

ARMY RESEARCH LABORATORY



# Analytical Blast Model Formulation With Computer Code

by Joseph Collins

ARL-TR-2009

July 1999

Approved for public release; distribution is unlimited.

DTIC QUALITY INSPECTED 4

19990827 032

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Citation of manufacturer's or trade names does not constitute an official endorsement or approval of the use thereof.

Destroy this report when it is no longer needed. Do not return it to the originator.

---

---

## Abstract

---

Overpressure time history data from warhead blast experiments yield peak overpressure  $P$  as a function of spatial position.

Dr. Owen Litt has proposed a model for  $P$  based on the peak-overpressure characteristics of a bare spherical charge. The direction-independent peak-overpressure function for a bare spherical charge is modified to have nonspherical level-surface structure by specifying surfaces of constant peak overpressure. This introduces a directional component into the model.

In this report, the original formulation is refined and generalized and a mathematical model and computer code are presented to evaluate the function. Such a computational device is required for model parameter estimation and experiment design.

## Acknowledgments

Thanks are extended to Ed Davisson for numerous enlightening discussions regarding mathematical modeling and the philosophy of mathematics in general. Andrew Thompson also reviewed the paper and provided guidance for improving the overall presentation of the topic. Without his input, this paper would certainly be less readable. Finally, thanks go to thank Dr. Owen Litt for sharing his concept of the peak-overpressure model.

INTENTIONALLY LEFT BLANK.

# Table of Contents

	<u>Page</u>
Acknowledgments . . . . .	iii
List of Figures . . . . .	vii
1. Background . . . . .	1
2. Level-Curve Model Specification . . . . .	1
2.1 An Example . . . . .	1
2.2 The General Construction . . . . .	2
3. Application to Blast Model Formulation . . . . .	3
3.1 Basic Model . . . . .	3
3.2 Enhanced Model . . . . .	5
4. Model Evaluation for Experiment Design . . . . .	8
5. Model Parameter Estimation . . . . .	8
6. References . . . . .	19
Appendix: Mathematica Code . . . . .	21
Distribution List . . . . .	27
Report Documentation Page . . . . .	29

INTENTIONALLY LEFT BLANK.

## List of Figures

<u>Figure</u>		<u>Page</u>
1.	Level Curves of $F_1$ From Section 1 . . . . .	10
2.	Level Curves of $F_2$ From Section 1 . . . . .	10
3.	Level Curves for Pressure Components $P_s$ and $P_n$ . . . . .	11
4.	Mass Scaling Functions $M$ and $s$ . . . . .	12
5.	Level Curves for Pressure Components $P_s$ and $P_n^*$ . . . . .	13
6.	$P$ at (2, 3, 4, 6, 9, 14, 20) psi . . . . .	14
7.	$P$ at (5, 8, 14, 22, 37, 61, 100) psi . . . . .	15
8.	$P$ at (10, 19, 37, 71, 136, 261, 500) psi . . . . .	16
9.	$P$ at (20, 41, 84, 173, 356, 730, 1,500) psi . . . . .	17
10.	$P$ at (50, 99, 196, 387, 766, 1,516, 3,000) psi . . . . .	18

INTENTIONALLY LEFT BLANK.

## 1. Background

Overpressure time history data from warhead blast experiments yield peak overpressure  $P$  as a function of spatial position. Dr. Owen Litt [1] has proposed a model for  $P$  based on the peak-overpressure characteristics of a bare spherical charge. In that model, the direction-independent peak-overpressure function for a bare spherical charge is modified to have a level curve structure with a specific nonspherical functional form. This induces a directional component into the model.

It is necessary to combine the peak-overpressure function representations of the bare spherical charge and an arbitrary level-curve structure to produce the required mathematical model. This report details the solution of that analytical problem and also an explicit solution to the problem of level-curve model specification in general, and so serves a twofold purpose. The development of a general theoretical framework for solving such model-specification problems appears in section 2. The rest of this report describes the application of the general principle to the specific problem of Dr. Litt's blast model. The original formulation is refined and generalized and a mathematical model and computer code are presented to evaluate the function.

## 2. Level-Curve Model Specification

This section contains a discussion of model formulation based on specifying a geometric level-curve structure. An example precedes the development of a general method for the formulation of such models.

**2.1 An Example.** Suppose  $F$  is a decreasing function on  $[0, \infty)$ . For example, take

$$F(r) = \frac{1}{1+r^2}. \quad (1)$$

The function  $F$  can be used to create function  $F_1: \mathbb{R}^2 \rightarrow \mathbb{R}^+$  by defining

$$F_1(r, \phi) = F(r) = \frac{1}{1+r^2}, \quad (2)$$

where the usual Cartesian coordinates on  $\mathbb{R}^2$  are  $(x, y)$  and polar coordinates  $(r, \phi)$  on  $\mathbb{R}^2$  are given by  $x = r \cos \phi$  and  $y = r \sin \phi$ . In the  $x$ - $y$  plane, level curves of  $F_1$  are concentric circles centered at the origin, since  $F_1$  is independent of  $\phi$ . For any  $L \in (0, 1]$ , the value of  $r$  that makes  $F_1(r, \phi) = L$  is given by  $F_1(r, \phi) = F(r) = 1/(1+r^2) = L$ , so that  $r = F^{-1}(L) = \sqrt{1/L - 1}$ . In other words,  $F(\sqrt{1/L - 1}) = L$ , so  $F_1 = L$  on the circumference of a circle with radius  $\sqrt{1/L - 1}$  and area  $\pi(1/L - 1)$ . It is possible to modify the definition of  $F_1$  and produce a function  $F_2$  that has elliptical level curves with a given eccentricity and

orientation. Furthermore, the value of  $F_1$  on a given circle should be equal to the value of  $F_2$  on an ellipse with the same area. The polar equation for an ellipse is

$$r^2 = ab \cdot \sqrt{1 - \varepsilon^2} \cdot \frac{1 + \tan^2(\phi - \phi_o)}{1 - \varepsilon^2 + \tan^2(\phi - \phi_o)}, \quad (3)$$

where the ellipse has eccentricity  $\varepsilon = \sqrt{1 - b^2/a^2}$ , major axis length  $2a$  in the direction  $\phi = \phi_o$ , minor axis length  $2b$  in the direction  $\phi = \phi_o + \pi/2$ , and area  $\pi ab$ . The function  $F_2$  is now defined by

$$F_2(r, \phi) = F \left( r \cdot \left[ \sqrt{1 - \varepsilon^2} \cdot \frac{1 + \tan^2(\phi - \phi_o)}{1 - \varepsilon^2 + \tan^2(\phi - \phi_o)} \right]^{-1/2} \right). \quad (4)$$

Then, it can be seen that  $F_2(r, \phi) = L$  when

$$r^2 = \left( \frac{1}{L} - 1 \right) \cdot \sqrt{1 - \varepsilon^2} \cdot \frac{1 + \tan^2(\phi - \phi_o)}{1 - \varepsilon^2 + \tan^2(\phi - \phi_o)}, \quad (5)$$

so that  $F_2(r, \phi) = L$  on the perimeter of an ellipse of area  $\pi(1/L - 1)$ . (See Figures 1 and 2 for a depiction of the level curves of these example functions.)

**2.2 The General Construction.** Consider a decreasing function  $F: [0, \infty) \rightarrow \mathbb{R}^+$ . This function  $F$  can be used to create a function  $F_1: \mathbb{R}^2 \rightarrow \mathbb{R}^+$  by defining

$$F_1(r, \phi) = F(r), \quad (6)$$

where  $r$  and  $\phi$  are polar coordinates. Since  $F_1$  is independent of  $\phi$ , the level curves of  $F_1$  are concentric circles centered at the origin of  $\mathbb{R}^2$ . And because  $F$  is decreasing, the value of  $F_1$  is smaller on a larger such circle.

It is possible to construct a version of  $F$  that has noncircular level curves with any specific functional form. In particular, say the level curves are to be given by

$$r(\phi) = g_\phi(u) \quad (7)$$

for various values of  $u$ . Suppose for all  $\phi$  that  $g_\phi(u)$  is a continuous function of  $u$ , that  $g_\phi(0) = 0$ , that  $g_\phi(u)$  is an increasing function of  $u$ , that  $g_\phi(u)$  is defined for all  $u \geq 0$ , and that  $\lim_{u \rightarrow \infty} g_\phi(u) = \infty$ . So,  $g_\phi$  is a bijective function on  $[0, \infty)$ , and  $g_\phi$  has an inverse in the following sense: for each fixed value of  $\phi$ , the inverse function  $g_\phi^{-1}$ , characterized by  $g_\phi(g_\phi^{-1}(u)) = g_\phi^{-1}(g_\phi(u)) = u$ , is well-defined for  $u \geq 0$ . In fact,  $g_\phi^{-1}(u)$  is also an increasing function of  $u$  on  $[0, \infty)$ .

A function  $F_2: \mathbb{R}^2 \rightarrow \mathbb{R}^+$  satisfying equation (7) can be defined in terms of polar coordinates by

$$F_2(r, \phi) = F(g_\phi^{-1}(r)). \quad (8)$$

To show this, let  $u > 0$  be constant. Then  $F(u)$  is also constant, and the locus of  $(r, \phi)$  which has  $F_2(r, \phi) = F(u)$  is given by  $F(u) = F_2(r, \phi) = F(g_\phi^{-1}(r))$ . This means that  $u = g_\phi^{-1}(r)$ , from which equation (7) follows.

### 3. Application to Blast Model Formulation

Here, the results of the previous section are applied to the formulation of models for the maximum peak overpressure of a detonating charge blast field. First, the basic formulation is discussed and then an enhanced model is presented.

**3.1 Basic Model.** This model works in two-dimensional polar coordinates  $(r, \phi)$  with the origin centered on the detonating charge. A three-dimensional spatial model for peak overpressure  $P$  can be obtained by rotating a two-dimensional model, defined in the half-plane  $0 \leq \phi \leq \pi$ , about the  $x$ -axis.

The development of a two-dimensional model for peak overpressure as a function of the polar coordinates  $(r, \phi)$  follows. In this report, models for peak overpressure are based on the function  $P_s(z)$ , which gives the maximum peak overpressure for detonation of a spherical TNT charge. In the definition of  $P_s$  and throughout this report, the normalized distance coordinate

$$z = \frac{r}{W^\alpha} \quad (9)$$

is used, where  $r$  is measured in feet,  $W$  is charge weight in pounds, and  $\alpha$  is a constant with nominal value  $\alpha = 1/3$ . The spherical charge pressure function  $P_s(z)$  itself is defined by

$$P_s(z) = \exp \left[ \frac{A}{z+B} - C \right], \quad (10)$$

where the constants have the empirical values  $A = 31.97$ ,  $B = 3.555$ , and  $C = 0.5$ . This function was derived from a fit to empirical data [1].

The modeling concept under consideration requires a pressure function component  $P_n$  with a nontrivial dependence on  $\phi$ , specified by a certain family of noncircular level curves. The function  $P_n$  is derived from  $P_s$  in the same way that  $F_2$  is derived from  $F$  in section 2, by the application of equation (8) to a specific level curve function. The level curve function for  $P_n$ , which gives an appropriate shape based on engineering considerations, is given by

$$g_\phi(u) = u \left[ \sin \frac{m\phi}{2} \right]^{4n(u)/m}, \quad (11)$$

where  $n(u)$  is defined shortly, and  $P_n$  is defined by

$$P_n(z, \phi) = P_s(g_\phi^{-1}(z)), \quad (12)$$

as in section 2.

Two more definitions complete the specification of  $P_n$ . The function  $f$  is defined to be a "smooth step function" with  $f(0) \simeq 0$ ,  $f$  increasing, and  $f(z) \rightarrow 1$  as  $z \rightarrow \infty$ . To be specific, set  $z_o = 10$  and take

$$f(z) = \frac{1}{2} + \frac{1}{\pi} \arctan(z - z_o). \quad (13)$$

The function  $n$  is defined by

$$n(u) = n_o(1 - f(u)), \quad (14)$$

where  $n_o$  is a positive constant. So  $n$  is a decreasing function with  $n(0) = n_o$  and  $n(u) \rightarrow 0$  as  $u \rightarrow \infty$ . The behavior of  $n$  along with the form of  $g_\phi$  implement the design objective that  $P_n$  looks like  $P_s$  at large distances; i.e., the level curves of  $P_n$  become circular for large  $z$ .

Now with the function  $P_n$  completely specified, the conditions on  $g_\phi(u)$  of section 2 are indeed satisfied. The exponent  $4n(u)/m$  is positive and decreasing with  $u$ , so  $\sin(m\phi/2)^{4n(u)/m}$  is a nondecreasing function of  $u$  for fixed  $\phi$ . Therefore,  $g_\phi(u)$  is increasing in  $u$  for any  $\phi$ . The conditions of section 2 are satisfied, so a level curve of  $P_n(z, \phi)$  is given by  $z = g_\phi(u)$  for  $u$  fixed, as required.

To specify the peak overpressure model, it remains only to combine the function components  $P_s$  and  $P_n$  in a certain way. The definition of the peak overpressure model function  $P$  is

$$P(z, \phi) = f(z)P_s(z) + (1 - f(z))P_n(z, \phi), \quad (15)$$

where the functions  $P_s$ ,  $P_n$ , and  $f$  are as previously discussed. Because of the nature of  $f$ , the pressure function looks like  $P_n$  for small  $z$  and like  $P_s$  for large  $z$ . Definition of the model is now complete. The quantities  $P$ ,  $z$ ,  $f$ ,  $P_s$ ,  $g_\phi$ , and  $n$  were specified by Dr. Owen Litt [1, 2, 3, 4, 5], as was an implicit characterization of  $P_n$ . The explicit representation of equation (12) for  $P_n$  is a product of this report.

In summary, the complete model is given by

$$\begin{aligned} P(z, \phi) &= f(z)P_s(z) + (1 - f(z))P_n(z, \phi), \quad \text{where} \\ z &= r/W^\alpha, \\ f(z) &= 1/2 + 1/\pi \cdot \arctan(z - z_o), \\ P_s(z) &= \exp(A/(z + B) - C), \\ n(u) &= n_o(1 - f(u)), \\ g_\phi(u) &= u (\sin m\phi/2)^{4n(u)/m}, \quad \text{and} \\ P_n(z, \phi) &= P_s(g_\phi^{-1}(z)). \end{aligned} \quad (16)$$

The quantities  $A$ ,  $B$ ,  $C$ , and  $W$  are constant;  $z_o = 10$ ,  $z_1 = 5$ , and  $z_2 = 15$  are fixed model parameters; and  $m$ ,  $n_o$ , and  $\alpha$  are model parameters to be estimated. Interpretations of

the parameters are as follows:  $m$  determines the direction of the  $P_n$  component, the value of  $n_o$  makes the  $P_n$  component more or less concentrated in the direction determined by  $m$ , and  $\alpha$  determines the dependence of normalized distance on charge weight.

Now some characteristics of the model can be examined in more detail. The extreme point on a level curve occurs when  $\phi = \pi/m$  and  $\sin(m\phi/2) = 1$ , in which case  $g_{\pi/m}(u) = u$  and also  $g_{\pi/m}^{-1}(u) = u$ . Then,  $P_n(z, \pi/m) = P_s(z)$ . So, in the direction of maximum peak overpressure,  $\phi = \pi/m$ , the  $P_n$  component has the same pressure value as spherical bare charge,  $P_s$ . In other directions, for fixed  $z$ , the value of  $P_n$  is lower than  $P_s$ .

For an example, set the charge weight to  $W = 1$  and set the function parameters to  $m = 1.75$ ,  $n_o = 2.0$ , and  $\alpha = 1/3$ . Figure 3 demonstrates the level curve characterizations of  $P_n$  and  $P_s$  at the same pressure value.  $P_n$  is evaluated at the point  $(z_o, \phi_o) = (4, \pi/3)$ . This point lies on the level curve  $z = g_\phi(u)$ , where  $g_{\phi_o}(u) = z_o$ , or  $u = g_{\phi_o}^{-1}(z_o)$ , so a general point on this level curve has coordinates  $(g_\phi(u), \phi)$ . The extreme point on this level curve, where  $\phi = \pi/m$ , has coordinates  $(g_{\pi/m}(u), \pi/m) = (u, \pi/m)$ . The value of  $P_n$  anywhere on this level curve is  $P_n((g_\phi(u), \phi)) = P_s(g_\phi^{-1}(g_\phi(u))) = P_s(u)$ . Particular values for this example are  $u \simeq 8.94$  and  $P_s(u) \simeq 8.49$ .

Once again, in the direction of maximum peak overpressure,  $\phi = \pi/m$ , the  $P_n$  component has the same pressure value as spherical bare charge,  $P_s$ . In other directions, for fixed  $z$ , the value of  $P_n$  is lower than  $P_s$ . This may not be realistic. In the maximum direction,  $P_n$  should have a higher value than  $P_s$ , since the blast modeled by  $P_n$  is focused in that direction. This additional feature is implemented by incorporating into the definition of  $P_n$  an equivalent spherical charge weight  $W_s$  that is greater than the actual charge weight  $W$ , effectively renormalizing the distance coordinate in equation (9), which is then used in equations (10) and (12). A conceptually equivalent approach is to directly reduce the distance argument  $z$  of  $P_s$  in equation (10), as it is used in the definition of  $P_n$ . Alternatively, the level curve function can be changed in the definition of  $P_n$  from  $g_\phi$ , equation (11), to a new function produces a higher pressure value on the level curve.

As shown in section 3.2, these three schemes are equivalent. The net effect of any of them is to force a  $P_n$  level curve to correspond to a smaller  $P_s$  level curve, on which the pressure is higher. The basic model can be modified to have this property.

**3.2 Enhanced Model.** The model of the section 3.1 is generalized by the introduction of a new pressure function  $P_n^*$ , which replaces  $P_n$ . The  $P_n^*$  component in the maximum direction ( $\phi = \pi/m$ ) has the pressure value of a spherical charge of arbitrary weight  $W_s(z)$ . To increase the generality and flexibility of the model,  $W_s$  is allowed to be a function of  $z$  rather than a constant. It is convenient to define the function  $M$  by

$$M(z) = \frac{W_s(z)}{W}, \quad (17)$$

so that  $M$  represents a dimensionless mass scaling ratio or magnification factor in the maximum direction, since  $W_s(z)$  is the equivalent sphere charge weight in that direction

$(\pi/m)$  at the distance  $r = zW^\alpha$ . Now, define  $P_n^*$  by

$$P_n^*(z, \phi) = P_s(s(g_\phi^{-1}(z))) \quad (18)$$

and proceed to solve for  $s$ . When  $\phi = \pi/m$ , the result is

$$P_n^*(z, \pi/m) = P_s(s(g_{\pi/m}^{-1}(z))) = P_s(s(z)). \quad (19)$$

Since  $z = rW^{-\alpha}$  and  $s(z) = rW_s(z)^{-\alpha} = rW^{-\alpha}M(z)^{-\alpha}$ , it follows that  $r = zW^\alpha = s(z)W^\alpha M(z)^\alpha$ , and then

$$M(z) = \left[ \frac{z}{s(z)} \right]^{1/\alpha} \quad \text{or} \quad s(z) = zM(z)^{-\alpha}. \quad (20)$$

This expresses the required function  $s$  in terms of the magnification factor  $M$  and, therefore, also in terms of the equivalent sphere weight  $W_s$ . Note, referring to equation (10), that the function  $s$  as it appears in  $P_s(s(z))$  amounts to a rescaling of distance in the function  $P_s$ . Also, the definition of  $P_n^*$  can be written as  $P_n^*(z, \phi) = P_s((g_\phi \circ h)^{-1}(z))$ , where  $h^{-1} = s$ , and thus making explicit the modification of the level curve function in the definition of  $P_n^*$ . So the three conceptual approaches (weight scaling, distance scaling, and level curve modification) to the derivation of  $P_n^*$  from  $P_n$  are equivalent.

Now, with  $W_s(z) \equiv W$ , then  $M(z) \equiv 1$  and  $s(u) = u$ , so that  $P_n^* = P_n$ . This reduces to the basic model of the section 3.1, where the  $P_n$  has the property that, in the maximum pressure direction  $\phi = \pi/m$ , the peak overpressure is equal to that of a spherical charge of the same weight.

A more realistic general formulation requires that  $M(0) > 1$ , that  $M(z)$  is a nonincreasing function of  $z$ , and that  $M(z) \rightarrow 1$  as  $z \rightarrow \infty$ . This behavior embodies the design criteria that  $P_n^*$  itself looks like  $P_s$  at large distances and that the pressure  $P_n^*$  is greater than  $P_s$  in the maximum direction at small distances.

It may be possible to completely specify  $M$  through energy conservation considerations, but, for illustrative purposes, a piecewise continuous version of  $M$  is used. This has corresponding  $s$  that is easy to calculate. Let  $M(z) = M_o > 1$  for  $z < z_1$ , let  $M(z) = 1$  for  $z > z_2$ , and let  $M(z)^\alpha$  be a linear function of  $z$  for  $z_1 \leq z \leq z_2$ . It is convenient to express piecewise function definitions in terms of the indicator function

$$I_T(t) = \begin{cases} 1, & t \in T \\ 0, & t \notin T. \end{cases} \quad (21)$$

First define the "linear step function"  $L$  with the characteristics that  $L(z) = b$  for  $z \leq z_1$ ,  $L(z) = 1$  for  $z \geq z_2$ ,  $L(z)$  is linear for  $z_1 \leq z \leq z_2$ , and  $L$  is continuous. The appropriate definition is

$$L(z; b, z_1, z_2) = b \cdot I_{[0, z_1)}(z) + (a_1 + a_2 z) \cdot I_{[z_1, z_2]}(z) + 1 \cdot I_{(z_2, \infty)}(z), \quad (22)$$

$$\text{where } a_1 = \frac{bz_2 - z_1}{z_2 - z_1} \quad \text{and} \quad a_2 = \frac{1 - b}{z_2 - z_1}. \quad (23)$$

The corresponding definition for  $M$  is then

$$M(z) = L(z; M_o^\alpha, z_1, z_2)^{1/\alpha}. \quad (24)$$

Solve for  $s$  in closed form to get

$$s(z) = z/b \cdot I_{[0, z_1)}(z) + z/(a_1 + a_2 z) \cdot I_{[z_1, z_2]}(z) + z \cdot I_{(z_2, \infty)}(z), \quad (25)$$

where  $b = M_o^\alpha$ , and  $a_1$  and  $a_2$  are given by equation (23).

A complete working model is then

$$\begin{aligned} P(z, \phi) &= f(z)P_s(z) + (1 - f(z))P_n^*(z, \phi) \quad \text{where} \\ z &= r/W^\alpha, \\ f(z) &= 1/2 + 1/\pi \cdot \arctan(z - z_o), \\ P_s(z) &= \exp(A/(z + B) - C), \\ n(u) &= n_o(1 - f(u)), \\ g_\phi(u) &= u (\sin m\phi/2)^{4n(u)/m}, \\ M(z) &= L(z; M_o^\alpha, z_1, z_2)^{1/\alpha}, \\ s(z) &= zM(z)^{-\alpha}, \quad \text{and} \\ P_n^*(z, \phi) &= P_s(s(g_\phi^{-1}(z))). \end{aligned} \quad (26)$$

The quantities  $A$ ,  $B$ ,  $C$ , and  $W$  are constant;  $z_o = 10$ ,  $z_1 = 5$ , and  $z_2 = 15$  are fixed model parameters; and  $m$ ,  $n_o$ ,  $M_o$ , and  $\alpha$  are model parameters to be estimated.

The example of section 3.1 illustrates the enhanced model. Again, the charge weight is  $W = 1$  and set the function parameters are  $m = 1.75$ ,  $n_o = 2.0$ , and  $\alpha = 1/3$ . The new function parameter for mass scaling is  $M_o = 4$ . Figure 4 depicts the functions  $M$  and  $s$ . Figure 5 demonstrates the level curve characterization of  $P_n^*$  in relation to that of  $P_s$  at the same pressure. As before,  $P_n^*$  is evaluated at the point  $(z_o, \phi_o) = (4, \pi/3)$ . This point lies on the level curve  $z = g_\phi(u)$  where  $g_{\phi_o}(u) = z_o$ , or  $u = g_{\phi_o}^{-1}(z_o)$ , so a general point on this level curve has locus  $(g_\phi(u), \phi)$ . The extreme point on this level curve, where  $\phi = \pi/m$ , has coordinates  $(g_{\pi/m}(u), \pi/m) = (u, \pi/m)$ .

Note that the  $P_n^*$  level curve is identical to the  $P_n$  level curve in Figure 3, which illustrates the example of section 3.1. The function value is different, however, to reflect the increased equivalent sphere charge weight or reduced distance in  $P_s$ . The corresponding  $P_s$  level curve in Figure 5 has a radius smaller than the extreme distance on the  $P_n^*$  level curve. The value of  $P_n^*$  anywhere on its level curve is  $P_n^*((g_\phi(u), \phi)) = P_s(s(g_\phi^{-1}(g_\phi(u)))) = P_s(s(u))$ , which is also the value of  $P_s$  on its level

curve in Figure 5. Particular values for this example are  $u \simeq 8.94$ ,  $s(u) \simeq 6.59$ , and  $P_s(s(u)) \simeq 15.6$ . The weight scaling factor is

$$M(u) = \left[ \frac{u}{s(u)} \right]^{1/\alpha} \simeq 2.49, \quad (27)$$

and, since  $W = 1$ , this is also the equivalent sphere charge weight in the direction  $\phi = \pi/m$  at the distance  $r = u$ . These values of  $u$ ,  $s(u)$ , and  $M(u)$  are distinguished in Figure 4.

#### 4. Model Evaluation for Experiment Design

Conducting an experiment to calibrate the model (estimate the parameters) involves placing pressure sensors in the detonation field of an explosive charge. The sensors must be placed so that optimal useful information is obtained from the experiment. Sensors cannot be overdriven. On the other hand, each sensor has a lower limit of resolution, beyond which the noise in the measurement system overrides any signal. Sensors must also be placed so that they register the nonspherical  $P_n$  component of the pressure field. It is therefore reasonable to “guess” what the model parameters are, evaluate  $P$ , and place the sensors accordingly.

A graphical display of the model response is useful in the design of an experiment for blast model parameter estimation. Since the model is a well-defined function, evaluation is conceptually simple: replace constants with numbers and evaluate the functions.

The form chosen for  $M(z)$  yields a closed-form representation for  $s$ . Generally,  $g_\phi^{-1}$  must be evaluated numerically, even if  $s$  has a closed-form representation. Choosing another form for  $M(z)$  may result in  $s$  having no closed-form representation, which will increase computational complexity.

Figures 6–10 are contour representations of  $P$  computed with parameter values  $W = 1$ ,  $m = 1.75$ ,  $n_o = 2.0$ ,  $M_o = 4$ , and  $\alpha = 1/3$  on various  $x - y$  grids. Spatial coordinates are equivalent to  $z$  units, since  $W = 1$ . Contour levels are indicated in the figure captions. Note that the logarithmic spacing of the level curves gives a better visual display than linear spacing would. Figure 6 has  $-20 \leq x \leq 20$  and  $0 \leq y \leq 20$  to show the far field, Figure 10 has  $-1 \leq x \leq 1$  and  $0 \leq y \leq 1$  to show the near field, and the intervening figures depict intermediate ranges. Computations and graphics were done with *Mathematica* [6]; the code necessary to reproduce these calculations are presented in the Appendix.

#### 5. Model Parameter Estimation

Data consist of empirical measurements of peak overpressure  $p$  at spatial location  $(r, \phi)$ , denoted as  $p_i$ ,  $r_i$ , and  $\phi_i$  for  $1 \leq i \leq N$ . Parameter estimates can be obtained, for example,

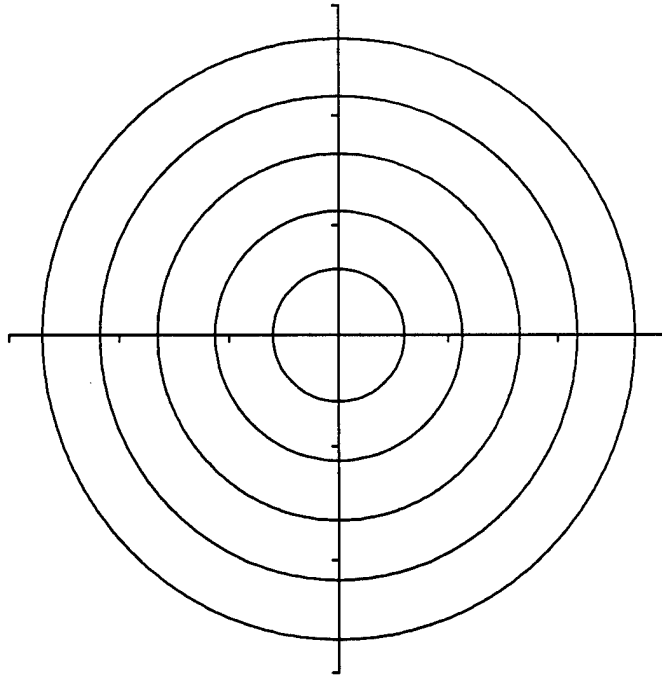
by least squares in the response or log response; i.e.,

$$\text{minimize } \sum_{i=1}^N [p_i - P(W^{-\alpha} r_i, \phi_i)]^2, \quad (28)$$

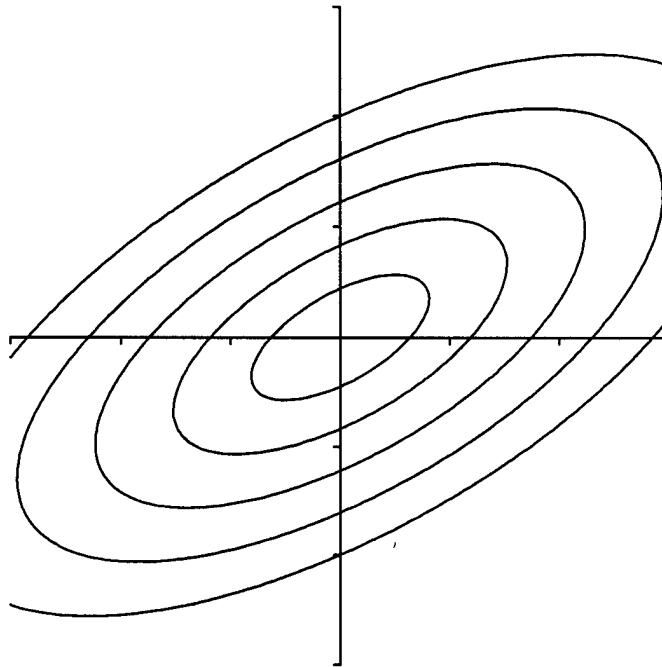
or

$$\text{minimize } \sum_{i=1}^N [\log p_i - \log P(W^{-\alpha} r_i, \phi_i)]^2, \quad (29)$$

where the minimizations are conducted over the parameter vector  $(m, n_o, M_o, \alpha)$ . Due to the exponential nature of  $P$  and the error characteristics of pressure sensors, estimation based on  $\log P$  will most likely yield more accurate results than estimation based on  $P$ .



**Figure 1.** Level Curves of  $F_1$  From Section 1.



**Figure 2.** Level Curves of  $F_2$  From Section 1.

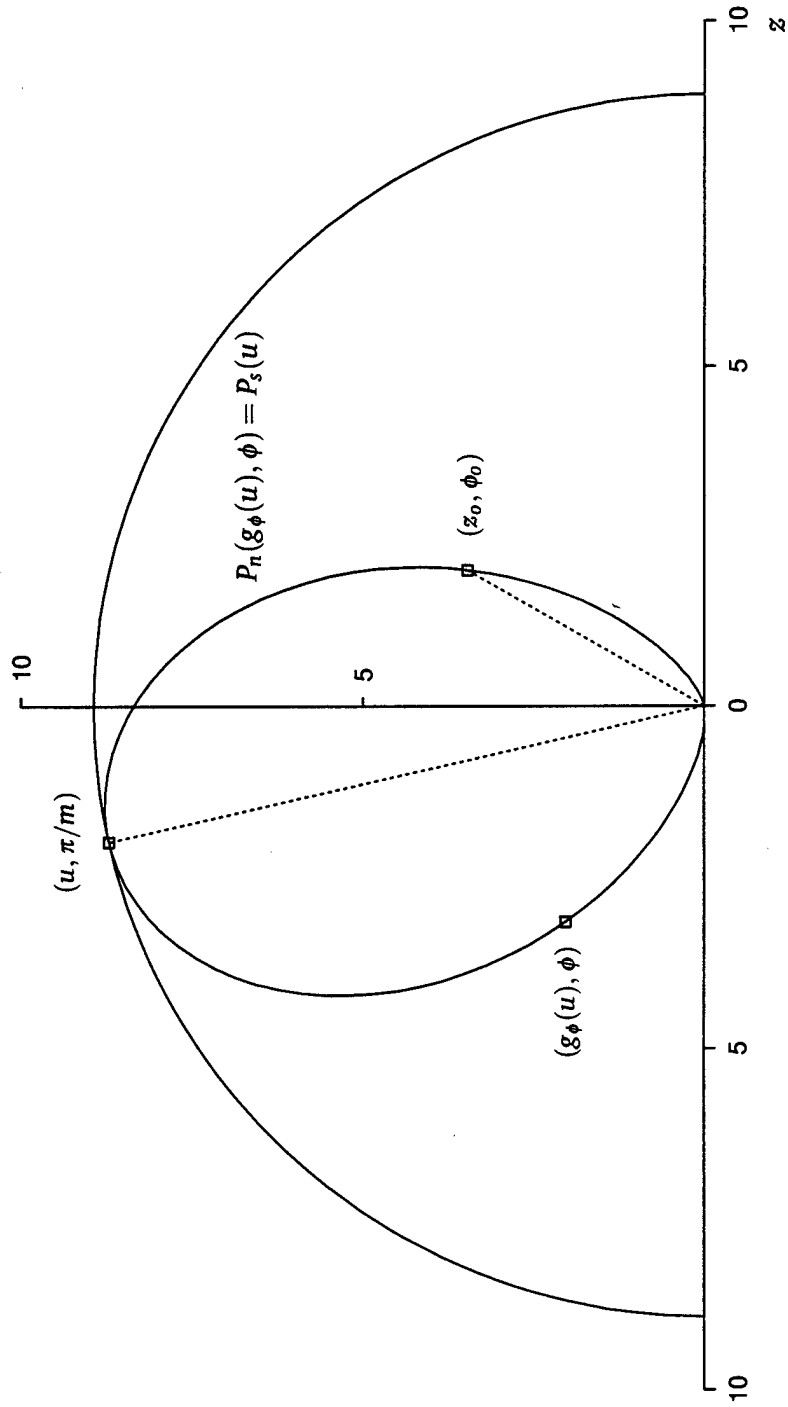


Figure 3. Level Curves for Pressure Components  $P_s$  and  $P_n$ .

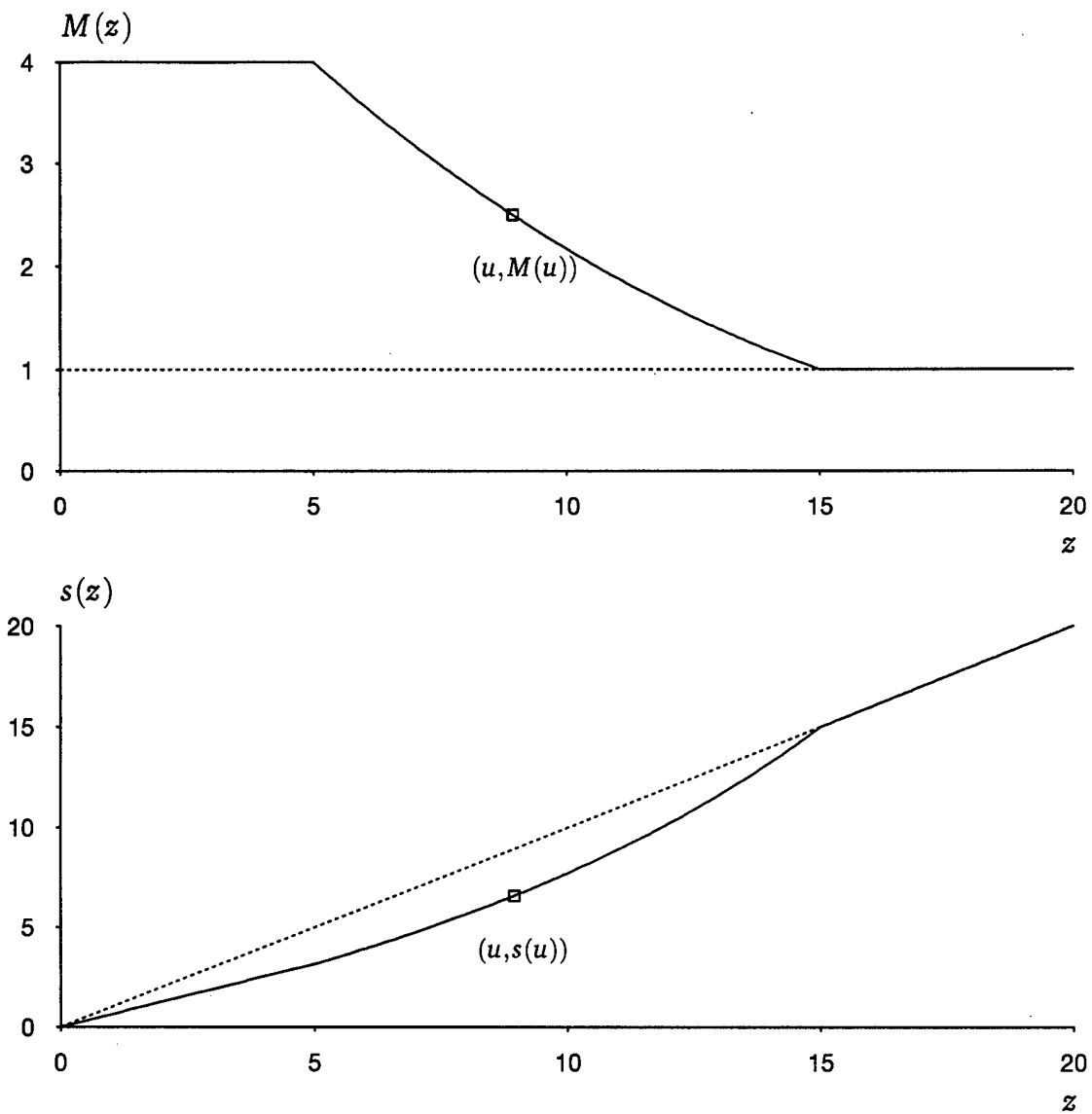


Figure 4. Mass Scaling Functions  $M$  and  $s$ .

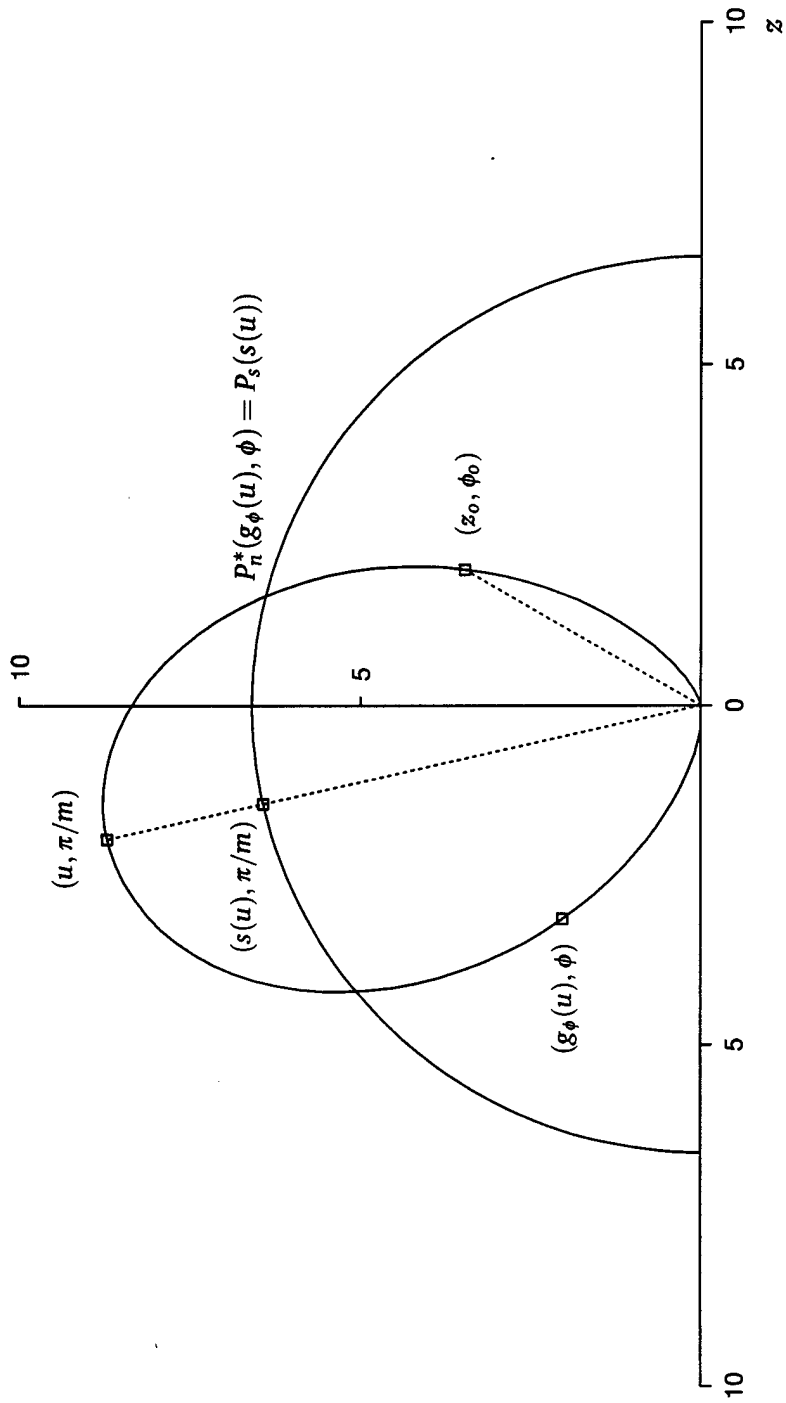


Figure 5. Level Curves for Pressure Components  $P_s$  and  $P_n^*$ .

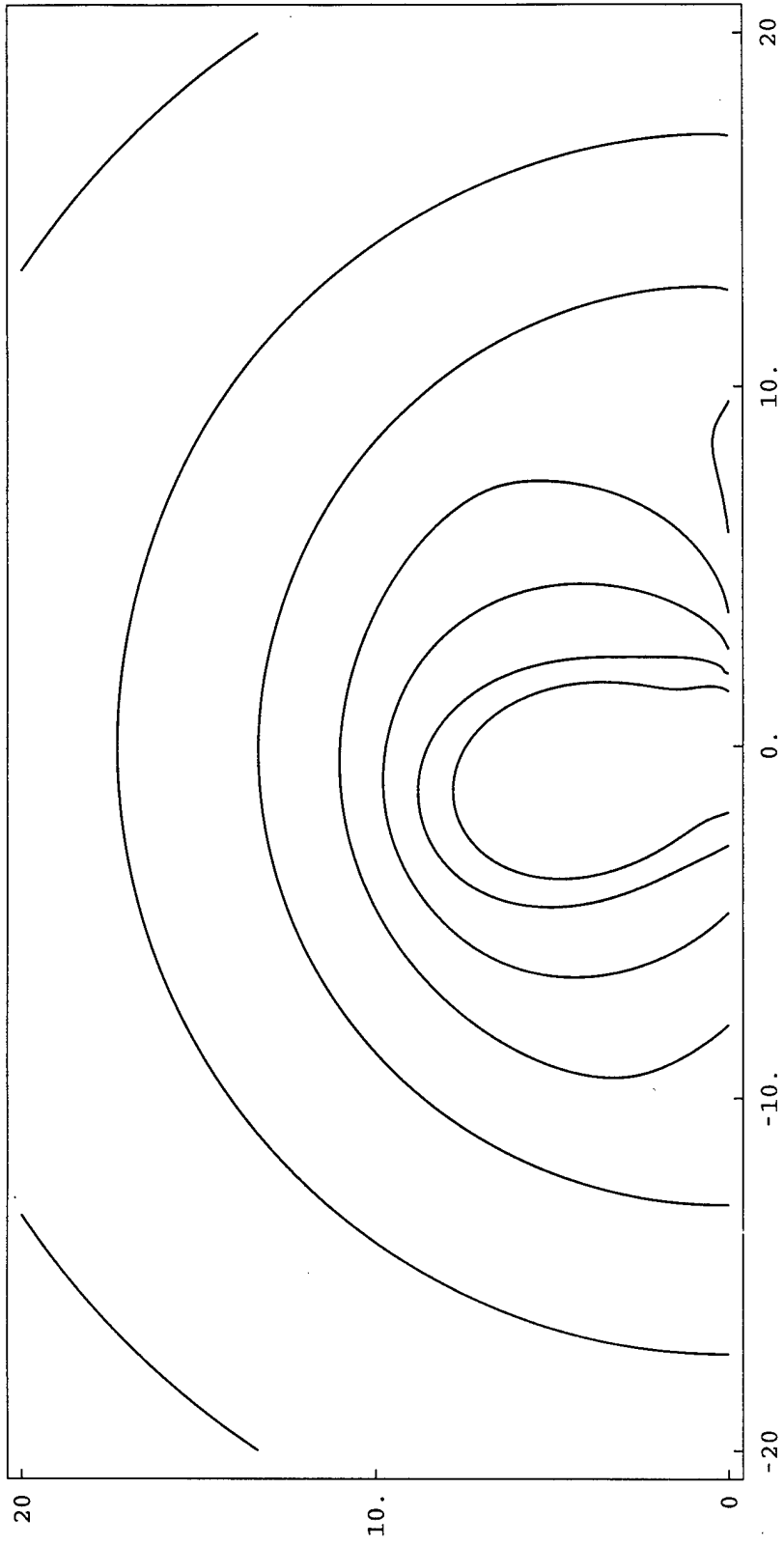


Figure 6.  $P$  at (2, 3, 4, 6, 9, 14, 20) psi.

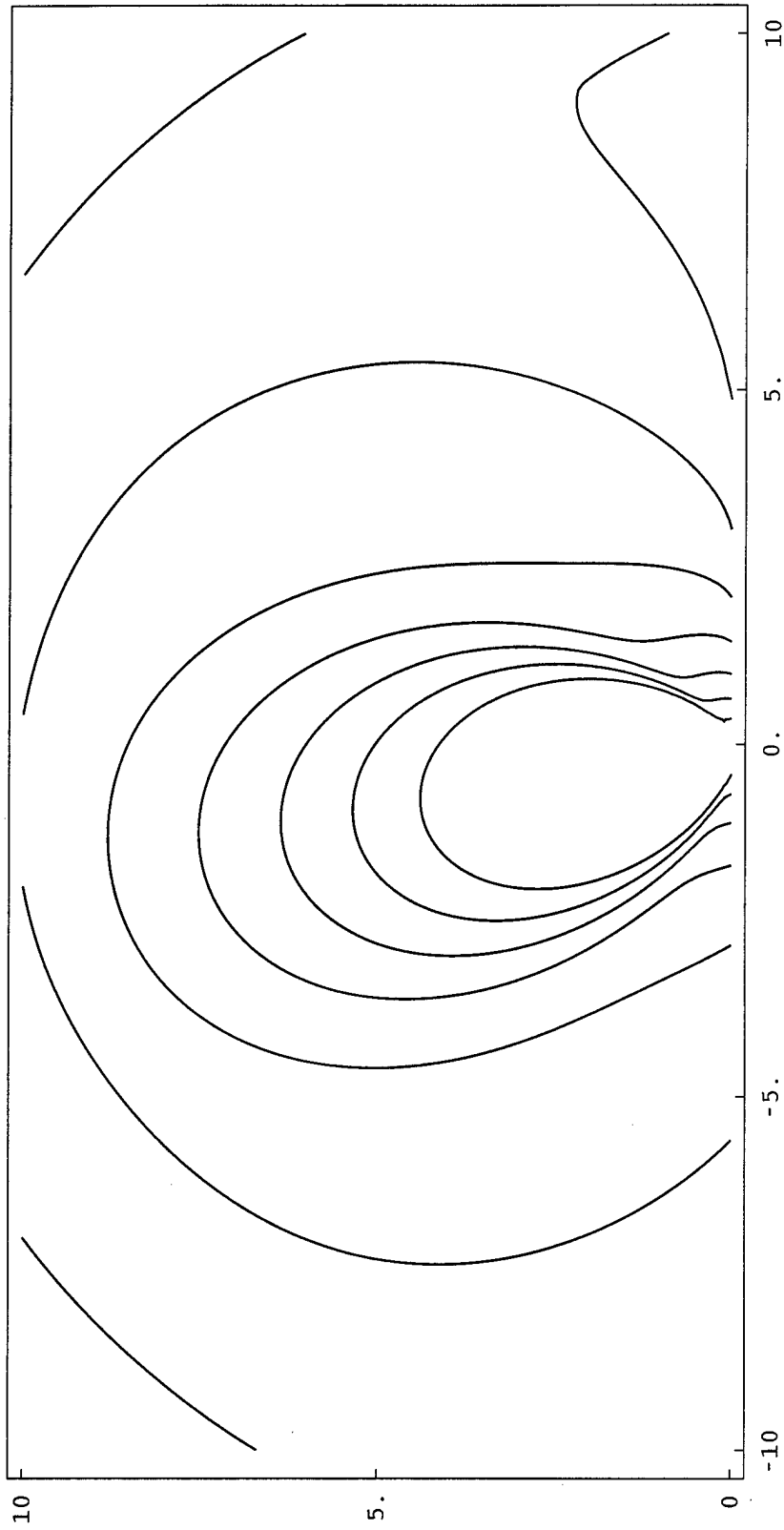


Figure 7.  $P$  at (5, 8, 14, 22, 37, 61, 100) psi.

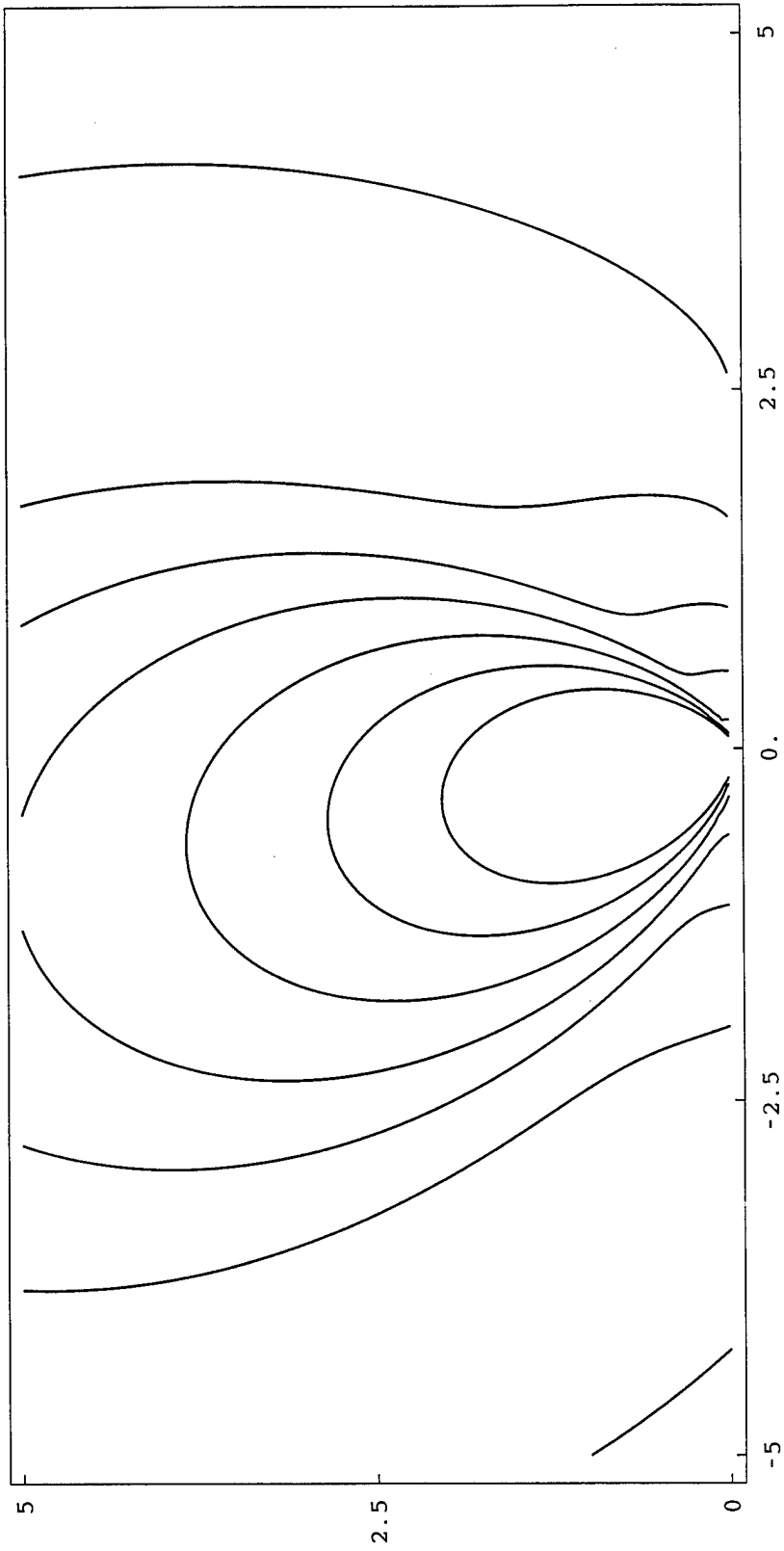


Figure 8.  $P$  at (10, 19, 37, 71, 136, 261, 500) psi.

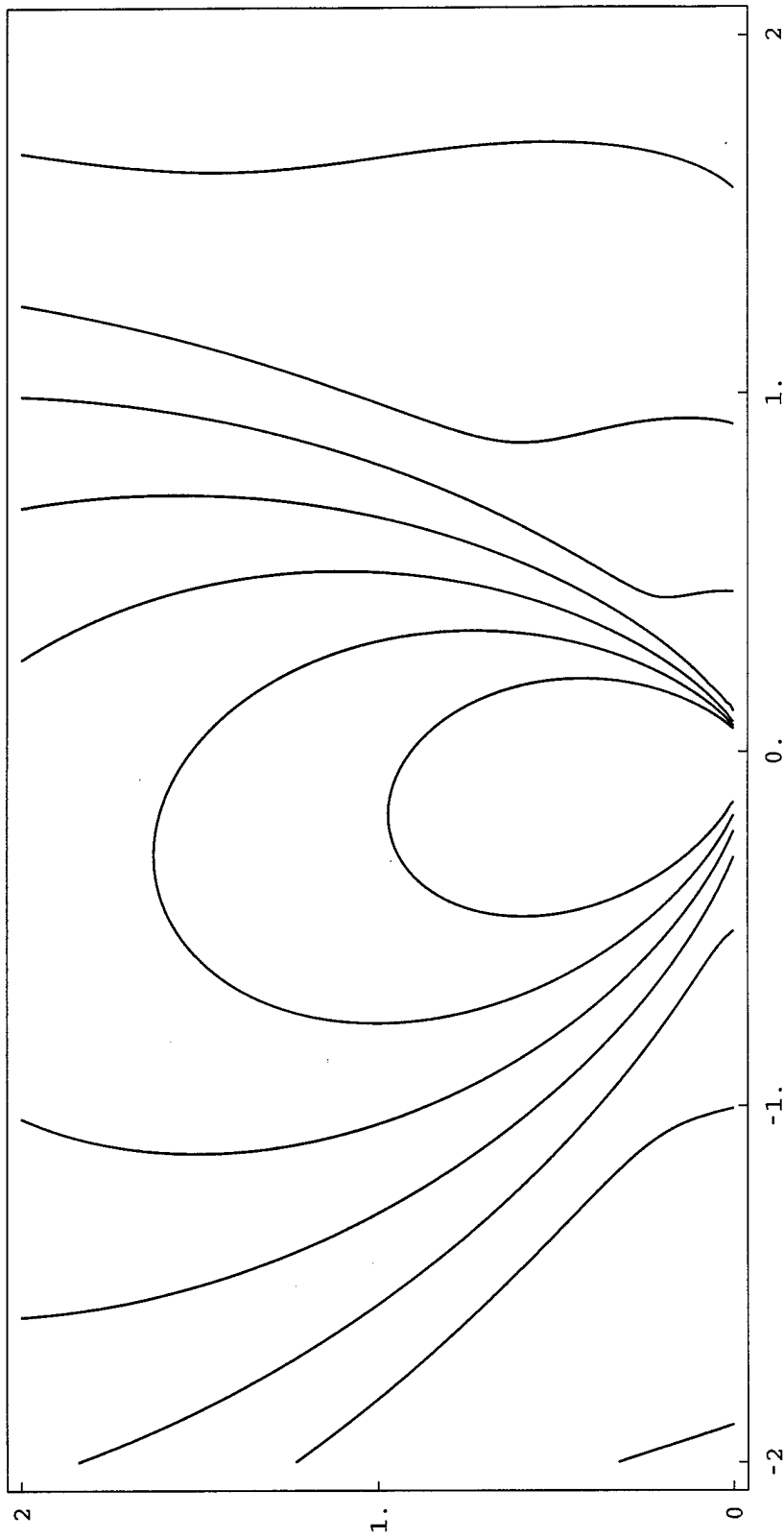


Figure 9. P at (20, 41, 84, 173, 356, 730, 1,500) psi.

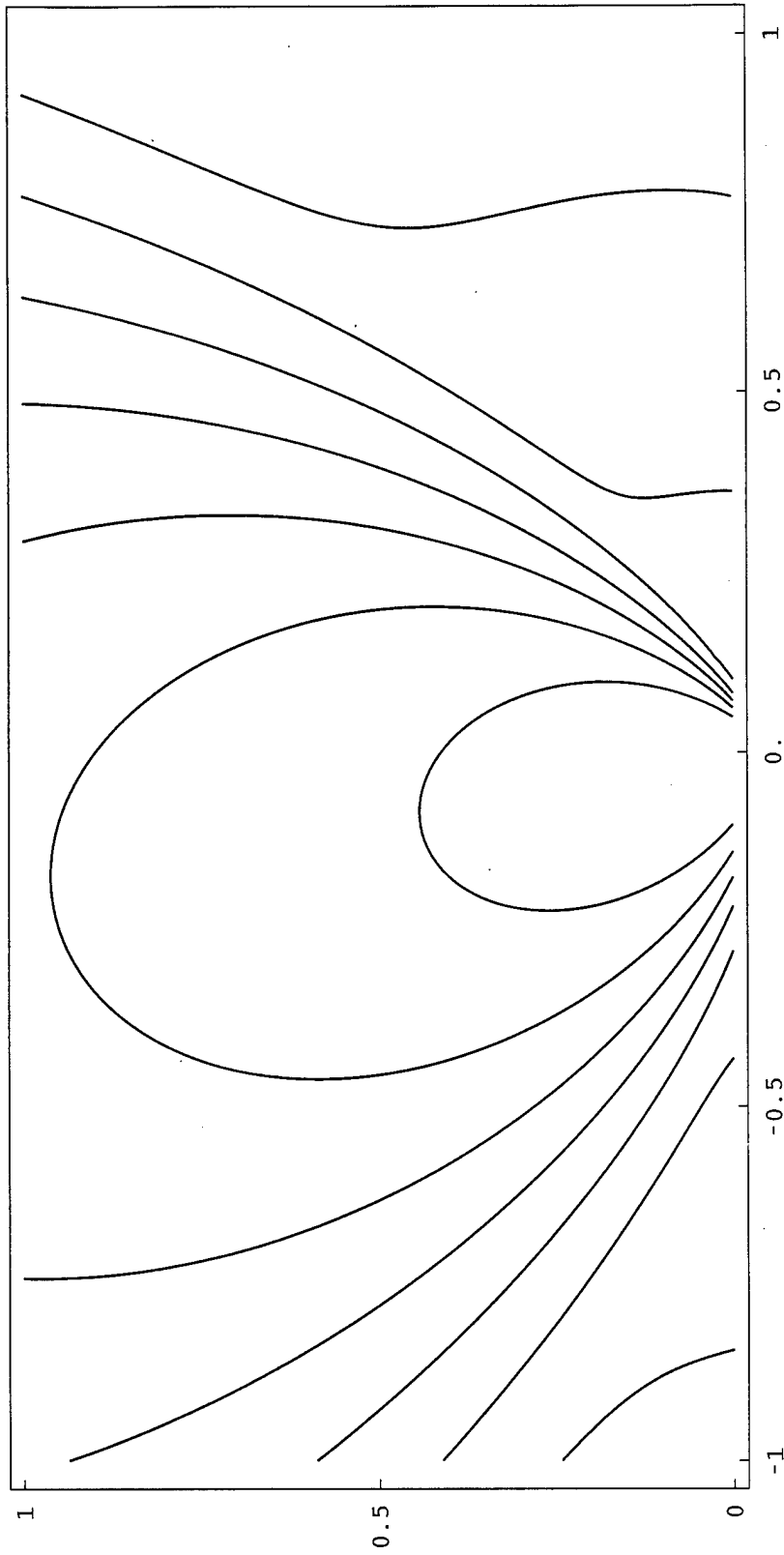


Figure 10.  $P$  at (50, 99, 196, 387, 766, 1,516, 3,000) psi.

## 6. References

1. Litt, O. Presentation at meeting. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, June 26, 1998.
2. Litt, O. Presentation at meeting. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, July 8, 1998.
3. Litt, O. Presentation at meeting. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, July 15, 1998.
4. Litt, O. Presentation at meeting. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, July 22, 1998.
5. Litt, O. Presentation at meeting. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, August 5, 1998.
6. Wolfram, S. *Mathematica, A System for Doing Mathematics by Computer*. 2nd edition, Addison-Wesley Publishing Company, Inc., Redwood City, CA, 1991.

INTENTIONALLY LEFT BLANK.

## **Appendix: Mathematica Code**

INTENTIONALLY LEFT BLANK.

This following Mathematica code is provided for function evaluation and visualization. This environment is useful for preliminary investigation, function selection, and experiment design. Estimation procedures are not provided. Mathematica names are generally consistent with names in the body of this report.

**A.1 Evaluation.** Define the utility functions Seq and Lseq to create linear and logarithmic sequences. The resulting sequences range from a to b and contain n elements.

```
(* UTILITY FUNCTIONS *)
Clear[Seq, Lseq];
Seq[a_, b_, n_] := Range[a, b, (b-a)/(n-1)];
Lseq[a_, b_, n_] := Exp[Seq[Log[a], Log[b], n]];
```

Define the model constants and parameters Ao, Bo, Co, w, alpha, m, No, Mo, z0, z1, and z2.

```
(* PARAMETERS & CONSTANTS *)
Clear[Ao, Bo, Co, w, alpha, m, No, Mo, z0, z1, z2];
(* sphere function parameters *)
Ao = 32.97;
Bo = 3.555;
Co = 0.5;
(* charge weight and scaling exponent *)
w = 1.0;
alpha = 1/3.0;
(* model parameters *)
m = 7/4;
No = 2.0;
Mo = 4;
(* transition function parameters *)
z0 = 10; z1 = 5; z2 = 15;
```

Define the model functions f01, f, n, k, g, gi, Psz, and P.

```
(* BLAST FUNCTIONS *)
Clear[f01, n, f, s, g, gi, Psz, P];
(* basic transition function *)
f01[x_] := 1/2 + ArcTan[x]/Pi;
(* generic transition function *)
f[z_, m_, r_] := f01[r(z-m)];
(* level curve exponent function *)
n[z_, No_] := No ( 1 - f[z, z0, 1]);
```

```
(* equivalent sphere weight function *)
s[z_, b_, z1_, z2_] := z/b /; z <= z1;
s[z_, b_, z1_, z2_] :=
  z/((b z2-z1)/(z2-z1) + (1-b)/(z2-z1)z) /; z1<z && z<z2;
s[z_, b_, z1_, z2_] := z /; z >= z2;
```

```
(* level curve function *)
g[z_, phi_, m_, No_, Mo_] := z Sin[m phi/2]^(4 n[z,No]/m);
```

```
(* level curve inverse function *)
gi[z_, phi_, m_, No_, Mo_] := Module[{u0, u1, u},
  If[g[z, phi, m, No, Mo]>z,
    For[u0=z, g[u0,phi,m,No,Mo] > z, u0=u0/3]; u1=3 u0,
    For[u1=z, g[u1,phi,m,No,Mo] < z, u1=3 u1]; u0=u1/3;
    u = FindRoot[g[u, phi, m, No, Mo] == z, {u, {u0, u1}},
      MaxIterations -> 25][[1]][[2]];
  u];
```

```
(* sphere charge function *)
Psz[z_] := Exp[Ao / ( z + Bo ) - Co];
```

```
(* peak overpressure model function *)
P[x_, y_] := Module[{r, phi, z, u0, u1, u, P},
  r = Sqrt[x^2+y^2];
  phi = ArcTan[x, y];
  z = r/w^alpha;
  u = gi[z, phi, m, No, Mo];
  P = f[z,z0,1]Psz[z]+(1-f[z,z0,1])Psz[s[u,Mo^alpha,z1,z2]];
  P]
```

**A.2 Visualization.** Define the function Ptable to evaluate  $P(x, y)$  on an  $n_x$  by  $n_y$  grid with  $-X1 \leq x \leq X1$  and  $dy \leq y \leq X1+dy$ .

```
Ptable[X1_, nn_] := Module[
  {nx, ny, dy = 0.05, X0, Y0, X, Y, Y1},
  X0 = -X1; Y0 = 0; Y1 = X1; nx = nn; ny = nn/2;
  X = Seq[X0, X1, nx];
  Y = dy+Seq[Y0, Y1, ny];
  XP = Table[P[X[[i]], Y[[j]]], {i, nx}, {j, ny}];
  {X0, X1, nx, Y0, Y1, ny, XP}]
```

Define the function Pshow to graph XP, the result of Ptable. The other arguments are the lowest contour level (L0), the highest contour level (L1), and the number of countour levels (NL).

```
Pshow[XP_, L0_, L1_, NL_] := Module[
  {X0, X1, nx, Y0, Y1, ny, Ftix, Mhue, Clevels, XF},
  X0 = XP[[1]]; X1 = XP[[2]]; nx = XP[[3]];
  Y0 = XP[[4]]; Y1 = XP[[5]]; ny = XP[[6]];
  PP = XP[[7]];
  Mhue[h_] = Hue[1, 0, 1];
  Ftix = {{{{1, X0}, {0.75 1 + 0.25 nx, 0.75 X0 + 0.25 X1},
            {0.5 1 + 0.5 nx, 0.5 X0 + 0.5 X1},
            {0.25 1 + 0.75 nx, 0.25 X0 + 0.75 X1}, {nx, X1}},
          {{1, Y0}, {0.5 1 + 0.5 ny, 0.5 Y0 + 0.5 Y1},
           {ny, Y1}}, None, None};
  Clevels = Log[Lseq[L0, L1, NL]];
  Print[N[Round[1 Exp[Clevells]] / 1]];
  XF = ListContourPlot[Transpose[Log[PP]],
    AspectRatio -> 1/2, ColorFunction -> Mhue,
    FrameTicks -> Ftix, Contours -> Clevels,
    ContourSmoothing -> 32];
  XF]
```

**A.3 Example Use.** The graphics in this report were produced by the following commands. First, create the numerical arrays. With  $nn=200$ , the functions are evaluated on a  $200 \times 100$  grid. This takes a while. Smaller values of  $nn$  can be used for quicker, lower resolution results.

```
nn = 200;
XP1 = Ptable[20, nn];
XP2 = Ptable[10, nn];
XP3 = Ptable[5, nn];
XP4 = Ptable[2, nn];
XP5 = Ptable[1, nn];
```

Then, construct the graphs.

```
XF1 = Pshow[XP1, 2, 20, 7]
XF2 = Pshow[XP2, 5, 100, 7]
XF3 = Pshow[XP3, 10, 500, 7]
XF4 = Pshow[XP4, 20, 1500, 7]
XF5 = Pshow[XP5, 50, 3000, 7]
```

Finally, export the graphics files.

```
IS = {600, 600};  
Display["xf1.ps", XF1, "EPS", ImageSize -> IS];  
Display["xf2.ps", XF2, "EPS", ImageSize -> IS];  
Display["xf3.ps", XF3, "EPS", ImageSize -> IS];  
Display["xf4.ps", XF4, "EPS", ImageSize -> IS];  
Display["xf5.ps", XF5, "EPS", ImageSize -> IS];
```

<u>NO. OF COPIES</u>	<u>ORGANIZATION</u>
2	DEFENSE TECHNICAL INFORMATION CENTER DTIC DDA 8725 JOHN J KINGMAN RD STE 0944 FT BELVOIR VA 22060-6218
1	HQDA DAMO FDQ D SCHMIDT 400 ARMY PENTAGON WASHINGTON DC 20310-0460
1	OSD OUSD(A&T)/ODDDR&E(R) R J TREW THE PENTAGON WASHINGTON DC 20301-7100
1	DPTY CG FOR RDE HQ US ARMY MATERIEL CMD AMCRD MG CALDWELL 5001 EISENHOWER AVE ALEXANDRIA VA 22333-0001
1	INST FOR ADVNCD TCHNLGY THE UNIV OF TEXAS AT AUSTIN PO BOX 202797 AUSTIN TX 78720-2797
1	DARPA B KASPAR 3701 N FAIRFAX DR ARLINGTON VA 22203-1714
1	NAVAL SURFACE WARFARE CTR CODE B07 J PENNELLA 17320 DAHLGREN RD BLDG 1470 RM 1101 DAHLGREN VA 22448-5100
1	US MILITARY ACADEMY MATH SCI CTR OF EXCELLENCE DEPT OF MATHEMATICAL SCI MAJ M D PHILLIPS THAYER HALL WEST POINT NY 10996-1786

<u>NO. OF COPIES</u>	<u>ORGANIZATION</u>
1	DIRECTOR US ARMY RESEARCH LAB AMSRL DD J J ROCCHIO 2800 POWDER MILL RD ADELPHI MD 20783-1145
1	DIRECTOR US ARMY RESEARCH LAB AMSRL CS AS (RECORDS MGMT) 2800 POWDER MILL RD ADELPHI MD 20783-1145
3	DIRECTOR US ARMY RESEARCH LAB AMSRL CI LL 2800 POWDER MILL RD ADELPHI MD 20783-1145
	<u>ABERDEEN PROVING GROUND</u>
4	DIR USARL AMSRL CI LP (305)

NO. OF  
COPIES

ORGANIZATION

ABERDEEN PROVING GROUND

17 DIR USARL  
AMSRL SL  
J WADE  
AMSRL SL B  
J SMITH  
W WINNER  
AMSRL SL BE  
W BAKER  
D BELY  
J COLLINS (5 CPS)  
O LITT  
L MOSS  
R SAUCIER  
R SHNIDMAN  
AMSRL SL BA  
M RITONDO  
E DAVISSON  
AMSRL SL BD  
L MORRISSEY

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE July 1999	3. REPORT TYPE AND DATES COVERED Final, June 1998 - July 1998	
4. TITLE AND SUBTITLE  Analytical Blast Model Formulation With Computer Code			5. FUNDING NUMBERS	
6. AUTHOR(S)  Joseph Collins				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  U.S. Army Research Laboratory ATTN: AMSRL-SL-BE Aberdeen Proving Ground, MD 21005-5068			8. PERFORMING ORGANIZATION REPORT NUMBER  ARL-TR-2009	
9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  Overpressure time history data from warhead blast experiments yield peak overpressure $P$ as a function of spatial position.  Dr. Owen Litt has proposed a model for $P$ based on the peak-overpressure characteristics of a bare spherical charge. The direction-independent peak-overpressure function for a bare spherical charge is modified to have nonspherical level-surface structure by specifying surfaces of constant peak overpressure. This introduces a directional component into the model.  In this report, the original formulation is refined and generalized and a mathematical model and computer code are presented to evaluate the function. Such a computational device is required for model parameter estimation and experiment design.				
14. SUBJECT TERMS  computer model, blast, mathematical modeling			15. NUMBER OF PAGES  32	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT  UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE  UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT  UNCLASSIFIED	20. LIMITATION OF ABSTRACT  UL	

INTENTIONALLY LEFT BLANK.

## USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. ARL Report Number/Author ARL-TR-2009 (Collins) Date of Report June 1999

2. Date Report Received \_\_\_\_\_

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

4. Specifically, how is the report being used? (Information source, design data, procedure, source of ideas, etc.) \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided, or efficiencies achieved, etc? If so, please elaborate. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.) \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

CURRENT  
ADDRESS

\_\_\_\_\_  
Organization

\_\_\_\_\_  
Name

\_\_\_\_\_  
E-mail Name

\_\_\_\_\_  
Street or P.O. Box No.

\_\_\_\_\_  
City, State, Zip Code

7. If indicating a Change of Address or Address Correction, please provide the Current or Correct address above and the Old or Incorrect address below.

OLD  
ADDRESS

\_\_\_\_\_  
Organization

\_\_\_\_\_  
Name

\_\_\_\_\_  
Street or P.O. Box No.

\_\_\_\_\_  
City, State, Zip Code

(Remove this sheet, fold as indicated, tape closed, and mail.)  
**(DO NOT STAPLE)**