

  
David A. Kienholz, Ph.D.  
Sr. Engineer

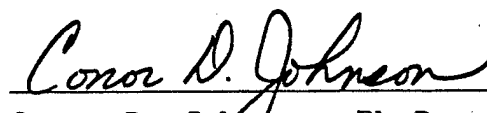
  
Conor D. Johnson, Ph.D.  
Principal Engineer

TABLE OF CONTENTS

	<u>Page</u>
1.0 INTRODUCTION . . . . .	1
2.0 THEORETICAL PROBLEM AREAS . . . . .	4
2.1 Normal Mode Methods for Structures with Frequency Dependent Material Properties . . . . .	4
2.2 Mechanics of Sandwich Structures . . . . .	5
2.3 Theoretical Basis of the Modal Strain Energy Method . . . . .	6
3.0 PROBLEMS IN CHARACTERIZATION OF MATERIALS . . . . .	9
3.1 Effect of Modeling Assumptions in Material Property Measurement . . . . .	9
3.2 Representation of General Constitutive Relations in Finite Element Modeling . . . . .	10
3.3 Modeling of High Temperature Ceramic Costings . . . . .	11
4.0 PROBLEMS IN FINITE ELEMENT MODELING . . . . .	12
4.1 Comparison to Experiment for Real Structures . . . . .	12
4.2 Development of a Special Purpose Finite Element Code . . . . .	13
4.3 Use of Advanced Elements in Modeling of Sandwich Structures . . . . .	14
4.4 Modeling for Preliminary Design . . . . .	14
4.5 Modeling of Sandwich Shells . . . . .	14
4.6 Damping Prediction in Very Large Models . . . . .	15
4.7 Modeling of Add-On Damping Treatments . . . . .	15
4.8 Analysis of Tuned Dampers . . . . .	16
4.9 Modeling of Damped Structures in a Project Environment . . . . .	16
5.0 EFFECTIVE DESIGN WITH VISCOELASTIC MATERIALS . . . . .	17
5.1 Identification of Applications . . . . .	17
5.2 Damping of Local Modes . . . . .	18
6.0 CONCLUSION . . . . .	19
REFERENCES . . . . .	20

## 1.0 INTRODUCTION

Vibration control in structures by means of viscoelastic damping material has gained wide acceptance, particularly in the aerospace industry. Significant weight, cost, performance, and reliability payoffs are possible in situations where resonant vibration cannot be avoided. Numerous examples of damping by constrained or unconstrained layers of viscoelastic material have appeared in the literature. However, the analysis of general structures to predict the damping due to an integral or add-on layered treatment or lumped viscoelastic elements is by no means a routine activity. The key to effective use of the well-developed technology of damping materials will be a parallel improvement in analytical capability. It will be necessary to make reliable and timely predictions of the system damping which can be expected when a given viscoelastic material is incorporated into a given structural configuration. It must also be possible for working engineers to produce these answers in a project environment using widely available analysis tools such as computer codes.

It is natural to look to finite element methods for damping predictions of general damped structures, just as they are used for routine calculation of the dynamic properties of undamped systems. Considerable work has already been done in this area, primarily aimed at prediction of damped response of simple structural elements such as sandwich beams, plates, etc. As participants in this work, it has been the authors' experience that current finite element tools show much promise for analysis of structures which include viscoelastic damping material. However, as with any emerging technology, there are specific questions, both theoretical and practical, which must be investigated and resolved in order to produce a reliable and useful design tool. The purpose of this paper is to describe the problem areas which, in the authors' view, must be addressed in the near term.

The problems are loosely arranged into four groups:

- (1) Theoretical problems relating to the basic mechanics of energy storage and dissipation in viscoelastic layers.
- (2) Questions relating to the adequacy of available data on viscoelastic materials.
- (3) Modeling questions relating to the capabilities which a finite element code must have, and how they should be used in damped structures.
- (4) Design questions of how a viscoelastic material can be effectively integrated into a structure for a given application.

As will be seen, many of the questions are closely related and the groupings are therefore somewhat arbitrary. They will serve simply as a framework for discussion.

Groups (1) and (2) above are not necessarily specific to finite element analysis. However, in finite element modeling of relatively simple sandwich structures, anomalies have been found which may be due to problems more basic than the models themselves. These difficulties, which will be explained shortly, must be resolved before meaningful application of finite element methods can proceed. Once this is done, the more applied question in Group (3) can be attacked.

References to earlier published work will be given where they are known. However, it should be pointed out that no thorough literature search has been performed in the area of finite element analysis of damped structures. Such a search should of course be part of any future organized work in the area. In general, it has been the authors' experience that published examples have been largely academic in nature. Little, if anything, has appeared which deals with the problems of

finite element analysis of viscoelastically damped structures on the scale which would be necessary in practical design work. It is this general area (Section 4.0) where advances are most urgently needed.

## 2.0 THEORETICAL PROBLEM AREAS

### 2.1 Normal Mode Methods for Structures with Frequency-Dependent Material Properties

It is well known that stiffness and damping properties of elastomers vary strongly with temperature and frequency. The variation with temperature does not present serious analysis problems since, at least for layered dampers, the coupling is only in one direction; temperature affects structural properties but not vice versa. One simply accepts the temperature or temperature range as given and inputs the corresponding material properties.

The variation with frequency is more serious. It is common practice (and an option in a few large finite element codes) to account for frequency variations by using a direct frequency response method to obtain a solution for sinusoidal input [1]. Stiffness matrices are simply recomputed at each frequency using a variable complex shear modulus for the viscoelastic material. Since this complex material parameter is itself obtained by sinusoidal test, the method by definition gives the correct frequency response so long as linearity with respect to amplitude holds for the material. However, it ignores the larger question of why one wanted the frequency response in the first place. It was not because solutions for sinusoidal loads are of any great importance in themselves. In most applications one must also be able to handle general periodic, transient, and random load cases. It is the theory of Fourier transforms which, for linear systems, allows one to compute solutions for all these cases given the sinusoidal solution. But for transform methods to be readily applied, the system matrices in the differential equations must be independent of time, which implies that the system matrices after transformation of the equations are independent of frequency.

This basic conflict surfaces when one attempts to employ normal mode methods, either real or complex, in order to take advantage of their computational efficiency. Normal mode

properties are obtained by solving an algebraic eigenvalue problem which requires that system mass and stiffness matrices be treated as constant. It is not clear at this point how (or even if) the variation of properties with frequency can be accommodated.

An ad hoc method of obtaining a normal mode representation for a structure containing a viscoelastic material has been suggested by several authors [2,3]. However, further work is clearly needed to investigate the meaning and consequences of the approximations and to determine the level of accuracy which can be expected.

## 2.2 Mechanics of Sandwich Structures

Virtually all analyses of sandwich beams, plates, shells, etc., assume either that the viscoelastic material behaves incompressibly (i.e., Poisson's ratio equal to .5) or implicitly assume that Poisson's ratio has no effect once the shear modulus is specified. Shear strain is taken to be the only significant mechanism of energy storage and dissipation in the sandwich core. The standard methods of measuring viscoelastic material properties are therefore designed to obtain only the shear modulus amplitude  $G$  and loss factor  $n$ . Young's modulus  $E$  and Poisson's ratio  $\nu$  are not usually measured.

Some recent unpublished work by the authors on predicting modal damping by the modal strain energy (MSE) method [2] has produced damping predictions for certain structures which varied significantly with the assumed value of Poisson's ratio in the core. This was disturbing in that the small amount of complete material data which was available indicated that typical elastomers are not truly incompressible. Rather, Poisson's ratio varies from nearly 0.5 in the rubbery region to about 0.3 in the glassy region. For sandwich beams it has been found that the value of  $\nu$  in the core has no effect as long as the modeling is done properly. While no finite element analysis of sandwich plates

has yet been completed, it is expected that only a slight effect will be observed. For doubly curved shells, the situation is more serious. The work done to date has indicated that natural frequencies, damping ratios, and mode shapes can all be significantly affected by the Poisson's ratio of the core material. The practical importance of this class of structure is clear motivation for further work.

To resolve this important issue several basic questions must be considered.

- (a) Is energy storage in the core due to dilational strain truly negligible for the range of structure types, material constants, sandwich geometries, and mode numbers of practical interest?
- (b) If not, what values of Poisson's ratio should be used for elastomers in the important transition region?
- (c) If the apparent sensitivity of core energy (and hence damping) to Poisson's ratio is only an artifact of the analysis model, how can it be suppressed and under what conditions will it need to be suppressed?

Questions (a) and (c) can be addressed by methods already in hand and should be part of any near term effort. Question (b) may be more difficult, but may not be important at all, depending on the outcome of (a). It should therefore be deferred for now. It is the authors' opinion that the difficulties observed so far are symptoms of a basic limitation in the analysis of sandwich structures by finite element methods. While the limitation may not be serious for applied work, it is clearly something that the analyst should be aware of and understand.

### 2.3 Theoretical Basis of the Modal Strain Energy Method

In Ref. [2], comparisons are given between modal loss factors obtained for uniform sandwich beams by two basically different methods. One method obtains the damping ratio in terms of the real and imaginary parts of a complex eigenvalue. The eigenvalue is that of a complex differential eigenvalue problem which describes the damped normal modes of the beam [4]. The other approach, called the modal strain energy (MSE) method, is based on a very simple conservation of energy argument. It obtains modal loss factors in terms of the core material loss factor and the strain energy distributions corresponding to undamped mode shapes. The two methods were found to be in excellent agreement.

However a basic question regarding the method was raised long ago by Ungar, one of its earliest proponents [9]. He pointed out that in a damped structure under sinusoidal load, stress and strain are out of phase and this relative phase is not the same at different points in the structure. Therefore, unlike the undamped case, one cannot define a unique potential energy of vibration, either for individual modes or for the structure as a whole. Thus the energy derivation of the modal strain energy principle is not entirely satisfactory.

The agreement observed between the two methods is therefore quite remarkable. It suggests a relation between damped and undamped modal properties which holds even for fairly high modal loss factors (in excess of 0.25). The agreement was so close and occurred for such a wide range of beam properties that one must suspect that some simple mathematical relation exists which could be rigorously derived. Such a proof would put the modal strain energy method on a better theoretical footing. More importantly, it would aid in defining the limits of the method and might allow a priori estimates of maximum error (relative to a true complex eigenvalue solution) to be made. Such estimates

would be highly useful in view of the practical and economic virtues of the MSE method.

It is therefore suggested that this theoretical problem should receive some attention. It could probably be approached most economically by simply comparing exact damped and undamped eigenvalue solutions for some simple case where both are available in closed form.

### 3.0 PROBLEMS IN CHARACTERIZATION OF MATERIALS

#### 3.1 Effect of Modeling Assumptions in Material Property Measurement

The present method for measuring shear moduli and loss factors of elastomers involves building a cantilever sandwich beam of specified dimensions with the material under test as the core. Modal frequencies and loss factors are then measured for a fairly large number (up to 15) of bending modes under low level sinusoidal excitation. Material shear moduli and loss factors are inferred from these data by using a particular mathematical model for the beam. The model is based on a fourth order flexural wave equation with a complex bending stiffness derived in terms of beam section properties and complex material constants. Transverse displacements of the upper and lower face sheets are assumed equal. Only this displacement and its gradient in the beam axis direction (slope) are retained as dependent degrees of freedom at each section. Boundary conditions are discarded in favor of assumed sinusoidal mode shapes in order to obtain simple invertible relations between material properties and modal properties.

Two high order theories are currently available for performing this same data reduction. It would be worthwhile to run a few test cases using the higher order theories to infer material properties and thereby investigate what scatter in those properties is being introduced by modeling assumptions. One would start with hypothetical but realistic values for modal properties and back out the material constants using each theory in turn.

The simpler of the higher order theories retains one additional dependent variable (a component of shear strain in the core) and yields a sixth order differential equation for transverse displacement. Solutions are available for a number of boundary conditions based on a complex shear modulus [4].

The remaining theory is that implemented in the finite element model. It could be called 10th order since three displacements and two rotations are held as dependent variables at each node. It has been demonstrated [2] to agree well with 6th order theory for the lowest 5-10 modes using fairly modest grid resolution and could readily be extended to higher modes as required. This is important since test data from modes as high as the 15th is sometimes used in obtaining material properties.

### 3.2 Representation of General Constitutive Relations in Finite Element Modeling

In order to use real normal mode methods, it must be assumed that viscoelasticity introduces nodal forces which are proportional to nodal velocities and that the matrix of proportionality constants is some linear combination of the mass and stiffness matrices. However, actual materials and structures are not usually so accommodating. More elaborate constitutive relations involving several orders [5] or even non-integer orders [6] of time derivatives of stresses and strains are required, and proportionality to mass or stiffness cannot be expected. It is probably beyond the scope of any near term work to attempt the inclusion of such general stress-strain relations in a production finite element code. Nonetheless, since they represent true material behavior, it is essential that the consequences of not including them be understood. To this end it would be worthwhile to pursue two subtasks.

(a) A modest theoretical study should be undertaken to determine if and how a general viscoelastic constitutive relation could be incorporated into the formulation of the describing matrices (mass, damping, stiffness, or whatever) for a solid element. If it can, some response prediction made using these general elements could be compared with a corresponding prediction made using a standard mass, stiffness, and proportional damping formulation (i.e., the modal strain energy method).

(b) Comparison to experiment should be carried out for both formulations.

### 3.3 Modeling of High Temperature Ceramic Coatings

Ceramic coatings have been shown to be effective dampers in high temperature situations such as turbine blades. In principle, techniques such as the modal strain energy method could be brought to bear in the design of such treatments. This would represent a step forward since current methods are based on simplifying assumptions as to mode shape, which may not be valid for important low order bending modes. A test case should be designed and carried out to evaluate the method for this type of application and to uncover any modeling problems peculiar to ceramic coatings.

#### 4.0 PROBLEMS IN FINITE ELEMENT MODELING

As might be expected, it is in this area that the most important questions lie. The realities of development budgets and schedules dictate that, for the foreseeable future, analysis of damped structures will be done with general purpose finite element codes which are already available. The real questions are how best to utilize existing codes such as NASTRAN and what level of accuracy can be expected in a given situation. The following specific activities are suggested as being worthwhile contributions.

#### 4.1 Comparison to Experiment for Real Structures

To date there is a frustrating scarcity of experimental data in the public domain which can be used in development and testing of modeling techniques for damped structures. A good test case must meet the following requirements:

- (a) The structural configurations and dimensions must be well documented.
- (b) The viscoelastic material must be one for which reliable material property data is available.
- (c) "Accidental" damping due to joint friction, radiation at boundaries, etc., must be small compared to dissipation in the viscoelastic.
- (d) Loading conditions must be accurately known and deterministic in space. Much vibro-acoustic data is unusable for this reason.
- (e) Structural mode shapes should be realistic. Highly uniform structures such as beams, plates, rings, etc., have their place as test cases. However, their modal strain energy distributions tend to be smoother than

is commonly encountered in applied work. A good model must also be capable of reproducing localized modes with attendant concentrations of strain energy. A sample problem must therefore have some modes of this type in order to be a realistic test of the modeling technique.

- (f) The test procedure and equipment must be described well enough such that, at least in principle, another independent investigator could reproduce the entire experiment.
- (g) Test results must be complete and in a format which can readily be compared to analysis results. Digital data which can be listed or replotted to different scales is particularly desirable, as is data such as modal properties or frequency responses which, in principle, are purely structural characteristics and independent of the excitation used to measure them.

Assembling a small number of test cases which meet these criteria, either via literature searching, contact with aerospace contractors, or original experiments, would be invaluable. Similarly a comprehensive set of academic test problems on uniform beams, plates, rings, etc. should be collected from the literature.

#### 4.2 Development of a Special Purpose Finite Element Code

In the authors' opinion, the development of a special purpose finite element code for analysis of integrally damped structures would not be a cost-effective approach. This is so simply because of the enormous fixed cost of any new finite element code which exists independent of the code's theoretical capabilities. It would, however, be worthwhile to investigate and catalog the

features which are essential, highly desirable, or simply convenient in modeling of viscoelastically damped structures. For example, the analysis of three layer sandwich structures by the modal strain energy method requires features such as membrane-bending coupling in plate elements and recovery of element energies. These are standard in MSC/NASTRAN (level 48 or above) but are not implemented in COSMIC/NASTRAN.

#### 4.3 Use of Advanced Elements in Modeling of Sandwich Structures

Level 60 of MSC/NASTRAN provides a new plate element (QUAD8) which could be highly useful in modeling of constrained or unconstrained layer dampers. This should be investigated in much the same way as was the QUAD4 element [2].

#### 4.4 Modeling for Preliminary Design

It is common to use fairly coarse models in preliminary design studies to predict undamped dynamic properties. It may be expected that the same will hold true for structures where it is known from the outset that damping will be required. Some effort should be allocated to determining what features of various generic structures must be preserved in order to obtain damping estimates of useful engineering accuracy.

#### 4.5 Modeling of Sandwich Shells

Sandwich shells, either singly or doubly curved, are a potentially important class of damped structures. However, little has been published about their analysis by finite elements. They should be investigated, particularly with respect to the adequacy of a single layer of core elements for various geometries and core material properties. The modeling problem mentioned in Section 2.2 first appeared in connection with a shell problem and is expected to be more severe in shells than in beams or plates.

#### 4.6 Damping Prediction in Very Large Models

It often occurs that very large models ( $10^4 - 10^5$  degrees of freedom) are needed to predict system dynamic response. In these cases the system must be divided into components whose normal mode properties are then obtained separately. This results in a substantial reduction in the number of d.o.f. required to characterize each component. The technique of component mode synthesis is then used to obtain the modal properties of the assemblage in terms of the modal properties of components. It is natural to inquire if the method can be extended to obtain estimates of system modal damping in terms of component modal damping, assuming dissipation at the connections can be ignored. The modal strain energy approach would seem appropriate here since, like component mode synthesis, it requires knowledge only of undamped mode shapes.

Some theoretical work will be required, although the practical question will probably reduce to the writing of DMAP instructions for MSC/NASTRAN to automate the procedure.

#### 4.7 Modeling of Add-On Damping Treatments

In the design cycle it often occurs that the need for engineered damping is not addressed until a structure reaches the prototype stage or beyond. The designer may wish to investigate add-on layered dampers to a structure for which a finite element model has already been prepared. In this situation the additional node and element data required to model the damping treatment will be largely determined by the original model. It may be possible to partially automate the process of altering the existing model to include added damping layers. A preprocessing code to accomplish this would be a significant step towards making damping technology accessible to the working structural engineer or designer.

#### 4.8 Analysis of Tuned Dampers

When unsatisfactory dynamic performance of a structure can be traced to a single vibration mode, the use of a simple add-on lumped damper tuned to the modes' natural frequency can often cure the problem. In this situation, the energy dissipation is largely confined to a known part of the system; namely, the damped spring of the tuned damper. This is the prime requirement for using the modal strain energy method. The accuracy of the method should be investigated for this class of problems. A test case should be run involving comparison to experiment where the damped spring is made of a viscoelastic whose material properties are accurately known.

#### 4.9 Modeling of Damped Structures in a Project Environment

It has been the authors' experience that problems will often exist with a new modeling technique which will not necessarily be uncovered in solving academic or controlled laboratory test problems. As the development of methods proceeds, it would be highly useful if the methods could be introduced in a controlled way into the design-analysis activities of some co-operating aerospace contractor. Ideally, this would produce one or more benchmark analysis cases meeting the requirements described in Section 4.1. Equally important, it would uncover at an early state the practical problems which often appear when new analytical methods are brought into an industrial setting.

## 5.0 EFFECTIVE DESIGN WITH VISCOELASTIC MATERIALS

As analysis techniques improve and the use of engineered material dampers increases, it may be anticipated that generic structural configurations will evolve which make effective use of viscoelastics. The identification of such configurations would fit naturally into the scope of work being described here. The following specific activities would be worthwhile.

### 5.1 Identification of Applications

Some informal survey work should be carried out to identify the applications where engineered damping would yield the greatest payoff and be most readily accepted. The specific problems and design constraints of these applications should be identified as they relate to the analysis methods being developed. With the nature and purpose of likely applications known in advance, the investigators can make better choices of methods to be developed and test problems to be executed.

For example, two likely applications are structural design of beam weapons [7] and large antennas [8] in space. Both represent cases where the orientation of an elastic structure must be controlled to microradian tolerances by an active servomechanism. Accuracy and even stability of the servo systems depend critically on structural damping.

A large space antenna (or any large space structure) would probably be constructed as a frame or truss because of the requirement of compact storage aboard the space shuttle. For this case, the well developed theory of sandwich beams or plates does not address the main problem. In the important global modes of vibration the individual frame members do not carry significant bending loads but rather act as simple tension-compression elements. Constrained layer damping might be useful to suppress the numerous high order local modes of individual beams but passive control of global modes would require the inclusion

of damped extensional links between carefully chosen locations. Design of these links and their placements would be the appropriate use of finite element methods in this case.

## 5.2 Damping of Local Modes

Practical structures often have highly non-uniform geometries which give rise to vibration mode shapes with large strain energy concentrations. The designer's challenge in these cases will be to find ways of introducing damping material such that it undergoes significant straining (the prerequisite for effective damping) and yet structural stiffness is not unacceptably reduced. It may be anticipated that this situation will occur repeatedly and that a systematic cataloging of case histories and solutions would be useful.

## 6.0 CONCLUSION

There are many areas in which basic methods development in finite element analysis of damped structures could produce significant payoffs in the near term. Because the avenues are so numerous and varied, it has not been attempted to set down a precise roadmap for a development program. Such a plan will, of course, be required, but it will inevitably be written around a known budget and time schedule and with more specific goals in mind. What has been presented here is more in the nature of a menu in that it probably contains more items than could be addressed in a single program. It may therefore be useful as a starting point in laying out a development plan.

REFERENCES

1. MSC/NASTRAN Applications Manual, Vol. 1, Jerrard A. Joseph, -  
Editor, MacNeal-Schwendler Corp.
2. Johnson, C. D., Kienholz, D. A., and Rogers, L., "Finite  
Element Prediction of Damping in Structures with Constrained  
Viscoelastic Layers," Proc. 51st Shock & Vibration Symposium,  
October.21-23, 1980, San Diego, California
3. Lu, Y. P., Douglas, B. E., and Thomas, E. V., "Mechanical  
Impedance of Damped Three-Layered Sandwich Rings,"  
AIAA Journal, Vol. 11, No. 3, March 1973, pp. 300-304.
4. Rao, D. K., "Frequency and Loss Factors of Sandwish Beams  
Under Various Boundary Conditions," J. Mechanical Engineering  
Science, Vol. 20, No. 5, 1978, pp. 271-282.
5. Christensen, R. M., "Theory of Viscoelasticity, An Intro-  
duction," Academic Press, New York, 1971.
6. Bagley, R. L., "Applications of Generalized Derivatives to  
Viscoelasticity," Air Force Materials Laboratory,  
AFML-TR-79-4103, Nov. 1979; also ADA 081 131.
7. Budde, C. L., Merritt, P. H., and Johnson, C. D., "Evaluation  
of Airborne Laser Beam Jitter Using Structural Dynamics  
Computer Codes and Control System Simulations," Bulletin 50,  
Part 3, Shock and Vibration Bulletin, September 1980.
8. Hughes, P. C., "Passive Dissipation of Energy in Large Space  
Structures," J. Guidance and Control, Vol. 3, No. 4,  
July-August, 1980.
9. Ungar, E. E., and Kerwin, E. M., "Loss Factors of Viscoelastic  
Systems in Terms of Energy Concepts," J. Acoustical Society of  
America, Vol. 34, No. 7, pp. 954-957, July, 1962.