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13. ABSTRACT (Maximum 200 words)

We have found the best solution to Duncan-Mortensen-Zakai (DMZ) equation for Yau filtering system, which is the most general filtering system and includes both linear filtering and exact filtering systems. We show that this equation can be solved explicitly with an arbitrary initial condition by solving a system of ordinary differential equations and a Kolmogorov type equation. Let n be the dimension of the state space. We show that we need only n sufficient statistics in order to solve DMZ equation.

In the other direction, we prove that if the estimation algebra is finite dimensional of maximal rank, then the Wang's matrix has constant structures. This theorem plays a fundamental role in the classification of finite dimensional estimation algebra of maximal rank. In fact we have classified all these Lie algebras for state space dimension less than or equal to 6.

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**PDE, DIFFERENTIAL GEOMETRIC, WAVELET AND ALGEBRAIC
METHODS IN NONLINEAR FILTERING**

Final Report

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A. Statement of problem studied.

The filtering problem considered here is based on the following signal observation model:

$$\begin{cases} dx(t) = f(x(t))dt + g(x(t))dv(t) & x(0) = x_0 \\ dy(t) = h(x(t))dt + dw(t) & y(0) = 0 \end{cases} \quad (1)$$

in which x , v , y and w are, respectively, \mathbf{R}^n , \mathbf{R}^p , \mathbf{R}^m and \mathbf{R}^m valued processes, and v and w have components which are independent, standard Brownian process. We further assume that $n = p$, f , h are C^∞ smooth, and that g is an orthogonal matrix. We will refer to $x(t)$ as the state of the system at time t and $y(t)$ as the observation at time t .

Let $\rho(t, x)$ denote the conditional probability density of the state given the observation $\{y(s); 0 \leq s \leq t\}$. It is well-known that $\rho(t, x)$ is given by normalizing a function, $\sigma(t, x)$, which satisfies the following Duncan-Mortensen-Zakai equation:

$$d\sigma(t, x) = L_0\sigma(t, x)dt + \sum_{i=1}^m L_i\sigma(t, x)dy_i(t), \sigma(0, x) = \sigma_0 \quad (2)$$

where

$$L_0 = \frac{1}{2} \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2} - \sum_{i=1}^n f_i \frac{\partial}{\partial x_i} - \sum_{i=1}^n \frac{\partial f_i}{\partial x_i} - \frac{1}{2} \sum_{i=1}^m h_i^2$$

and for $i = 1, \dots, m$, L_i is the zero degree differential operator of multiplication by h_i . σ_0 is the probability density of the initial point, x_0 .

Equation (2) is a stochastic partial differential equation. In real applications, we are interested in constructing robust state estimators from observed sample paths with some property of robustness. Davis studied this problem and proposed some robust algorithms. In our case, his basic idea reduces to refining a new unnormalized density

$$u(t, x) = \exp\left(-\sum_{i=1}^m h_i(x)y_i(t)\right)\sigma(t, x).$$

It is easy to show that $u(t, x)$ satisfies the following time varying partial differential equation

$$\begin{aligned} \frac{du}{dt}(t, x) &= L_0u(t, x) + \sum_{i=1}^m y_i(t)[L_0, L_i]u(t, x) \\ &+ \frac{1}{2} \sum_{i,j=1}^m y_i(t)y_j(t)[[L_0, L_i], L_j]u(t, x) \\ u(0, x) &= \sigma_0 \end{aligned} \quad (3)$$

where $[L_0, L_i]$ denotes the Lie bracket of L_0 and L_i .

Definition. The estimation algebra E of a filtering problem (1), is defined to be the Lie algebra generated by $\{L_0, L_1, \dots, L_m\}$, or $E = \langle L_0, L_1, \dots, L_m \rangle_{L.A.}$

If in addition there exists a potential function ψ such that $f_i = \frac{\partial \psi}{\partial x_i}$ for all $1 \leq i \leq n$, then the estimation algebra is called exact.

The estimation algebra is said to be with maximal rank if $x_i + c_i$ is in E for all $1 \leq i \leq n$ where c_i is a constant.

The problem is to solve explicitly equation (3) in real-time. In particular, we want to construct all possible finite dimensional filters via direct method or Wei-Norman approach. This includes solving the Brockett problem on classification of finite dimensional estimation algebras.

B. Summary of the most important results.

(I) In [Mi 1], Mitter pointed out that the innovation approach to nonlinear filtering theory is not, in general, explicitly computable. It was first proposed by Brockett and Clark [Br-Cl], Brockett [Br], and Mitter [Mi 1] to use estimation algebras to construct finite-dimensional filters. The idea is to use the Lie algebraic method to solve the Duncan-Mortensen-Zakai (DMZ) equation, which is a stochastic partial differential equation. By working on the robust form of the DMZ equation, we can reduce the complexity of the problem to that of solving a time-variant partial differential equation.

In the past decade, the Lie algebraic viewpoint has been remarkably successful, and recent works [T-W-Y], [D-T-W-Y], [Ya], [Ch-Ya], [Ch-Ya1], [Ch-Ya2], [C-L-Y1], [C-L-Y2], [Hu-Ya2], [Hu-Ya3], [Wo-Ya] have given us a deeper understanding of the DMZ equation, which was essential for progress in nonlinear filtering as well as in stochastic control. In fact it was Yau (cf. [Ya1], [Ya2]) who first used Lie algebraic method to discover the most general class of finite dimensional filters which include linear filters and exact filters as special cases. In [Ch], this general class of finite dimensional filters are called Yau filters.

In spite of the success of the Lie algebra method, it is extremely desirable to treat the DMZ equation by a direct method.

Despite its usefulness, the Kalman-Bucy filter is not perfect. One of its weaknesses is that it needs a Gaussian assumption on the initial data. The situation is more complex when the statistics of the initial condition are modeled by an arbitrary distribution. In the case where the linear filtering system (i.e., f , g , and h are linear functions in (1) is completely reachable and completely observable, Hazewinkel observed on p. 115 of [Ha] that the estimation algebra E (i.e., a Lie algebra generated by differential operators $L_0, h_1(\frac{\partial}{\partial x_1}), \dots, h_m(\frac{\partial}{\partial x_n})$) is the $2n + 2$ dimensional Lie algebra with basis $L_0, \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}, x_1, \dots, x_n, 1$. Even in this case, the Wei-Norman approach used to find the solution of (3) is more complicated than the procedure in Theorem 1 below because not only must one solve a finite system of ordinary differential equation and a Kolmogorov equation, but also one has to integrate n partial differential equations corresponding to the operators $\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}$. More important, if the linear system is not completely reachable or completely observable, then the basis of the estimation algebra is not explicitly known (although it can be computed). As a result, there is an additional disadvantage of the Wei-Norman approach, namely, one cannot write down the finite system of ordinary differential equation explicitly in the non-maximal rank case. The novelty of theorem 1 is

that our finite system of ordinary differential equations is explicitly written down and our procedure to get the solution of (3) is simpler than the Lie algebra approach. Most importantly, our theorem works for Yau filtering system which includes linear filtering system and exact filtering system as special cases.

Theorem 1. Consider the Yau filtering system (1) with arbitrary initial condition, i.e.,

$$(Y1) \quad f_i(x) = \ell_i(x) + \frac{\partial F}{\partial x_i}(x), \quad 1 \leq i \leq n, \text{ where } \ell_i = \sum_{j=1}^n d_{ij}x_j + d_i \text{ for } 1 \leq i \leq n \text{ and } F \text{ is a } C^\infty \text{ function.}$$

$$(Y2) \quad h_i(x) = \sum_{j=1}^n c_{ij}x_j + c_i, \quad 1 \leq i \leq m.$$

$$(Y3) \quad \eta(x) := \sum_{i=1}^n f_i^2(x) + \sum_{i=1}^n \frac{\partial f_i}{\partial x_i} + \sum_{i=1}^m h_i^2(x) = \sum_{i,j=1}^n \eta_{ij}x_i x_j + \sum_{i=1}^n \eta_i x_i + \eta_0, \text{ where } \eta_{ij}, \eta_i \text{ and } \eta_0 \text{ are constants.}$$

Choose a C^∞ function $G(x)$ such that

$$\Delta G(x) + |\nabla G|^2(x) + 2 \sum_{i=1}^n \ell_i(x) \frac{\partial G}{\partial x_i}(x) = \eta(x) - \sum_{i=1}^n \ell_i^2(x) - \sum_{i=1}^n \frac{\partial \ell_i}{\partial x_i}(x).$$

Then the solution $u(t, x)$ for the Duncan-Mortensen-Zakai equation (3) is reduced to the solution $\tilde{u}(t, x)$ for the Kolmogorov equation

$$\frac{\partial \tilde{u}}{\partial t}(t, x) = \frac{1}{2} \Delta \tilde{u}(t, x) - \sum_{i=1}^n \left(\ell_i(x) + \frac{\partial G}{\partial x_i}(x) \right) \frac{\partial \tilde{u}}{\partial x_i}(t, x) - \sum_{i=1}^n \left(\frac{\partial \ell_i}{\partial x_i}(x) + \frac{\partial^2 G}{\partial x_i^2}(x) \right) \tilde{u}(t, x)$$

where

$$\tilde{u}(t, x) = e^{c(t)+G(x)+\sum_{i=1}^n a_i(t)x_i - F(x+b(t))} u(t, x + b(x))$$

and $a_i(t)$, $b_i(t)$, and $c(t)$ satisfy the following system of ODEs: For $1 \leq i \leq n$

$$b'_i(t) - a_i(t) - \sum_{j=1}^n d_{ij}b_j(t) + \sum_{j=1}^m c_{ji}y_j(t) = 0 \quad (4)$$

$$a'_i(t) + \sum_{j=1}^n d_{ji}b'_j(t) - \frac{1}{2} \sum_{j=1}^n (\eta_{ij} + \eta_{ji})b_j(t) = 0 \quad (5)$$

$$\begin{aligned} c'(t) + \frac{1}{2} \sum_{i=1}^n (b'_i(t))^2 - \sum_{i=1}^n a_i(t)b'_i(t) - \frac{1}{2} \sum_{k,j=1}^n \eta_{kj}b_k(t)b_j(t) \\ - \frac{1}{2} \sum_{i=1}^n \eta_i b_i(t) + \sum_{i=1}^n d_i b'_i(t) = 0. \end{aligned} \quad (6)$$

Theorem 2. Consider the Yau filtering system (1) with arbitrary initial condition satisfying (Y1), (Y2) and (Y3) in Theorem 1. Then the solution $u(t, x)$ for the Duncan-Mortensen-Zakai equation (3) is reduced to the solution $\tilde{u}(t, x)$ for the Kolmogorov equation

$$\begin{aligned} \frac{\partial \tilde{u}}{\partial t}(t, x) &= \frac{1}{2} \Delta \tilde{u}(t, x) - \sum_{i=1}^n \ell_i(x) \frac{\partial \tilde{u}}{\partial x_i}(t, x) \\ &+ \frac{1}{2} \left(\sum_{i=1}^n \ell_i^2(x) - \sum_{i=1}^n \frac{\partial \ell_i}{\partial x_i}(x) - \eta(x) \right) \tilde{u}(t, x) \end{aligned}$$

where

$$\tilde{u}(t, x) = e^{c(t) + \sum_{i=1}^n a_i(t)t_i - F(x+b(t))} u(t, x + b(t))$$

and $a_i(t)$, $b_i(t)$ and $c(t)$ satisfy ODEs (4), (5) and (6).

Theorem 3. Consider the Yau filtering system (1) with arbitrary initial condition satisfying (Y1), (Y2) and (Y3) in Theorem 1. Then the solution $u(t, x)$ for the Duncan-Mortensen-Zakai equation (3) is reduced to the solution $\tilde{u}(t, x)$ for the Kolmogorov equation

$$\begin{aligned} \frac{\partial \tilde{u}}{\partial t}(t, x) &= \frac{1}{2} \Delta \tilde{u}(t, x) - \sum_{i=1}^n f_i(x) \frac{\partial \tilde{u}}{\partial x_i}(t, x) \\ &+ \frac{1}{2} \left(\sum_{i=1}^n f_i^2(x) - \sum_{i=1}^n \frac{\partial f_i}{\partial x_i}(x) - \eta(x) \right) \tilde{u}(t, x) \end{aligned}$$

where

$$\tilde{u}(t, x) = e^{c(t) + \sum_{i=1}^n a_i(t)x_i + F(x) - F(x+b(t))} u(t, x + b(t))$$

and $a_i(t)$, $b_i(t)$ and $c(t)$ satisfy ODEs (4), (5) and (6).

Notice that the explicit recursive filter for exact filtering system was previously derived only for maximal rank case (i.e., rank of (c_{ij}) is n), see [Be] for a particular maximal rank case and [T-W-Y] for general maximal rank case. Our result is an important breakthrough for the last 20 years because our method works for most general filtering systems and we no longer need maximal rank condition to construct explicit recursive filter and we can allow arbitrary initial condition. The significance of our results are that our algorithm is universal for any Yau filtering system with arbitrary initial condition and that this estimation problem have been factored into two parts: (1) the off-line calculation of the Kolmogorov type equation which does not depend on the observations and (2) the on-line solution of a finite linear system of ordinary differential equations, which can be realized in real time.

(II) Although the concept of estimation algebra has proven to be an invaluable tool in the study of nonlinear filtering problems [Ma], until recently very little was known about

estimation algebras. Beginning in the late 1980s, however, the structure and classification of finite-dimensional exact estimation algebras were studied in detail [T-W-Y] [D-T-W-Y]. In [Wo], the concept of Ω was introduced, which is defined as the matrix whose (i, j) element is $\frac{\partial f_j}{\partial x_i} - \frac{\partial f_i}{\partial x_j}$, where f is the drift term of the state evolution equation. For the class of exact filtering systems, Ω is identically zero. More recently, Yau [Ya1], [Ya2] has studied filtering systems such that all entries of Ω are constants. He was able to classify all finite-dimensional estimation algebras of maximal rank in such filtering systems. Chiou-Yau [Ch-Ya] and Chen-Leung-Yau [C-L-Y1], [C-L-Ya2] have shown respectively that if the dimension of the state space is two or three or four, all entries of Ω are constants as long as the estimate algebra is of maximal rank and finite dimensional. Thus, finite-dimensional estimation algebra of maximal rank is completely classified if the dimension of state space is at most three. The novelty of their theorems is that there are no a priori assumptions on the drift term of the nonlinear filtering system.

Our approach for the complete classification of finite-dimensional estimation algebras of maximal rank consists of two steps. The first step is to prove that for such an estimation algebra, all the entries in the Ω -matrix are degree one polynomials. The second step is to prove that in fact all the entries in Ω are constants. Then we can apply the result of Yau [Ya] to give a complete classification of finite-dimensional estimation algebras of maximal rank. Recently we have completed the first step (cf. [Ch-Ya1]). The following theorem is due to Hu and Yau [Hu-Ya2].

Theorem 3. If E is a finite-dimensional estimation algebra of maximal rank, then all the entries $\omega_{ij} = \frac{\partial f_j}{\partial x_i} - \frac{\partial f_i}{\partial x_j}$ of Ω are degree one polynomials. Let k be the maximal rank of quadratic forms in E . Then ω_{ij} are constants for $1 \leq i, j \leq k$ and $k+1 \leq i, j \leq n$; ω_{ij} are degree one polynomials in x_1, \dots, x_k for $1 \leq i \leq k$ or $1 \leq j \leq k$.

Let n be the dimension of the state space. In case $n = 3$, there are three unknowns: ω_{12} , ω_{13} , and ω_{23} . It is easy to see that they are all degree two polynomials in view of Ocone's theorem. In [Ya-Le], Leung and Yau showed that the coefficients of the quadratic parts of ω_{12} , ω_{13} , and ω_{23} have to satisfy 90 quadratic equations. It was also shown in that paper that this system of 90 quadratic equations has only a trivial solution. Later Chen, Leung, and Yau proved that Ω is a matrix of constants [C-L-Y1]. The novelty of our main theorem is that it holds for arbitrary n . Thus, it is the fundamental step in the classification of finite-dimensional estimation algebras of maximal rank.

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C. List of all Publications and Technical Reports

1. The wavelet application to Kolmogorov equation, (with Zhigang Liang) Proceeding of International Conference on Control and Information, (1995), 271-276.
2. A report on explicit formulas for $\exp(tA)$, (with Hon-wing Cheng) Proceeding of International Conference on Control and Information, (1995), 69-75.
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5. Explicit construction of finite dimensional nonlinear filters with state space dimension 3, (with Jie Chen and Chi-Wah Leung) Proceedings of the 34th IEEE Conference on Decision and Control, New Orleans, Louisiana, Dec. 13-15, (1995). 4030-4034.
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Report of Inventions

The best algorithms for Yau filtering system with arbitrary initial conditions were found. These algorithms are of practical importance.