

**Workshop**  
**Future Directions in Systems and**  
**Control Theory**  
**Tuesday , June 22**  
**Viewgraphs - Volume 1**

# ***DIRECTIONS IN SYSTEMS AND CONTROL THEORY***

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## ***Control is Vital Technology***

- Half a century of significant advances in theory, methodology, and software
- Mushrooming applications
- Exciting opportunities for future research
- More and more people discover the value and benefits of feedback control
  - The "Joy of Feedback"

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## ***Workshop Objectives***

- Define a research agenda for the next decade and beyond
- Survey research needs for theories, methodologies, algorithms in
  - System theoretic issues for deterministic and stochastic systems
  - Linear and nonlinear robust control
  - System identification
  - Robust adaptive control
  - Fault tolerant and reconfigurable systems
  - Hybrid control
  - Hierarchical, decentralized, and distributed large scale systems
  - Intelligent systems
  - Chaotic and complex systems ( *a la* Santa Fe Institute ...)
  - Novel concepts

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## ***Optimal Control Theory***

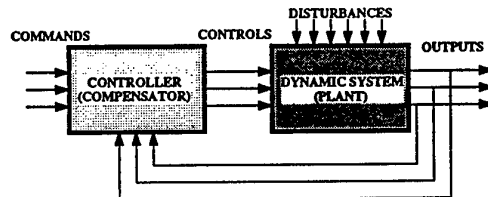
- Maximum Principle of Pontryagin and Bellman's Dynamic Programming are the foundations of optimal control theory
  - why no new powerful theorems in the last 35 years?
- Neurodynamic programming uses iterative methods to approximate cost-to-go function and store it in a neural network
- One could also use repetitive numerical solutions of the two-point-boundary-value problem to train neural network for approximate implementation of optimal nonlinear feedback control law
- Combination of genetic algorithms with classical algorithms to handle issues of locally optimal solutions
- Efficient and reliable numerical methods for approximating solution of Hamilton-Jacobi-Bellman equation would lead to major advances in several domains
- Are there new advances in dynamic team and dynamic game theory and algorithms?

## *Estimation Theory*

- Classical Kalman filters and (suboptimal) extended Kalman filters remain the workhorses of applied estimation
- Blending of hypothesis-testing concepts and filtering algorithms can lead to significant performance payoffs
  - driven by multi-sensor multi-object tracking problems that include changes in maneuvers, temporary occlusion of target, motion constraints etc.
  - performance vs. combinatorial complexity tradeoffs are required
- Distributed/decentralized estimation problems are much more difficult than their centralized counterparts
  - example: decentralized team-theoretic hypotheses-testing problems

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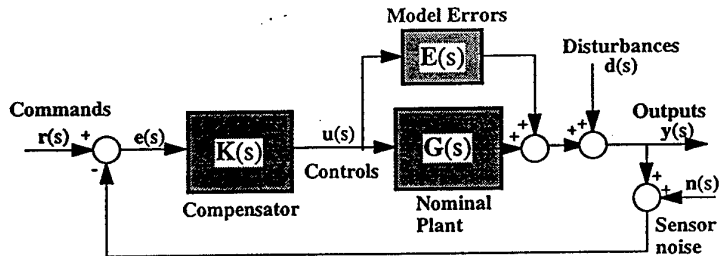
## *Robust Multivariable Control*



- Feedback imperative to
  - stabilize open-loop unstable plants and/or
  - deliver superior performance in the presence of plant and exogenous signal uncertainties
- Robust control issues: in the presence of parametric and unmodeled dynamic errors, guarantee
  - stability-robustness
  - performance-robustness

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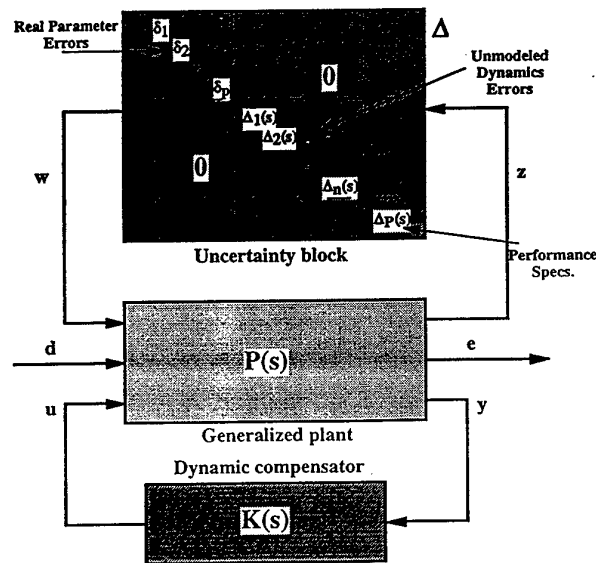
## Robust Linear Control Designs



- Structured errors: known bounds on uncertain parameters
- Unstructured errors: unmodeled dynamics (possibly infinite-dimensional) bounded in the frequency domain
- For all legal structured and unstructured uncertainty must guarantee
  - stability-robustness
  - performance-robustness to posed specifications (command-following, disturbance-rejection, insensitivity to sensor noise)

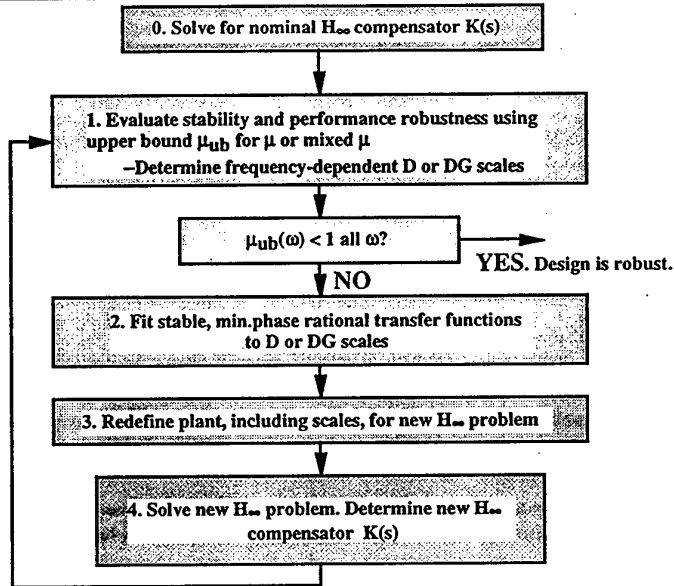
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## Mixed $\mu$ -synthesis Setup



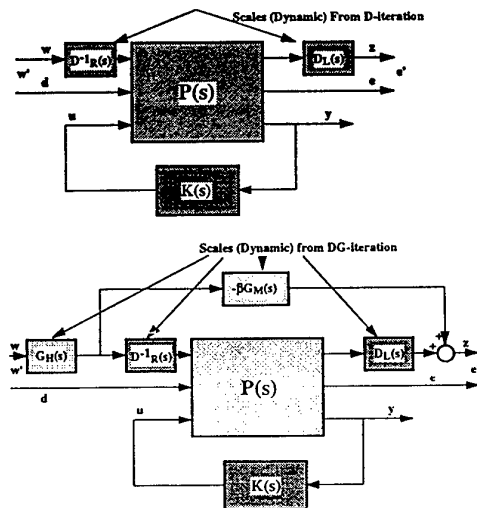
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## The DK or DGK Iteration



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## $H_\infty$ Problem in DK and DGK Iteration



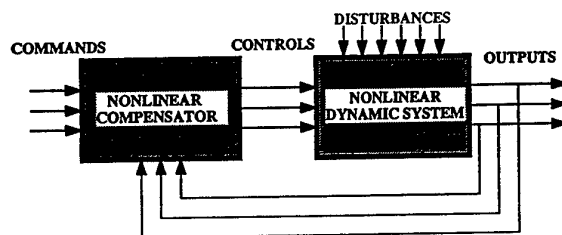
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## *Directions in Linear Robust Control*

- Mixed  $\mu$ -synthesis methodology well established
  - computationally intensive for several uncertain parameters, leading to compensators of increasing dimensionality
- Integration with adaptive control to reduce parameter uncertainty bounds?
- Alternatives for robust synthesis (must be valid for both structured and unstructured uncertainty)
  - view parameters as random variables?
  - other?

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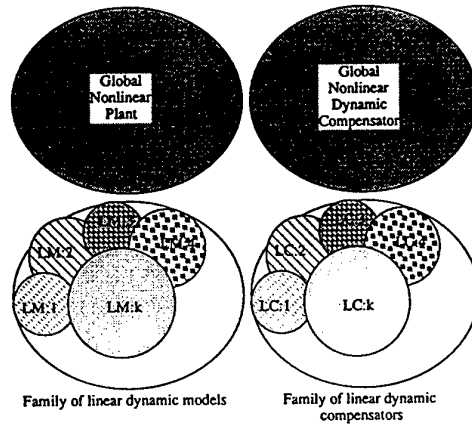
## *Nonlinear Feedback Control Methods*



- Research goal is to design for stability and performance robustness
- Common present methodologies
  - Lyapunov-based designs
  - Gain-scheduling
  - State-feedback linearization
  - Sliding-mode controllers
  - Backstepping methods
- Do we need a new paradigm?

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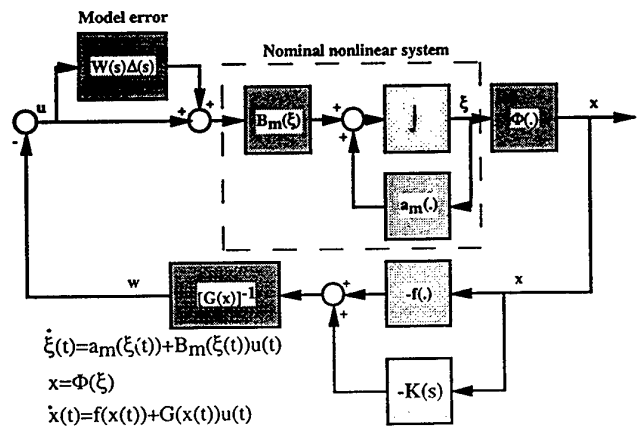
# Gain-Scheduling



- Stability-robustness requirements place additional constraints on how gain scheduling is implemented

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# Nonlinear Robust Stability with Feedback Linearization



- To obtain non-conservative stability-robustness results, one needs to routinely solve Hamilton-Jacobi-Bellman partial differential equation inequalities

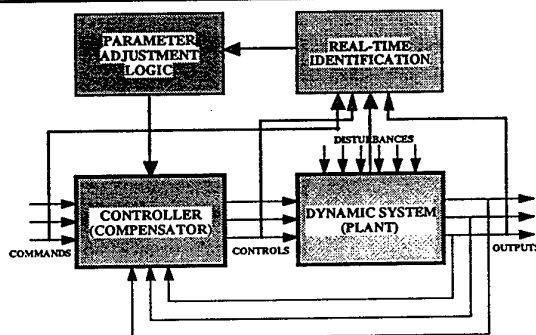
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## System Identification

- Open-loop vs. closed-loop
  - role of excitation signals
- Classical methods have difficulty when there are several poles and zeroes near the  $j\omega$ -axis, especially in MIMO settings
- Identification for robust control
- Identification in behavioral modeling setting

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## Robust Adaptive Control



- Classical adaptive MIMO designs awkward. Stability-robustness to unmodeled dynamics still a big problem! Identification for robust control
- Sequential identification/robust-controller resynthesis?
- Robustified Multiple-Model Adaptive Control (MMAC) methods

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## ***Classical Adaptive Control***

- Traditional methods (MRAC, STURE, ...) developed for plant parameter estimation; do not take into account unstructured uncertainty
  - focus on command-following, not disturbance-rejection
  - difficult to generalize to MIMO systems
  - stability-robustness not guaranteed, ad-hoc fixes still do not lead to global stability guarantees
  - unmeasurable plant disturbances and sensor noise can create stability and performance problems
- Benefits of adaptive control should be quantified
  - must quantify tradeoffs of using persistent excitation that increase output disturbances vs. longer-term performance improvements due to learning
- Operation in closed-loop not well understood

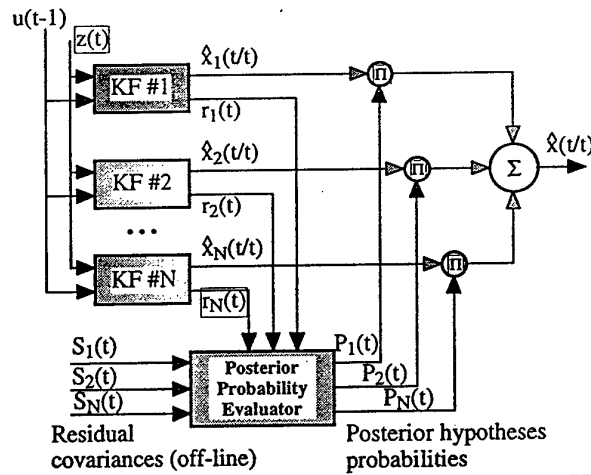
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## ***Sequential Identification/Compensator Redesign***

- Continuous readjustment of compensator parameters seems to be wrong way to go
- Sequential closed-loop identification/compensator redesign may be superior engineering approach
  - advances in identification for robust control
  - needs rethinking of proper persistent excitation signals
  - should take advantage of major advances in the design of robust linear controllers (mixed  $\mu$ -synthesis,...)
  - open closed-loop bandwidth slowly, monitor for instabilities so as to revert to prior stabilizing compensator
  - should improve performance degradation caused by "detuning" of fixed compensators because of large parameter errors

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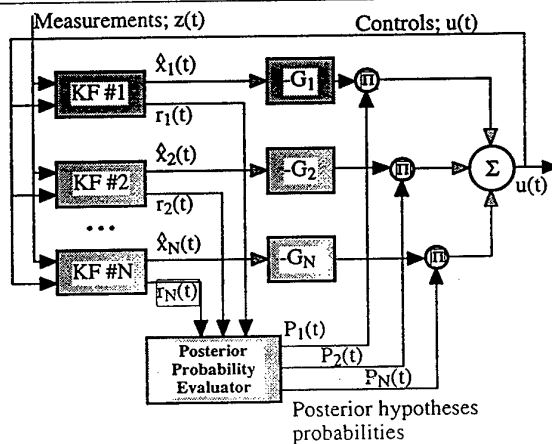
## Multiple Model Adaptive Estimation (MMAE)



- If true system is included in bank of Kalman filters, it is identified with probability one. Otherwise, we identify nearest "probabilistic neighbor"

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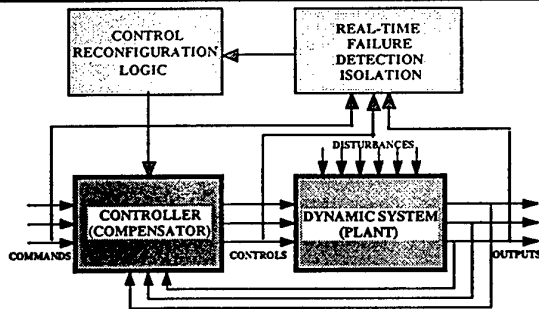
## Multiple Model Adaptive Control (MMAC)



- Applied control is the probabilistic average of "matched LQG controls"
- Other variants possible, e.g. apply control with the largest current posterior probability (switching...)

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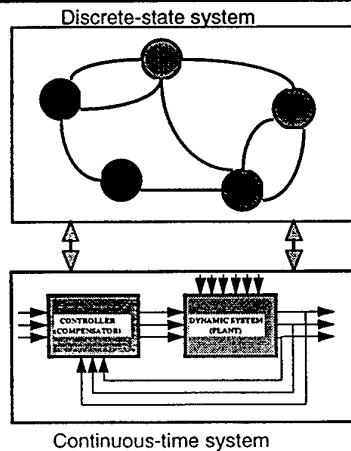
## Fault-Tolerant Control



- Detect soft/hard failures in sensors and actuators, abrupt dynamic changes
- Impact of parametric and dynamic errors on FDI algorithms, e.g. GLR
- Controller reconfiguration algorithms
- Role of hybrid system theory (mode switching...)
- Role of "intelligent" supervisory methods
- Robust stability and performance guarantees

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## Hybrid Control



- Architectures involving interactions between a finite-state event-driven system and a continuous-state continuous-time system
- Discrete level can establish different modes of operation for feedback system based upon feedback of behavior of continuous variables

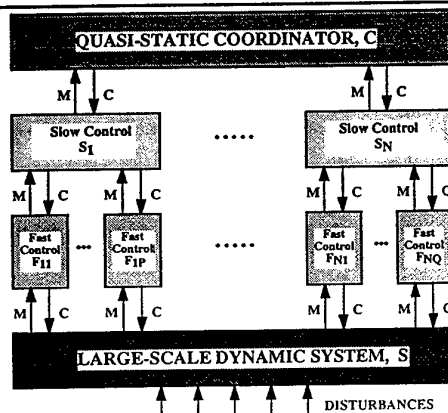
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## Issues in Hybrid Control

- Deals with switched dynamical systems, software enabled control, and multimodal control
- Impact on several disciplines and applications, e.g.
  - fault-tolerant reconfigurable control
  - hierarchical systems
- Formal modeling leads to some qualitative properties (reachability,...)
  - difficult to establish existence, uniqueness and sensitivity of solutions
  - some verification software is available
- Design-directed theories, methodologies, software are at their infancy. They seem to require advances in
  - optimal control theory
  - dynamic team theory
  - dynamic game theory
- Complexity and combinatorics are main obstacles

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## Decentralized Control



- Weak dynamic coupling
- Multiple time-scales
- Stability and performance robustness guarantees?

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## ***Issues in Decentralized Control***

- Non-classical information patterns “fight” with classical optimization
  - signaling strategies
  - second-guessing strategies
  - cost of communications and communications delays
  - “easy” convex centralized problems transform into “nasty” nonconvex combinatorial ones in decentralized setting
- WHO should know WHAT, WHEN, WHERE and WHY
  - Performance impact of enhanced information patterns in teams
- Cooperative multiperson dynamic game theory can be exploited
  - Nash strategies
  - Stackelberg (leader-follower) strategies
- Can we isolate superior alternative hierarchical and spatial decompositions and architectures?
- Is there a decentralized “maximum principle?”

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## ***Intelligent Systems***

- Fuzzy systems
  - fuzzy feedback control
  - fuzzy supervisory layers (fuzzy hybrid?)
  - uncertainty modeling?
- Neural networks
  - training and learning
  - specific means of approximating nonlinear static maps
- Genetic/evolutionary algorithms
  - minimal assumptions on problem structure
  - used for bypassing local optima
  - require extensive number of iterations for convergence
  - hard convergence and rate of convergence proofs
- All three have attracted a lot of attention in computer science community. Normative vs. empirical approaches.

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## ***Chaotic and Complex Systems***

- Chaotic systems: consequence of solution properties for classes of systems of ordinary differential equations
  - analysis by simulation studies
- Complex systems: macroscopic behavior of systems that contain a very large number of interacting simple agents with specific survival, reproductive, etc. characteristics
  - artificial life: studies involving large-scale simulations providing trends of species survival, migration, ...
- What is role of systems and control theory (apart from simulations with sexy graphics?)
- Recent research on "Highly Optimized Tolerance" by J.C. Doyle

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## ***Concluding Remarks***

- Our heritage is fifty years of exciting and important developments in theory, design methodologies and algorithms
- There exist numerous opportunities for relevant basic theoretical and methodological research
  - adaptive, multimodal, complex systems
  - my favorite: software that routinely solves Hamilton-Jacobi-Bellman partial differential equations
- Expanding applications that benefit from theoretical developments
- Should always be on lookout for significant engineering/scientific/socioeconomic applications that suggest novel theory development and extensions
- Expect continuation of "clashes" between ad-hoc approaches from the computer science community and the quantitative model-based methodologies of our discipline
  - we shall overcome!

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# Fundamental Problems In Adaptive Control

Workshop on Future  
Directions in Systems  
and Control Theory

Cascais, Portugal  
22 June 1999

Brian D O Anderson  
Australian National University

# OUTLINE

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- THREE FUNDAMENTAL PROBLEMS OF ADAPTIVE CONTROL
  - » Changing experimental conditions accurate but inexact models
  - » Impractical control objectives
  - » Transient instability
- VINNICOMBE METRIC
- ITERATIVE CONTROL AND IDENTIFICATION
- ADDRESSING THE FUNDAMENTAL PROBLEMS
- MULTIPLE MODEL ADAPTIVE CONTROL AND VINNICOMBE METRIC
- CONCLUSIONS

# A Result with Practical Difficulties

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- Let  $P$  be unknown linear system.
- Let  $P$  have known degree and be minimum phase.
- [Statements regarding operation of an adaptive controller.]
- Then as  $t \rightarrow \infty$  all signals in the adaptive loop remain bounded, the controller converges and the performance index is minimized (i.e. asymptotically, all is right with the world).
- This result sidesteps three problems which arise in adaptive control, including adaptive control of nonlinear, distributed etc systems.

## Problem Of Changing Experimental Conditions with Accurate but Inexact Models

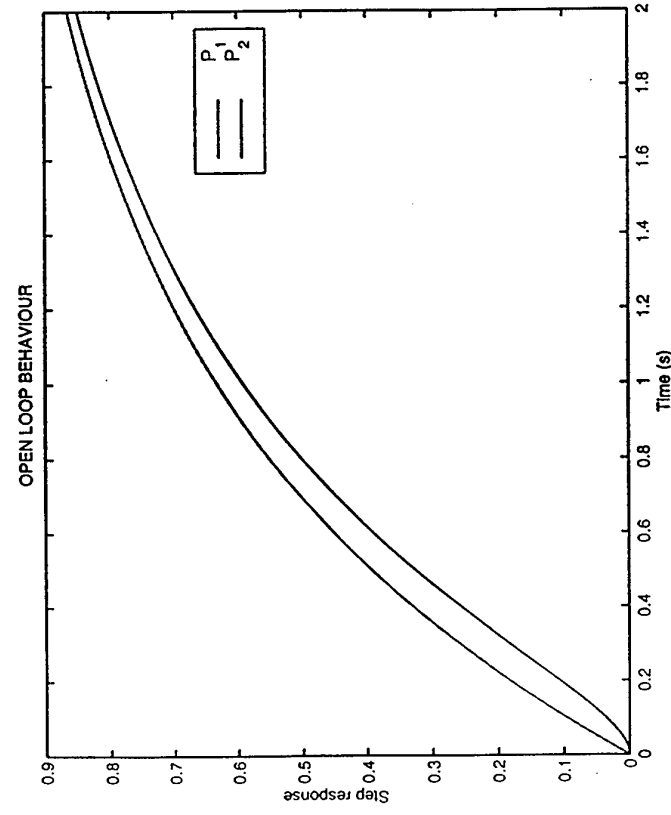
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- Arbitrarily small open-loop modelling errors can lead to arbitrarily bad closed-loop performance, when a nice controller is designed for the model and connected to the real plant.
- Large open-loop modelling errors may give small closed-loop errors.

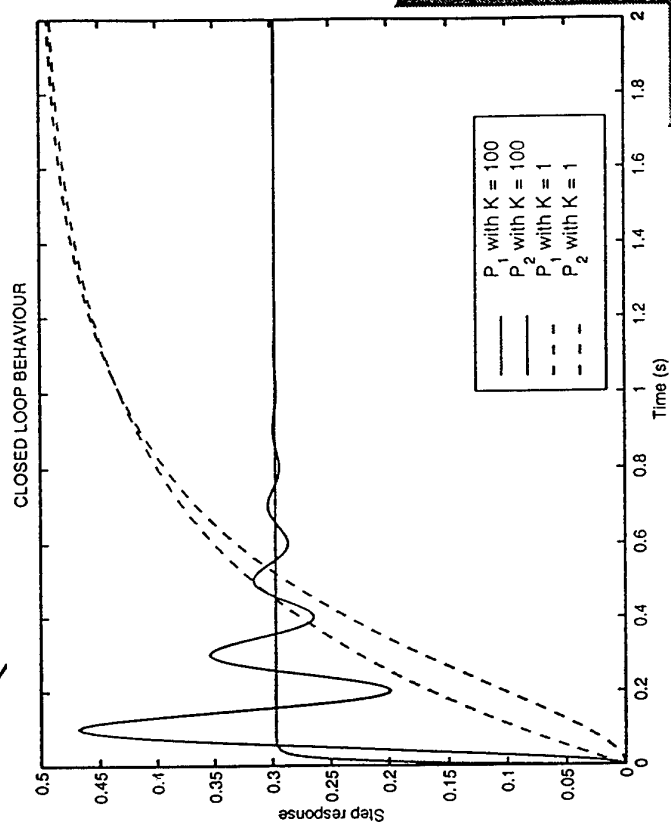
*Example:*  $P_1 = P_0 +$  high frequency resonance,  $C$  stabilizes  $P_0$  with closed-loop bandwidth excluding the resonance.

- Problem is that closeness of models is evaluated with particular experimental conditions.

- Similar open-loop behaviours:  $P_1 = \frac{1}{s+1}$  and  $P_2 = \frac{1}{(s+1)(0.1s+1)}$



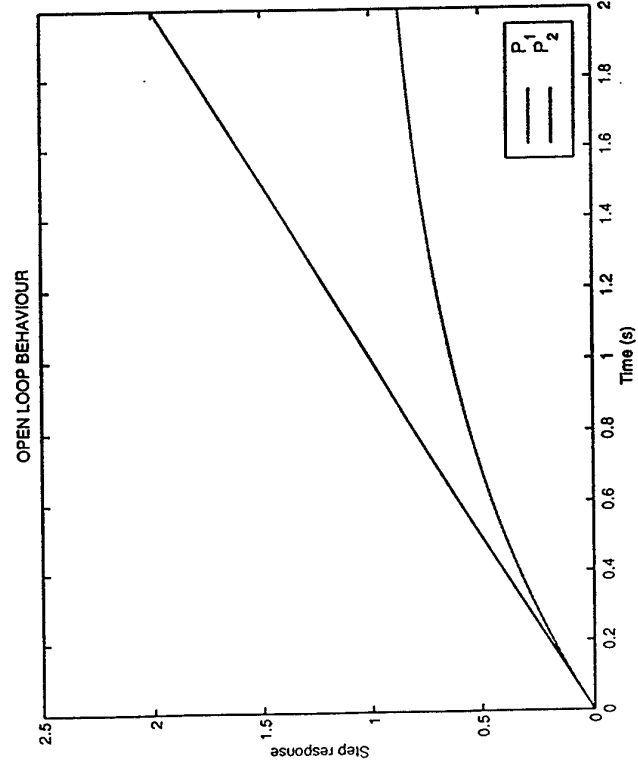
open-loop



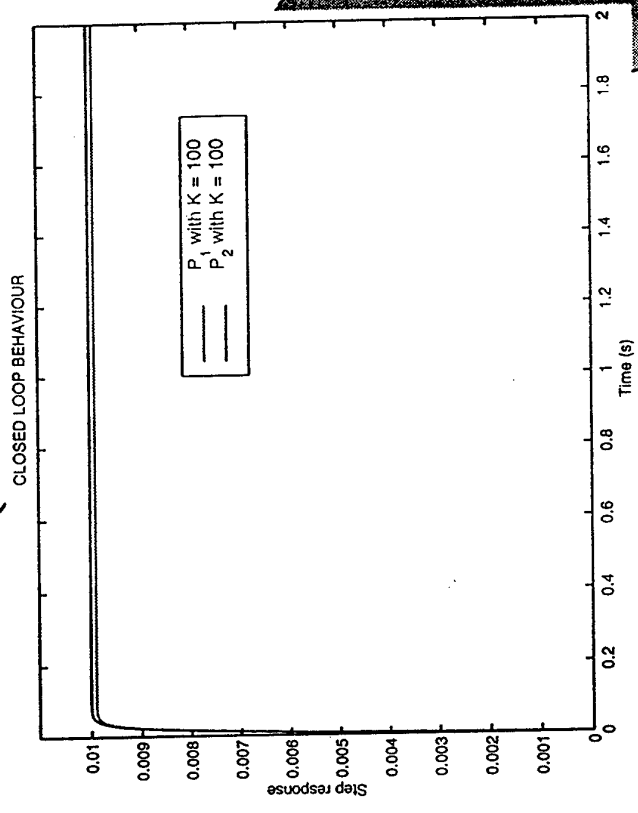
closed-loop

$$K = 100 \text{ (---)} / K = 1 \text{ (- - -)}$$

- Different open-loop behaviours:  $P_1 = \frac{1}{s+1}$  and  $P_2 = \frac{1}{s}$



Plants in open-loop



Plants in closed-loop with  $K = 100$

## Problem Of Changing Experimental Conditions with Accurate but Inexact Models

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- Insertion, removal or change of a controller is a change of experimental conditions.

*When in adaptive control the controller is changed, the assumed model of the plant may change from effective to ineffective.*

- Qualitative conclusion: controller changes should be small.
- Question: How can we measure smallness?

# Problem of Impractical Control Objective

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- Some non-adaptive control problems, while theoretically soluble, have impractical solutions.

*Example: LQG problem yielding  $\epsilon$  degrees phase margin for the loop.*

- If one tried an adaptive control solution, not knowing the plant initially, there would be trouble (large transient signals, failure to converge etc.)

*Since plant is initially unknown, practical solubility of the adaptive control problem is unknown*

- *Question:* How can we arrange that adaptive control algorithms flag over-ambitious control objectives, and back off in seeking to achieve them?

# Problem of Transient Instability

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- “All signals in the adaptive closed-loop remain bounded” permits  $10^{12}$  amps motor current.
- A property of one famous adaptive control *convergence* result is the following:

Let  $\xi_0$  denote the vector of initial conditions and  $Z(t)$ , the vector of all interesting signals at time  $t$ .

For arbitrary  $N > 0$ , there exists  $\xi_0$  with  $\|\xi_0\| < 1$

and  $t_1 > 0$  such that  $\|Z(t_1)\| > N$

*The frozen plant-controller connection  
may be unstable at any finite time.*

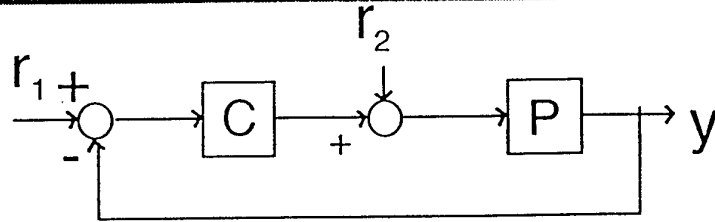
- Suppose that at  $t = 0$ , we know how to connect a stabilizing controller. How can we adapt it - not knowing the plant - to retain frozen closed-loop stability?

# OUTLINE

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- THREE FUNDAMENTAL PROBLEMS OF ADAPTIVE CONTROL
- VINNICOMBE METRIC
  - » Generalized stability margins
  - » Vinnicombe Metric and link to stability margin
  - » Consequences of the metric/margin connection
- ITERATIVE CONTROL AND IDENTIFICATION
- ADDRESSING THE FUNDAMENTAL PROBLEMS
- MULTIPLE MODEL ADAPTIVE CONTROL AND VINNICOMBE METRIC
- CONCLUSIONS

# Generalized Stability Margin



- $$\begin{pmatrix} y \\ u \end{pmatrix} = T(P, C) \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$$
- $$T(P, C) = \begin{bmatrix} P(I + CP)^{-1}C & P(I + CP)^{-1} \\ (I + CP)^{-1}C & (I + CP)^{-1} \end{bmatrix} = \begin{bmatrix} P \\ I \end{bmatrix} (I + CP)^{-1} [C \ I]$$
- Define generalized stability margin by
 
$$b_{P,C} = \|T(P, C)\|_{\infty}^{-1} \quad \text{if } (P, C) \text{ is stable}$$

$$= 0 \quad \text{otherwise}$$
  - small if close to instability, ( $1 + PC$  close to zero)
  - small if closed loop bandwidth exceeds open loop  $[C(1 + PC)^{-1}$  large]

# Generalized Stability Margin

---

- Measure of difficulty of controlling a

particular plant  $P$  is  $b_{opt}(P) = \sup_C b_{P,C}$

- Nice formula available, showing that right half plane poles or zeros make  $b_{opt}(P)$  smaller,

i.e.  $\|T_{opt}(P)\|_{\infty}$  bigger

- nonlinear version available

# Vinnicombe Metric and Link to Stability Margin

- Introduce distance measure between two plants

For scalar plants,

$$\delta_v(P_1, P_2) = \max_{\omega} \left| \frac{P_2 - P_1}{(1 + |P_2|^2)^{\frac{1}{2}} (1 + |P_1|^2)^{\frac{1}{2}}} \right|$$

(provided a winding number condition is satisfied), and

$\delta_v(P_1, P_2) = 1$  otherwise.

- nonlinear and multivariable version available.

- Suppose  $(P_1, C)$  is stable. Then

$(P_2, C)$  is stable for all  $P_2$  with  $\delta_v(P_1, P_2) \leq \beta$

iff

$$b_{P_1, C} > \beta$$

# Vinnicombe Metric and Link to Stability Margin

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- Suppose  $(P, C_1)$  is stable. Then

$(P, C_2)$  is stable for all  $C_2$  with  $\delta_v(C_1, C_2) \leq \beta$

iff

$$b_{P, C_1} > \beta$$

# Consequences of the Metric/Margin Connection

---

- $P_1$  hard to control  $\Leftrightarrow b_{opt}(P_1)$  is small
  - $\Leftrightarrow b(P_1, C)$  is even smaller for some stabilizing  $C$
  - $\Leftrightarrow$  little scope to change  $C$  without risking instability
  - $\Leftrightarrow$  little scope for modelling errors in  $P_1$ , or satisfactory behaviour with  $C$  should  $P_1$  drift
- Small changes in  $C$  (= safe changes in  $C$ ) require knowledge of  $b_{P,C}$  for their determination.
- Suppose  $C_1$  stabilizes  $P_1$  and  $C_2$  replaces  $C_1$  with  $\delta_v(C_1, C_2) < b_{P,C_1}$   
 $C_2$  may give less robust design than  $C_1$  :  

$$\sin^{-1} b_{P,C_2} \geq \sin^{-1} b_{P,C_1} - \delta_v(C_1, C_2)$$
 and equality is possible.

# Consequences of the Metric/Margin Connection

---

- $\delta_v(C_1, C_2) \leq \|T(P, C_1) - T(P, C_2)\|_\infty \leq \frac{\delta_v(C_1, C_2)}{b_{P, C_1} b_{P, C_2}}$

$$\delta_v(C_1, C_2) \leq \|T(P, C_1) - T(P, C_2)\|_\infty \leq \frac{\|T(P, C_1)\|_\infty^2 \delta_v(C_1, C_2)}{1 - \|T(P, C_1)\|_\infty \delta_v(C_1, C_2)}$$

# OUTLINE

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- THREE FUNDAMENTAL PROBLEMS OF ADAPTIVE CONTROL
- VINNICOMBE METRIC
- ITERATIVE CONTROL AND IDENTIFICATION
  - » Algorithm framework
  - » Acceptable cautious controller adjustments
  - » Desirable cautious controller adjustments
- ADDRESSING THE FUNDAMENTAL PROBLEMS
- MULTIPLE MODEL ADAPTIVE CONTROL AND VINNICOMBE METRIC
- CONCLUSIONS

# Iterative Control and Identification Algorithm Framework

---

- 1 Assume controller  $C_i$  is connected to real plant  $P$  and gives stable closed loop.
  - 2 Identify a model  $\hat{P}_j$  of  $P$  using closed-loop data.
  - 3 Using  $\hat{P}_j$  redesign controller to obtain  $C_{i+1}$  - ensuring  $C_{i+1}$  is not too different to  $C_i$ .
  - 4 Check that  $(\hat{P}_j, C_{i+1})$  behaves like  $(P, C_{i+1})$ . If so, update  $i$  and go back to step 3. If not, go to step 5.
  - 5 Reidentify  $P$  to obtain a new model  $\hat{P}_{j+1}$ , connected to a controller  $C_{i+1}$ .
- Repeat cycle until desired performance is attained or is seen to be unattainable.

# Iterative Control and Identification Algorithm Framework

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- For plants with only unstable poles at origin, and with  $C_i$  obtained by IMC design (with iterative closed-loop bandwidth broadening), windsurfer approach to adaptive control results.

## Cautious Controller Adjustment: What is Acceptable?

---

---

- Assumption: the statement that " $\hat{P}_j$  is a good model of  $P$  using closed-loop data with controller  $C_i$ "

means

$$T(P, C_i) \cong T(\hat{P}_j, C_i)$$

- Hence

$$b_{P, C_i} \cong b_{\hat{P}_j, C_i} = \|T(\hat{P}_j, C_i)\|_{\infty}^{-1}$$

- Hence we should select  $C_{i+1}$  such that

$$\delta_v(C_i, C_{i+1}) < kb_{\hat{P}_j, C_i} \quad k \in (0, 1)$$

## Cautious Controller Adjustment: What is Acceptable?

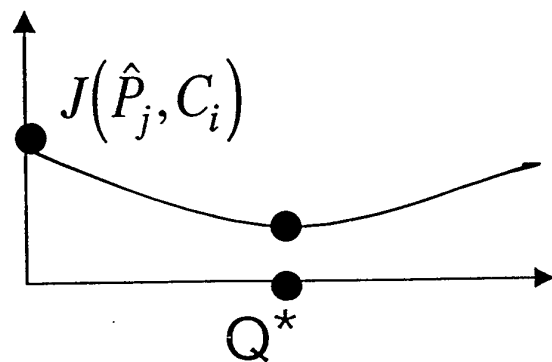
- Assumption: for unknown  $P$ , the design goal is to obtain stabilizing  $C$  to minimize  $J(P, C)$ .
- Let  $\hat{P}_j$  have right coprime description  $ND^{-1}$  and  $C_i$  right coprime description  $UV^{-1}$ .

$$\mathcal{C} = \{C(Q) : C(Q) = (U - DQ)(V + NQ)^{-1}\}$$

for  $Q$  stable delineates set of stabilizing compensators

- Assumption:  $J(\hat{P}_j, C(Q))$  for  $C \in \mathcal{C}$  depends on  $Q$  in a convex manner.

[ In  $H_2$  and  $H_\infty$ , dependence is convex ]



$$C_{i+1}^* = \arg \min_{C(Q)} J(\hat{P}_j, C(Q))$$

## Cautious Controller Adjustment: What is Acceptable?

---

- If  $\delta_v(C_i, C_{i+1}^*) \leq kb_{\hat{P}_j, C_i}$ , choose  $C_{i+1} = C_{i+1}^*$

- Else look at set for  $\alpha \in [0, 1]$

$$C(\alpha Q^*) = (U - \alpha DQ^*)(V + \alpha NQ^*)^{-1}$$

$$\alpha = 0 : C = C_i \quad \alpha = 1 : C = C_{i+1}^*$$

Choose  $\alpha \in (0, 1)$  with

$$\delta_v(C_i, C(\alpha Q^*)) = kb_{\hat{P}_j, C_i}$$

$$C_{i+1} = C(\alpha Q^*)$$

and

$$J(\hat{P}_j, C_{i+1}) < J(\hat{P}_j, C_i)$$

(Safety and performance improvement)

# OUTLINE

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# Addressing the Fundamental Problems

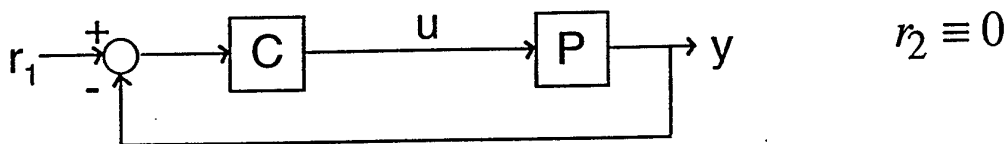
- Changing experimental conditions given accurate but inexact models

- Limit  $\delta_v(C_i, C_{i+1})$  using  $\|T(\hat{P}_j, C_i)\|_\infty$

- Rely on  $\|T(P, C_i)\|_\infty \cong \|T(\hat{P}_j, C_i)\|_\infty$

- Impractical control objectives

*Possibility 1:* Identification may not be sufficiently reliable to conclude  $T(\hat{P}_j, C_i) \cong T(P, C_i)$



- Suppose closed loop bandwidth is  $\omega_0$
- Suppose estimate  $PC(1+PC)^{-1}$  at  $10\omega_0$ , SNR = 1
- $P(1+PC)^{-1}$  at  $10\omega_0$  estimate will have SNR = 1
- If  $P$  has resonance at  $10\omega_0$  not captured in  $\hat{P}_j$ ,  $\|T(P, C)\|_\infty$  could be due to  $P(1+PC)^{-1}$  at  $10\omega_0$  and misestimated.

# Addressing the Fundamental Problems

---

- *Possibility 2:*  $T(\hat{P}_j, C_i)$  may be very large, so scope for adjusting  $C_i$  is negligible.
- Transient instability
  - Assume stabilizing compensator is known for the unknown plant.
  - Then transient instability is avoided (so long as controllers are not switched too fast).

- 
- THREE FUNDAMENTAL PROBLEMS OF ADAPTIVE CONTROL
  - VINNICOMBE METRIC
  - ITERATIVE CONTROL AND IDENTIFICATION
  - ADDRESSING THE FUNDAMENTAL PROBLEMS
  - MULTIPLE MODEL ADAPTIVE CONTROL AND VINNICOMBE METRIC
  - CONCLUSIONS

# Multiple Model Adaptive Control and Vinnicombe Metric

---

- Consider a set of plants

$$P_\lambda(s) = \lambda \frac{s-1}{(s-2)(s+1)} \quad 0.3 \leq \lambda \leq 3$$

or an even bigger set

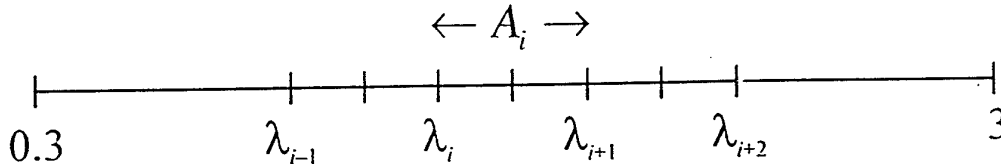
$$\Pi = \{P : P = P_\lambda(s)[1 + \Delta(s)] \text{ with } \|\Delta(j\omega)\|_\infty < \delta\}$$

- Multiple model adaptive control assumes there exist  $N$  and  $\lambda_1 < \lambda_2 < \dots < \lambda_N$  such that
  - a nice controller  $C_{\lambda_i}$  exists for  $P_{\lambda_i}$
  - any plant in  $\Pi$  is controlled nicely by some  $C_{\lambda_i}$
- Multiple model control provides an algorithm to switch among the  $C_{\lambda_i}$ . Safety is an issue.

# Multiple Model Adaptive Control and Vinnicombe Metric

Key Question : Apart from safety, what should  $N$  be chosen as and how does one choose  $\lambda$ ?

- Answer for  $P_\lambda(s)$



- All plants in  $A_i$  obey

$$\delta_v(P_\lambda, P_{\lambda_i}) \|T(P_{\lambda_i}, C_{\lambda_i})\|_\infty \leq 0.1$$

- This ensures

$$\|T(P_\lambda, C_\lambda) - T(P_{\lambda_i}, C_{\lambda_i})\|_\infty \leq 0.1 \|T(P_{\lambda_i}, C_{\lambda_i})\|_\infty$$

# OUTLINE

---

- THREE FUNDAMENTAL PROBLEMS OF ADAPTIVE CONTROL
- VINNICOMBE METRIC
- ITERATIVE CONTROL AND IDENTIFICATION
- ADDRESSING THE FUNDAMENTAL PROBLEMS
- MULTIPLE MODEL ADAPTIVE CONTROL AND VINNICOMBE METRIC
- CONCLUSIONS
  - » Main messages
  - » Shortcomings and future possibilities

# Main Messages

---

- Adaptive control should have modest steps only in the controller.
- The Vinnicombe metric plus closed-loop measurements indicate what is modest.
- A desired control objective may be impractical; good algorithms need to flag this and adjust their goal. Some do this.
- Transient instability should be avoided, and can be avoided.

# Shortcomings and Future Possibilities

---

- The Vinnicombe metric is a single number. It would be better to look at the whole imaginary axis
- There is no protection against step changes in the plant (Hypothesis testing rather than parameter estimates should be used if possible to identify change)
- There is a lack of methodology for optimally using closed-loop data obtained for one plant and more than one controller in identifying that plant
- The ideas look fruitful for nonlinear problems. Much has already been done on the Vinnicombe metric / robust stability question.

# Identification of Complex Systems

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## Motivation

- Robust Control Theory: Major development in the past 20 years.
- Design tools
  - Start with the nominal process, a description of the unmodeled dynamics and exogenous inputs.
  - Provide performance tradeoffs.

- An example of an I/O description of a process:

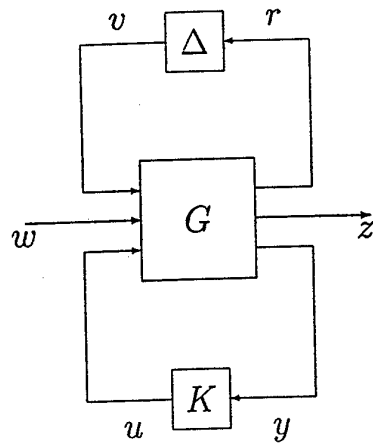
$$y = Gu + \delta Gu + W\Delta u + w.$$

- System identification has not had a parallel development.
- Existing theory did not address control-oriented modeling.

## Objective

- Develop a paradigm for identification of complex systems using limited-complexity models.
- Motivation: consistency with existing robust control paradigms.
- Main considerations
  - Explicit under modeling.
  - Error measures.
  - Noise (deterministic or stochastic).
- Standard issues
  - Experiment Design.
  - Algorithms and Computations.
  - Error bounds.
  - Complexity.
- Reference: S.R. Venkatesh. Identification for complex systems. Ph.D. Thesis # LIDS-TH-2394, MIT, Cambridge, MA.

## Robust Control



- Interconnection:
  - $G$ : Nominal system.
  - $\Delta$ : Describes plant uncertainty.
  - $K$  stabilizing feedback controller.
  - $w$ : exogenous input.
  - $z$ : regulated output.
- Uncertainty:
  - Uncertainty description is linear fractional.
  - Example: Additive or multiplicative uncertainty.
  - $\Delta$  is an unknown system, with bounded “induced” norm ( $\mathcal{H}_\infty, \ell_1$ ).

## (MPE) System Identification

- Minimum Prediction Error Paradigm:

$$y = f_{\theta}(u, w) \quad \theta \in \Theta$$

- $w$  is a stochastic Process (generally white noise).
  - A parametrized set of models.
  - Actual process is in the parametrization.
  - Estimate the parameters from Data by minimizing the “prediction error”.
- Positive results.
    - Consistency, if actual process is in the model parametrization.
    - Asymptotic distribution of the parameters.
    - Sample complexity issues.
  - Limitations
    - Stationarity assumptions on the noise.
    - Results are asymptotic.
    - Under modeling is not accounted for a priori. Equivalently, noise is the only source of mismatch.
    - Difficult to integrate with existing control paradigms.

## Set-Membership System Identification

- Prior Information

- The Process:

$$y = G(\theta_0)u + \Delta(u) + w$$

- $w :=$  output noise,  $w \in \mathcal{W}$ .

- $\Delta :=$  Plant uncertainty,  $\Delta \in \Delta$ .

- model parametrization  $\mathcal{M}(\theta) = \{G(\theta) + \Delta, \Delta \in \Delta\} \subset X$ ,  $X :=$  model set.

- $\theta \in \Theta \in \mathbb{R}^n$

- $u :=$  Input,  $u \in \mathcal{U}$  ( $\|u\|_\infty \leq 1$ ).

- Positive issues

- Consistent with robust control.

- Explicit under modeling.

- Negative issues

- No clear separation between the finite models and the unmodeled.

- $\mathcal{W} = Bl_\infty$  is a conservative model for noise.

## Problem Description

- Process class,  $\mathcal{T}$ , contains stable LTI systems,
  - Set is complex; not approximable by a finite dimensional set.
  - equipped with a metric.
  - Real process is in this set.
- Model parametrization  $\mathcal{G}$ 
  - Finite dimensional set (subspace).
  - $\mathcal{G} \subset \mathcal{T}$
- Noise
  - Stochastic or deterministic.
  - Deterministic: set is not too rich or too constrained.
  - Set description of white noise.
- Data: finite observations of the input and output.

$$y = T_0 u + w$$

## Problem Formulation

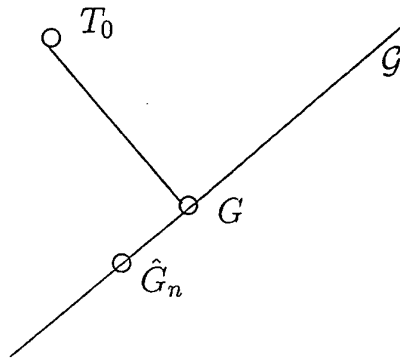
- Output equation:

$$y = T_0 u + w \quad T_0 \in \mathcal{T}, w \in \mathcal{W}$$

- Define  $G_0$  as

$$G_0 = \arg \min_{G \in \mathcal{G}} \|T_0 - G\|$$

- Prior: There exists a  $\gamma$  such that  $\text{dist}(T_0, \mathcal{G}) \leq \gamma$ .
- Problem: Does there exist an input  $u$ , and an algorithm  $\hat{G}_n$  so that:
  1.  $\lim_{n \rightarrow \infty} \sup_{T_0, w} \|G_0 - \hat{G}_n\| = 0$ ,
  2. Time complexity of convergence is polynomial.



## Comments

- Choice of  $\mathcal{T}$

- Operator spaces  $(H_\infty, \ell_1)$ .
- The space  $\ell_2$  more convenient mathematically.
- Hardy-Sobolev space  $\mathcal{H}$ ; Hilbert space with inner product

$$\langle f, g \rangle_{\mathcal{H}} = \langle f, g \rangle + \langle f', g' \rangle$$

- The following inequality holds:

$$\|T\|_\infty \leq \|T\|_1 \leq c\|T\|_{\mathcal{H}}$$

- Robust control can deal with  $\mathcal{H}$ .

- Prior structure is additive. Need more complicated structures.

- Outcome of identification:

- An estimate  $\hat{G}_n$ .
- A (parametric) estimate of the error  $\|G_0 - \hat{G}_n\|, D_n$ .
- An estimate  $\hat{\gamma}_n$  of the unmodeled part  $\hat{\gamma}_n \leq \gamma$ .
- Additional structure on the unmodeled part,  $\Delta = \mathcal{G}^\perp$ .
- The model

$$\{\hat{G}_n + \Delta \mid \|G - \hat{G}_n\| \leq D_n, \Delta \in \Delta, \|\Delta\| \leq \hat{\gamma}_n\}$$

## Description of the Unmodeled

- $G_0$  is the best approximation of  $T_0$  in  $\mathcal{T} = \mathcal{H}$ .

- Express  $T_0$  as

$$T_0 = G_0 + \Delta_0$$

$$\Delta_0 \in \mathcal{G}^\perp.$$

- Example:  $\mathcal{G}$  is the set of FIR of order  $M$ , then

$$\mathcal{G}^\perp = \{\lambda^{M+1} f(\lambda), f \text{ arbitrary}\}.$$

- Example:  $\mathcal{G} = \{\theta/1 - a\lambda, \theta \in \mathbb{R}\}$ , then

$$\mathcal{G}^\perp = \{\Delta \mid \sum_{k=0}^{\infty} (1+k^2) a^k \delta_k = 0.\}$$

- If  $\mathcal{G}$  is a subspace, then  $\Delta$  is nice set!

## Diameter of Uncertainty

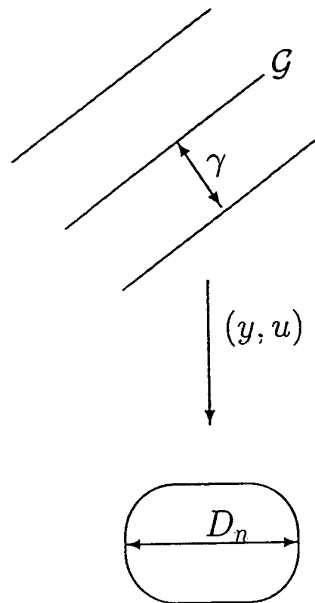
- Set of unfalsified models:

$$S_n(y, u) = \{G \in \mathcal{G} \mid y(n) = Gu(n) + \Delta u(n) + w(n), \Delta \in \Delta, \|\Delta\| \leq \gamma, w \in \mathcal{W}\}$$

- Diameter of uncertainty at time  $n$ :

$$D_n(\mathcal{T}, \mathcal{G}, \mathcal{W}, u) = \sup_{T \in \mathcal{T}} \sup_{w \in \mathcal{W}} \text{diam}(S_n)$$

- Asymptotic diameter of uncertainty  $D = \lim_{n \rightarrow \infty} D_n$ .
- Problem: Does  $D_n$  converge to zero for some input  $u$  in polynomial time?



## Results

- Exact modeling
  - Bounded-but-unknown noise.
  - Low correlated noise models.
  - Bridge the gap between stochastic and deterministic formulations.
- Under modeling
  - FIR model structures
  - General finite dimensional subspaces of finite-dimensional systems.
- Identification in practice.
- Connection to robust control.

## Correlation Methods: Exact Modeling

- Problem Set-up:

- $y = G_0 u + w$

- $G_0 \in \mathcal{G}$  is the subspace of FIR models of order  $M$ .

- The estimate is computed as follows:

$$r_{yu}^N(\tau) = \hat{G}_N r_u^N(\tau), \quad \tau \leq M$$

- When does  $\hat{G}_N$  converge to  $G_0$ ?

- Input is persistently exciting of order  $M$ .

- Noise:

- quasi-stationary, uncorrelated with  $u$  (e.g., white noise).

- Set-valued with Correlation Constraints: ( $\gamma_N < \frac{1}{\sqrt{N}}$ )

$$\mathcal{W}_N = \left\{ d \in \mathbb{R}^N \mid \frac{1}{N} \sum_{\tau=1}^N |r_d^N(\tau)| \leq C \gamma_N \right\}$$

- Set-valued with Periodogram Constraints:

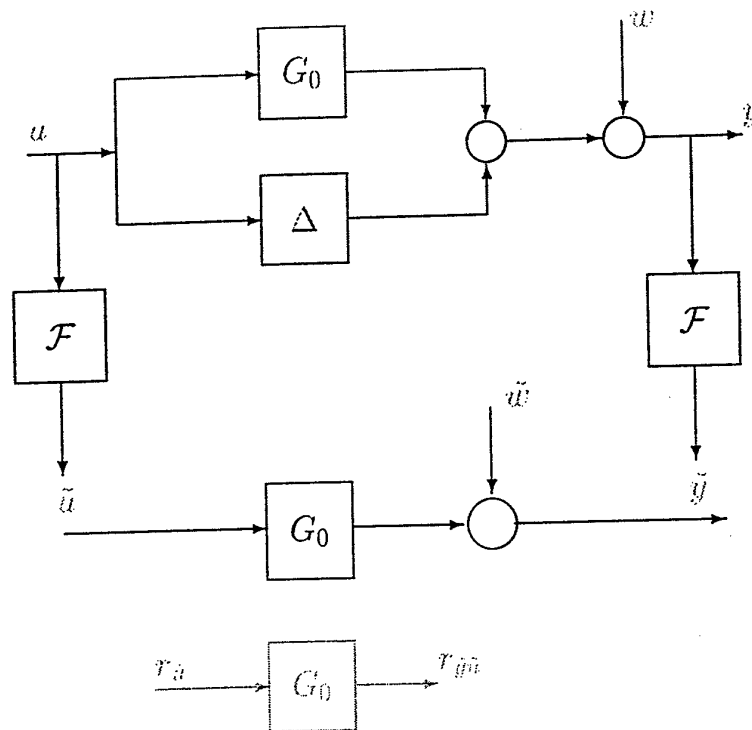
$$\mathcal{W}_N^F = \left\{ d \in \mathbb{R}^N \mid \frac{1}{\sqrt{N \log N}} \left| \sum_{k=0}^N d_k e^{i w k} \right| \leq W(w), w \in [0, 2\pi] \right\}$$

- Both  $\mathcal{W}_N, \mathcal{W}_N^F$  contain white noise with high probability.

- Convergence is uniform with polynomial sample complexity.

- Does not converge if noise is only “bounded-but-unknown”.

## Under Modeling: Basic Idea



- Two-step algorithm
  1. Filter the data to eliminate the unmodeled part.
  2. Use previous correlation method.
- $\mathcal{F}$  is constructed from the annihilators of  $\Delta$ .
- What about the input?

## Input Design: Motivation

- $\mathcal{G}$  is the subspace of FIR models of order 0 (constants).

- $\Delta = (0, f_1, f_2, \dots)$ .

- $y(k) = g_0 u(k) + \sum_{j=1}^k f_j u(k-j) + w(k)$

- Tradeoff

- If  $w = 0$ , then the best  $u$  is the unit pulse.

- If  $w \neq 0$ , then  $u$  needs to be longer.

- Periodic inputs will not work:

$$y(k) = (g_0 + f_1 + \dots)u(k) + (f_1 + f_2 + \dots)u(k-1) + \dots + w(k)$$

- Obtain first order correlations with  $u$

$$r_{yu}^N = g_0 r_u^N(0) + \sum_{i=1}^N f_i r_u^N(i) + r_{wu}^N(0)$$

- Conditions on the input (Robust Input)

1.  $\sup_{1 \leq i \leq N} \left| \frac{r_u^N(i)}{r_u^N(0)} \right| \rightarrow \text{zero}$ .

2.  $\left| \frac{r_{wu}^N(0)}{r_u^N(0)} \right| \rightarrow \text{zero}$ .

## Input Design

- Not robust inputs

- Periodic

- Standard Chirp  $u(t) = \exp(i\alpha t^2)$ ,  $\alpha \in \mathbb{R}$

$$\max_{1 \leq i \leq N} \left| \frac{r_u^N(i)}{r_u^N(0)} \right| \geq \frac{1}{2} - \epsilon \quad \text{for any } \epsilon \text{ and some } N$$

- PBRs: For large period  $m$ , correlations up to  $m$  can be made small.  
Not sufficient.

- Robust Inputs

- High order chirp  $u(t) = \exp(i\alpha t^3)$ ,  $\alpha = \pi(1 + \sqrt{5})$ :

$$\max_{1 \leq i \leq N} \left| \frac{r_u^N(i)}{r_u^N(0)} \right| \leq C \frac{\log(N)}{\sqrt{N}}$$

- i.i.d. Bernoulli process with zero mean and variance 1

$$\mathcal{P} \left\{ \max_{1 \leq i \leq N} r_u^N(i) \geq \alpha \right\} \leq n \exp(-nf(\alpha))$$

- For both signals

$$\left| \frac{r_{wu}^N(0)}{r_u^N(0)} \right| \rightarrow \text{zero}$$

## FIR Models

- Assumptions:

- $\mathcal{T}$  is  $\ell_1$  or  $\mathcal{H}$ .
- $\mathcal{G}$  is the set of FIR systems of order  $M$ .
- Noise is in  $\mathcal{W}$ .
- Prior  $\mathcal{I}$ :  $\text{dist}(T_0, \mathcal{G})$  is bounded by  $\gamma$ .
- Input: High order chirp.

- Results

- The least squares estimator converges to  $G_0$ :

$$\sup_{T \in \mathcal{I}} \sup_{w \in \mathcal{W}} \|\hat{G}_n - G_0\| \leq (C_1 \gamma + C_2 M) \frac{\log(n)}{\sqrt{n}}$$

- The estimator is untuned
- Complexity is polynomial.

## Example: One-Pole Model

- $\mathcal{G} = \left\{ \frac{\theta}{1-a\lambda}, \theta \in \mathbb{R} \right\}$ .
- $\mathcal{T} = \ell_2$  (for simplicity).
- $\Delta = \{(\lambda - a)f(\lambda), f \in \ell_2\}$ .
- Equivalent output equation:

$$y(k) = \theta \tilde{u}(k) + (\lambda - a)f(\lambda)u + w, \quad \tilde{u} = \frac{1}{1-a\lambda}u$$

- Comments

- There is no direct separation between the model and the unmodeled dynamics.
- Least squares estimator will not converge. The unmodeled part affects  $r_u^N(0)$ .
- Need to exploit the structure of the unmodeled.

- A two-step approach

- Pre-filter the data using the non-causal filter  $a^{-k}$ . This corresponds to multiplying the data by

$$\begin{pmatrix} 1 & a & a^2 & \dots & a^N \\ \frac{1}{a} & 1 & a & \dots & a^{N-1} \\ \vdots & \vdots & \ddots & & \\ & & & \dots & 1 \end{pmatrix}$$

- Apply a least squares estimator to the filtered Data.
- Can interpret as a weighted LS.

## General Result

- Assumptions

- $\mathcal{T} = \mathcal{H}$ .
- $\mathcal{G}$  A finite dimensional subspace of stable systems, with dimension  $M$ .
- Noise is in  $\mathcal{W}$ .
- Prior  $\mathcal{I}$ :  $\text{dist}(T_0, \mathcal{G})$  is bounded by  $\gamma$ .
- Input: High order chirp.

- Results

- There exists a filter  $\mathcal{F}$  such that the two-step algorithm converges to  $G_0$

$$\sup_{T \in \mathcal{I}} \sup_{w \in \mathcal{W}} \|\hat{G}_n - G_0\| \leq (C_1 \gamma + C_2 M) \frac{\log(n)}{\sqrt{n}}$$

- The estimator is untuned
- Complexity is polynomial.

## MPE Revisited

- If  $w$  is WN, then it can be shown that for a fixed  $T_0 \in \mathcal{T}$ , the LS estimator satisfies:

$$\lim_{N \rightarrow \infty} \hat{G}_N = \tilde{G} = \min_{G \in \mathcal{G}} \|(T_0 - G)\Phi_u^{\frac{1}{2}}\|_2 \quad w.p.1$$

where  $\Phi_u$  is the spectral density function of  $u$ .

- Comments:

- Convergence is only pointwise (w.p.1).
- There is no uniform convergence rate.
- Results are asymptotic.
- Results require quasi-stationarity.
- $\tilde{G} \neq G_0$  unless the input is robust.
- No robust inputs are constructed.
- Theory addresses  $\ell_2$  error bounds.

- Contrast with previous results:

- Convergence is uniform (w.p.1 if  $w$  is WN).
- A uniform convergence rate is derived.
- Estimates are based on finite data.
- No stationarity assumptions.
- Robust inputs are constructed.
- Theory addresses more general error bounds.

## Identification In Practice

- The set of unfalsified models,  $S_N$  is convex.
  - Define  $z_w(t) = y(t) - Gu(t) - w(t)$  for  $t = 0, \dots, N$
  - Let  $\mathbf{z}_w$  be the vector constructed from  $z_w(t)$ .
  - The set  $S_N$  can be written as:

$$S_N = \{G \mid \mathbf{z}_w = \mathcal{U}\delta, \langle g^i, \Delta \rangle = 0, i = 1, \dots, M, w \in \mathcal{W}\}$$

$\mathcal{U}$  is the Toeplitz matrix constructed from  $u$  and  $g^i$ 's are the annihilators of  $\Delta$ .

- Eliminating  $\Delta$

$$S_N = \{G \mid \mathbf{z}_w^T V \mathbf{z}_w \leq \gamma, w \in \mathcal{W}\}$$

$V$  can be computed from the resulting QP.

- Let  $D_N(\gamma) = \text{diam}(S_N)$ .
- Estimate  $\hat{\gamma}_N$  as follows:

$$\min \gamma$$

$$D_N(\gamma) \leq (C_1\gamma + C_2M) \frac{\log(N)}{\sqrt{N}}$$

- Estimates capture tradeoff between parametric and non parametric errors.

## Example

- Process:

$$T(y) = \frac{1}{1 - 0.8\lambda} + \Delta(\lambda),$$

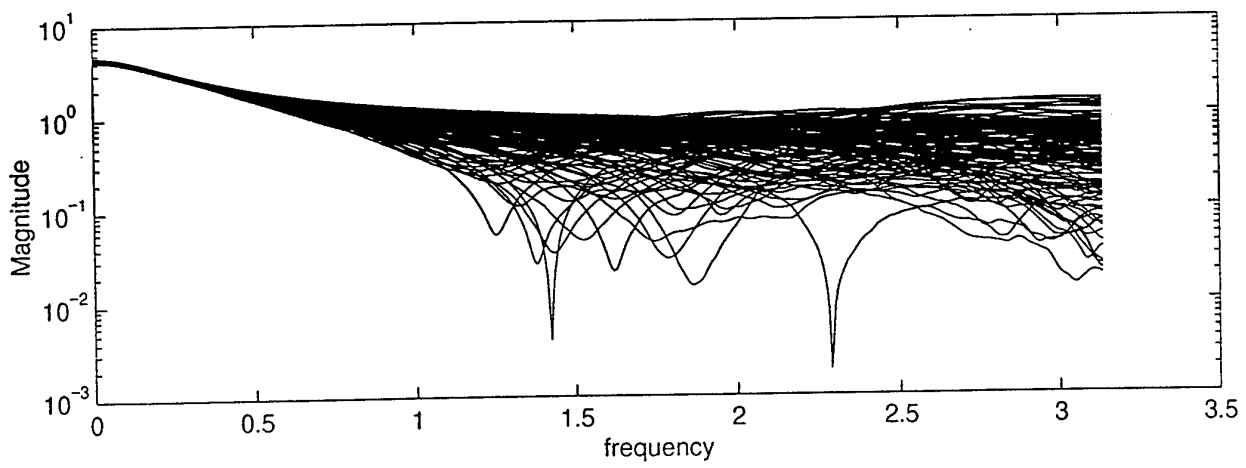
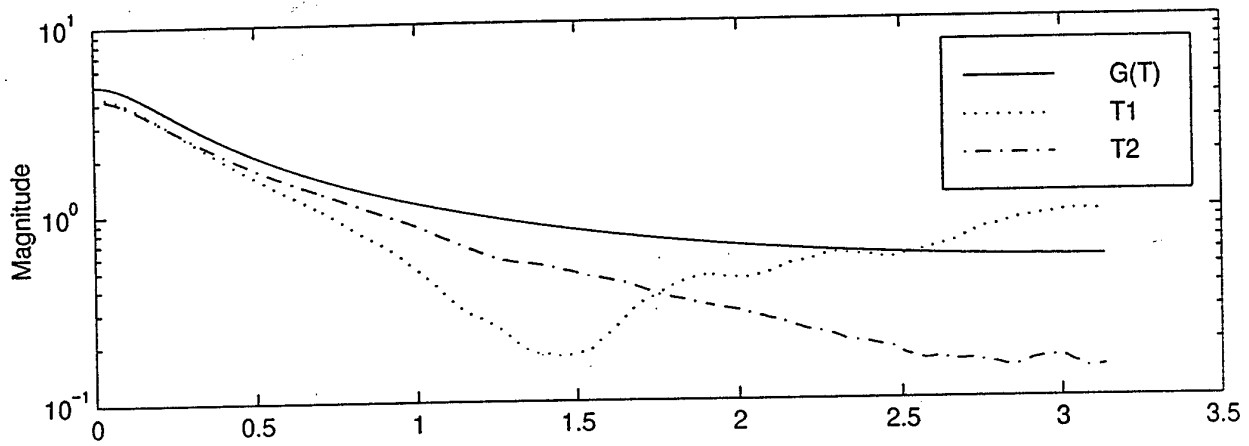
- The unmodeled:

$$\Delta(\lambda) = \sum_{k=0}^{200} \delta(k)\lambda^k$$

$$\delta(k) = \left( \sum_{k=0}^{200} \delta_1^2(k)(k^2 + 1) \right)^{-1/2} \frac{\delta_1(k)}{k^2 + 1}$$

where  $\delta_1(k) = (\lambda - a)v(k)$  and  $v$ , a vector of length 200, was selected using a random number generator.

- Noise:  $N(0, .3)$
- Input: High order chirp.



## Example

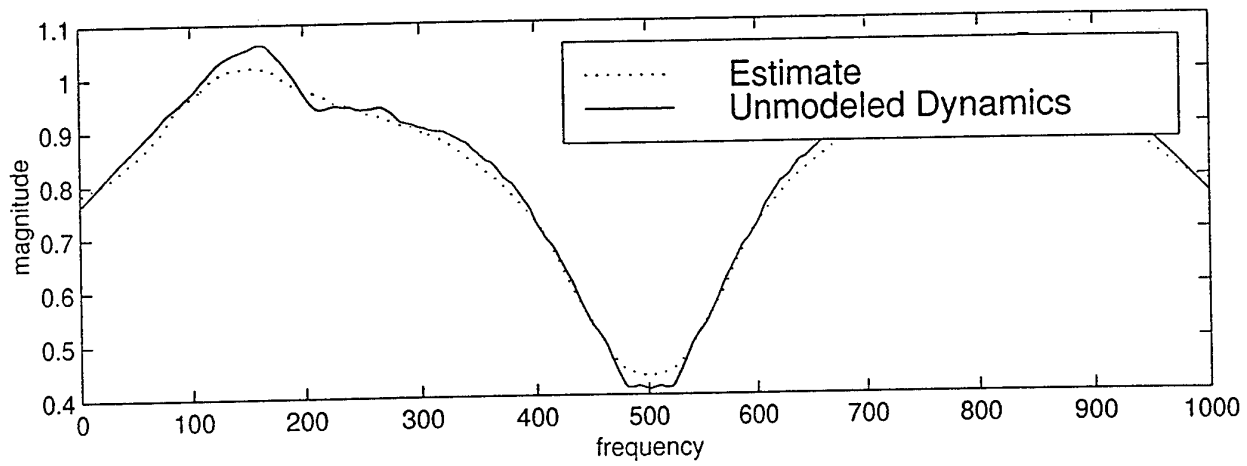
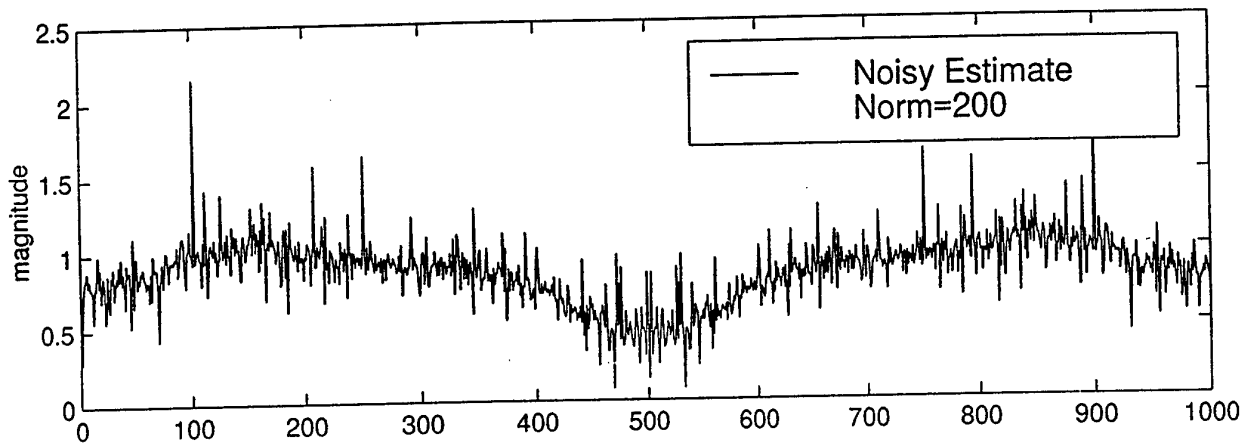
- Comparisons:

Process $T_1$					
Algorithm	$\theta$	% Error	Actual Error	Estimated-Error	unmodeled-error
MUD	.97	3	.03	.06	1.0
LS	.67	33	.33	.0002	1.9
WLS	.60	40	.4	.0002	2.2

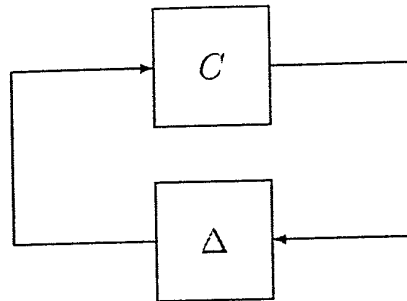
  

Process $T_2$					
Algorithm	$\theta$	% Error	Actual Error	Estimated-Error	unmodeled error
MUD	.99	1	.01	.06	1.01
LS	.64	36	.36	.0002	1.95
WLS	.58	42	.42	.0001	2.3

- Estimating the unmodeled:



## Robust Control



- Standard robustness results extend for perturbations in  $\mathcal{H}$ .
- Stability if and only if for all  $\theta \in [0, 2\pi)$  and  $\Delta \in \Delta$

$$\left( 1 + (\Delta_r \quad \Delta_i) \begin{pmatrix} C \\ iC \end{pmatrix} \right) (e^{i\theta}) \neq 0$$

- Idea is to compute the range of  $\Delta$  at each frequency:

$$\mathcal{S}(\theta) = \left\{ \begin{pmatrix} \Delta_r(e^{i\theta}) \\ \Delta_i(e^{i\theta}) \end{pmatrix}; \sum_{k=0}^{\infty} (1 + k^2 \delta_k^2) \leq 1 \right\}$$

- Through the singular value decomposition, we can find a continuous function in  $\theta$   $W(\theta)$ , such that

$$\mathcal{S}(\theta) = \left\{ W(\theta) \begin{pmatrix} \eta \\ \xi \end{pmatrix}; \|\eta \quad \xi\|_2 \leq 1 \right\}$$

- Stability if and only if for all  $\theta \in [0, 2\pi)$  and  $\|[\eta \quad \xi]\|_2 \leq 1$

$$\left( 1 + [\eta \quad \xi] W^T \begin{pmatrix} C \\ iC \end{pmatrix} \right) (e^{i\theta}) \neq 0$$

- Above is a standard one-rank problem. Both analysis and synthesis are equivalent to convex problems.
- A theory for the general robustness problem (including parametric uncertainty) has been developed with such error bounds.

## Conclusions

- A new paradigm for system identification
  - Bridges the gap between stochastic and deterministic formulations.
  - Under modeling is explicit in the formulation.
  - Based on sample-path analysis.
  - Captures tradeoffs in error estimates.
- Formulation compatible with robust control
  - Derive error bounds in terms of  $\|\cdot\|_{\mathcal{H}}$ .
  - Derive robustness results for this norm.
  - Exploit the Hilbert space structure.
- Future directions.

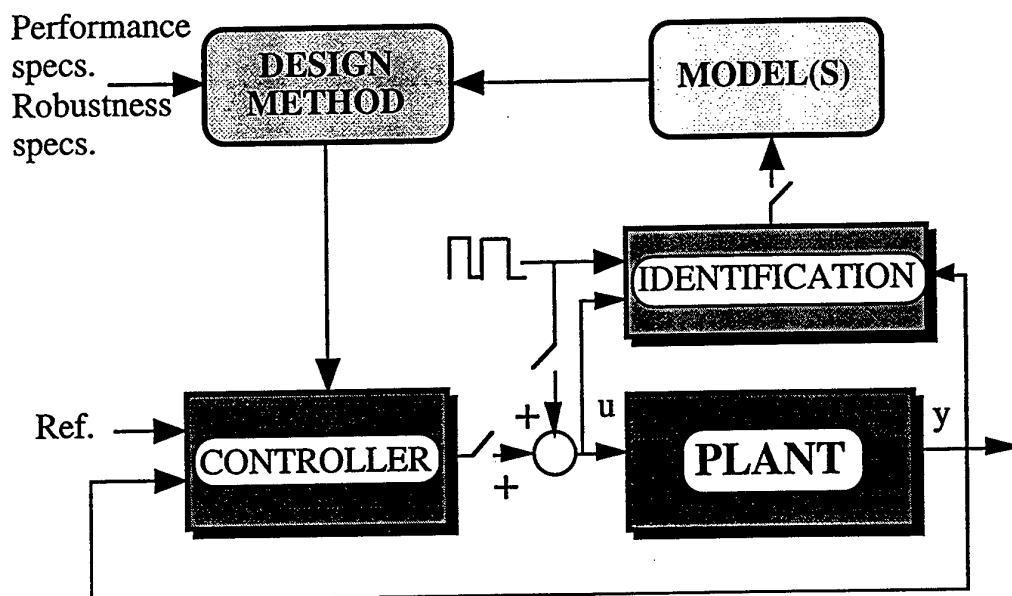
**ISSUES IN CONTROL DESIGN :**

**THE IMPORTANCE OF SYSTEM  
IDENTIFICATION IN CLOSED LOOP**

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**June 1999**

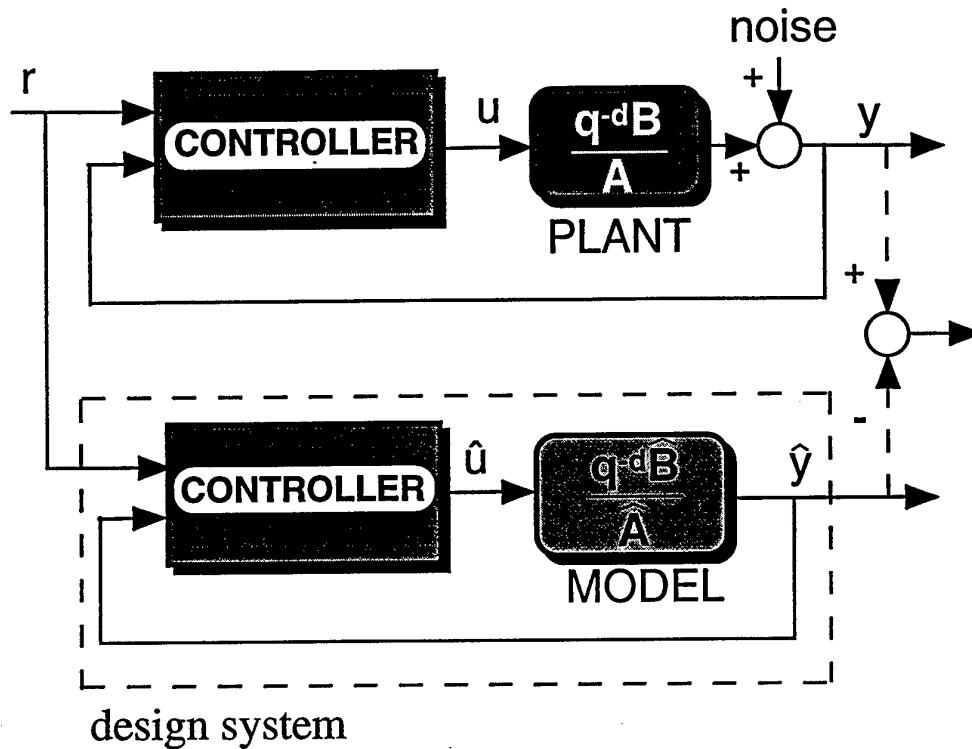
# CONTROLLER DESIGN AND VALIDATION



- 1) Identification of the dynamic model(s)
- 2) Performance and robustness specifications
- 3) Compatible controller design method
- 4) Controller implementation
- 5) Real-time controller validation  
(and on site re-tuning)
- 6) Controller maintenance (same as 5)

(5) and (6) require *closed-loop identification*

# REAL-TIME CONTROLLER VALIDATION



**Comparison of  
“achieved” and “desired” performances**

**A useful interpretation :**

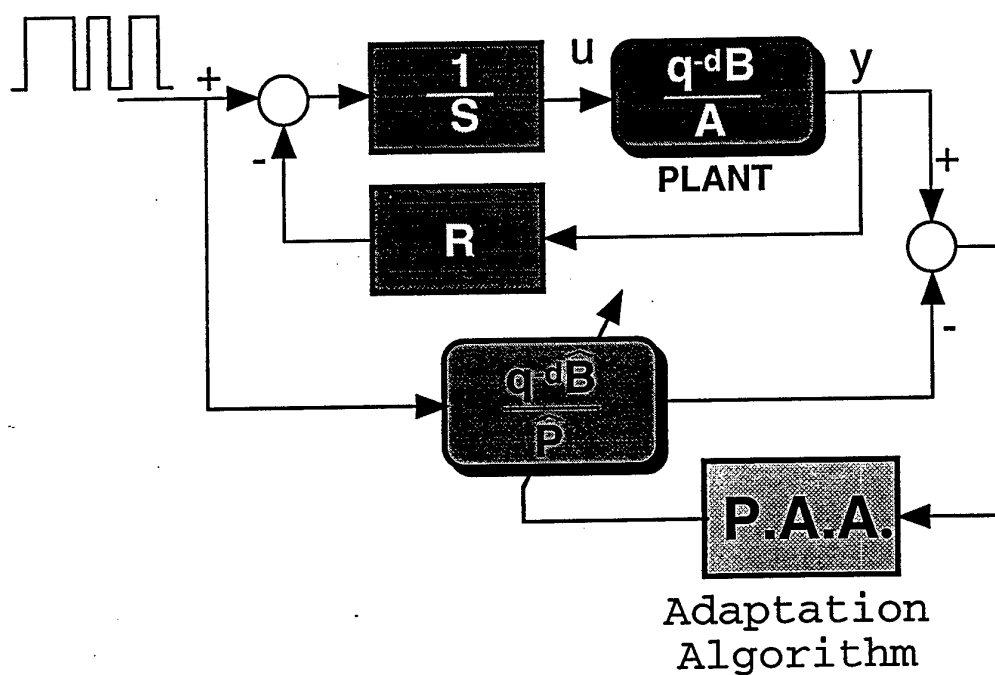
Check to what extent the model used for design allows achievement of :

- desired nominal performance
- desired robustness specs. (sensitivity fct.)

# REAL-TIME CONTROLLER VALIDATION

Time domain tests  
( too inaccurate in some cases )

Closeness tests of achieved and desired  
*poles* and/or *sensitivity functions*  
by **identification of the closed loop**



**If the results are not satisfactory :**

Plant model identification in *closed loop*  
+  
Controller redesign

# CLOSED LOOP PLANT IDENTIFICATION

## Why ?

There are systems where open loop operation is not suitable ( instability, drift, .. )

A controller may already exist ( ex . : PID )

Re-tuning of the controller

- a) to improve achieved performances
- b) controller maintenance

Iterative identification and controller redesign

**Cannot be dissociated from the controller and robustness issues**

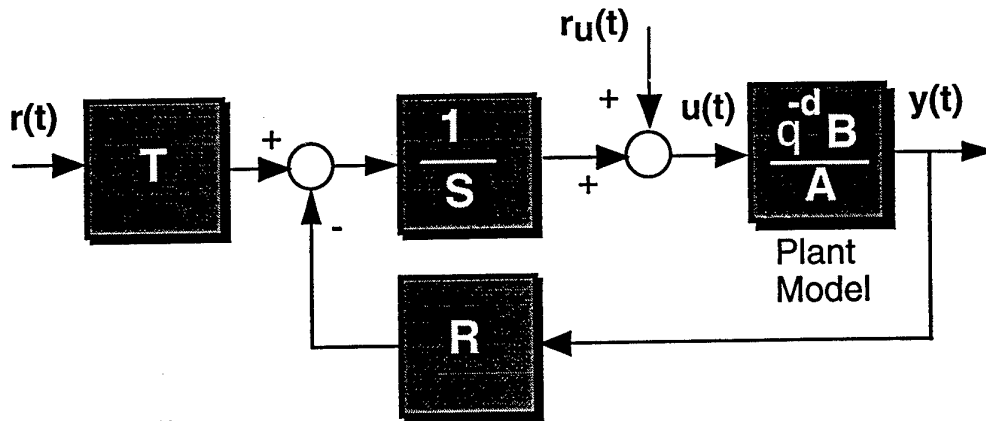
*DUAL PROBLEM*  
**CONTROLLER REDUCTION**

## OUTLINE

- Introduction
- Background in robust digital control
- Identification in Closed loop
  - Some facts
  - Closed loop output error (CLOE) algorithms
- Validation of models identified in closed loop
- Iterative identification in closed loop and controller re-design
- Experimental results
- Controller reduction by identification in closed loop
- Experimental results
- Identification in closed loop of nonlinear plants
- Conclusions and Perspectives

# **Background in Robust Digital Control**

## The R-S-T Digital Controller



$$(y(t-1) = q^{-1}y(t))$$

**Plant Model :**

$$G(q^{-1}) = \frac{q^{-d} B(q^{-1})}{A(q^{-1})}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_{n_A} q^{-n_A}$$

$$B(q^{-1}) = b_1 q^{-1} + \dots + b_{n_B} q^{-n_B}$$

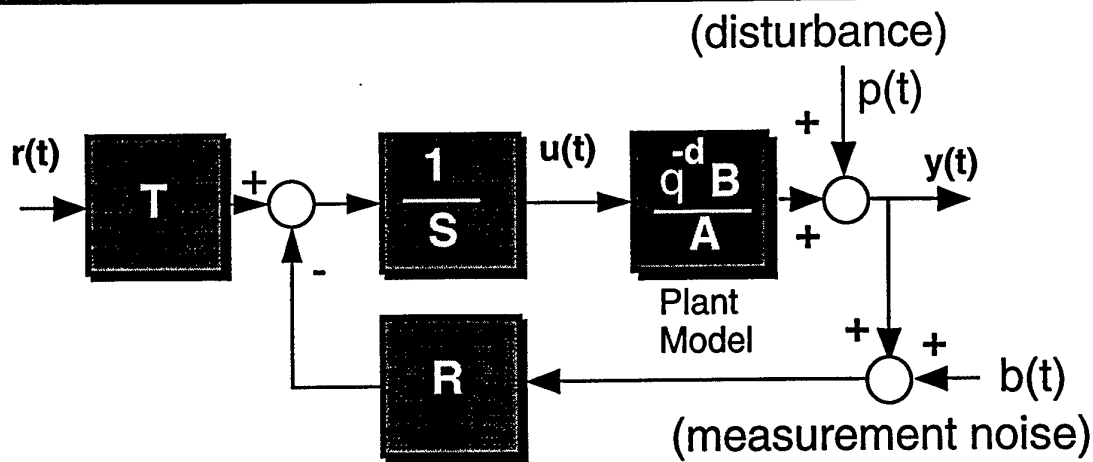
**R-S-T controller :**

$$S(q^{-1}) u(t) = T(q^{-1}) r(t) - R(q^{-1}) y(t)$$

**Closed loop poles :**

$$P = AS + z^{-d}BR$$

# THE SENSITIVITY FUNCTIONS



**Output Sensitivity Function ( $S$ ) :**

$$S_{yp}(z^{-1}) = \frac{AS}{AS + z^{-d}BR} = \frac{AS}{P}$$

**Input Sensitivity Function ( $U$ ) :**

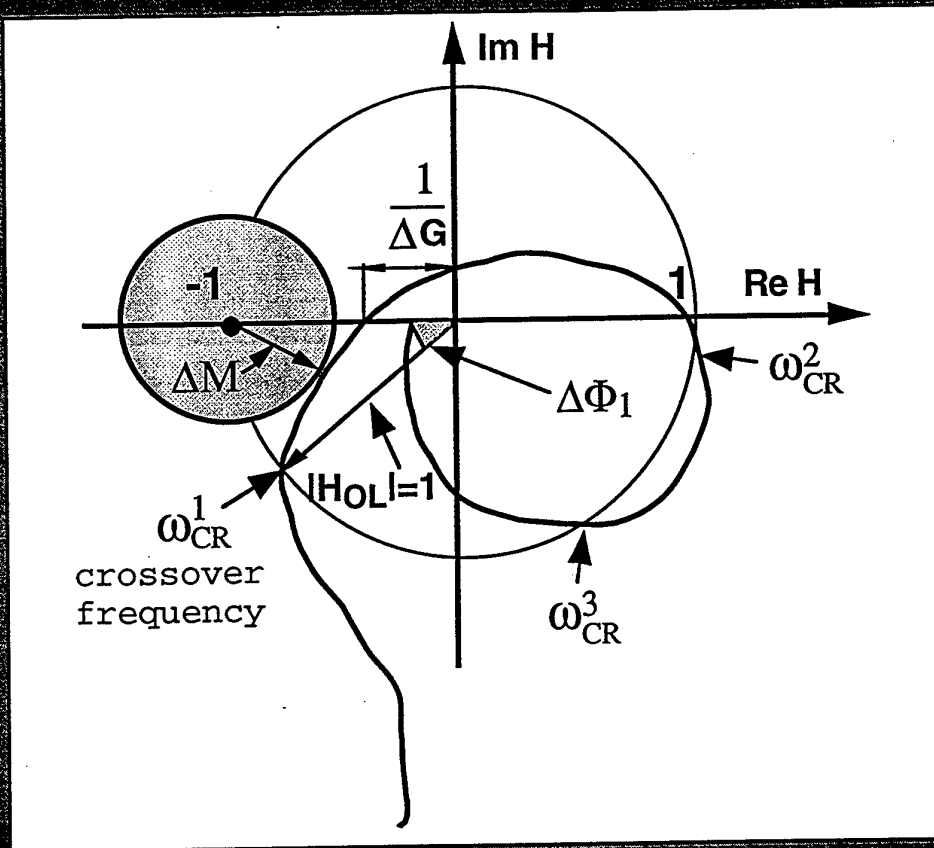
$$S_{up}(z^{-1}) = - \frac{AR}{AS + z^{-d}BR} = - \frac{AR}{P}$$

**Noise Sensitivity Function ( $-T$ ) :**

$$S_{yb}(z^{-1}) = - \frac{z^{-d}BR}{AS + z^{-d}BR} = - \frac{z^{-d}BR}{P}$$

$$S_{yp} - S_{yb} = 1$$

# ROBUSTNESS MARGINS



**Modulus Margin :**

$$\Delta M = |1 + H_{OL}(z^{-1})|_{\min} = (\|S_{yp}(z^{-1})\|_{\max})^{-1} = (\|S_{yp}\|_{\infty})^{-1}$$

**Delay Margin :**

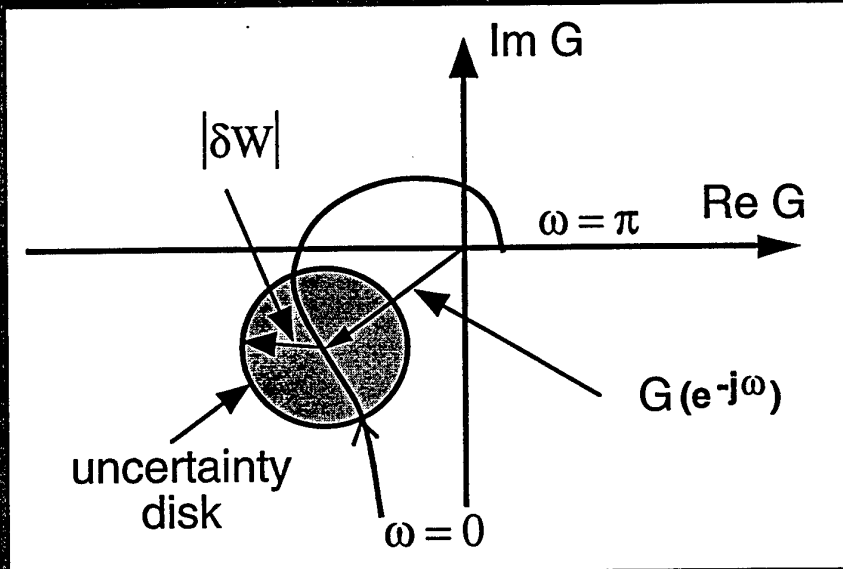
$$\Delta \tau = \min_i \frac{\Delta \Phi_i}{\omega_{CR_i}}$$

**Typical values :**

$$\Delta M \geq 0.5 \text{ (-6dB)}, \quad \Delta \tau \geq T_s$$

$$\Delta M \geq 0.5 \Rightarrow \Delta G \geq 2, \quad \Delta \Phi > 29^\circ$$

# ROBUST STABILITY



Family of Plant Models :  $G' \in F(G, \delta, W_x)$

$G$  - nominal model ;  $\|\delta(z^{-1})\|_\infty \leq 1$

$W_x(z^{-1})$  - size (and type) of uncertainty

## Robust Stability Condition :

a related sensitivity function

a type of uncertainty

$$\|S_x W_x\|_\infty \leq 1$$

defines an upper template for the modulus of the sensitivity function

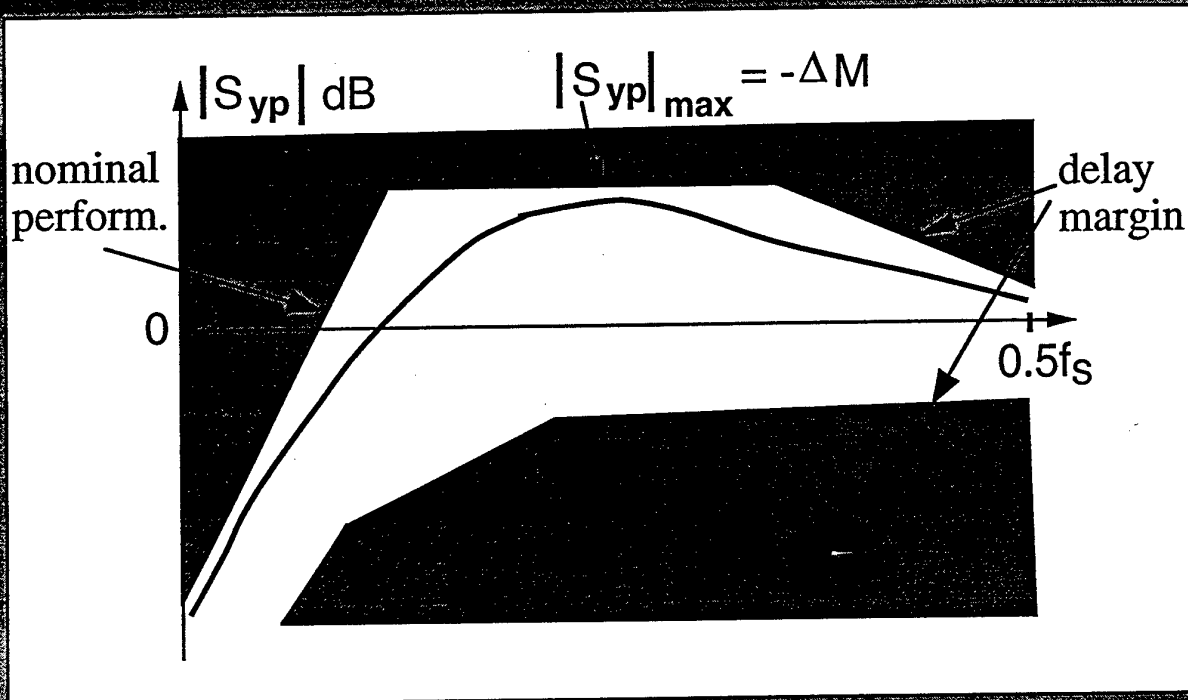
$$|S_x| \leq |W_x|^{-1}$$

defines the size of the tolerated uncertainty

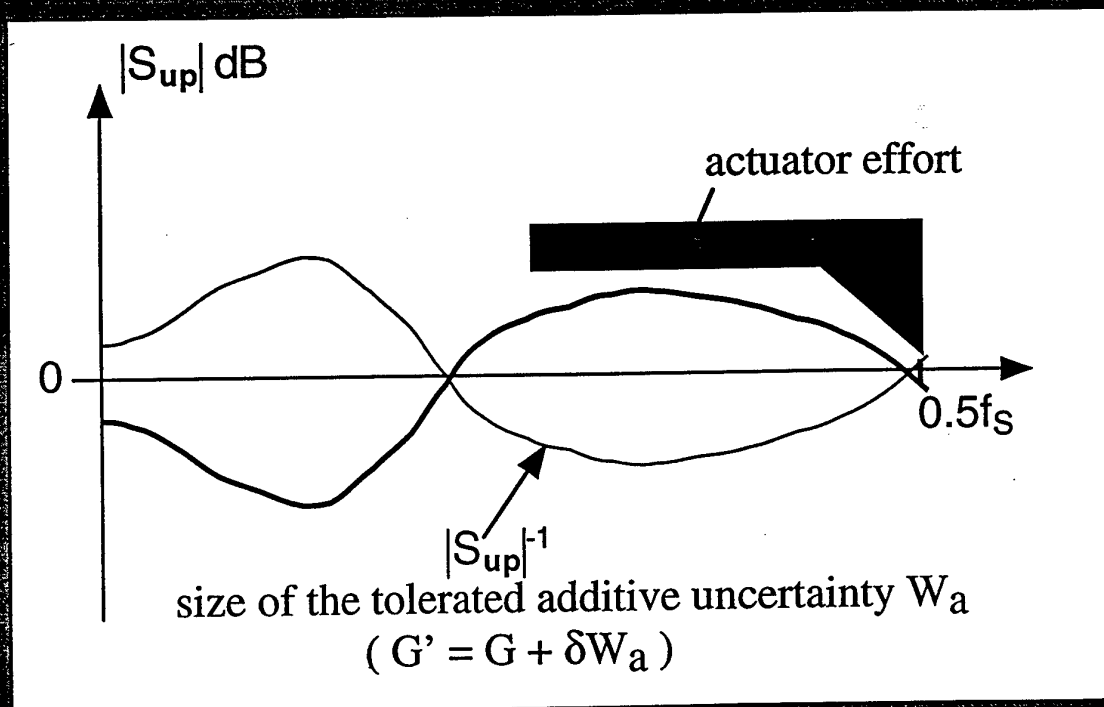
*There are also lower templates (because of the relationship between various sensitivity fct.)*

# Templates for the Sensitivity Functions

## Output Sensitivity Function



## Input Sensitivity Function



## ROBUST CONTROLLER DESIGN

Combining nominal performance design  
with shaping of the sensitivity functions

Several approaches exist

An efficient method :

Pole placement with sensitivity function shaping  
using convex optimization

*uses convex parametrization w.r. to  
auxiliary poles and controller parameters.*

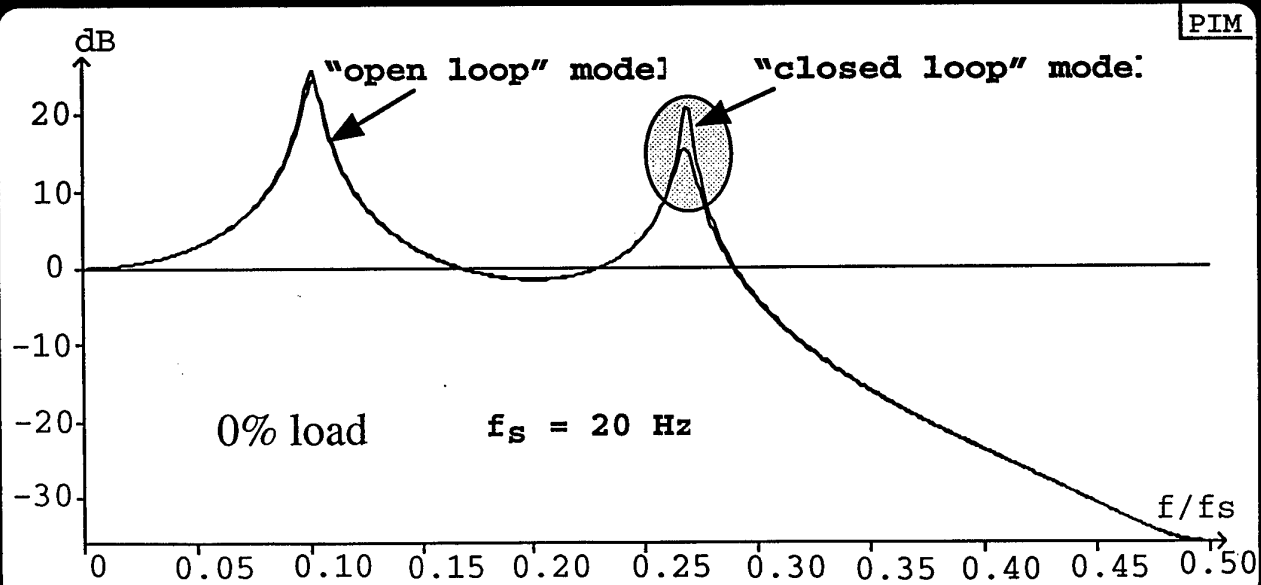
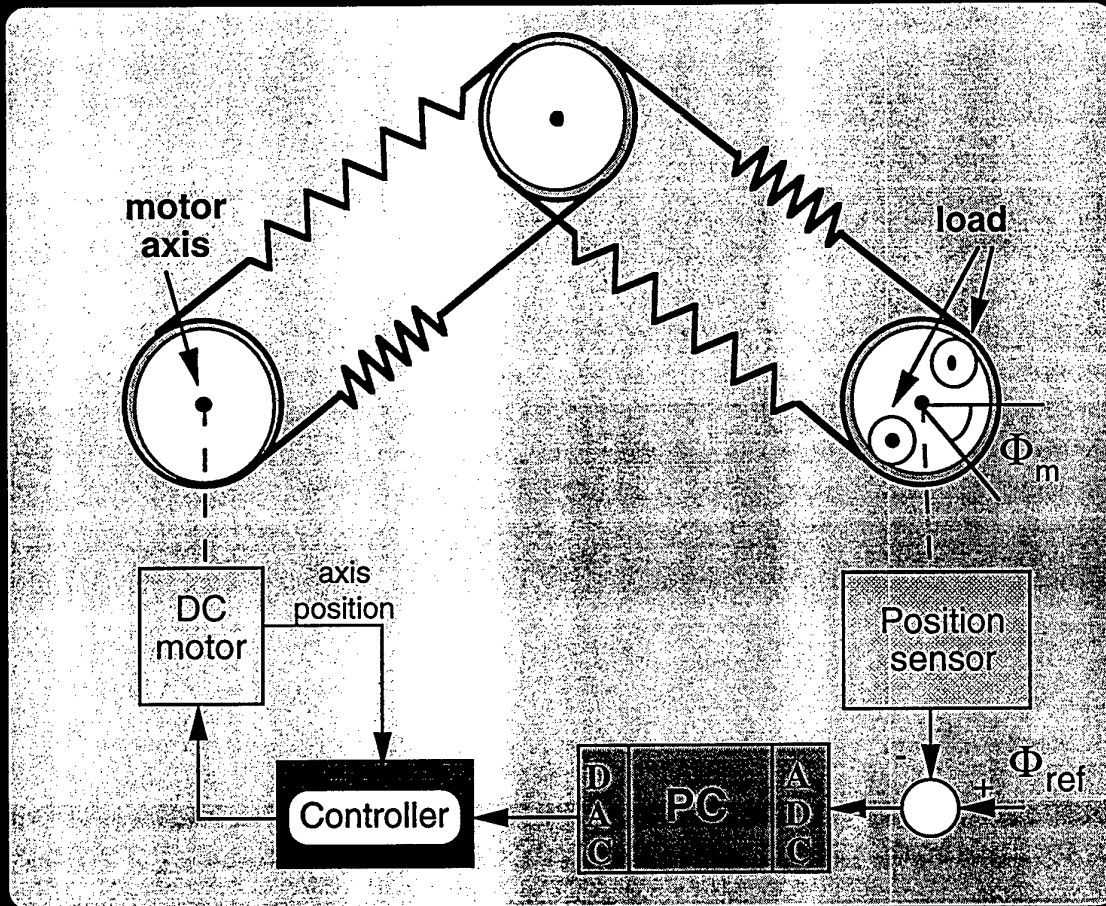
(ECC 97, AUTOMATICA 6/99 - J. LANGER)

Illustrates the benefit of using  
*convex optimization for robust control*  
(S. Boyd, A. Rantzer, S. Megretski)

# Identification in Closed Loop

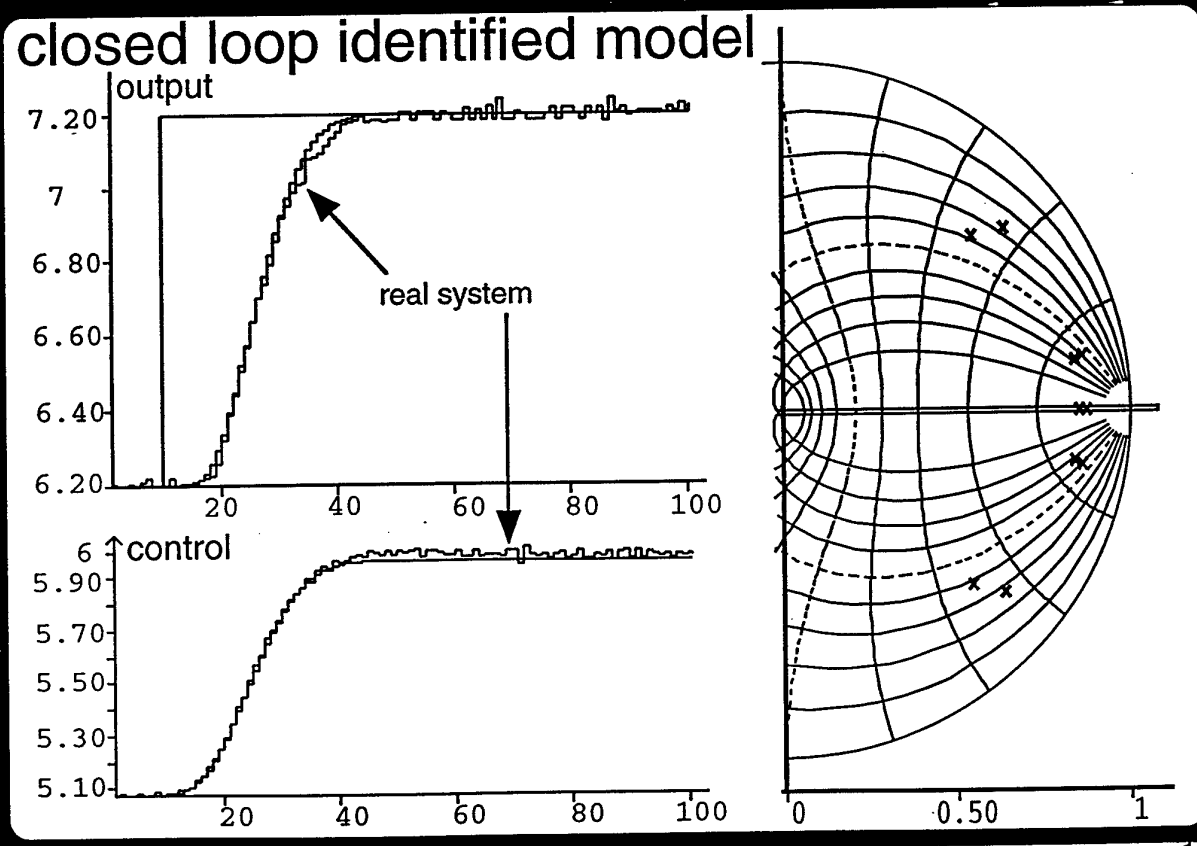
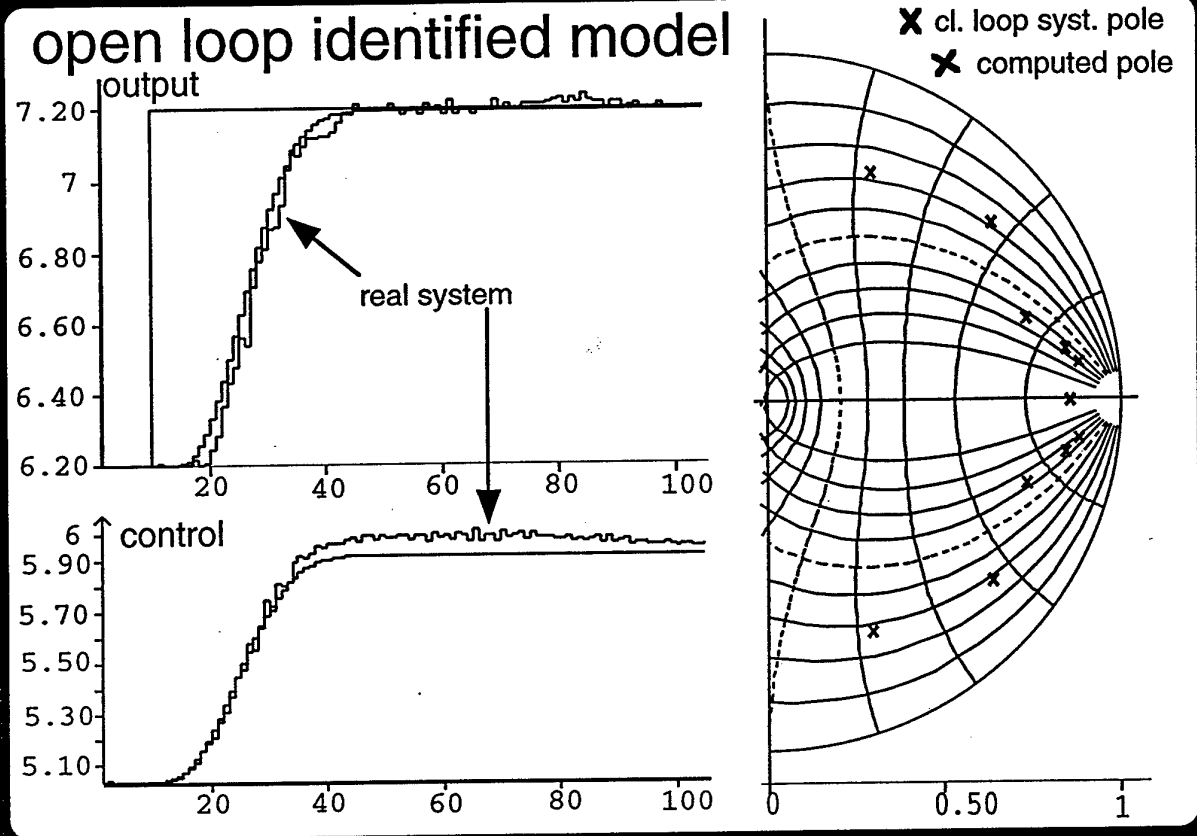
# Identification in Closed Loop

## The Flexible Transmission

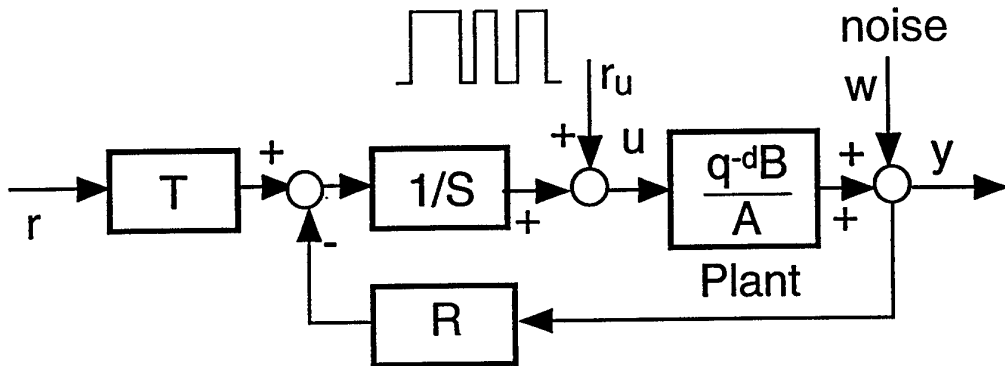


# Benefits of Identification in Closed Loop

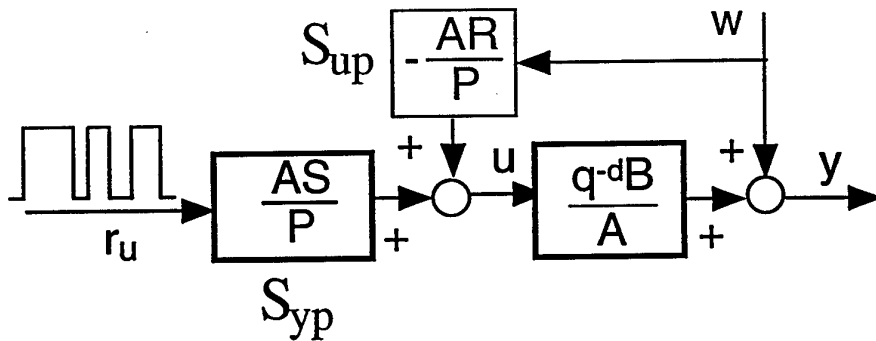
## The Flexible Transmission



## IDENTIFICATION IN CLOSED LOOP



## OPEN LOOP INTERPRETATION

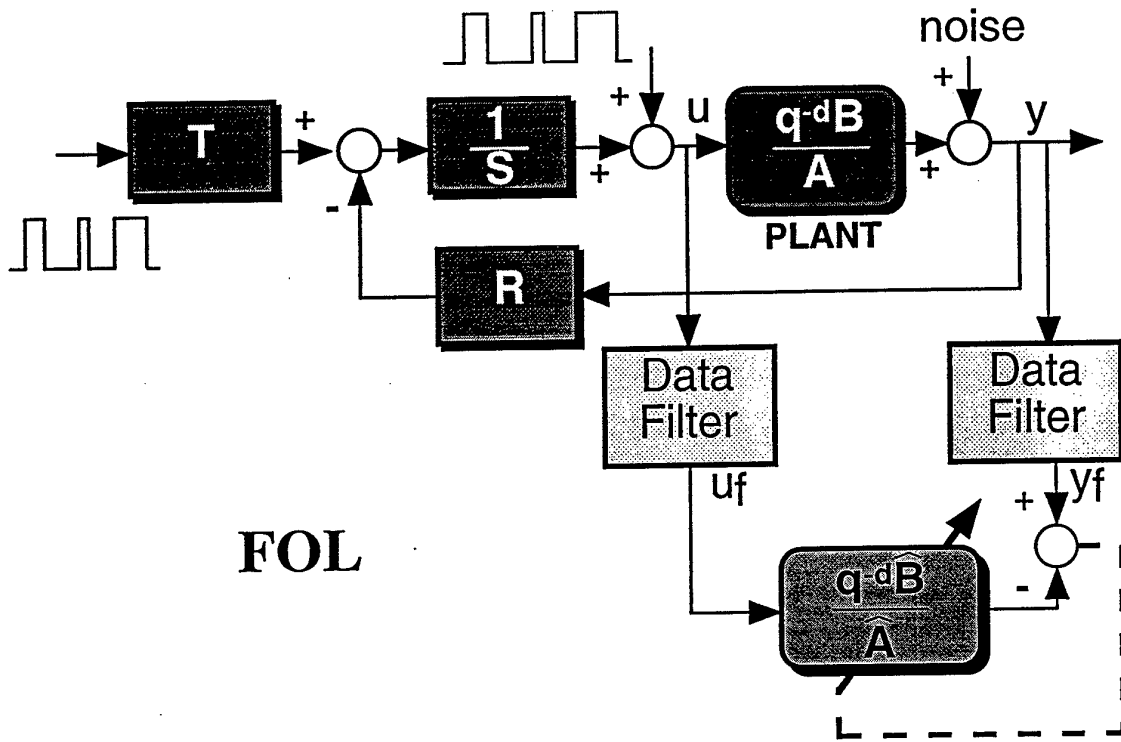


**Obj. : Development of Algorithms which :**

- take advantage of the “improved” input spectrum
- are insensitive to noise in closed loop operation

# CLOSED LOOP PLANT IDENTIFICATION

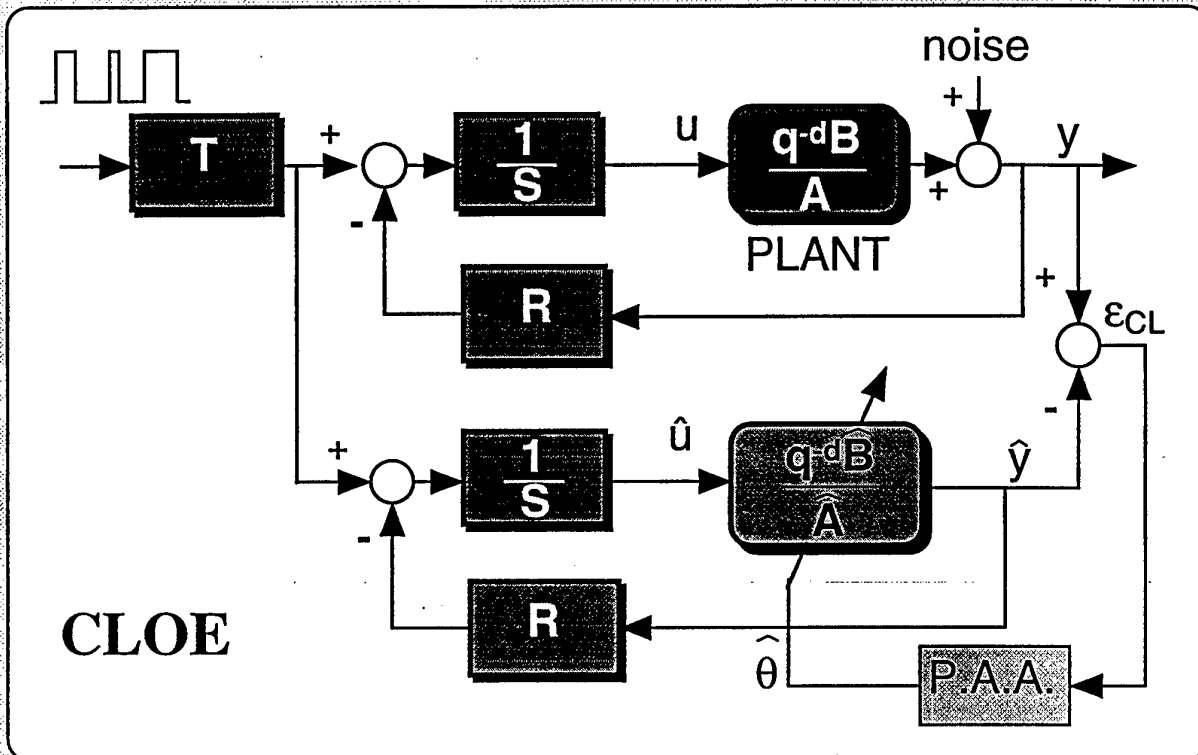
## FILTERED OPEN LOOP ALGORITHMS



I/O data filters :  $S/\hat{P}$  or  $\hat{A}S/\hat{P}$  or ....  
(may be related to the control objective)

- biased estimates
- require (theoretically) time varying filters
- FOL can be seen as approx. of CLOE alg.
- are used in standard indirect adaptive control

# CLOSED LOOP OUTPUT ERROR ALGORITHMS



$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\epsilon_{CL}(t+1)$$

$$\Phi(t) = \phi(t) = [-\hat{y}(t), \dots, \hat{u}(t), \dots] \quad (\text{CLOE})$$

or

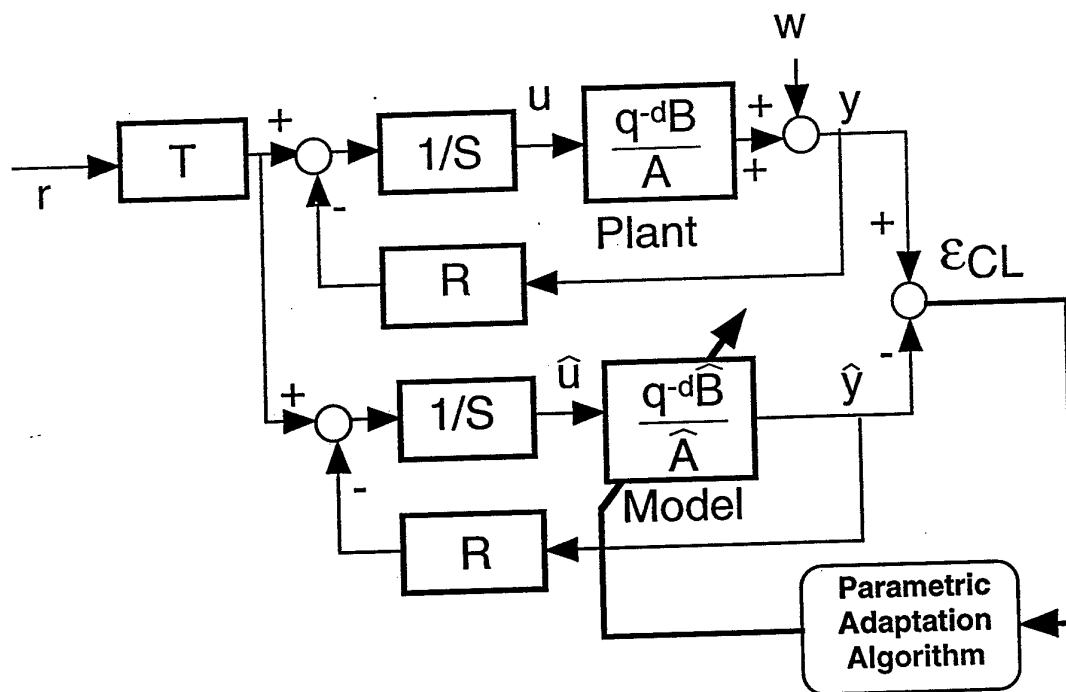
$$\Phi(t) = \frac{S}{\hat{P}}\phi(t) \quad (\text{F-CLOE})$$

- Unbiased parameter estimation (asympt.)
- Mild sufficient positive real condition for convergence ( S/P or  $\hat{P}/P$  )
- Insensitive to plant disturbances

## Objective of the Identification in Closed Loop

( Identification for Control )

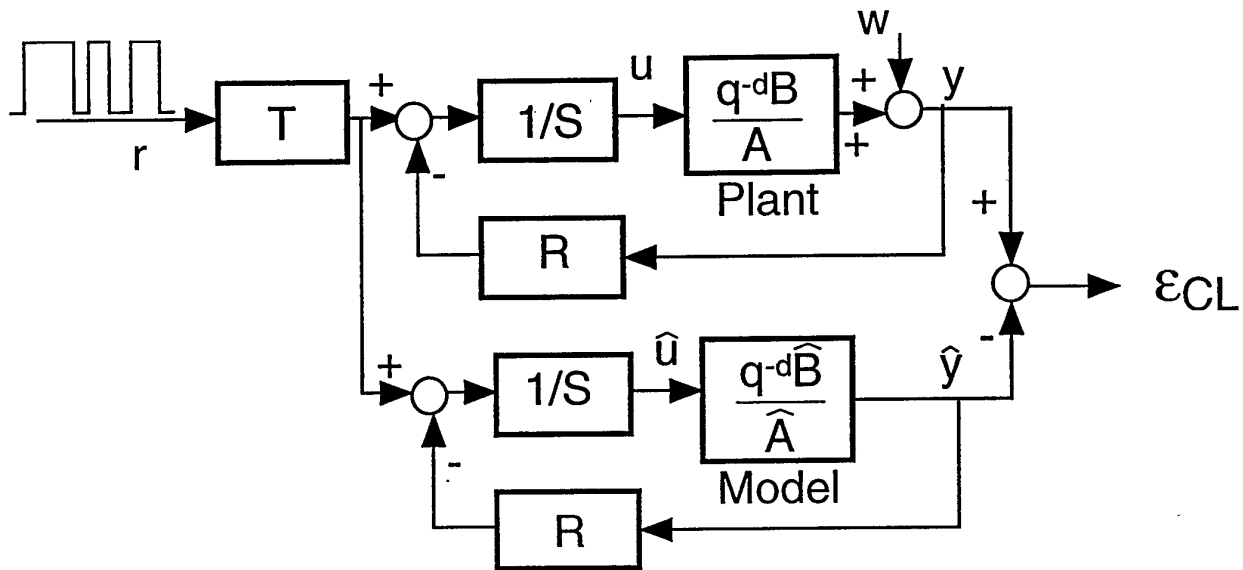
Find the “plant model” which minimizes the discrepancy between the “real” closed loop system and the “simulated” closed loop system



**CLOE Algorithms**  
(Closed Loop Output Error)

# IDENTIFICATION IN CLOSED LOOP

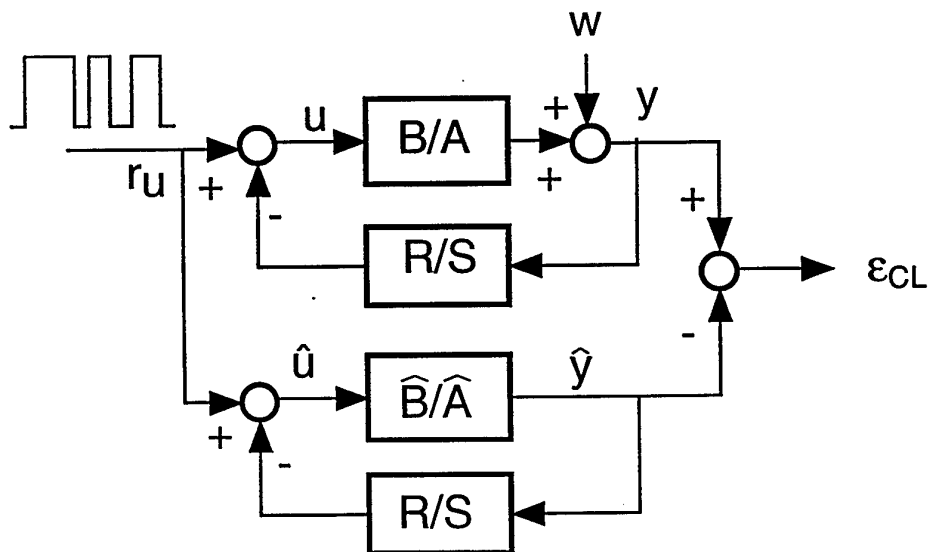
Excitation on the reference



$$u = -\frac{R}{S}y + \frac{T}{S}r$$

$$\hat{u} = -\frac{R}{S}\hat{y} + \frac{T}{S}r$$

Excitation added to the controller output



$$u = -\frac{R}{S}y + r_u$$

$$\hat{u} = -\frac{R}{S}\hat{y} + r_u$$

Closed loop system:

$$(1) \quad y(t+1) = -A^*y(t) + B^*u(t) = \theta^T \phi(t)$$

$$\theta^T = [a_1, \dots, a_{nA}, b_1, \dots, b_{nB}]$$

$$\phi^T(t) = [-y(t), \dots, -y(t-nA+1), u(t), \dots, u(t-nB+1)]$$

$$u(t) = -\frac{R}{S}y(t) + r_u$$

Closed loop adjustable predictor:

$$(2') \quad \hat{y}^0(t+1) = -\hat{A}^*(t)\hat{y}(t) + \hat{B}^*(t)\hat{u}(t) = \hat{\theta}^T(t)\phi(t)$$

$$(2'') \quad \hat{y}(t+1) = -\hat{A}^*(t+1)\hat{y}(t) + \hat{B}^*(t+1)\hat{u}(t) = \hat{\theta}^T(t+1)\phi(t)$$

$$\hat{\theta}^T = [\hat{a}_1, \dots, \hat{a}_{nA}, \hat{b}_1, \dots, \hat{b}_{nB}]$$

$$\phi^T(t) = [-\hat{y}(t), \dots, -\hat{y}(t-nA+1), \hat{u}(t), \dots, \hat{u}(t-nB+1)]$$

$$\hat{u}(t) = -\frac{R}{S}\hat{y}(t) + r_u$$

Closed loop prediction( *output* ) error

$$\varepsilon_{CL}^0(t+1) = y(t+1) - \hat{y}^0(t+1) \quad \text{a priori}$$

$$\varepsilon_{CL}(t+1) = y(t+1) - \hat{y}(t+1) \quad \text{a posteriori}$$

## CLOE

Adjustable Predictor :

$$\hat{y}^0(t+1) = \hat{\theta}^T(t)\phi(t)$$

$$\hat{y}(t+1) = \hat{\theta}^T(t+1)\phi(t)$$

$$\hat{\theta}^T(t) = [\hat{a}_1(t), \dots, \hat{a}_{nA}(t), \hat{b}_1(t), \dots, \hat{b}_{nB}(t)]$$

$$\phi^T(t) = [-\hat{y}(t), \dots, -\hat{y}(t-nA+1), \hat{u}(t), \dots, \hat{u}(t-nB+1)]$$

The Algorithm

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon_{CL}(t+1)$$

$$F(t+1)^{-1} = \lambda_1(t)F(t) + \lambda_2(t)\phi(t)\phi^T(t)$$

$$0 < \lambda_1(t) \leq 1 ; \quad 0 \leq \lambda_2(t) < 2$$

$$\varepsilon_{CL}(t+1) = \frac{\varepsilon_{CL}^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)}$$

$$\varepsilon_{CL}^0(t+1) = y(t+1) - \hat{\theta}^T(t)\phi(t)$$

## CLOE Analysis

( the “plant model” is in the “model set” )

$$\varepsilon_{CL}(t+1) = \frac{S}{P} [\theta - \hat{\theta}(t+1)]^T \phi(t) + \frac{AS}{P} w(t+1)$$

Deterministic environment ( $w=0$ )

$$\lim_{t \rightarrow \infty} \varepsilon_{CL}(t+1) = \lim_{t \rightarrow \infty} \varepsilon_{CL}^0(t+1) = 0$$

$$\|\phi(t)\| < \infty ; \forall t$$

Stochastic Environment ( $w \neq 0$ )

$$\text{Prob}\left\{\lim_{t \rightarrow \infty} \hat{\theta}(t) \in D_C\right\} = 1$$

$$\text{where : } D_C = \left\{\hat{\theta} : \phi^T(t, \hat{\theta})[\theta - \hat{\theta}] = 0\right\}$$

*If:*

$$H'(z^{-1}) = \frac{S(z^{-1})}{P(z^{-1})} - \frac{\lambda}{2} = \text{S.P.R.}$$

$$\text{where : } \max_t \lambda_2 \leq \lambda < 2$$

## F - CLOE and AF-CLOE

(Adaptive) Filtered Cloded Loop Output Error

$$\varepsilon_{CL}(t+1) = \frac{S}{P} \frac{\hat{P}}{S} [\theta \quad -\hat{\theta}(t+1)]^T \frac{S}{\hat{P}} \phi(t) = \frac{\hat{P}}{P} [\theta \quad -\hat{\theta}(t+1)]^T \phi_f(t)$$

**F-CLOE:**

$$\hat{P} = \hat{A}S + \hat{B}R ; \quad \phi_f(t) = \frac{S}{\hat{P}} \phi(t)$$

**AF-CLOE:**

$$\hat{P}(t, q^{-1}) = \hat{A}(t, q^{-1})S + \hat{B}(t, q^{-1})R \quad \phi_f(t) = \frac{S}{\hat{P}(t, q^{-1})} \phi(t)$$

The Algorithm

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi_f(t)\varepsilon_{CL}(t+1)$$

$$F(t+1)^{-1} = \lambda_1(t)F(t) + \lambda_2(t)\phi_f(t)\phi_f^T(t)$$

$$0 < \lambda_1(t) \leq 1 ; \quad 0 \leq \lambda_2(t) < 2$$

$$\varepsilon_{CL}(t+1) = \frac{\varepsilon_{CL}^0(t+1)}{1 + \phi_f^T(t)F(t)\phi_f(t)} \quad (\text{approx.})$$

$$\varepsilon_{CL}^0(t+1) = y(t+1) - \hat{\theta}^T(t)\phi(t)$$

## F - CLOE (AF-CLOE) analysis

- Deterministic Stability Condition :
- Stochastic Convergence Condition (O.D.E.):

$$\frac{\hat{P}(z^{-1})}{P(z^{-1})} - \frac{\lambda}{2} = \text{S.P.R.}$$

$$\max_t \lambda_2 \leq \lambda < 2$$

## Asymptotic BIAS - Frequency Distribution

(“plant model” not necessarily in the “model set”)

### CLOE/F-CLOE/AF-CLOE algorithms

$$\hat{\theta}^* = \operatorname{argmin}_{\theta} \int_{-\pi}^{\pi} |S_{yp}|^2 [ |G - \hat{G}|^2 |\hat{S}_{yp}|^2 \phi_{r_u}(\omega) + \phi_w(\omega) ] d\omega$$

(excitation added to the controller output)

excitation on the reference :

$$|\hat{S}_{yp}|^2 \phi_{r_u}(\omega) \rightarrow |\hat{T}_{ur}|^2 \phi_r(\omega) ; \hat{T}_{ur} = T\hat{A}/\hat{P}$$

$\hat{S}_{yp}$  = Sensitivity of the closed loop predictor

- noise does not affect parameter estimation (as.)
- precision of estimated model enhanced in the critical frequency regions for design

# **Validation of Models Identified in Closed Loop**

## **Validation of Models Identified in Closed Loop**

**Controller dependent validation !**

### **- Statistical Validation**

### **- Pole Closeness Validation**

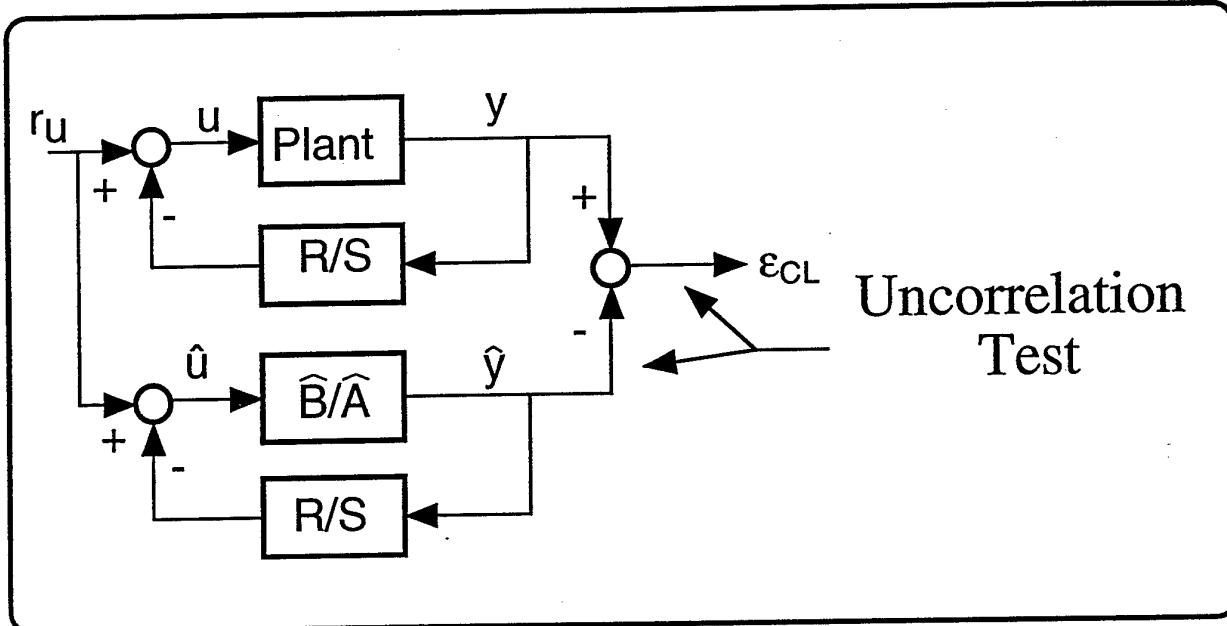
- 1) Identify the closed loop poles  
( identification of the closed loop system)
- 2) Compute the closed loop poles based on  
the identified model
- 3) Compare (visually and using v-gap)

### **- Time Domain Validation**

- 1) Compare real time results and simulation  
results

# Identification in Closed Loop

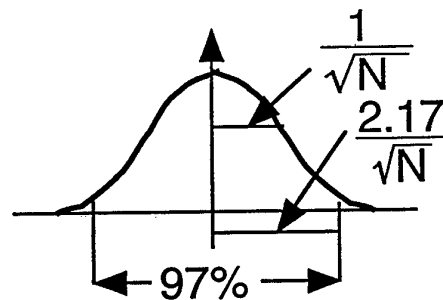
## STATISTICAL MODEL VALIDATION



$$|RN(i)| \leq \frac{2.17}{\sqrt{N}} ; i \geq 1$$

normalized  
crosscorrelation.

number of data



$$N = 256 \rightarrow |RN(i)| \leq 0.136$$

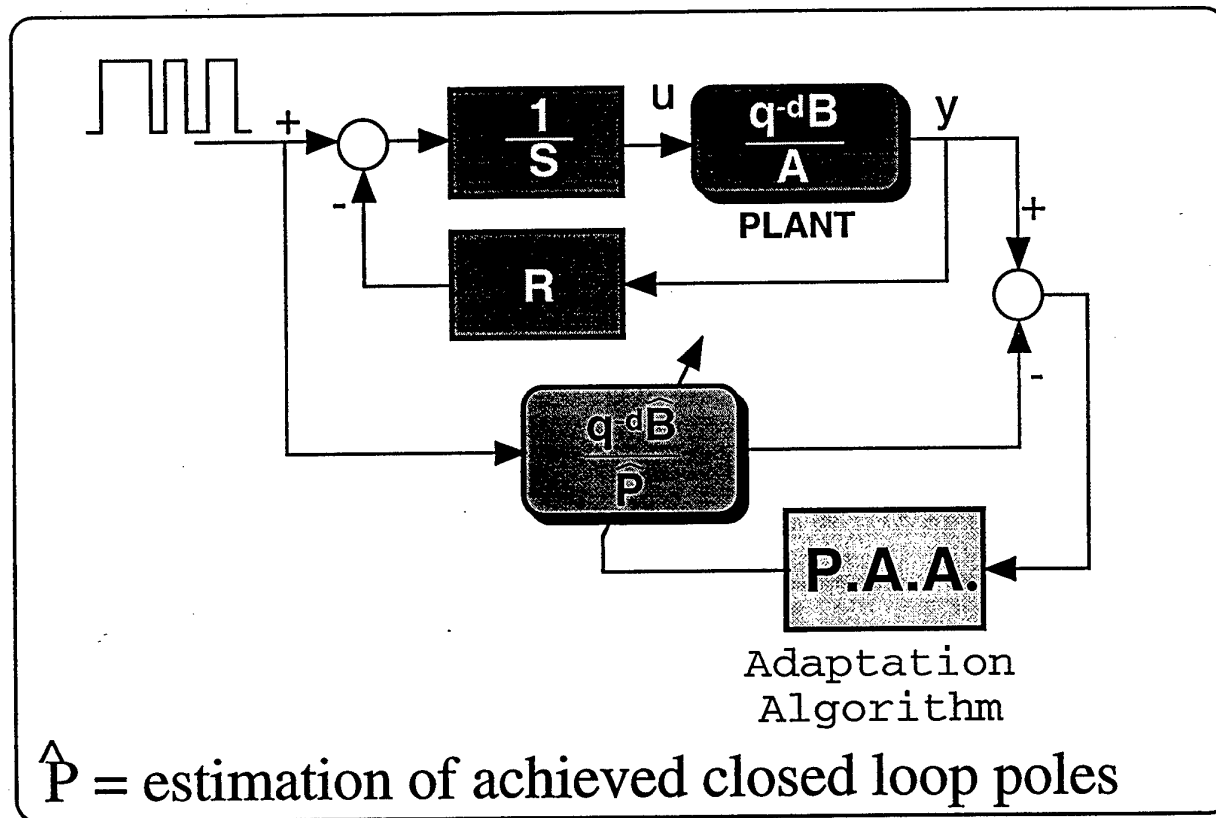
$$|RN(i)| \leq 0.15$$

**Controller dependent validation !**

## Identification in Closed Loop

### MODEL VALIDATION by POLE CLOSENESS

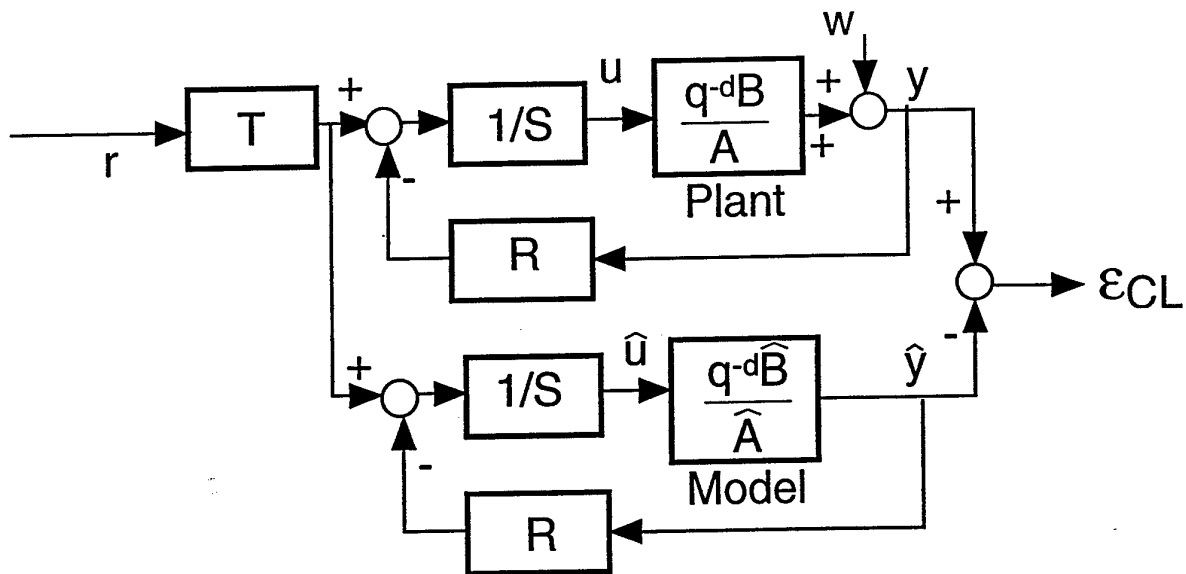
- 1) Identification of *achieved poles* and/or *sensitivity functions* by **identification of the closed loop**



The same signals are used for the the identification of the “plant model” in closed loop and for the identification of the “closed loop”

- 2) Computation of the closed loop poles and/or the sensitivity functions based on the identified model
- 3) Comparison (visual or using  $v$ -gap)

## Iterative Identification in Closed Loop and Controller Re-Design



### *Step 1* : Identification in Closed Loop

- Keep controller constant
- Identify a new model such that:  $\epsilon_{CL}$   $\rightarrow$
- Validate the new model

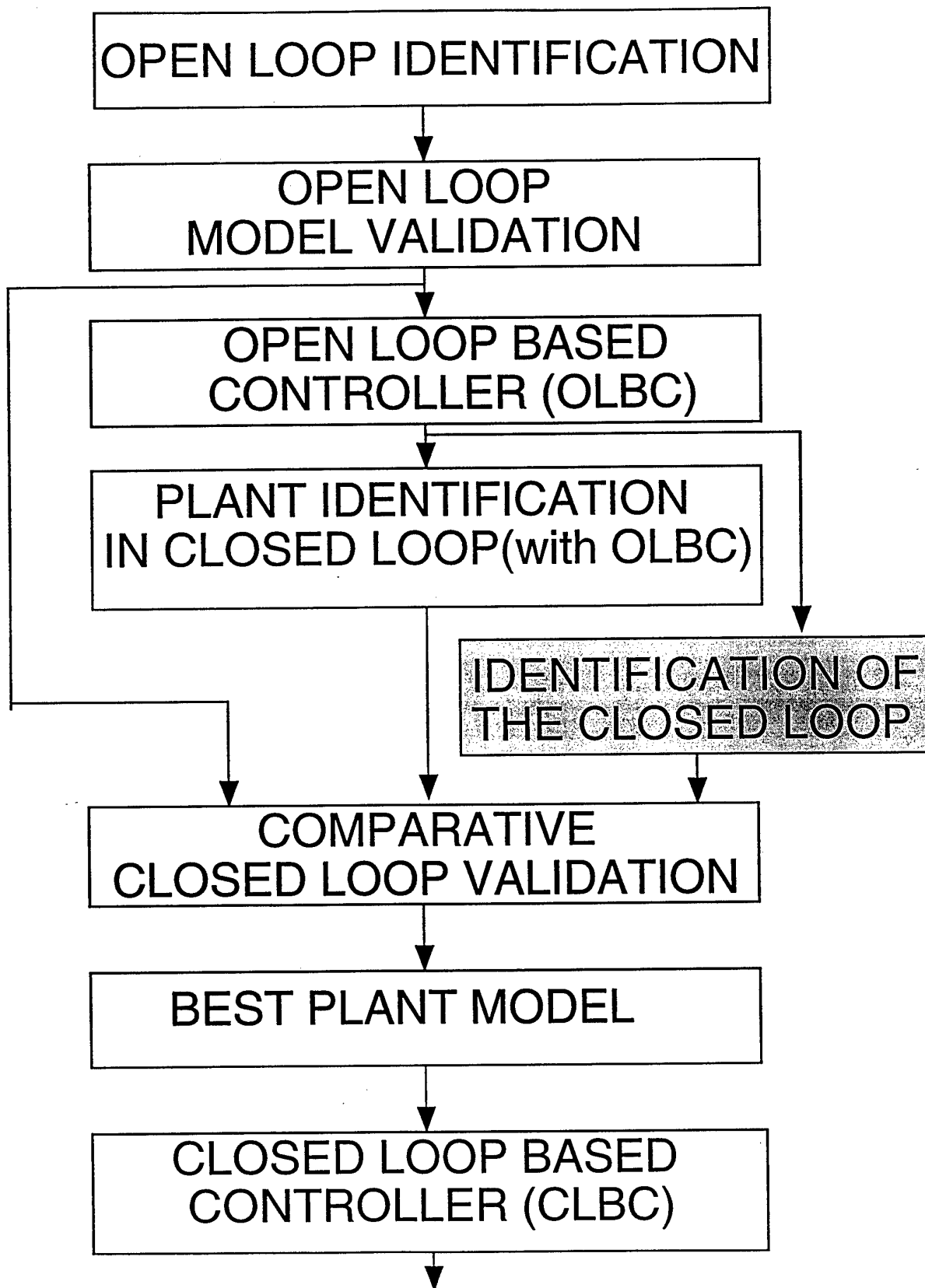
### *Step 2* : Controller Re - Design

- Compute a new controller such that :  $\epsilon_{CL}$   $\rightarrow$

*Repeat* 1, 2, 1, 2, .....

Rem.: the first iteration gives the most significant improvement

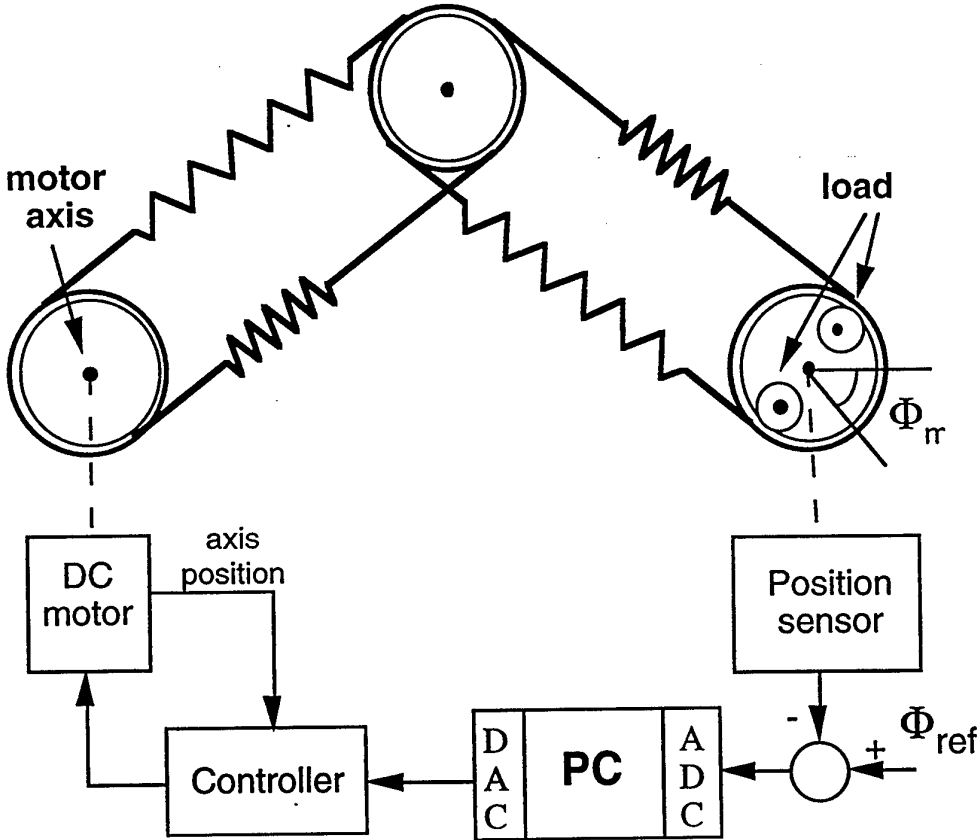
## How to use OL and CL Identification

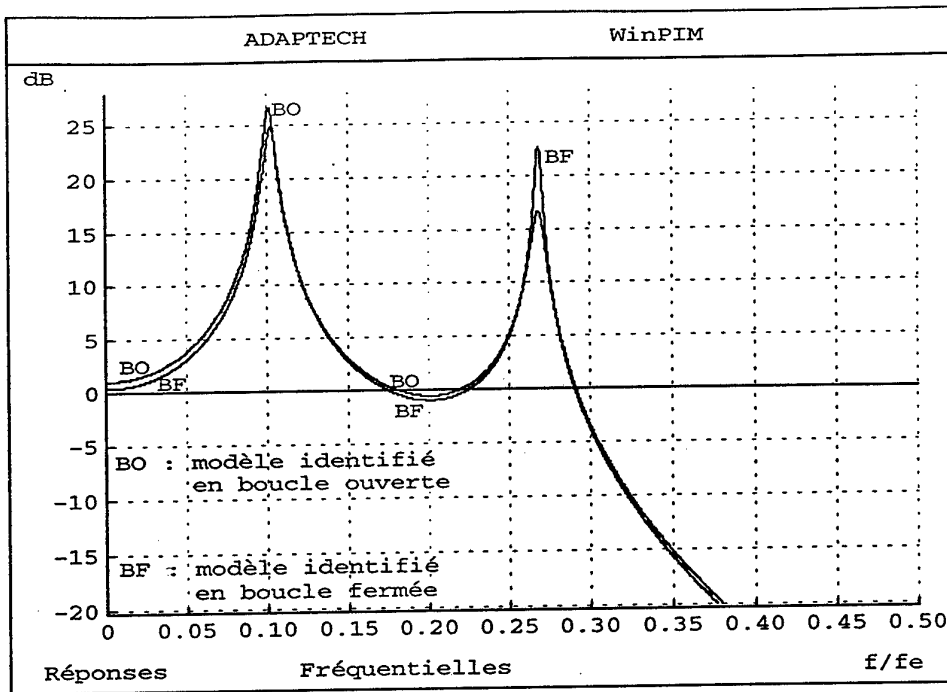


**Experimental Results:  
Identification in Closed Loop  
and Controller Re-design**

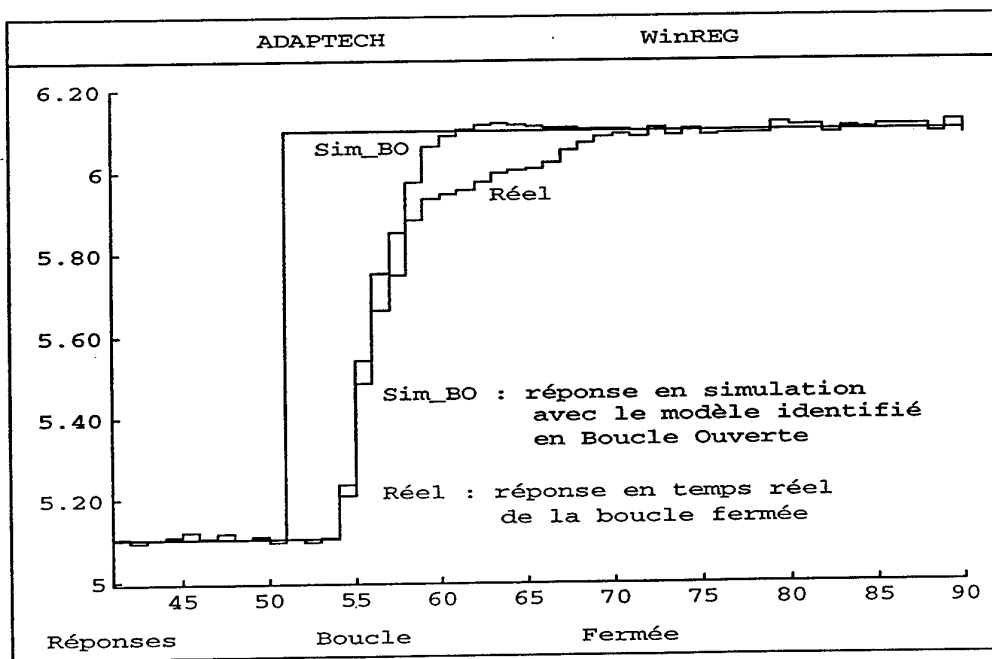
# Identification in Closed Loop + Controller Re-design

## The Flexible Transmission

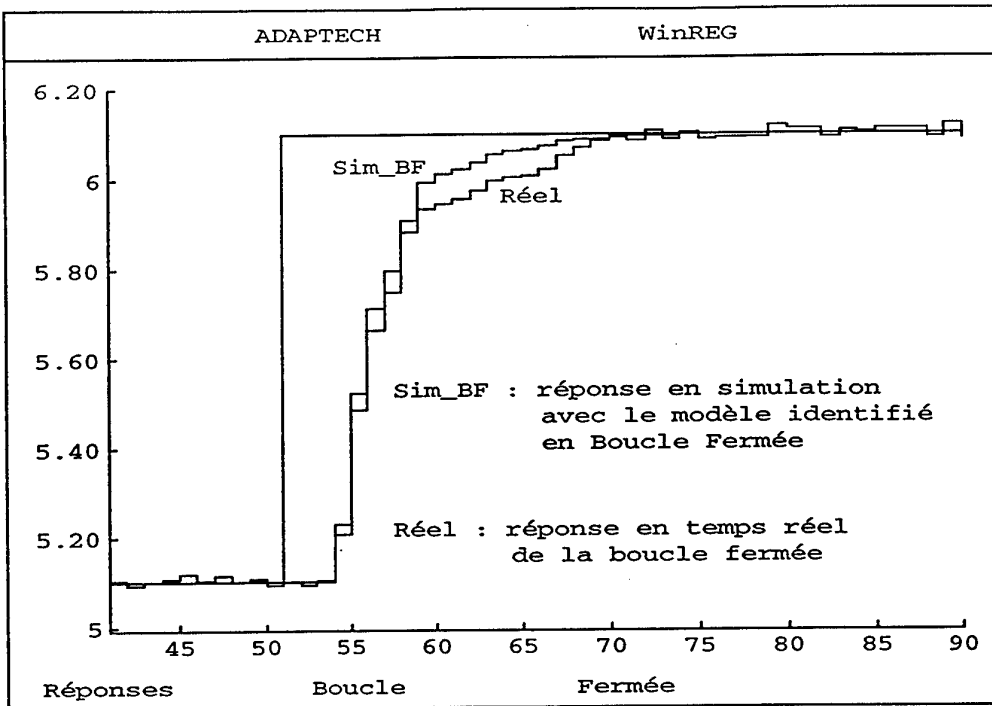




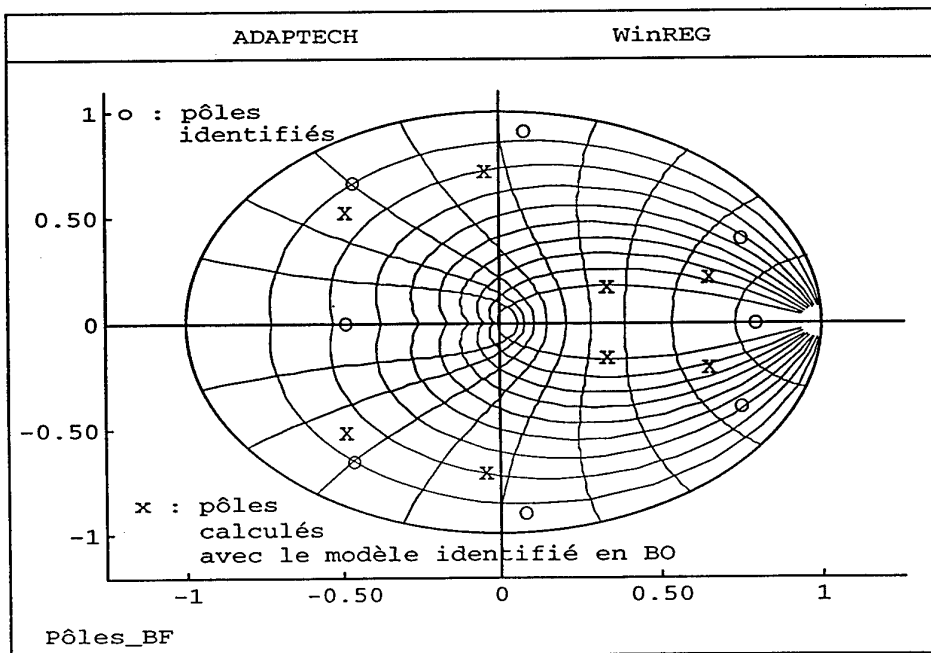
Frequency response of the flexible transmission (identified models)



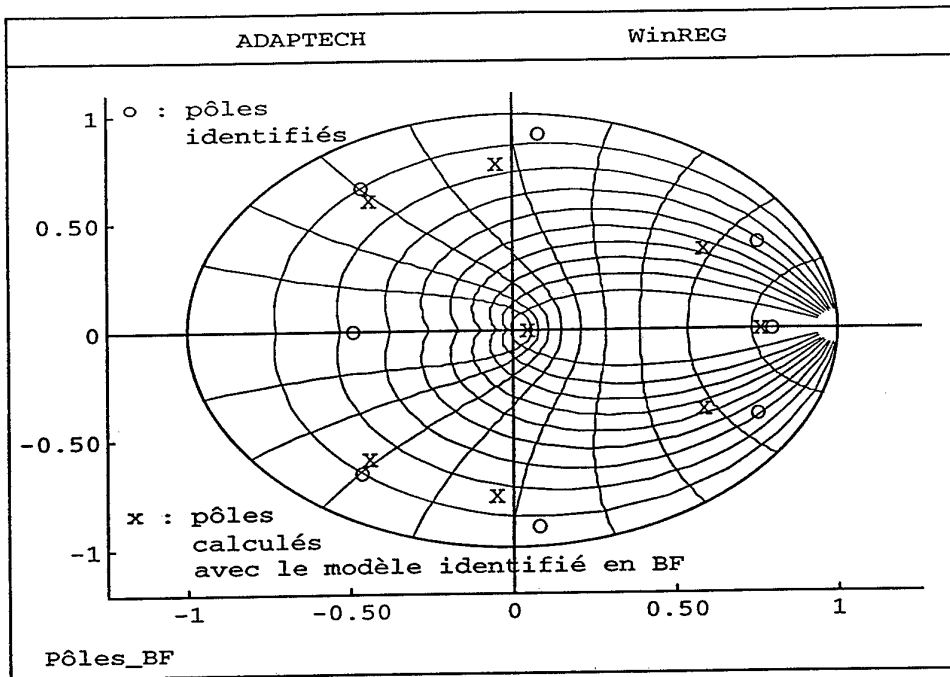
Step response of the closed loop system (model identified in open loop(BO).)



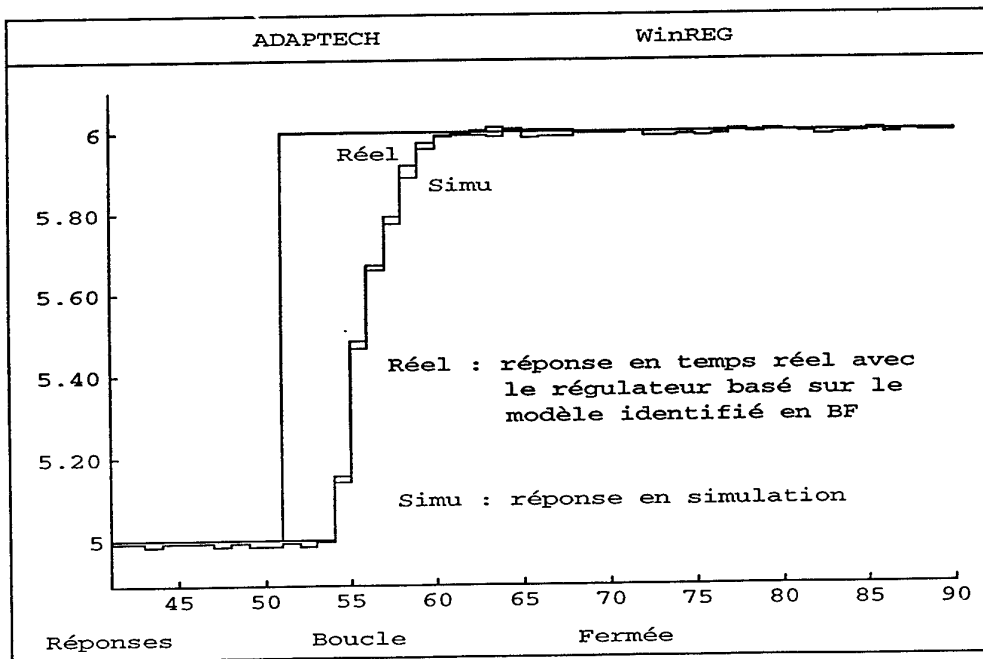
Step response of the closed loop system  
(model identified in closed loop ( B.F.))



Poles of the Closed Loop  
(model identified in open loop (B.O.))



Poles of the Closed Loop  
(model identified in closed loop (B.F.))



Step response of the closed loop system.  
(controller based on the model identified in closed loop)

**CONTROLLER REDUCTION  
USING  
IDENTIFICATION IN  
CLOSED LOOP**

## CONTROLLER REDUCTION

- It is an important issue in many applications
- Several approaches can be considered
- The most important : controller reduction should be done with the aim to preserve the closed loop properties

### **Identification in closed loop offers two useful approaches for controller reduction :**

- 1) (indirect) Identify a reduced order model in closed loop operation  
(will capture the essentials characteristics of the model in the critical frequency regions for design)
- 2)(direct) Reduced order controller identification in closed loop  
(will preserve the closed loop characteristics in the critical frequency regions)



## Asymptotic BIAS - Frequency Distribution

$$K = \frac{R}{S} \quad (\text{nominal controller})$$

$$\widehat{K} = \frac{\widehat{R}}{\widehat{S}} \quad (\text{estimated reduced order controller})$$

$$\widehat{\theta}^T = [\widehat{r}_0, \widehat{r}_1, \dots, \widehat{r}_{n_{\widehat{R}}}, \widehat{s}_1, \widehat{s}_2, \dots, \widehat{s}_{n_{\widehat{S}}}]$$

$$\widehat{\theta}^* = \operatorname{argmin}_{\theta} \int_{-\pi}^{\pi} |S_{yp}|^2 [ |K - \widehat{K}|^2 |\widehat{S}_{yp}|^2 \phi_r(\omega) + \phi_p(\omega) ] d\omega$$

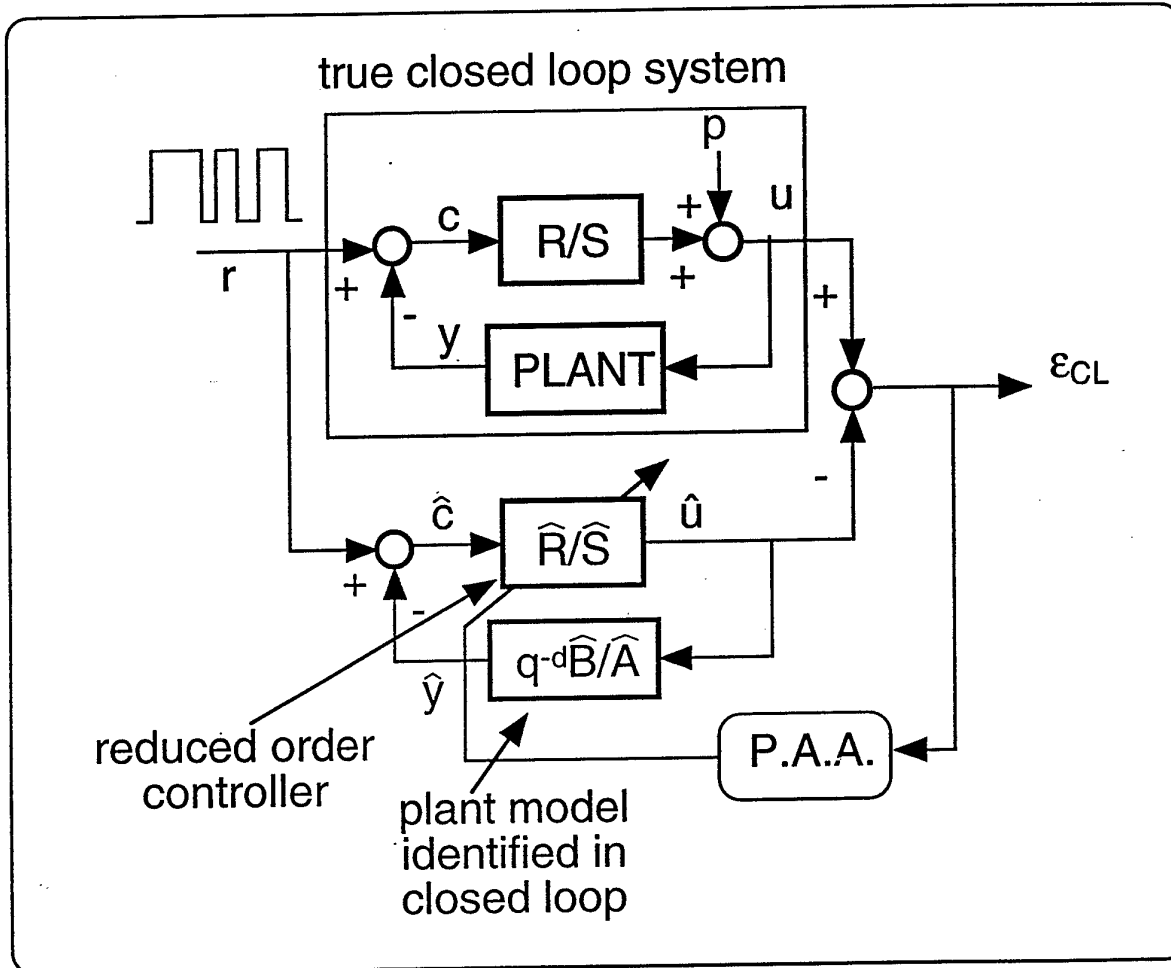
- noise does not affect parameter estimation (as.)
- precision of estimated model enhanced in the critical frequency regions for design
- precision can be further “tuned” by the choice of  $r(t)$

## Evaluation of the reduced order controllers

- Comparison of the sensitivity functions
- Comparison of the controller frequency characteristics

*Rem.: The v-gap seems very useful*

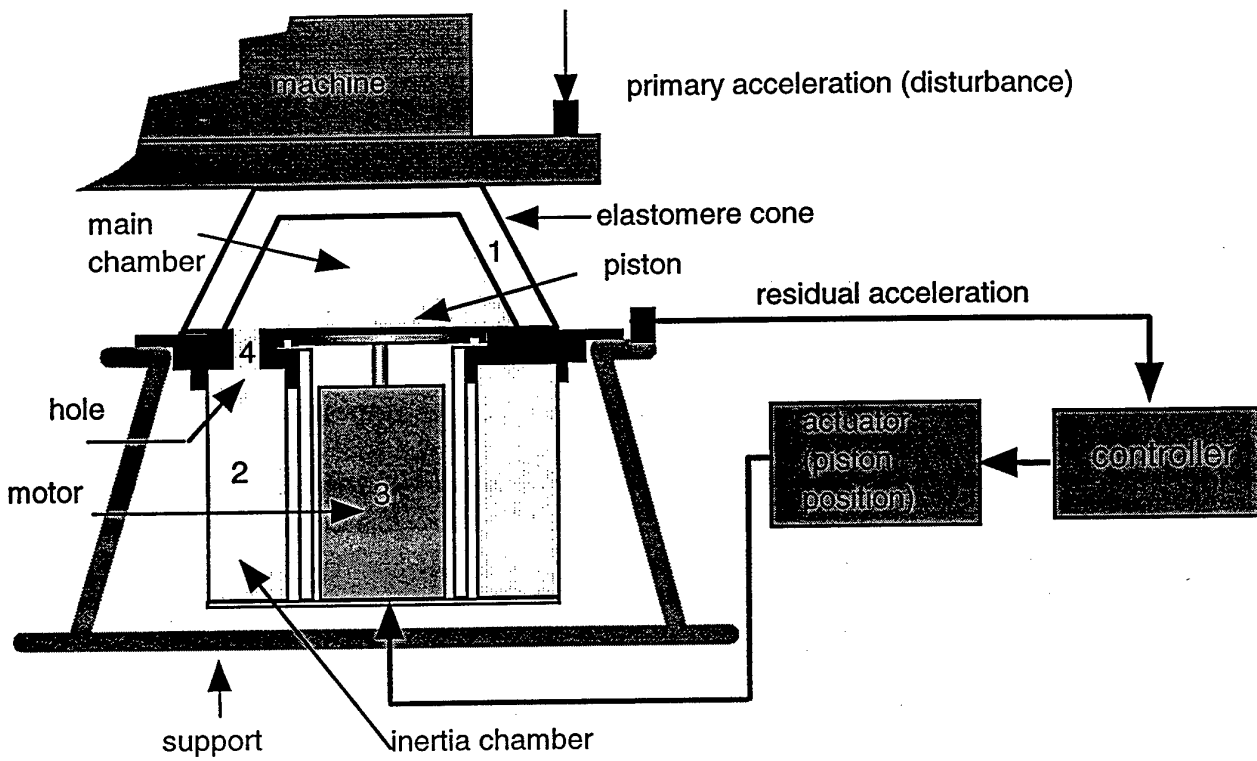
## Use of Real Data for Controller Reduction



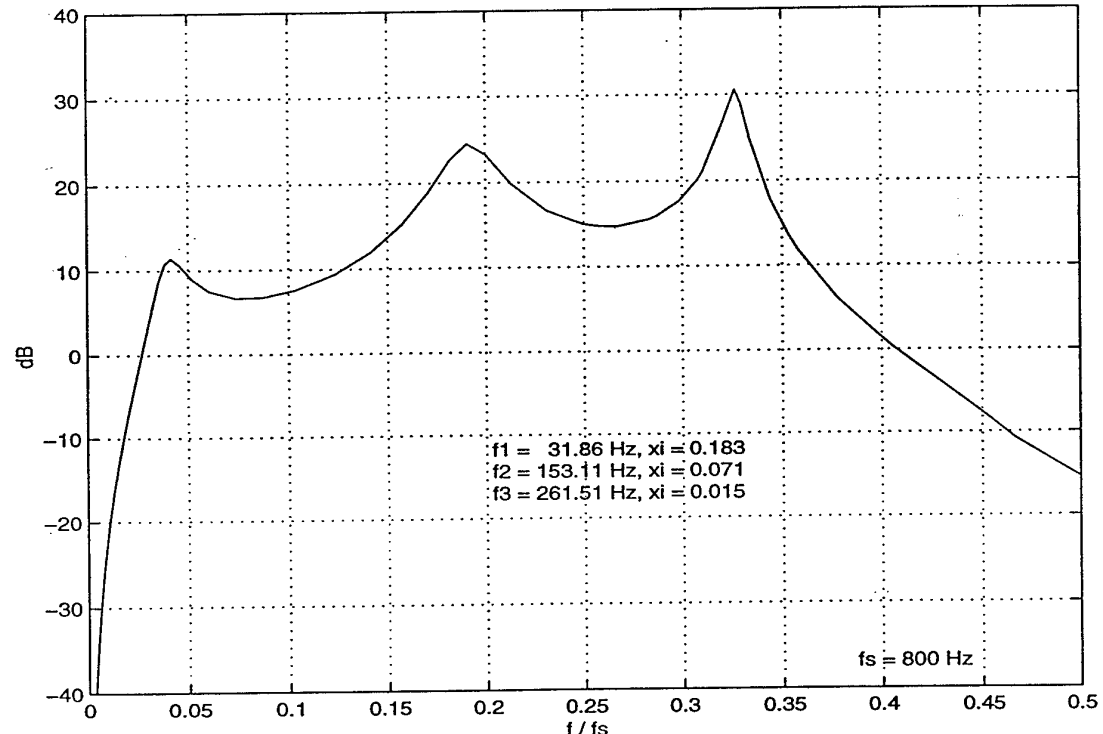
- Validation of the estimated reduced controller using similar tests as for plant model validation in closed loop

**Experimental results :  
Controller Reduction using  
Identification in Closed Loop**

# ACTIVE SUSPENSION



Active suspension identified model



## CONTROL OF THE ACTIVE SUSPENSION

Model :

$n_A = 6, n_B = 8, d = 0$  ; (order of the system : 8)

Fixed part of the controller :

$R = R' H_R \quad H_R = 1 + q^{-1}$  (opening the loop at 0.5fs)

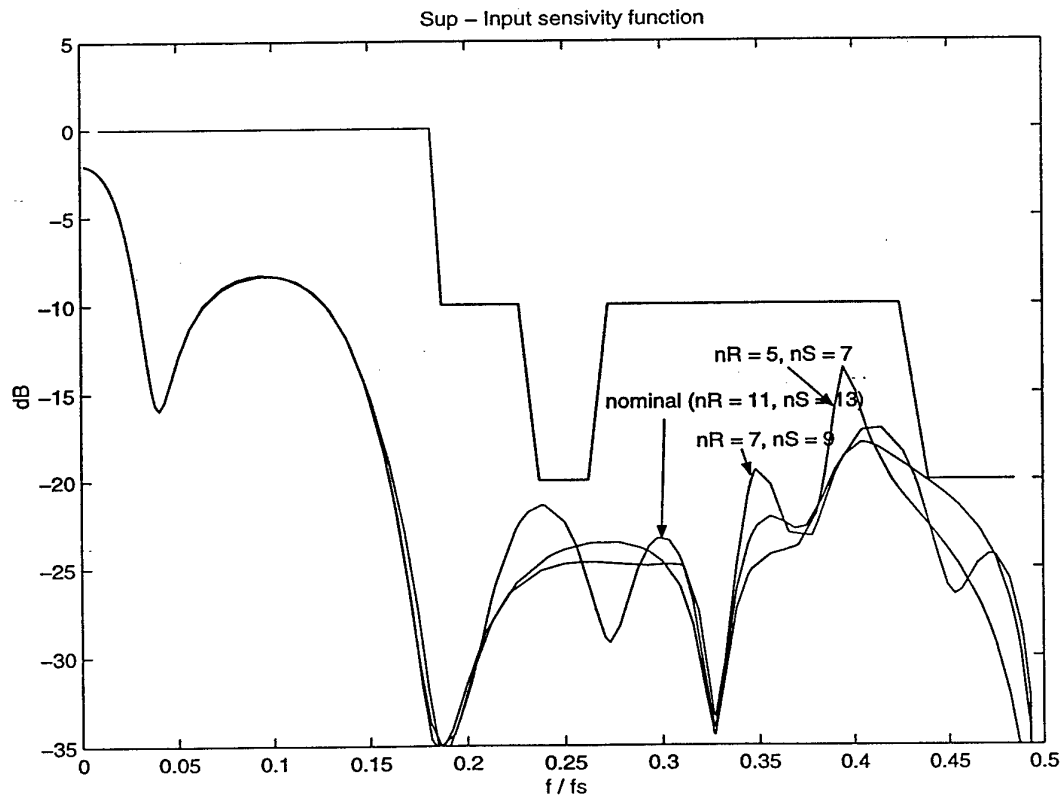
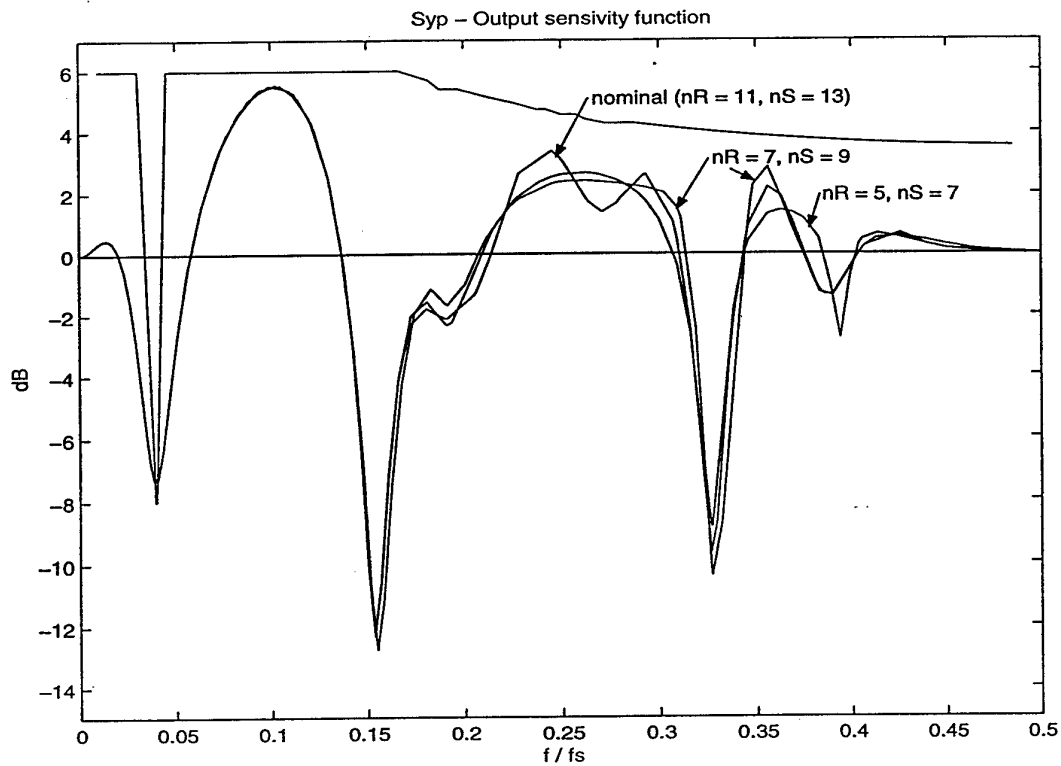
Design : *Pole placement with sensitivity functions shaping* (see templates next viewgraph)

Resulting controller (nominal) :  $n_R = 11, n_S = 13$

Theoretical dimension of pole placement controller :

$n_R = 5, n_S = 7$

# CONTROL OF THE ACTIVE SUSPENSION

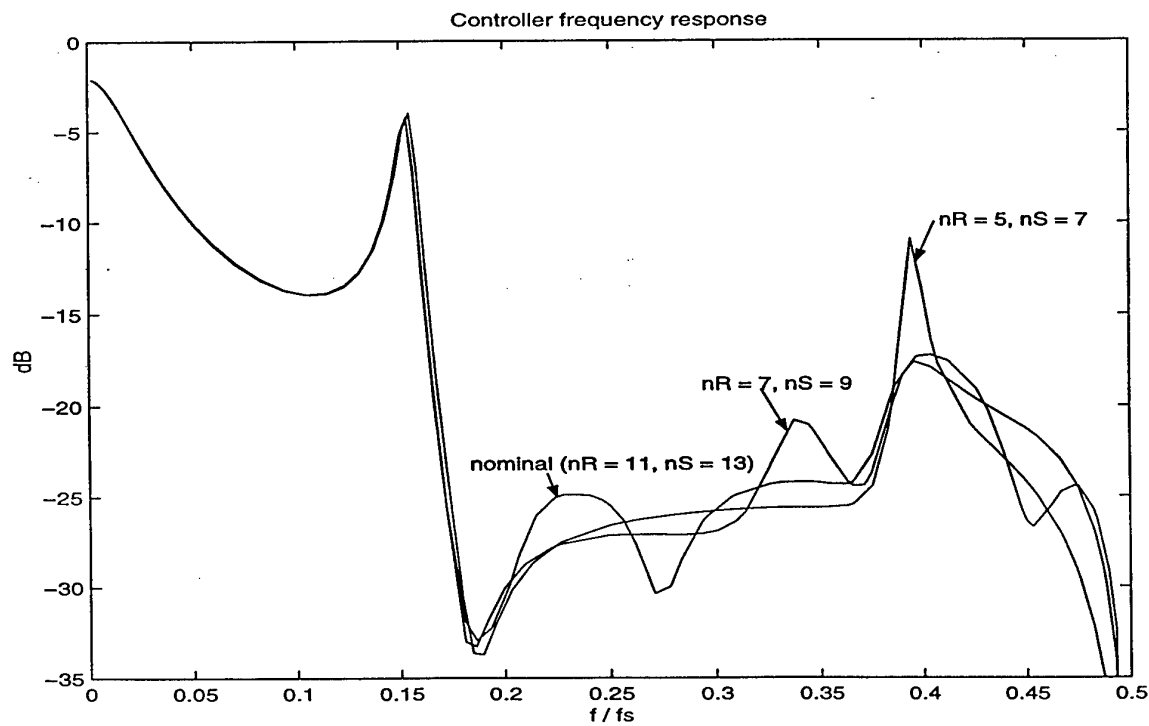


## Dual Vinnicombe stability criterion :

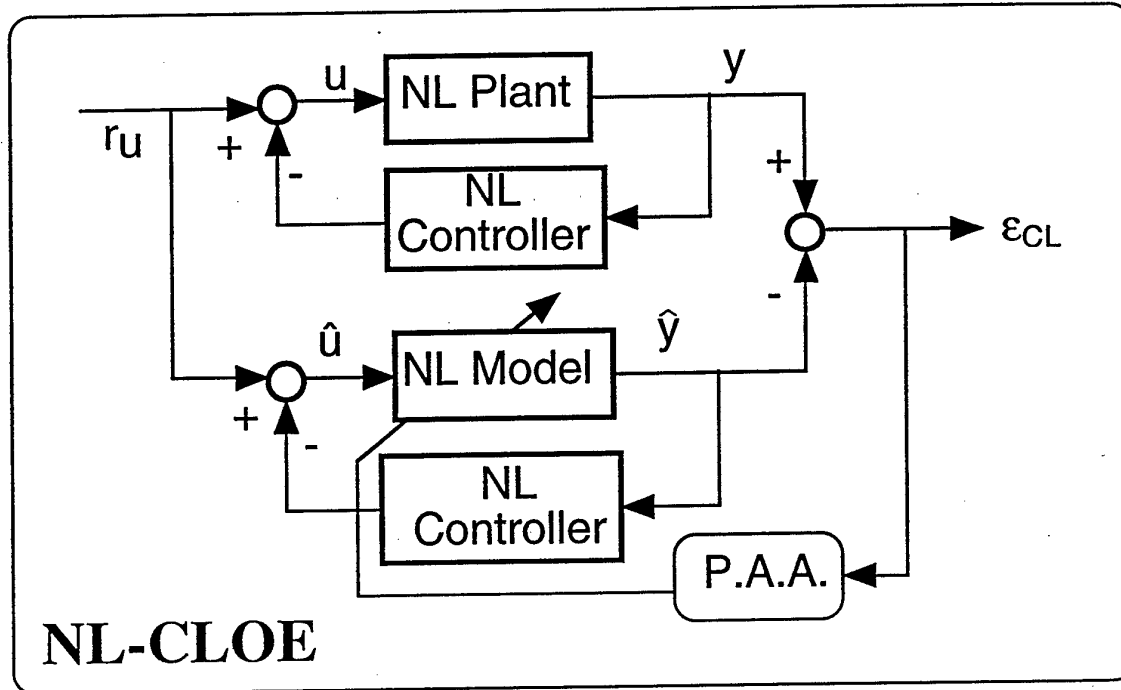
$$\delta(K_n, K_i) < b(K_n)$$

v-gap  $\nearrow$   $\delta(K_n, K_i)$   $\nwarrow$  stability margin  $b(K_n)$

Controller	$\delta(K_n, K_i)$	$\delta(S_{ypn}, S_{ypi})$	$\delta(S_{upn}, S_{upi})$	$b(K)$
Kn nR=11, nS=13	0	0	0	<b>0.079</b>
K1 nR=7, nS=9	<b>0.039</b>	0.122	0.041	0.069
K2 nR=5, nS=7	0.187	0.119	0.135	0.072



## Identification of Continuous Time Nonlinear Plant Models in Closed Loop Operation



- Applicable to nonlinear plant models which are nonlinear in the parameters
- P.A.A. with a similar structure as for the linear case
- $\Phi$  contains computable gradient expressions (functions of  $\hat{u}$  and  $\hat{y}$ )
- The noise problems are for the moment an open issue

## CONCLUSIONS and PERSPECTIVES

- Identification in closed loop can provide better plant models for design
- Identification in closed loop can not be dissociated from the controller and robustness issues
- Specific closed loop identification algorithms with convenient properties have been developed
- Identification in closed loop has started to make its way in practice
- Algorithms for identification in closed loop have been successfully applied for controller reduction
- Algorithms for identification of nonlinear plant models in closed loop operation have been developed
- Comparative evaluation of the controller reduction techniques (to be done)
- The nonlinear case raise numerous and interesting problems (to be explored)

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# Vision Based Navigation and Control of Robotic Systems

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Instituto Superior Técnico  
Instituto de Sistemas e Robótica  
Lisbon, PORTUGAL

*VisLab - Computer Vision Lab*



## Vision Based Control: First Steps

- Shirai and Inoue (1973) : visual feedback for precise motion control of a manipulator.
- Hill and Park (1979) : “continuous use” of image information; term **Visual Servoing** first used
- Weiss, (1980) : dynamic visual control of robots, stabilizing initial concepts.

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## Initial Difficulties

- Limited Computing Power
- Expensive and complex image digitizers and processors
- Slow sampling frequencies and dynamics



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## Why vision based control of robots

- Many robots used in (*constrained /changed*) industrial environments
- Need for dealing with more general environments
- Vision allows non-contact measurements (perception) of the environment (*as humans*)



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# Challenges for Control Science

- Noisy image measurements
- time consuming (still!) image processing
- large amount of data to process
- non-linearities of camera-lens system
- difficulty of inferring 3D information from 2D images



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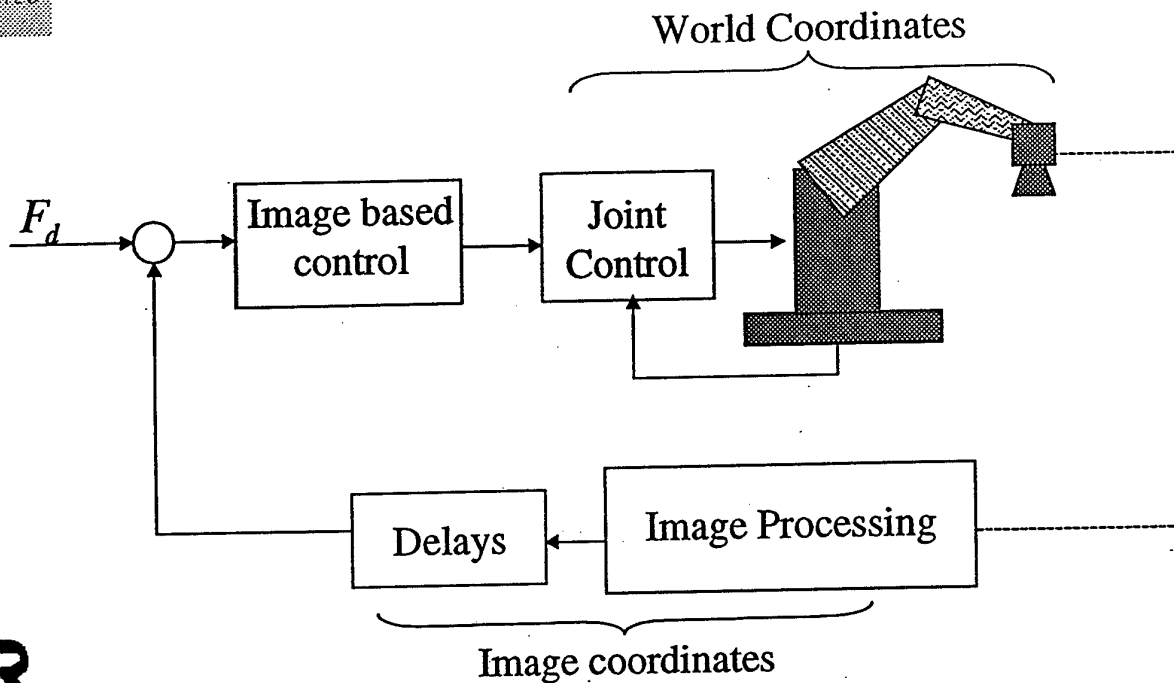
# Challenges for Control Science

- Large delays in the feedback loop
- Unreliable sensor data
- Inadequately modeled or difficult to model plants or systems
- Poorly conditioned transformations

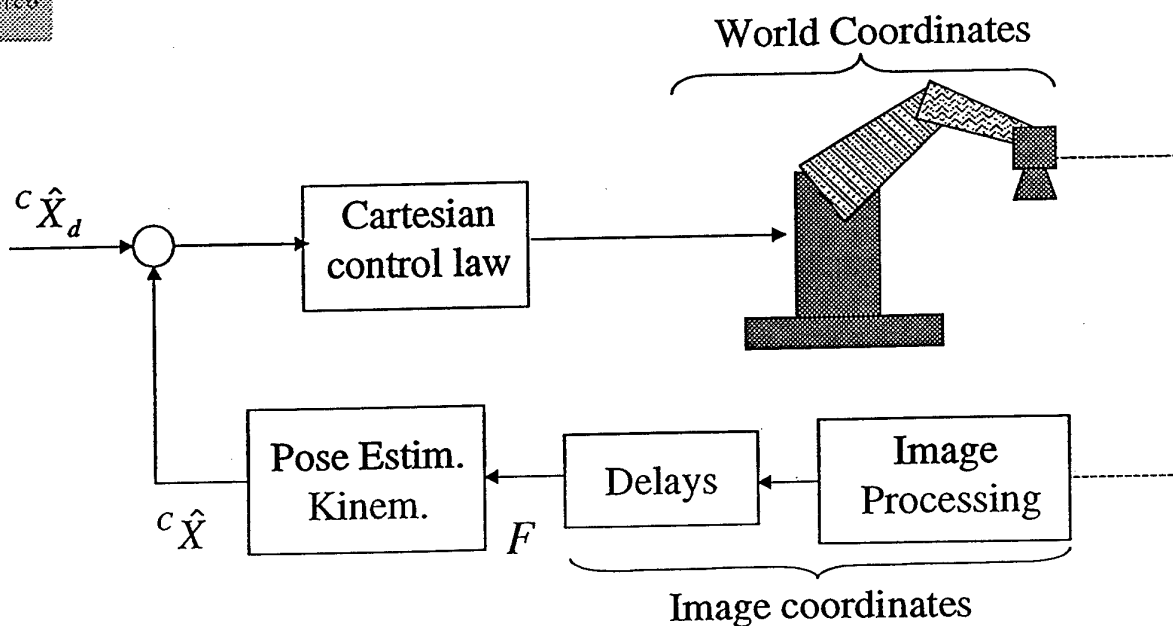


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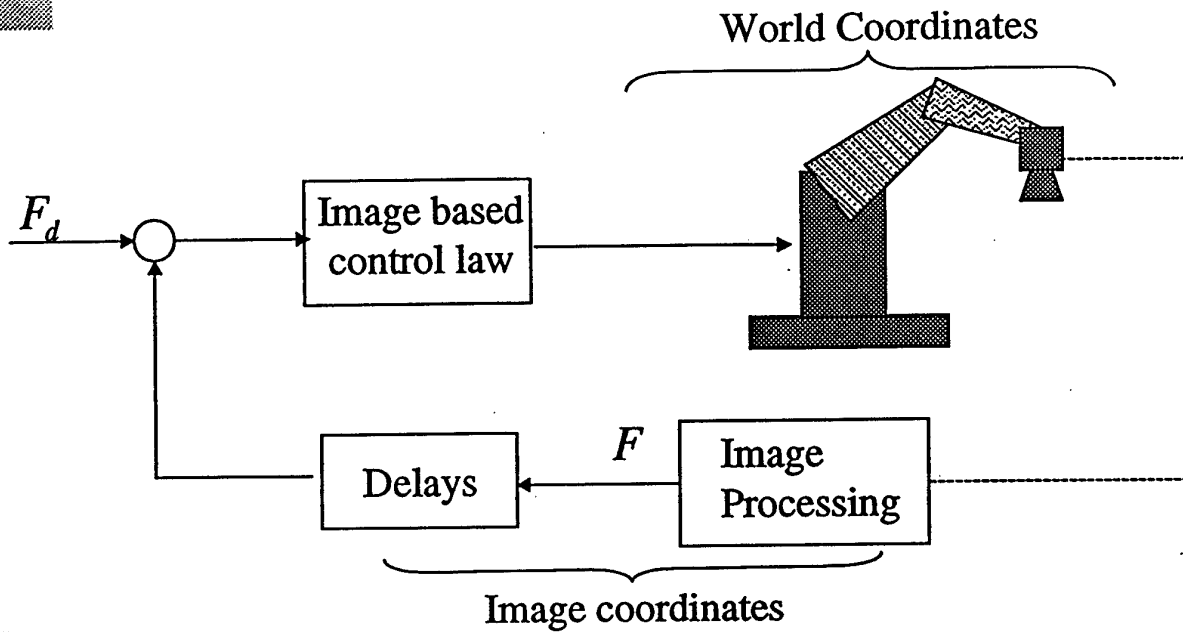
# Image-based look and move structure



# Position-Based Visual Servoing



# Image-Based Visual Servoing



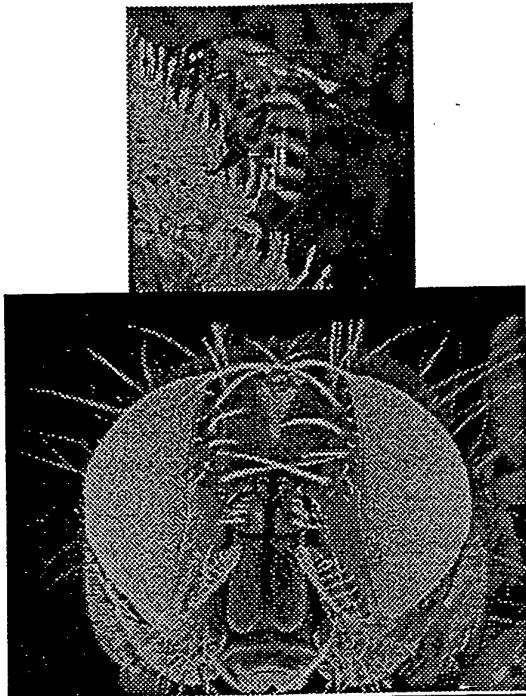
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## Talk Outline (example cases)

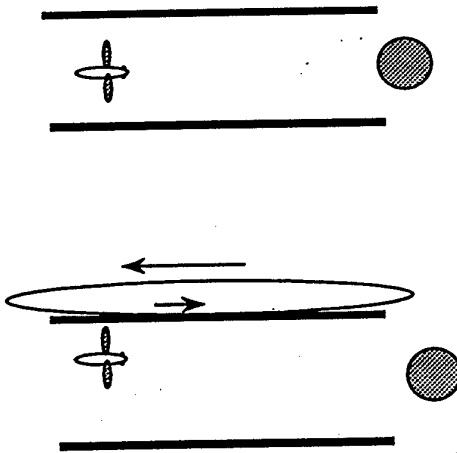
- Visual Navigation: Learning from bees
  - Problem
  - Motivation
- Visual Docking
- Remote control of Cellular Robots
- Binocular Tracking with a robot Head
- Conclusions

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# Visual Navigation



(Srinivasan 91)




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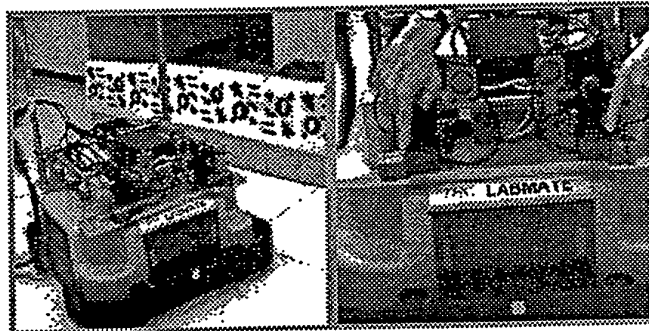
## Peripheral flow

$$u_L = \frac{1}{Z_L} [x_a \rho \dot{\theta} \sin \theta + \rho \dot{\theta} \cos \theta - T_M] + (1 + x_a^2) \dot{\theta}$$

$$u_R = \frac{1}{Z_R} [-x_a \rho \dot{\theta} \sin \theta + \rho \dot{\theta} \cos \theta + T_M] - (1 + x_a^2) \dot{\theta}$$

No rotation

$$e = T_M \left( \frac{1}{Z_R} - \frac{1}{Z_L} \right)$$

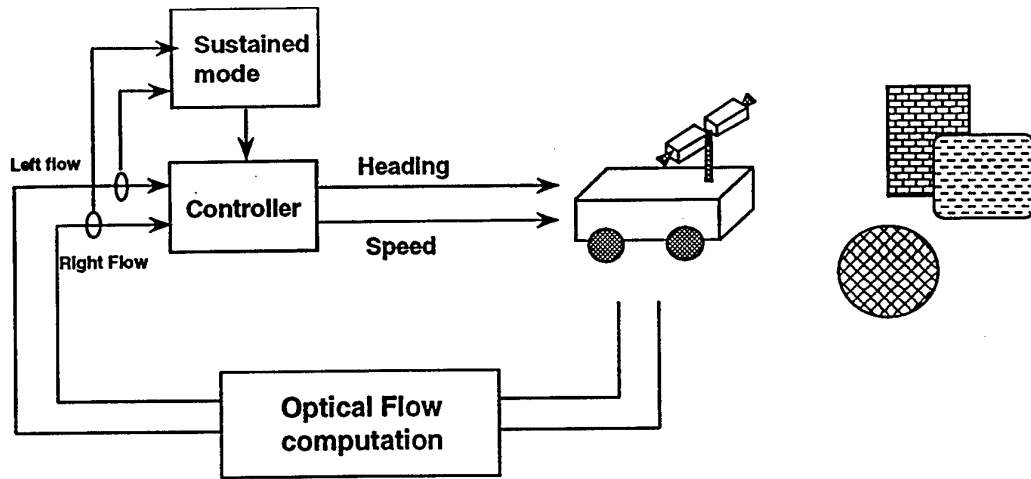


## Flow difference

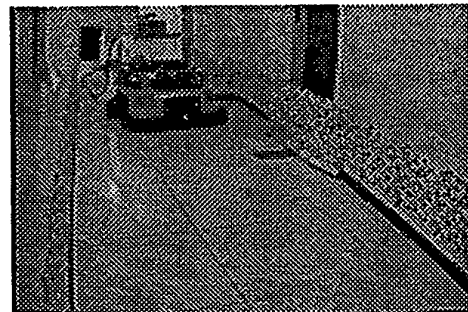
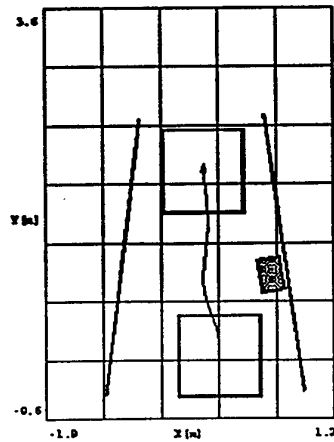
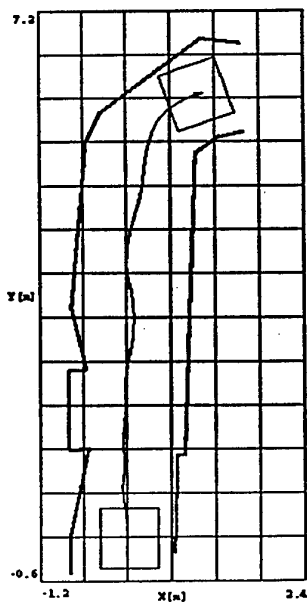
$$e = u_L + u_R = [T_M - x_a \rho \dot{\theta} \sin \theta] \left( \frac{1}{Z_R} - \frac{1}{Z_L} \right) + \rho \dot{\theta} \cos \theta \left( \frac{1}{Z_R} + \frac{1}{Z_L} \right) - 2(1 + x_a^2) \dot{\theta}$$



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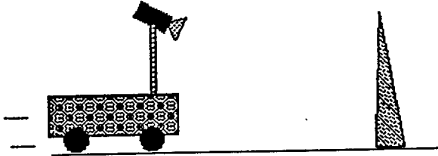
# Visual Navigation



# Visual Docking



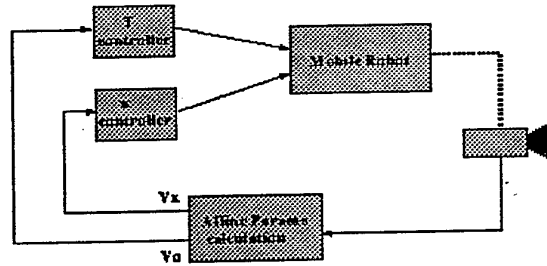
Ego-Docking



Eco-Docking



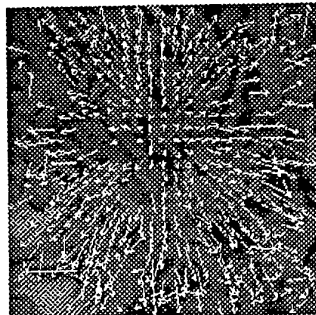
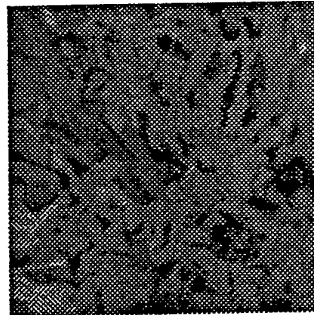
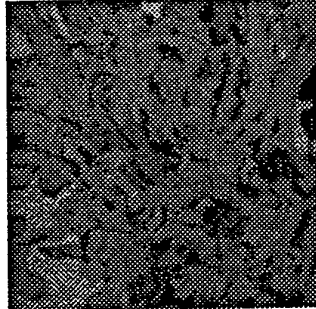
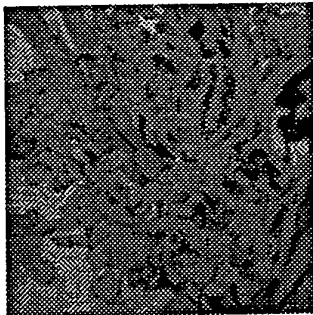
Control : Heading  
Speed



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What is the relative camera motion and scene structure ??



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# Motion Field

Rigid Body Motion : T, W  
Perspective Projection

## Optical Flow

$$u(x,y) = f_x \left[ \frac{\frac{x}{f_x} T_z - T_x}{Z(x,y)} + \omega_x \frac{xy}{f_x f_y} - \omega_y \left( 1 + \frac{x^2}{f_x^2} \right) + \omega_z \frac{y}{f_y} \right]$$

$$v(x,y) = f_y \left[ \frac{\frac{y}{f_y} T_z - T_y}{Z(x,y)} + \omega_x \left( 1 + \frac{y^2}{f_y^2} \right) - \omega_y \frac{xy}{f_x f_y} - \omega_z \frac{x}{f_x} \right]$$

## Planar Surfaces

$$Z(X,Y) = Z_0 + \gamma_x X + \gamma_y Y$$

$$Z(x,y) = \frac{Z_0}{1 - \gamma_x x / f_x - \gamma_y y / f_y}$$

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# Motion Model

## Global Parametric Model

$$u(x,y) = u_0 + u_x x + u_y y + u_{xy} xy + u_{xx} x^2$$

$$v(x,y) = v_0 + v_x x + v_y y + v_{xy} xy + v_{yy} y^2$$

## Affine Model

$$u(x,y) = u_0 + u_x x + u_y y$$

$$v(x,y) = v_0 + v_x x + v_y y$$

## Parameters

$$u_0 = -f_x \left[ \frac{T_x}{Z_0} - \omega_x \right]$$

$$u_x = \frac{T_x + \gamma_x T_z}{Z_0}$$

$$u_y = \frac{f_x}{f_y} \left[ \frac{T_x \gamma_y}{Z_0} + \omega_x \right]$$

$$u_{xy} = v_{xy}$$

$$u_{xx} = v_{yy}$$

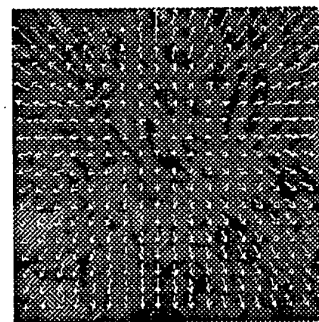
$$v_0 = f_y \left[ -\frac{T_y}{Z_0} + \omega_y \right]$$

$$v_x = \frac{f_y}{f_x} \left[ \frac{T_y \gamma_x}{Z_0} - \omega_y \right]$$

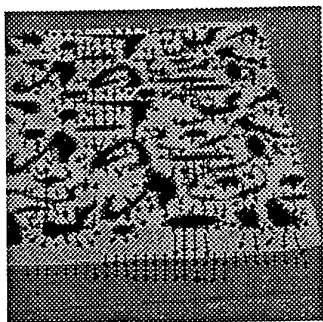
$$v_y = \frac{T_y + \gamma_y T_z}{Z_0}$$

$$v_{xy} = \frac{1}{f_x} \left( \frac{\gamma_x T_y}{Z_0} - \omega_y \right)$$

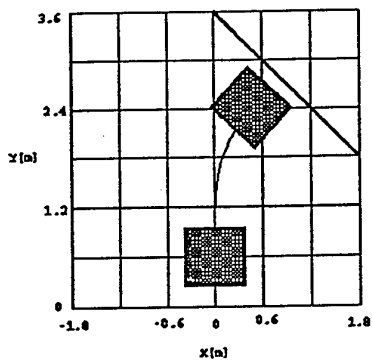
$$v_{yy} = \frac{1}{f_y} \left( -\frac{\gamma_y T_y}{Z_0} + \omega_y \right)$$



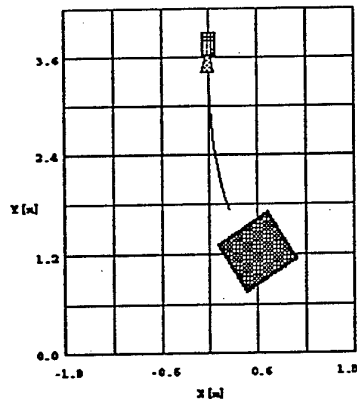
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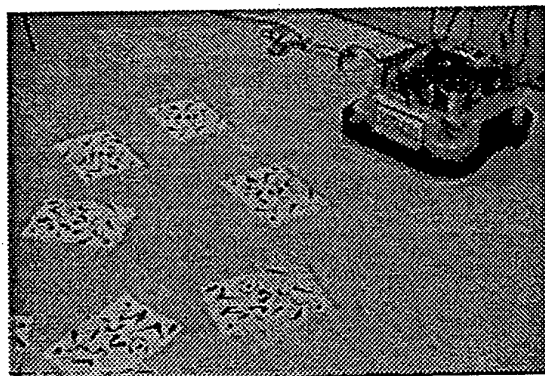
**Ego-Docking**



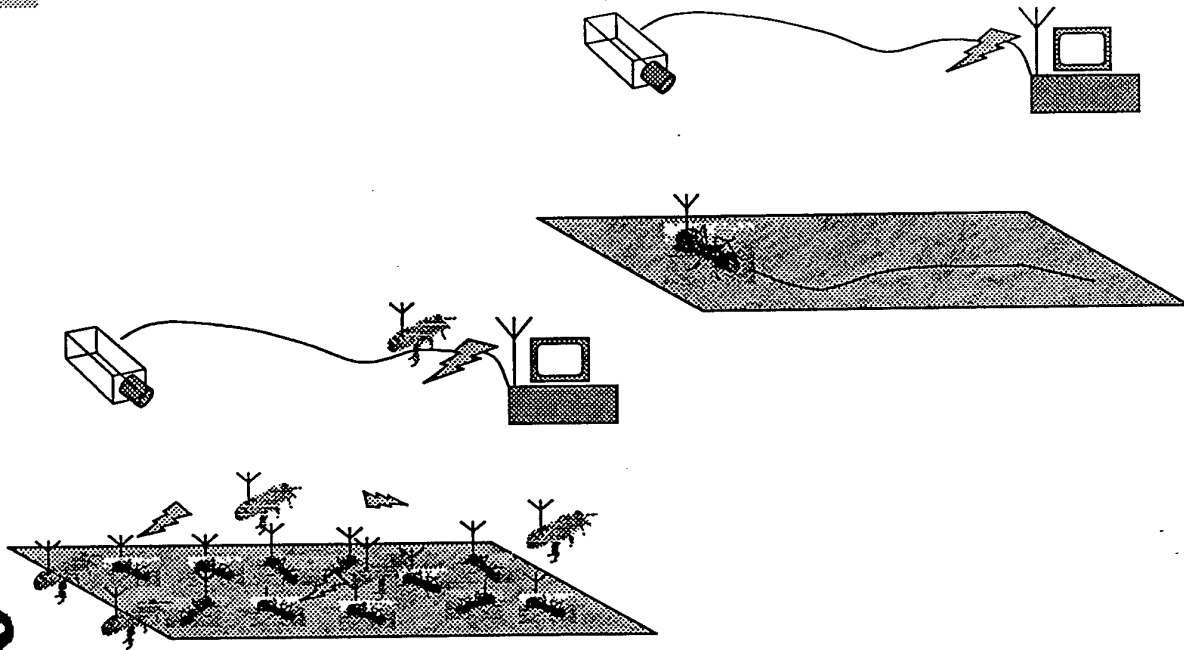
**Eco-Docking**



## RESULTS



# Visual Control of Cellular Robots



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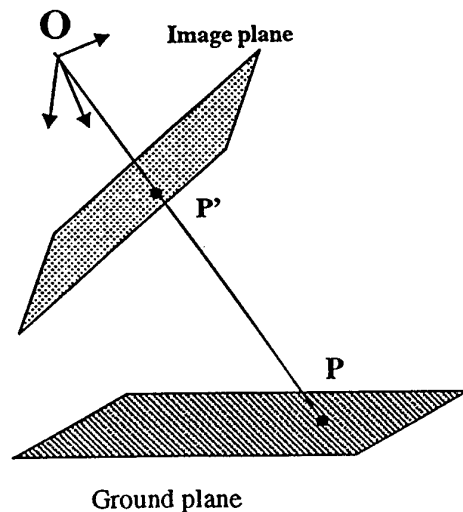
# Robot-Eye Coordination

$$m = M m'$$

$$\begin{bmatrix} \lambda x' \\ \lambda y' \\ \lambda \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X & Y & 1 & 0 & 0 & 0 & -x'X & -x'Y & -x' \\ 0 & 0 & 0 & X & Y & 1 & -y'X & -y'Y & -y' \end{bmatrix} [m_v] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

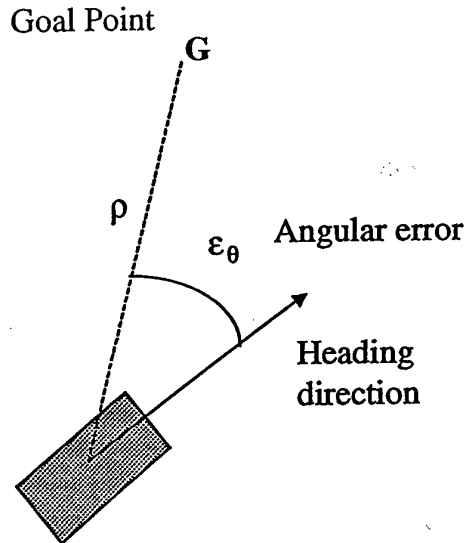
Image and plane coordinates of 4 points (minimum) required



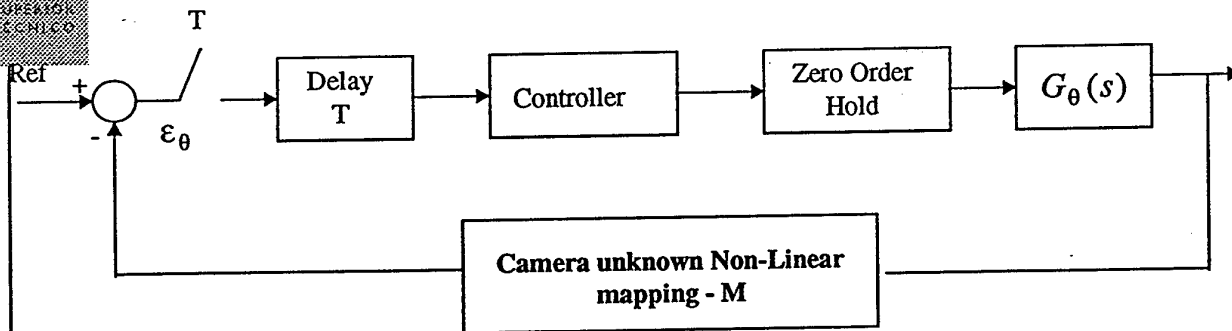
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# Control Strategy

- Decoupled angular and linear velocities dynamic systems
- Separate *Heading* and *speed* control using the *angular error* and *distance* to target point.

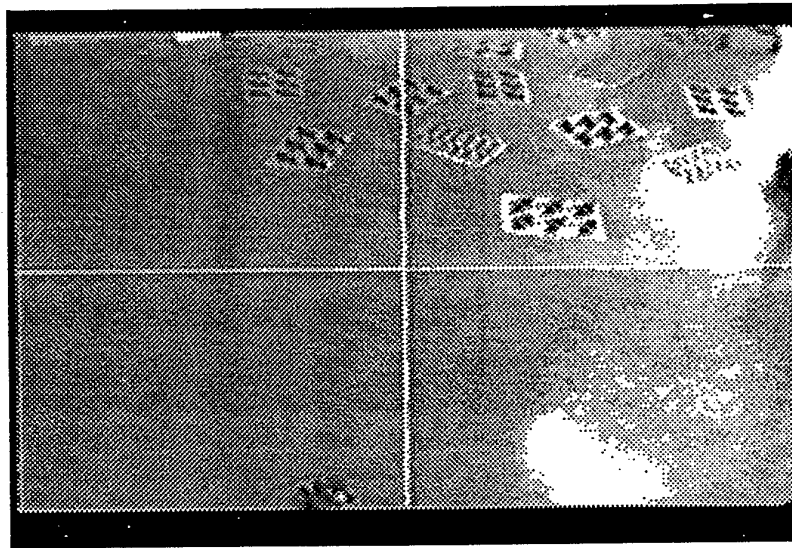


# Control Problem structure



- Measure angular error between robot heading and direction to target
- Generate differential voltages to control the robot heading direction
- Similar loop can be used to control the linear speed as a function of the distance to goal

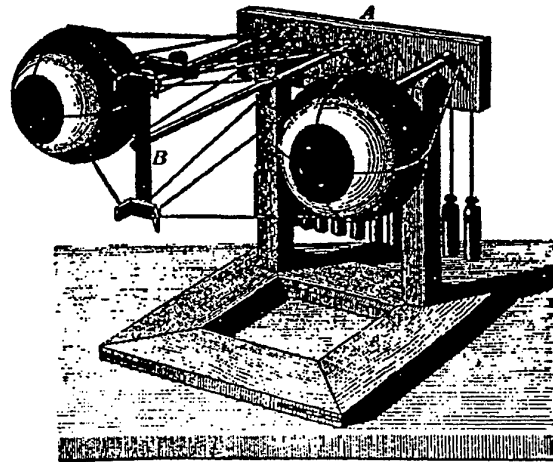
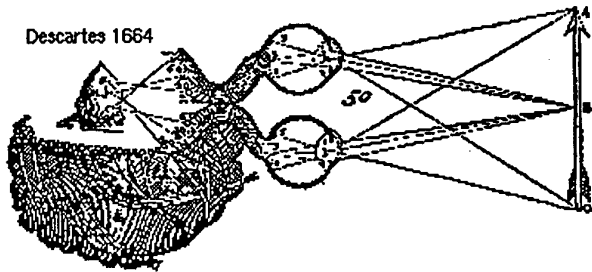




SR

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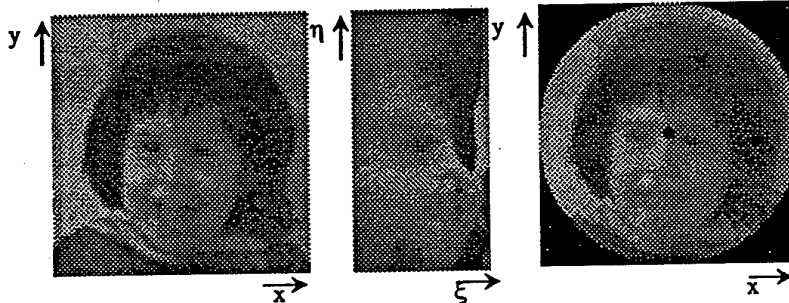
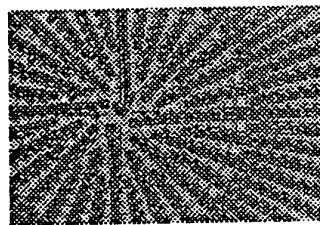
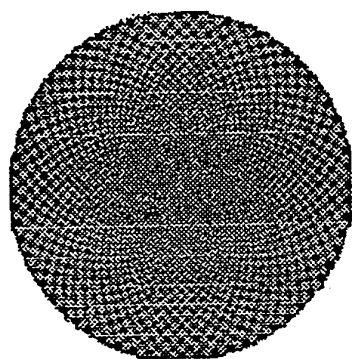
# Binocular Visual Tracking



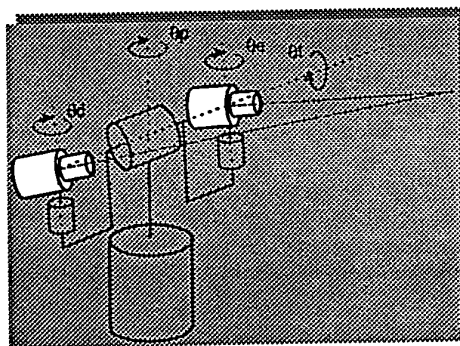
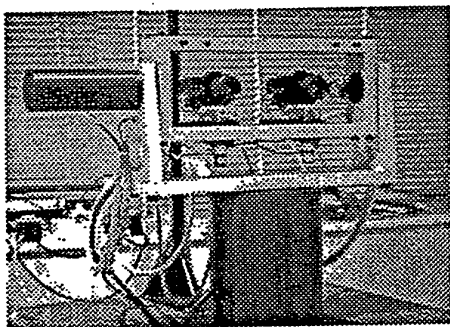
SR

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# Foveal Vision - log-polar mapping



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## Head/Object Motion

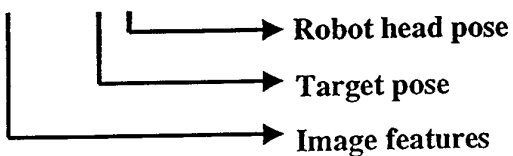
$$\delta \bar{f} = J_q \delta q + J_p \delta p$$

## Kinematic controller

$$\delta q = -J_q^{-1} \delta \bar{f} + J_q^{-1} J_p \delta p$$

$$p = K(q, \bar{f}) \quad \text{Forward kinem.}$$

$$\bar{f} = \Psi(p, q)$$



Robot head pose

Target pose

Image features

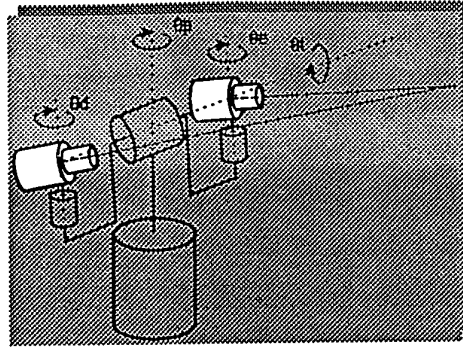


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## Fixation Constraint

Small errors:

$$\bar{f} \approx \bar{f}^0$$

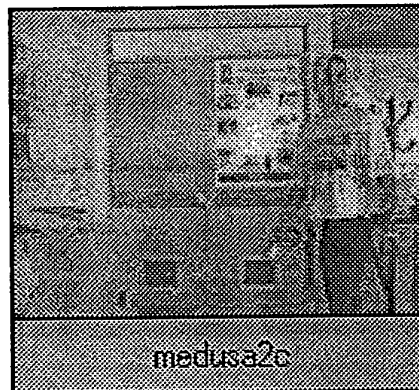
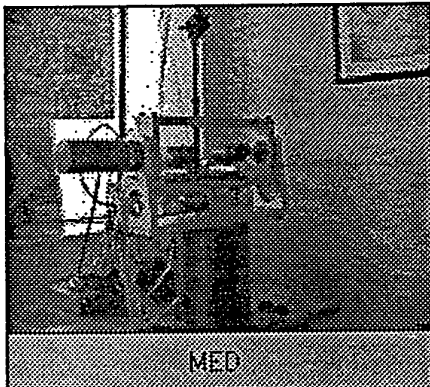


Introduces a *linkage* between the camera and target poses

$$p^0 \approx K(q, \bar{f}^0)$$

This constraint can be used for simplify the system kinematics (Jacobians) and dynamics (decoupling various DOFs) and one can easily introduce motion models for the target.

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# Conclusions

- “Opportunistic” use of visual information for control purposes.
- Challenging image geometries
- Ill-conditioned (perspective) observations
- Many issues of stability and performance analysis remain to be addressed.



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