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Ground-Based Intercept of a Ballistic Missile  
Exoatmospheric Kill Vehicle Design and Simulation Considerations

By

SCOTT LEONARD KLEMPNER

B.S., United States Air Force Academy, 1998

A creative investigation submitted to the Graduate Faculty of  
the University of Colorado at Colorado Springs  
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Klempner, Scott Leonard (ME, Space Operations)

Ground-Based Intercept of a Ballistic Missile:

Exoatmospheric Kill Vehicle Design and Simulation

Considerations

Creative Investigation directed by Dr. Don Caughlin

This paper details the simulation process followed to design a model of the Kinetic Kill Vehicle portion of a Ballistic Missile Defense simulation. In order to have full knowledge of the system specifications, we went through the design process to find vehicle's propulsion characteristics, center of gravity, and moments of inertia. Using the simulation modeling goals as a guide, the system was then abstracted into a set of desired behaviors. These include ballistic motion, rotational motion, propulsion, vehicle control, communication with the ground, and operation of an on-board sensor. Finally, the paper briefly defines the governing equations used to implement the model in the simulation. Overall, when the presented approach was followed, the simulation process provided a clear progression between steps and a robust method for system modeling.

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## I. INTRODUCTION

The scope of this creative investigation is two-fold. The primary focus of the project was to learn and implement the system simulation process as part of a managed team. This included both taking responsibility for a system component and the role of software engineer. However, due to the nature of the project, a secondary focus of this investigation became the design of the component with particular focus on designing and testing the vehicle autonomous control system. The design problem, while not the point of the investigation, was still integral to the simulation process.

This paper presents the Exoatmospheric Kinetic Kill Vehicle (EKV or KV) model, which implies a number of major sub-tasks within the simulation process framework. First, the design of the vehicle must be fully specified and understood before any simulation can take place. Although it currently exists as the Theater High Altitude Area Defense system, access to the Kill Vehicle design (or system designers) was not given to the simulation team as it would have been were this a real-world simulation contract. Where information does not exist, it must be supplied through analysis and design. Second, the team needed to decide which behaviors would be included in the

Kill Vehicle model and what level of abstraction the model would include. Third, using physical principles, the system behaviors needed to be mathematically or logically described. Fourth, the conceptual model needed to be translated into code following the system norms established in the simulation architecture.

The Kill Vehicle model is a product of topical research, a non-rigorous application of the design process, applying appropriate governing equations to the design variables, and a formal application of the simulation process. Although the results are preliminary they are the groundwork for a solid piece of the team effort.

## II. UNDERSTANDING THE SYSTEM

In order to simulate the Exoatmospheric Kinetic Kill Vehicle, as with the other component systems, we must fully understand the system design before proceeding further into the simulation process. The Kill Vehicle is patterned after the existing design of the Theater High Altitude Area Defense system's intercept missile. Therefore, some of the design parameters are fixed while others were left for analysis because system documentation is not accessible to the public. Also, for the sake of simulation simplicity, some aspects of the system were not fully designed and instead assumed or picked based on typical or other existing systems.

The baseline system is a single ground-based interceptor missile. Nominally its initial mass is approximately 650 kg and is 6.2 m (20.3 ft) long<sup>1</sup>. The missile has a single stage booster that uses solid propellant and a Divert and Attitude Control System (DACS) that uses both liquid bi-propellant and cold-gas thrusters. Four liquid propellant divert thrusters make precision trajectory corrections while six cold-gas attitude control thrusters position the vehicle so it can view the target<sup>2</sup>. The vehicle is launched by the Battle Manager C<sup>4</sup>I system and

receives target updates from the Battle Manager/on-board radar combination along a dedicated 20/44 GHz communication link<sup>3</sup>. The missile has an on-board IR seeker head which also produces target updates. Finally, the system has inertial measurement units that provide attitude information and solid-state accelerometers to determine vehicle position and velocity<sup>4</sup>.

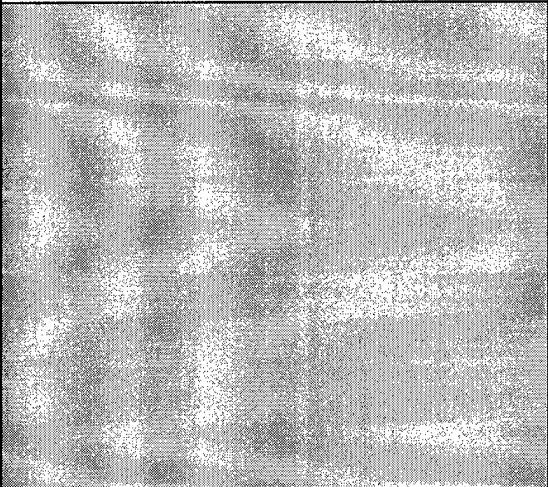
For academic purposes, the basic Kill Vehicle configuration was altered slightly. First, the solid-state accelerometers were replaced by an on-board Global Positioning System (GPS) receiver. Additionally, the IR seeker head was replaced by a radar seeker head similar to the one found in the Air Intercept Missile-120 Advanced Medium Range Air-to-Air Missile (AIM-120 AMRAAM). Because the Lockheed team is having difficulties getting the IR seeker head to correctly update the Kill Vehicle, we decided to redesign the system with a radar seeker and evaluate its performance. Clearly the problems with the IR seeker have been hardware-related, and unless we are going to model hardware faults of a radar seeker, this does not make the simulation a decision-making tool for comparing radar seeker performance to IR seeker performance<sup>5</sup>. However, if we assume that this simulation is a preliminary product and that we have created capability to expand the

radar seeker model to include hardware failure issues at an unspecified later date, then this decision becomes justified.

## SYSTEM DESIGN DECISIONS

As was stated earlier, the remainder of the missile design parameters are unspecified. Therefore, we needed to go through a design process to determine the rest of the system characteristics. However, it must be clear that this was not a formal design process. For example, in researching and analyzing the propulsion system, analysis of assumed starting parameters produced a baseline thrust level. The formal design process would begin with range and energy requirements, which derive a  $\Delta V$  requirement, which drives a thrust/impulse requirement, which leads to trade studies of various fuels and types, thrust chamber temperatures, pressures, structural data, and nozzle geometries. Instead, our goal was to estimate as best we could the unspecified parameters so that we could characterize and understand the system we need to simulate. Table 1 shows a summary of the Kill Vehicle's parameters before any design decisions were made. Greyed-out boxes indicated information that was not available.

Table 1. Preliminary Design Information

| Configuration Item                         | Value                                                                                                                 |
|--------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| Mass                                       | 650 kg                                                                                                                |
| Length                                     | 6.2 m                                                                                                                 |
| Thrusters and propellant                   | 1 main rocket engine, solid<br>4 course correction, liquid<br>bi-propellant<br>6 attitude control, cold<br>gas        |
| Seeker head                                | radar                                                                                                                 |
| navigation and guidance                    | inertial measurement units<br>and accelerometers                                                                      |
| trajectory control                         | thrust vectoring about<br>longitudinal axis at rear<br>nozzle, DACS lateral<br>thruster positioning near<br>nose cone |
| vehicle diameter                           |                                    |
| fuel type and quantity                     |                                                                                                                       |
| solid fuel web design                      |                                                                                                                       |
| motor/engine thrust<br>and Isp             |                                                                                                                       |
| booster $\Delta V$ and<br>burnout velocity |                                                                                                                       |
| inertia tensor                             |                                                                                                                       |
| center of gravity<br>location              |                                                                                                                       |
| thrust vector control<br>authority         |                                                                                                                       |
| vehicle control law                        |                                                                                                                       |

A description of the required analysis for each item of interest, including assumptions, follows.

#### Vehicle diameter

The diameter was estimated using a picture of the missile and a CAD program to measure the length and width.

The ratio of length to width is 16.6. With a length of 6.2 meters, the estimated diameter is 0.3735 m.

### **Fuel type, quantity, and shape**

Designing the solid rocket motor is the next logical step because most of the resulting parameters depend on the propulsion system. The general approach is to use typical values for solid rocket motors and scale them down to the size of our vehicle. In the absence of better information, it is the best approach to take.

### **Fuel volume**

Fuel mass and type are major system drivers, but we do not know these, nor can we find them directly without making gross assumptions. However, we can infer them from the vehicle dimension and some careful assumptions.

By using graphical analysis used to find diameter, we can determine maximum fuel volume. The ratio of the thrust chamber length to the total length is 0.578, thus the length of the thrust chamber is  $0.578 * 6.2 \text{ m} = 3.59 \text{ m}$ . The maximum volume of the chamber is  $0.393 \text{ m}^3$ .

We can use an average fuel density and determine the mass of the fuel, but first we need to determine the actual volume of the chamber. The rocket motor includes the motor case, internal insulation, and nozzle, but these can be accounted for by assuming a typical volumetric loading of 0.906. This means that 10% of the volume is taken up with inert mass, and the resulting fuel volume is  $0.90 * 0.3933 \text{ m}^3$  or  $0.3540 \text{ m}^3$ . We need to subtract the volume of the port from this result for a first approximation of the fuel volume (see figure 1).

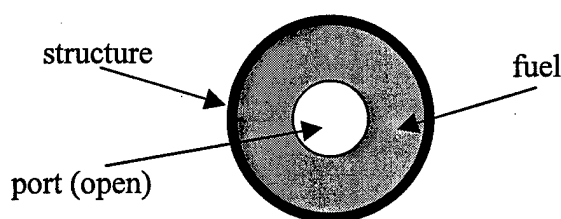


Figure 1.

### Web design

The shape of the solid propellant determines the burn rate and resulting thrust profile. For simplicity, we chose a design that results in theoretically constant thrust such as a star pattern.

The exact volume adjustment of the port depends on its size, so we can use the fact that web ratios (ratio of web

radius to total inner radius) are generally close to 30% to size our star<sup>7</sup>.

A circular port with radius  $r_{\text{port}} = 0.3 r_{\text{inner}}$  would have an area of  $A = \pi(0.05453)^2 = 0.00934 \text{ m}^2$ . A regular pentagram-shaped port has an area given by the following:

$$A = \frac{\frac{5}{4}L^2}{\tan\left(\frac{\pi}{5}\right)} + \frac{\frac{5}{4}L^2}{\tan\left(\frac{\pi}{10}\right)} \quad (1)$$

where:  $L$  = the length of a pentagon side.

The final web design can be visualized in figure 2.

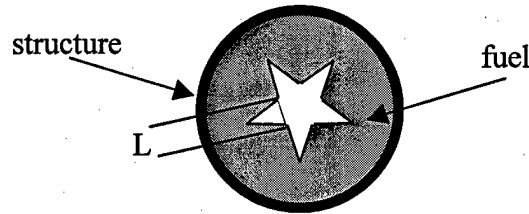


Figure 2.

If we want our star-shaped port to have the same area, we require  $L = 0.0410 \text{ m}$ . The corresponding volume adjustment is given as:

$$\Delta Volume = A_{\text{port}} L_{\text{stage1}} = 0.0395 \text{ m}^3 \quad (2)$$

$$Volume = A_{\text{stage1}} L_{\text{stage1}} \eta_{\text{volumetric}} - \Delta Volume = 0.3145 \text{ m}^3 \quad (3)$$

## Fuel Mass

Assuming a fuel/oxidizer/binder combination is the next step to sizing the thrust. The most common fuel and oxidizer are aluminum and ammonium perchlorate<sup>8</sup>. The oxidizer-to-fuel ratio ranges anywhere from 1 to 12 for all types of fuel, with 4 to 6 being typical for Aluminum<sup>9</sup>. The solid loading ratio, or the ratio of fuel/oxidizer to crystalline binder ranges from 0.84-0.90. We chose 0.85 because the larger commercial solid rocket boosters generally have more efficient grain designs than our small missile would. There are a number of binders we can choose, but Hydroxy-terminated Polybutadiene (HTPB) is by far the most common so it will be our choice. We determine the propellant mass using the following equations and given values<sup>10</sup>:

$$\rho_{AL} = 2700 \text{ kg/m}^3 \quad (4)$$

$$\rho_{NH_4ClO_4} = 1950 \text{ kg/m}^3 \quad (5)$$

$$\rho_{composite} = \sum_{i=1}^n \rho_i \text{ fraction}_i = 0.15 \rho_{AL} + 0.7 \rho_{NH_4ClO_4} \approx 1800 \text{ kg/m}^3 \quad (6)$$

$$\text{mass} = \text{volume} \cdot \rho_{composite} = 566 \text{ kg} \quad (7)$$

This mass is a rough estimate, and it is quite high. The resulting booster mass is 629 kg, leaving the remainder

of the mass, 21 kg, for the Kill Vehicle and its propellant. One possible explanation for this is the fact that we picked too dense a fuel in the design process, which was patterned after space vehicle design.

Regardless, we left this figure as the baseline for the Kill Vehicle mass. Understanding this part of the system is key to timing the boost phase and separation of the boost stage and the Kill Vehicle and before the model can seriously be validated, that is we say it models the real-world system with some degree of accuracy, the assumptions and choices made in analyzing vehicle mass need to be revisited.

### **SRM Propulsion Performance**

We need to know how to characterize the vehicle thrust and also predict the total  $\Delta V$  we can expect out of the KV during boost phase. Thrust is a function of motor efficiency and mass flow rate. We use the equations below to derive expected values for mass flow rate and thrust.

Throat area is defined in terms of chamber pressure and various combustion parameters:

$$A_t = \frac{a \rho_{prop} A_b c^*}{p_c^{(1-n)}} \quad (8)$$

where:  $A_t$  = nozzle throat area (m)  
 $p_c$  = chamber pressure (Pa)  
 $c^*$  = exhaust velocity (m/s)  
 $A_b$  = burn area (surface area of the SRM port) ( $m^2$ )  
 $a$  = regression burn rate coefficient (cm/s/MPa $^n$ )  
 $n$  = regression burn rate exponent

With the exception of chamber pressure and burn area, which we picked, these physical variables are empirically determined for each fuel/oxidizer combination. The following values correspond to 18% Al, 71% AP, and 11% HTPB, a mixture very similar to the one we are using (15% Al / 70% AP / 15% HTPB)<sup>11</sup>. Included are parameters we need to solve the above equations as well as some that will be useful later on:

$$\rho_{\text{prop}} = 1800 \text{ kg/m}^3$$

$$c^* = 1527 \text{ m/s}$$

$$a = 0.399 \text{ cm/s/MPa}^n$$

$$n = 0.3$$

$$T_c \text{ (thrust chamber temperature)} = 3392 \text{ K}$$

$$\gamma \text{ (ratio of specific heats)} \approx 1.2$$

Additionally, we know chamber pressures for this type of fuel range from 2-5 MPa. Chamber pressure is usually left up to the designer, who must consider the trades

involved. Low pressures result in lower thrust and the possibility of subsonic exhaust (not desirable), while high pressures require larger (heavier) structure and a possible drop in  $\Delta V$ . We selected a lower pressure at 2.25 MPa primarily because missiles tend to have lower chamber pressures than space motors. Also, there is no analysis of the structural support characteristics of the motor case to support an extremely high chamber pressure selection, so for safety reasons we used the lower figure.

We must also know the burn area, but this is simple geometry. For the web designed above, the burn area is the linear dimension of the web multiplied by the length of the thrust chamber, or  $2.382 \text{ m}^2$ .

After solving equation (8) above for throat area, we know throat area must be  $0.0148 \text{ m}^2$  or  $\sim 1.5 \text{ cm}^2$ . From here we solve for mass flow and thrust. Mass flow comes from equation (9), below, but we cannot solve for thrust directly. Using equations (10) - (15) we specify the motor's characteristics in more detail until we can solve for thrust in equation (16). We first found expansion ratio, the ratio of the exit area to the nozzle throat area. Next came exit mach number,  $M_{\text{exit}}$ , from which we calculated exit temperature, exit velocity, exit pressure, and finally thrust.

$$\dot{m} = \frac{p_c A_t}{c^*} = \frac{2.25 \times 10^6 \cdot 0.0148}{1529} = 21.8155 \text{ kg/s} \quad (9)$$

$$\varepsilon = \frac{A_e}{A_t} = \frac{\pi \left( \frac{0.3735}{2} \right)^2}{0.01482} = 7.39 \quad (10)$$

$$\varepsilon = \frac{1}{M_{exit}} \sqrt{\left( \frac{2}{\gamma+1} \left( 1 + \frac{\gamma-1}{2} M_{exit}^2 \right) \right)^{\left( \frac{\gamma+1}{\gamma-1} \right)}} \quad (11)$$

Solving numerically,  $M_{exit} = 3.066$ .

$$T_{exit} = \frac{T_c}{1 + \frac{\gamma-1}{2} M_{exit}^2} = 1748.5 \text{ K} \quad (12)$$

$$V_{acoustic, chamber} = c^* \left( \gamma \left( \frac{2}{\gamma+1} \right)^{\left( \frac{\gamma+1}{2\gamma+2} \right)} \right) = 1747.125 \text{ m/s} \quad (13)$$

$$V_e = \sqrt{\frac{2\gamma R T_c}{\gamma-1} \left( 1 - \frac{T_{exit}}{T_c} \right)} = \sqrt{\gamma R T_c} \cdot \sqrt{\frac{2}{\gamma-1} \left( 1 - \frac{T_{exit}}{T_c} \right)} = V_{acoustic, chamber} \sqrt{\frac{2}{\gamma-1} \left( 1 - \frac{T_{exit}}{T_c} \right)} \quad (14)$$

$$V_e = 3845.75 \text{ m/s}$$

$$p_{exit} = \frac{p_c}{\left( 1 + \frac{\gamma-1}{2} \right)^{\left( \frac{\gamma}{\gamma-1} \right)}} = 1.27 \text{ MPa} \quad (15)$$

$$F_{t, vacuum} = \dot{m} V_e + A_e p_e = 223051.5 \text{ N} \quad (16)$$

This vacuum thrust is only valid for this particular burn area. Burn area will fluctuate throughout the motor firing. However, our chosen web shape gives more or less a

constant burn, and we use this assumption to obtain a good estimate of the level of thrust we can expect to see.

We can do a quick analysis of the ideal rocket equation to determine what  $\Delta V$  we can expect (assumed  $I_{sp}$  of 290s). Note the effect of atmospheric drag losses.

$$\Delta V = I_{sp} g_0 \ln \left( \frac{m_i}{m_f} \right) - L_{atm} = 5821 m/s - 500 m/s = 5321 m/s \quad (17)$$

### Inertia tensor

We assumed an axis-symmetric body with 0 products of inertia. If the longitudinal axis is x, then  $I_{yy} = I_{zz}$ . Using a uniformly dense cylinder approximation, the resulting moments of inertia calculation are as follows:

$$m = 650 kg$$

$$r = d/2 = 0.1867 m$$

$$I_{yy} = I_{zz} = \frac{1}{4} m r^2 + \frac{1}{12} m r^2 \quad (18)$$

$$I_{xx} = \frac{1}{2} m r^2 \quad (19)$$

$$I = \begin{bmatrix} 11.34 & 0 & 0 \\ 0 & 7.56 & 0 \\ 0 & 0 & 7.56 \end{bmatrix} kg \cdot m^2 \quad (20)$$

Uniform material density is generally not a valid assumption, but it provides a good initial guess. The

solid rocket fuel can be approximated as a uniform material, and while the motor casing is a higher-density cylindrical shell around the fuel, the difference in mass moments is assumed to be small. However, the Kill Vehicle mass is not uniformly distributed. The degree to which this affects the moment of inertia estimate is bounded by the extreme scenarios where most of the mass is located around the outside or most of the mass is bunched around the rotational axis of symmetry.

#### **Center of gravity location**

We assumed the center of gravity (CG) is along the longitudinal axis of the missile. As fuel is consumed, the CG will move forward along the longitudinal axis. A CG that is further forward is less stable because the moment arm of the thrust vectoring increases, but this is not a parameter under direct control of the system designers. By calculating the first mass moments of the vehicle along the longitudinal axis, we were able to find the initial center of gravity and rate of change in center of gravity in terms of fuel flow rate,  $\dot{m}$ . The reference datum ( $x=0$ ) is the nozzle end of the missile with  $+x$  pointing through the nose.

$$x_{CG} = \frac{x_{CG,stage1} m_{stage1} + x_{CG,KV} m_{KV}}{m_{total}} = \frac{\frac{3.59}{2} * 629 + \left(3.59 + \frac{6.2 - 3.59}{2}\right) * 21}{650} = 1.90m \quad (21)$$

$$\dot{x}_{CG} = \frac{m_{total} (x_{CG,stage1} \dot{m}_{stage1}) - \dot{m}_{total} (x_{CG,stage1} m_{stage1} + x_{CG,KV} m_{KV})}{(m_{total})^2} \quad (22)$$

$$\dot{m}_{total} = \dot{m}_{stage1} \quad (23)$$

### Thrust Vector Authority and Control Law

Designing a control law for the interceptor was one of the straight design problems of this project, and in the end it proved to be one of the simpler ones. Given a desired intercept point, "Proportional Navigation" is an efficient and simple way to have the interceptor vehicle to maintain a collision course with the target. There are two ways of implementing a proportional navigation loop, and while they seem the same, they control different vehicle states. With proportional nav, the interceptor seeks to keep the relative angular position of the target fixed. If there is no apparent relative motion, the interceptor will eventually hit the target. One way of accomplishing this type of control is determine a desired Line of Sight (LOS) angle, defined as the angle between the interceptor's velocity vector and the line of sight vector to the target, and maintain that angle. If the actual LOS angle is

greater than or less than the desired, the control inputs change the angular pointing (Euler angles) of the Kill Vehicle to correct the angle. If the angle is too shallow, then the Kill Vehicle turns away from the target, and if the angle is too large, the Kill Vehicle turns towards it. The other way of accomplishing this is to keep the angular rate of the LOS vector close to zero. This requires a differential measurement over subsequent time steps (rate gyros), and the LOS angular rate is fed back as the error instead of the LOS angle residual.

In either case, the block diagrams are similar. The actual value of the transfer functions depends on with what detail we wanted to model the sensors used. Blakelock adds noise filters to his model<sup>12</sup>, but the two are essentially the same. Figure 3 shows two potential control laws:

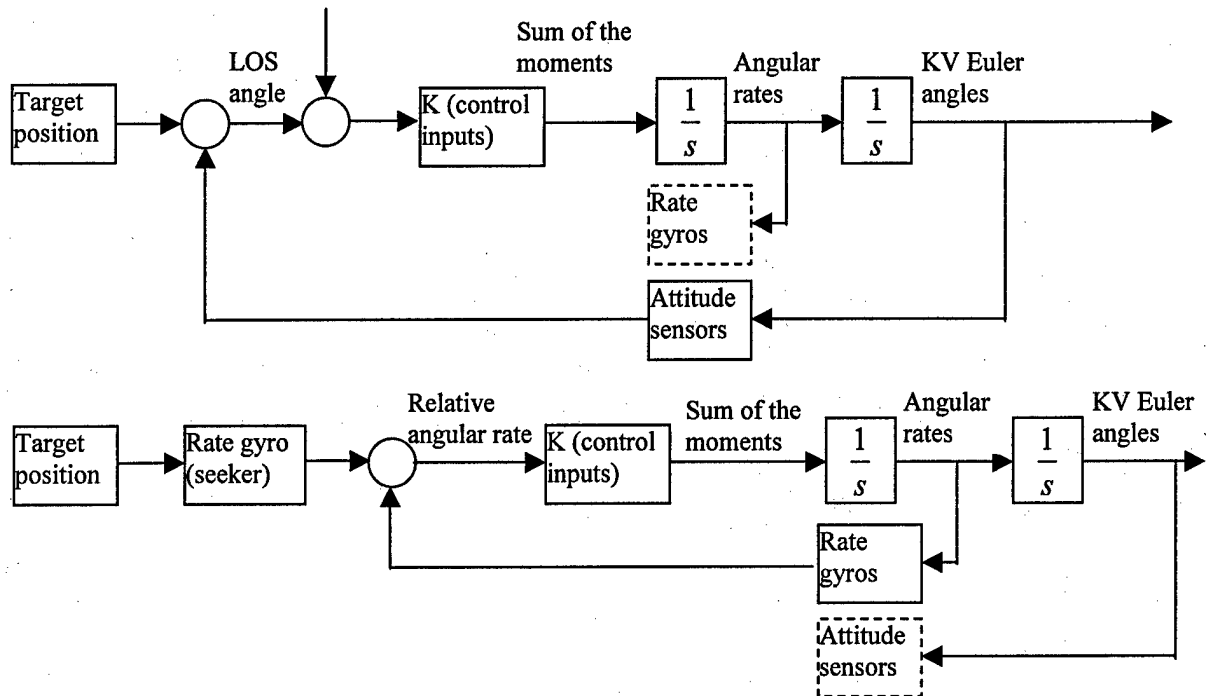


Figure 3.

The values for K will be gains (rather than a function of s), but their value is not yet determined. The control inputs will be in the form of thrust-vectoring gimbal angles for the booster and in lateral thrusting combined with fine attitude control for the Kill Vehicle.

#### DESIGN SUMMARY

The results of the design process are shown below. The greyed out boxes indicate starting values, and the rest of table 1 is now shown filled in as table 2.

Table 2. Post-Analysis Design Information

| Configuration Item                         | Value                                                                                                           |
|--------------------------------------------|-----------------------------------------------------------------------------------------------------------------|
| Mass                                       | 650 kg                                                                                                          |
| Length                                     | 6.2 m                                                                                                           |
| Thrusters and propellant                   | 1 main rocket engine, solid<br>4 course correction, liquid bi-<br>propellant<br>6 attitude control, cold gas    |
| Seeker head                                | Radar                                                                                                           |
| navigation and guidance                    | inertial measurement units and<br>accelerometers                                                                |
| trajectory control                         | thrust vectoring about longitudinal<br>axis at rear nozzle, DACS lateral<br>thruster positioning near nose cone |
| vehicle diameter                           | 0.3735 m                                                                                                        |
| fuel type and quantity                     | Al/AP/HDTP; 556 kg; 629 w/ stage 1                                                                              |
| solid fuel web design                      | star pattern (steady burn)                                                                                      |
| solid rocket motor thrust<br>and $\dot{m}$ | 2.2E5 N, 21 kg/s                                                                                                |
| booster $\Delta V$ and burnout<br>velocity | 5.321 km/s                                                                                                      |
| inertia tensor                             | $I_{xx} = 11.34 \text{ kg-m}^2$<br>$I_{yy}=I_{zz} = 7.56 \text{ kg-m}^2$                                        |
| center of gravity<br>location              | Initial: 1.90m from exhaust nozzle                                                                              |
| vehicle control law                        | Proportional navigation                                                                                         |

### III. SYSTEM ABSTRACTION AND MODEL BUILDING

Once the system was fully described, we started choosing what we wanted to model in the simulation. Identifying and selecting Kill Vehicle behaviors is a major driver of the model's detail. Selecting system behaviors to model is in turn driven by our modeling and simulation goals. Keeping these considerations in focus prevents modeling an unnecessary level of detail.

The following simulation goals apply to modeling the EKV:

- The Kill Vehicle must fly according to the Battle Manager-provided trajectory
- The Kill Vehicle must attempt to acquire the target with its on-board sensor
- The Kill Vehicle must attempt to intercept the target by impacting it with hit-to-kill energy

These goals sublimated into the following system behaviors:

- KV communicates with the Battle Manager
- KV flight
  - movement based on non-equilibrium forces

- KV generates force of thrust
- KV flies through an atmosphere
- KV is subject to gravity
- KV maneuvering
  - KV directs the thrust or fires divert thrusters
  - Thrust creates a moment
- KV autonomous control
  - KV receives information inputs: its own state, target state
  - KV computes line-of-sight angle
  - KV computes control inputs and sends them to the actuators
- KV seeker head
  - KV sends a radar pulse
  - Radar pulse hits the target
  - Radar returns to seeker attenuated by some amount
  - Radar energy which is above the seeker sensor threshold causes the seeker to register a return
  - Seeker interprets the energy and tries to determine the angle to the origin of the radar return and range

Each of these behaviors is modeled in the simulation. For each modeled behavior, we selected a level of detail and fidelity appropriate to the entire simulation. For example, after the initial thrust calculation, we used a thrust table to generate thrust rather than simulate the thrust chamber, which is possible with computational fluid dynamics principles, but not necessary for this exercise.

Once our simulation behaviors were defined, we translated behaviors into models. For a physical system, this is researching and applying basic principles and equations of motions. It is key to ensure that our equations and logic are sufficient to describe the system at the selected level of fidelity and detail. Each major subdivision of system behavior has a sub-model as follows:

- Ballistic Motion
- Rotational Motion
- Thrusting
- Radar Seeker
- Controller
- Communications

From a modeling standpoint, each set of behaviors is driven by governing equations, which are summarized below. Further explanation past this point would require

presentation of software-specific models, which are contained entirely in Simulink.

#### Ballistic Modeling

$$\begin{aligned}\sum \bar{F}|_{inertial} &= m\ddot{\bar{x}}|_{inertial} \\ \sum \bar{F}|_{local} &= m\ddot{\bar{x}}|_{local} + \bar{\omega}_{frame} \times \bar{r}_{frameorigin}|_{inertial} \\ \sum \bar{F} &= \bar{F}_{gravity} + \bar{F}_{drag} + \bar{F}_{thrust}\end{aligned}$$

#### Rotational Modeling

$$\begin{aligned}\sum \bar{M}|_{inertial} &= I\dot{\bar{\omega}}|_{inertial} \\ \sum \bar{M}|_{local} &= I\dot{\bar{\omega}}|_{local} + \bar{\omega}_{frame} \times I\dot{\bar{\omega}}|_{inertial} \\ \sum \bar{M} &= \bar{M}_{thrusting}\end{aligned}$$

#### Thrusting

$$\begin{aligned}\bar{F}_{thrust} &= 2.2 \times 10^5 N \\ \dot{m} &= \frac{-F_{thrust}}{I_{sp} g_0}\end{aligned}$$

#### Radar Seeker

The governing equations for the radar seeker are the same as for the ground radar component of the simulation.

#### Controller

$$\Delta\theta_{gimbal} = -K(\alpha_{LOS} - \alpha_{LOS,desired})$$

## Communications

Communications between the Kill Vehicle and Battle Manager are assumed instantaneous. Uncorrupted information is available to the Kill Vehicle as soon as the Battle Manager makes it available.

## V. CONCLUSION

The model building process requires, above all, a comprehensive understanding of the system needing to be modeled. Without fully specifying all design variables needed to be modeled, it is impossible to do any simulation at all. For a normal simulation project, the system is already designed or in the process of being designed, but for this project the system design was not accessible. This required detailed analysis before any steps of the simulation process could begin.

The simulation process itself is fairly straightforward if the steps are followed. As with a design problem, the simulation has a set of requirements for what it needs to show. The selected system behaviors must map to at least one requirement, or else the simulation will include either unnecessary or insufficient levels of detail. This becomes the plan of action for when abstraction takes place. Abstraction requires a thorough understanding of the system governing principles, whether they be physical laws of motion or some other principles. The mathematical model must also match the level of fidelity and detail selected for the simulation. Coding the model can be straightforward, provided the simulation

architecture is carefully-planned out. Overall, solidly following the process will result in a solid model.

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