Progress report on the grant ONR N00014-98-1-0070
"Modeling Swell High Frequency Spectral and Wave Breaking"

Principal Investigator, V. E. Zakharov

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Multiple Nonlinear Diffusion Equation

In the previous report we proposed Nonlinear Diffusion Equation (NDE) which is diffusion approximation to Hasselmann equation (HE) for gravity waves. NDE is the second-order partial differential equation describing the diffusion of the density of action for gravity waves $n(k,\phi)$

$$\frac{\partial n}{\partial t} = \frac{2a}{k}\frac{\partial}{\partial k}k^{\frac{1}{2}}\frac{\partial}{\partial k}k^{12}n^3 + k^{\frac{19}{2}}\frac{\partial^2}{\partial \phi^2}n^3 + \Gamma n, \quad n = n(k,\phi), \quad \Gamma = \Gamma(k,\phi) \qquad (1)$$

where k and ϕ are the modulus of wavenumber and polar angle in Fourier space, $\Gamma(k)$ is the coefficient responsible for external forcing and viscous damping; a is the constant to be found from the comparison of numerical simulation of NDE with numerical simulation of HE.

In the absence of external forcing and viscosity NDE has the same motion integrals as HE - wave action, energy and momentum; it also has similar to HE asymptotic Kolmogorov spectra corresponding to given energy, momentum and wave fluxes.

After change of variables $\omega = k^{\frac{1}{2}}$ equation (1) takes the form

$$\frac{\partial n}{\partial t} = \frac{a}{\omega^3}\left[\frac{1}{2}\frac{\partial^2}{\partial \omega^2} + \frac{1}{\omega^2}\frac{\partial^2}{\partial \phi^2}\right]\omega^{24}n^3 + \Gamma n \qquad (2)$$

The procedure of comparison of NDE and HE consists in comparison of the right-hand side (collision term) for two model being calculated on $JONSWAP$

spectrum. Results of such calculation for the equation (2) have shown an excellent agreement with the results of HE in directional distribution. Agreement between angular distributions was not, however, very good [2].

To improve an agreement between HE and NDE we propose Multiple Nonlinear Diffusion Equation ($MNDE$)

$$\frac{\partial n}{\partial t} = \frac{a}{\omega^3}\left[\frac{1}{2}\frac{\partial^2}{\partial \omega^2} + \frac{1}{\omega^2}\frac{\partial^2}{\partial \phi^2}\right]\omega^{24}(\alpha_1 n^3 + \alpha_2 n^2\langle n\rangle + \alpha_3 n\langle n^2\rangle + \alpha_4 n\langle n\rangle^2) + \Gamma n \quad (3)$$

where brackets mean angle averaging in ϕ-direction:

$$\langle n\rangle = \frac{1}{2\pi}\int_0^{2\pi} n d\phi$$

This equation is identical to the equation (2) if $\alpha_1 \neq 0$ and $\alpha_i = 0,\ i = 2,3,4$. Extra diffusion terms model non-locality of interaction in HE (that's where the word "multiple" stem from in $MNDE$).

The value of the constants α_i ($i = 1,2,3,4$), as usual, has to be defined from the comparison of the results of calculation of the right-hand sides for $MNDE$ (3) and HE being calculated on the $JONSWAP$ spectrum. Such multi-parameter optimization procedure is nontrivial and is the subject of future research.

In the current report we represent the results of "informal" choice of the coefficients α_i ($i = 1,2,3,4$) which demonstrate that even simple modification of the model (2) gives dramatically better agreement in the angular distribution between HE and NDE.

We developed numerical solver of the equation (3) allowing to calculate the temporal evolution of the gravity surface wave spectra. To get the "feeling" of the effect of the terms in the right-hand side of (3) corresponding to different α_i, ($i = 1,2,3,4$) we have made four independent runs of $MNDE$ solver corresponding to the sets:

begin eqnarray* $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$
$\alpha_1 = 0, \alpha_2 = 1, \alpha_3 = 0, \alpha_4 = 0$
$\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 1, \alpha_4 = 0$
$\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 1$

It is interesting that three first runs gave close results (up to the scaling constant). This fact prompted us to make the conclusion that simple choice of $\alpha_1 \neq 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 \neq 0$ could produce better agreement in angular distribution with the results of simulation of HE by Resio and Tracy [2]. The choice of the constants $\alpha_1 = 0.467 \cdot 10^{-2}$ and $\alpha_4 = 0.14$ gives directional and angular distributions represented on the Fig.1 and Fig.2 (solid line corresponds to the results of $MNDE$ and crosses to the results of HE by Resio and Tracy). It is seen that an agreement is excellent for directional spectrum and quite good for angular distribution at $f = 0.148$.

We hope to make the angular distribution correspondence even better as a result of calculation of the coefficients α_i $(i = 1, 2, 3, 4)$ from multi-parameter optimization.

References

[1] V.E. Zakharov, A.N. Pushkarev, Diffusion model of interacting gravity waves on the surface of deep fluid, *Nonlinear Processes in Geophysics*, Vol. 6, No. 1, 1999.

[2] WISE meeting in Annapolis, March 21-25, 1999.

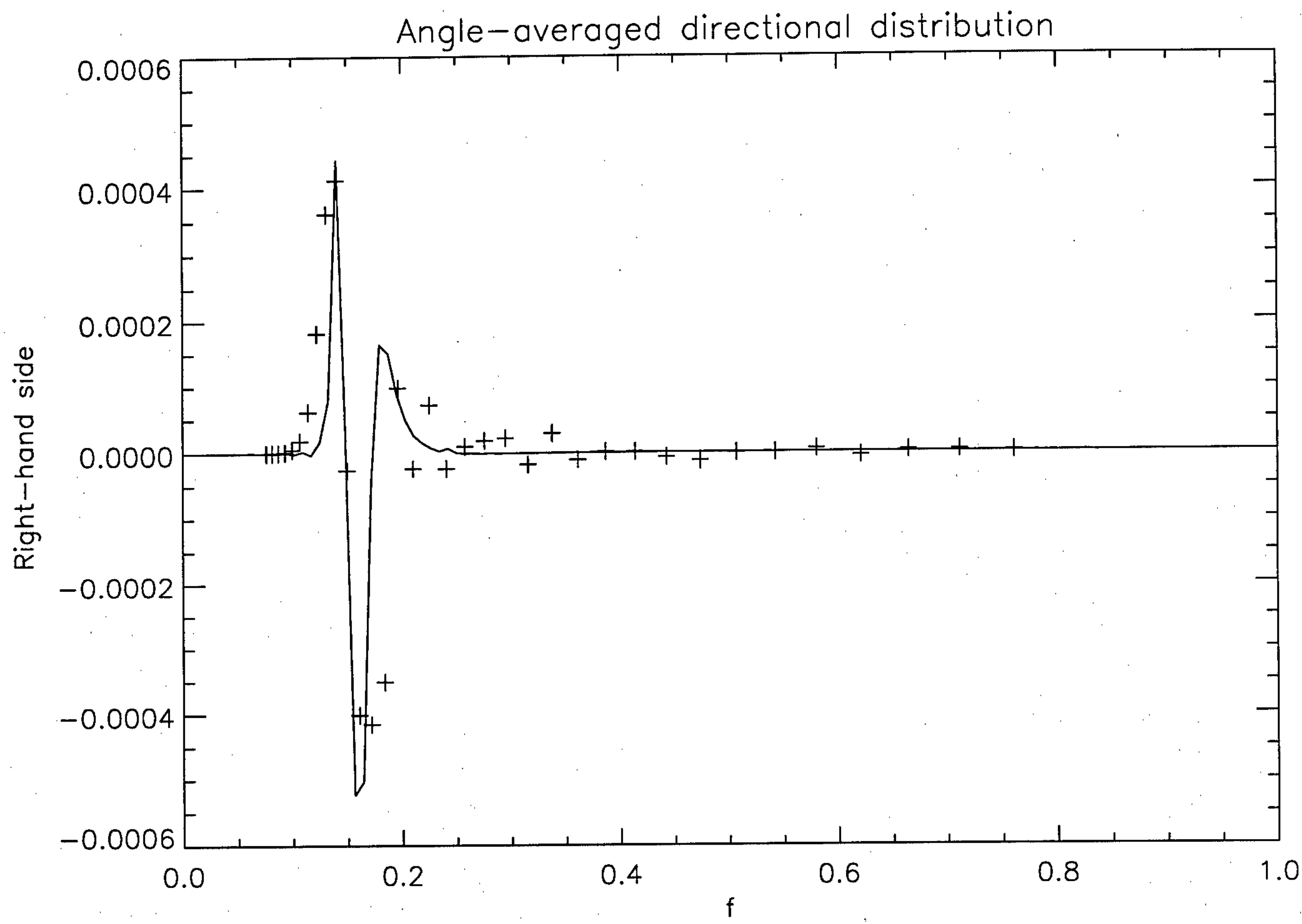
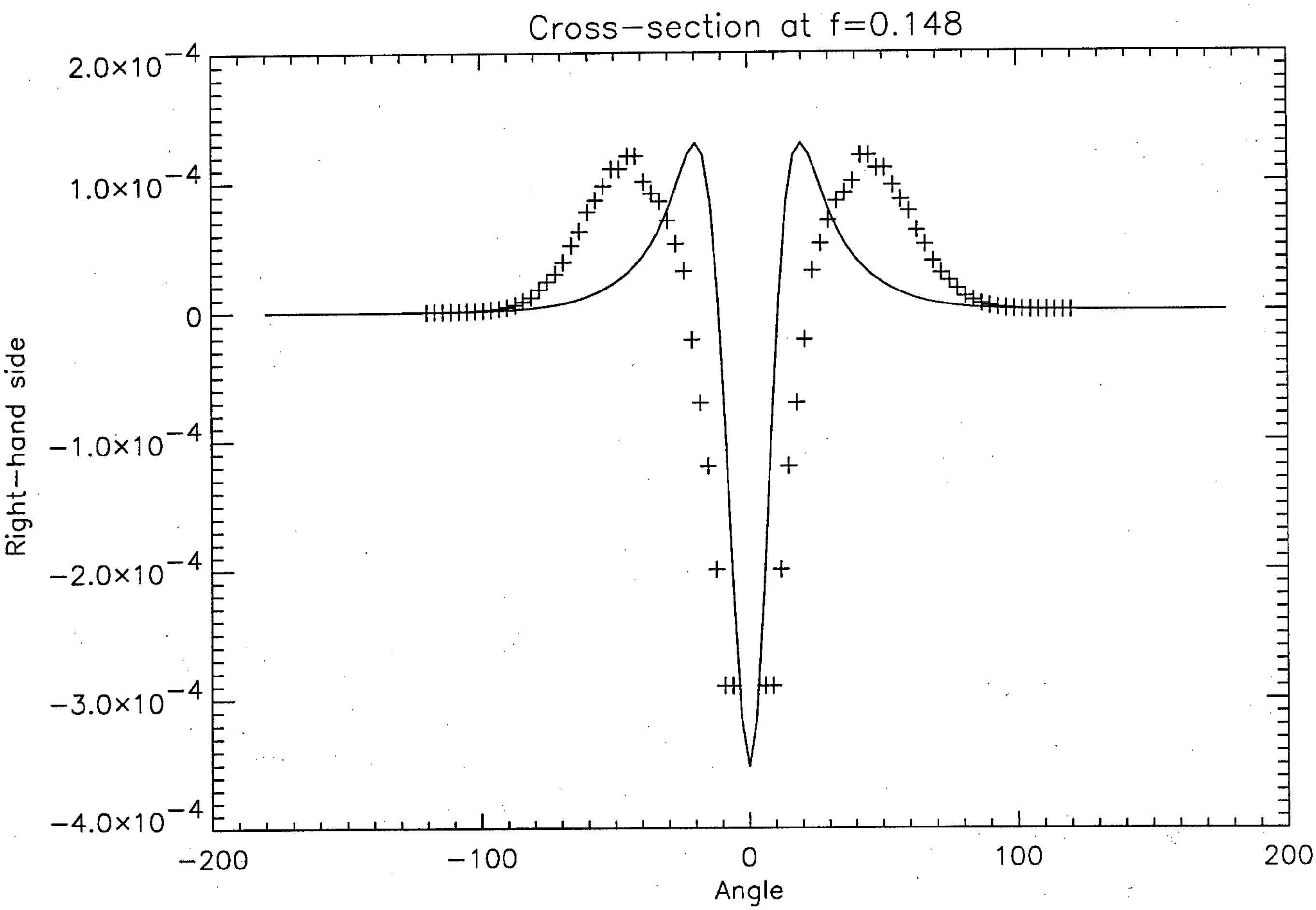

Angle-averaged directional distribution
Right-hand side
0.0006
0.0004
0.0002
0.0000
-0.0002
-0.0004
-0.0006
0.0
0.2
0.4
0.6
0.8
1.0
f

Fig. 1



Cross-section at f=0.148
2.0×10^{-4}
1.0×10^{-4}
0
−1.0×10^{-4}
−2.0×10^{-4}
−3.0×10^{-4}
−4.0×10^{-4}
Right-hand side
−200
−100
0
100
200
Angle

Fig. 2