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STUDY OF EXCEEDANCE CURVE SPREAD BY AIRCRAFT TAIL NUMBER BY MISSION TYPE & BASE FOR F-4 AND A-37B FLIGHT DATA

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TECHNICAL MEMORANDUM 75-37-FBE

FOREWARD

This report was prepared by Mr. David L. Banaszak of the Structural Integrity Branch, Structures Division, Air Force Flight Dynamics Laboratory. This effort was performed under Project 1367, "Structural Integrity for Military Aerospace Vehicles," Task 136701, "Structural Flight Loads."

This exercise was initiated largely in response to determinations reached in a series of meetings with personnel in ASD/ENF; namely Messrs. W. J. Crichlow, C. W. Luchsinger, and Troy King. These meetings were convened to discuss methods and techniques for analyzing and utilizing flight loads data in modifying and updating MIL-A-8866B specification.

Appreciation is extended to Messrs. W. J. Crichlow, C. W. Luchsinger, and Troy King for their guidance and suggestions in these data analyses.

This technical memorandum report has been reviewed and is approved.

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SUMMARY

This report presents the results of the analysis of exceedance curves by aircraft tail number for the F-4 and A-37B aircraft.

The aim of this report is to provide a statistical bound for the different exceedance curves that are obtained from various aircraft tail numbers on a given aircraft type. The F-4 and A-37B aircraft were selected only because load factor (nz) exceedance data by tail number were available in table formats in References 1 and 2.

Results of this report indicate that reasonable bounds on exceedance curves may be obtained by tolerance limits which assume that for a fixed load factor (nz), the log of the exceedances per thousand flight hours has a normal distribution. Also, tolerance limits could be obtained using nonparametric techniques when the quantity of tail numbers is sufficiently large.

Finally, curves were fitted to the composite data. The designer may extrapolate these curves to determine design limit load factor and change the shape of the curves by varying values of coefficients.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

LIST OF TABLES

DEFINITION OF SYMBOLS

nz	vertical load factor in g's.
MED (nz)	the median value of exceedances per 1,000 hours for a fixed value of nz.
Ei(nz)	the number of exceedances per 1,000 hours for a fixed value of nz for the ith aircraft tail number.
LCI(nz)	the lower bound of the 90% confidence interval for MED(nz).
UCI(nz)	the upper bound of the 90% confidence interval for MED(nz).
C	the composite exceedance curve.
MAX(nz)	the maximum number of exceedances per 1,000 hours for a fixed value of nz.
MIN(nz)	the minimum number of exceedances per 1,000 hours for a fixed value of nz.
P	the percent of tail numbers that lie between 2 given limits.
γ	confidence, i.e. the probability that a given statement is true.
M(nz)	mean of exceedances per 1,000 hours for fixed nz.
s(nz)	standard deviation of exceedances per 1,000 hours for fixed nz.
NorUL(nz)	upper bound of 90% two-sided tolerance limit with 90% confidence assuming normal distribution.
NorLL(nz)	lower bound of 90% two-sided tolerance limit with 90% confidence assuming normal distribution.
$M3=\log_{10}(C(nz))$	used as the mean for log normal assumptions.

DEFINITION OF SYMBOLS (cont'd)

s'(nz)	standard deviation of the logs of the Ei(nz) for a fixed nz.
LNUL	Upper bound on log scale of 90% two-sided tolerance limits with 90% confidence of exceedances per 1,000 hours.
LNLL	lower bound on log scale of 90% two-sided tolerance limits with 90% confidence of exceedances per 1,000 hours.
LgNorUL	$10^{(LNUL)}$
LgNorLL	$10^{(LNLL)}$
F	curve fitted to nz composite exceedance per 4,000 hours.
b(nz)	best fit polynomial that log F = b(nz).
M_w	Weighted mean.

SECTION I
INTRODUCTION

As a result of a series of meetings with Mr. W. J. Crichlow, ASD/ENF, it appears that some changes are necessary in the presentation of flight loads data. This is expected to improve the development of more definitive structural design criteria. Mr. Crichlow, who has prime responsibility for MIL-A-8866B revision, explained the need for re-examining the statistical variations in vertical load factor (n_z) exceedance curves and this distribution's impact on more precise aircraft design, and fatigue, and fracture analyses. Accordingly, AFFDL/FBE initiated a program to analyze flight loads data, categorized by mission segment, type, and tail number as available, to determine the load's distribution mean exceedance curve and the statistical spread about the mean.

Load factor (n_z) data from A-37B (Reference 1) and F-4 (Reference 2) aircraft were segregated by tail number, base, and mission type as available in the reports. The data from these subdivisions were analyzed and the results plotted in the form of exceedance curves.

These methods provide a precise procedure for criteria development by computing exceedance curves with a mean and tolerance limits for all possible exceedance curves in the given category. The results appear promising in providing a more descriptive USAF military specification, which in turn, necessarily results in more refined structural design for future aircraft.

SECTION II

DISCUSSION

To achieve the objectives stated above, several methods of analyzing the flight loads data as extracted from References 1 and 2 were employed. For each set of data by aircraft tail numbers, the following statistical functions were computed for the vertical load factor (n_z) exceedance curves:

a. Maximum, minimum and median exceedance curves with a 90% confidence interval for the median.

b. 90% two-sided tolerance limits with 90% confidence assuming the exceedances are normally distributed.

c. 95% two-sided tolerance limits with 95% confidence assuming the log of the exceedances are normally distributed.

d. Least squares curve fits to the log of the composite data.

Plots of the above results are shown in Figures 1 through 36 in Appendix I of this report. Detailed discussions of the flight data, computer programs, and assumptions used to obtain the above results are presented in the following paragraphs of this report.

1. Flight Data

The load factor (n_z) peak data used for this study were obtained from References 1 and 2. Reference 1 contained operational flight loads data from the A-37B aircraft while performing training flights from May 1969 to September 1971. A total of 4,001 hours of data was analyzed and is summarized in Table I. For a detailed description of the data, the reader is referred to Reference 1.

TABLE I

SUMMARY OF A-37B DATA USED[1]

Base and Year	Hours	Tail Numbers	Hours/Tail Number
England AFB (1969)	541.47	11	49.22
Bien Hoa AB (1970)	2038.23	12	169.85
Binh Thuy AB (1971)	913.84	8	114.23
England AFB (1971)	507.87	4	126.97
	4,001.41		

TABLE II

SUMMARY OF F-4 DATA USED[2]

Mission Type	Hours	Tail Numbers	Hours/Tail Number
Air-Ground	2308.6	46	50.19
Air-Air	149.0	23	6.48
In-Nav	481.2	40	12.03
Recon	515.2	19	27.12
Test	19.1	15	1.27
	3,473.1		

Reference 2 contained F-4 aircraft Southeast Asia (SEA) load factor data that were segregated by tail number, and mission types. These data were collected by Technology, Incorporated (TI), as part of Aircraft Structural Integrity Program (ASIP) during the period of 15 August 1969 through 31 December 1970. The 3,473 total hours analyzed are summarized in Table II.

The above referenced reports were selected since the data are in terms of aircraft tail number by mission type or base. However, these data are not in terms of mission segment as desired. It seems reasonable to assume that segregation by base, and mission type would be as applicable to the analysis methods as segregation by mission segment.

Before proceeding, some of the problems encountered in using the above data should be considered. One problem that appeared in Tables I and II was that both aircraft types have what appeared to be a large total number of hours, but may have too small a number of hours for any given tail number and category of mission or base. For example, there is 4,001 hours of F-4 data but only an average of 50 hours per tail number in the air-ground mission category, with other mission categories having even less hours. This raised the question about the validity of an exceedance curve for a given tail number. A small number of hours for a tail number would probably give the exceedance curves a wider spread than is actually the case; this would tend to make any estimates about the variability to be unnecessarily large. If the criteria, that 1,000 hours of data are needed to plot a valid exceedance curve is used, it would be necessary to have at least 46(1,000) + 40(1,000) + 19(1,000) + 15(1,000) = 143,000 total hours of F-4 data to obtain valid exceedance curves for all the aircraft tail numbers and categories listed in Table II. This is a large data requirement.

Another question concerns the type of distribution the exceedances have for a given value of n_z. It will be shown that normality should probably be ruled out, but the log of the exceedances being normally distributed is a plausible assumption. The assumption of a nonparametric (not normal) distribution would also provide some usable results, but more tail numbers would be required, and hence more data.

Lastly, the problem of zero exceedances at high n_z values caused a number of interrelated problems, for example zero peaks in 50 hours implies zero peaks per 1,000 hours is not a valid assumption. That is, the real number of peaks per 1,000 hours at some high load factor may have really been 10 peaks implying .5 peaks in 50 hours. Since there can be no fractional count of a peak, 0.5 peaks in 50 hours of flight cannot occur. Even with over 1,000 hours of data, the result is questionable since occurrences of n_z peaks at high n_z values (above 7 g's) are rare. Handling of this zero occurrences problem is discussed throughout this report.

It was decided that these data would suffice for this study, since there did exist variability of the exceedance curves for each aircraft tail number. However, for better results, more data hours per aircraft and more tail numbers should be used.

2. Computer Programs

Several computer programs (See Figures 37-40) were used on the Hewlett-Packard (HP) 9830 calculator system to verify the validity of some new approaches to analyzing flight loads data. The computer programs were used to generate plots, tolerance limits, confidence intervals and curve fits for the n_z exceedance data used herein.

One computer program, which handles a maximum of 49 tail numbers and 16 different values of n_z along the abscissa, used the n_z peak data obtained from the references, and stored it on a file of a tape cassette. Hence, each file of the tape cassette contained n_z data by tail number for a particular mission or base as shown in Tables I and II, and also the composite data which includes all tail numbers.

The tape cassette was used to input the n_z data in an analysis program. The analysis program was used to analyze the data as follows:

a. Plot n_z exceedance points per 1,000 hours for each tail number.

b. Plot n_z exceedance curves for each aircraft tail number.

c. Compute and plot the minimum, maximum and median exceedance per 1,000 hours for each of the 16 n_z values.

d. Compute and plot a 90% confidence interval for the median of the exceedances per 1,000 hours.

e. Compute and plot the mean and composite exceedances for each of the 16 n_z values.

f. Compute and plot tolerance limits on the exceedances per 1,000 hours for each n_z stratum assuming that the exceedances or the log of the exceedances are normally distributed.

A third computer program, which was a revision of a HP Plot Pac Program, performs a least square fit of a polynomial to the log of the exceedance data. Composite data for all tail numbers for each category were input through the keyboard to fit curves of the form

$$F = 10^{b(n_z)},$$

where F is the number of exceedances per 4,000 hours and $b(n_z)$ is the best polynomial that estimates log F.

3. Data Presentation

Results of the analysis are presented in Figures 1 through 36 and Table III in Appendix I. A detailed discussion of the various methods that were studied is presented below.

3.1 Exceedance Curves by Aircraft Tail Numbers

For each of the nine categories listed in Tables I and II, the exceedance curve for each individual tail number has been plotted in Figures 1 through 9. These exceedance curves give a rough idea of the type of spread that might be expected for different aircraft tail numbers. The following sections primarily investigate the means by which statistical statements can be made in describing this spread in the exceedance curves.

3.2 Confidence Intervals for the Median

The median of the exceedances per 1,000 hours ($MED(n_z)$) for each given value of n_z have been plotted in Figures 10 through 18 for each of the nine cases listed in Tables I and II. Straight line segments were also drawn connecting the $MED(n_z)$ values. The median was calculated so that one can say that about 1/2 of the exceedances per 1,000 hours for each n_z (E_i (n_z) where i is the tail number) lie above and below the $MED(n_z)$ value.

Next, a 90% two-sided confidence interval (CI) for the median was found using a distribution free procedure based on the sign test as outlined in Reference 3. The procedure was applied to each $E_i(n_z)$ for a given n_z so that the 90% confidence interval for the median corresponds to the statement: the probability

$$LCI(n_z) \leq MED(n_z) \leq UCI(n_z) = 0.90$$

where $LCI(n_z)$ is the lower limit of the CI and $UCI(n_z)$ is the upper limit of the CI (i.e. the probability that the true median lies between $LCI(n_z)$ and $UCI(n_z)$ is 0.90).

All $E_i(n_z) = 0$ cases were included for the above computation. In addition, the composite was also plotted and found within the 90% CI for the median for the lower values of n_z. Figures 10 through 18 showed that a difference exists between the median and the composite (C) for the F-4 data whereas they are very similar for the A-37 data. In fact, for the F-4 data the median usually lies below the composite. This is probably due to the greater frequency of $E_i(n_z) = 0$, for the F-4 data which was caused by an inadequate number of hours per tail number as shown in Table II and discussed earlier in data presentation. It should be noted that 1 peak in 100 hours of data would imply 10 peaks per thousand and may overestimate a theoretical value of 1 peak per thousand hours by quite a margin. Intuitively, it then seems that for the F-4, the composite would usually lie below any exceedance per tail number due to the overestimation of the exceedance for a limited number of hours.

3.3 Maximum and Minimum Values

The maximum and minimum values of $E_i(n_z)$ are also plotted in Figures 10 through 18. The maximum ($MAX(n_z)$) and minimum values ($MIN(n_z)$) are plotted for each n_z value and a line drawn connecting the points. These curves may then be used to make distribution free two-sided tolerance limit statements which means that with some confidence gamma (γ), a percentage of the tail numbers (P) lies between the two limit values. By using Table A-32 in Reference 4, values of γ are obtained so that 90% of the tail numbers lie between the maximum and minimum values of n_z exceedances per 1,000 hours.

In Figures 10 through 18, the values of γ have been included on the plots. Note that gamma was higher as the number of tail numbers increased. For example, $\gamma = .30$ for 11 tail numbers in Figure 10 and $\gamma = 0.95$ for 46 tail numbers in Figure 14. Hence, we can use the maximum and minimum value of exceedances per 1,000 hours as statistical bounds. This method would give a very conservative estimate of the bounds on the exceedance curves since there are no assumptions made about the underlying distribution. For Figures 10 through 14, the bounds determined in this manner appear to be relatively narrow. However, in Figures 15 through 18, the minimum is zero too often since these cases have a small average hours per tail number.

3.4 Tolerance Limits Assuming Normal Distribution

In Figures 19 through 27, a plot of a 90% two-sided tolerance limit with 90% confidence is shown with the assumption that for a given n_z, the exceedances per 1,000 hours have a normal distribution.

The tolerance limit was computed by considering the $E_i(n_z)$s for each given n_z. The mean $M(n_z)$ was calculated using

$$M(n_z) = \sum_{i=1}^{N} \frac{E_i(n_z)}{N}$$

where $E_i(n_z)$ is the exceedances per 1,000 hours for a fixed n_z and N is the number of aircraft tail numbers. Then, the sample standard deviation(s) were computed by

$$s(n_z) = \left(\frac{n \sum_{i=1}^{n} E_i(n_z)^2 - \left(\sum_{i=1}^{n} E_i(n_z) \right)^2}{n(n-1)} \right)^{1/2}$$

which was obtained from Reference 1 on page 1-10.

The upper NorUL and lower limits NorLL were

$$NorUL(n_z) = M(n_z) + K \cdot s\ (n_z)$$

and

$$\text{NorLL} = M(n_z) - K \cdot S\ (n_z)$$

where K is obtained from Table A-6 of Reference 4 with $\gamma = .90$, $P = 90\%$ and N = number of tail numbers. Before proceeding, it should be noted that all zero exceedances per 1,000 hours were included in the computations.

The lower limit did not appear in most of the figures, because the assumption, that the exceedances are normally distributed, allows the lower limit to take on negative values. This is not possible. Another factor to consider concerns plotting of limits on a log scale. Since

$$|\log (M + KS) - \log M| \leq |\log (M-KS)-\log M|$$

the upper limits would be closer to the mean than the lower limits on the log scale.

As was the case for the median, the mean and composite are approximately equal at low n_z values but differ at higher n_z values. This is again probably due to errors in exceedances per 1,000 hours that occur at the higher n_z values.

It was concluded from the above discussions and Figures 19 through 27 that the assumption of a normal distribution was not valid.

3.5 Tolerance Limits Assuming a Log Normal Distribution

Plots of 90% two-sided tolerance limits with 90% confidence are shown in Figures 19 through 27 with the assumptions that the log of the exceedances per 1,000 hours for each n_z has a normal distribution. The mean M3 was computed as the common logarithm of the composite $(C(n_z))$ at each n_z. Note, the composite is the same as the weighted mean (M_w) defined by

$$M_w = \sum_{i=1}^{N} \frac{E_i(n_z)}{\Sigma t_i}$$

which equals the total number of occurrences greater than n_z divided by total time. This result is the composite. Next, a standard deviation $s'(n_z)$ is defined as

$$s'(n_z) = \left(\sum_{i=1}^{N} \frac{(\log E_i(n_z) - \log C(n_z))^2}{N} \right)^{1/2}$$

Since, the log of zero is undefined, zero values of $E_i(n_z)$ were not included in the solution for $s'(n_z)$. N was decreased by 1 each time a zero was excluded for a fixed value of n_z.

Then, as in the previous section, upper (LNUL) and lower (LNLL) tolerance limits were computed by formulas:

$$LNUL(n_z) = \log C(n_z) = K \cdot S'(n_z)$$

and

$$LNLL(n_z) = \log C(n_z) - K \cdot s'(n_z).$$

These upper and lower tolerance limit (LNUL, LNLL) curves are then equidistant from the composite when plotted on semilog paper as is evident from Figures 19 through 27. For most cases, the log normal assumption gave some reasonable bounds but deviations do occur. For instance, at the extreme n_z values, the limits go to infinity and zero for the upper and lower bounds respectively. This is because at the extreme n_z values there are many zeroes which make N small, and K which was obtained from Table A-6 becomes very large. Hence, LNUL and LNLL become very large and small respectively as can be seen clearly in Figure 26. The spike at n_z = 6 g's on Figure 22 resulted from one aircraft tail number exceedance curve being farther from the mean than the other three; thus, making a higher than probable standard deviation. It was decided, that overall, the log normal assumption did provide reasonable bounds for n_z exceedance curves.

3.6 Composite Exceedance Curve Fitting

The final computer program provided for the fitting of polynomial equations to the log of the exceedances per 4,000 hours. Four thousand hours was chosen because it is the design life for most fighter-type aircraft. The fitted curves that were obtained are presented in Figures 28 through 36 and summarized in Table III.

In most instances, a third degree polynomial provided the best fit, but in instances where this was not a good fit, (e.g. the curve goes to plus infinity as n_z goes to infinity) the best straight line was fitted through the data points. In most cases when the straight line fit was used it proved to be adequate.

These fitted curves to the n_z exceedances per 4,000 hours can be used as a uniform method of extrapolation to one exceedance per 4,000 hours (i.e. the aircraft life to obtain an estimate of a realistic design limit load factor).

In addition, these equations may be used to change the magnitude and slope of the exceedance curves for test purposes. The equation has the form

$$F(x) = P.10^{B(x) \cdot x}$$

where F is the curve function, x corresponds to n_z and B is a function of x of the form

$$B(x) = b_2x^2 + b_1x + b_0$$

For the case of the cubic equation for positive n_z in Figure 32, the equation for the exceedance curve for F-4 air-ground data was found to be

$$F(x) = 10^{(6.9742-1.4829x + .2914x^2-0.0249x^3)}$$

(See Table II in Appendix I)

This equation can be reduced to

$$F(x) = 10^{6.9742}10^{x(-.0249x + .2914x^2 0.0249x^3)}$$

This means we can place

$$P = 10^{6.9742} = 9,423,234$$

and

$$B(x) = (-.0249x^2 + .2914x-1.4829)$$

P will define the point at which the curve intercepts the $n_z = 0$ axis and B(x) defines an instantaneous slope to the exceedance curve on the log scale. This takes into account the changes in slope which is typical for fighter n_z exceedance curves. As long as b_2 is a negative coefficient (-.0249 for the example cited above), we know that the fitted curve will approach zero as n_z increases without bounds.

With these equations, a series of curves could be obtained by varying the magnitude P and coefficients of the slope B(x) such that they do not exceed the bounds determined by using one of the previous techniques. The effect of varying the P and the B(x) has not been studied in this report, but might be an approach that may prove effective.

Extrapolation of these curves to design limit load factor is now a simple procedure. Since F is the exceedances per 4,000 hours, find the value of n_z such that the log F = log 1 = 0. For example in Figure 28 for A-37B, England AFB, 1969, we would find a design limit load factor of 6.8 g's.

SECTION III
CONCLUSIONS

The plotting of n_z exceedance curves by aircraft tail number to obtain statistical bounds did appear to be feasible. Finding two-sided tolerance limits for exceedance curves with the assumption that the log of the exceedances per 1,000 hours are normally distributed appeared to give reasonable bounds. Nonparametric techniques gave reasonable, but more conservative bounds. The assumption that the exceedances per 1,000 hours were normally distributed appeared to give the widest bound and hence were not as useful. Curve fitting polynomials to the log of the exceedances per 4,000 hours for composite n_z data provided usable equations for representing n_z exceedance curves in most cases for the two aircraft types that were considered. In addition, the polynomials provide a standard mean of extrapolating n_z exceedance curves to design limit load factor.

SECTION IV

RECOMMENDATIONS

As noted earlier, the data used for the F-4 represented a small amount of flight time per tail number. A further study should be made to determine what minimum time on a tail number is required in each category to ensure that a representative exceedance curve can be obtained for that particular aircraft. The A-37B data indicated that 100 hours per aircraft tail number would probably be sufficient, but more information would have to be studied before this statement could be made with certainty.

Separating the data into mission segments would increase the need for a larger data base. For example, if nonparametric techniques are used, then at least 50 tail numbers should be used for each mission segment. If 100 hours per tail number are required and five mission segments for a typical aircraft type are assumed, there would need to be at least 25,000 hours of data on the particular aircraft type. If the data were not categorized by mission segment, 5,000 hours would suffice, and hence, it is seen that the total number of hours needed is directly proportional to the number of categories within the data base. Thus, it should be remembered that segregating data into mission segment would greatly increase the amount of data that needs to be collected.

Also, the uncertainty of the number of exceedances per 1,000 hours at higher n_z values is of concern. A minimum of 100 hours per tail number would be a minimum requirement; however, there is still a problem when zero exceedances per 1,000 hours occur, because exceedance per 1,000 hours approaches zero as n_z increases, but does not really reach zero. These problems at high n_z values could easily form the major topic of further study.

Lastly, the aircraft for this report were both fighter-type aircraft. For other aircraft, like transport, other techniques may be more applicable than those cited in this study.

APPENDIX I

DATA RESULTS

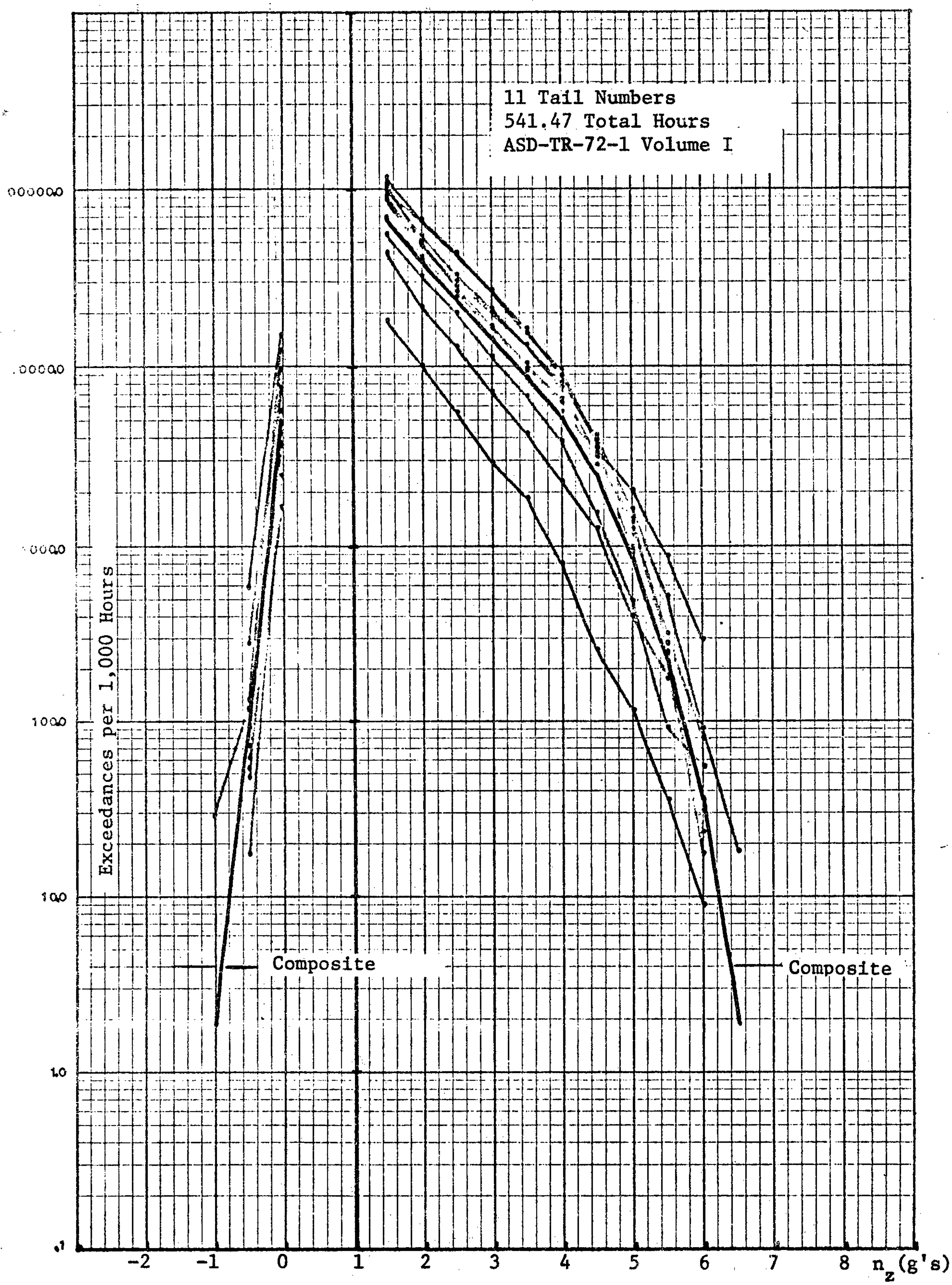

FIGURE 1. N_z Exceedance Curves by Tail Number for A-37B at England AFB (1969)

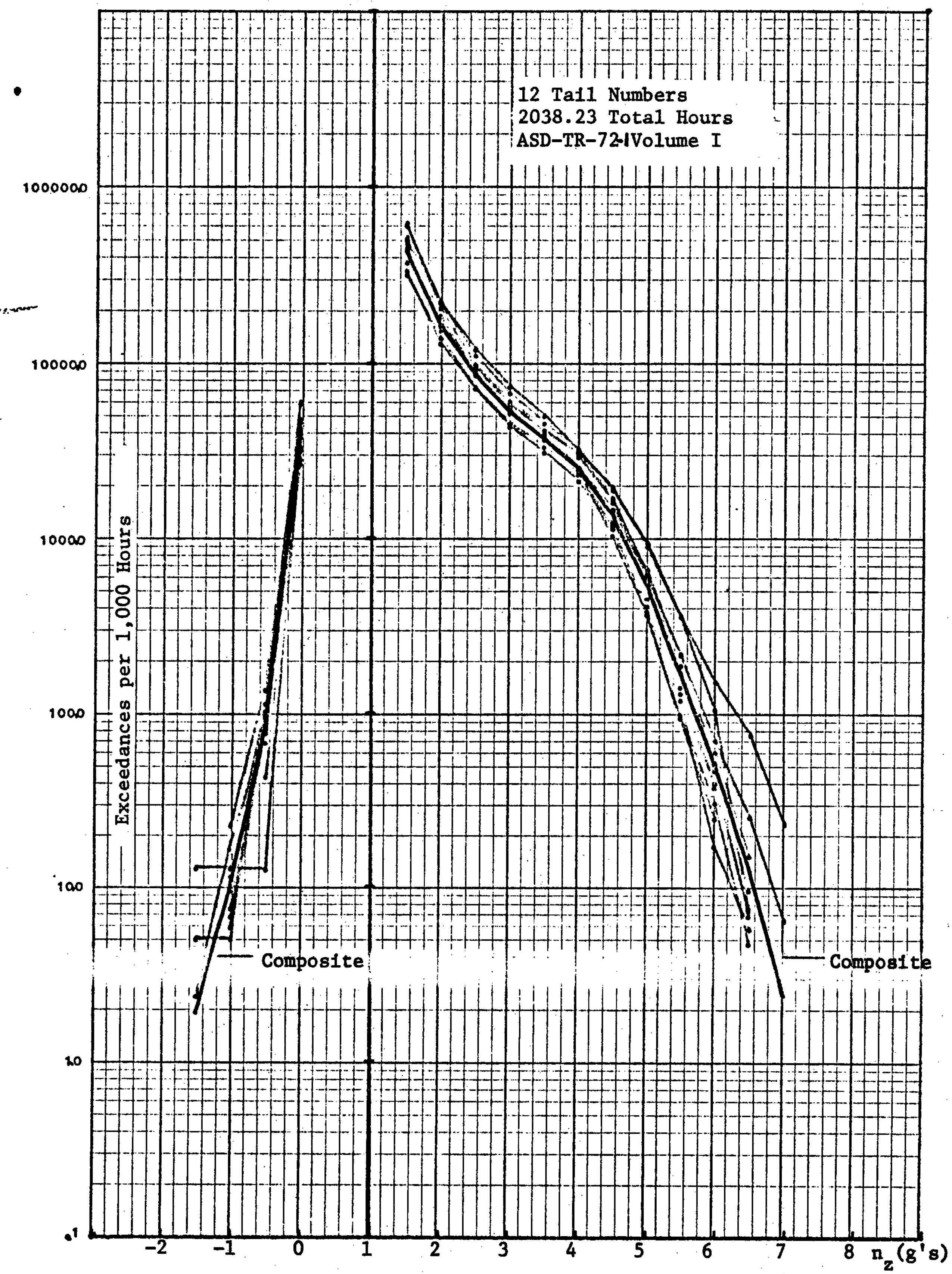

FIGURE 2. N_z Exceedance Curves by Tail Number for A-37B at Bien Hoa AB (1970)

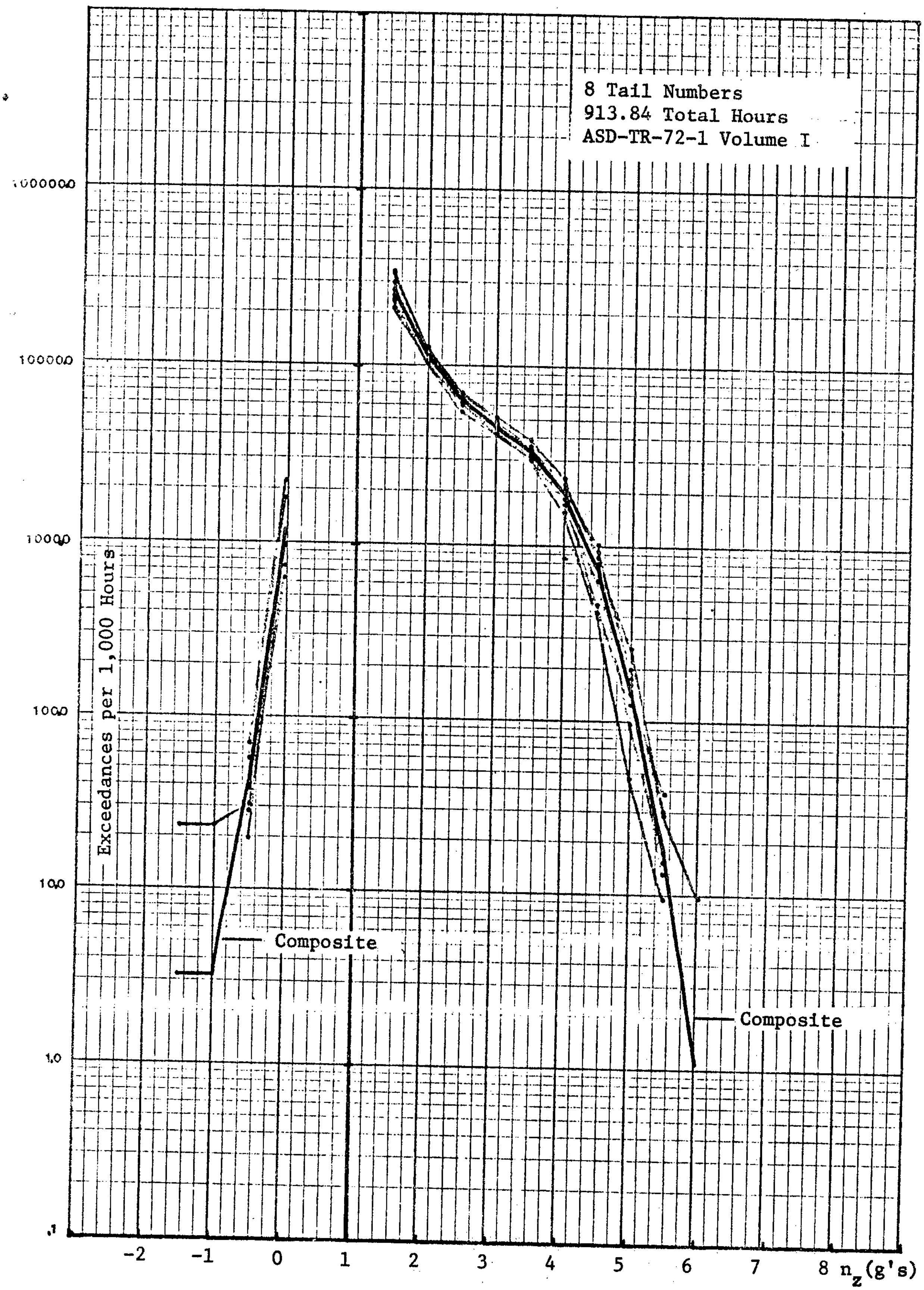

FIGURE 3. N_z Exceedance Curves by Tail Number for A-37B at Binh Thuy AB (1971)

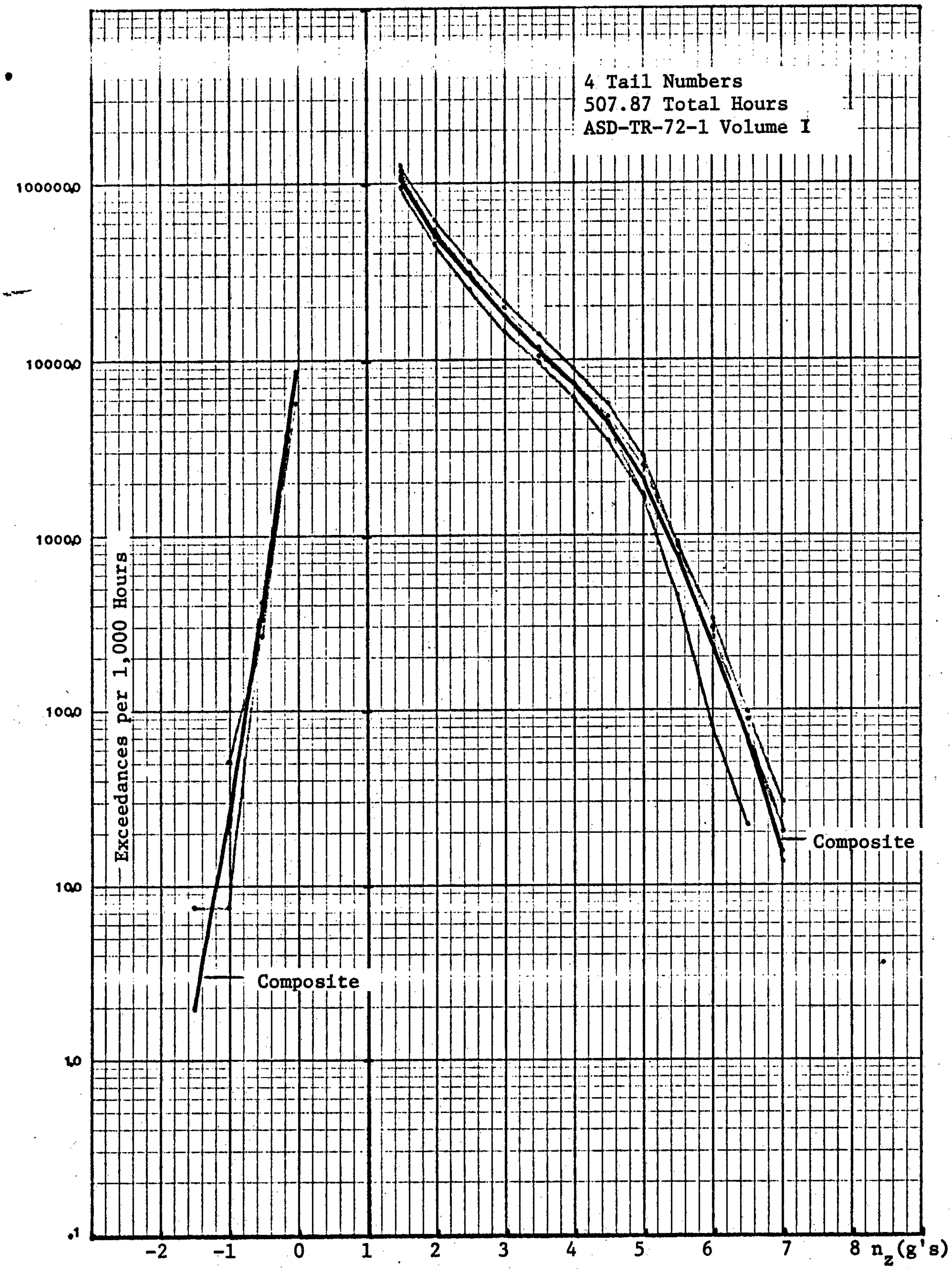

FIGURE 4. N_z Exceedance Curves by Tail Number for A-37B at England AFB (1971)

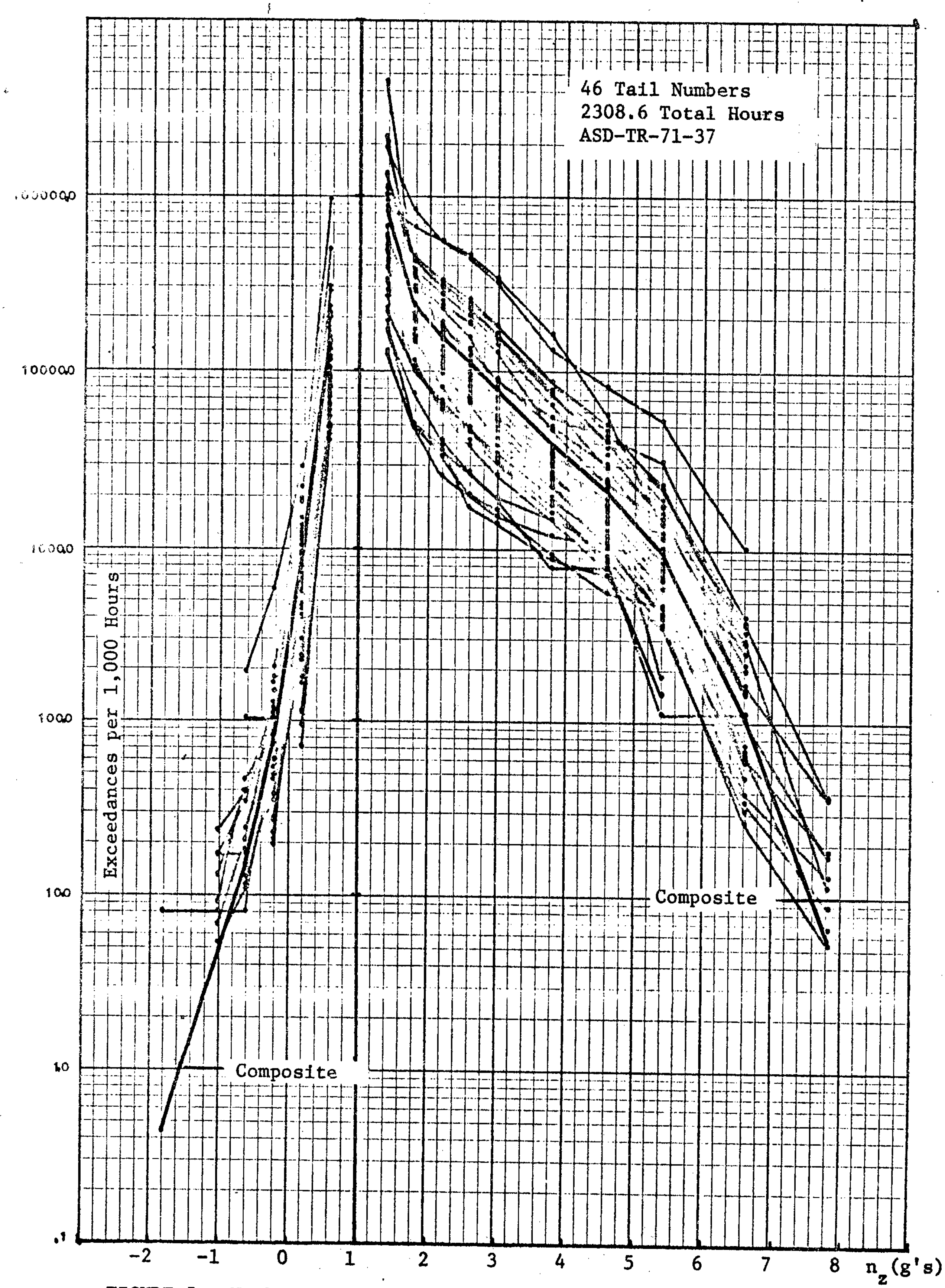

FIGURE 5. N_z Exceedance Curves by Tail Number for F-4 Air-Ground

FIGURE 6. N_z Exceedance Curves by Tail Number for F-4 Air-Air

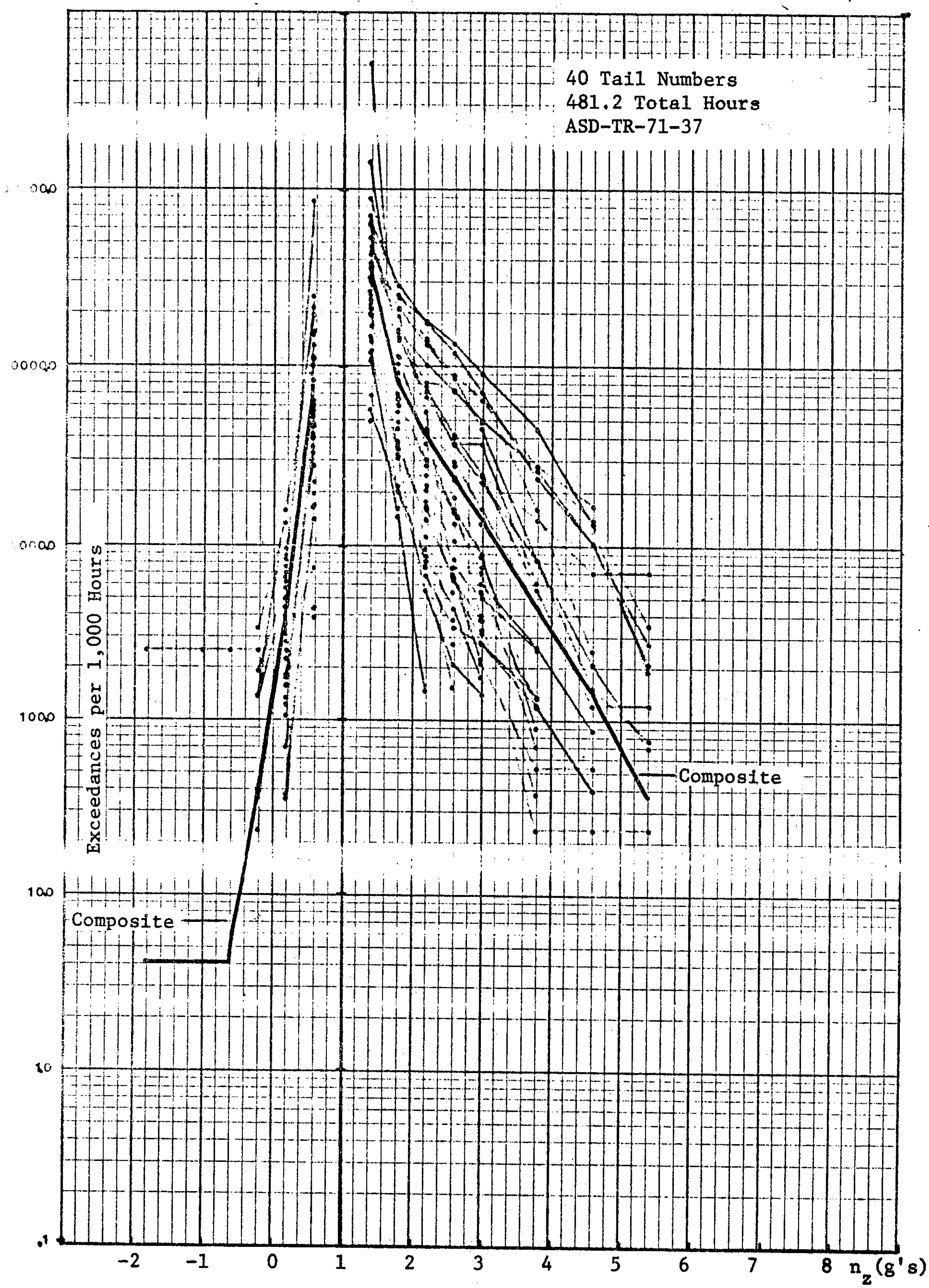

FIGURE 7. N_z Exceedance Curves by Tail Number for F-4 Inst. & Nav.

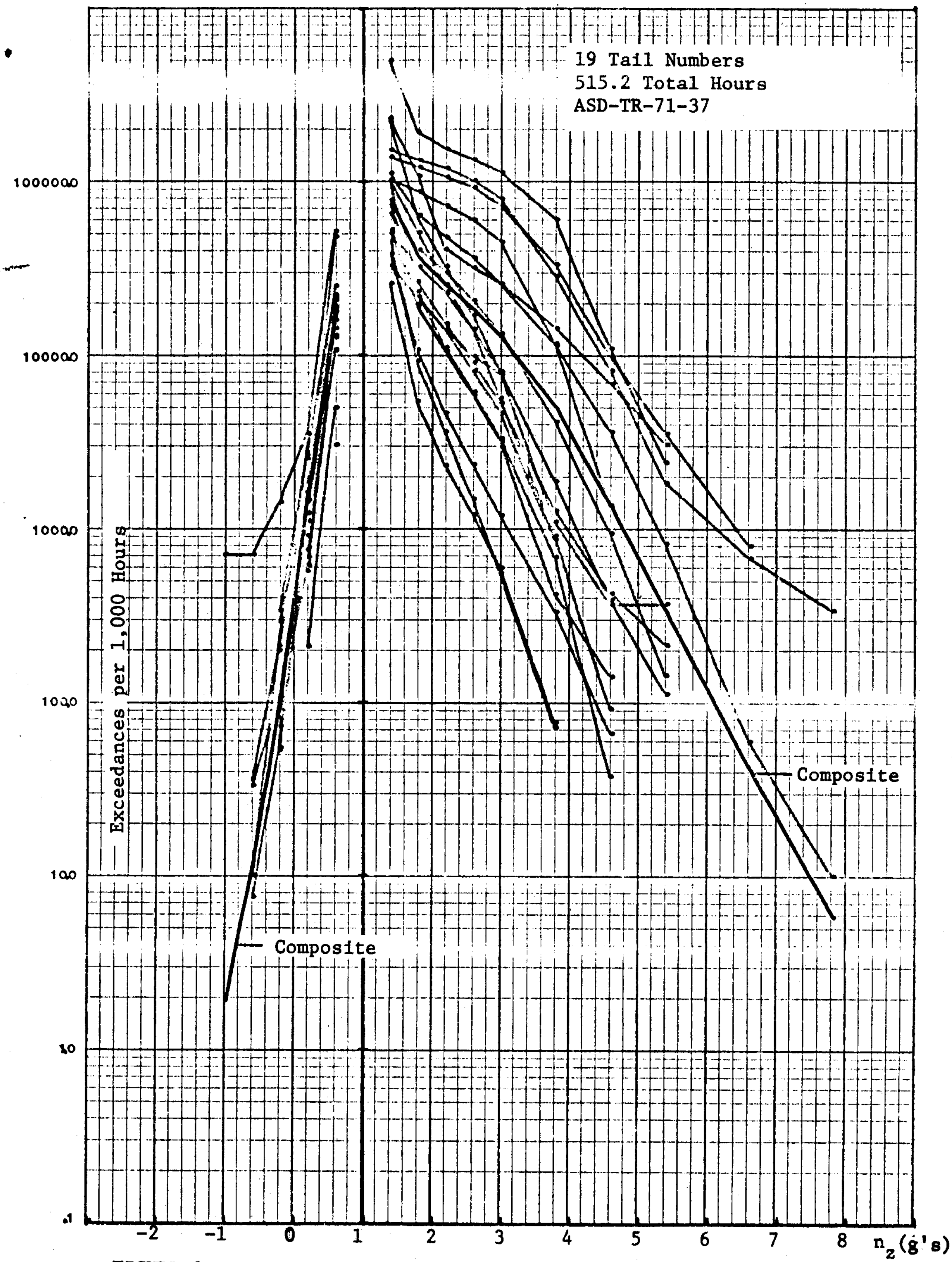

FIGURE 8. N_z Exceedance Curves by Tail Number for F-4 Reconnaissance

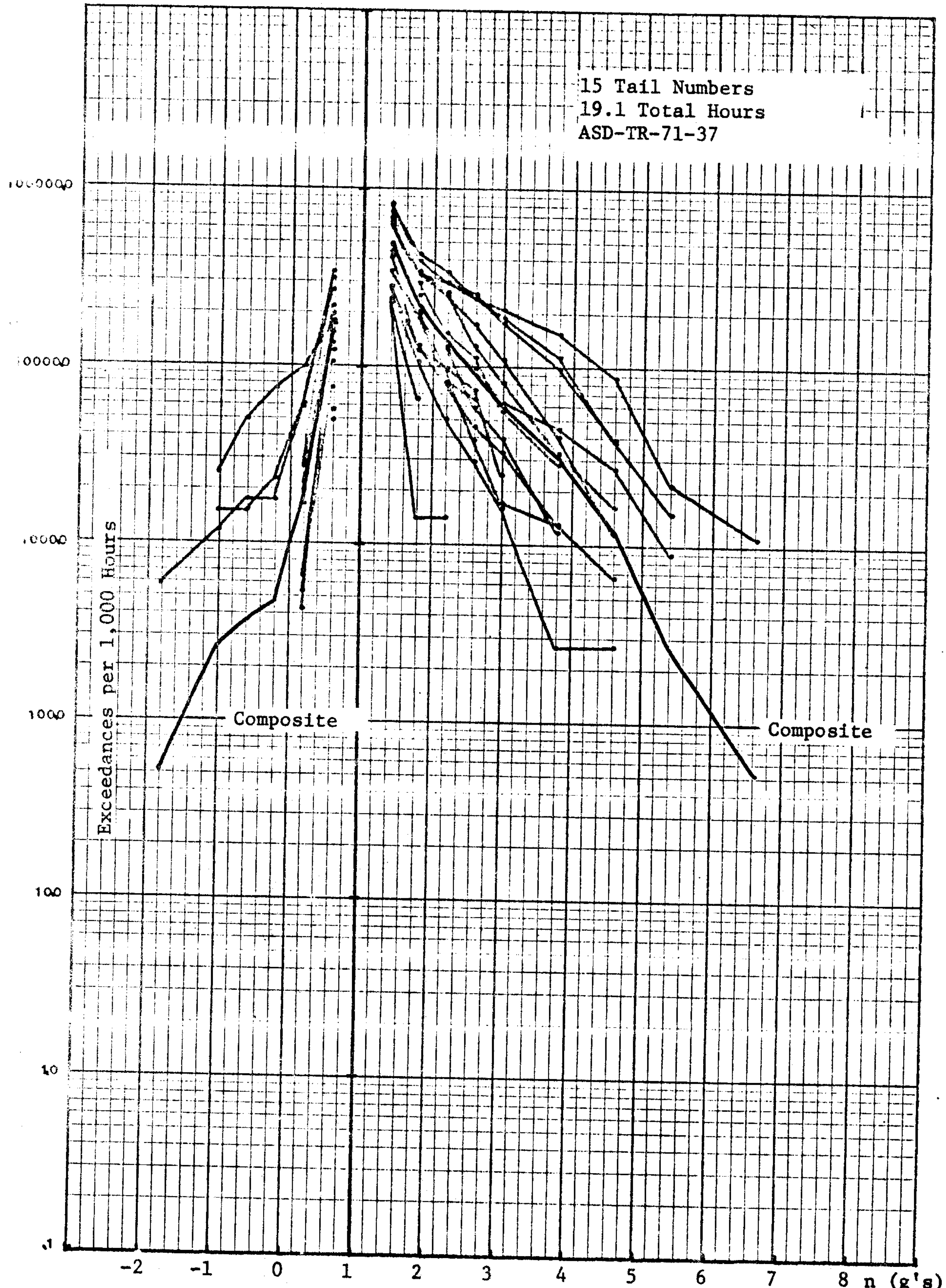

FIGURE 9. N_z Exceedance Curves by Tail Number for F-4 Test

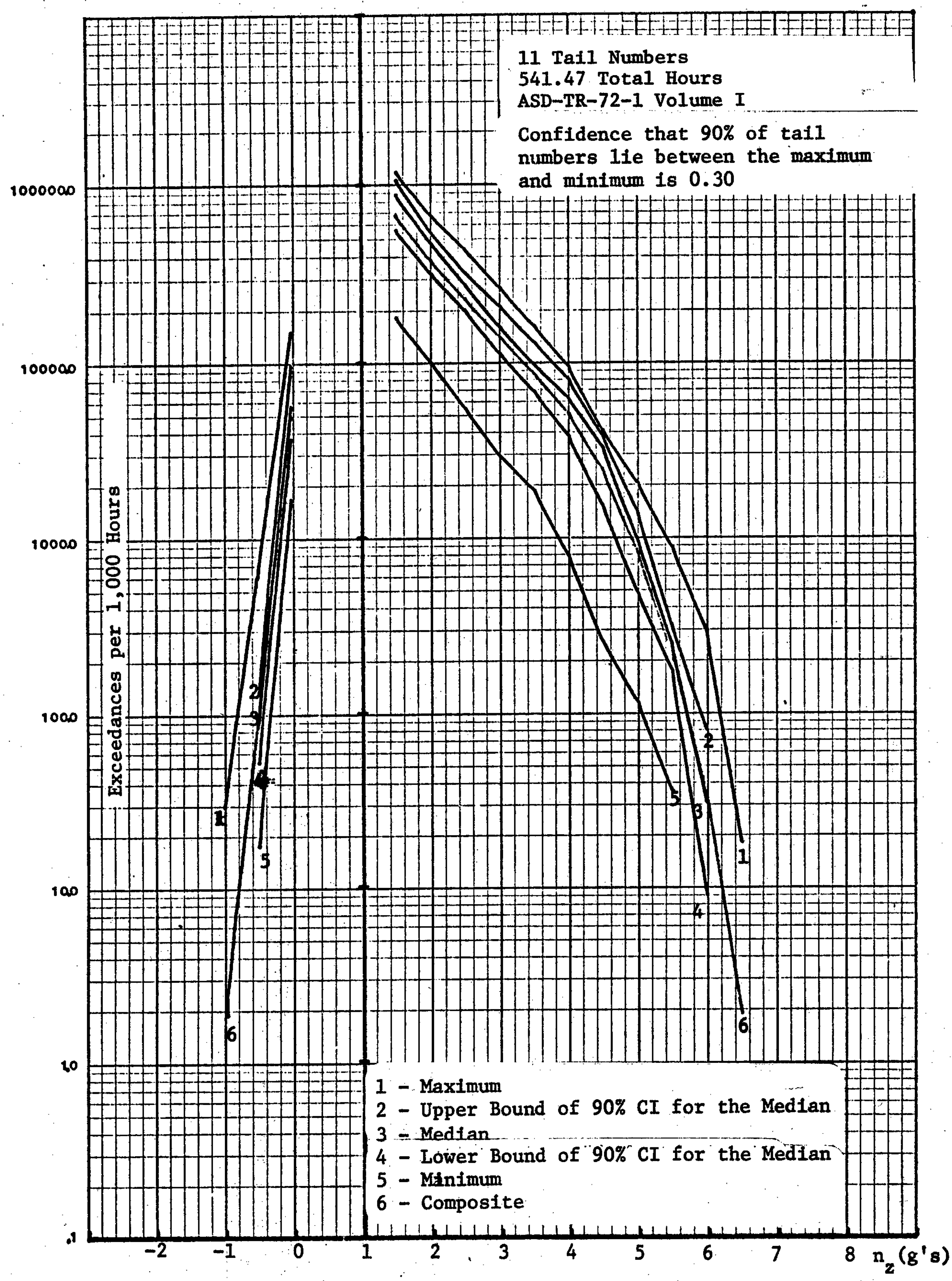

FIGURE 10. Nonparametric Bounds for A-37B at England AFB (1969) Exceedances

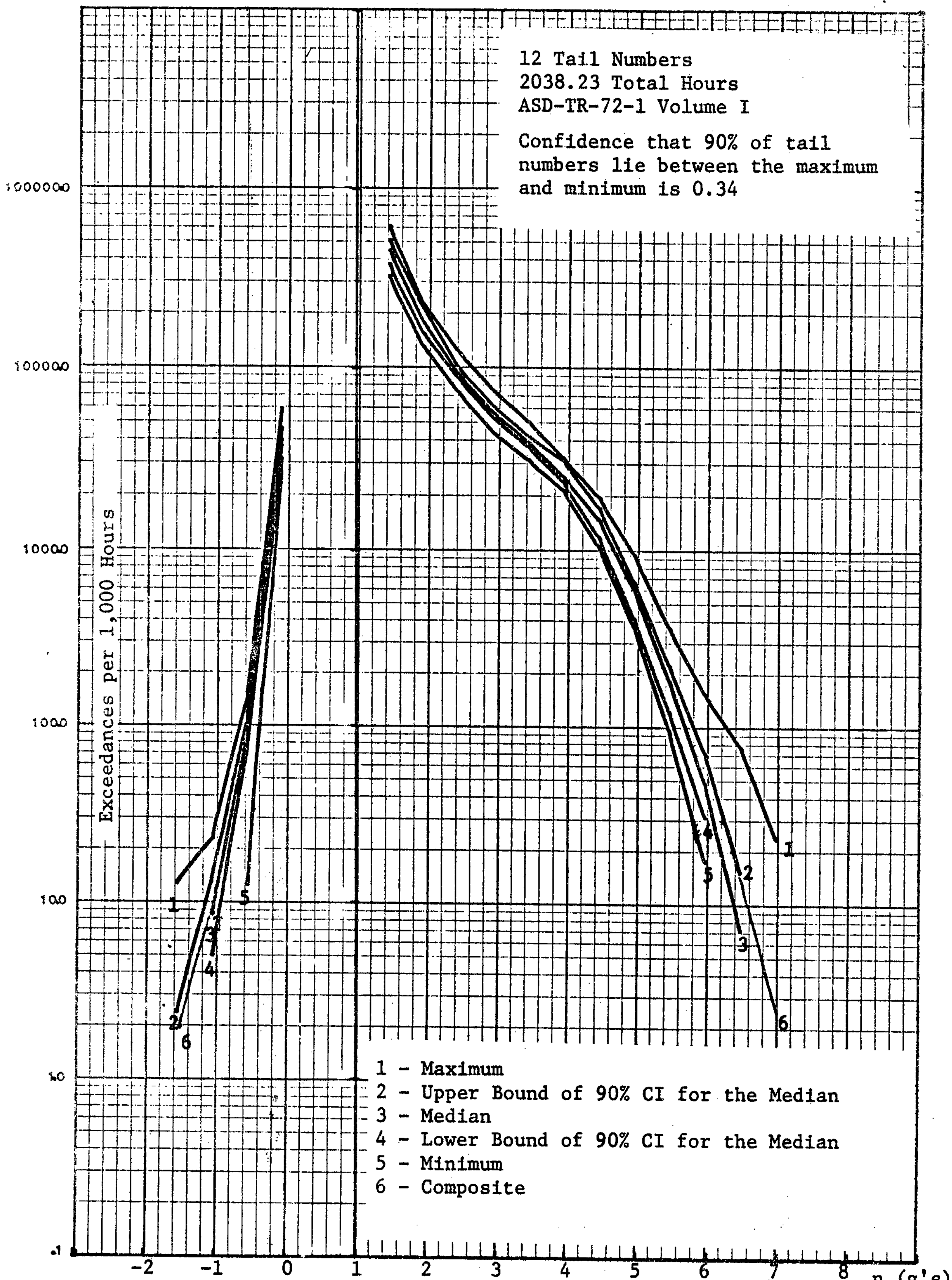

FIGURE 11. Nonparametric Bounds for A-37B at Bien Hoa AB (1970) Exceedances

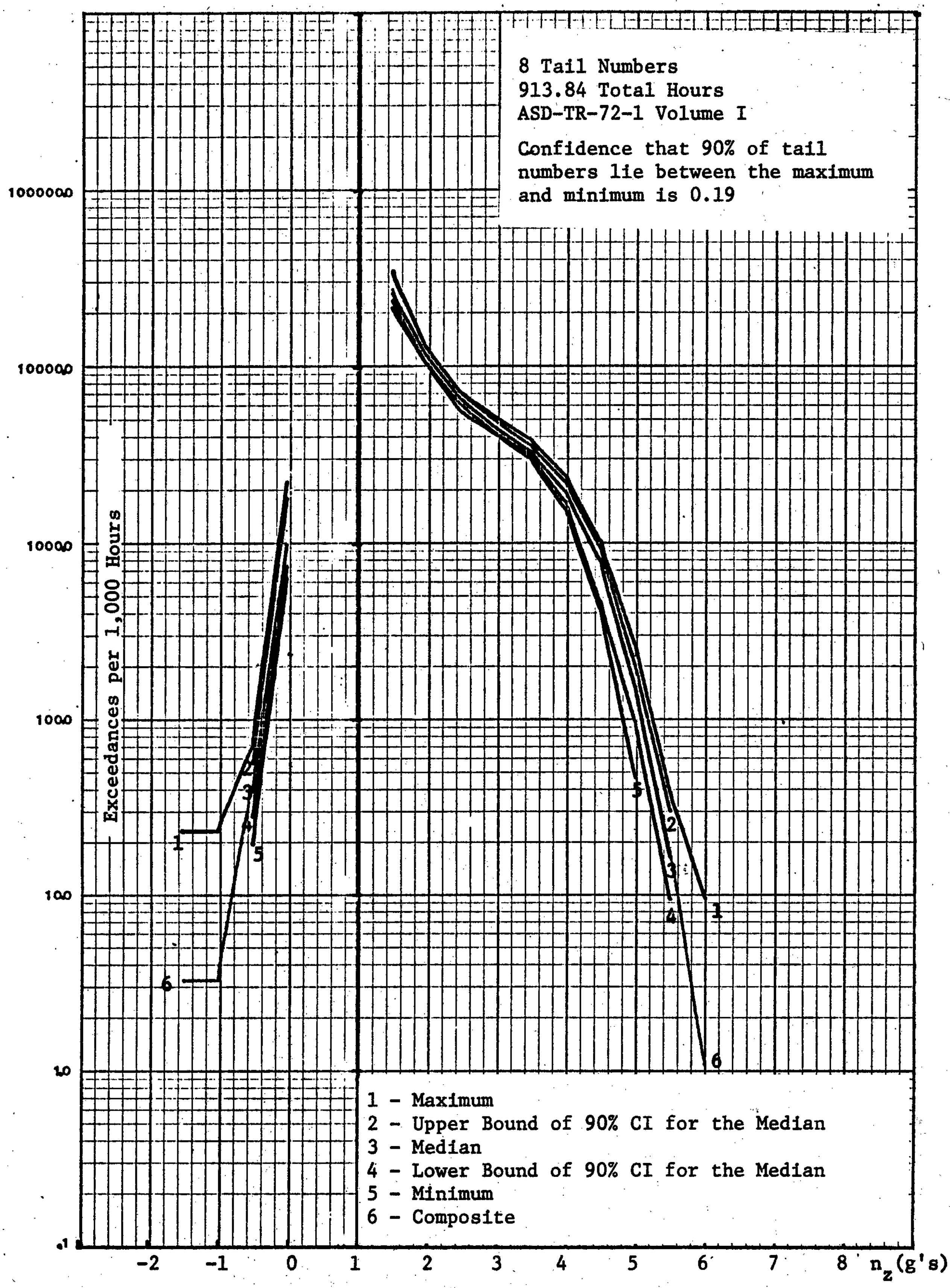

FIGURE 12. Nonparametric Bounds for A-37B at Binh Thuy AB (1971) Exceedances

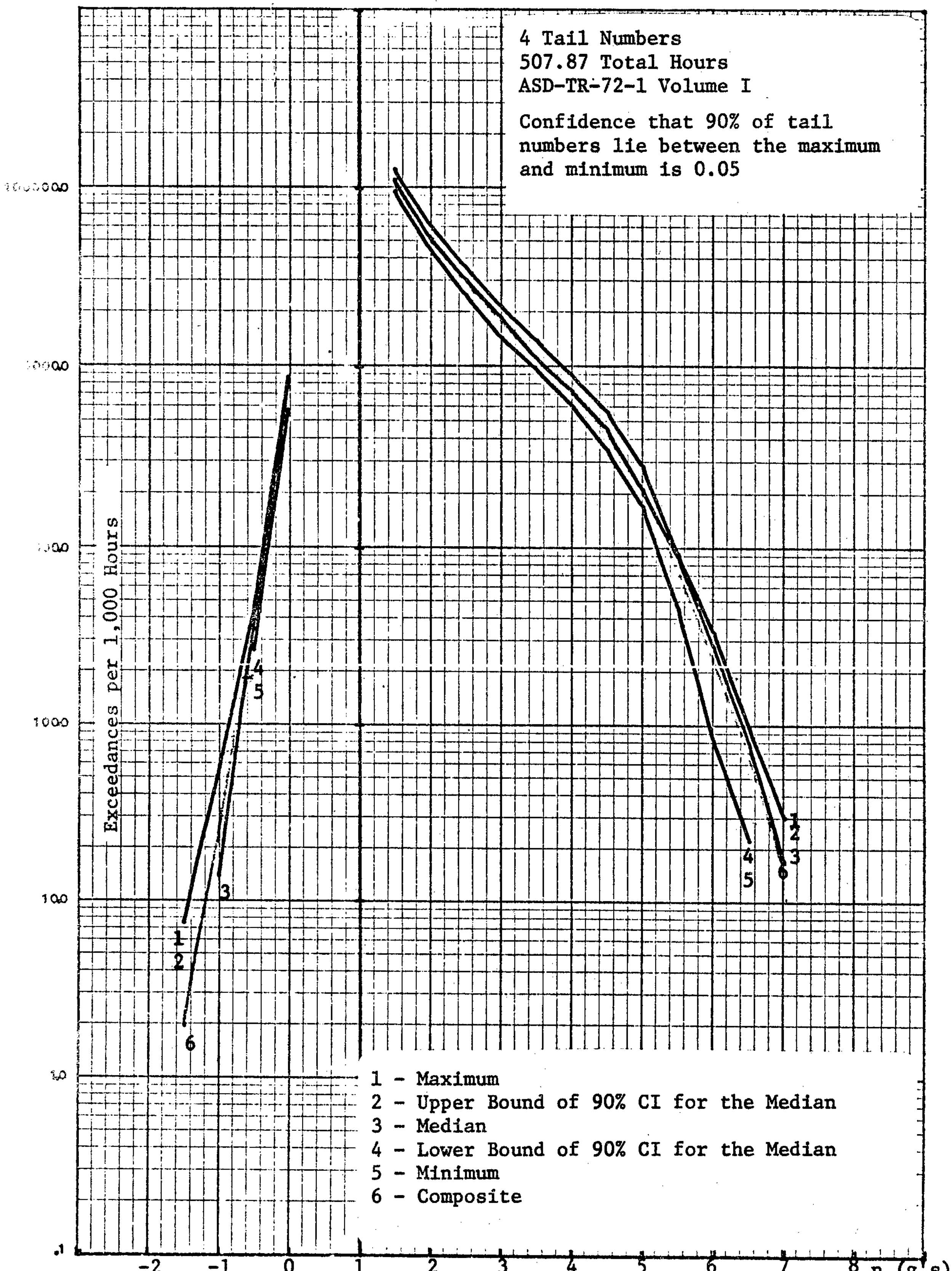

FIGURE 13. Nonparametric Bounds for A-37B at England AFB (1971) Exceedances

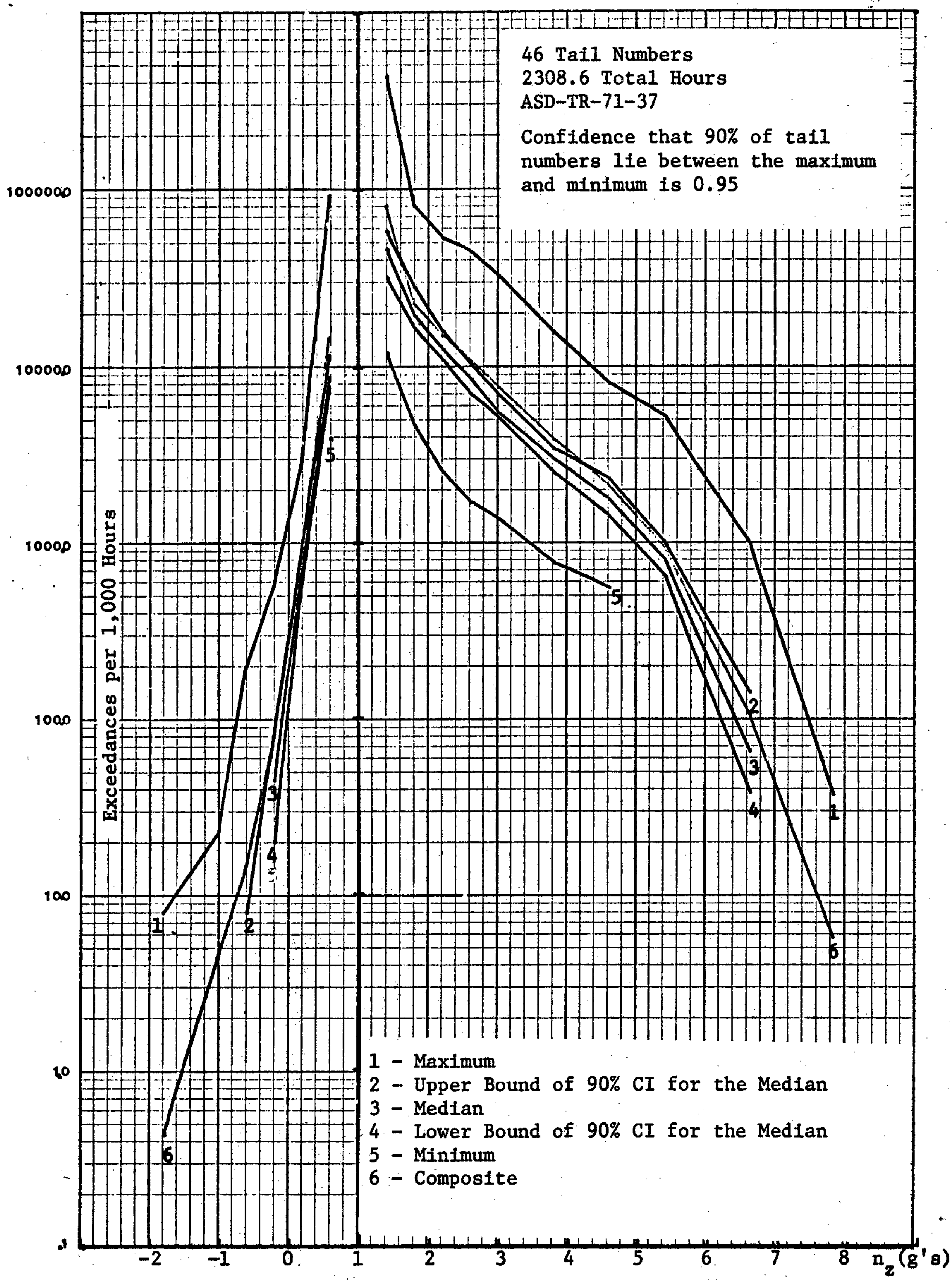

FIGURE 14. Nonparametric Bounds for F-4 Air-Ground Exceedances

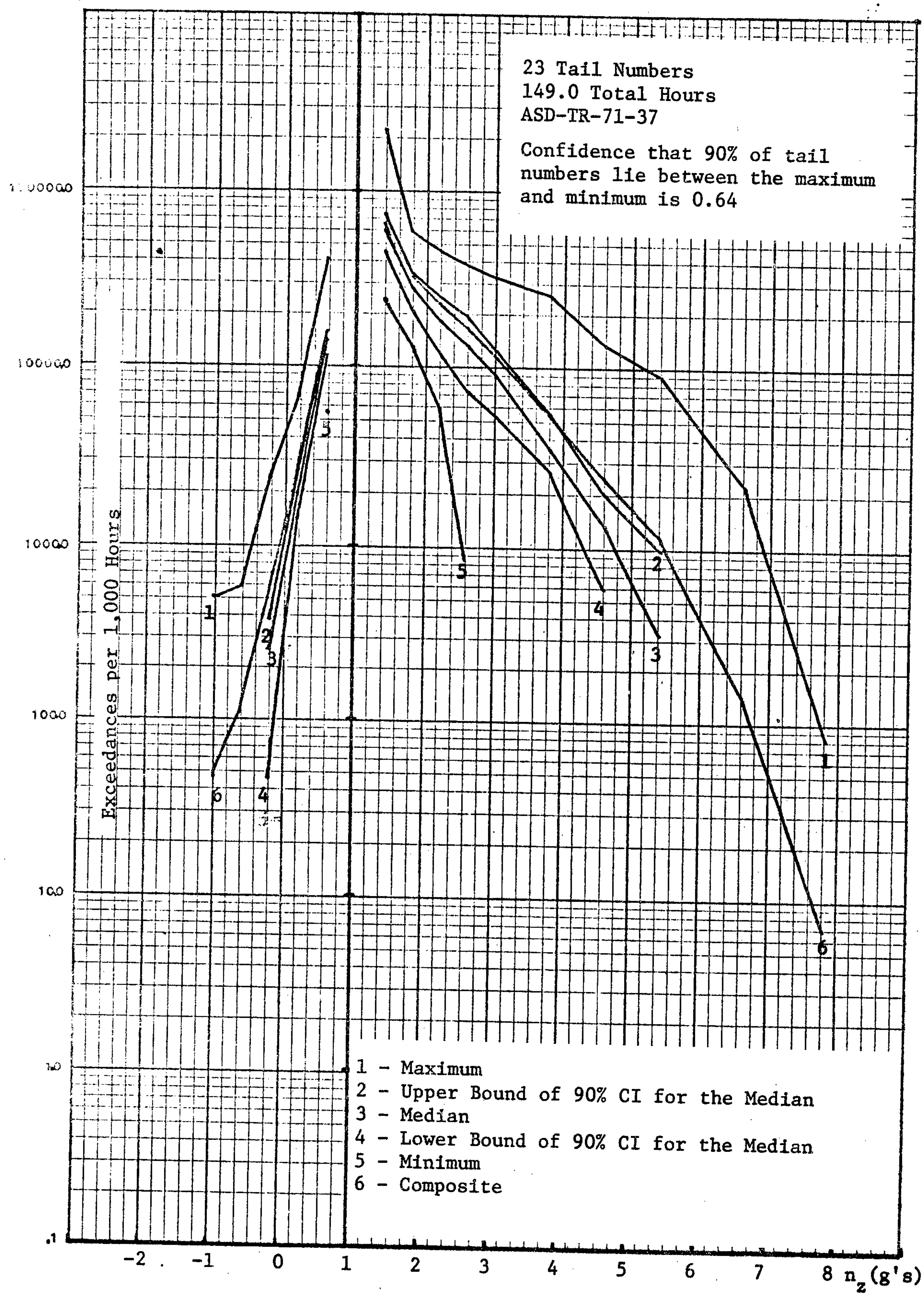

FIGURE 15. Nonparametric Bounds for F-4 Air-Air Exceedances

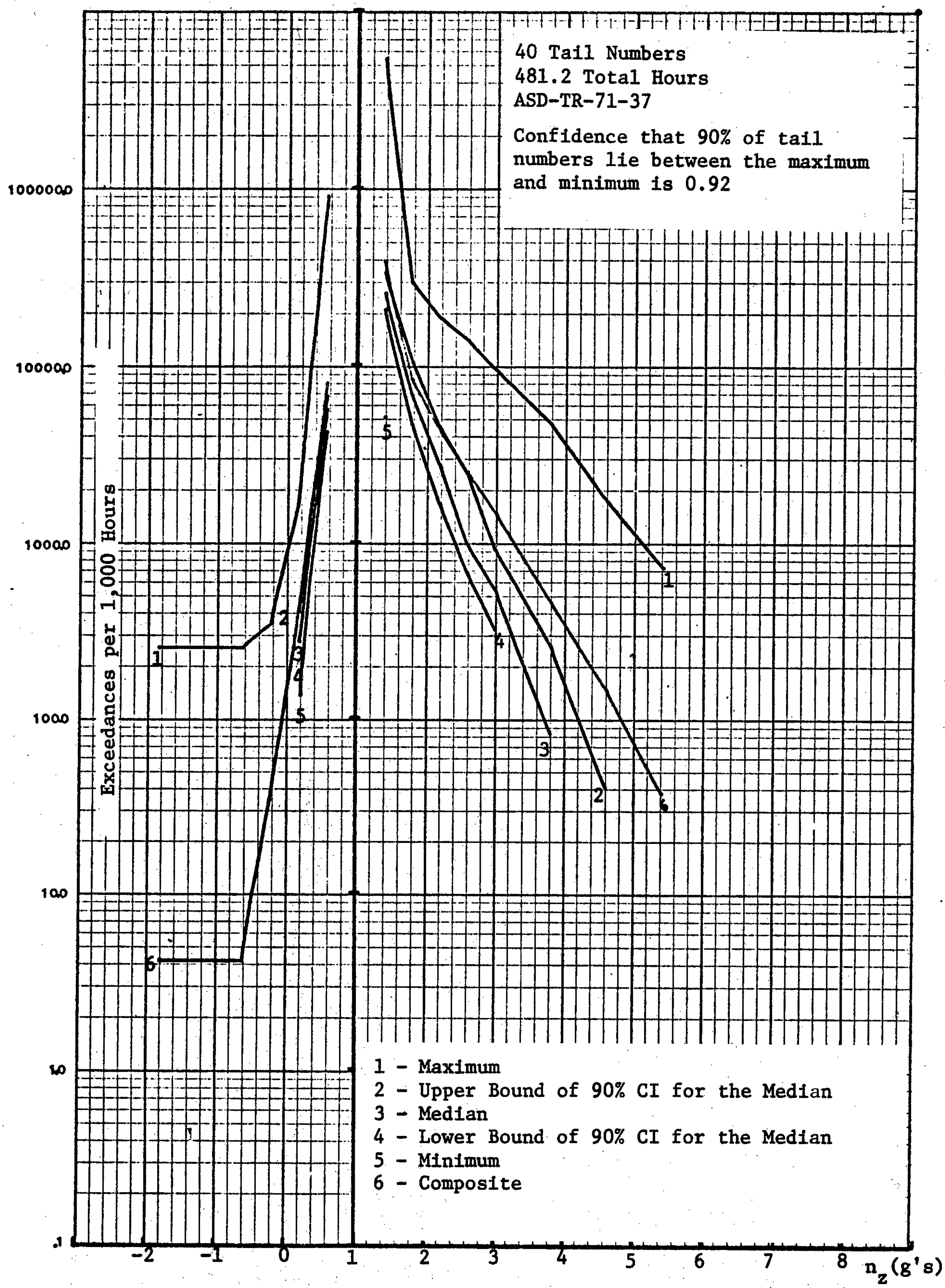

FIGURE 16. Nonparametric Bounds for F-4 Inst. & Nav. Exceedances

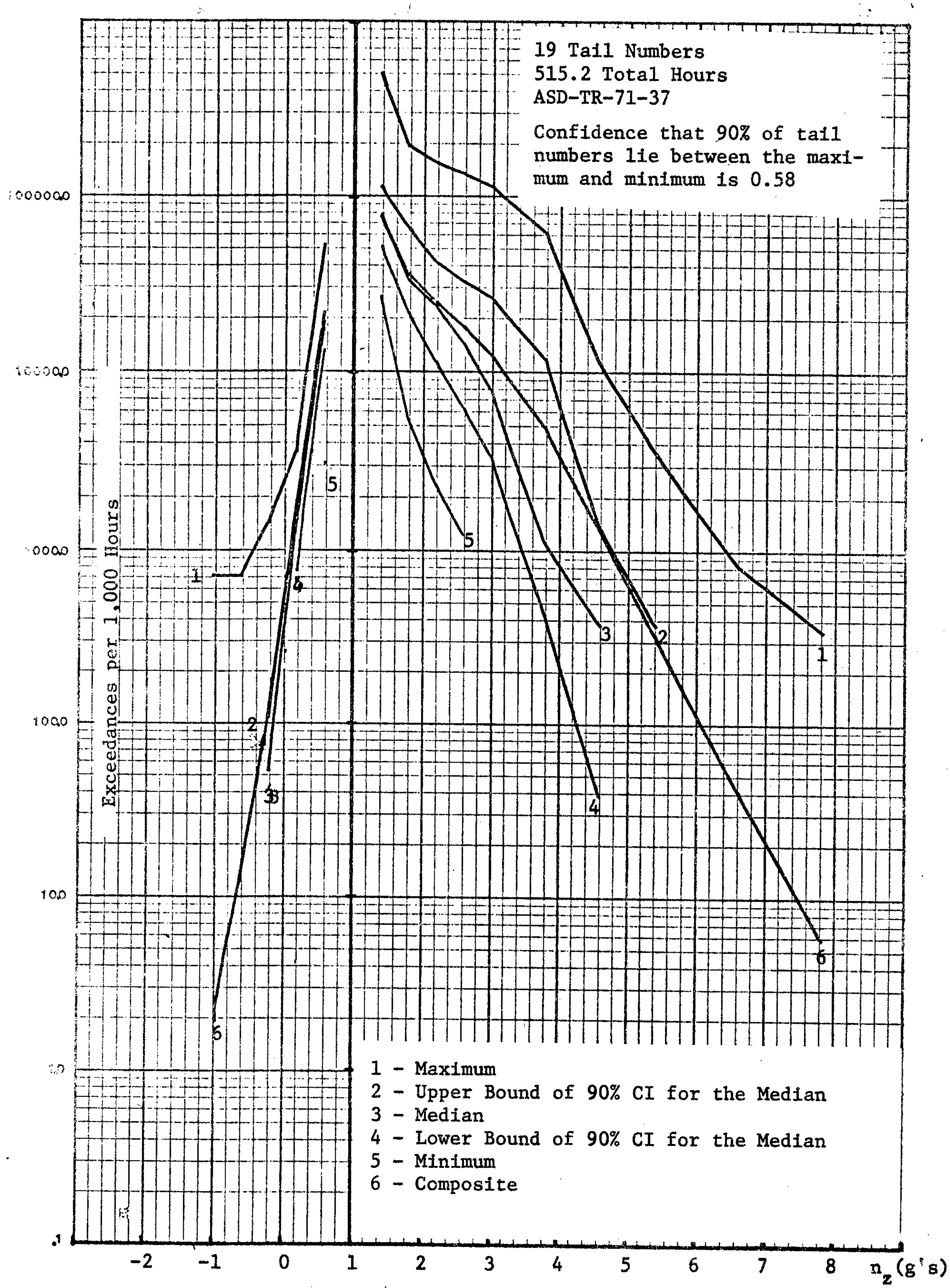

FIGURE 17. Nonparametric Bounds for F-4 Reconnaissance Exceedances

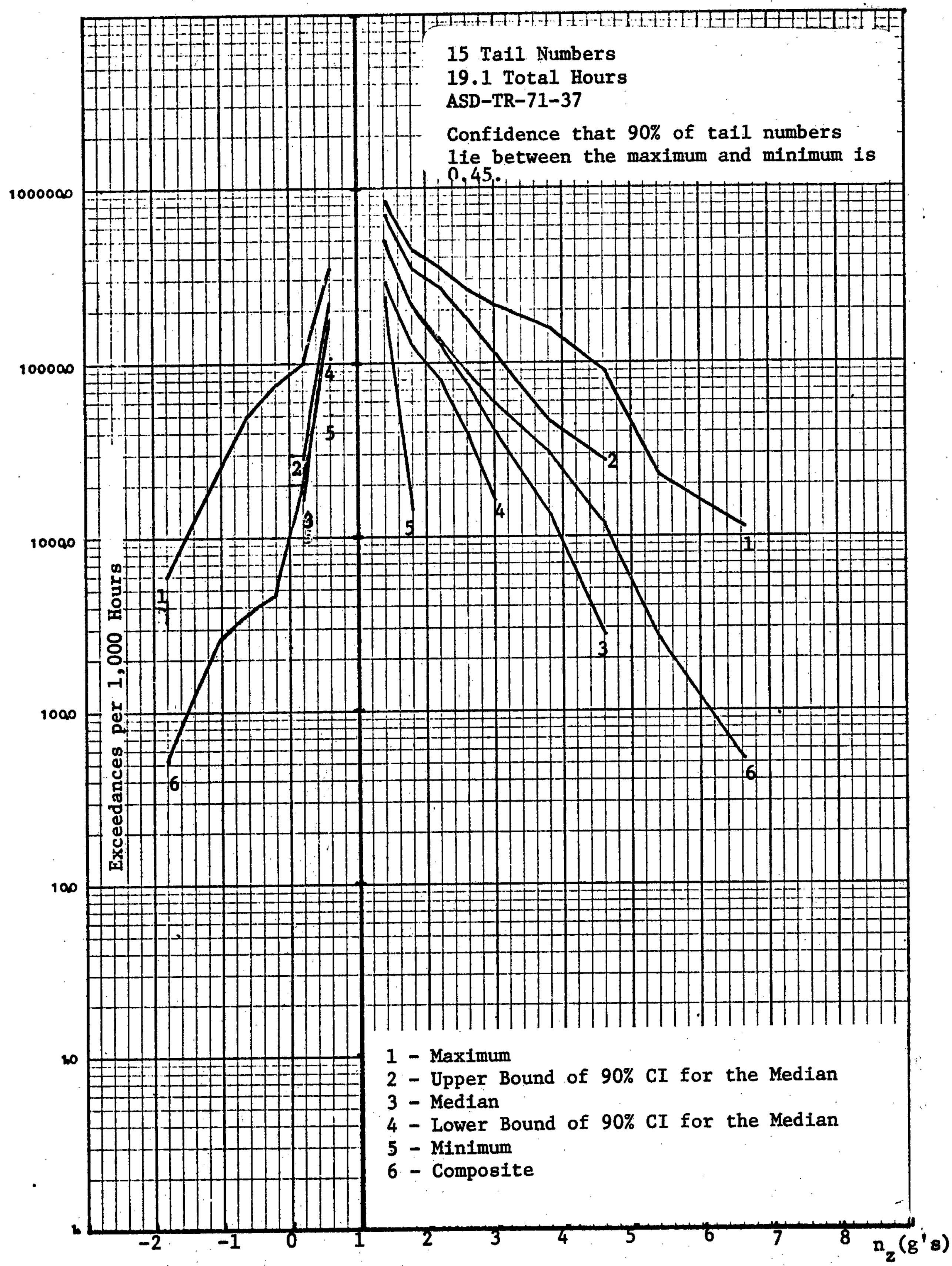

FIGURE 18. Nonparametric Bounds for F-4 Test Exceedances

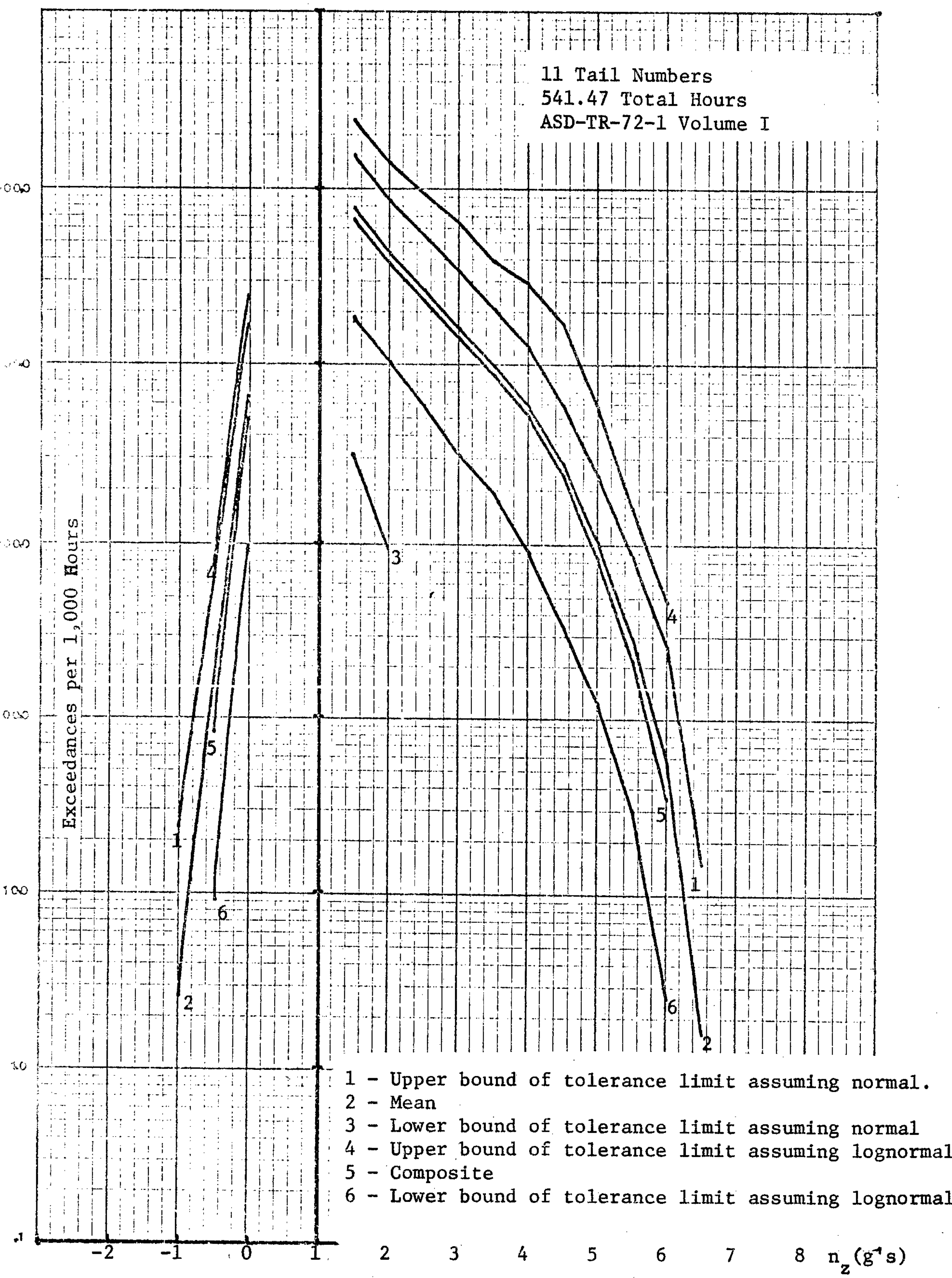

FIGURE 19. 90% Tolerance Limits with 90% Confidence for A-37B at England AFB (1969).

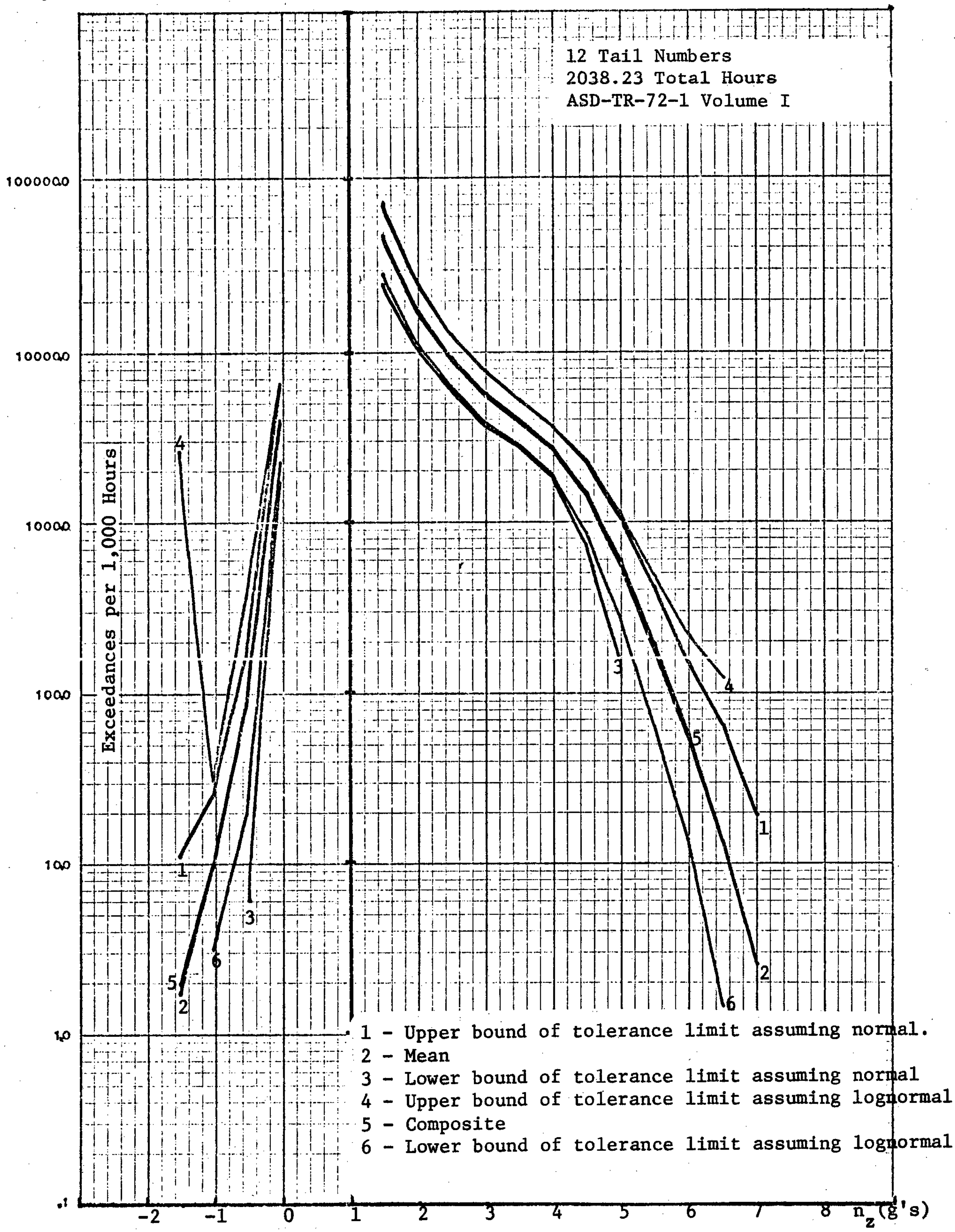

FIGURE 20. 90% Tolerance Limits with 90% Confidence for A-37B at Bien Hoa AB (1970)

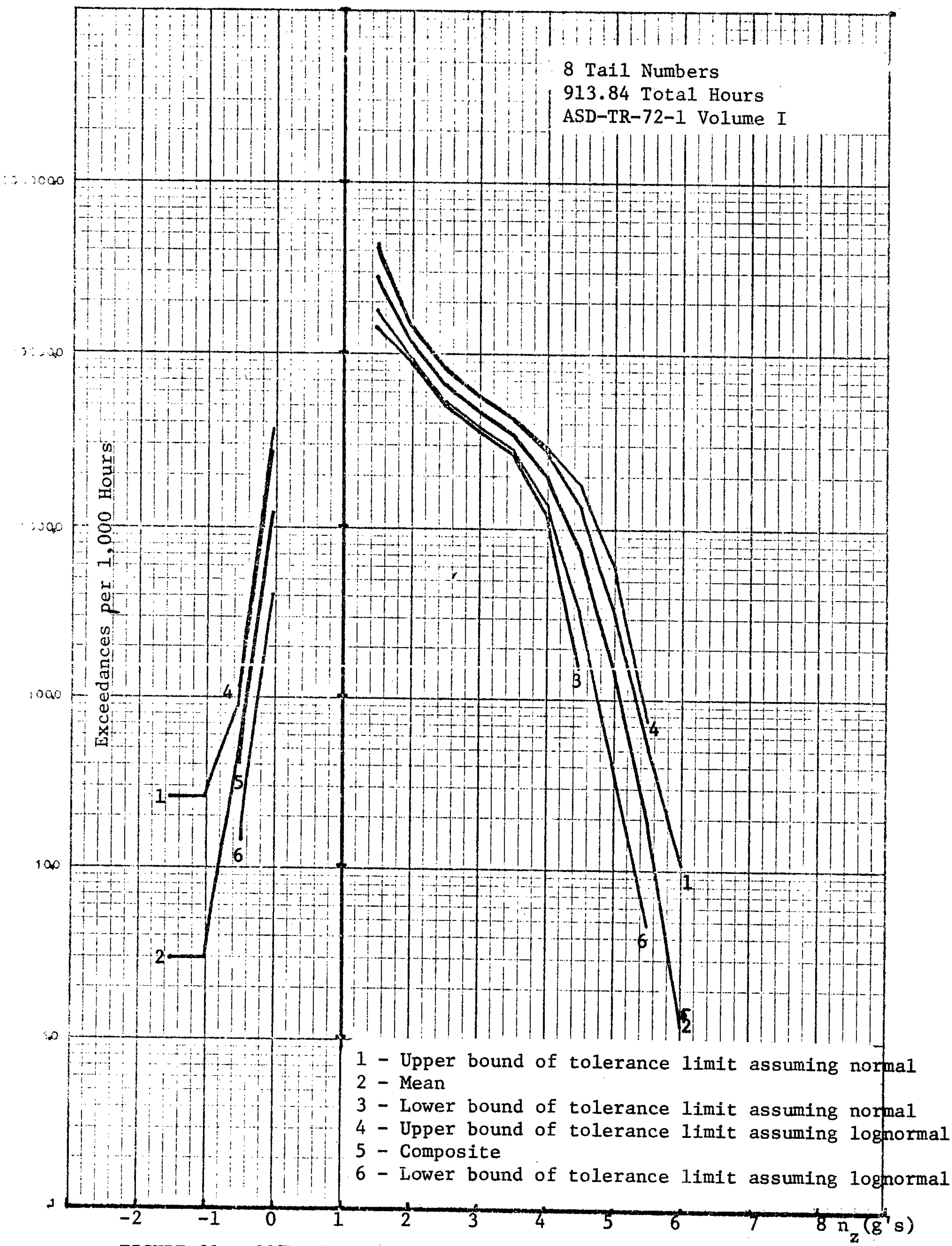

FIGURE 21. 90% Tolerance Limits with 90% Confidence for A-37B at Binh Thuy AB (1971).

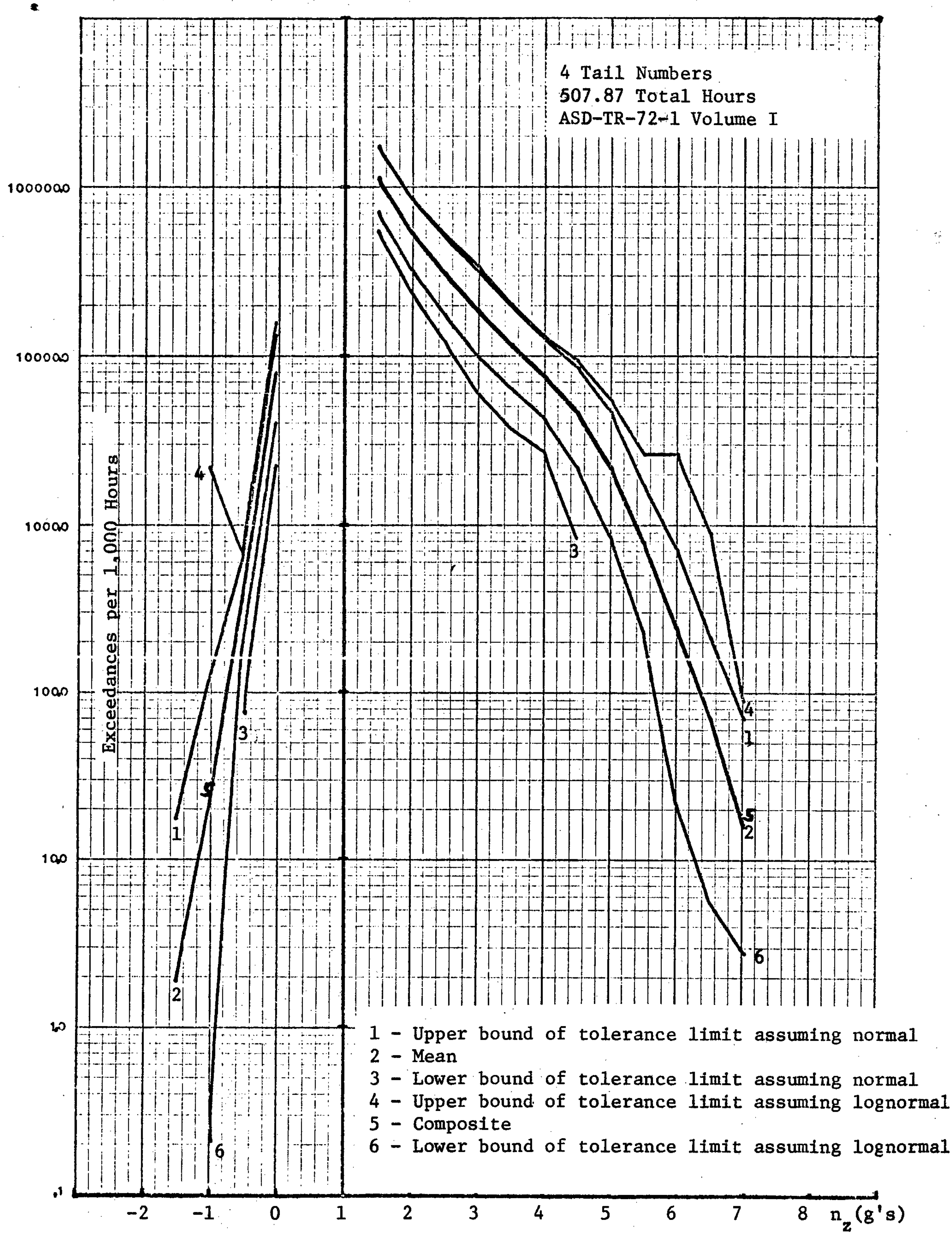

FIGURE 22. 90% Tolerance Limits with 90% Confidence for A-37B at England AFB (1971).

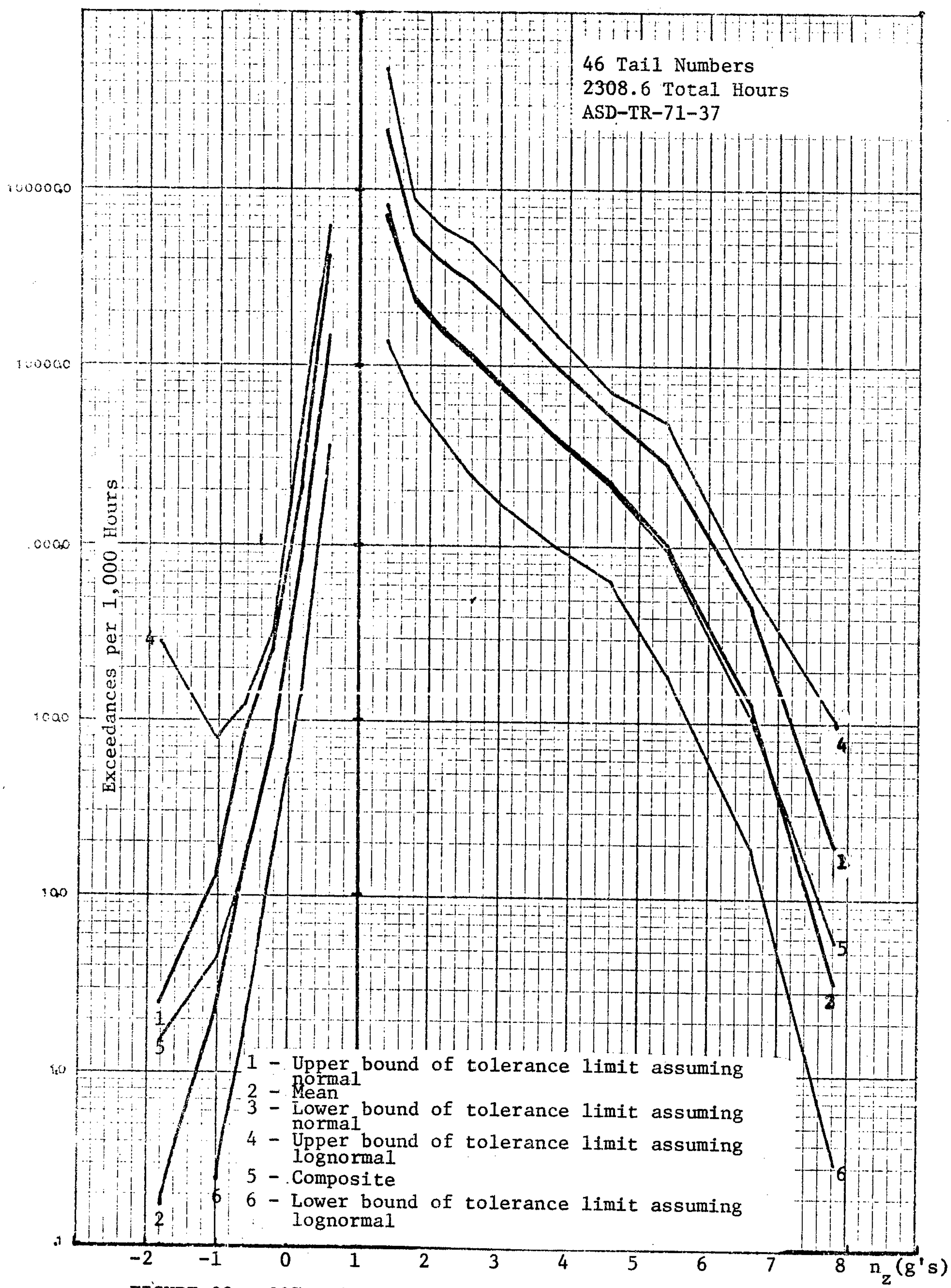

FIGURE 23. 90% Tolerance Limits with 90% Confidence for F-4 Air-Ground.

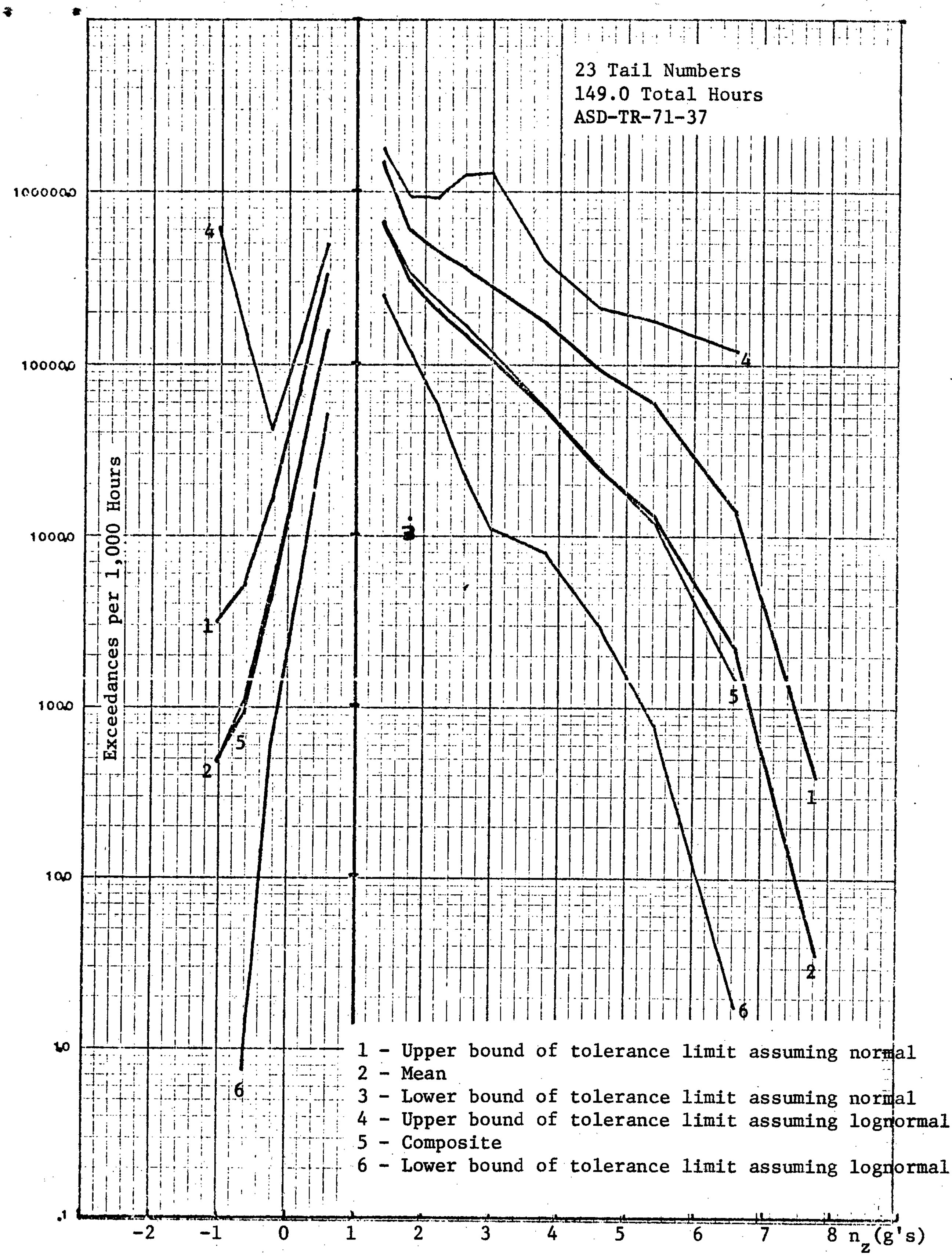

FIGURE 24. 90% Tolerance Limits with 90% Confidence for F-4 Air-Air .

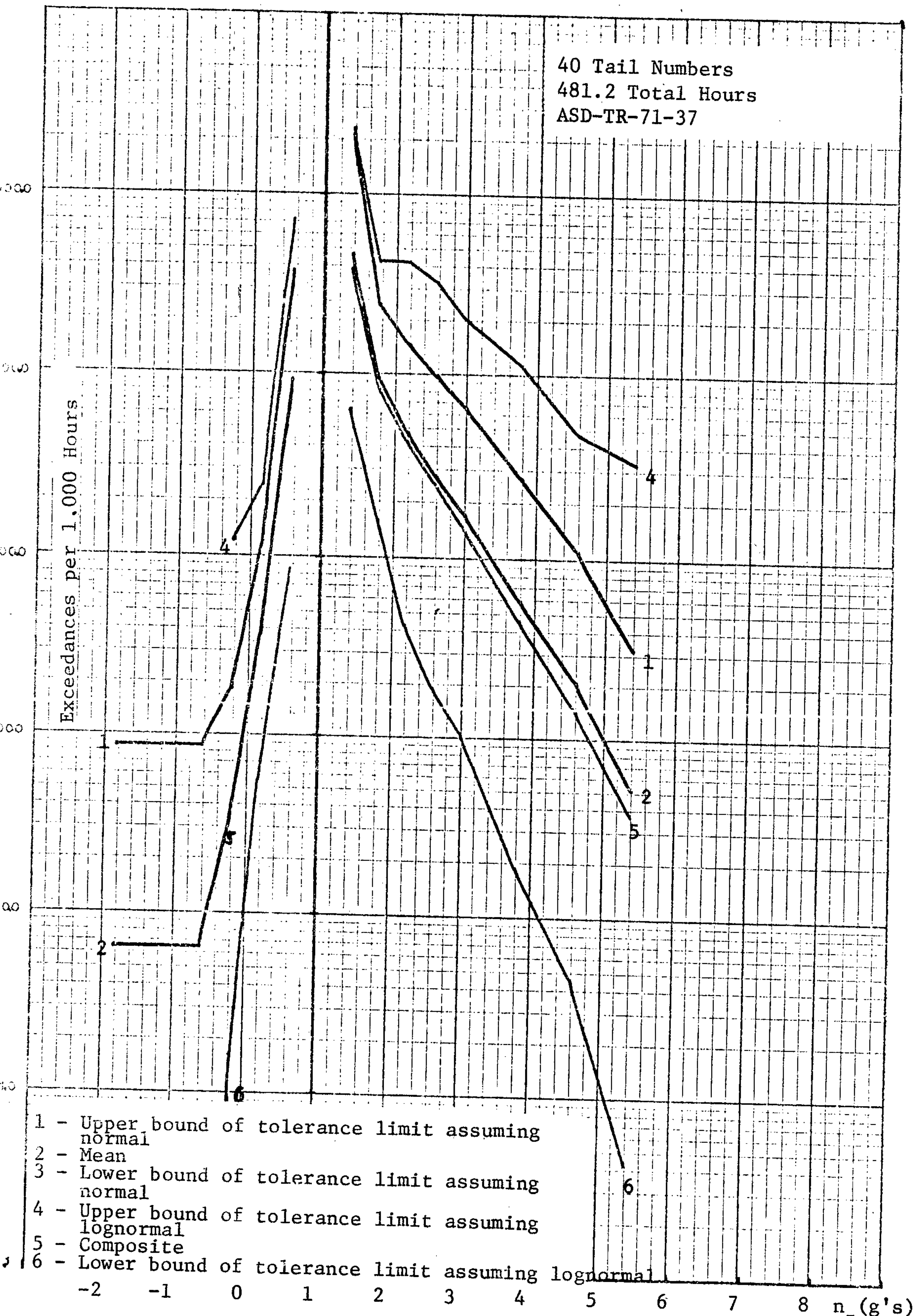

FIGURE 25. 90% Tolerance Limits with 90% Confidence for F-4 Inst. and Nav.

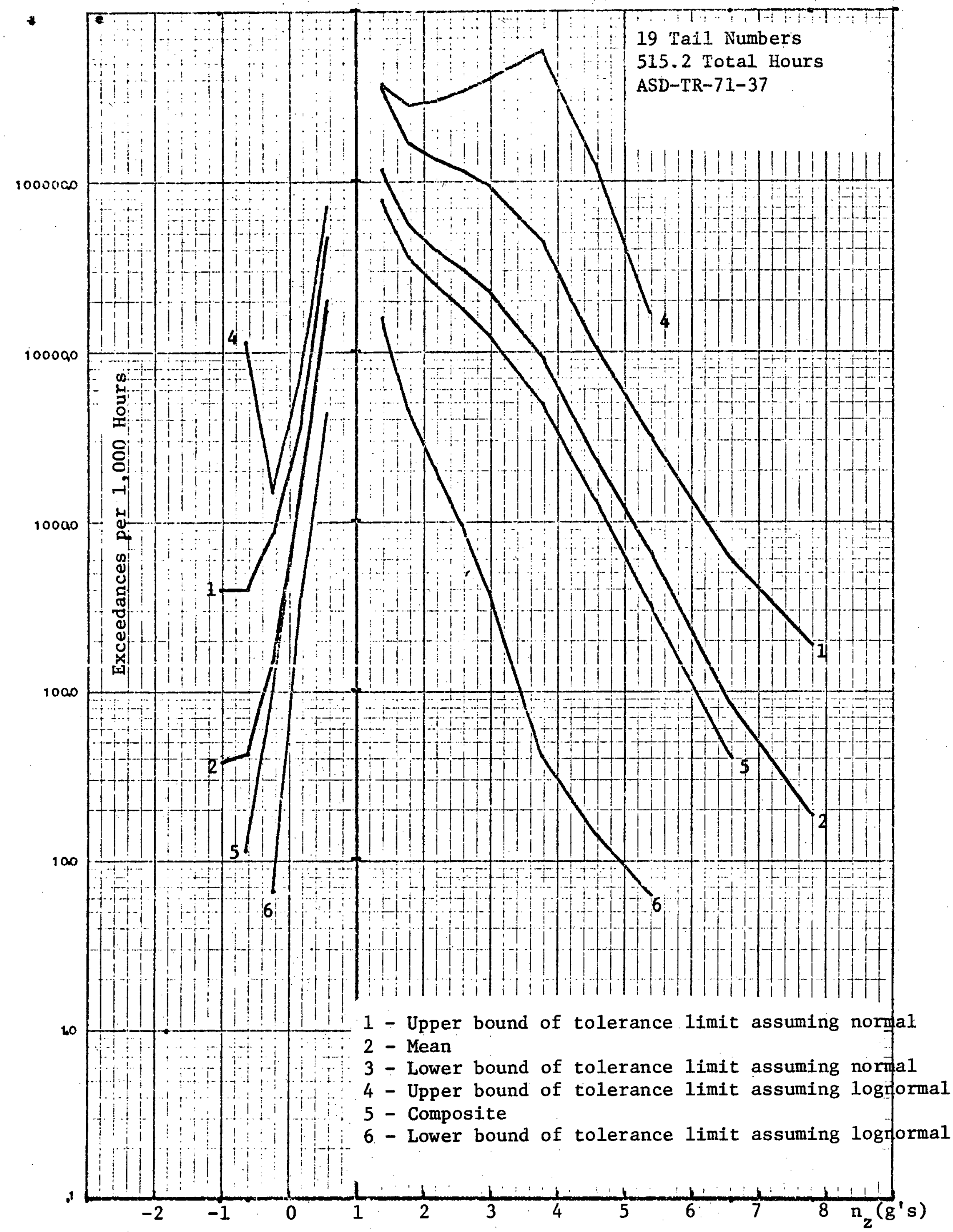

FIGURE 26. 90% Tolerance Limits with 90% Confidence for F-4 Reconnaissance.

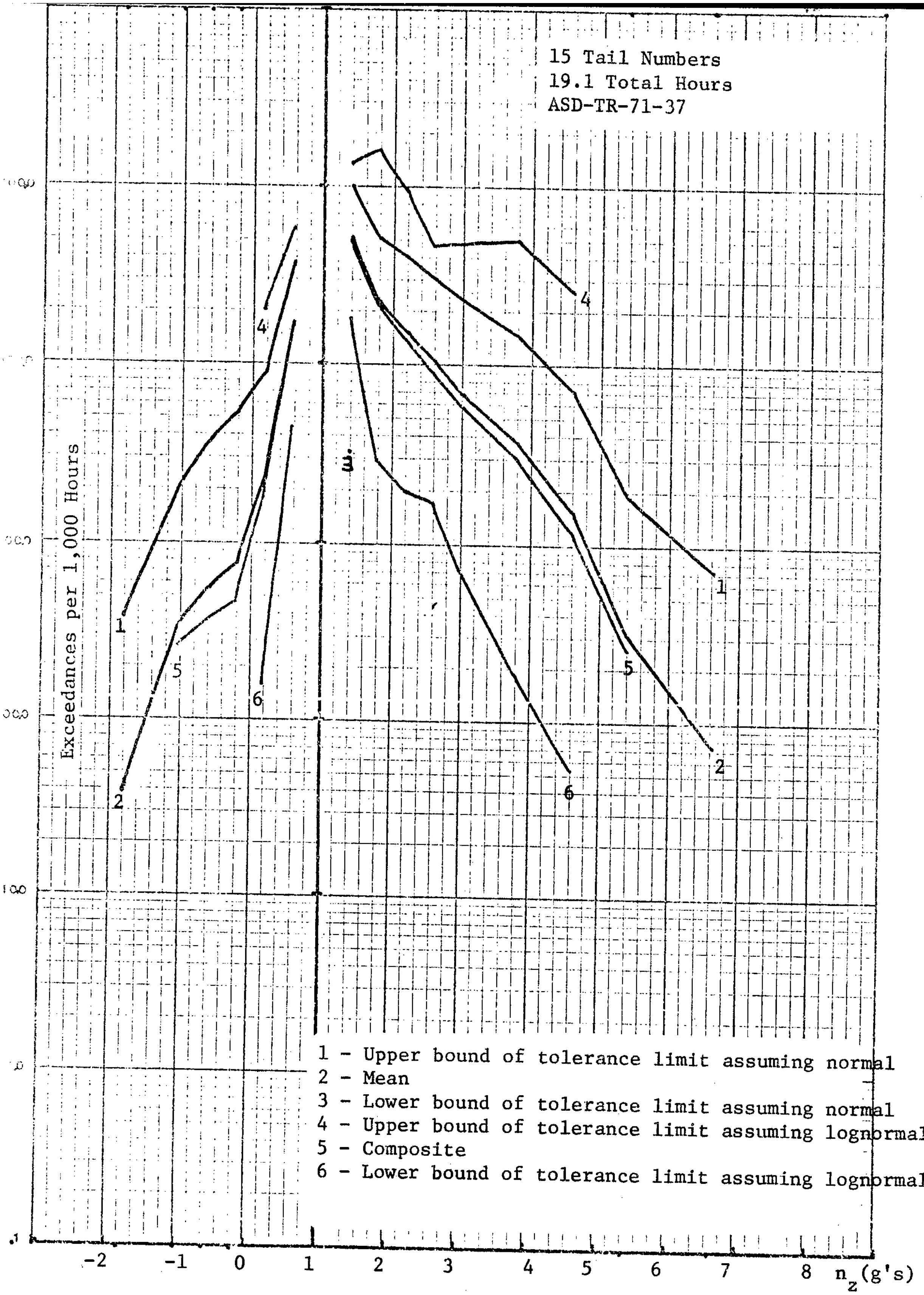

FIGURE 27. 90% Tolerance Limits with 90% Confidence for F-4 Test.

TABLE III. CURVE FIT COEFFICIENTS

$F(X)=10^{(a_3X^3+a_2X^2+a_1X+a_o)}$ where $x=n_z$ and F(X)= Exceedances/4,000 Hours

A. Coefficients for n_z >1g

Figure	a_3	a_2	a_1	a_o	R-Square	Hours
28	-0.0592	.5281	-1.9186	7.3527	.99908	541.47
29	-0.0305	.2907	-1.3611	6.6859	.99796	2038.23
30	-0.0954	.8393	-2.7681	7.6166	.99956	913.84
31	-0.0254	.2390	-1.1635	6.9181	.99913	507.87
32	-0.0249	.2914	-1.4829	6.9742	.99681	2308.6
33	-0.0177	.1833	-1.0033	6.4857	.99945	149.0
34	-0.0525	.5726	-2.6019	7.7763	.99583	401.2
35			-.6364	6.4918	.98916	515.2
36	-0.0117	.1180	-.8723	6.3584	.99442	19.1

B. Coefficients for n_z < 1g

Figure	a_3	a_2	a_1	a_o	R-Square	Hours
28			3.4216	4.2709	.99962	541.47
29	.6777	2.4887	4.4009	4.1837	1	2038.23
30			1.7549	3.3475	.87189	913.84
31			2.4042	4.4226	.99650	507.87
32	.2336	.9398	2.4319	2.9379	.99968	2308.6
33			1.6055	3.7242	.98488	149.0
34			2.7188	2.7525	.99223	401.2
35			2.5057	3.2584	.99435	515.2
36	.5177	1.2371	1.2208	3.5295	.99835	19.1

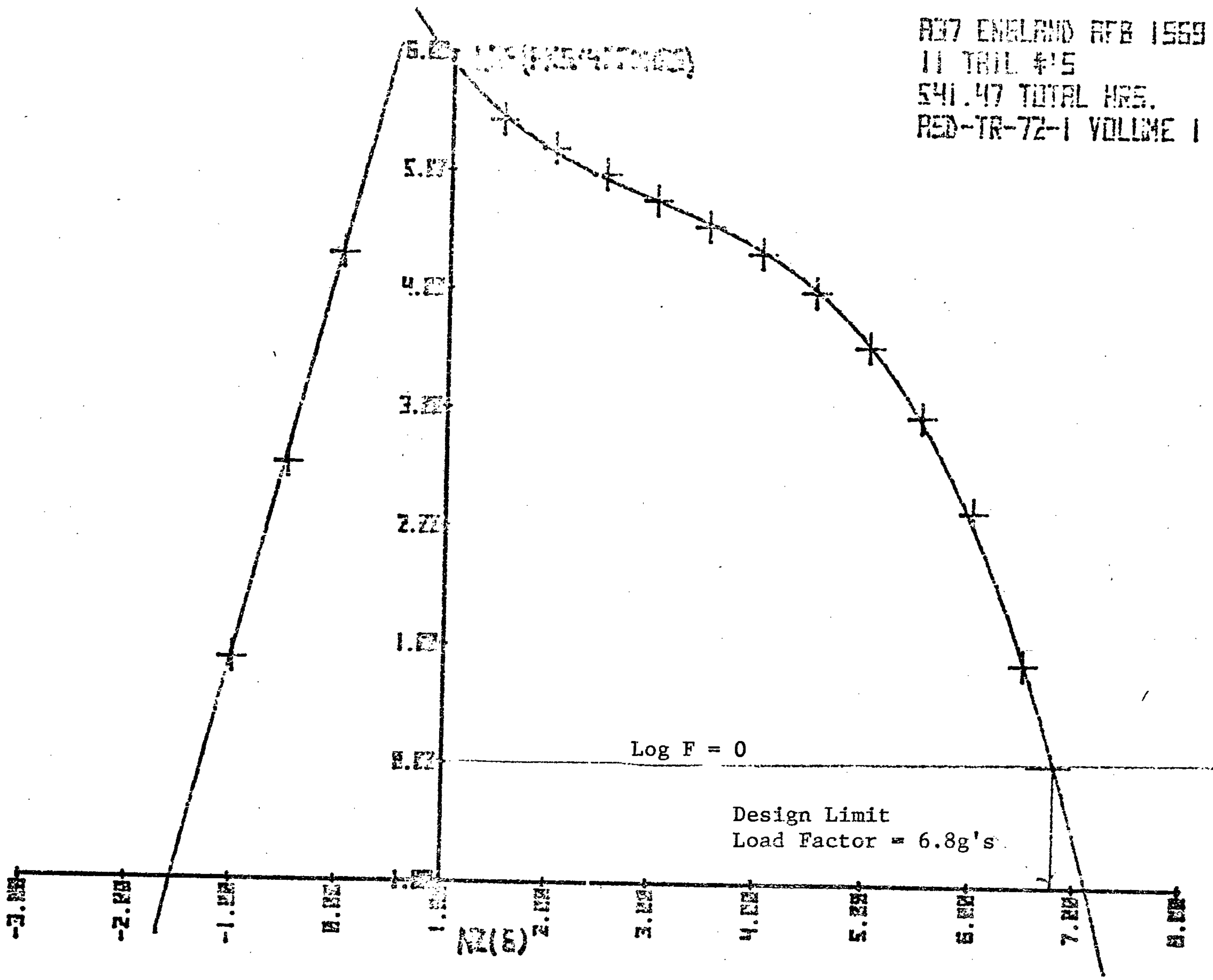

FIGURE 28. Composite Data Curve Fit for A-37B at England AFB (1969)

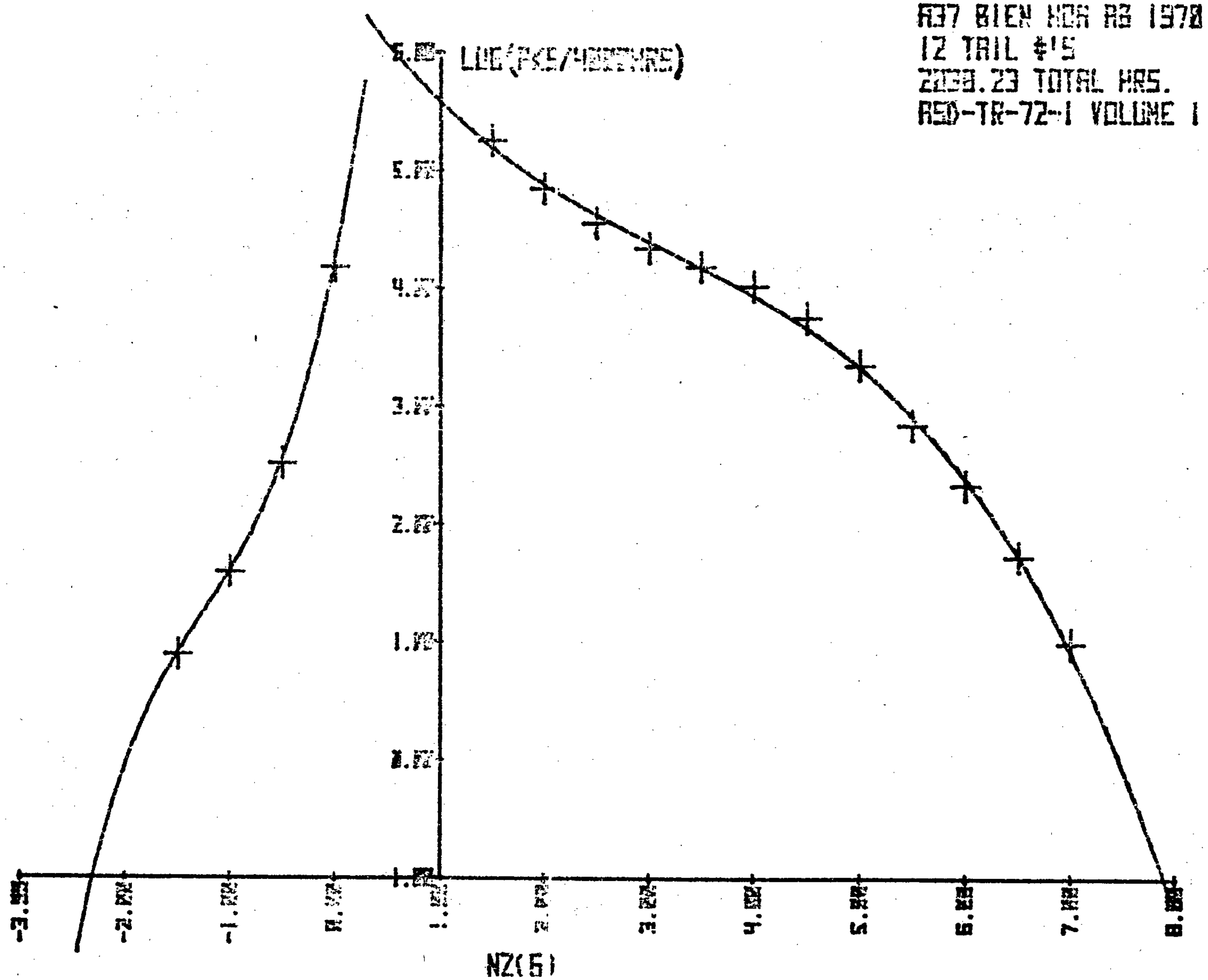

FIGURE 29. Composite Data Curve Fit for A-37B at Bien Hoa AB (1970)

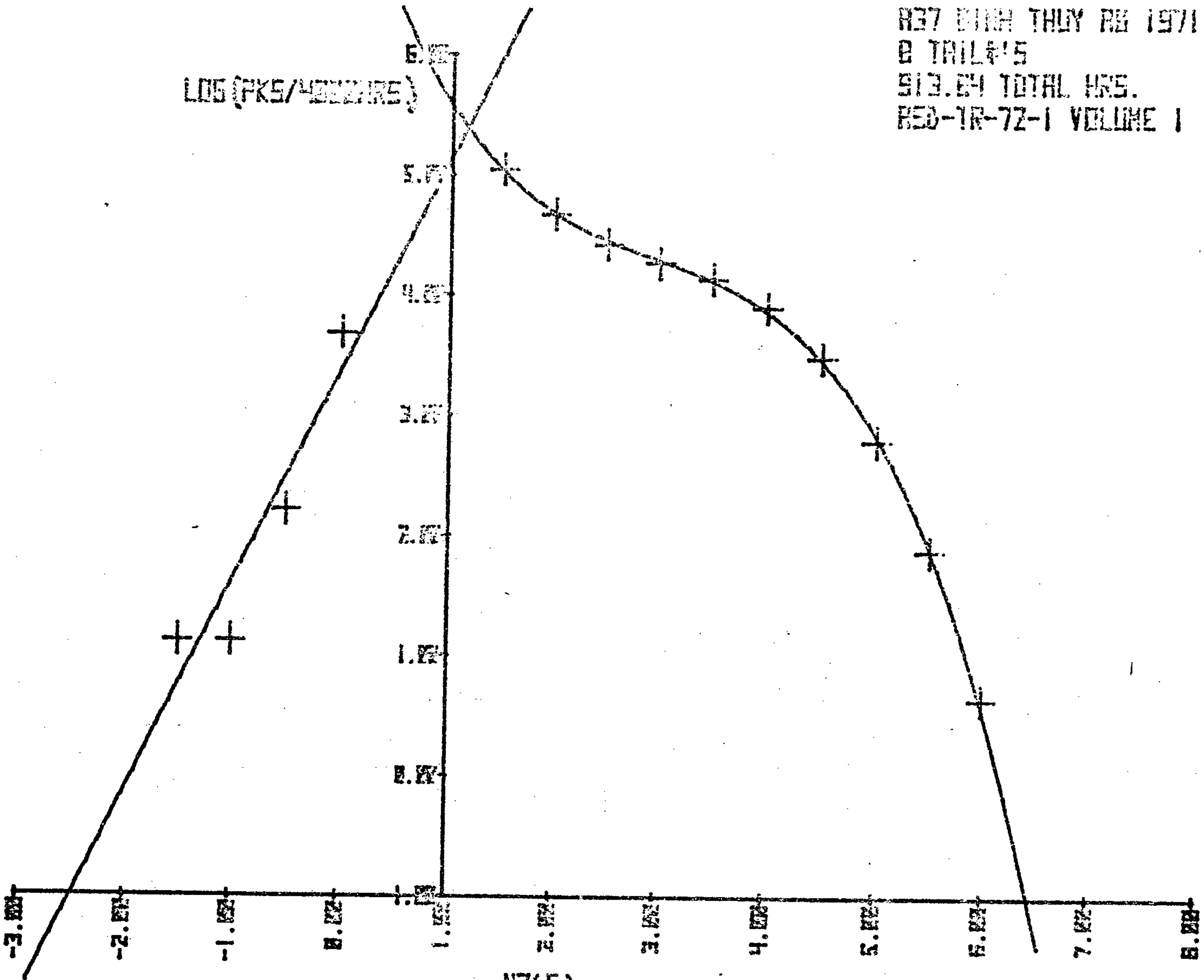

FIGURE 30. Composite Data Curve Fit for A-37B at Binh Thuy AB (1971)

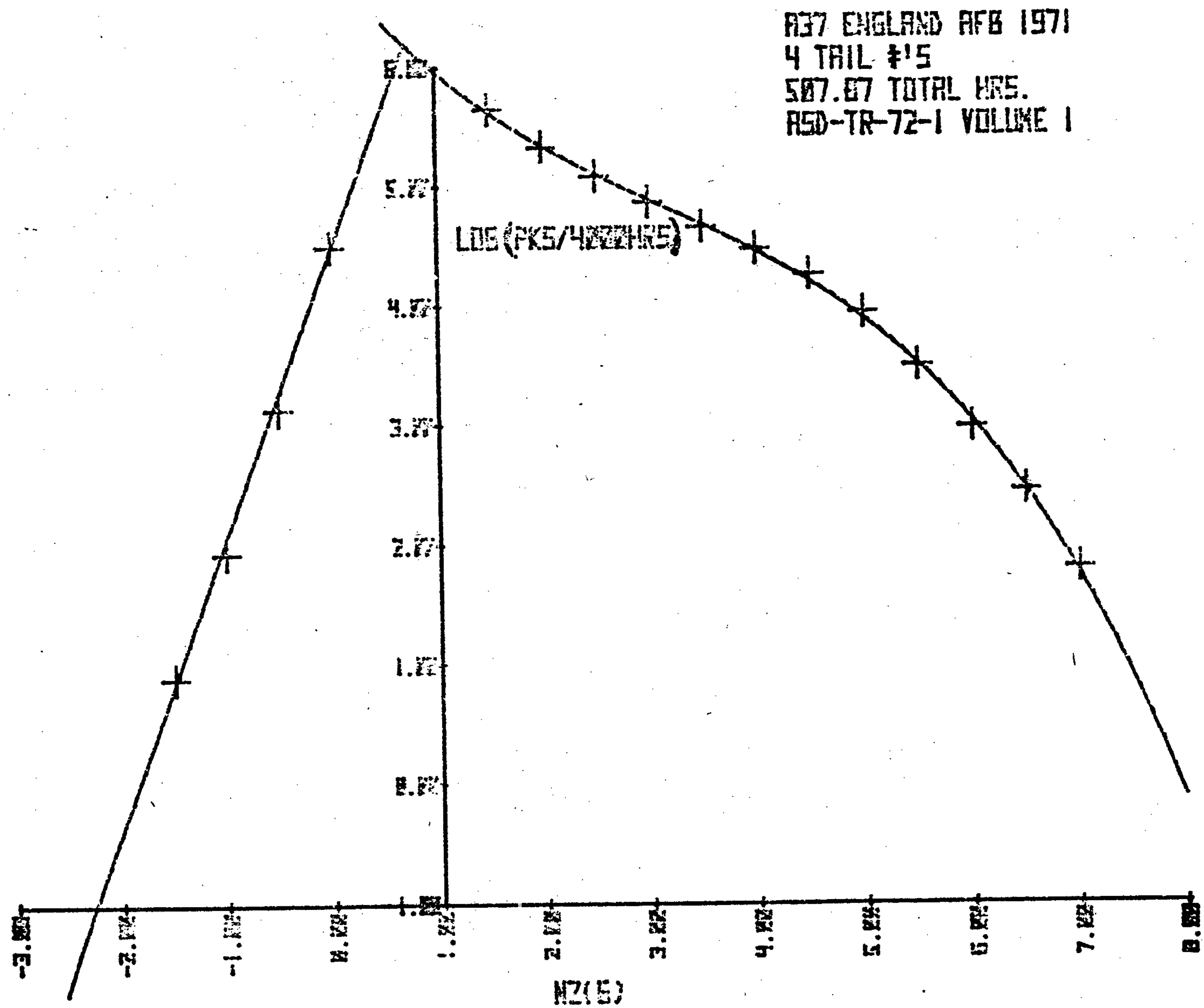

FIGURE 31. Composite Data Curve Fit for A-37B at England AFB (1971)

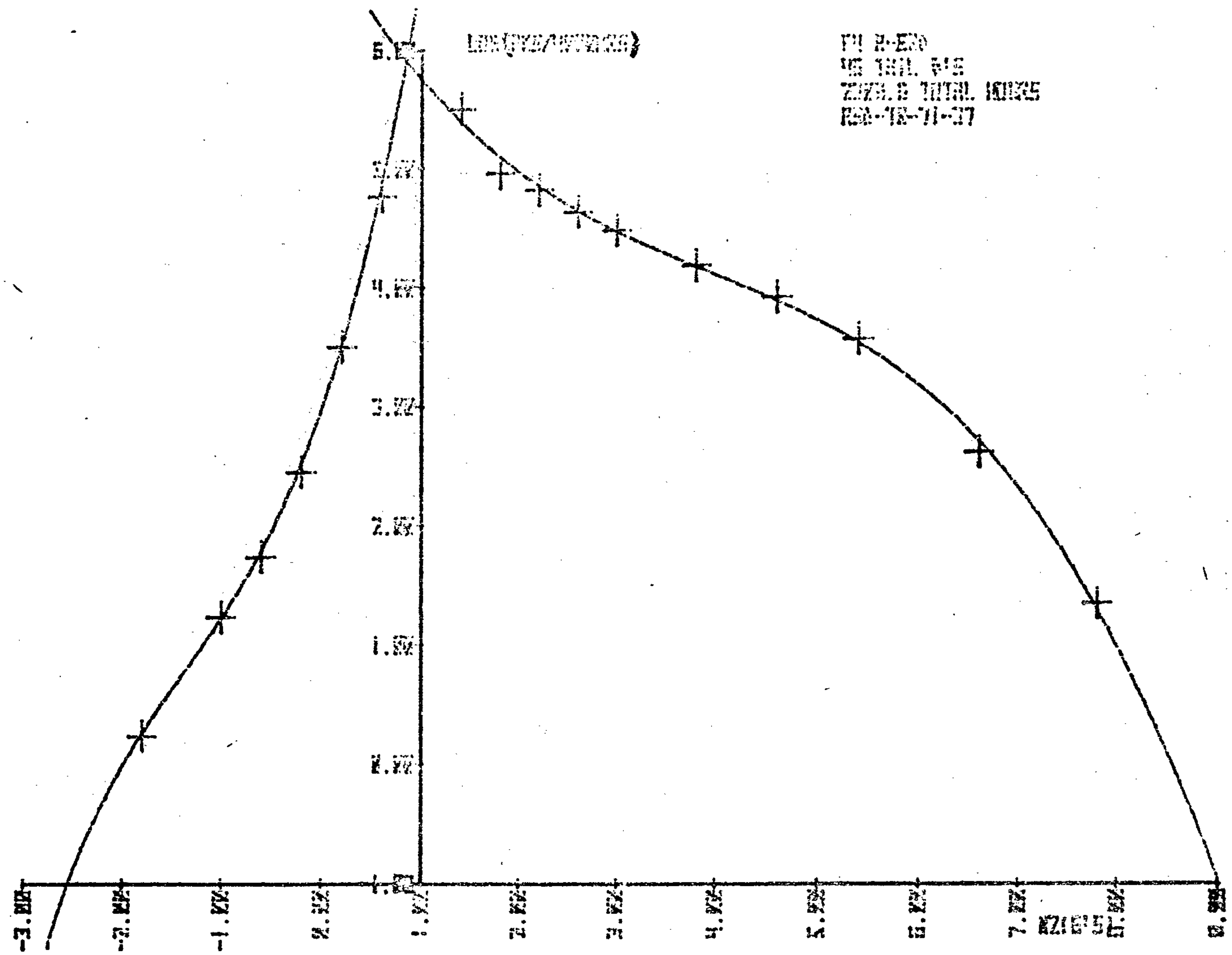

FIGURE 32. Composite Data Curve Fit for F-4 Air-Ground

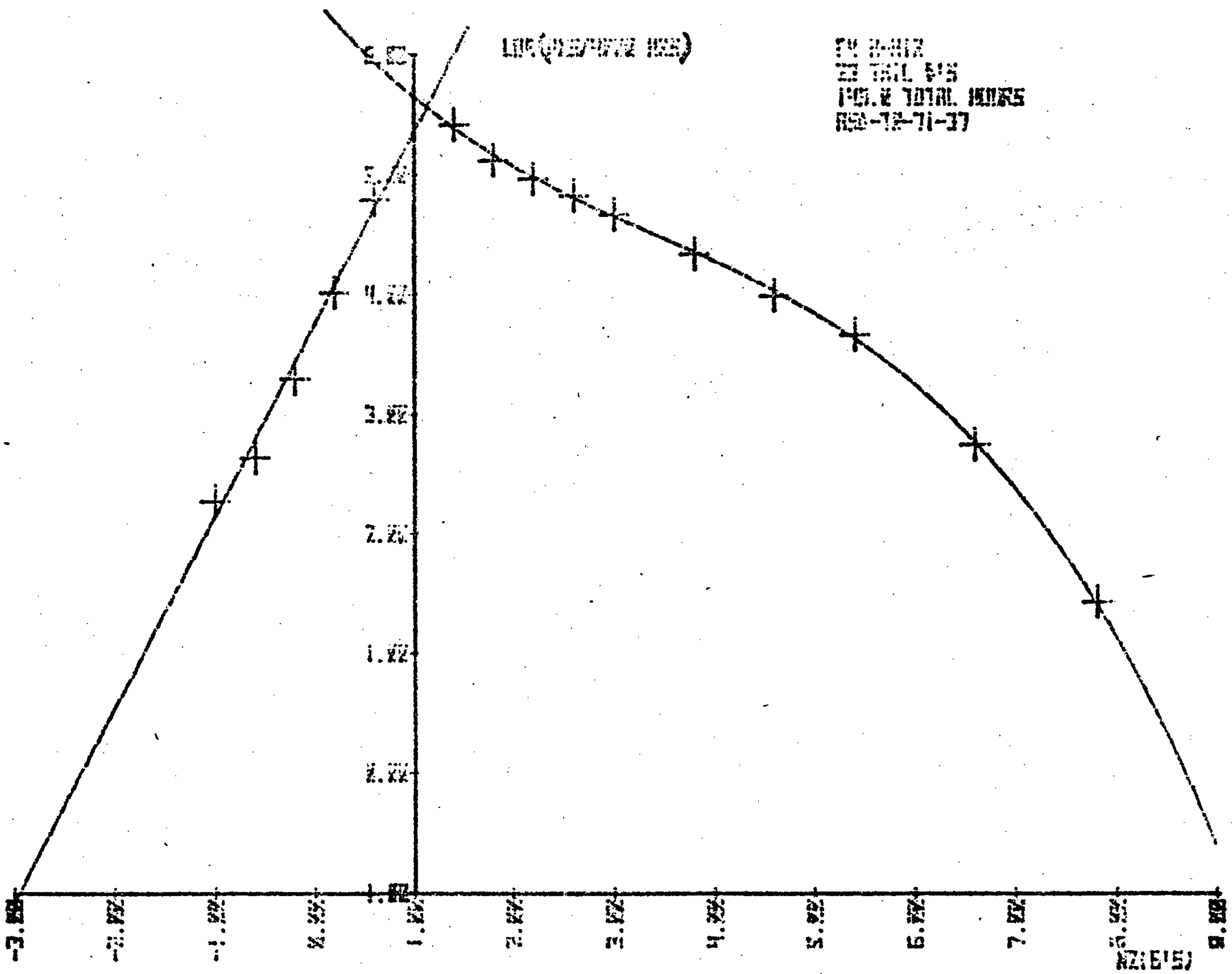

FIGURE 33. Composite Data Curve Fit for F-4 Air-Air

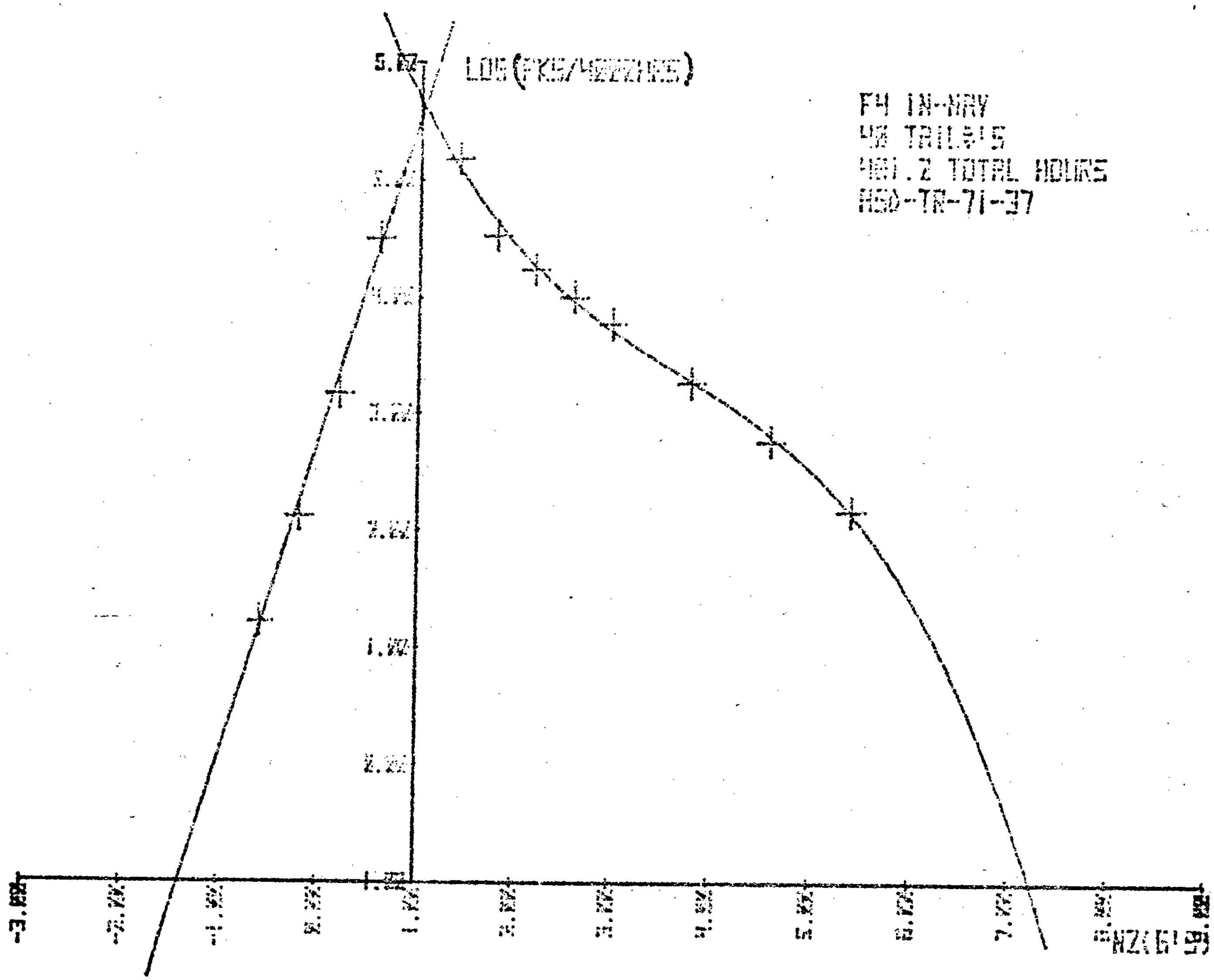

FIGURE 34. Composite Data Curve Fit For F-4 Inst. and Nav.

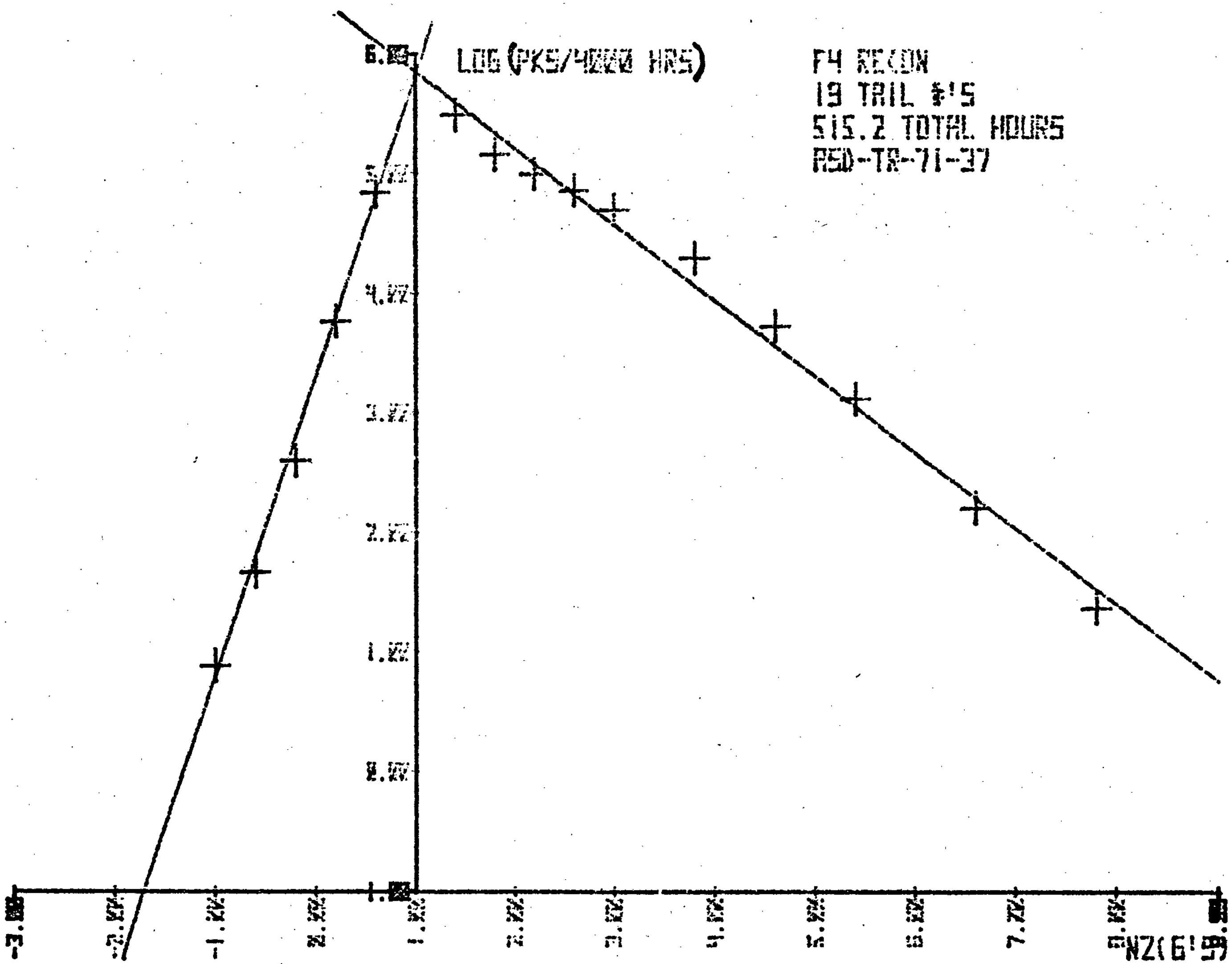

FIGURE 35. Composite Data Curve Fit For F-4 Reconnaissance

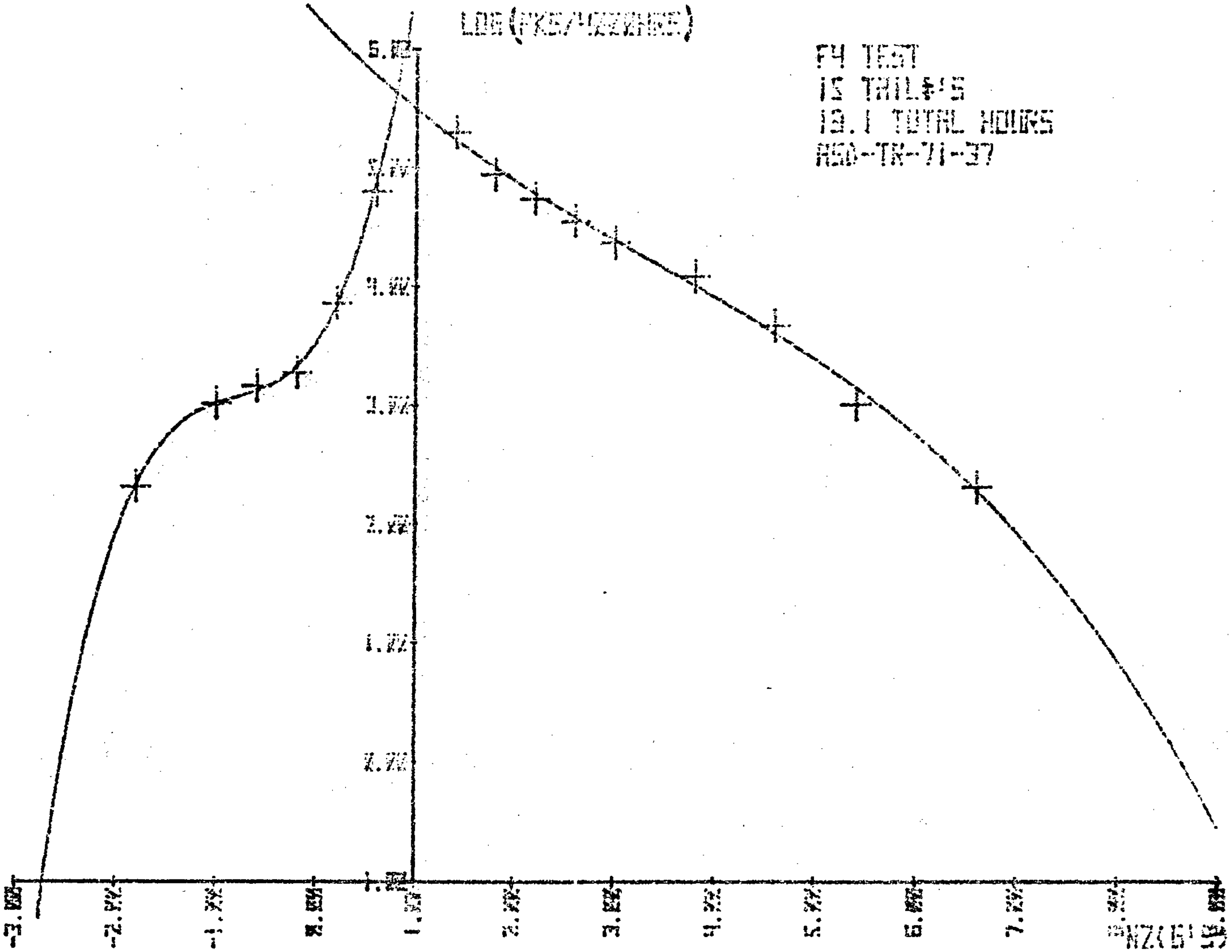

FIGURE 36. Composite Data Curve Fit For F-4 Test

APPENDIX II

COMPUTER PROGRAMS USED

FIGURE 37. Program to Store Data on Tape Cassette

```
10 COM AS[50],XS[50,16],TS[50],YS[16]
11 FOR J=1 TO 50
12 FOR K=1 TO 16
13 A[J]=0
14.X[J,K]=0
15 T[J]=0
16 Y[K]=0
17 NEXT K
18 NEXT J
20 DISP "ROWL=,COLUMNM=";
30 INPUT L,M
40 FOR I=1 TO L
50 DISP "A("I")=?";
60 INPUT A[I]
70 FOR J=1 TO M
80 DISP "X("I,",","J")=?";
90 INPUT X[I,J]
100 NEXT J
110 DISP "T("I")=?";
120 INPUT T[I]
130 NEXT I
140 FOR J=1 TO M
150 DISP "Y("J")=";
160 INPUT Y[J]
170 NEXT J
180 FOR J=1 TO L
190 PRINT
200 PRINT A[J];
210 FOR K=1 TO 8
220 PRINT X[J,K];
230 NEXT K
235 PRINT T[J];
240 NEXT J
245 PRINT
250 FOR I=1 TO 8
260 PRINT Y[I];
270 NEXT I
273 PRINT
277 PRINT
280 FOR J=1 TO L
285 FOR K=9 TO 16
```

FIGURE 37. Continued

```
290 PRINT X[J,K];
300 NEXT K
310 NEXT J
315 PRINT
320 FOR I=9 TO 16
330 PRINT Y[I];
340 NEXT I
345 PRINT
350 DISP "ENTER 1 IF DATA IS GOOD";
360 INPUT T1
370 IF T1=1 THEN 440
380 DISP "DATA BAD IN ROW#,COLUMN#";
390 INPUT L1,M1
400 DISP "A("L1,"X("L1","M1"),T("L1,")Y("M1")";
410 INPUT A[L1];X[L1,M1];T[L1];Y[M1]
420 PRINT A[L1];X[L1,M1];T[L1];Y[M1]
430 GOTO 350
440 DISP "FILE NO.=";
450 INPUT Z
460 STORE DATA Z
470 STOP
480 END
```

FIGURE 38. Program to Read Data on Tape Cassette

```
10 COM A$[50],X$[50,16],T$[50],Y$[16]
20 DISP "FILE NO.,#ROWS,#COLUMNS";
30 INPUT T,L,M
40 LOAD DATA T
50 DISP "COLUMN TO PRINT";
60 INPUT M1
70 PRINT Y[M1]
80 FOR J=1 TO L
90 PRINT A[J];X[J,M1];T[J]
100 NEXT J
110 GOTO 50
120 END
```

FIGURE 39. Program to Analyze the Data by Tail Number

```
[illegible]
[illegible]
[illegible]
[illegible]
[illegible] X2,Y2
SCALE -1,,3
AXIS 1,Y%
DISP "FILENO.,ROWS,COLUMNS=";
INPUT T,L,M
LOAD DATA T
N=L-1
FOR K=1 TO M
FOR R1=1 TO 5
U[R1,K]=-100
NEXT R1
FOR J=1 TO L
170 X[J,K]=1000*X[J,K]
180 NEXT J
190 NEXT K
DISP "NEG MAX #";
INPUT K1
FOR J=1 TO L
X[J,1]=X[J,1]/T[J]
FOR K=2 TO K1
X[J,K]=X[J,K]/T[J]
X[J,K]=X[J,K]+X[J,K-1]
NEXT K
X[J,M]=X[J,M]/T[J]
FOR K2=1 TO M-K1-1
K=M-K2
X[J,K]=X[J,K]/T[J]
X[J,K]=X[J,K]+X[J,K+1]
NEXT K2
NEXT J
DISP "ENTER1 TO PLOT ALL EXCEEDANCES";
INPUT T1
IF T1#1 THEN 430
FOR J=1 TO N
S=J
GOSUB 730
NEXT J
PEN
430 DISP "ENTER 1 TO PLOT ALL EXCEEDANCE POINTS";
440 INPUT T1
450 IF T1#1 THEN 540
460 FOR J=1 TO N
470 FOR K=1 TO M
PEN
IF X[J,K]=0 THEN 510
PLOT Y[K],LGT(X[J,K]),
NEXT K
NEXT J
PEN
```

FIGURE 39. Continued

```
540 DISP "ENTER 1 TO PRINT A COLUMN";
550 INPUT T1
560 IF T1#1 THEN 640
570 DISP "COLUMN TO PRINT";
580 INPUT M1
590 PRINT Y[M1]
600 FOR J=1 TO L
610 PRINT X[J,M1]
620 NEXT J
630 GOTO 540
640 DISP "ENTER 1 TO PLOT COMPOSITE";
650 INPUT T1
660 IF T1#1 THEN 800
670 REM PLOT COMPOSITE
680 R1=L
690 GOSUB 730
700 PEN
710 GOTO 800
720 STOP
730 FOR K=1 TO M
740 IF X[R1,K]=0 THEN 770
750 PLOT Y[K],LGT(X[R1,K])
760 IF K#K1 THEN 780
770 PEN
780 NEXT K
790 RETURN
800 DISP "ENTER 1 TO FIND CON.INT.";
810 INPUT T1
820 IF T1#1 THEN 1350
830 REM THE ELEMENTS OF EACH COLUMN ARE RANKED
840 IF N>25 THEN 880
850 DISP "INPUT B(A/2,1/2;N) FOR N="N;
860 INPUT B
870 GOTO 900
880 DISP "Z(A/2)=";
890 INPUT Z
900 FOR K=1 TO M
910 FOR J=1 TO N
920 R[J]=X[J,K]
930 NEXT J
940 FOR J=1 TO N-1
950 FOR J1=J+1 TO N
960 IF R[J]>R[J1] THEN 980
970 GOTO 1010
980 T5=R[J]
990 R[J]=R[J1]
1000 R[J1]=T5
1010 NEXT J1
1020 NEXT J
1030 C[1,K]=R[1]
1040 C[5,K]=R[N]
1050 [illegible]
1060 C5=INT(C[illegible])
1070 IF C5=C9 THEN 1110
1080 C8=C5+1
1090 C[3,K]=R[C8]
1100 GOTO 1120
1110 C[3,K]=(R[illegible]+R[illegible])/2
```

FIGURE 39. Continued

```
[illegible] IF N>25 THEN 1100
[illegible] [illegible]
[illegible] [illegible]
[illegible] GOTO 1190
[illegible] [illegible]/2-2*(N/4)↑(1/2)
[illegible] [illegible]INT(C2)
[illegible] [illegible]=N+1-C2
[illegible] [illegible][2,K]=R[C2]
[illegible] [illegible][4,K]=R[C4]
[illegible] NEXT K
[illegible] DISP "ENTER 1 TO PLOT CONF.INT.";
[illegible] INPUT T1
[illegible] IF T1#1 THEN 1300
[illegible] [illegible]2=0
[illegible] FOR R1=1 TO 5
[illegible] GOSUB 2400
[illegible] NEXT R1
[illegible] PEN
[illegible] DISP "ENTER 1 TO PRINT C'S";
1310 INPUT T1
1320 IF T1#1 THEN 1390
1330 FOR K=1 TO N
1340 GOSUB 2530
1350 FOR R1=1 TO 5
1360 GOSUB 2560
1370 NEXT R1
1380 NEXT K
1390 REM M2-MEAN,V2-VARIANCE,S2-ST.DEV.
1400 REM L1=LOWER TOL LIM,L2=UPPER TOL LIM
1410 DISP "ENTER 1 TO FIND M2,V2,S2;ASSUMING NORMAL DISTRIBUTION";
1420 INPUT T1
1430 IF T1#1 THEN 1610
1440 FOR K=1 TO N
1450 Q=X[1,K]
1460 FOR J=2 TO N
1470 Q1=X[J,K]+Q
1480 Q=Q1
1490 NEXT J
1500 M2=Q/N
1510 W=X[1,K]↑2
1520 FOR J=2 TO N
1530 W1=X[J,K]↑2+W
1540 W=W1
1550 NEXT J
1560 V2=((N*W)-Q↑2)/(N*(N-1))
1570 S2=SQRV2
1580 DISP "ENTER 1 TO FIND L1,L2,NORMAL DISTR.";
1590 INPUT T1
1600 IF T1#1 THEN 1660
1610 DISP "INPUT K* FOR P=.90,C=.95,N=";N;
[illegible] INPUT K*
[illegible] L1=M2-(K*S2)
[illegible] L2=M2+(K*S2)
[illegible] GOTO 1750
[illegible] DISP "ENTER 1 TO FIND [illegible] NORMAL D[illegible]";
[illegible] INPUT T1
[illegible] IF T1#1 THEN 1000
```

FIGURE 39. Continued

```
1690 DISP "INPUT [illegible] FOR P=[illegible]
1700 INPUT Z1,Z2
1710 A1=1-(Z2↑2/(2*(N-1)))
1720 B=Z1↑2-(Z2↑2/[illegible])
1730 [illegible]=(Z1+SQR(Z1↑2-(A1*B)))/A1
1740 L2=M2+(K5*S2)
1750 C[1,K]=V2
1760 C[2,K]=L1
1770 C[3,K]=M2
1780 C[4,K]=L2
1790 C[5,K]=S2
1800 NEXT K
1810 DISP "ENTER 1 TO FIND M2,V2,S2,ASSUMING LOG NORMAL DISTRIBUTION";
1820 INPUT T1
1830 IF T1#1 THEN 2040
1840 FOR K=1 TO M
1850 IF X[L,K]=0 THEN 2160
1860 M2=LGTX[L,K]
1870 FOR J=1 TO N
1880 IF X[J,K]=0 THEN 1920
1890 N1=N-J+1
1900 Q=(LGTX[J,K]-M2)↑2
1910 GOTO 1930
1920 NEXT J
1930 FOR J=J+1 TO N
1940 IF X[J,K]=0 THEN 1980
1950 Q1=(LGTX[J,K]-M2)↑2+Q
1960 Q=Q1
1970 GOTO 1990
1980 N1=N1-1
1990 NEXT J
2000 IF N1=1 THEN 2160
2010 IF N1=2 THEN 2160
2020 V2=Q/N1
2030 S2=SQRV2
2040 DISP "ENTER 1 TO FIND L1,L2,LOG NORMAL DISTR.";
2050 INPUT T2
2060 IF T2#1 THEN 2170
2070 DISP "K5=? FOR P=.90,C=.95,n=";N1;
2080 INPUT K5
2090 L1=M2-K5*S2
2100 L2=M2+K5*S2
2110 C[1,K]=V2
2120 C[2,K]=L1
2130 C[3,K]=M2
2140 C[4,K]=L2
2150 C[5,K]=S2
2160 NEXT K
2170 DISP "ENTER 1 TO PLOT L1,M2,L2";
2180 INPUT T1
2190 IF T1#1 THEN 2340
2200 FOR R1=2 TO [illegible]
2210 GOSUB 2[illegible]
2220 NEXT R1
2230 GOTO 2300
2240 DISP "ENTER 1 TO [illegible] STRED L2";
2250 INPUT T1
2260 IF T1#1 THEN [illegible]
```

FIGURE 39. Continued

```
2270 FOR R1=3 TO 4
2280 GOSUB 2400
2290 NEXT R1
2300 DISP "ENTER 1 TO PRINT V2,L1,M2,L2,S2";
2310 INPUT T1
2320 IF T1#1 THEN 2580
2330 FOR K=1 TO M
2340 GOSUB 2530
2350 FOR R1=1 TO 5
2360 GOSUB 2560
2370 NEXT R1
2380 NEXT K
2390 STOP
2400 FOR K=1 TO M
2410 IF T2=1 THEN 2450
2420 IF C[R1,K] <= 0 THEN 2480
2430 PLOT Y[K],LGT(C[R1,K])
2440 GOTO 2470
2450 IF C[R1,K]=-100 THEN 2480
2460 PLOT Y[K],C[R1,K]
2470 IF K#M1 THEN 2490
2480 PEN
2490 NEXT K
2500 PEN
2510 RETURN
2520 GOTO 2580
2530 PRINT
2540 PRINT Y[K]
2550 RETURN
2560 PRINT C[R1,K];
2570 RETURN
2580 END
```

FIGURE 40. Revised HP Plot Pac Program

```
10 DIM C[63],B[11],R[101]
20 FOR I=1 TO 11
30 C[I]=R[I]=0
40 NEXT I
50 FOR I=12 TO 63
60 C[I]=0
70 NEXT I
80 B[1]=1
90 W=H=S1=S2=S3=S4=S5=0
100 DISP "MAX.DEGREE=";
110 INPUT D2
120 IF D2>9 THEN 100
130 DISP "XMIN,XMAX,INCRM.=";
140 INPUT X1,X2,X3
150 DISP "YMIN,YMAX,INCRM.=";
160 INPUT Y1,Y2,Y3
170 I=(X2-X1)/27
180 J=(Y2-Y1)/17
190 Y5=Y1-2*J
200 Y6=Y2+J
210 SCALE X1-2*I,X2+I,Y5,Y6
220 PLOT X2,Y1
230 PLOT X1,Y1
235 PLOT 1,Y1
240 PLOT 1,Y2,-1
250 U=Y1
260 V=Y2
270 T=Y3
280 Z=FNL0
290 U=X1
300 V=X2
310 T=X3
320 Z=FNL1
330 X3=I
340 Y3=J
350 Z=FNK0
360 LABEL (*,3,1,0,2/3)
370 DISP "ENTER 1 TO PRINT DATA";
380 INPUT P9
390 IF P9#1 THEN 430
400 PRINT
410 PRINT "PT.NO."TAB14"X"TAB[illegible]"Y"
420 PRINT
430 DISP "PRESS 'ENTER DATA' KEY"
440 END
450 FORMAT 2F7.2
460 DEF FNL(Z)
470 X=ABSU
480 Y=ABSV
490 P=INT(LGT(X+(Y-X)*(Y>X)))
500 P0=(P<-1 OR P>2)
520 LABEL (*,1.5,2,Z*ATN[illegible]P+9[illegible]2/[illegible])
530 FOR K=U TO V STEP T
540 PLOT 1/ NOT Z*K+Z*K, NOT Z*Y1/Z,1
550 CPLOT -7.3,-9.3
560 LABEL (450)K/( NOT P0+P0*10^P),"-";
570 NEXT K
580 IF P0=0 THEN 600
590 LABEL (*)" [illegible]"P;
600 RETURN 9
```

FIGURE 40. Continued

```
10 IF W THEN 60
15 DISP "T(HOURS)";
16 INPUT T
20 DISP "X,Y"
30 INPUT B[2],Y
32 PRINT "NZ="B[2],"CUMULATIVE PEAKS ="Y
34 Y=Y/T
35 Y=Y*4000
36 PRINT "NZ="B[2],"CUMULATIVE PEAKS PER 4000 HR.="Y
37 Y=LGT(Y)
40 IF FNX1 THEN 20
50 END
60 DISP "NOT ALLOWED"
70 END
```

```
10 IF W THEN 60
20 DISP "WRONG X,Y=";
30 INPUT B[2],Y
40 IF FNX(-1) THEN 20
50 END
60 DISP "NOT ALLOWED"
70 END
```

```
10 S8=SQR((S2-S1↑2/N)/(N-1))
20 S9=SQR((S4-S3↑2/N)/(N-1))
30 R9=(S5-S1*S3/N)/(N-1)/S8/S9
40 PRINT
50 PRINT "NO. POINTS ="N
60 PRINT
70 PRINT "X:  MEAN="S1/N;TAB25"ST.DEV.="S8
80 PRINT "Y:  MEAN="S3/N;TAB25"ST.DEV.="S9
90 PRINT
100 PRINT "CORR.COEFF.="R9
110 PRINT
120 END
```

```
10 IF N <= D2-W THEN 260
20 DISP "DEG.REG.=";
30 INPUT D1
40 IF D1 <= D2-W THEN 70
50 DISP "MAX DEG=";D2-W
60 END
70 IF W=0 THEN 250
80 T=0
90 FOR I=1 TO D1+1
100 B[I]=0
110 FOR J=1 TO D1-I+2
120 R=(I+J-1)*(D2+2-0.5*(I+J))
130 B[I]=B[I]+C[T+J]*C[R]
140 NEXT J
150 T=I*(D2+(3-I)/2)
```

FIGURE 40. Continued

```
160 NEXT I
170 R1=0
180 FOR I=2 TO D1+1
190 R1=R1+C[I*(D2+(3-I)/2)]↑2
200 NEXT I
210 T0=C[(D2+1)*(D2+2)/2]
220 T0=T0-C[D2+1]↑2
230 DISP "DONE"
240 END
250 IF N>D2 THEN 280
260 DISP "NOT ENOUGH POINTS"
270 END
280 P=W=1
290 D2=D2+1
300 FOR J=1 TO D2
310 C[P]=SQRC[P]
320 FOR I=1 TO D2-J+1
330 C[P+I]=C[P+I]/C[P]
340 NEXT I
350 R=P+I
360 S=R
370 FOR L=1 TO D2-J
380 P=P+1
390 FOR M=1 TO D2+2-J-L
400 C[R+M-1]=C[R+M-1]-C[P]*C[P+M-1]
410 NEXT M
420 R=R+M-1
430 NEXT L
440 P=S
450 NEXT J
460 T=(D2+1)*(D2+2)/2
470 FOR I=1 TO D2-1
480 T=T-1-I
490 C[T]=1/C[T]
500 FOR J=1 TO D2-I
510 P=D2+1-I-J
520 P=P*(D2+1-(P-1)/2)-I
530 R=P-J
540 S=0
550 U=I+J+1
560 V=P
570 FOR K=1 TO J
580 V=V+U-K
590 S=S-C[R+K]*C[V]
600 NEXT K
610 C[P]=S/C[R]
620 NEXT J
630 NEXT I
640 C[1]=1/C[1]
650 GOTO 80
```

FIGURE 40. Continued

```
10 IF W=0 THEN 120
20 PRINT
30 PRINT "COEFFICIENTS"
40 PRINT
50 FORMAT F3.0,F12.4
60 FOR I=1 TO D1+1
70 WRITE (15,50)"B("I-1")="B[I]
80 NEXT I
90 PRINT
100 PRINT "R SQUARE = "R1/T0
110 PRINT
120 END
```

```
10 FOR X=X1 TO X2 STEP (X2-X1)/100
20 Y=FNZX
30 IF Y<Y5 OR Y>Y6 THEN 60
40 PLOT X,Y
50 GOTO 70
60 PEN
70 NEXT X
80 Z=FNK0
90 END
```

```
10 DISP "CHARACTER HEIGHT(%)";
20 INPUT H
30 LABEL (*,H,2,0,2/3)
40 LETTER
50 Z=FNK0
60 END
```

```
10 DISP "X=";
20 INPUT X
30 DISP "Y(CALC)="FNZX
40 END
```

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