

**UNITED STATES AIR FORCE
RESEARCH LABORATORY**

**THE BENEFITS FROM BETTER TESTING
AND TRAINING:
A COMPREHENSIVE FRAMEWORK**

Burke Burreight

**AIR FORCE RESEARCH LABORATORY
HUMAN EFFECTIVENESS DIRECTORATE
MISSION CRITICAL SKILLS DIVISION
7909 Lindbergh Drive
Brooks AFB, Texas 78235-5352**

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**BURKE BURRIGHT
Project Scientist**

**R. BRUCE GOULD, Ph.D.
Acting Chief
Mission Critical Skills Division**

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Summary

This paper puts forward a new paradigm for thinking about how better ways for assessing candidates and for training employees increase efficiency. It transcends the limitations of the two currently most popular paradigms for doing so: the psychologists' "utility" models and economists' decentralized matching models. In contrast to these models, this new paradigm identifies and includes three distinct ways by which improved assessment methods increase efficiency. It also points the way towards a method for quantifying all the benefits of using improved methods of assessment candidates. Moreover, this new paradigm can deal with the benefits of improved training methods as well as those of improved assessment methods. So, it provides a contest in which an analyst can trade-off expenditures for improving assessment methods against expenditures for improving training methods.

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The paper begins with the concept of an applicant's expected first-term productivity. Then, it lays out the structure of the mathematical programming model. Next, the paper explains how improvements in assessment methods and training programs can be linked to the model. Finally, it shows how the dual of the programming model and its shadow prices can be used to estimate the benefits of the improvement.

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The Benefits from Better Testing and Training:

A Comprehensive Framework

I. Introduction

A traditional example ...of comparative advantage is the case of the best lawyer in town who is also the best typist in town. Will she not specialize in law and leave typing to a secretary? How can she afford to give up precious time from the legal field, where her comparative advantage is very great, to perform typing activities in which she is efficient but in which she lacks comparative advantage? Or look at it from the secretary's point of view. She is less efficient than the lawyer in both activities; but her relative disadvantage compared with the lawyer's is least in typing. Relatively speaking, the secretary has a comparative advantage in typing (Samuelson, p. 669)

Better methods of evaluating applicants and predicting their performance in Air Force jobs can increase the efficiency with which the Air Force accomplishes its mission in three distinct ways.

1. Selecting More Productive Applicants. Better methods increase the Air Forces' chances of selecting more productive individuals from its applicant pool.
2. Capitalizing on Individuals' Relative Strengths. As the opening quotation from Samuelson's *Economics* suggests, the Air Force can increase its efficiency by assigning selected individuals to jobs in which they have comparative advantages.
3. Cutting Staffing Safety Margins. Some recruits will fail to perform as well as expected. So, to ensure that its can carry out its missions, the Air Force has to provide for staffing safety margins. Predicting individuals' job performance for accurately means that the staffing safety margins required to ensure mission accomplishment could be cut.

Industrial psychologists have addressed the benefits of testing within their "utility model" framework (Casio, 1987, pp. 270-296; Roach. 1984). Their framework assumes one or more openings for a single kind of job. It deals with the

interrelationships between five variables: (1) the ratio of openings to applicants, (2) the job performance measure, (3) the correlation of the selection score and job performance, (4) the cutoff test score, and (5) the cost of testing.

The utility model framework deals only with the first of the three ways of increasing efficiency—selecting more productive applications. Since it assumes a single kind of job, it cannot deal with the efficiency gains available through more fully exploiting selected individuals' comparative advantages. Moreover, the framework has not been used to address the required size of the staffing safety margins.

In the 1960's, economists began to develop another framework for studying assignment problems (Gale & Shapley, 1992). Dale Mortensen (1988) surveyed their approach and contributions. Their framework, which has been applied to both the job market and the marriage market (Becker, 1973), assumes a large number of agents who are searching for employees and jobs or husbands and wives.

Within this framework, economists have focused on identifying the conditions under which decentralized search processes would result in an optimal set of matches and on the incident of search and information production costs (MacDonald, 1980). Most the specific models within this general framework reflect extreme assumptions concerning what is known about job applicants and potential mates; these models assume either job performance and marital happiness can be predicted perfectly and costlessly when a contact is first made or they cannot be known until the persons hired or married. Both its focus of decentralized decision making and its extreme assumption about what is known about applicants and potential mates limits its usefulness as a bases for estimating the benefits to the Air Force of better ways for evaluating applicants.

This paper puts forward a new framework for understanding the benefits provided by better ways for evaluating applicants. This new framework transcends the limitations of both psychologists' utility model and economists' decentralized matching models. It encompasses all three ways by which better assessment methods can increase the Air Force's efficiency and points towards a method for quantifying the comprehensive benefits of better assessment methods. In addition, the new framework is general enough so that one can deal with the benefits of better training methods as well as those of better assessment methods. So, it provides an analytic paradigm in which an analyst can compare the benefits and costs of better assessment methods with those of better training methods.

This framework extends a fifty-year old tradition in applying mathematical programming techniques to improve job assignment processes in the military services (D. F. Votaw & J. T. Daily, 1952; J. H. Ward, Jr., 1959; R.L. Fernandez & J. B. Garfinkle, 1985; R. L. Fernandez, 1987; M. Pina, M. S. Emerson, D. L. Leighton, & W. Cummings, 1988). Its used both new ways of representing quantitatively on-the-job performance and stochastic programming techniques to

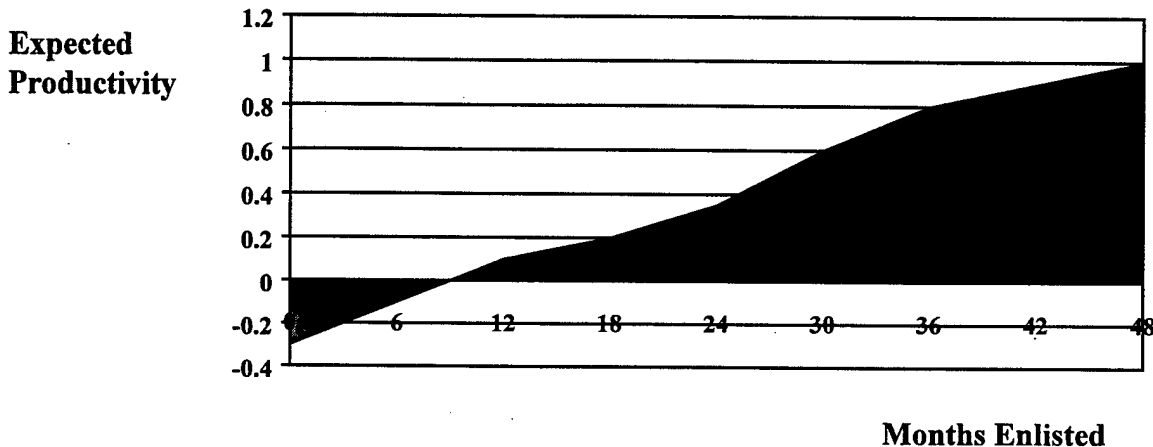
deal with improvements in assessment and training methods in a mathematical programming structure.

The next sections develop the analysis. First, the concept of an applicant's expected first- term productivity is described and explained. Then, the structure of the mathematical programming model is laid out. The paper then turns to how improvements in assessment methods and training programs could be linked to the model. Next, the use of the dual of the programming model and its shadow prices to estimate the benefits of the improvement are explained. Finally, the implications of the new framework for assessing the benefits of improving Air Force selection, classification, and training processes are discussed.

II. Expected First-Term Productivity.

Once a recruit is trained and given his first assignment, his productivity will increase as he acquires experience. For an Air Force Specialty Code (AFSC), an applicant's expected total first-term productivity can be represented by the area under the curve in Figure 1.

Figure 1
Expected First-Term Productivity
of Applicant i in AFSC i



In Figure 1, applicant j 's productivity in AFSC i is expressed at a specific point in this first-term career— $y(t)_{ij}$ —as a fraction of some standard. The standard used in this study is the average productivity of airmen in AFSC i with journeyman skills and with four years of service. Other benchmarks are possible and could have been chosen without altering the logic of this study.

As shown in Figure 1, an initial section of applicant j 's first-term productivity curve might lie below the horizontal axis. Such an initial below-the-axis segment implies negative productivity during the initial months of applicant j 's first job assignment in AFSC i . Negative productivity would occur if other members of his first unit had to reduce their contributions to unit output to train him and if their reductions exceeded his initial contribution to his unit's output.

The productivity curve concept has strong empirical support. Haggstrom, Chow, and Gay (1984) fitted curves for 48 enlisted occupations in the Army, Navy, and Air Force using data from almost 30,000 supervisor surveys; the surveys ask supervisors to provide their subjective evaluations of productivity contributions of specific individuals. Carpenter and Monaco (1987, pp. 32-45) derived a productivity curve for the Avionics Communications Maintenance Specialty based on supervisors' assessments of the times that airmen required to perform representative clusters of tasks. Fernandez (1987, pp. 23-25) refers to data

collected by the RAND Corp. on the actual times required to complete specific tasks in the Aerospace Ground Equipment Specialty; it also supports the hypotheses of a rising productivity curve.

The height of the first-term productivity curve has been related to measures of ability. Marcus and Quester (1984) demonstrated a positive relationship for Navy using subjective evaluation data from surveys. Carpenter and Monaco (1987) demonstrated a positive overall relationship for the Avionics Communications Maintenance Specialty using subjective estimates of completion times for task clusters,

Total first-term productivity is defined as the area between the productivity curve and the horizontal axis. In Haggstrom, Chow, and Gay's results (1984, pp. 72-93), first-term productivity averaged 1.9q equivalent man-years of five "high-skill" Air Force Specialties (AFSs), 2.34 equivalent man-years for eleven "medium skill" AFSs, and 2.56 equivalent man-years for six "low skill" AFSs.

Many selectees will not remain in the Air Force for all four years of their first terms. Define $r(t)_{ij}$ as the likelihood that applicant j will be on active duty in month t if he is assigned to AFSC i . Then the expected first term productivity of applicant j if he would be assigned to AFSC i -- m_{ij} —is defined by the integral:

$$m_{ij} = \int_0^{48} r(t)_{ij} y(t)_{ij} dt \quad (1)$$

III. An Analytic Framework

This section outlines a framework for estimating the benefits of better assessment methods and training programs. It applies to a cohort of applicants for accession to the Air Force. Each of n applicants can be selected for one of M AFSC's or can be rejected. Table 1 summarizes the definitions of the symbols used to express this framework.

Setting the Goal: Minimum Cost. This framework assumes that the Air Force wants to minimize the discounted total first term costs --- C^* ---of meeting productivity capacity requirements for each AFSC with specific confidence levels. Total first term costs is defined as:

$$C^* = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \quad (2)$$

Here C_{ij} stands for the total discounted first-term costs expected if applicant j is selected and assigned to AFSC i . It includes the costs of recruiting, training, transporting, and maintaining applicant j if he is selected and assigned to AFSC i and remains on active duty. The total cost for each month is multiplied by the probability that applicant j will still be on active duty during that month. While the task of bringing together the data to calculate the C_{ij} 's might require substantial effort, it does not pose significant conceptual or data-availability problems.

The variable x_{ij} is restricted to the values zero and one. If applicant j is selected and assigned to AFSC i , then $x_{ij} = 1$. Otherwise, $x_{ij} = 0$. From a mathematical perspective, the problem is to identify the sets of $x_{ij} = 1$ that minimizes Eq. 2 while ensuring that the constraints discussed below are met.

Identifying the Constraints. The framework assumes that the Air Force needs to obtain R_i of total productive capacity from the cohort of each AFSC i . It also assumes that, recognizing the random error inherent in forecasting the first-term productivities, the Air Force wants the probabilities of not obtaining the R_i 's be no greater than some set of p_i 's ($0 < p_i < 1$). Drawing on stochastic programming methods, (Taha, 1982, pp. 784-788), the m constrains can be written in the form:

$$P_i \left\{ \int_{j=1}^n m_{ij} x_{ij} \geq R_i \right\} \geq 1 - p_i \quad (3)$$

for $i = 1, \dots, m$.

Table 1

Symbols Used in Mathematical Expressions

<u>Symbol</u>	<u>Definition</u>
a	The expected increase in journeyman-equivalent man-years during the first term in specialty I associated with one unit increase in the on the test of evaluation criteria. $k = 0, 1, \dots, K$
b^l	The values at the l breakpoints of the approximation internal for the specialty $l = 0, L$. They are required for the separable programming solution method used in this paper.
C^*	The total first-term cost of satisfying journeyman-equivalent man-year requirements $R_1, R_2, \dots, R_i, \dots, R_m$ with indicated confident levels
C_{ij}	The expected first-term cost of assigning applicant j to specialty i . It includes the costs of recruitment, training, and transportation as well as the costs of pay and allowances. Each month's costs in multiplied by the probability that airman j will still be on active duty in that month if assigned to specialty i .
H_i	The matrix of test scores used to estimate the journeyman-equivalent man-years equation for specialty i .
h^k_{ij}	The score of the j th applicant on the k th test or factor used to predict journeyman-equivalent man-years for specialty i . $k = 1, \dots, K$.
h_{ij}	The vector of scores for applicant j expressed as the difference from the mean of the scores used to estimate the equation for specialty i .
m_{ij}	Expected first-term productive capacity: the journeyman-equivalent man-years during the first term if applicant j is assigned to specialty i .
R_i	The number of journeyman-equivalent man-years required in specialty I from members of the cohort.
$r(t)_{ij}$	The probability that applicant j will be on active duty at t months if assigned to AFSC i .

S^a	The slack variable associated with applicant j. If applicant j is selected to enter the Air Force, $S^a = 0$. If applicant j is not accepted, $S^a = 1$.
S^s	The surplus variable associated with the man-years requirement for the ith specialty.
s^2	The common variance of the linear regression equation used to predict journeyman-equivalent man-years for specialty i.
t^p	The standard normal value required to ensure that the journeyman-equivalent man-year requirement (R_i) is met or exceeded with a probability of $1-p$ in specialty i.
$VAR(m_{ij})$	The variance of the jth applicant's journeyman-equivalent man-years during his first term.
W^l	The non-negative weights associated with the l breakpoints of the approximation interval for specialty i. No more than two adjoining weights can be positive. They are required for a separable programming solution method. $l = 0, \dots, L$.
x_{ij}	Indicates whether applicant j is assigned to specialty i. If applicant j is assigned to specialty i, the $x_{ij} = 1$. Otherwise, $x_{ij} = 0$.
$y(t)_{ij}$	The expected productivity of applicant j in AFSC i at t months on the job as a fraction of the average productivity of journeymen airmen with four years of service.
Z^a	The change in total first term cost if a second applicant identical to applicant j was available to enlist; the "shadow price" of applicant j.
Z^p	The change in the total first-term costs associated with an increase in the sum of the variance for the ith specialty by one journeyman-equivalent man-year.
Z^r	The change in the total first-term cost associated with increasing the specialty's requirement (R_i) by one journeyman equivalent man-year. The "shadow cost" of the ith specialty's requirement.
Z^w	The change in total first-term costs associated with the ith specialty's staffing safety margin.

Assuming that each m_{ij} is normally distributed and defining

$$d_i = \sum_{j=1}^n m_{ij} x_{ij}$$

Then, d_i is normally distributed with :

$$E\{d_i\} = \sum_{j=1}^n E\{m_{ij}\} x_{ij}$$

and

$$VAR\{d_i\} = M^T D_i M$$

where

$$M_i = (m_{i1}, \dots, m_{in})$$

$$D_i = \begin{pmatrix} VAR(m_{i1}) & \dots & COV(m_{i1}, m_{in}) \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ COV(m_{in}, m_{i1}) & \dots & VAR(m_{in}) \end{pmatrix}$$

Now,

$$P_i\{d_i \geq R_i\} = P_i\left\{\frac{d_i - E(d_i)}{\sqrt{VAR(d_i)}} - \frac{R_i - E(d_i)}{\sqrt{VAR(d_i)}}\right\} \geq 1 - P_i$$

Where

$$\left(d_i - E(d_i) / \sqrt{VAR(d_i)}\right)$$

is the standard normal variable with mean zero and variance one. This means:

$$P_i\{d_i \geq R_i\} = \Phi\left[\left(\frac{R_i - E(d_i)}{\sqrt{VAR(d_i)}}\right)\right]$$

where Φ is the CDF of the standard normal function. Let t_i^p be the standard normal variable such that:

$$\Phi(t_i^p) = 1 - p_i$$

Then, the statement

$$P_i\{d_i \geq R_i\} \geq 1 - p_i$$

Is realized if and only if

$$\frac{R_i - E_i(d_i)}{\sqrt{VAR(d_i)}} < t_i^p$$

This yields the following nonlinear constraints:

$$\sum_{i=1}^n E\{m_{ij}\} x_{ij} - t_i^p \sqrt{M^T DiM} \geq R_i \quad (5)$$

for $i = 1, \dots, m$. These nonlinear constraints are equivalent to the original stochastic. Since the distributions of the m_{ij} are independent,

$$\text{COV}(m_{ij}, \dots, m_{ij}) = 0$$

And Eq. (5) can be written,

$$\sum_{j=1}^n E \{ m_{ij} \} x_{ij} - t_i^p \sqrt{\sum_{j=1}^n \text{VAR} (m_{ij}) x_{ij}^2} \geq R_i \quad (6)$$

For $i = 1, \dots, m$.

Eq. 6 makes clear an important implication of the framework. When person j is assigned to AFSC i , he will contribute both positively and negatively towards satisfying the total product capacity requirement for his AFSC. His positive contribution is represented by the expression $E(m_{ij})$ —his expected equivalent man-years. The negative aspect is represented by his

$$\text{VAR} (m_{ij}) x_{ij}^2$$

portion of the second term on the left-hand side of Eq 6. The second term stands for the staffing safety margin necessary to ensure the R_i is met or exceeded $1 - p_i$ percent of the time. Selecting and assigning individuals whose productivity capacities can be forecast only with large errors is more costly because the required staffing safety margins must be larger. A smaller $\text{VAR}(m_{ij})$ is as much a cost-reducing attribute as is a larger m_{ij} .

One can solve a programming problem with nonlinear constrains like Eq. 6 with separable programming methods. Following Tana (1982, pp. 768-774), one can put the constraints in separable programming form with the substitution:

$$q_i = \sqrt{\sum_{j=1}^n \text{VAR} (m_{ij}) x_{ij}^2}$$

So, each of the original constraints are equivalent to the following

$$\sum_{j=1}^n E (m_{ij}) x_{ij} - t_i^p q_i \geq R_i \quad (7 a)$$

and

$$\sum_{j=1}^n \text{VAR} (m_{ij}) x_{ij}^2 - q_i^2 = 0$$

for $i = 1, \dots, m$. Since $x_{ij} = 0, 1$, one can write the second constraint:

$$\sum_{j=1}^n \text{VAR} (m_{ij}) x_{ij} - q_i^2 = 0 \quad (7b)$$

One can eliminate the nonlinearities arising from the q_i^2 terms by dividing the possible ranges for the q_i 's into l segments. Let b_i^l stand for the values of q_i at the limiting values of its range and at its breakpoints. Then,

$$q_i \cong \sum_{l=0}^L b_i^l w_i^l$$

and

$$q_i^2 \cong \sum_{l=0}^L (b_i^l)^2 w_i^l$$

where the w_i^l 's are weights. The larger the number of breakeven points, the better the approximation. No more than two adjoining w_i^l 's can be positive and their sum must equal one, e.g.;

$$\sum_{l=0}^L w_i^l = 1 \quad (8)$$

for $i = 1, \dots, m$.

Substituting for q_i and q_i^2 in Eqs. 7a and 7b, respectively,

$$\sum_{j=1}^n E(m_{ij}) x_{ij} - t_i^p \sum_{l=0}^L b_i^l w_i^l \geq R_i \quad (9a)$$

and,

$$\sum_{j=1}^n VAR (m_{ij}) x_{ij} - \sum_{l=0}^L (b_i^l)^2 w_i^l = 0 \quad (9b)$$

for $i = 1, \dots, m$.

Another set of constraints is necessary to ensure that applicant i is not assigned to more than one position,

$$\sum_{i=1}^m x_{ij} \leq 1 \quad (10)$$

Assembling the Analytic Framework. We are not ready to assemble the analytic framework. It involves minimizing the expression at Eq. 2 subject to the expressions at Eqs. 9a, 9b, 8, and 10. The framework can be restated in standard form as follows:

$$MinC^* = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} + \sum_{i=1}^m \sum_{l=0}^L (0) w_i^l + \sum_{i=1}^m (0) S_i^s + \sum_{j=1}^n (0) S_j^a \quad (2')$$

subject to:

$$\sum_{j=1}^n E(m_{ij}) x_{ij} - t_i^p \sum_{l=0}^L b_i^l w_i^l - S_i^s = R_i \quad (9a')$$

for $i = 1, \dots, m$.

$$\sum_{j=1}^n VAR (m_{ij}) x_{ij} - \sum_{l=0}^L (b_i^l)^2 w_i^l = 0 \quad (9b)$$

for $i = 1, \dots, m$.

$$\sum_{l=0}^L w_i^l = 1 \quad (8)$$

for $i = 1, \dots, m$.

$$\sum_{i=1}^m x_{ij} + S_j^a = 1 \quad (10')$$

for $j = 1, \dots, n$.

$$x_{ij} = 0, 1; w_i^l, S_i^s, S_j^a \geq 0$$

Here, the S_i^s 's are the surplus variables for the AFSC required productive capacity constraints and the S_j^a 's are the slack variables in the applicant constraints. A particular S_i^s is positive if the cost minimizing selection and assignment of applicants involves assigning more productive capacity to the i th AFSC than is required to meet R_i with the required confidence level. A particular $S_j^a = 1$ if applicant j is not selected for Air Force service.

Solving the Problem. The above framework represents a zero-one mixed integer-programming problem. Taha (1975, pp. 113-119, 154-162, and 346-347) has identified and described four possible algorithms for solving zero-one mixed integer programs.

However, an off-the-shelf computerized algorithm might not be available for solving the above problem. The restriction on the w_i^l weights mentioned above reduce the likelihood that such an off-the-shelf algorithm exists. Recall that for a particular AFSC i , no more than two w_i^l 's can be positive and that the two positive w_i^l 's must be adjacent. This restriction means that the corresponding continuous linear programming problem, which usually must be solved to provide a starting point for the zero-one mixed integer algorithm, must be solved under the conditions of restricted bases (Taha, 1975, pp. 771-774). The implications of restricted bases for solving the zero-one mixed integer problem does not seem to have been fully explored in the operations research literature.

IV. The Benefits and Better Testing and Training Methods

The analytic framework constructed above provides ways of estimating the benefits to the Air Force of improvements in assessment methods and in training programs. It provides a single context in which the benefits of alternative improvements in assessment methods and training programs can be compared.

The Benefits of Improving Assessment Methods. Better assessment methods predict job performance more accurately. The measure of job performance used in this paper is journeyman-equivalent man-years (m_{ij}). The simplest improvement in an assessment method is one limited to a single AFSC. Within the analytic framework, this kind of improvement would translate into reductions in the $\text{VAR}(m_{ij})$'s for the i th AFSC. So, the $\sum \text{VAR}(m_{ij})x_{ij}$ term in the Eq. 9b for AFSC i above would fall. To maintain the equality of this Eq. 9b, the two positive adjacent w_I^1 weights must shift "downward towards (b^1)²'s with lower values. As a result, the $t^p \sum b^1 w_I^1$ term in the Eq. 9a' for AFSC i also must decline in value.

If the Eq. 9a' for the i th AFSC is binding (i.e., $S_i^s = 0$), then the term $\sum \text{VAR}(m_{ij})x_{ij}$ must fall. The Air Force will be able to satisfy at a $1-p_i$ level of confidence the total equivalent man-years requirement— R_i —by assigning fewer applicants to the i th AFSC. Since fewer applicants are required, the value of the objective function— C^* —will decline. The approximate benefit of the assessment method improvement is the difference between the values of C^* with the original assessment method and with the improved one.

If the constraint for AFSC i represented by Eq. 9a' is not binding (i.e., $S_i^s > 0$), f fall in the $\sum \text{VAR}(m_{ij})x_{ij}$ term might still lead to a decline in C^* . A decline would occur if the zero/one nature of the x_{ij} 's made it necessary to assign more than enough applicants to the i th AFSC in order to equal or exceed R_i . If the $\sum \text{VAR}(m_{ij})x_{ij}$ term falls enough to allow one or more "whole" applicants to be saved, then C^* could decline. However, a decline would not result if Eq. 9a's fails to be binding for some other reason.

This analytic framework can deal with more complex improvements in assessment methods.¹ It can deal with improvements for a group of AFSCs or for all AFSC's. In all cases, the benefit is the difference in the values of the objective function— C^* —before and after the improvement.

The Benefits of Improving Training Programs. In terms of this analytic framework, a new training program could have impacts via three routes.

- A new training program would alter the C_{ij} coefficients in Eq. 2 for the AFSC's served by the training program. A lower cost training program would reduce the C_{ij} 's and a higher cost one would increase them.
- A new training program would impact the expected journeyman-equivalent man-years term ---the $E(m_{ij})$'s---in Eqs 9a' for the impacted AFSC's. An increase in the $E(m_{ij})$'s would mean that fewer applicants would have to be to the AFSC's to ensure that their total equivalent man-year requirements (their R_T 's) were achieved with the desired level of confidences. For example, Nichols, Pokorny, Jones, Gott, and Alley (1989) have shown that an intelligent tutoring system can substantially increase trouble-shooting efficiency in an AFSC. Such an increase would represent an increase in expected journeyman-equivalent man-years.
- The outcomes of different training programs can have different variances (Bloom, 1984). The impact of a new training program with a lower variance would follow a path similar to the one described above for an improved assessment method.

A new training program that lowered the C_{ij} 's, the increased the $E(m_{ij})$'s, and that reduced the $VAR(m_{ij})$ could clearly provide cost avoidance benefits. Just as clearly, one that increased the C_{ij} 's, the reduced the $E(m_{ij})$, and that increased the $VAR(m_{ij})$'s would be wasteful. Whether other new programs would produce new positive benefits would depend on the directions and relative magnitudes of their three impacts.

V. Useful Information from the Dual Problem

Linear programming problems have associated dual problems, which can provide additional useful information. In this paper, the main or "primal" problem involves minimizing costs. If a primal a primal problem minimizes something, it dual problem will maximize something else; moreover, primal problem's minimum will equal its dual problem's maximum. Following Taha (1982, pp. 106-110), the dual problem can be written as follows:

$$MaxZ^* = \sum_{i=1}^m R_i Z_i^r + \sum_{i=1}^m (0) Z_i^p + \sum_{i=1}^m Z_i^w + \sum_{j=1}^n Z_j^a \quad (11)$$

subject to:

$$E(m_{ij}) Z_i^r + VAR(m_{ij}) Z_i^p + Z_j^a \leq C_{ij} \quad (12)$$

for all combinations of i and j, and subject to:

$$- t_i^p b_i^l Z_i^r + (b_i^l)^2 Z_i^p + Z_i^w \leq 0 \quad (13)$$

for each l within each i, and subject to :

$$Z_i^r \geq 0; Z_i^p, Z_i^w \text{ unrestricted ed}; Z_j^a \leq 0$$

The primal problem represents an attempt to minimize costs subject to constraints imposed by staffing and staffing safety margin requirements for each specialty, by our capability to predict on the job performance in each specialty, and by characteristics of the individuals in the applicant pool. In the dual problem, Z^* represents that sum of the implied values of the specialty staffing and staffing safety margin requirements, of the capability to predict one the job performance in each specialty, and of particular members of the applicant pool. The dual problem maximizes the implied values of the constraints. In other words, the solution that minimizes the costs of meeting the constraints maximizes the marginal values of "loosing" the constraints that have proven to be binding.

The other four Z variables are the implied or "shadow" marginal cost of meeting each of the constrains in the primal problem. There is a Z variable for each constraint in the primal problem; the these four Z variables correspond to the four kinds of constraints equations in the primal problem. The can be interpreted as follows:

- Z_i^r the change in the total first-term costs that would result from increasing the *i*th speciality's staffing requirement (R_i) by one journeyman-equivalent man-year. It represents the marginal shadow cost of the *i*th speciality's man-year requirement.
- Z_i^p the change in the total first-term costs that would result from increasing the sum of the variances in predicted performance in the *i*th specialty by one journeyman-equivalent man-year. The negative of this value provides an estimate of the incremental value of predicting on-the-job performance in the *i*th specialty more accurately.
- Z_i^w the change in total first-term costs that would result from increasing the *i*th speciality's staffing safety margin.
- Z_i^a the change in total first term cost that would result if another applicant identical to applicant *j* in all ways was available to enlist. It represents the incremental value of adding another applicant like applicant *j* to the applicant pool.

The presence of the x_{ij} integer variables in the primal problem can cause the values of some of the shadow cost variables to be misleading (Baumol, 1965, pp.163-166). The shadow cost variables corresponding to primal problem constraints that contain integer variables could provide misleading information about costs. Procedures have been developed to recalculate such shadow cost variables so that they reflect better economic costs (Alcaldy & Klevorick, 1966). However, they would not be needed for our analysis, since it uses the Z_i^w shadow costs and the Z_i^w 's correspond to constraints in the primal problem that contain only continuous w_i^l variables.

Above we interpreted the Z_i^w shadow costs as the changes in the total first-term costs associated with specialty *i*'s staffing safety margin. This interpretation needs a better explanation. Please review Eq.8, which corresponds to a particular Z_i^w . For the moment, assume that it equals zero. But, if all the w_i^l weights for the *i*th AFSC were zero, then the Eq. 9b corresponding to the *i*th AFSC would require $\sum \text{VAR}(m_{ij})x_{ij} = 0$, and. So, the staffing safety margin term in Eq9a' corresponding to the *i*th AFSC would disappear. As a result, the total first term cost of satisfying the journeyman-equivalent man-year requirement (C^*) would fall since the Air Force no longer would have to select and assign additional applicants to satisfy the *i*th speciality's staffing safety. So, the Z_i^w variable must reflect the cost of providing the staffing safety margin for the *i*th AFSC.

In the last section, we saw that an improvement in assessment methods for an AFSC led to a reduction in its required staffing safety margin. Let Z_i^w represent the shadow costs of the required staffing safety margin with the old

assessment methods and Z_i^w represent the shadow cost of the required staffing safety margin with the new assessment methods. Then, the cost-saving benefit resulting from the improvement in assessment methods for the i th AFSC is given by

$$C_i = Z_i^w - Z_i^w \quad (14)$$

In the last section, we say that the difference in the values of the primal problem's objective function before and after an improvement in assessment methods or a training program for the i th AFSC represented the total cost-saving benefit. Eq 14 gives the cost-saving benefit associated with a reduction in the required staffing safety margin. It is a component of total benefit. The difference between the two benefits can be viewed as the cost-saving benefit due to improved selection and greater exploitation of the individuals' comparative advantages.

VI. Concluding Comments

The framework put forward in this paper has several advantages:

- It represents the only framework that includes all three sources of benefits—the selection of more productive applicants, the exploitation of their comparative advantages in different jobs, and the reduction of staffing safety margins.
- By using the journeyman-equivalent man-year concept and structuring the problem as a cost minimizing one, it avoids the industrial psychologist's problem of having to define and quantify "utility"
- The framework can deal with the benefits of both improvements in assessment methods and improvements in training programs; it is the only framework that puts improvements assessment methods and training programs on the same footing and allows their benefits to be compared.
- It builds on demonstrated mathematical and statistical methods.

These advantages have a price. The analytic framework makes large demands of quantitative inputs; using the framework to analyze meaningful problems would require major data collection and analysis efforts. Also, the analytic framework draws on advanced operation research techniques—stochastic programming, separable programming, and mixed integer programming. Their application to an actual problem is likely to involve much more than running an off-the-shelf computer program (Taha 1975, pp. 346-47). Existing algorithms might have to be modified or new ones might have to be written.

Viewed from another perspective, an important function of a analytic paradigm is to clarify concepts and to identify research problems. Moreover, a set of researchable problems can represent a research agenda. Hopefully, the analytic framework described in this paper has been useful in doing both.

Notes

1. An interesting special case would arise if journeyman-equivalent man-years could be predicted with a linear regression equation like

$$m_{ij} = a_i^0 + a_i^1 h_{ij}^1 + \dots + a_i^k h_{ij}^k$$

where h_{ij}^k ($k=1, \dots, K$) stands for the score of the j th candidate on the k th assessment test of method and a_i^0, \dots, a_i^k are estimated regression coefficients. Substituting the predicted journeyman-equivalent man-years of candidate j if selected and assigned to AFSC i for $E(m_{ij})$ and the standard errors of prediction (Intriligator, p. 112) for the variances in the i th AFSC in Eqs 9a and 9b, respectively,

$$\sum_{j=1}^n \hat{m}_{ij} x_{ij} - t_i^p \sum_{l=0}^L b_i^l w_i^l \geq R_i \quad (9a'')$$

$$\hat{S}_i^2 \sum_{j=1}^n [1 + h_{ij} (H^T H) h_{ij}^T] x_{ij} - \sum_{l=0}^L (b_i^l)^2 w_i^l = 0 \quad (9b)$$

where

S_i^2 is the common variance of the linear regression equation used to predict journey-man equivalent man-years for AFSC i .

h_{ij} is the vector of scores for candidate j expressed as differences from the mean of the population used to estimate the regression equation for AFSC i .

H_i is the matrix of scores used to estimate the regression equation for AFSC i .

Note that the standard error of prediction will be larger for applicants with very high and very low assessment scores. In this special case, assigning an applicant with extreme assessment scores is inherently riskier than assigning an applicant with "average" assessment scores. If regression equations were used to predict equivalent man-years, applicants with very low assessment scores would be burdened in the competition for Air Force jobs not only by their low predicted performance but also by the greater risk associated with realizing their predictions. At the other extreme, candidates with very high assessment scores would experience some "drag" on their predictions of superior job performance due to the larger risks associated with realizing their predictions.

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