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- COMMUNIST CHINA -

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FERMI'S UNIVERSAL THEORY OF INTERACTION

AND

NUCLEAR CAPTURE OF MINUS MU-MESON*

-COMMUNIST CHINA-

[Following is a translation of an article by Chou Kuang-chou (0719 0342 0664) and V. Mayevsky, Moscow Combined Nuclear Research Institute, in Wu-li Hsueh-pao, Volume 15, Number 7, July 1959, pp. 377-388.]

1. Preface

The V-A weak coupling interaction -- pointed out by Feynman and Gell-Mann, and Sudarshan and Marshak -- has been verified by Beta-decay experiments. In our work, we (the authors) feel that all weak coupling Hamiltonians have the same form; therefore, the (Fermi) Universal Theory of Interaction is presented.

Some particles -- such as nucleon -- interact rather strongly with π meson and K meson; other particles -- such as mu-meson -- do not manifest this type of interaction. Due to the strong effects of this interaction the physical nuclei and the mu-meson will have different kinds of decay. Although the form of the Hamiltonian equation is the same in both instances, the weak coupling constant might be different. The change of the coupling constant due to these strong interactions is termed the regulated effect.

Let the Hamiltonian density of the mu-meson capture by the nucleus be $H(x) = J_\alpha \bar{\psi}_\nu \gamma_\alpha (1+\gamma_5)\psi_\mu,$ (1)

where $J_\alpha = \frac{g_0}{\sqrt{2}} \bar{\psi}_n \gamma_\alpha (1+\gamma_5)\psi_p$

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is the electric current vector of the weak nuclear interaction; g_0 is the unregulated coupling constant; the indices of the lower right-hand corner of the field value ψ indicate the different particles with μ, v, p, n; γ_a is the Dirac representation. In Eq. (1), the Hamiltonian density is balanced with a.

Let the initial state of the atomic nucleus be $|i\rangle$, and let the final state be $|f\rangle$. The μ -meson starts at the lowest Bohr orbit (disregard the change of the μ meson wave function $\varphi(x)$ on the atomic nucleus), from which we can formulate the μ meson capture by the atomic nucleus due to the shifting of orbits as shown below [see Note]:

$$T_{fi}=\int\langle f|J_a(x)|i\rangle\frac{1}{(2\pi)^3}\bar{u}_\nu(k)\gamma_a(1+\gamma_5)u_\mu\varphi(0)\cdot e^{-i(k-p_\mu)\cdot x}d^4x, \qquad (2)$$

where u_μ is the μ meson's Dirac spin moment, its relative momentum $\underline{p}_\mu=0$, energy $p_{\mu 0}=m_\mu$ (m_μ is the mass of the meson at rest); $\bar{u}_\nu(k)$ is the Dirac spin value of the neutrino, and its kinetic energy vector is k. Eq. (2) is derived from the first order process of Eq. (1), with respect to the weak interaction. In order to integrate all the results of the strong interactions in the calculation, the $J_a(x)$ in Eq. (2) is from Heisenberg's expression (for strong interactions), and $|i\rangle$ and $|f\rangle$ are the state vectors of the atomic nucleus (including the meson cloud interaction).

([Note] From now on, we use the unit system $\hbar=c=1$, a•b $\underline{a}\cdot\underline{b}-a_0b_0$, $\underline{a}$ and a_0, respectively, are the space and time components of the four-dimensional vector a.).

Based on the property of the strong interaction, there is no effect on the moving group; therefore, we have

$$J_a(x)=e^{-i\hat{p}\cdot x}J_a(0)e^{i\hat{p}\cdot x}, \qquad (3)$$

where $\hat{p}$ is the kinetic energy of the atomic nucleus. The initial state and final state of $\hat{p}$ and p_i and p_f, respectively. Substituting Eq. (3) into Eq. (2), we obtain

$$T_{fi}=(2\pi)\delta^4(p_f+k-p_i-p_\mu)\cdot\langle f|J_a(0)|i\rangle\bar{u}_\nu(k)\gamma_a(1+\gamma_5)u_\mu\varphi(0). \qquad (4)$$

Let us first examine a situation concerning μ^- meson capture by the proton. The initial state is the physical proton, and the final state is the physical neutron. According to the invariance of the theory of relativity, $\langle f|J_a|i\rangle$, should be a four-dimensional vector, and its most common form is

$$(2\pi)^3\langle n|J_a|p\rangle=\frac{g}{\sqrt{2}}\bar{u}(p_n)\Big\{\gamma_a+\lambda\gamma_a\gamma_5-i\frac{\mu}{4M}[\gamma_a(\hat{p}_n-\hat{p}_p)-(\hat{p}_n-\hat{p}_p)\gamma_a]+$$
$$+c(p_n-p_p)_a+d[\gamma_a(\hat{p}_n-\hat{p}_p)-(\hat{p}_n-\hat{p}_p)\gamma_a]\gamma_5+\frac{f}{im_\mu}(p_n-p_p)_a\gamma_5\Big\}u(p_p). \qquad (5)$$

where $u(p)$ is the Dirac spin momentum, p_p and p_n are the kinetic energy vectors of the initial state proton and the final state neutron; M is the mass of the nucleus; g, λ, μ, c, d and f are the quantities of the function

$$(p_n-p_p)^2=(\mathbf{p}_n-\mathbf{p}_p)^2-(p_{n0}-p_{p0})^2 \; ; \; \hat{a}=\mathbf{a}\cdot\gamma+ia_0\gamma_4.$$

From the current density of the original Hamiltonian $J_a(x)$, there are the following symmetrical properties: let p_c and n_c represent the charge conjugates of the proton and neutron, respectively (or their their anti-particles). After exchange $p \to n_c, n \to p_c, x \to -x$, the vector of the weak interaction current $\bar{\psi}_n(x)\gamma_a\psi_p(x)$ changes sign, but the pseudo-vector weak interaction current $\bar{\psi}_n(x)\gamma_a\gamma_5\psi_p(x)$ does not change. So, in $\langle n|J_a(x)|p\rangle$, the same properties should be present. This is due to the relationship between the strong interaction and the above exchange. From this we can prove that c=d=0.

If the strong interaction is not present, from Eq.(1) and Eq. (2), we should get

$$(2\pi)^3\langle n|J_a(0)|p\rangle=\frac{g_0}{\sqrt{2}}\bar{u}(p_n)\gamma_a(1+\gamma_5)u(p_p). \qquad (6)$$

In other words, μ and f should equal zero, and $g=g_0$, $\lambda=1$. Generally speaking, the strong interaction not only changes the value of g_0, it could also add new terms of current.

If we consider the presence of a strong interaction, then it should not be too surprising if we notice the existence of a very large anomalous magnetic moment in the nucleus. During the electro-magnetic interaction, we have not only the normal charge-induced current, but also a current induced by the anomalous magnetic moment. Similar phenomena appear in the decay problem too, Since the electro-magnetic interaction current is a vector quantity, many properties of the current vector in the weak interaction are similar to the electro-magnetic interaction current. We can designate g as the Beta-charge, and μ as the Beta-moment (anomalous).

From the law of constant current, the existence of a strong interaction in the electro-magnetic interaction does not effect the charge of the atomic proton and the positron. Likewise, the charge of the other particles not possessing a strong interaction -- such as electrons -- remains unchanged with the strong interactions. The strong interaction only changes the magnetic moment of the physical nucleus. Likewise, in the weak interaction, Gell-Mann and Feynman discovered from decay of the μ meson that they could theorize that the coupling constant and the coupling constant of the Beta-decay vector were very close to each other. This states that the strong interaction has little effect on the coupling constant of the vector. For the explanation of this phenomenon, F-G suggested the hypothesis that the current vector in the weak interaction is constant. Under this assumption, the current vector in the weak interaction and the current vector in the electro-magnetic interaction have completely similar properties. They are dissimilar components of similar vectors, occupying the same orbit. So, when $(p_n-p_p)^2=0$, Beta-charge g lacks strong interaction effect $g=g_0$; moreover, μ should equal the difference of the anomalous magnetic moment between the proton and the neutron, $\mu=3.7$. In addition, g and μ are a function of $(p_n-p_p)^2$, and the same as the charge of the nucleus and the magnetic moment distribution function.

We have calculated the effect of Beta-magnetic moment on the process of capture by the proton. The result has shown that if the Gell-Mann and Feynman hypothesis is accurate, then the Beta-magnetic moment effect should be approximately 20%. Due to the low probability of μ^- capture by the proton, we have already mentioned the effects of the Beta-magnetic moment on μ meson capture in the nucleus, so we can predict that this effect would not be too small. The purpose of this paper is to complete this problem. In order to arrive at the most general result, in this paper we will consider not only the effect of the Beta-magnetic moment, but also, we will consider the effect on the f term.

In section two, we shall approach the general formulae of capture probability, the angular relationships between neutrons and the polarized μ, and polarized neutrons. In section three, we shall take up the formulae for the probability of the μ^- capture by the proton, angular relationships, and the polarized neutrons. In section four, we shall discuss the μ meson capture by the Radon nucleus. In section five, we shall discuss the μ meson capture by the atomic nucleus.

2. General Formulae for Mu Minus Meson Capture by Nuclei

If we represent the weak nuclear interaction current in the nuclear self-spin space, we obtain

$$(2\pi)^3\langle n|J_\alpha|p\rangle\bar{u}_\nu\gamma_\alpha(1+\gamma_5)u_\mu=A+\mathbf{B}\cdot\sigma, \qquad (7)$$

where σ represents the nuclear self-spin:

$$A=\frac{g}{\sqrt{2}}\left(\bar{u}_\nu\beta(1+\gamma_5)u_\mu+\frac{\mathbf{k}}{2M}\cdot\bar{u}_\nu\beta\rho_1\sigma(1+\gamma_5)u_\mu\right),$$

$$\mathbf{B}=\frac{g}{\sqrt{2}}\left[\lambda\bar{u}_\nu\beta\rho_1\sigma(1+\gamma_5)u_\mu+\frac{\mu+1}{2M}i\mathbf{k}\Lambda\bar{u}_\nu\beta\rho_1\sigma(1+\gamma_5)u_\mu-\right.$$
$$\left.-\frac{f-\lambda}{2M}\mathbf{k}\bar{u}_\nu(1-\gamma_5)u_\mu\right]. \qquad (8)$$

In obtaining Eq. (8), disregard all the square terms of the nuclear velocity, and use the conditions $\underline{P}_\mu=0$, $\underline{P}_n-\underline{P}_p=-P_\nu=-\underline{k}$. For the sake of convenience, let μ' represent $\mu+1$, and let $f'=f-\lambda$.

In representing the μ meson and neutrino self-spin space, Eq. (8) can be expressed in the following form:

$$A=\frac{g}{2}\left(1+\frac{k}{2M}\right)(1-\sigma_\mu\cdot\mathbf{n}),$$
$$\mathbf{B}=\frac{-g}{2}(1-\sigma_\mu\cdot\mathbf{n})[\lambda\sigma_\mu+i\frac{k}{2M}\mu'\mathbf{n}\Lambda\sigma_\mu+\frac{f'}{2M}k\mathbf{n}], \qquad (9)$$

where $\underline{n}$ is the unit vector parallel with the neutrino momentum $\underline{k}$. In order to distinguish the nuclear self-spin, in Eq. (9), σ_μ is used to represent the self-spin of the μ meson and neutrino. From Eq. (9), we see that A and $\underline{B}$ are the self-spin spaces of the μ meson and neutrino, respectively. They are independent of the nuclear self-spin and co-ordinates.

So far we have only considered μ^- meson capture by the proton. When we treat the nuclear structure of the atom and assume as only nominal the effects of other nuclei on the meson cloud of the physical nucleus, then the μ^- meson capture by the atomic nucleus is proportional to the following formula:

$$R=\int\psi_f^*\sum_j\tau^{(-)}(j)(A+\mathbf{B}\cdot\sigma_j)e^{-i\mathbf{k}\cdot\mathbf{r}_j}\psi_i dV, \qquad (10)$$

where $\tau^{(-)}(j)$ is the isotopic spin of a neutron which is changed from a proton. If the j^{th} nucleon is a proton, then it changes to a neutron after the $\tau^{(-)}(j)$ interaction. If it is a neutron, then $\tau^{(-)}(j)$ would produce a result of zero. ψ_i and ψ_f are the wave functions of the initial state and the final state, respectively, of the atomic nucleus. In general, neutrons have enough energy to escape from the atomic nucleus. So the final state wave function includes the escaped neutrons and residual nuclear wave functions. Since we have partitioned out the central mass moment in Eq. (4), ψ_f consists only of the relative co-ordinates of the neutrons and the residual nucleus. Therefore, it depends only on the magnitude of the relative momentum $\underline{P}=2\underline{P}_n+\underline{k}$.

Considering the property of anti-symmetry of the wave function, Eq. (10) can be written in the following form:

$$R = Aa + \mathbf{B}\cdot\mathbf{b}, \tag{11}$$

where

$$a = N\int \psi_f^* \tau^{(-)}(1) e^{-i\mathbf{k}\cdot\mathbf{r}_1} \psi_i dV,$$
$$\mathbf{b} = N\int \psi_f^* \tau^{(-)}(1) \sigma_1 e^{-i\mathbf{k}\cdot\mathbf{r}_1} \psi_i dV, \tag{12}$$

where N is the number of the nuclei in the atomic nucleus. Let σ_1, be the self-spin value of the escaped neutrons.

Using Eq. (11), we can write Eq. (4)

$$T_{fi} = (2\pi)^{-3}\delta^4(p_f + k - p_i - p_n) R\varphi(0). \tag{13}$$

Notice that $\varphi(x)$ is the wave function of the μ meson on the lowest Bohr orbit; therefore,

$$|\varphi(0)|^2 = \frac{Z^3}{\pi a_0^3}. \tag{14}$$

where a_0 is the Bohr radius of the hydrogen atom, Z is the charge of the atomic nucleus. From Eq. (13) and Eq. (14), it is easy to arrive at the probability and angular relationships of the μ^- meson capture by the atomic nucleus. If the atomic nucleus is in the initial and non-polarized state, we obtain

$$dw = (2\pi)^{-3}\frac{Z^3}{\pi a_0^3}\frac{1}{2(2J+1)}\mathrm{Sp}R(1+\sigma_\mu\cdot\mathbf{P}_\mu)R^+ dp_n d\mathbf{k}/dE, \tag{15}$$

where $\underline{P}_\mu$ is the polarized vector of the μ^- meson at the initial state; J is the sum of the angular momenta of the atomic nucleus at the initial state; Sp is the average magnetic exponent between the initial and final states.

We can express the polarized vector $\langle\sigma_1\rangle$ of the neutron in the following formula:

$$[\mathrm{Sp}R(1+\sigma_\mu\cdot\mathbf{P}_\mu)R^+]\langle\sigma_1\rangle = \mathrm{Sp}R\sigma_1(1+\sigma_\mu\cdot\mathbf{P}_\mu)R^+. \tag{16}$$

The next problem is to find Sp. We average the magnetic exponent between the initial and final states, based on the property of invariance of rotation and reflection space,

$$\frac{1}{2J+1}\mathrm{Sp}\, aa^+ = F_1(E_N),$$
$$\frac{1}{2J+1}\mathrm{Sp}\, a\mathbf{b}^+ = \frac{1}{2J+1}\mathrm{Sp}\, \mathbf{b}a^+ = 0,$$
$$\frac{1}{2J+1}\mathrm{Sp}\, \mathbf{b}\mathbf{b}^+ = F_2(E_N)\,\mathbf{I} + F_3(E_N)\mathbf{p}\mathbf{p}. \tag{17}$$

In Eq. (17), the second form should be zero, since Spab$^+$ should be a pseudo-vector. However, one vector $\underline{P}$ (the relative momentum of the neutron and the residual nucleus) cannot form a pseudo-vector, so it can only be zero. F_1, F_2, and F_3, are functions of the neutron energy E_N. They have different forms because of the different properties of their atomic nuclei.

In the same way , we obtain

$$\left|\begin{array}{l} \frac{1}{2J+1}\mathrm{Sp}\, a\sigma_1 a^+ = 0, \\ \frac{1}{2J+1}\mathrm{Sp}\, a\sigma_1 \mathbf{b}^+ = F_4(E_n)\mathbf{I} + F_5(E_n)\mathbf{PP}, \\ \frac{1}{2J+1}\mathrm{Sp}\, \mathbf{b}\cdot\mathbf{B}\sigma_1\mathbf{b}^+\cdot\mathbf{B}^+ = iF_6(E_N)\mathbf{B}\times\mathbf{B}^+ + iF_7(E_N)\mathbf{PP}\cdot(\mathbf{B}\times\mathbf{B}^+), \end{array}\right. \qquad (18)$$

where F_4, F_5, F_6, and F_7, are also functions related to the properties of atomic nuclei. From Eq. (15) and Eq. (17), we see that we must find the following forms,

$$\left|\begin{array}{l} (1)\ \mathrm{Sp}\, A(1+\sigma_\mu\cdot\mathbf{P}_\mu)A^+, \\ (2)\ \mathrm{Sp}\, \mathbf{B}\cdot(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+, \\ (3)\ \mathrm{Sp}\, \mathbf{B}\cdot\mathbf{P}(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}\cdot\mathbf{p}^+; \end{array}\right. \qquad (19)$$

Also, we have to find the following -- for the polarization of the neutron

$$\left|\begin{array}{l} (4)\ \mathrm{Sp}\, A(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+, \\ (5)\ \mathrm{Sp}\, A(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}\cdot\mathbf{p}^+, \\ (6)\ \mathrm{Sp}\, \mathbf{B}\times(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+, \end{array}\right. \qquad (20)$$

All these forms are presented in Appendix I.

Substituting the results obtained from Appendix I into Eq. (17), Eq. (18), and Eq. (15), we obtain the probability and the angular relationships of the μ^- meson capture by atomic nucleus. From Eq. (16), we can get the polarization of the nuclear neutron.

3. Minus Mu Meson Capture by the Proton

When we consider the μ^- capture by the proton, we obtain especially simple results. Here, ψ_i and ψ_f consist of only the self-spin wave function of the initial state of the proton and the final state of the neutron. From the nuclear self-spin expression, it is a simple matter to get

$$\tilde{a} = 1, \quad \mathbf{b} = \sigma_1, \qquad (21)$$

From Eq. (21), we immediately arrive at the F_1 function of Eq. (17) and Eq. (18):

$$\left|\begin{array}{l} F_1 = F_3 = 1, \quad F_2 = 0, \\ F_4 = F_6 = 1, \quad F_5 = F_7 = 0, \end{array}\right. \qquad (22)$$

In Eq. (15), we still must make a minor correction -- i.e., when the μ^- meson is captured by the proton, the final state consists only of a neutron and a neutrino (no residual nuclei remain), and its energy satisfies the relationship

$$\mathbf{p}_n = -\mathbf{k}. \tag{23}$$

When we calculate the state number of the final state, we should not re-integrate into it the term $d\underline{P}_n/(2\pi)^3$. The probability formula which should be used is,

$$dw = (2\pi)^{-3}\frac{1}{\pi a_0^3}\frac{1}{4}\mathrm{Sp}R(1+\sigma_\mu\cdot\mathbf{P}_\mu)R^+ dk/dE. \tag{24}$$

From Eq. (17), Eq. (18), Eq. (24), and Appendix I, we can get the probability of the μ^- meson capture by the proton.

$$dw = (2\pi)^{-3}(\pi a_0^3)^{-1}2^{-1}g^2 I[1-a\mathbf{P}_\mu\cdot\mathbf{n}]k^2 d\Omega, \tag{25}$$

where

$$I = 1 + 3\lambda^2 + \beta(1-\lambda f') + \beta\mu'(2\lambda + \beta\mu'/2) + \frac{\beta^2}{4}f'^2, \tag{26}$$

$$Ia = 1 - \lambda^2 + \beta((1-\lambda f') - \beta\mu'(2\lambda + \beta\mu'/2) + \frac{\beta^2}{4}f'^2, \tag{27}$$

and, therein,

$$\beta = \frac{k}{M}. \tag{28}$$

If, in Eq. (25), Eq. (26), and Eq. (27), we let f' equal f-λ, and disregard all terms which relate to f, we will get the results in our earlier work. In this paper we consider not only the effects of the Beta-magnetic moment, but also the effects of the pseudo-scalar coupling due to the regulated effect. From Eq. (26) and Eq. (27), we can see that if f' and μ' are positive, then the effect would be to increase the co-efficient of angular relationships, and to decrease the probability of capture. From the present experimental materials, we know the value of I is greater than $1+3\lambda^2$ (approximately 0.4-1 times greater). -- [See Note]. This shows that f' is either quite small, or it might even be negative. This does not match the estimate of f' in Eq. (2) by using the spectrum. The estimate of f' by using spectral analysis is a very large positive value. Simultaneous experiments to study the probability of capture and the angular relationships -- i.e., determine the value of I and att the same time -- would help to clear up this problem.

([Note] In the comparison with experiments, we assume the "Fermi" Universal Interaction to be correct, and the coupling constant due to change of energy not large. Then, the values of g and λ are the same as those developed in the

In Eq. (17), the second form should be zero, since $\mathrm{Sp}\,a\underline{b}^+$ should be a pseudo-vector. However, one vector $\underline{P}$ (the relative momentum of the neutron and the residual nucleus) cannot form a pseudo-vector, so it can only be zero. F_1, F_2, and F_3, are functions of the neutron energy E_N. They have different forms because of the different properties of their atomic nuclei.

In the same way, we obtain

$$\begin{aligned} &\frac{1}{2J+1}\mathrm{Sp}\, a\sigma_1 a^+ = 0, \\ &\frac{1}{2J+1}\mathrm{Sp}\, a\sigma_1 \mathbf{b}^+ = F_4(E_n)\mathrm{I} + F_5(E_n)\mathbf{PP}, \\ &\frac{1}{2J+1}\mathrm{Sp}\, \mathbf{b}\cdot\mathbf{B}\sigma_1\mathbf{b}^+\cdot\mathbf{B}^+ = iF_6(E_N)\mathbf{B}\times\mathbf{B}^+ + iF_7(E_N)\mathbf{PP}\cdot(\mathbf{B}\times\mathbf{B}^+), \end{aligned} \qquad (18)$$

where F_4, F_5, F_6, and F_7, are also functions related to the properties of atomic nuclei. From Eq. (15) and Eq. (17), we see that we must find the following forms,

$$\begin{aligned} &(1)\ \mathrm{Sp}\, A(1+\sigma_\mu\cdot\mathbf{P}_\mu)A^+, \\ &(2)\ \mathrm{Sp}\, \mathbf{B}\cdot(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+, \\ &(3)\ \mathrm{Sp}\, \mathbf{B}\cdot\mathbf{P}(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}\cdot\mathbf{p}^+; \end{aligned} \qquad (19)$$

Also, we have to find the following -- for the polarization of the neutron

$$\begin{aligned} &(4)\ \mathrm{Sp}\, A(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+, \\ &(5)\ \mathrm{Sp}\, A(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}\cdot\mathbf{p}^+, \\ &(6)\ \mathrm{Sp}\, \mathbf{B}\times(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+, \end{aligned} \qquad (20)$$

All these forms are presented in Appendix I.

Substituting the results obtained from Appendix I into Eq. (17), Eq. (18), and Eq. (15), we obtain the probability and the angular relationships of the μ^- meson capture by atomic nucleus. From Eq. (16), we can get the polarization of the nuclear neutron.

3. Minus Mu Meson Capture by the Proton

When we consider the μ^- capture by the proton, we obtain especially simple results. Here, ψ_i and ψ_f consist of only the self-spin wave function of the initial state of the proton and the final state of the neutron. From the nuclear self-spin expression, it is a simple matter to get

$$\bar{a} = 1, \quad \mathbf{b} = \sigma_1, \qquad (21)$$

From Eq. (21), we immediately arrive at the F_1 function of Eq. (17) and Eq. (18):

$$\begin{aligned} &F_1 = F_2 = 1, \quad F_3 = 0, \\ &F_4 = F_6 = 1, \quad F_5 = F_7 = 0, \end{aligned} \qquad (22)$$

In Eq. (15), we still must make a minor correction -- i.e., when the μ^- meson is captured by the proton, the final state consists only of a neutron and a neutrino (no residual nuclei remain), and its energy satisfies the relationship

$$\mathbf{p}_n = -\mathbf{k}. \qquad (23)$$

When we calculate the state number of the final state, we should not re-integrate into it the term $d\underline{P}_n/(2\pi)^3$. The probability formula which should be used is,

$$dw = (2\pi)^{-2}\frac{1}{\pi a_0^3}\cdot\frac{1}{4}\mathrm{Sp}R(1+\sigma_\mu\cdot\mathbf{P}_\mu)R^+ dk/dE. \qquad (24)$$

From Eq. (17), Eq. (18), Eq. (24), and Appendix I, we can get the probability of the μ^- meson capture by the proton.

$$dw = (2\pi)^{-2}(\pi a_0^3)^{-1}2^{-1}g^2 I[1-a\mathbf{P}_\mu\cdot\mathbf{n}]k^2 d\Omega, \qquad (25)$$

where

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$$Ia = 1-\lambda^2+\beta((1-\lambda f')-\beta\mu'(2\lambda+\beta\mu'/2)+\frac{\beta^2}{4}f'^2, \qquad (27)$$

and, therein,

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If, in Eq. (25), Eq. (26), and Eq. (27), we let f' equal f-λ, and disregard all terms which relate to f, we will get the results in our earlier work. In this paper we consider not only the effects of the Beta-magnetic moment, but also the effects of the pseudo-scalar coupling due to the regulated effect. From Eq. (26) and Eq. (27), we can see that if f' and μ' are positive, then the effect would be to increase the co-efficient of angular relationships, and to decrease the probability of capture. From the present experimental materials, we know the value of I is greater than $1+3\lambda^2$ (approximately 0.4-1 times greater). -- [See Note]. This shows that f' is either quite small, or it might even be negative. This does not match the estimate of f' in Eq. (2) by using the spectrum. The estimate of f' by using spectral analysis is a very large positive value. Simultaneous experiments to study the probability of capture and the angular relationships -- i.e., determine the value of I and att the same time -- would help to clear up this problem.

([Note] In the comparison with experiments, we assume the Fermi Universal Interaction to be correct, and the coupling constant due to change of energy not large. Then, the values of g and λ are the same as those developed in the

Beta-decay.).

Similarly, in finding the formula for neutron polarization, we get

$$I[1-a\mathbf{P}_\mu\cdot\mathbf{n}]\langle\sigma_1\rangle=[a+b\mathbf{P}_\mu\cdot\mathbf{n}]\mathbf{n}+c\mathbf{P}_\mu, \tag{28}$$

where

$$a=2\left[\lambda(\lambda+1)+\frac{\beta}{2}\lambda+\beta\mu'(\lambda+\beta\mu'/4)-\frac{\beta}{2}f'-\frac{\beta^2}{4}f'\right], \tag{29}$$

$$b=\beta\left[f'(\lambda+1)+\mu'(\lambda+1)+\frac{\beta^2}{2}\mu'(f'+\mu')+\frac{\beta}{2}(f'+\mu')\right]=$$
$$=\beta(f'+\mu')\left(\lambda+1+\frac{\beta}{2}+\frac{\beta^2}{2}\mu'\right), \tag{30}$$

$$c=2\left[\lambda^2-\lambda+\frac{\beta}{2}\mu'(\lambda-1)-\frac{\beta}{2}f'\left(\lambda+\frac{\beta}{2}\mu'\right)-\frac{\beta}{2}\left(\lambda+\frac{\beta}{2}\mu'\right)\right]=$$
$$=2\left[(\lambda-1)\left(\lambda+\frac{\beta}{2}\mu'\right)-\left(\frac{\beta}{2}f'+\frac{\beta}{2}\right)\left(\lambda+\frac{\beta}{2}\mu'\right)\right]=$$
$$=2\left(\lambda+\frac{\beta}{2}\mu'\right)\left(\lambda-1-\frac{\beta}{2}-\frac{\beta}{2}f'\right). \tag{31}$$

Notice in Eq. (28), n represents the neutrino's moving direction. It has direction opposite to that of the neutron. So, the sign for a is not the same here as that in equations developed by Mayevsky and myself in 1958. At that time we used n to represent the neutron's moving direction. Furthermore, Eq. (29) and Eq. (31) are different from our 1958 work. This is the result of different handling techniques for terms of higher powers. Therefore, the formulae we have presented in this paper are accurate up to β^2. This is then more accurate than what we have done previously.

We can see that if f' and μ' have positive values, then their interaction will mutually increase in the co-efficient b, and will partially cancel each other out in a and c.

4. Mu Minus Meson Capture by the Radon Nucleus

Before we go into the complicated nucleus, we will first discuss a simple nucleus of the Radon atom. Under these simplified circumstances, we see certain properties of the μ^- meson capture by the nucleus.

In the case of a radon nucleus, since the sum of self-spin is constant, the two neutrons of the final state are either in the self-spin triple states or in the self-spin single state. Let φ_s, φ_t, φ_d, be the final state neutron in the single state, triple states, and the radon's space wave functions, respectively; let X_s, X_t, X_d, be their self-spin wave functions, respectively. We will disregard the tension force (i.e., the D wave in the basic state of the radon nucleus. From Eq. (12), we get

$$a = X_s^* X_d J_s,$$
$$b = X_t^* \sigma_1 X_d J_t + X_t^* \sigma_2 X_d J_s, \qquad (32)$$

where

$$J_s = \int \varphi_s^* e^{-i\mathbf{k}\cdot\mathbf{r}/2} \varphi_d \, dV,$$
$$J_t = \int \varphi_t^* e^{-i\mathbf{k}\cdot\mathbf{r}/2} \varphi_d \, dV, \qquad (33)$$

r is the relative co-ordinates of the nucleus. From the self-spin space, we can show that

$$X_s^* X_d = \frac{1}{4}(3 + \sigma_1 \cdot \sigma_2),$$
$$X_t^* \sigma_1 X_d = \frac{1}{16}(3 + \sigma_1 \cdot \sigma_2)\sigma_1(3 + \sigma_1 \cdot \sigma_2) = \frac{1}{16}(8\sigma_1 + 2\sigma_2 + 4i\sigma_1 \times \sigma_2),$$
$$X_s^* \sigma_1 X_d = \frac{1}{16}(1 - \sigma_1 \cdot \sigma_2)\sigma_1(3 + \sigma_1 \cdot \sigma_2) = \frac{1}{16}(4\sigma_1 - 2\sigma_2 - 3i(\sigma_1 \times \sigma_2)). \qquad (34)$$

Substituting Eq. (32) and Eq. (34) for Eq. (17) and Eq. (18), we get

$$F_1 = |J_s|^2,$$
$$F_2 = \frac{1}{3}(2|J_t|^2 + |J_s|^2) \qquad (35)$$

$$F_3 = 0,$$
$$F_4 = \frac{4}{6}\left(|J_t|^2 + \frac{1}{2} J_s^* J_t\right),$$
$$F_5 = F_7 = 0,$$
$$F_6 = \frac{1}{3}(|J_t|^2 + 2\,\mathrm{Re}\, J_s^* J_t). \qquad (36)$$

Substituting Eq. (35) and Eq. (36) into Eq. (17), Eq. (18), and Eq. (15), observe that

$$E = \frac{P_0^2}{2M} + \frac{(p_0 + k)^2}{2I} + k$$

and

$$M\, dE = k p_0 \sin\theta \, d\theta.$$

After integrating the energy of the neutrino, we get

$$dw=(2\pi)^{-4}(\pi a_D^2)^{-1}\frac{g^2}{2}M^2 I_D(1+a_D\mathbf{P}_\mu\cdot\mathbf{p}_n/p_n)\cdot dE_n d\Omega_n, \tag{37}$$

where

$$I_D=(1+\beta)I_{ii}+\frac{1}{3}\left[3\lambda^2+\beta\lambda(2\mu'-f')+\frac{\beta^2}{2}\left(\mu'^2+\frac{1}{2}f'^2\right)\right][2I_{ii}+I_{ss}], \tag{38}$$

$$a_D=(1+\beta)I'_{ii}-\frac{1}{3}\left[\lambda^2+\beta\lambda(2\mu'+f')+\frac{\beta^2}{2}\left(\mu'^2-\frac{f'^2}{2}\right)\right]\cdot(2I'_{ii}+I'_{ss}), \tag{39}$$

$$I_{ij}=\int_{k_{min}}^{k_{max}} J_i^* J_j kdk,$$
$$I'_{ij}=-\int_{k_{min}}^{k_{max}} J_i^* J_j \frac{\mathbf{k}\cdot\mathbf{p}_n}{kp_n}kdk. \tag{40}$$

Since the values of $\beta=k/M$ and k are related, in Eq. (38), and Eq. (39), we should replace the β with its average value $\bar{\beta}$.

Überall and Wolgenstein disregarded the case of Beta-magnetic moment in the calculation of the μ^- meson capture by the radon nucleus. If we let $\mu=0$, in Eq. (38) and Eq. (39), and disregard the higher order of infinitesimals -- i.e., all terms proportional with β^2 -- then our formulae agree with Überall and Wolgenstein, so we need not repeat them here. Notice that the Beta-magnetic moment can alter the values of I_D and a_D up to approximately 20%. I_{ij} and I'_{ij} have been integrated into the Überall-Wolgenstein formula.

In the same way, we can find the neutron polarization equation

$$I_D(1+a_D\mathbf{P}_\mu\cdot\mathbf{p}_n/p_n)\langle\sigma_1\rangle=a\mathbf{p}_n/p_n+b\mathbf{P}_\mu\cdot\mathbf{p}_n\mathbf{p}_n/p_n^2+c\mathbf{P}_\mu+d\mathbf{P}_\mu\times\mathbf{p}_n/p_n, \tag{41}$$

where

$$a=-\frac{2}{3}\left\{\left(\lambda^2+\beta\mu'\lambda+\frac{\beta^2}{4}\mu'^2\right)(I'_{ii}+2\,\mathrm{Re}\,I'_{is})+\left(1+\frac{\beta}{2}\right)\left(\lambda-\frac{\beta}{2}f'\right)\mathrm{Re}(2I'_{ii}+I'_{ss})\right\} \tag{42}$$

$$b=\frac{1}{3}\beta\left\{(\mu'+f')\left(\lambda+\frac{\beta}{2}\mu'\right)(I''_{ii}+2\,\mathrm{Re}\,I''_{is})+\left(1+\frac{\beta}{2}\right)(\mu'+f')\mathrm{Re}(2I''_{ii}+I''_{ss})\right\} \tag{43}$$

$$c=\frac{2}{3}\left(\lambda+\frac{\beta}{2}\mu'\right)\left\{\left(\lambda-\frac{\beta}{2}f'\right)(I_{ii}+2\,\mathrm{Re}\,I_{is})+\left(1+\frac{\beta}{2}\right)\mathrm{Re}(2I_{ii}+I_{ss})\right\}, \tag{44}$$

$$d=-\frac{2}{3}\left(1+\frac{\beta}{2}\right)\left(\lambda+\frac{\beta}{2}\mu'\right)\mathrm{Im}(2I'_{ii}+I'_{ss}), \tag{45}$$

$$I'_{ij} = \int_{k_{min}}^{k_{max}} J_i J_j \frac{(\theta_R \cdot k)^2}{2k^2} k\,dk, \tag{46}$$

In Eqs. (42) to (45), let $\bar{\beta}=0$, and we will obtain the results which have already been achieved earlier.

In the same way, we can calculate the difference of the probability of μ^- meson capture under dissimilar hyperfine conditions. Let λ_+ and λ_- be, respectively, the probabilities of μ^- meson capture in F 3/2 and F 1/2. F is the total angular momentum of the μ^- meson. Then, $\bar{\lambda}=2/3\lambda_- + 1/3\lambda_+$ can be taken as the average capture probability, and we easily arrive at

$$\frac{\lambda_+ - \lambda_-}{3\bar{\lambda}} = -\frac{2b_1 \int I_{ss}\,dE_N + b_2 \int (I_{ss}+I_{tt})\,dE_N}{\int I_{ss}\,dE_N}, \tag{47}$$

where

$$b_1 = \left(1+\frac{B}{2}\right)\left(\lambda+\frac{B}{2}\mu'\right),$$
$$b_2 = \left(\lambda+\frac{B}{2}\mu'\right)\left(\lambda-\frac{B}{2}f\right). \tag{48}$$

If we do not consider the mutual interaction of neutrons in the final state, then I_{ss} I_{tt}. Hence,

$$\frac{\lambda_+ - \lambda_-}{3\bar{\lambda}} = \frac{-b}{a}, \tag{47'}$$

where

$$b = 2b_1 + 2b_2 = 2\left(\lambda+\frac{B}{2}\mu'\right)\left(1+\lambda+\frac{B}{2}-\frac{B}{2}f\right),$$
$$a = 1+3\lambda^2+B+B\lambda(2\mu'-f)+\frac{B^2}{2}\left(\mu'^2+\frac{1}{2}f^2\right). \tag{48'}$$

If we let $\mu=0$, we will arrive at the conclusions of the Bernstein Equation.

For other atomic nuclei, if we use the Fermi gas model, we will get

$$\frac{\lambda_+ - \lambda_-}{\bar{\lambda}} = -\frac{b}{aZ'}\frac{2I+1}{I}, \quad I = L+1/2,$$
$$\frac{\lambda_+ - \lambda_-}{\bar{\lambda}} = \frac{b}{aZ'}\frac{2I+1}{I+1}, \quad I = L-1/2, \tag{49}$$

where I is the self-spin of the atomic nuclei, L is the angular momentum of the proton in the initial orbital state, b and a are given in the Appendix II:

$$Z' = (Z-1)\xi + 1,$$

Z is the charge of the atomic nuclei, ξ is one factor in the Pauli interaction. In cases where we disregard the Beta-magnetic moment, we will see how Eq. (49) derives from Überall and Wolgenstein. When we find Eq. (49), we can use a technique similar to that employed by Bernstein-Primakoff.

From Eq. (48'), we can see that if f' is very small or negative, then because of interaction of the Beta-magnetic moment, both a and b will increase about 20%, and their ratio will remain relatively unchanged. If βf' is positive and very large, then it is possible to diminish this difference.

5. Minus Mu Meson Capture by Atomic Nuclei

Since we lack a complete understanding of the structure of atomic nuclei, and without accurate wave functions of the atomic nuclei, it is impossible to accurately calculate the μ^- meson capture by atomic nuclei.

There has already been some work done in hypothesizing that we can use the Fermi gas model or the independent-particle model as approximations upon which to calculate the probability of μ^- meson capture in atomic nuclei. In none of this work were the effects of the Beta-magnetic moment considered.

In this section we shall use the independent-particle model. Blokhintsev and Dolinsky used this model to calculate the μ meson capture. We will use their computational results in our calculations.

Based on the work of Blokhintsev and Dolinsky, we have the following conclusions about the F_i functions:

$$F_1 = F_2, \quad F_3 = 0,$$

The inter-relationships of the Blokhintsev-Dolinsky functions A_o, A_1, A_2, B_o, B_1, and B_2, and the functions F_1, are as follows:

$$A_l(E_N) = (2\pi)^{-3}(m\alpha\xi)^{-1}Z^3(N-1)M^3\int_{k_{min}}^{k_{max}} F_1 k\left(\frac{k}{2M}\right)^l dk,$$

$$B_l(E_N) = (2\pi)^{-3}(m\alpha\xi)^{-1}Z^3(N-1)M^3\int_{k_{min}}^{k_{max}} F_1 k\left(\frac{\mathbf{k}\cdot\mathbf{p}_e}{kp_e}\right)\left(\frac{k}{2M}\right)^l dk,$$

$$l = 0, 1, 2. \qquad (50)$$

From Eq. (50) and Eq. (49), we can express the probability and the angular relationships of the μ^- meson capture by the atomic nucleus as follows: $$d\omega = \frac{\xi^2}{2} I(1 + a\mathbf{P}_\mu\cdot\mathbf{p}_e/p_e)\,dE_N d\Omega_N/(2\pi), \qquad (51)$$

where

$$I = (1+3\lambda^2)A_0(E_N) + (1+2\lambda\mu' - \lambda f')A_1(E_N) + \left(\frac{1}{2}\mu'^2 + \frac{1}{4}f'^2 + \frac{1}{4}\right)A_2(E_N). \qquad (52)$$

$$-I\alpha = (1-\lambda^2)B_0(E_N) + (1-2\lambda\mu' - \lambda f')B_1(E_N) + \left(\frac{1}{4} - \frac{1}{2}\mu'^2 + \frac{1}{4}f'^2\right)B_2(E_N). \qquad (53)$$

Blokhintsev and Dolinsky calculated the $A_1(E_N)$ and $B_1(E_N)$ functions of the Ca^{40} and O^{16} nuclei. We need only change the co-efficients preceding A_1 and B_1 in order to determine the contribution of the Beta-magnetic moment.

APPENDIX I

$$(1)\quad \frac{1}{2}\mathrm{Sp}\ A(1+\sigma_\mu\cdot\mathbf{P}_\mu)A^+ = \frac{g^2}{2}\left(1+\frac{\beta}{2}\right)^2(1-\mathbf{P}_\mu\cdot\mathbf{n}),$$

$$(2)\quad \frac{1}{2}\mathrm{Sp}\ \mathbf{B}\cdot(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+ = \frac{g^2}{2}\left\{\left[3\lambda^2+\lambda\beta(2\mu'-f')+\frac{\beta^2}{2}\left(\mu'^2+\frac{1}{2}f'^2\right)\right]+\right.$$
$$\left.+\left[\lambda^2+\lambda\beta(2\mu'+f')+\frac{\beta^2}{2}\left(\mu'^2-\frac{1}{2}f'^2\right)\right]\mathbf{P}_\mu\cdot\mathbf{n}\right\},$$

$$(3)\quad \frac{1}{2}\mathrm{Sp}\ [A(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+ + \mathbf{B}(1+\sigma_\mu\cdot\mathbf{P}_\mu)A^+] =$$
$$= -\frac{g^2}{2}\left(1+\frac{\beta}{2}\right)\left[2\left(\lambda+\frac{\beta}{2}\mu'\right)\mathbf{P}_\mu-\beta(\mu'+f')\mathbf{P}_\mu\cdot\mathbf{n}\mathbf{n}-2\left(\lambda-\frac{\beta}{2}f'\right)\mathbf{n}\right\},$$

$$(4)\quad \frac{1}{2}\mathrm{Sp}\ [A(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+ - \mathbf{B}(1+\sigma_\mu\cdot\mathbf{P}_\mu)A^+] = i\,\frac{g^2}{2}\left(1+\frac{\beta}{2}\right)\left(\lambda+\frac{\beta}{2}\mu'\right)\mathbf{P}_\mu\times\mathbf{n},$$

$$(5)\quad \frac{1}{2}\mathrm{Sp}\ \mathbf{B}\times(1+\sigma_\mu\cdot\mathbf{P}_\mu)\mathbf{B}^+ = -g^2 i\left\{\left(\lambda+\frac{\beta}{2}\mu'\right)\left(\lambda-\frac{\beta}{2}f'\right)\mathbf{P}_\mu+\right.$$
$$\left.+\frac{\beta}{2}(\mu'+f')\left(\lambda+\frac{\beta}{2}\mu'\right)\mathbf{P}_\mu\cdot\mathbf{n}\mathbf{n}+\left(\lambda+\frac{\beta}{2}\mu'\right)^2\mathbf{n}\right\}.$$

APPENDIX II

Under varying experimental conditions, the atomic weight of the μ^- meson in the hyperfine nuclear structure may be different. In addition, in the polarized target experiment, the $(\mu^- p)$ system in the initial state may also vary according to differing conditions and experimental techniques. Under the most usual circumstances, the initial state density of the orbitals would have the following form:

$$\rho = 1/4\,(1+\sigma_\mu\cdot\mathbf{P}_\mu+\sigma_p\cdot\mathbf{P}_p+\zeta(\sigma_\mu\cdot\sigma_p)),$$

where P_p is the polarized vector of the proton in the initial state. The magnitude of ζ is related to the atomic weight of the $(\mu^- p)$ system in different hyperfine structures. We shall not discuss the magnitudes of P_μ, P_p, and ζ. But, we will use the ρ to calculate the probability of μ^- meson capture by the proton, angular relationships, and the neutron polarization. When we compare the theoretical and the experimental aspects, we should judge the magnitudes of P_μ, P_p, and ζ, based on the whole body of experimental conditions. This is a complicated problem and we shall not discuss it here.

The computation is as follows:

1. Probability of capture and angular relationships.

$$dw = (2\pi)^{-2}(\pi a_0^3)^{-1} 2^{-1} g^2 I(1 - \alpha \mathbf{P}_\mu \cdot \mathbf{n} - \gamma \mathbf{P}_p \cdot \mathbf{n}) E^2 d\Omega,$$

where

$$I = 1 + 3\lambda^2 + \beta + \beta\lambda(2\mu' - f') + \frac{\beta^2}{4}(2\mu'^2 + f'^2) - 6\zeta\Big[\lambda^2 + \Big(1 + \frac{\beta}{2}\Big)\lambda + \frac{\beta}{2}(\mu'\lambda - f'\lambda + \mu') + \frac{\beta^2}{4}(\mu' - \mu' f')\Big] - \frac{1}{3}\beta\zeta(\mu' + f')\Big(1 + \lambda + \frac{\beta}{2}\mu'\Big),$$

$$I\alpha = 1 - \lambda^2 + \beta - \beta\lambda(2\mu' + f') + \frac{\beta^2}{4}(f'^2 - 2\mu'^2),$$

$$I\gamma = 2\Big[\lambda^2 + \Big(1 + \frac{\beta}{2}\Big)\Big(\frac{\beta}{2}f' - \lambda\Big) + \beta\mu' f' + \frac{\beta^2}{4}\mu'^2\Big].$$

2. Neutron Polarization

$$I(1 - \alpha \mathbf{P}_\mu \cdot \mathbf{n} - \gamma \mathbf{P}_p \cdot \mathbf{n}\langle\sigma\rangle = [a + b\mathbf{P}_\mu \cdot \mathbf{n} + c\mathbf{P}_p \cdot \mathbf{n}]\mathbf{n} + d\mathbf{P}_\mu + e\mathbf{P}_p$$

where

$$a = 2\Big[\lambda(\lambda + 1) + \frac{\beta}{2}\lambda + \beta\mu'(\lambda + \beta\mu'/4) - \frac{\beta}{2}f' - \frac{\beta^2}{4}f'\Big] - \zeta\Big[\Big(1 + \frac{\beta}{2}\Big)^2 + 7\lambda^2 + 4\beta\mu'\lambda - 3f'\beta\lambda - \beta^2 f'\mu' + \frac{\beta^2}{4}f'^2 - \frac{\beta^2}{2}\mu'^2\Big],$$

$$b = \beta(f' + \mu')\Big(\lambda + 1 + \frac{\beta}{2} + \frac{\beta}{2}\mu'\Big),$$

$$c = 2\beta(\mu' + f')\Big(\lambda + \frac{\beta}{4}\mu' - \frac{\beta}{4}f'\Big),$$

$$d = 2\Big(\lambda + \frac{\beta}{2}\mu'\Big)\Big(\lambda - 1 - \frac{\beta}{2}(1 + f')\Big),$$

$$e = \Big(1 + \frac{\beta}{2}\Big)^2 + \Big(\beta\lambda f' - \frac{\beta^2}{4}f'^2 - \lambda^2\Big),$$

Under normal circumstances, dw denotes the factor of probability $(1/4\pi)(\pi a_0^3)^{-1}$ of the μ meson in the nucleus which should be replaced by $|\varphi(0)|^2$, where $\varphi(\underline{r})$ is the wave function of the μ meson.

When the μ^- meson atom is in the hyperfine state F=1, its capture probability λ_+ can be ascertained from the above equation -- if we let $\zeta = 1/3$.

$$\lambda_+ = (1 - \lambda)^2 + \beta(1 - 2\lambda + \mu'(\lambda - 1) - \frac{1}{g}(\mu' + f')(1 + \lambda)).$$

When expressing λ_+, we disregard all those terms proportional to β^2. Let $\zeta=-1$, and we will get the probability of λ_- of the μ^- meson capture in the state of F=0;

$$\lambda_- = (1+3\lambda)^2 + \beta\left(1+3\lambda+3\mu'+5\lambda\mu'-4\lambda f' + \frac{1}{8}(\mu'+f')(1+\lambda)\right).$$

Using the above-mentioned λ_+ and λ_- we now obtain

$$\frac{\lambda_+ - \lambda_-}{\bar{\lambda}} = -\frac{b}{a}\cdot 4,$$

where

$$b = 2\lambda(1+\lambda) + \beta\left(\lambda - f'\lambda + \mu'(1+\lambda) + \frac{1}{g}(\mu'+f')(1+\lambda)\right),$$

$$a = 1 + 3\lambda^2 + \beta(1+2\mu'\lambda - f'\lambda).$$

In the above equations, we disregarded all terms proportional to β^2. In the above, although we only obtain the probability of the μ^- capture by the proton in the hyperfine state, if we assume the accuracy of the Fermi gas model approximation, then we can obtain Eq. (49) in section four. The a and b in Eq. (49) agree with the a and b found above in the μ capture by the proton.

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