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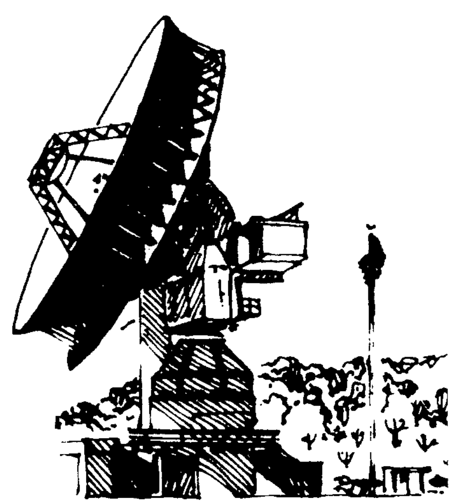
Principles of Air Defense And Air Vehicle Penetration



Frank Heilenday

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13. ABSTRACT *(Maximum 200 words)*

This text presents straightforward methods to analyze air defense and air vehicle penetration. Unique expected value models are developed with frequent numerical examples. Radar (masking, multipath, clutter and low RCS) and electro-optics processing are analyzed, as are electronic warfare, lethal self defense, and AWACS, SAM and AI one-on-one P_k. An integrated air defense system is used to explore relationships among the many factors and inputs. Results from these simple models compare well with far more sophisticated models. Expected target damage, compounding damage and outcome variability (with dependence in factors and inputs) are also addressed. This text was published in 1988. Included in this copy are: one correction (on Page 5-5) and, six replacement pages (17-10 through 17-15).

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PRINCIPLES OF AIR DEFENSE AND AIR VEHICLE PENETRATION

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My thanks also to Joan for all her help and encouragement.

Preface

This text provides a basic understanding of air defense and air vehicle penetration. The fundamentals are introduced, and a simplified approach is used to analyze attrition through various defensive threat environments. Frequent numerical examples of practical applications are provided.

The book is designed to assist those wishing to attain or improve professional competence in assessing air defense performance and penetration aids. Sensitivity analyses are presented to illustrate dominant factors in air vehicle attrition, and to provide insight on penetration issues. The interaction of offensive and defensive factors (such as electronic countermeasures and electronic counter counter-measures) are stressed in performance evaluations.

The text is divided into five parts which build on each other. The parts are:

- Part I - Basic Concepts
- Part II - Radar and Electro-optics Fundamentals
- Part III - Offensive/Defensive Interactions
- Part IV - Attrition Methodology
- Part V - Effects on Target Damage

Basic Concepts includes both defensive concepts in conducting the air defense mission to increase air vehicle attrition, and offensive concepts for mission planning to reduce attrition. (Chapters 1 and 2)

Radar and EO Fundamentals explores the detection of air vehicles by radar and by electro-optics (EO) sensors. A basic understanding of radar sensitivity, masking, multipath and clutter is provided to highlight their roles in evaluating air defense effectiveness. Infrared and optical detection are also discussed. (Chapters 3 through 8)

Preface

<u>Offensive/Defensive Interactions</u> begins with a discussion of the basic interactions, and then analyzes penetrator time under defense radar coverage. Electronic warfare is next explored, with both electronic countermeasures and electronic counter countermeasures highlighted. Lethal self defense range requirements are described for penetrator attacks against a Surface-to-Air Missile (SAM) site, an Airborne Warning and Control System (AWACS), an airborne interceptor (AI) and an approaching missile. (Chapters 9 through 15)

<u>Attrition Methodology</u> presents simple methods to analyze air vehicle penetration - starting with one penetrator against one threat (one-on-one) and ending with many penetrators against many threats (many-on-many) in an air vehicle campaign. The critical factors which can dominate attrition are highlighted in the methodology development and sensitivity analyses. (Chapters 16 through 19)

<u>Effects on Target Damage</u> examines the effects of penetration probability on expected target damage, and on confidence in damage. (Chapter 20)

This text is believed to be unique in providing a basic understanding of the entire penetration process and simplified models to illustrate results. Far more complex models can be used to evaluate attrition, but they seldom provide a fundamental understanding of the air defense process. The purpose of this text is to provide that understanding in a simple, straightforward way.

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PART I

BASIC CONCEPTS

Chapter 1

Defense Concepts

The basic concepts used by air defense systems to detect, track, identify, intercept and down/destroy hostile air vehicles are described in this chapter. Various defense options are identified, with forward, barrier, point and area deployments and defense movements highlighted.

The Air Defense Mission

The air defense mission is 1) to detect, track and identify all air vehicles entering the assigned air space, 2) to intercept unidentified objects, and 3) to down or turn back hostiles (also called penetrators or intruders) soon after they enter defense sensor coverage. A discussion of each of these actions (detection, tracking, identification, interception and destruction) and basic requirements for a command, control and communications (C^3) network follows.

Detection

A penetrator can be detected by active sensors (e.g. active radar) or by passive sensors (e.g. passive radar or infrared). Detection occurs when an operator finds a new return on a scope, or if automated, when a computer discerns a new return from the data stream. In later chapters the problems of detecting a penetrator in the presence of spurious returns from internal and external noise will be discussed.

Tracking

With sufficient sensor information to determine heading and speed, a track is initiated. Current position and the past few scans are usually displayed to aid the next step - identification.

Identification

The track is compared to all known tracks to ascertain if it matches any expected friendly air vehicle routes and times. The track might match an airliner, a friendly military aircraft or a general aviation flight plan. If no match is found, the track is usually declared to be an unknown. Various identification procedures are available to determine whether the track is friendly or hostile (e.g. an Identification Friend or Foe or IFF system). However, a vehicle might automatically be declared a hostile if it displayed jamming or flew over restricted air space.

Interception

Once a track is declared to be an unknown, the defense needs to find out as soon as possible if the track is hostile. One positive method to verify a hostile is to intercept the track with an airborne fighter or interceptor and make visual detection of an enemy aircraft. However, during a conflict, any unknown might be placed in the same category as a hostile, and orders to destroy the unknown might be given without positive identification.

Destruction

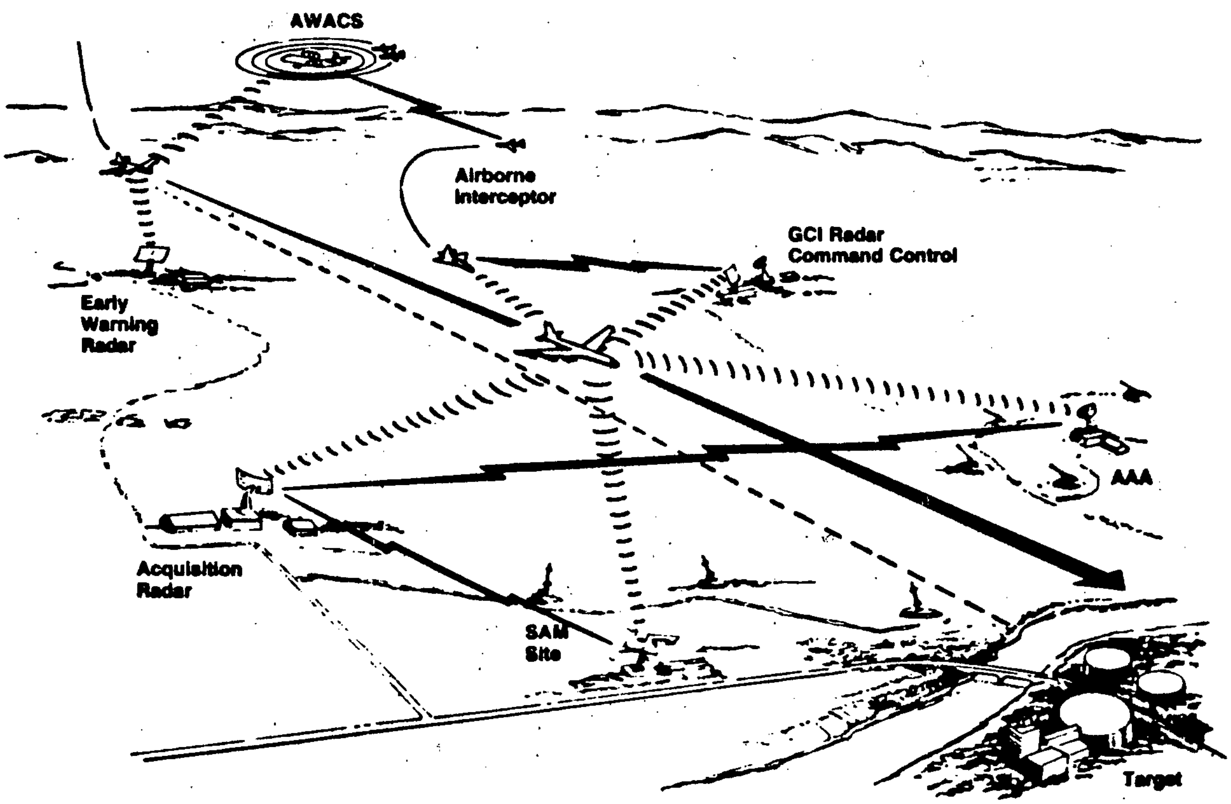
Once a track is considered hostile, the defense assigns weapon systems (e.g. airborne interceptors, surface-to-air missiles and antiaircraft artillery) to down or destroy the intruder. The assignment doctrine is usually for the earliest possible intercept. If the intruder is within surface weapon system intercept range, one or more of these systems will be assigned. If not under surface weapon system coverage, airborne interceptors will likely be assigned, since penetrators might purposely avoid known locations of surface weapon system defenses.

C^3 System

A defense system needs to tie together all its sensors and weapon systems to a command and control network to set priorities, assign weapon systems, conserve defense assets and inflict the greatest attrition possible to the penetrating force. A reliable/redundant communications system is needed for this network.

Typical Defense Elements

The tie-in between some typical defense elements is illustrated in Figure 1.1.



(Boeing)

Figure 1.1 Typical Defense Elements

An AWACS (Airborne Warning and Control System) aircraft is shown operating at high altitude to extend radar coverage hundreds of miles beyond surface system limits. With long range airborne interceptors (AIs) flying Combat Air Patrol (CAP), intercepts can occur in territorial airspace far beyond the border. This AWACS/AI combination is illustrated at the upper left of Figure 1.1.

In the illustration, five different ground sensors track the aircraft. An Early Warning (EW) ground-based radar alerts the defense network. A Ground Control Intercept (GCI) radar site provides command and control to direct an Airborne Interceptor to the penetrator. An acquisition radar site communicates with a Surface-to-Air Missile (SAM) site and an Antiaircraft Artillery (AAA) battery, alerting these weapon systems.

Only one of each of these five different sites is shown, but many will be found throughout a heavily defended area. With many sites to control, an integrated and redundant warning/command, control and communications C^3 network is required to pass information from the sensors to the AI, SAM and AAA weapon systems.

Defense Tactics and Options

Attrition can be increased if the defense can improve the effectiveness of its sensors to detect, and its weapon systems to destroy, the penetrators. The defense also needs to survive enemy attacks, in order to carry out its mission. Some of the key defense options in these three tactics (detect, destroy and survive) are illustrated in the table on the next page.

Defensive Options

Tactic	Attrition will increase, If
Detect	Sensors are alerted and fully prepared when penetrators arrive.
	Deployment and mobility of sensors leave few gaps in coverage
	Diversity of active/passive sensors stress penetration.
Destroy	Good C^3 and intelligence allow near optimum employment of forces.
	Deployment and mobility of weapon systems leave few areas undefended.
	Diversity of AI, SAM and AAA weapon systems stress penetration.
Survive	Deployment, mobility and/or hardening allow sensors and weapon systems to survive.

Defense deployment and mobility are included in all three options and can be a major factor in attrition of the penetrating force. Three deployment options: 1) forward deployments of airborne and shipboard systems, 2) barriers, and 3) point and area defenses will be highlighted. After these discussions, defense mobility will be examined.

Forward Deployments

One forward deployment is an AWACS aircraft flying an orbit (or race-track pattern) hundreds of miles from the border. Radar coverage against both low and high altitude penetrators is illustrated in Figure 1.2.

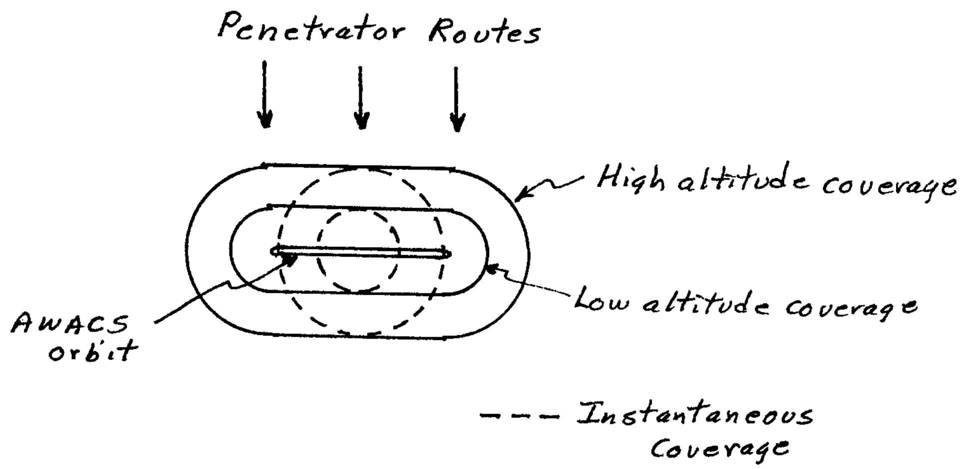


Figure 1.2 AWACS Coverage During Orbit

The large area scanned by this airborne radar platform increases penetrator exposure significantly and makes AWACS a major factor in air vehicle operations.

Ships at sea can also extend the defended airspace. Ships can provide floating platforms for SAM batteries or for mobile air bases (aircraft carriers). Radar coverage of a five ship group (dispersed in expectation of an air attack) is illustrated in Figure 1.3:

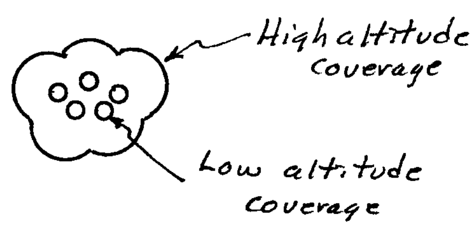


Figure 1.3 Radar Coverage from Ship Formation

Penetrators with adequate warning receivers might be able to avoid active SAM batteries by descending and/or detouring around ships at sea. AIs operating from aircraft carriers are not so easy to avoid, particularly if the defense includes a sophisticated airborne radar platform to direct these airborne interceptors.

Barriers

The defense can set up a sensor barrier across likely penetration routes. Defense elements should be spaced close together so that no gaps or weak points can be exploited. Considerable overlap is required, so that after detection the penetrator's exposure time will be sufficient to allow intercept before surveillance ends. An example of a simple barrier (four sites in flat, barren terrain) is shown in Figure 1.4:

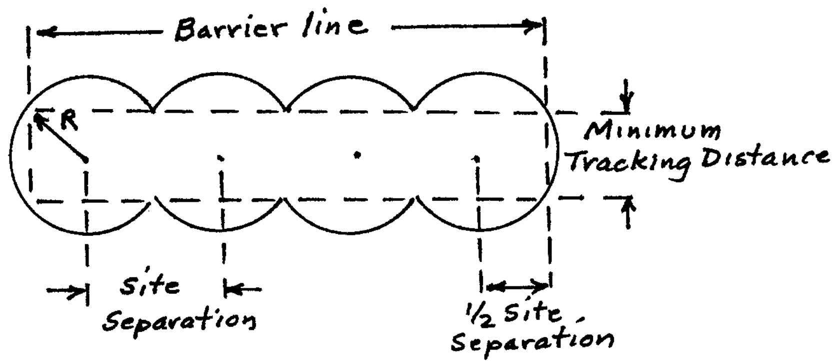


Figure 1.4 Barrier in Flat, Barren Terrain

For example, the detection radius, R, might be about 20 nautical miles (NM) against low altitude penetrators. If the minimum tracking distance required for intercept of a penetrator were 26 NM (1.3R), a 30 NM (1.5R) spacing would be required between sites. This four site barrier would insure intercept along a 3(1.5)R + 2(.75R) = 6R or 120 NM line.

In hilly or built-up areas, site spacing is usually determined by more practical considerations - availability of clear, unobstructed views and local siting restrictions. A barrier in this case might appear as in Figure 1.5 below:

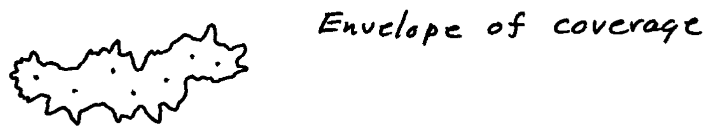


Figure 1.5 Barrier in Hilly or Built-up Area

A barrier is only as strong as its weakest link. Faced with a barrier, intruders may choose to create a gap (or corridor for penetration) by destroying or degrading one or more sites. The defense must consider this possibility, perhaps increasing the overlap to minimize the effects of such an attack.

Point and Area Defenses

Other deployment options are point or area defenses, as illustrated in Figure 1.6 below:

Number	1 site	3 sites		5 sites		Many
Target Size	small	small	medium	medium	large	large
Vulnerability	hard	soft	hard	soft	hard	soft

Figure 1.6 Point and Area Coverage

The single defense site case can be called a point defense, since a small, hard target area is being defended. The terms for size (small, medium or large) refer to the dimensions of the target area to be defended relative to the defense's intercept radius. The terms for vulnerability (hard or soft) refer to the area's vulnerability to damage. To damage a hard area, a weapon must detonate close to the area. Thus, it can be defended with a small intercept or keep-out radius. (High value areas may be hardened both to improve survivability to weapon effects and for ease of defense). A soft area can be damaged by detonations farther away, requiring the defense to maintain a larger keep-out radius. Defending a large, soft area requires a very large keep-out radius, and a very large number of sites.

Due to these requirements for target area coverage, SAM deployments are usually limited to high value targets and ground sensor area coverage limited to areas containing high value targets.

A diagram of full area coverage would show overlapping circles (in flat terrain with no interfering land cover features). For this ideal case, a sensor spacing of 1.73 R would provide solid cover. (This spacing was determined by using hexagonal patterns to represent the circles). In rougher terrain or in urban areas, much closer spacing is required to avoid gaps. Gaps which do occur can be exploited by careful offense routing (as illustrated in Figure 1.7), if radar locations and local terrain masking are known.

Figure 1.7 Exploiting the Gaps

Careful routing usually requires many turns to snake through defense weak points. (This adds to the total flight time of the penetrator and to the fuel load required to complete the mission). Since each turn must be detected and a new heading determined, tracking delays occur. False tracks also abound (from dead reckoning along previous paths). Frequent turns (to take advantage of gaps) can degrade, confuse and even overload defense capability within penetration corridors. This offense tactic, combined with large numbers of intruders in the corridor and their penetration aids, can seriously degrade the number of penetrators which a defense net can handle.

If the defense faces standoff weapons, coverage may have to be expanded to intercept air vehicle carriers before they can launch these weapons. This may be a far more effective defense tactic than attempting to intercept cruise or rocket propelled missiles after launch.

This completes the discussion of defense deployments. Now consider defense movement or mobility.

Defense Mobility

Movement of defense elements during, or just prior to, a penetration mission can affect attrition. Defense relocations may be ordered to 1) increase defense survival during wartime - while still protecting the assigned area, 2) accompany forces on the move - such as a field army, or 3) fill a gap or strengthen a weak point.

Two examples of defense movements are of particular interest. The first involves interceptor survival. If all known main operating bases (MOBs) were attacked early in a mission, interceptors caught on the ground might suffer considerable damage. This should reduce attrition. However, if the AIs flushed on warning and landed at whatever surviving bases were available, attrition might be close to the case when no MOBs are attacked.

The second example is SAM movement during, or just prior to, a mission. Penetrator routes might have been based on careful study of SAM locations from information a few hours, days or weeks before the operation. The routes chosen may have avoided known SAM sites, but might enter the intercept zone of some of the SAMs which moved.

Even if the offense discovered defense movements, mission plans might not be flexible enough to react with route or target changes in time. Defense relocation can be a major factor if penetration plans do not include both timely discovery of defense movements and timely reaction.

Summary

The air defense mission is to detect, track, identify, intercept and destroy (or turn back) hostile vehicles. Defense options can be categorized into detect, destroy and survive actions. Defense deployments and defense movements are key defense actions to increase attrition.

Three deployments are of particular interest - 1) Forward defenses with AWACS/AI or shipboard SAMs and AIs, 2) Ground barriers, 3) Point and area defenses. Defense movements prior to, or during, a mission can also increase attrition on a penetrating force.

Chapter 2

Offense Concepts

The basic concepts used to plan air vehicle penetration missions are described in this chapter. Options available to the offense to improve penetration are highlighted, along with three basic tactics - avoid, degrade and destroy.

Definitions

An air vehicle is any aircraft or missile which flies within the earth's atmosphere. This includes bombers, cruise missile carriers, air transports, fighters, cruise missiles and other ground, sea and air launched missiles which are propelled and guided within the atmosphere.

A sortie is the flight of a single air vehicle from launch to end of of flight.

A mission consists of one or more sorties flown to achieve a specific short range goal or advantage. Mission and operation will be used synonymously.

A campaign consists of one or more missions flown to achieve a strategic or long range goal.

Tactics are actions taken by forces in combat to achieve their objectives in the battle.

An air vehicle is destroyed if weapon effects cause immediate cessation of flight.

An air vehicle is lost if it does not land at a base (friendly or neutral) from which it can be recovered or reused.

An air vehicle is damaged if weapon effects prevent the mission from being completed as scheduled.

An air vehicle survives if the remainder of the mission can be completed as scheduled.

The Probability of Prelaunch Survival (PLS) is the probability that a sortie will survive the effects of enemy action through launch. For an aircraft, this includes takeoff and escape from attacks against its base. For air launched vehicles, this includes survival of the carrier from all enemy actions through launch, and vehicle escape from attacks on the carrier immediately after launch.

Weapon System Reliability (WSR) is the probability that the sortie will arrive at its goal (e.g. target area, combat location, or landing base), neglecting the effects of enemy actions. For air launched vehicles, this includes the WSR of the carrier up to launch, as well as the launch and flight reliability of the vehicle.

The Probability of penetration survival (Ps) is the probability that a sortie entering the defense area survives all defenses up to some stated location or event time along its route (e.g. exit defenses, last target or complete combat operations).

The Probability of Arrival (PA) is the probability that the sortie will survive launch, have a reliable weapon system and successfully penetrate to some stated location or event time along its route. When the three probabilities (PLS, WSR and Ps) are independent of each other, they can be multiplied together to yield the expected PA:

$$PA = PLS \times WSR \times Ps$$

Mission Planning

The objective of a non-combat mission may be transport, deployment, reconnaissance, search and rescue or demonstration of air superiority. Missions involving combat generally include one or more of the following objectives:

Close Air Support (CAS) of surface forces.
Battlefield Interdiction (BI) to interfere with enemy movements and cut lines of communication.
Air-to-Air Combat with enemy forces.
Suppression of surface defenses.
Attacks against fixed targets, such as airfields.
Attacks against relocatable targets, such as garrison areas.
Attacks against moving targets, such as ships or tanks.

A mission plan is limited by the resources available. These resources include the air vehicles, their payloads and support facilities. Each air vehicle type has unique range/payload capabilities based on such factors as:

Take off base - runway length, altitude, temperature.
Payload - offensive and defensive weapons and equipment.
Flight route - to each mission event point (e.g. weapon release, combat location, loiter or turn point).
Weather en route - winds aloft, cloud cover.
Flight profile - altitude/speed for cruise, penetration, withdrawal.
Fuel load - fuel consumption during each leg, reserve at scheduled landing base, alternate bases available.
Air refueling - off-load, distance from take off base.

Survival within enemy defended airspace is a critical factor in mission planning, often dictating the flight route and profile, and sometimes the tasks assigned to a sortie. Penetration analyses attempt to predict probability of survival for each sortie and mission. Results are of prime importance to mission planners, as well as to air vehicle designers and force structure planners.

Offense Options

Survival can be improved within hostile airspace, if the offense knows the enemy's Consolidated Air Defense Order of Battle (CADOB). This includes information on detection, tracking, identification, C^3 and all defensive weapon systems (AIs, SAMs and AAAs). With this knowledge the offense can reduce attrition by selecting from various offense options, such as:

Option	Attrition Reduced, If
Routing	All known key defenses are avoided.
Standoff weapons	Carrier launches weapons outside of defenses.
Air vehicle type	Intruder has higher survivability - faster, lower altitude, lower observables.
Date/time	Defense is unprepared (surprised), or less effective (at night or in bad weather).
Number of vehicles	Massed attack overloads or exhausts defense. A few vehicles can slip by undetected.
Pen Aids	Electronic countermeasures are effective. Support force dilutes/confuses defenses
Suppression	Key defenses can be destroyed. A penetration corridor can be developed.

Offensive Tactics

Offensive tactics can be categorized into Avoid, Degrade and Destroy actions.

Avoid

A penetrator seeks to avoid all defense threats which can lead to loss or damage to the air vehicle. If all threats cannot be avoided, priority goes to avoiding, or at least minimizing exposure to, those threats which place the vehicle at the greatest risk (e.g. AWACS/AI systems, or the most lethal SAM threats).

Tactics which can be used by the offense to avoid or minimize exposure to the defense include: 1) routing around defenses (described in the previous chapter), 2) low altitude to minimize exposure, 3) high speed to decrease time in cover, 4) choice of low observable vehicles to avoid or minimize exposure, and 5) standoff weapons which may allow the carrier to avoid defenses entirely.

Degrade

Degrades reduce the effectiveness of the defense by non-lethal means - by dilution, saturation, deception or confusion. Electronic counter-measures (ECM) and other tactics can be very effective, if detailed information on the threat is available. Some of this information may be known well before take off time. A warning receiver on the vehicle can provide more up-to-date information on the particular active threat ahead.

Tactics to degrade the defense include: 1) effective use of ECM, 2) effective use of support forces to confuse, dilute and saturate the defense, 3) surprise, 4) choice of attack time to take advantage of night or bad weather, and 5) massing intruders in a penetration corridor.

Destroy

Defense installations and weapon systems can be destroyed or damaged by attacks aimed at the defenses themselves, or by attacks aimed at nearby targets. Targets which are damaged as a by-product of an attack against other installations are said to receive collateral damage. Collateral damage against defenses can be increased (while still achieving the desired level of damage on the primary target) by varying the offensive weapon's burst location, yield or height of burst. Attacks against defenses can occur before or during the mission.

Tactics to destroy defenses include attacks on: 1) air bases, 2) SAM sites, 3) AAA sites, 4) C^3 facilities, and 5) AIs and defense missiles using self-defense weapons on the penetrators.

Summary

The basic concepts used in planning air vehicle sorties are described. Penetrator losses can be decreased using knowledge of the defensive order of battle to take advantage of surprise, nighttime penetration, low observables, low altitude, high speed, careful routing, standoff weapons and penetration aids. Offensive tactics can be categorized into Avoid, Degrade and Destroy actions.

PART II

RADAR AND ELECTRO-OPTICS FUNDAMENTALS

Chapter 3

Radar Sensitivity

This chapter describes a simple method to estimate radar sensitivity, or maximum detection range when internal radar noise is the only interference. The signal return from a target, the receiver's internal noise level and the signal-to-noise ratio (S/N) required for target detection are combined to estimate radar sensitivity.

Signal Return from a Target

The signal return from a target will be determined by examining:

1) the radar illumination of a target,
2) the target reflection, and
3) the antenna reception.

Illumination

Radar is an acronym for Radio Detection and Ranging. A pulse radar measures target range by the elapsed time between pulse transmission and reception of the echo reflected back from a target. Figure 3.1 below illustrates this measurement:

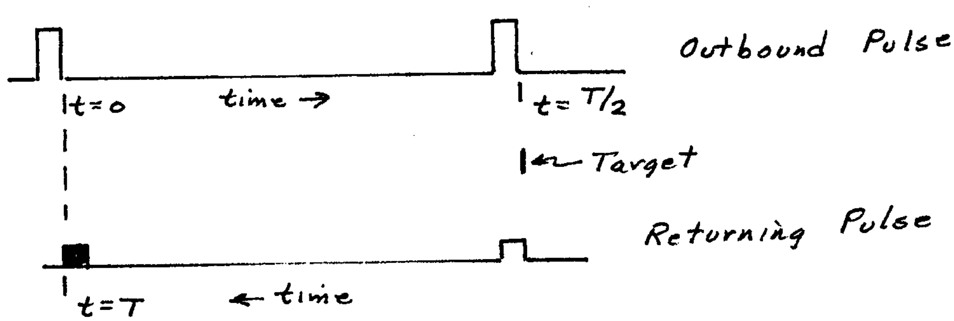


Figure 3.1 Radar Ranging

A pulse travels a round trip distance of 2R in time t at the speed of light c. Since c is essentially constant, the distance is just the velocity multiplied by time (i.e. 2R = ct). The target range, R, is:

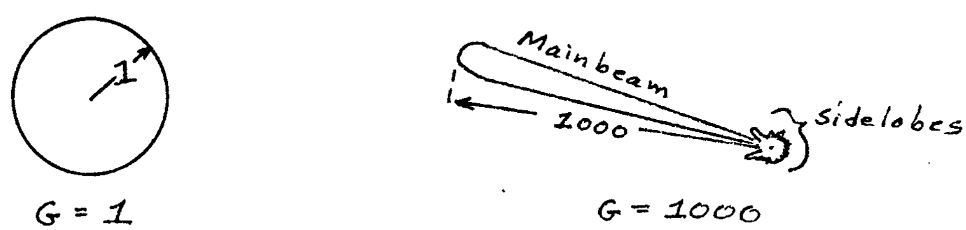
$$R = ct/2$$

The speed of light in air is near 3×10^8 meters per second, or about 16.2×10^4 Nautical Miles (NM) per sec. The pulse will travel out 1 NM and back in 12.355 microseconds, or 12.355 µsec. Thus, 12.355 µsec is often called a radar mile. (A 1 µsec round trip means a target is 492' away).

The power density in watts/m^2 at a distance of R meters from a radar with a peak power of P_p watts and transmitter antenna gain of G_t is :

$$\text{Power density} = P_p G_t / 4\pi R^2 \ (\text{watts}/m^2)$$

Note that $4\pi R^2$ is just the surface area of a sphere of radius R over which the radar energy spreads. The transmitter antenna gain, G_t, is the relative gain in the direction of the target compared to an isotropic or omnidirectional antenna (or an antenna which has the same gain in all directions). An isotropic antenna and a high gain antenna are illustrated in Figure 3.2 below:



Isotropic Antenna High Gain Antenna

Figure 3.2 Antenna Patterns

Note the sidelobes of the high gain antenna. These lobes play an important role in clutter rejection and in electronic countermeasures (ECM), as will be shown in later chapters.

If a radar has a transmitter peak power of 1 Megawatt (10^6 watts), and an antenna gain of 1000 (10^3), the power density as a function of distance is:

$$\text{Power density} = 10^6 \times 10^3/4\pi R^2 = 7.96 \times 10^7/R^2$$

At a range of 100 NM (185 Km) the power density is 0.0023 watts per square meter in this example.

A decibel scale is often used to express the ratio between widely different power levels. The ratio of any two power levels (P_1/P_2) can be expressed in decibel (dB) form as:

$$dB = 10 \log_{10} (P_1/P_2)$$

Any ratio can be expressed in dB's as:

$$dB = 10 \log_{10} (\text{ratio})$$

If a value is given in dB's, the ratio can be obtained by:

$$\text{ratio} = 10^{dB/10}$$

For example 0.0023 → -26.4 dB, and:

$10^6 \rightarrow 60$ dB	$1 \rightarrow 0$ dB
$4\pi \rightarrow 11$ dB	$0.1 \rightarrow -10$ dB
$10 \rightarrow 10$ dB	$R^2 \rightarrow 2$(R in dB)

Note that when meters are expressed in decibels, dBm is used. Also,

$m^2 \rightarrow$ dBsm, watts $\rightarrow$ dBw and milliwatts $\rightarrow$ dBmw.

Reflection

A target reflects the radar signal in all directions. Part of the energy is sent back in the direction of the radar. Assume that the transmitting and receiving antenna are colocated - a monostatic case. (Two antennas not colocated form a bistatic case.) The target's backscatter radar cross section (RCS), labeled σ_t, is a measure of the reflecting area of the target in the direction of the radar. Since the returning energy spreads, this backscatter reflection must be divided by the surface area of a sphere of radius R to obtain the fraction of the energy reflected back to the vicinity of the radar. Thus:

$$\text{Fraction reflected} = \sigma_t/4\pi R^2$$

For example, if $\sigma_t = 10\ m^2$, the fraction reflected at 185 Km (100 NM) is 2.3×10^{-11} or -106.4 dB.

Reception

The power density <u>outside</u> the radar receiving antenna is equal to the power density at the target multiplied by the fraction of the energy reflected back in the radar's direction. The receiving antenna captures some of this returning energy. The effective receiving antenna aperture (or capture area) is labeled A_e. A commonly used expression for A_e is:

$$A_e = G_r\ \lambda^2/4\pi$$

where λ = radar wavelength.

The wavelength is the distance between successive crests of a wave, or the length of one complete cycle. The frequency, f, is the number of cycles which pass a fixed point in a given time. The wavelength is related to frequency as:

$$\lambda = c/f$$

Frequency is expressed in cycles per second, or Hertz (Hz). Some examples of the correspondence of f and λ are shown below:

Frequency	300 MHz	3 GHz	10 GHz	30 GHz	300 GHz
Wavelength	1 m	.1 m	3 cm	1 cm	1 mm

where MHz = MegaHertz or Million cycles/sec
and GHz = GigaHertz or Billion cycles/sec.

The antenna aperture area for a λ = 0.1 m (3 GHz) radar with the same transmitting and receiving antenna gain of 1000 (in the example) is:

$$\text{Antenna aperture} = 1000\ (0.1)^2/4\pi = 0.8\ \text{m}^2\text{, or } - 1\ \text{dBsm}$$

Signal Received

Combine the three terms - radar illumination, target reflection and antenna reception - to obtain the signal received by a radar when there are no losses:

Signal received = Power density at Range R x Fraction reflected x Effective receiving antenna aperture

$$S = \frac{P_p\ G_t}{4\pi R^2} \times \frac{\sigma_t}{4\pi R^2} \times \frac{G_r\ \lambda^2}{4\pi}$$

$$S = \frac{P_p \, G_t \, G_r \, \lambda^2 \, \sigma_t}{(4\pi)^3 \, R^4}$$

This equation on the decibel scale is:

$$S = P_p + G_t + G_r + 2\lambda + \sigma_t - (33 + 4R)$$

since $(4\pi)^3$ is 3(11) dB = 33 dB

For example, with the following values:

P_p = 60 dBw, G_t= G_r= 30 dB, λ = -10 dBm, σ_t = 10 dBsm
and R = 52.7 dBm (for 185 Km or 100 NM)

$$S = 60 + 30 + 30 + 2(-10) + 10 - \{33 + 4(52.7)\}$$

$$S = -134 \text{ dBw, (or } 4 \times 10^{-14} \text{ watts)}$$

This answer can also be obtained by combining the three terms - radar illumination, target reflection and antenna reception as:

$$S = -26.4 - 106.4 - 1 = -134 \text{ dBw}$$

Radar Noise

One competing signal to the target return is the internal noise in the receiver. Competing signals outside the radar include ground and sea returns, rain and other reflectors of radar energy. The dominant interference in free space is the receiver internal noise, N. This interference can be expressed in terms of Boltzmann's constant, k, as:

$$\text{Internal noise} = N = k\ T\ B\ F_n$$

where k = Boltzmann's constant = 1.38×10^{-23} watt-sec/°K or -228.6 dB

T = Temperature, in °Kelvin (a 21°C earth temperature = 294° K or 24.7 dB)

B = Bandwidth of the receiver, in Hz

F_n = Noise factor of the receiver (non-dimensional)

On the decibel scale, this equation is:

$$N = k + T + B + F_n$$

Note: F_n is called a noise <u>figure</u> when expressed in dB's.

For example, assume that N is to be found for a radar with a bandwidth of 1 MHz (60 dB) and a noise figure of 10 dB, on a nominal 21°C day. For this case:

$$N = -\ 228.6 + 24.7 + 60 + 10 = -133.9 \text{ dBw, or} \sim 4 \times 10^{-14} \text{ watts}$$

Maximum Detection Range

For reliable detection, the target signal must reach some minimum or threshold value called S_{min}. When internal noise is the only interference, S_{min} can be determined as:

$$S_{min} = (S/N)_{reqd} \times N$$

The range at which the signal received is just equal to this minimum signal is called the maximum detection range, R_{max}. The maximum detection range is a measure of the radar's sensitivity, or target detection capability under free space conditions. To find R_{max}, modify the signal received equation by substituting S_{min} for S, and R_{max} for R. Now:

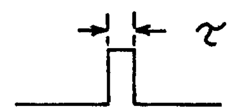
$$S_{min} = \frac{P_p \; G_t \; G_r \; \lambda^2 \; \sigma_t}{(4\pi)^3 \; R_{max}{}^4}$$

Setting S_{min} equal to $(S/N)_{reqd}$ x N, and solving for R_{max} for the case where internal noise is the limiting interference:

$$R_{max}{}^4 = \frac{P_p \; G_t \; G_r \; \lambda^2 \; \sigma_t}{(4\pi)^3 \; kTBF_n \; (S/N)_{reqd}}$$

When the radar's output S/N is optimized, B can be expressed in terms of τ as:

$$B = 1.2/\tau$$



where τ is the pulse width, in sec

Substituting this value, and adding radar losses (L), the following expression is obtained:

$$R_{max}{}^4 = \frac{P_p \; G_t \; G_r \; \lambda^2 \; \tau \; \sigma_t}{1.2 \; (4\pi)^3 \; kTF_n L \; (S/N)_{reqd}}$$

Then,

$$R_{max} = \left\{ \frac{P_p \; G_t \; G_r \; \lambda^2 \; \tau \; \sigma_t}{1.2(4\pi)^3 \; kTF_n L \; (S/N)_{reqd}} \right\}^{1/4}$$

Note that R_{max} varies as $\sigma_t{}^{1/4}$. The above equation can be rewritten as:

$$R_{max} = K \; \sigma_t{}^{1/4}$$

K is the <u>K Value</u> or maximum detection range of a radar against a 1 m^2 target. On the decibel scale, the K Value is:

$$K = 1/4 \; (P_p + G_t + G_r + 2\lambda + \tau - \{- 170 + F_n + L + (S/N)_{reqd}\})$$

where $1.2\ (4\pi)^3$ kT is - 170 dB at 21°C earth temperature

For example, assume that P_p = 60 dBw, $G_t = G_r$ = 30 dB, λ = - 10 dBm, τ = - 60 dB, F_n = 10 dB and L = 10 dB. K can be determined in terms of $(S/N)_{reqd}$ as:

$$K = 47.5 \text{ dBm} - (1/4)(S/N)_{reqd}$$

If the signal required was just equal to the noise (S/N = 1, or 0 dB), K would be equal to 47.5 dBm, which is 56.2 Km or about 30 NM.

K Values can be used to estimate maximum detection ranges for different radar cross sections, if a change in RCS does not change S/N or other factors in the equation. For example, maximum detection ranges against vehicles with radar cross sections from .01 to 100 square meters are shown below, for the K = 30 NM example:

RCS =	.01	.1	1	10	100	m^2
R_{max} =	10	17	30	54	96	NM

Changes in many radar parameters (e.g. frequency, power, antenna gain or target characteristics) will change the K Value of a radar.

The simple methods shown above assume linear relationships and stable parameters. When nonlinearities or instabilities occur, detection ranges will be degraded from values predicted by these equations.

Required Signal-to-Noise Ratio

The S/N ratio required for reliable detection is a dominant factor in determining detection range. For example, with a non-fluctuating target return viewed by a scanning radar, 13 dB (or a 20 to 1 S/N ratio) may be required for detection. A 13 dB S/N would lower the K Value calculated above by 13/4 or 3.25 dB (about a factor of two), from 30 NM to 14.3 NM.

The standard non-fluctuating radar target is a sphere. A sphere would not only have a constant return, but would also have the same radar cross section (RCS) in all directions. An air vehicle's RCS fluctuates at a constant viewing angle, and also varies as the viewing angle changes. A classic categorization of targets by RCS fluctuations was reported by Swerling[1]. Five Swerling cases were identified to account for five different types of backscatter return fluctuations. These are shown below:

Swerling Case	Scan-to-Scan Fluctuations	Pulse-to-Pulse Fluctuations	Example
0	No	No	Sphere
1	Yes-Complex	No	Jet Aircraft
2	Yes-Complex	Yes	Prop Aircraft
3	Yes-Simple	No	Small Missile
4	Yes-Simple	Yes	Small Prop Aircraft

Jet or rocket propelled air vehicles might be categorized as either Swerling cases 1 or 3, depending upon their size.

Swerling cases can be used to estimate S/N_{reqd}, for any given detection probability and false alarm probability. The single pulse detection probability is the probability of the target return exceeding the detection threshold on any pulse. The false alarm probability is the probability of noise exceeding the detection threshold level on any pulse.

Ideally, the detection probability, P_d, would be 1.0 and the false alarm probability 0. However, if one attempted to obtain a P_d of 1.0, the detection threshold must be set very low to find the target on every pulse. But noise fluctuations would occasionally exceed this low threshold - causing false alarms. If instead, one attempted to eliminate false alarms by raising the threshold, the detection probability would be reduced. Thus, there must be a compromise.

One set of compromises between P_d and false alarm rate (FAR) values is shown below:

Radar Type	P_d	FAR
Scanning radar	.5	10^{-7}
Tracking radar	.9	10^{-8}

A scanning (acquisition or search) radar can tolerate a higher false alarm rate, since there is time to reject the random noise false alarms that occur. A lower P_d is also allowed, since there is usually time to detect the target on a later pulse or scan, if missed earlier.

A target tracking radar usually has very little time to find a target, so P_d must be high. This radar can ill afford to lock on to a false target, so the FAR must be held very low. Fortunately, a tracking radar is normally directed by a scanning radar. Thus, the tracking radar can limit its search for a target to a very small range, azimuth and elevation sector at a time. This speeds the detection process and limits false alarms.

With this criteria and Blake's charts[2] (for single pulse detection and optimum receiver processing), S/N values required for a scanning and a tracking radar for three Swerling cases are as shown:

Swerling Case	Scanning radar	Tracking radar
0	S/N = 12 dB	S/N = 14 dB
3	S/N = 12.5	S/N = 18
1	S/N = 13	S/N = 22

The greater the target's RCS fluctuation, the higher the S/N required for high probability of detection. Complex fluctuations (case 1) require higher S/N ratios than simpler fluctuations (case 3). Note the marked increase in the S/N required to detect targets with a tracking radar. This is caused by the higher P_d and lower false alarm rate criteria.

The K Values corresponding to these S/N ratios are shown below, for the example:

Case	Scanning	Tracking
0	15.2 NM	13.5 NM
3	14.8	10.8
1	14.3	8.5

The change in K Value from case 0 to case 1 for a scanning radar is only 6% (from 15.2 to 14.3 NM). Changes for a tracking radar are far greater. With a 8 dB change in S/N (from 14 to 22 dB), the maximum detection range is reduced by 37% (from 13.5 to 8.5 NM). This is the price of seeking high P_d and low FAR against a target with complex fluctuations in RCS.

Values for S/N_{reqd}, target RCS and radar losses are difficult to obtain in practical applications. S/N_{reqd} and L should be derived from radar test data. Target RCS values and fluctuations should be based on test measurements at various elevation and azimuth angles, distances, radar frequency and polarization desired. (Polarization is the direction of the electrical axis of a wave - e.g. horizontal, vertical or circular/-rotating). Actual target fluctuations may not fit a specific Swerling case, but fit other distributions, such as a log normal.

Pulse Integration

A radar may not have sufficient sensitivity to make a detection decision, based on one pulse return. Integrating many pulses before attempting detection can improve the S/N level, and so markedly increase radar range.

The detection decision may be based on the number of pulses illuminating the target during one scan of a radar. This number depends on the amount of time that the radar beam takes to sweep by the target (called the dwell time) <u>and</u> on the pulse rate (called the pulse repetition frequency or PRF of the radar). The PRF is the number of pulses transmitted by the radar each second. The dwell time for a scanning radar is just the width of the beam (measured at the half power points) divided by the scan rate:

$$\text{Dwell time} = \frac{\text{Beamwidth}}{\text{Scan rate}}$$

For example, assume a 1° azimuth beamwidth, a 360° azimuth scan (repeated each 10 seconds) and a PRF of 360 pulses per second. The scan rate and dwell time are:

$$\text{Scan rate} = 360°/10 \text{ sec} = 36°/\text{sec}$$

$$\text{Dwell time} = 1°/36°/\text{sec} = 1/36 \text{ sec}$$

The number of pulses in a scan = Dwell time x PRF.

$$\text{Pulses/scan} = 1/36 \times 360 = 10 \text{ pulses}$$

A coherent radar is more efficient in integrating pulses than a noncoherent radar. A coherent radar transmitter insures that each pulse starts nearly in phase with adjacent pulses. One common measure for coherence is that three adjacent pulses must be within 15° phase of each other. Coherence can also be achieved in a non-coherent radar by measuring the phase of each transmitted pulse and using this phase measurement in processing the returned pulses.

Blake's charts[2] provide signal-to-noise ratios required to detect a target as a function of: 1) the number of pulses integrated, 2) the probability of detection, 3) the false alarm rate and 4) the Swerling case.

For example, a scanning radar (P_d = .5 and FAR = 10^{-7}) in case 1 requires 13 dB S/N for single pulse detection. Blake's charts indicate only 6 dB S/N is required for 10 pulse non-coherent detection, or a 7 dB improvement factor. This 7 dB improvement translates into a 7/4 or 1.75 dB improvement, or a factor of 1.5 range improvement over the single pulse case. For coherent integration, the improvement in this example would be somewhat better - about 8 dB instead of 7 dB.

When more than 30 pulses are integrated, a simple approximation to the improvement due to pulse integration for coherent and non-coherent integration of n pulses can be used, as shown below:

Coherent radars	Improvement = n (in dB) - 3 dB
Non-coherent radars	Improvement = n (in dB)/2 + 3 dB

Using this approximation, the following improvements would be noted:

n	30	100	300 pulses
Coherent radrars	12 dB	17 dB	22 dB
Non-coherent radars	10.5 dB	13 dB	15.5 dB

Other Factors

Before ending this chapter on radar sensitivity, three terms should be discussed:

1) Maximum unambiguous range,
2) Range resolution, and
3) Duty cycle.

Unambiguous Range

The unambiguous range of a radar depends upon the interval between radar pulses. This relationship is illustrated in Figure 3.3 below:

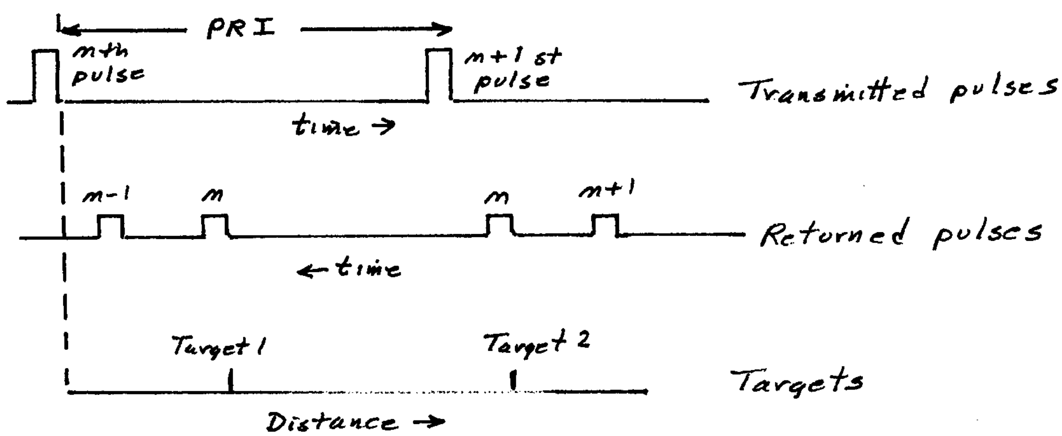


Figure 3.3 Maximum Unambiguous Range

The interval between adjacent pulses is called the pulse recurrence interval (PRI). It is just equal to 1/PRF. If the time for a target return to be received exceeds the PRI, then the target return will not arrive until after the next pulse is sent out, and so cannot be distinguished from a target at a much shorter range. The maximum unambiguous range, R_{ua}, is just the range corresponding to this PRI, that is:

$$R_{ua} = c\ PRI/2 = c/2\ PRF$$

For example, a radar with a PRF of 360 pulses per sec would have a maximum unambiguous range of $3 \times 10^5/2 \times 360 = 417$ Km, or 225 NM.

In order to increase this range, the PRF could be decreased. But this would decrease the number of pulses on the target during a scan, and thus decrease target detection probability. There is a method called multiple PRFs to increase the unambiguous range without decreasing the PRF. To illustrate this, note Figure 3.4 below showing two different PRFs:

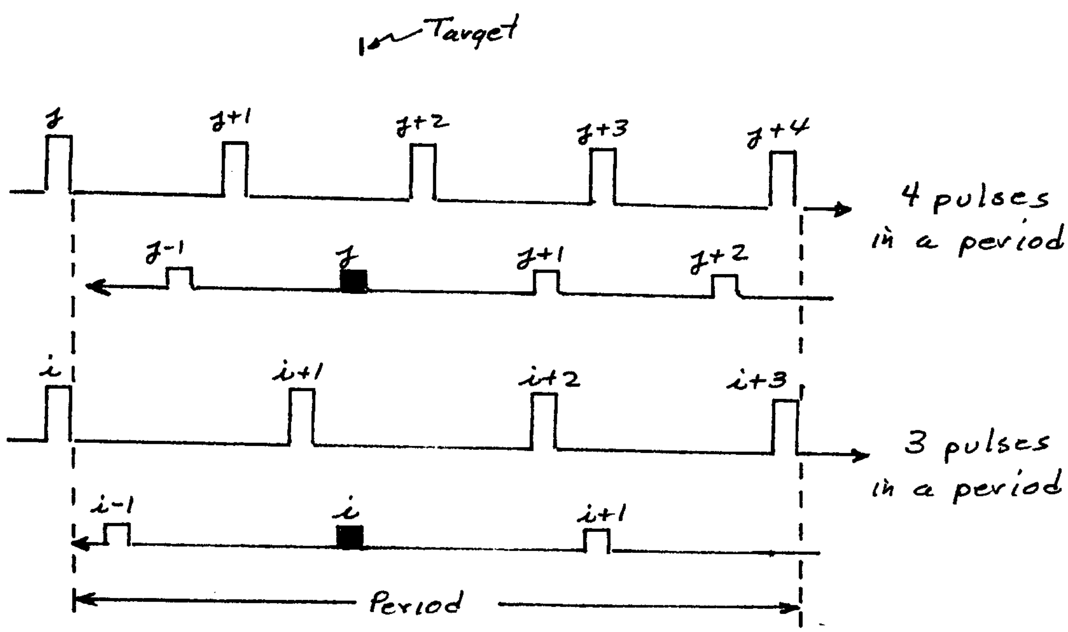


Figure 3.4 Multiple PRFs

The top of the figure illustrates the higher of the two PRFs and the bottom shows the lower PRF. The minimum interval between the time that two transmitted pulses line up in time is defined as the period. There are 3 pulses within this period for the lower PRF, and 4 pulses for the higher PRF.

The time for the pulse to return to the radar from the single target is shown by the black rectangle. Note that the ith pulse of the lower PRF is synchronized with the jth pulse of the higher PRF, and that the target returns from these two pulses are received at the same time (they line-up).

The i-1 pulse will provide a return from this target soon after the ith pulse is transmitted, as will the j-1 pulse. However, these two returns will not line up (since these pulses were not synchronized when transmitted) and can be rejected. Only the ith and the jth pulse returns line up during this period, and are accepted. The net effect of multiple PRFs is to allow higher PRFs without decreasing the maximum unambiguous range of a radar. In this example, the equivalent PRF for determining the maximum unambiguous range is equal to 1/3 of the lower PRF or 1/4 of the higher PRF.

The maximum unambiguous range equation is now:

$$R_{un} = c/2\ (PRF_1/n_1) = c/2\ (PRF_2/n_2)$$

where n_1 and n_2 are the number of pulses in the period.

In the example, the unambiguous range is increased by a factor of 3 over PRF_1, or by a factor of 4 over PRF_2.

The advantage of multiple PRFs is an increase in maximum unambiguous range. However, since only a fraction of the pulses can be used for detection, higher PRFs do not provide initially the full pulse integration benefits discussed earlier.

Range Resolution

The range resolution, R_{res}, of a radar is the minimum distance that two targets can be separated radially (along a radar's line of sight), and still be resolved as separate returns. The range resolution is:

$$R_{res} = c\tau/2$$

The numerator, $c\tau$, is just the range dimension of the pulse in space. The factor of 2 accounts for the two way path of the pulse between the radar and the target. For example, a 1 μsec pulse would provide a 492' range resolution.

The choice of a pulse width, τ, in radar design involves a trade off. Low values improve range resolution. High values increase the energy in the pulse, and thus improve target detection range.

Pulse compression offers a way to improve target detection range using longer pulses, while maintaining the range resolution of short pulses. Pulse compression breaks down a pulse into separate coded parts, each of which can be processed individually by the radar receiver. For example, assume that a pulse of 10 μsec total width was broken up into 10 different parts, each 1 μsec wide. As long as the radar can process each part individually, the range resolution is 1 μsec or 492'. The equation for range resolution with pulse compression ratio, PC, is:

$$R_{res} = c\tau/2\ PC$$

Pulse compression improves detection by increasing the energy transmitted in a pulse without degrading the radar's range resolution. It also greatly improves radar performance in clutter and in the presence of barrage jamming ECM as will be shown later.

Duty Cycle

The duty cycle of a radar is the fraction of the time that pulses are being transmitted. It is the pulse width (in seconds) times the number of pulses per second (the PRF):

$$\text{Duty cycle} = \tau \text{ PRF}$$

Low PRF radars usually have low duty cycles. For example, a radar with a PRF of 400 pulses per second and a 1 μsec pulse width would have a 0.04% duty cycle. A radar with a 100,000 PRF and the same pulse width would have a 10% duty cycle

The relationship between duty cycle, peak power (P_p) and average power (P_{av}) is noted below:

Energy $= P_{av} \times t_o$ for coherent integration over time t_o or for continuous wave (CW) radars.

Energy $= P_p \times \tau \times \text{PRF} \times t_o$ for non-coherent radars.

Or:

$$P_{av} = P_p \times \tau \times \text{PRF} = P_p \times \text{duty cycle.}$$

For CW and coherent radars using integration, $P_{av}\ t_o$ expresses the energy over the integration interval t_o. For non-coherent systems, $P_p\ \tau$ (expressing the energy in one pulse) is of prime interest.

Summary

Simple equations for the signal reflected from a target were developed for conditions where the signal competes only with the internal noise in the radar receiver. The signal-to-noise ratio required for target detection depends upon the desired probability of detection, the false alarm rate that can be tolerated and upon fluctuations in the target return. Pulse integration can be used to decrease the required S/N ratio and so improve detection range. The sensitivity of a radar can be estimated by its maximum detection range against a 1 m^2 target of any fluctuating class, called a radar's K Value.

Equations for the maximum ambiguous range and range resolution cell of a radar were shown. Examples of multiple PRFs increasing unambiguous range and pulse compression improving range resolution were presented.

Chapter 4

Radar Propagation, Masking and Multipath

This chapter describes how 1) the propagation of radar waves over the earth affects line of sight limits, 2) the earth, terrain and land cover masks penetrators, and 3) multipath interferes with radar tracking.

Wave Propagation

The propagation of radar waves over the earth's surface can be approached by imagining a cue ball earth - an ideal, smooth sphere. The straight line from a point at a height h above the earth that is tangent to the earth's surface is illustrated below in Figure 4.1:

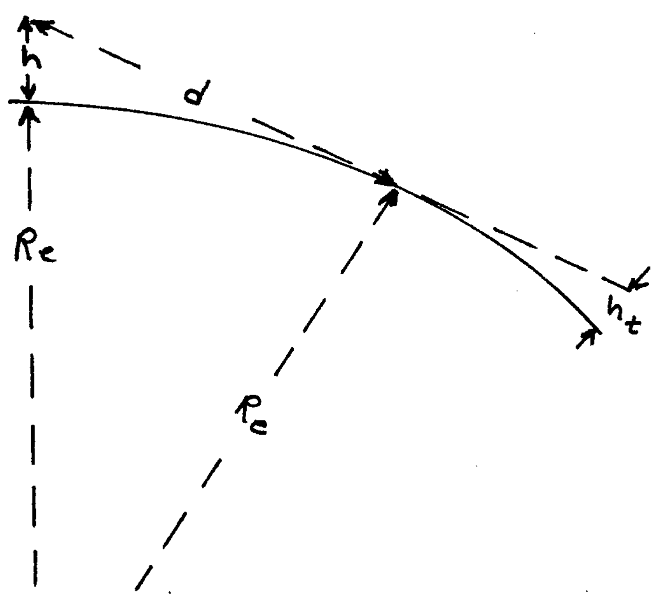


Figure 4.1 Tangent Line

The distance d to the tangent point is one side of a right triangle. Its value is found from:

$$d^2 = (R_e + h)^2 - R_e^2$$

From which:

$$d = (2\ R_e\ h + h^2)^{1/2}$$

If the height h is very small compared to R_e, an excellent approximation is:

$$d = (2\ R_e\ h)^{1/2}$$

Substituting the earth's radius as 21 x 10^6 ft, and expressing h in feet and d in Nautical Miles (NM):

$$d = 1.07\ h^{1/2}$$

This is the distance to the tangent point from a position h feet above the surface of a cue-ball earth. The distance, d_t, to a farther point along this tangent line at a height h_t above the earth's surface can be determined by adding a second term, as noted below:

$$d_t = 1.07\ (h^{1/2} + h_t^{1/2})$$

The radar line of sight (LOS) distance differs only in the bending of radar waves over the earth's surface. This bending extends the radar horizon, just as optical wave bending extends the optical horizon. A commonly accepted way to account for this increased radar line of sight is to increase the earth's radius in the above equations by a factor of 4/3rds. Making this change, and using h_a to represent the radar antenna's height, the radar line of sight (LOS) is:

$$\text{Radar LOS} = 1.23\ (h_a^{1/2} + h_t^{1/2})$$

This is the basic equation to estimate radar line of sight in NM for any antenna or target height in feet. Note that if both heights are expressed in meters, the constant changes from 1.23 to 2.23 with LOS still measured in NM.

For example, assume a 50' antenna height and a 200' target. The radar line of sight is then:

$$\text{Radar LOS} = 1.23\ (50^{1/2} + 200^{1/2}) = 26\ \text{NM}$$

A similar equation exists for the optical line of sight. Optical waves are at a higher frequency, and do not bend as much as radar waves. Using a 7/6th earth radius to account for refraction, the constant changes from 1.23 to 1.15, as noted below:

$$\text{Optical LOS} = 1.15\ (h_o^{1/2} + h_t^{1/2})$$

where h_o is the observer height and h_t is the target height

he above equations assume normal bending of the waves. Anomalous conditions - where the atmosphere bends the wave in markedly different ways - can change the detection range dramatically. Figure 4.2 illustrates a case of radar ducting - bending of the radar beam downwards in elevation. This ducting extends the radar horizon against low altitude targets, but severely limits the detection range against targets at medium altitudes.

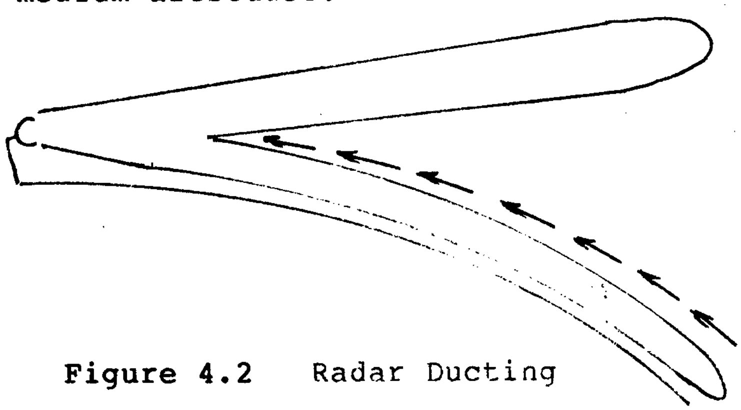


Figure 4.2 Radar Ducting

If this ducting existed, a penetrator staying at low altitude would be under radar cover for a longer period than normal. With prediction of the duct, however, a penetrator could minimize radar coverage by flying above the duct as shown by the dotted line in Figure 4.2.

Masking Fundamentals

The earth itself masks targets beyond the radar horizon. For normal propagation conditions (4/3rds earth radius), a penetrator would be essentially masked from the radar beyond this LOS. Since radar waves are diffracted beyond the horizon, some energy reaches targets beyond this LOS limit. But very little energy is returned to the radar due to diffraction losses in both directions. Only high power radars working against large RCS targets can overcome these losses.

The earth mask, therefore, will be considered to be at the radar line of sight. Any target beyond this radar LOS will be assumed to be outside radar coverage. Masking around a radar on a cue ball earth would show as a circle of radius LOS, as in Figure 4.3 below:

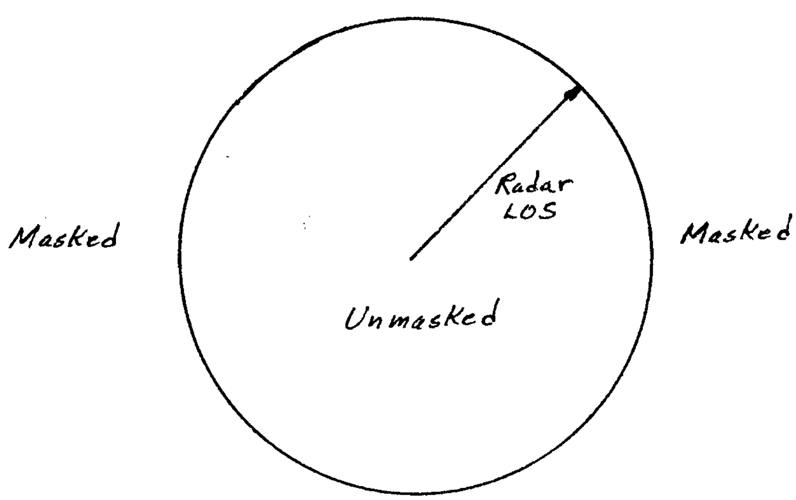


Figure 4.3 Masking for a Cue Ball Earth

A cue ball earth has no terrain, vegetation or cultural features. A radar antenna location shows far different masked regions as shown in Figure 4.4 below (for a particular penetrator altitude):

Figure 4.4 Radar Mask

Buildings, trees and the unevenness of the terrain combine to provide this irregular masking pattern.

Terrain alone has a dramatic effect on masking. For example, with a 50' antenna and 200' target, some <u>average</u> mask radii (as a fraction of the cue ball LOS) are illustrated on the next page.

Terrain	Fraction of LOS
Flat	0.85
Rolling	0.65
Hilly	0.50

For example, if the cue ball radar line of sight is 26 NM against a 200' target, the average unmask radius in hilly terrain would be 13 NM.

Trees around a radar antenna can also have a dramatic effect upon masking. A simple rule of thumb is to reduce the radar antenna height one foot for every foot of tree height. For example, with 25' trees surrounding a 50' high antenna, the unmask range for a 200' target would be:

$$\text{Radar LOS} = 1.23\ \{(50 - 25)^{1/2} + 200^{1/2}\} = 23.5\ \text{NM}$$

This compares to 26 NM without trees. This is a reduction of 18% in area covered, in this example.

Multipath Fundamentals

Multipaths are indirect paths between the radar and the target. They include a bounce off an intervening surface before they return to the radar. Figure 4.5 illustrates the direct and one indirect path from the target back to the radar:

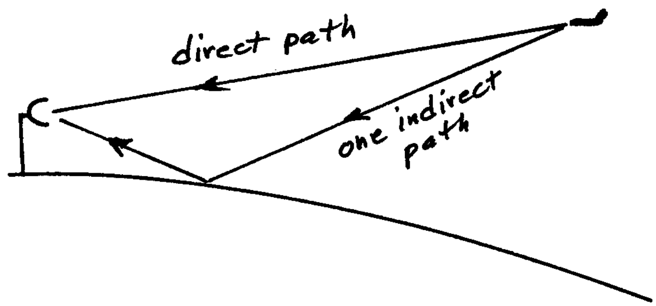


Figure 4.5 Multipath Illustration

Note that the angle of arrival differs between the direct and the indirect path. The multipath signal arrives at a lower elevation angle than the direct signal. If the indirect signal enters the main-beam of the radar, this signal will share the same receiver antenna gain as the direct signal.

Radar returns are combinations of direct and multipath signals. Since all indirect paths include a reflection, signals on these paths will usually lose energy in the bounce. The bounce loss will depend upon the specular reflection coefficient of the reflecting surface.

The indirect path is longer than the direct path, and thus this signal can return at a different phase than the direct signal. Figure 4.6 illustrates both returns as vectors, with one multipath return shown as a rotating vector (with its phase dependent upon the bounce path length):

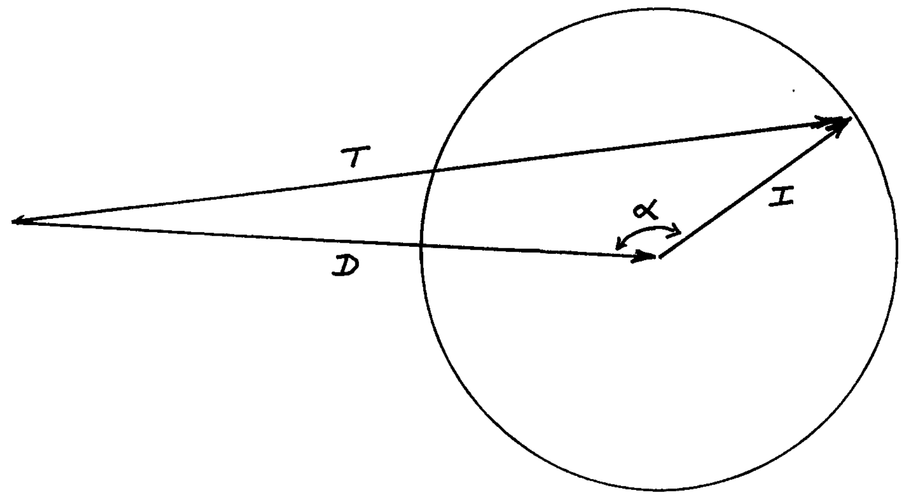


Figure 4.6 Vector Combination with Multipath

At times the multipath return reinforces the direct return, and at other times it will reduce this return, as the phase relationship changes. The equation for this combined return is:

$$T^2 = D^2 + I^2 + 2\,D\,I\cos\alpha$$

where T = total signal vector
D = the direct path vector
I = the indirect path vector
α = the angle between D and I

The net effect is interference in the radar receiver, with both reinforcement and fading of the signal as the target approaches due to changes in the phase of the multipath vector. Since the interfering signal arrives at a different angle than the direct signal, a radar with a narrow beam will markedly reduce multipath returns which do not enter the mainbeam of the radar. Whenever multipath occurs in the mainbeam, however, a null can appear on the horizon with lobing patterns illustrated by Figure 4.7.

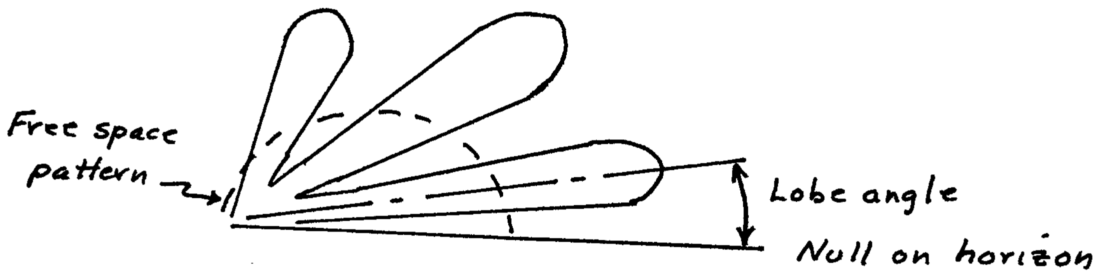


Figure 4.7 Lobing Pattern

The angles to the center of the lobes are calculated as:

$$\text{ith lobe angle} = (2i - 1)\lambda/4h_a$$

where λ is the radar wavelength
and h_a is the antenna height.

For example, for a 0.1 m wavelength and a 50' (15.2 m) antenna height the first lobe will be at 0.094°, the second at 0.282° and the third at 0.470°. The impact of multipath can be noted by translating these angles to ranges from a radar for a given target altitude. Ranges at which a 200' target enters the 1st-3rd null/lobe are shown below:

Number	Null	Lobe
1st	26 NM	16 NM
2nd	10 NM	6.5 NM
3rd	4.8 NM	3.7 NM

These ranges were calculated by the following approximation by Cunningham[3] for the range, R, as a function of elevation angle α:

$$R = \{(R_E \sin\alpha - \{2R_E h_a\}^{1/2})^2 + 2R_E(h_t - h_a)\}^{1/2}$$

$$- \{R_E \sin\alpha - (2R_E h_a)^{1/2}\}$$

where R_E = 4/3 earth radius

Ideal multipath conditions exist whenever the surface exhibits high specular reflection coefficients (e.g. over smooth, highly reflecting surfaces such as over calm water or smooth deserts). A low altitude penetrator will remain within a degree of the horizon from a surface radar for most of its coverage time and thus can take advantage of multipath, particularly against lower frequency radars. The lobing and fading of the radar return can cause serious degrades so that a low altitude penetrator may fly in very close to a radar before being accurately tracked.

Summary

The radar horizon for 4/3rd earth refraction on a cue ball earth is:

$$\text{Radar LOS} = 1.23(h_a^{1/2} + h_t^{1/2})$$

where LOS is in NM, with h_a, h_t in ft.

Masking from terrain, trees or cultural features can markedly reduce radar line of sight limits.

Ducting of the radar beam can significantly increase detection range against low altitude targets. Knowledge of this ducting allows a penetrator to choose an altitude profile to minimize radar coverage.

Multipath can be severe over highly reflecting surfaces (such as calm water or smooth desert). Multipath returns interfere with the target return - delaying detection/tracking of penetrators. The main effect is limited to multipaths arriving in the mainbeam of the radar, and so is restricted by the radar beamwidth. However, low altitude penetrators remain within a degree of the horizon for most of their flight past a radar, and so can benefit the most from multipath degrades.

Chapter 5

Clutter

This chapter describes clutter (unwanted surface and airborne returns) entering the radar receiver. The emphasis will be on surface clutter, its area and backscatter coefficient, and on the signal-to-clutter (S/C) ratio.

Resolution Cell

Before examining clutter areas, the size of the range and angle resolution cell of a radar should be defined.

If two targets are within one beamwidth, the radar can not separate (or resolve) them <u>in angle</u> (without special processing such as Doppler beam sharpening). Two targets, which are separated by more than one beamwidth, can be resolved in angle. The antenna beamwidth defines the angular resolution of a radar. When this is translated to a distance R away from the radar, the angular resolution width becomes:

$$\text{Angular resolution width} = 2R \tan(\text{beamwidth}/2)$$

Range resolution depends on the pulse width (τ) of a radar, as described in Chapter 2. Recall that:

$$\text{Range resolution} = c\tau/2$$

Now define a volume (called a resolution cell) enclosing the range and angle resolutions of a radar. For example, assume that a radar has a 1 μsec pulse width and a 1° azimuth and elevation beamwidth. The closest spacing between two targets (averaging 100 NM away) that can be resolved by the radar, and the volume defined by these resolutions is sought.

The closest distance, measured along the radar line of sight is $c\tau/2$. For a 1 μsec pulse, the range dimension of the cell is 492'. At greater range separations, two targets can be resolved, even if they are within the same beamwidth.

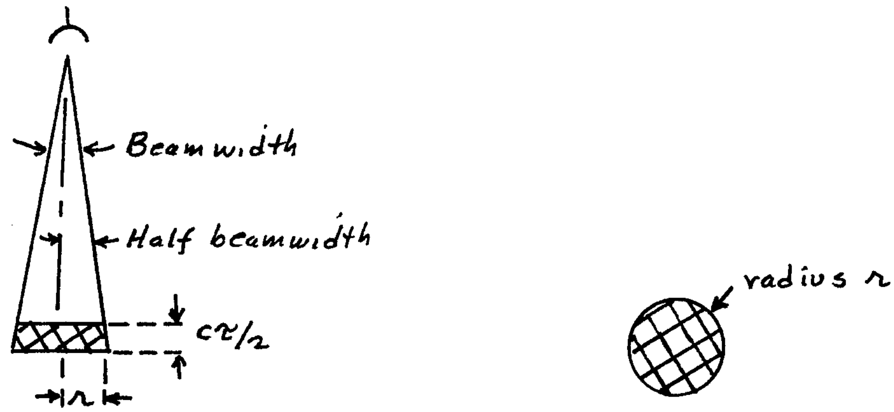
The closest angular spacing is one beamwidth. The extent of one beamwidth in azimuth is 2R tan (Az/2). At 100 NM with a 1° beam, the minimum azimuth spacing is 1.75 NM (measured perpendicular to the radar LOS) for the radar to separate two targets. Since the elevation beamwidth is also 1°, the minimum vertical separation is 1.75 NM or 10,600'.

The volume enclosed is found by multiplying the three orthogonal dimensions, i.e. 492' x 1.75 NM x 1.75 NM = 0.248 NM cubed. This is the volume of the radar resolution cell.

Clutter Area

Clutter can be defined as unwanted returns which originate from airborne scatterers (e.g. rain, snow and chaff) and surface scatterers (e.g. the ground, sea and cultural features). The emphasis here is on surface clutter.

The area of the surface illuminated by a radar depends on the orientation of the beam to the surface and the radar's range and angle resolution. If the center of a radar beam is perpendicular to a flat surface, the surface area illuminated can be measured with the help of Figure 5.1.



Side View   Top View

Figure 5.1 Beam Perpendicular to a Surface

If the beam has equal elevation and azimuth beamwidths, the intersection of the beam and the surface forms a circle of radius r. The radius of this circle is a function of the beamwidth and the range, R, from the antenna to the surface. The radius, r, is:

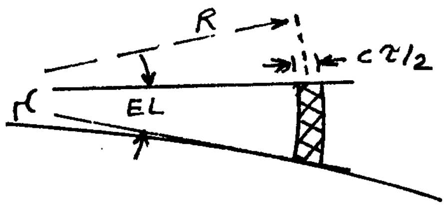
$$r = R \tan (\text{Beamwidth}/2)$$

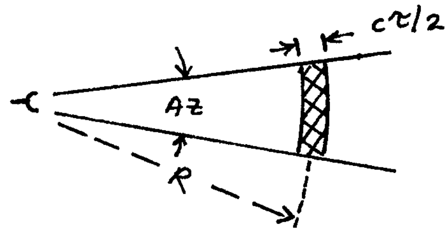
For example, with a 1° beamwidth at 5 NM range, r would be 0.044 NM, and the intersection area would be about 0.006 NM^2.

If the elevation (EL) and azimuth (AZ) beamwidths are not equal, the intersection will be an ellipse with an area of $\pi r_1 r_2$, where r_1 is R tan (EL/2) and r_2 is R tan (AZ/2).

Targets or surface clutter in the resolution cell (the cross hatched volume) can not be separated by location differences. A radar, looking directly down on the earth, will receive strong surface clutter returns. Any air vehicles within the beam and flying within $c\tau/2$ of the surface will be in the same resolution cell as these strong clutter returns.

Now consider a radar beam parallel to a intersecting surface, as shown in Figure 5.2 below:





Side view

Top view

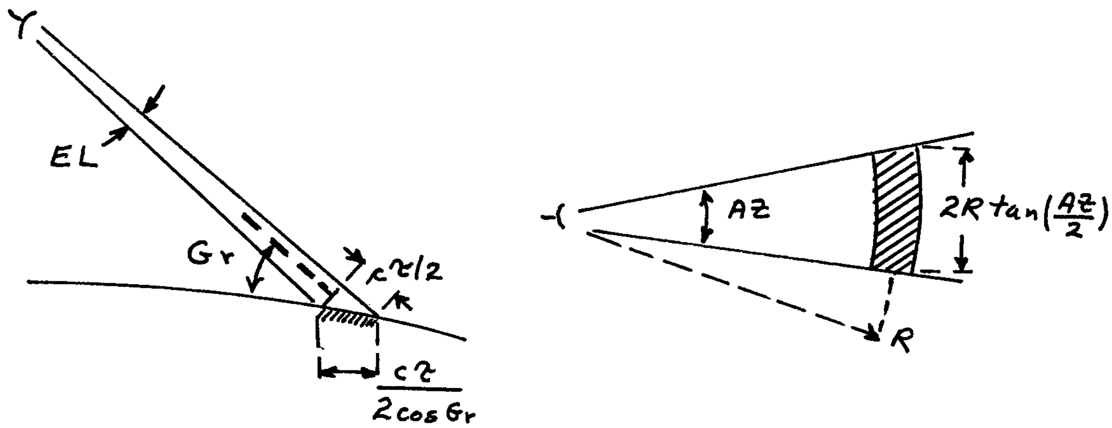
Figure 5.2 Parallel Beam

The volume defined by the range and angle resolutions of the radar is noted by the cross hatching. Returns within this volume can not be resolved from each other. The solid bar shows the surface illuminated by the radar within this cell. (This surface area is often called a patch, or clutter patch). The area of this clutter is the range resolution multiplied by the azimuth resolution width at the specified range:

$$\text{Clutter area} = c\tau/2 \times 2R \tan(AZ/2)$$

Clutter from this area will compete with air vehicle targets that are within the same resolution cell.

Thus far only two cases (a beam parallel and a beam perpendicular to the surface) have been considered. The general case of a radar beam being at some grazing angle, Gr, from the surface will now be described. For <u>low</u> grazing angles, the intersection with the ground is limited by the range and azimuth resolution, as shown in Figure 5.3:



Side view Top view

Figure 5.3 Low Grazing Angle

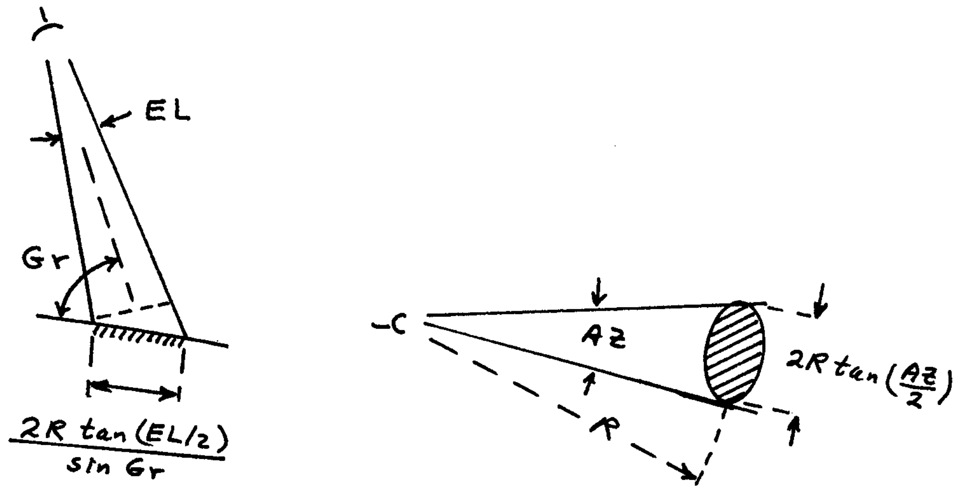
The range resolution is $c\tau/2$ and the angular width is 2R tan (AZ/2). The only complicating factor is that the surface is no longer perpendicular to the center of the beam, but is now at an angle Gr to the beam. This spreads the range cell from $c\tau/2$ to $c\tau/2\cos Gr$. The clutter surface area can be approximated by:

$$\text{Clutter area} = (c\tau/2\cos Gr) \times 2R \tan (AZ/2)$$

Low grazing angles can be defined as:

$$\tan Gr < \frac{2R \tan (EL/2)}{c\tau/2}$$

For high grazing angles, the beam intersection with the surface changes to that of Figure 5.4.



Side view Top view

Figure 5.4 High Grazing Angle

Now the patch is defined by the azimuth and elevation beams. The intersection of the azimuth beam on the surface is 2R tan (AZ/2). The elevation beam now spreads along the surface due to the grazing angle, so that its extent is 2R tan (EL/2) divided by the sine of Gr.

However, for most surface and airborne radar applications, the low grazing angle case (Figure 5.3) is the geometry of interest. The clutter area for this case is:

$$\text{Clutter area} = A_c = c\tau R \tan(AZ/2)/\cos Gr$$

Whenever both angles are small (no more than a few degrees) this equation can be approximated by:

$$\text{Clutter area} = (c\tau/2)\ R\ AZ$$

This is just the range resolution multiplied by an approximation to the azimuth resolution width at a range R. For example, with a 1 µsec pulse and a 1° beam radar which is 10 NM from beam intersection of the surface, the clutter area is:

$$\text{Clutter area} = (3\text{x}10^5 \text{x } 1\text{x}10^{-6}/2) \text{ x } 18.5 \text{ x } 1/57.3 = 0.048 \text{ Km}^2$$
$$\text{or } 0.014 \text{ NM}^2$$

Backscatter Coefficients

The clutter cross section (σ_c) is defined as:

$$\sigma_c = \sigma° \text{ x } A_c$$

where $\sigma°$ = clutter backscatter coefficient
A_c = clutter area

Two ideal earth surfaces will be described to develop the basic concepts of clutter backscatter coefficients. First a cue ball earth, and then a sandpaper earth will be considered.

A cue ball earth reflects radar signals as a mirror reflects light. All reflections are specular (the angle of incidence equals the angle of reflection). Surface returns reach the radar only when the antenna is within one beamwidth of being perpendicular to the earth's surface.

For the cue ball earth, the clutter backscatter coefficient varies with antenna grazing angle as illustrated in Figure 5.5.

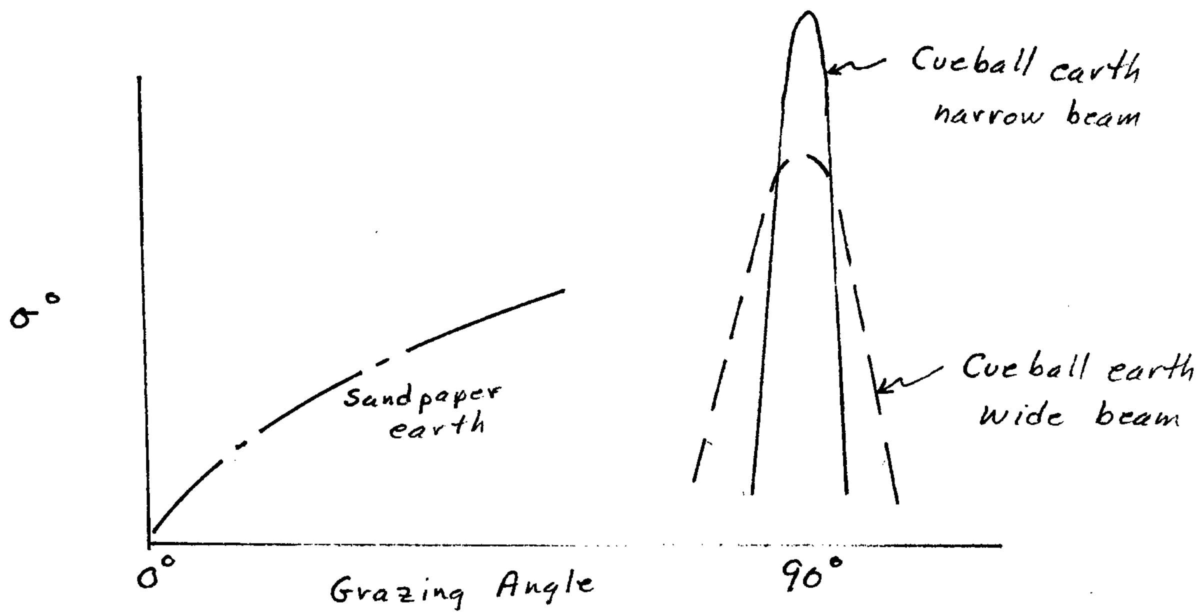


Figure 5.5 Ideal Surfaces

Clutter backscatter coefficients vary with the beamwidth. High $\sigma°$ values come from specular reflections of narrow (high gain) antennas (shown by the solid line) when the beam is perpendicular to the surface. A wider beam (dotted line) has a lower $\sigma°$ at 90° due to its lower gain, but has a broader pattern in angle.

The second idealized surface is a sandpaper earth, with uniformly distributed surface clutter providing diffuse returns in all directions. The variation of the clutter backscatter coefficient with grazing angle is shown by the dotted line in Figure 5.5. This pattern has a maximum value at 90°, with $\sigma°$ varying as the sine of the grazing angle at lower angles. Theoretically, $\sigma°$ drops to zero at 0°.

Backscatter coefficients of many ground surfaces vary as the sine of the grazing angle for angles over 2°. Below 2°, the height of the clutter sources (e.g. trees, buildings and even plowed fields) cause far higher returns (due to reflections from vertical surfaces) than the sine relationship would predict. One simple approximation for these very low grazing angles is to maintain the $\sigma°$ level at 2° for all lower grazing angles.

Clutter data might be categorized into many different landforms and different land covers in any development of ground clutter models. One possible category set is shown below:

Landforms	Land Covers
Level	Rangeland - herbaceous
Undulating	- shrub + mixed
Hummocky	Forest - deciduous
Inclined	- coniferous
Broken	- mixed
Rolling	- clear cut
Ridged	- block cut
Moderately steep	Wet land - forested
Steep	- non-forested
	Barren
	Urban

This list might be reduced to just three categories:

Rural low relief
Rural high relief
Urban low/high relief

All land covers except urban can be considered to be rural. Low relief landforms range from level to broken, generally with less than 2° slope or less than 100' variation in height within each patch. High relief landforms (from rolling to steep) generally have more than 2° slope and more than 100' height changes within each resolution cell.

Within each of these three categories, σ° varies with the radar frequency, antenna polarizations, the grazing angle of the beam to the surface, the antenna height and the range to the clutter, as well as with radar resolution cell size.

Antenna polarization (orientation of the electric vector of an electromagnetic wave) does affect the clutter received. Vertically polarized antennas generally receive higher backscatter returns, but lower forward scatter. Rain can be eliminated by transmitting one particular circular polarization and receiving with the same antenna polarization. Sea clutter exhibits larger σ° levels with vertical polarization, but has a spikier appearance (causing more false alarms) with horizontal.

With urban clutter, σ° decreases from UHF to X band. For low relief, rural cases, the clutter backscatter coefficient increases over this same frequency range. There is less variation of σ° with frequency for high relief, rural cases.

The backscatter coefficient can be provided as an average value, a median or as a probability distribution. The distribution may be needed to reveal the presence of a few strong discrete returns which can dominate radar false alarms.

σ° data should specify the frequency, polarization, resolution cell size, antenna height, range and the landform/cover. Distributions of σ° might appear as in Figure 5.6.

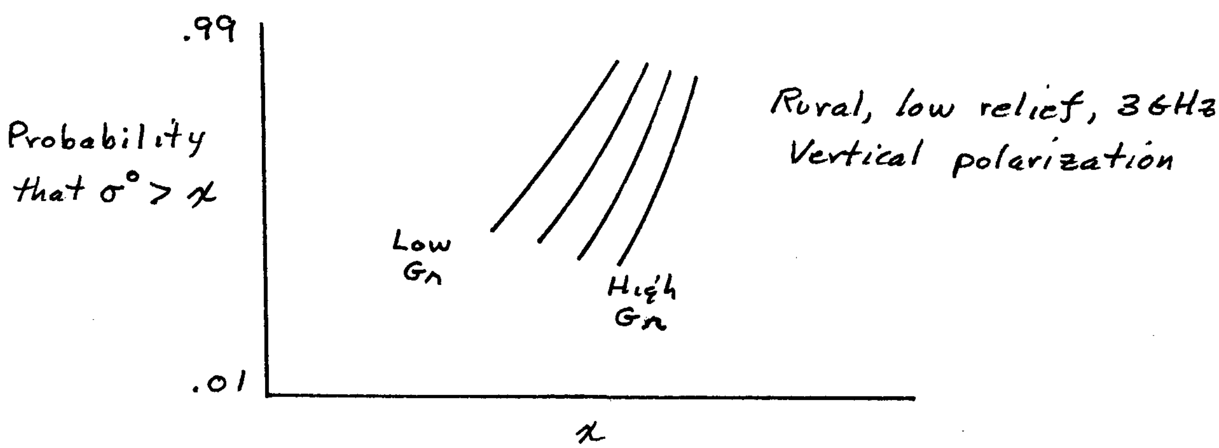


Figure 5.6 Typical σ° Distribution

Average values for σ° are typically in the -20 to -35 dB area for ground clutter at low grazing angles (0 to 2°). However, discrete clutter returns can far exceed these values.

Clutter Around a Site

The sandpaper earth model would cause radar clutter returns out to the radar horizon, as illustrated in Figure 5.7.

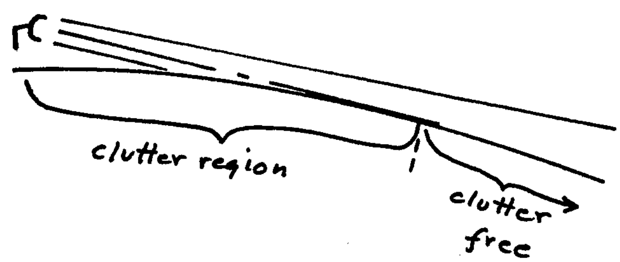


Figure 5.7 Sandpaper Earth - Clutter Horizon

The maximum range of this sandpaper earth clutter is the radar horizon.

$$\text{Radar horizon} = 1.23\ h_a^{1/2}$$

For example, the clutter region around a radar with a 50' antenna would be a circle of radius $1.23 \times 50^{1/2} = 9$ NM.

If an air vehicle target were flying over this sandpaper earth at 200' altitude, the radar LOS would be 26 NM. Thus, the radar coverage would extend from 26 NM to 9 NM with no clutter interference, and from 9 NM to the radar site with clutter interfering with the target return. This ideal sandpaper earth case is illustrated in Figure 5.8.

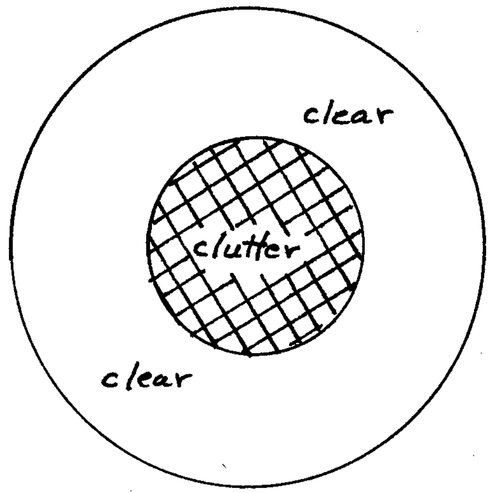


Figure 5.8 Sandpaper Earth Clutter

The antenna height affects both the target LOS and the clutter horizon. For example, a 6' antenna height would reduce target range in the example from 26 NM to about 20 NM, but would also reduce the clutter horizon from 9 NM to 3 NM from the site. The target LOS would be reduced 22%, with the clutter radius decreased by 65%. The target observation region is reduced 39%, but the clutter region is reduced 88% for the sandpaper earth model. When clutter prevents target detection, a lower antenna height may be desired to prevent clutter from obscuring the target at critical close-in ranges, if the loss in LOS range is acceptable.

Clutter and masking around a site are shown in Figure 5.9, for one particular clutter threshold level, target altitude and antenna height.

Figure 5.9 Clutter and Masking at a Site

The extent of the clutter about any site varies with the specific clutter threshold level used. As the threshold is raised, clutter areas will decrease until they finally disappear. Conversely, as the threshold is lowered, clutter areas will enlarge.

Signal-to-Clutter Ratios

The clutter return in a radar receiver is normally much greater than the receiver's internal noise level. Thus, when clutter competes with the target return, the detection range is determined by the ratio of the target signal, S, to the clutter signal, C. These can be expressed as:

$$S = \frac{P_p\ G_t\ G_r\ \lambda^2\ \sigma_t}{(4\pi)^3\ R^4\ L_s}$$

$$C = \frac{P_p\ G_t\ G_r\ \lambda^2\ A_c\ \sigma°}{(4\pi^3)\ R^4\ L_c}$$

If the losses are equal, or $L_s = L_c$, the S/C ratio is:

$$S/C = \sigma_t/A_c\sigma°$$

Recall that A_c was found to be:

$$A_c = (c\tau/2\cos Gr) \times 2R \tan(AZ/2)$$

Whenever the grazing angle and the azimuth beamwidth is small (no more than a few degrees), this was approximated by:

$$A_c = (c\tau/2)x(R\ AZ)$$

As mentioned earlier this is just the range resolution multiplied by the azimuth resolution width at the target range.

S/C is then:

$$S/C = \frac{\sigma_t}{\sigma°\ (c\tau/2)\ R\ AZ}$$

Or, on the decibel scale:

$$S/C = \sigma_t - (\sigma° + \tau + R + AZ + 81.8)$$

The term 81.8 dB is just c/2. σ_t is in square meters, τ in seconds, R in meters and AZ in radians.

For example, assume a 10 m^2 target at 10 Km range from a radar with a 1° azimuth beamwidth and a 1 μsec pulse width. If $\sigma°$ were -20 dBsm, the S/C would be:

$$S/C = 10 - (- 20 - 60 + 40 - 17.5 + 81.8) = -14 \text{ dB}$$

Thus, the target signal in this example is 14 dB below the clutter.

Subclutter Visibility and Improvement Factor

Two different terms are used to measure clutter rejection - subclutter visibility (SCV) and improvement factor (I.F.). Both terms measure a radar's ability to detect moving targets in a clutter background.

SCV is the clutter suppression measured between the clutter and the target signal.

I.F. is the clutter suppression measured between the clutter and the radar's internal noise.

On the decibel scale, the two terms are related as:

$$I.F. = SCV + S/N_{reqd}$$

This relationship is illustrated in Figure 5.10 below:

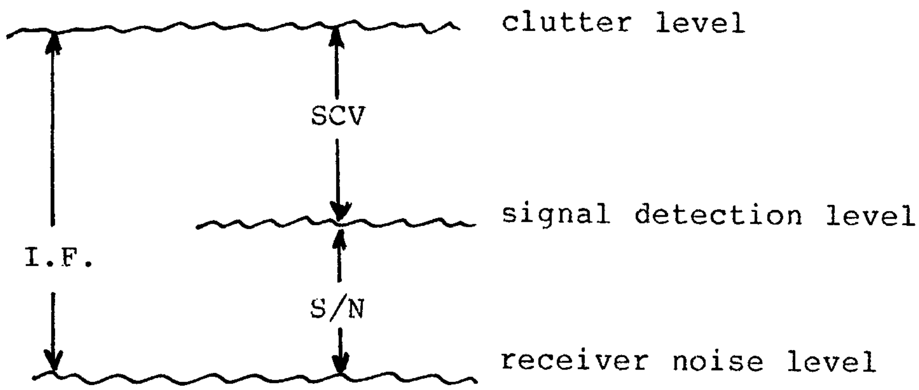


Figure 5.10 Subclutter Visibility

For example, if 13 dB S/N was required, a subclutter visibility of 14 dB is equivalent to an I.F. of 27 dB. An improvement factor of 27 dB means that the clutter is suppressed to 1/500 of its original value.

Note that the improvement factor reduces clutter to a residual value of:

$$\text{Residual clutter} = C/\text{I.F.}$$

As the residual clutter approaches the receiver noise level (N), clutter is no longer the predominant factor. Now the S/C ratio becomes:

$$S/(C + N) = S/\{(C/\text{I.F.}) + N\}$$

Summary

The range resolution of a radar is defined by $c\tau/2$. The angular resolution width, a distance R away, is defined by 2R tan(beamwidth/2). For small beamwidth and grazing angles, the clutter patch, or surface area, is $(c\tau/2) \times R \times$ beamwidth.

The clutter return C is the clutter area multiplied by σ°, the clutter backscatter coefficient. σ° varies with many factors, including radar frequency and polarization, grazing angle and landform/cover.

When clutter enters a radar's resolution cell, it can far exceed the receiver's internal noise level. In those cases, the S/N ratio should be replaced by the S/C ratio - to determine detection range. Either subclutter visibility or clutter improvement factor can be used to measure a radar's clutter rejection capability.

Chapter 6

Clutter Processing

This chapter describes the Doppler shift, velocity ambiguities and methods of suppressing clutter by velocity discrimination. Moving Target Indicator (MTI) processing in a ground based radar, and Doppler processing in a ground and airborne radar, will be explained.

Velocity Discrimination

Clutter returns entering a radar receiver can be a major problem in target detection. These returns can far exceed the internal noise level and the target return. Fortunately, velocity differences may allow discrimination between clutter and air vehicle targets.

To a surface-based radar, surface clutter returns are almost stationary. Land clutter motion varies up to 0.5 m/sec on a windy day in wooded areas. Sea clutter can vary up to 1.5 m/sec, while normal rainfall might be as high as 4 m/sec (~ 8 knots). However, birds and motor vehicles can exceed 50 knots.

Since air vehicle speeds are usually well above 100 knots, they may be discriminated by their higher closure speeds. (Closure speed is the radial component of velocity along the radar's line of sight.) A closure speed of 100 knots is often used as a threshold to distinguish between clutter and air vehicle target returns.

When air vehicles pass abeam of a radar, the closure speed drops to zero. The sector defined by closure speeds less than the velocity threshold is called the Doppler notch. A Doppler notch is illustrated in Figure 6.1.

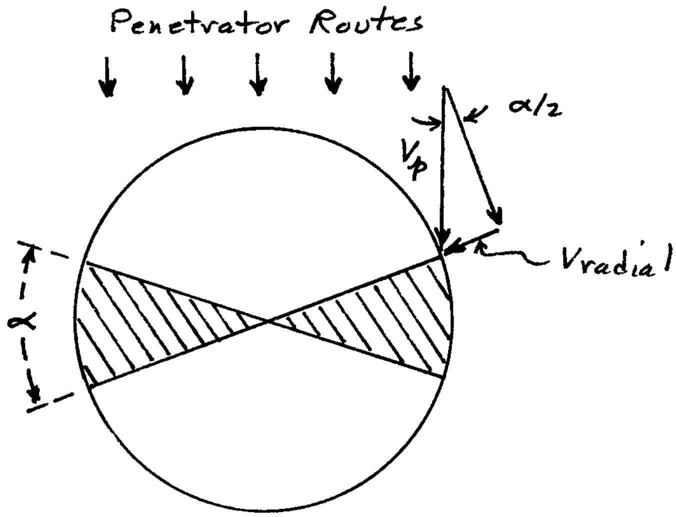


Figure 6.1 A Doppler Notch

The notch angle on each side can be determined by:

$$\alpha = 2 \sin^{-1}(\text{Threshold velocity}/V_p)$$

where V_p = penetrator velocity

For example, with a 100 knot threshold and a 300 knot target velocity, the Doppler notch is 39° wide on each side of the radar.

Doppler Shift and Velocity Ambiguities

The Doppler frequency shift for a target with closing velocity V_c is:

$$\text{Doppler shift} = 2V_c/\lambda$$

A Doppler, or frequency, diagram for the clutter about a ground based radar is shown in Figure 6.2 .

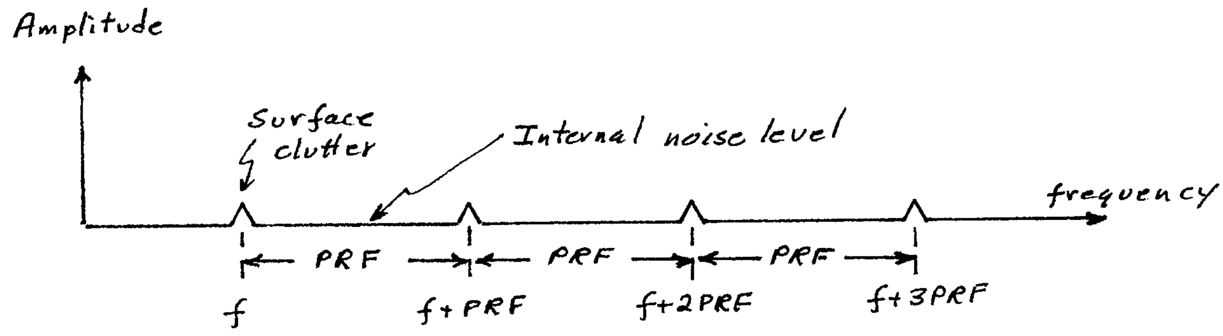


Figure 6.2 Surface Clutter Doppler Diagram

This diagram shows the signal power level as a function of frequency when only surface clutter returns are present. The clutter not only appears centered at the radar frequency, f, but also at f plus and minus multiples of the PRF due to the frequency spectrum of coherent pulses. The Doppler spread of the surface returns is primarily due to clutter motion. The internal noise in the receiver determines the base level.

An air vehicle target that is approaching a surface radar will be displayed within each PRF interval, as shown in Figure 6.3 below:

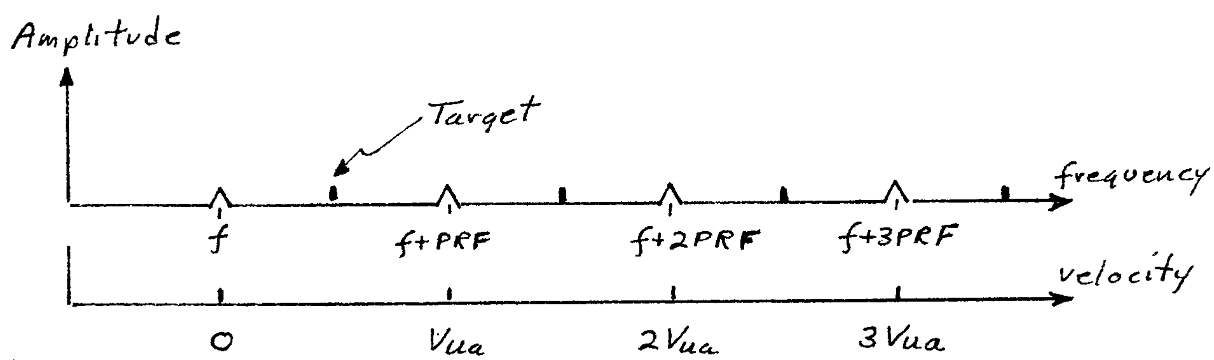


Figure 6.3 Approaching Target on a Doppler Diagram

Two scales are shown. One is the Doppler frequency shift. The other is the corresponding closing velocity. The target closing velocity which corresponds to a PRF is called the maximum unambiguous velocity or V_{ua}. This can be determined from:

$$V_{ua} = \lambda \ PRF/2$$

Closing speeds higher than this value will be ambiguous, since they will lap over into the next PRF interval. For example, the maximum unambiguous velocity for a 3 GHz radar frequency (λ = 0.1 m) and a PRF of 10,000 pulses per second is:

$$V_{ua} = 0.1 \times 10{,}000/2 = 500 \text{ m/sec, or 970 knots.}$$

This means that no ambiguity in target velocity will occur if all target closing speeds are less than 970 knots.

A radar with the same λ of 0.1 m, but with a 360 PRF will have a V_{ua} of only 0.1 x 360 /2 = 18 m/sec, or 35 knots. Now only closing speeds up to 35 knots can be measured unambiguously. For example, targets with closing speeds that differ by integer multiples of 35 knots (e.g. 30, 65, 100 knots) can not be distinguished from each other. This is not a desirable situation, since approaching automobiles can not be distinguished in velocity from faster closing air vehicles.

V_{ua} can be increased by raising the radar's PRF. However, raising the PRF will decrease the maximum unambiguous range.

Staggered PRFs

The effective PRF of a low pulse radar can be increased by a technique called <u>staggered</u> PRFs. This technique will increase V_{ua} without decreasing R_{ua}. Figure 6.4 illustrates a three PRI stagger.

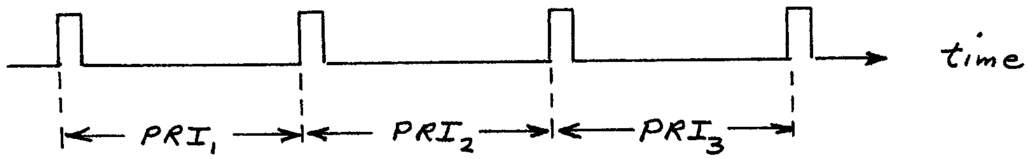


Figure 6.4 A Three PRI Stagger

The ratios between the three pulse recurrence intervals (PRIs) shown are 31/32/33. This stagger will change the maximum unambiguous velocity to 32 times that of the original single PRI radar, as shown in Figure 6.5 from Skolnik[4]

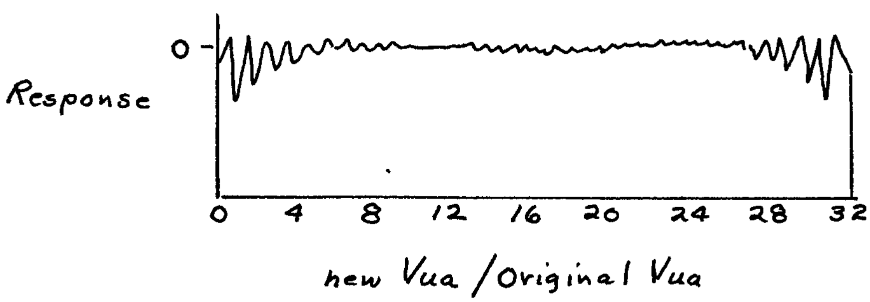


Figure 6.5 A 31, 32, 33 PRI Stagger Response

Some signal attenuation losses (noted by the dips at the top of the diagram) appear due to this stagger. But, V_{ua} is raised to 32 times 35 knots, or to a 1120 knots closing speed in the example - without reducing the maximum unambiguous range.

Moving Target Indicator

One method of setting a clutter velocity threshold in a pulse radar is by a Moving Target Indicator (MTI). In a single delay MTI, the returns from one pulse are delayed and then compared to the returns on the next pulse.

Any returns with range rates (measured by the difference in range over the time between the two pulses) exceeding the threshold are processed as target signals. If not, the returns are eliminated from the MTI display. This method is called a single delay, since the comparison is based on a single PRI delay between two successive pulses. Single delay MTI systems can provide a clutter Improvement Factor (I.F.) of 20 to 25 dB.

A double delay (comparing three pulses) can provide 30 to 35 dB improvement to a pulse radar system. Further improvements for low PRF radars are difficult to achieve, due to scanning motion, PRF stagger, limiting, analog-to-digital (A-to-D) conversion and equipment instabilities. Greater improvements are possible with continuous wave (CW) or high PRF radars which employ Doppler processing.

A Comparison of Pulse and Doppler Radar Types

A simple comparison between low PRF pulse radars and high PRF pulse Doppler and CW radars is shown below:

	Low PRF Pulse Radar	High PRF Doppler Radar
Average power	Low	High
Duty cycle	Low	High
Unambiguous	Range	Velocity
Clutter supression	Fair	Excellent

Note that low PRF pulse radars generally have low average power (but may have high peak power). They depend upon range measurement as the primary detection mode and usually have only fair target detection in clutter, even with MTI velocity discrimination.

Doppler radars use high average power (but relatively low peak power). They depend upon velocity measurement as the primary detection mode, and usually have excellent target detection capability in clutter.

Doppler Processing

Figure 6.6 illustrates Doppler bands or filters in a surface-based radar.

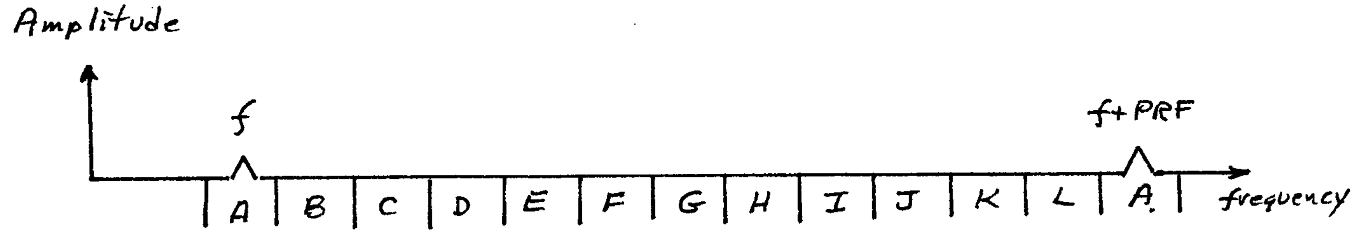


Figure 6.6 Doppler Filters

The first filter contains all the surface clutter around the site. It may extend +/- 100 knots. Targets with greater closing or receding speeds will appear in other filters which are free of surface clutter.

Figure 6.7 illustrates the airborne radar case. For this illustration, assume that the radar platform is moving over the surface at 250 meters per second velocity:

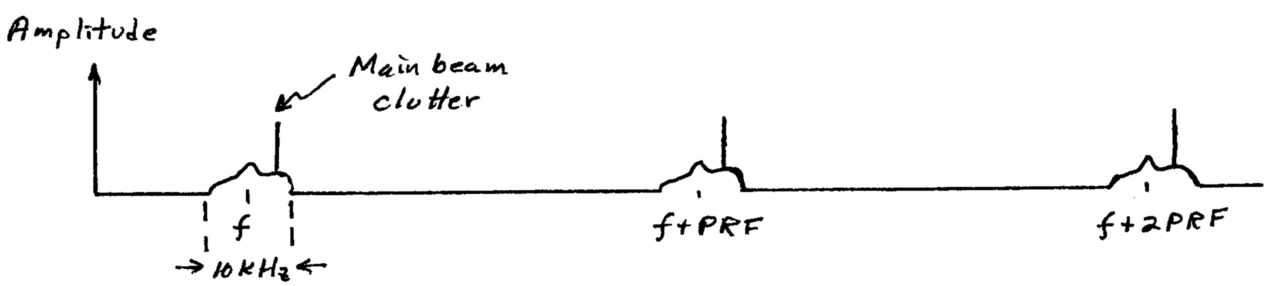


Figure 6.7 Airborne Radar Doppler Diagram

The strongest return shown comes from the mainbeam of the radar as it intersects the surface. Other surface returns are received in the side-lobes of the antenna, and are attenuated by the decreased gain outside the mainbeam. The center return comes from surface clutter directly below the radar, where there is little closing velocity but many strong specular reflections.

The Doppler extremes of the clutter region are determined by the radar platform's surface speed. For example, a 250 m/sec ground speed for a 0.1 m wavelength radar translates into a Doppler shift of 2 x 250 / 0.1 = 5,000 Hz from ground clutter at the radar's horizon - directly ahead of the platform. The clutter at f - 5,000 Hz comes from the ground at the rear horizon. The total Doppler spread of clutter is +/- 5,000 Hz, or 10,000 Hz, corresponding to a +/- 250 m/sec ground speed at this radar frequency.

High Closing Speed Targets

Consider now an air vehicle target which is approaching an airborne interceptor head-on. The relative velocity between these two airborne vehicles will be greater than the radar's surface speed, and the target return will fall outside the clutter region (for a high PRF radar), as shown in Figure 6.8 below:

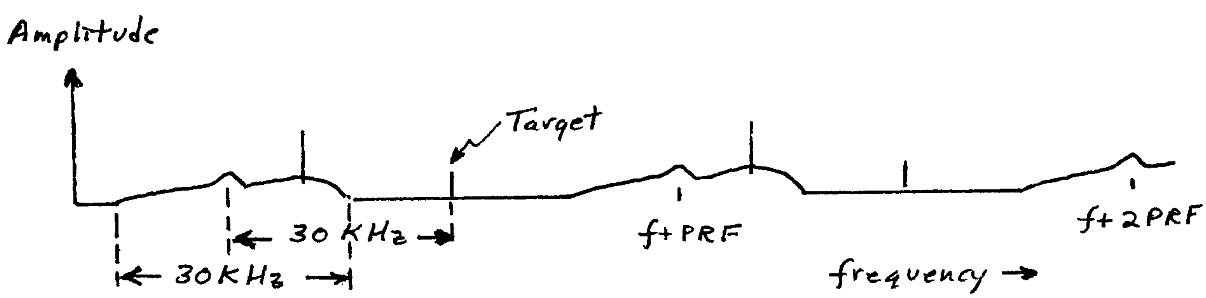


Figure 6.8 Head-on Target Doppler Diagram

For example, assume that the target and the interceptor are both flying at a ground speed of 250 m/sec head-on, for a 500 m/sec closing speed. If the interceptor's radar operates at 10 GHz (a wavelength of 1/30 meters), the Doppler shift will be:

$$\text{Doppler shift} = 2V_c/\lambda = 2 \times 500 \;/\; 1/30 = 30{,}000 \text{ Hz.}$$

The target appears outside of the sidelobe clutter, which extends +/- 15,000 Hz around the radar frequency. The only competing signal shown is the radar's internal noise level.

A high closing rate alone (or being outside the surface return Doppler filter) does not guarantee that the target will be detected. Many factors complicate target detection, such as:

Dynamic range - The radar's dynamic range must process both the strongest clutter and weakest target return. This is illustrated in Figure 6.9 below:

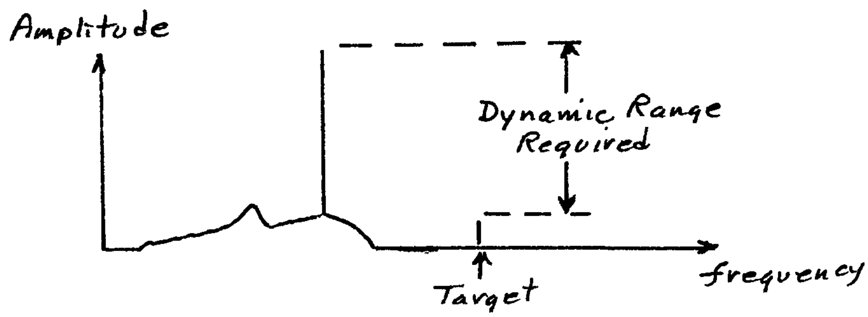


Figure 6.9 Dynamic Range

Clutter improvements - The radar's Improvement Factor (I.F.) must provide the clutter suppression needed.

Radar instabilities, non-linearities and sidebands may limit detection, particularly against low RCS targets.

Low Closing Speed Targets

Consider the case of an airborne radar attempting to detect a target which has a closing speed less than the airborne radar platform's surface speed. One such case, called a tail-on situation, is illustrated in Figure 6.10 below:

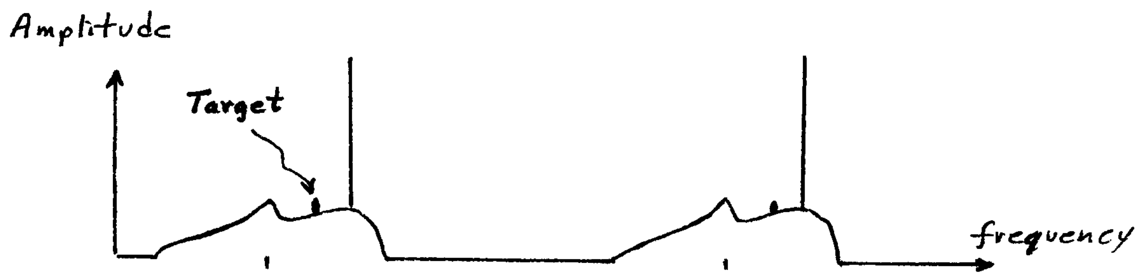


Figure 6.10 Low Closing Speed Doppler Diagram

Now the target competes with the sidelobe clutter. The intensity of the sidelobe clutter (within the same range gate and Doppler filter band as the target) will now be examined to illustrate why a decrease in PRF reduces the sidelobe clutter competing with a target's return. The approach used to calculate sidelobe clutter is based on Cunningham[5].

Most of the clutter within the same range gate as the target comes from a circular ring of average radius R_o (the closest clutter with the same ambiguous range as the target range from the radar) with width $cT/2$ (the range gate size). The clutter backscatter coefficient, $\sigma°$, will be assumed to vary as the sine of the grazing angle, or $\sigma° = \gamma \sin Gr$. The sine of the grazing angle is the radar's height divided by the clutter range. Thus, the potential clutter based on the range gate alone is:

$$\sigma_c = A_c \, \sigma° = \{2\pi R_o \, (cT/2)\} \, \{\gamma \, h_a/R_o\} = \pi \, cT \, \gamma \, h_a$$

However, only a fraction of this area will return energy in the same

Doppler filter band as the target. The locus of points having the same Doppler shift is an isodoppler, sometimes called an isodop. The intersections of the range gate and Doppler filter lines are shown in Figure 6.11:

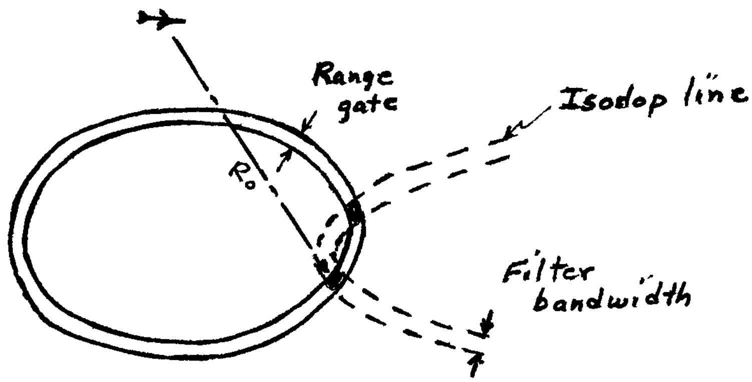


Figure 6.11 Isodop Lines

The effective clutter F σ_c is the clutter cross section with the same Doppler filter bandwidth, B_d, as the target. The effective clutter can be determined as the summation of F σ_c for all the pulses during the target observation time, t. Assume that the Doppler bandwidth, B_d is 1/t. The effective clutter cross section can be estimated from the closest clutter in the target's range gate as:

$$\text{Effective } \sigma_c = \frac{\pi\ T\ \gamma\ h_a\ \lambda\ B_d\ PRF\ R^4}{4\ R_o^{\ 3}\ V_R\ \{1 - V_c/V_r)^2\}^{1/2}}$$

where B_d = the Doppler filter bandwidth

V_R = the radar's velocity over the surface

R_o = range to the closest clutter interfering with the target.

R = range to the target

From which the S/C ratio is:

$$S/C = \left[\frac{G}{G_{sl}}\right]^2 \frac{L_c \quad 4\, R_o^{\,3} \quad V_R \,\{1-(V_c/V_R)^2\}^{1/2} \quad \sigma_t}{L_s \quad \pi\, T\, \gamma\, h_a\, \lambda\, B_d \; PRF \quad R^4}$$

The ratio G/G_{sl} accounts for the clutter entering the sidelobe of the antenna, while the target is in the mainlobe. L_c/L_s accounts for differences in losses between the target signal and the clutter. The range to the closest clutter competing with the target (R_o) depends upon the target range and the unambiguous range (which is a function of the PRF). The system can be said to be isodop limited when the pulse repetition frequency is greater than:

$$PRF > (c/2h_a)\ \{1 - (V_c/V_R)^2\}^{1/2}$$

so that:

$$R_o = h_a/\ \{ 1 - (V_c/V_R)^2\}^{1/2}.$$

When isodop limited: the closest range of sidelobe clutter does not vary with the PRF, but the signal-to-clutter ratio (S/C) does vary as 1/PRF.

For PRFs below this limit, a system can be said to be range limited. Now the range to the closest competing clutter will be a function of the PRF. If the PRF is varied until the range gate with the target includes the sidelobe clutter from the maximum unambiguous range:

$$R_o = R_{ua} = c/2\ PRF.$$

In this case of range limited sidelobe clutter (medium PRFs):

$$S/C \sim 1\ /G_{sl}^{\,2}\ \lambda\ T\ B_d\ PRF^4$$

S/C can be increased by lowering the sidelobe gain (G_{sl}), the wavelength, the range gate or Doppler filter bandwidth, or the PRF. As long as radar sidelobes can not be decreased, lowering the PRF is the dominant factor in reducing sidelobe clutter competing with the target.

For example, assume that the target's closing speed is 20% of an airborne interceptor's surface velocity and that the AI is at 3 Km altitude. The transition between the range and the isodop limit is at:

$$PRF = (3 \times 10^5/2 \times 3) \{1 - 0.2^2\}^{1/2} = 48{,}000 \text{ pulses/sec}$$

The range (R_o) to the nearest sidelobe interference for PRFs of 48,000 and above is:

$$R_o = 3/(1- 0.2^2)^{1/2} = 3.1 \text{ Km}$$

The range to the nearest sidelobe clutter interference for PRFs below 48,000 pulses per second can be varied with PRF. For cases where the PRF is adjusted so that the range gate contains both the clutter at the maximum unambiguous range (R_{ua}) and the target at range R (i.e. R is a multiple integer value of R_{ua}):

$$R_o = R_{ua} = 3 \times 10^5/2 \times PRF$$

Since the range to the nearest clutter is increased as the PRF is decreased, the clutter level competing with a target will be reduced. This reduction can be noted by calculating S/C improvements for several PRFs compared to the transition 48,000 pulses per second:

PRF	15,000	30,000	48,000 pulses/sec
R_o	10 Km	5 Km	3.1 Km
Δ S/C	15 dB	6 dB	Baseline

The table illustrates that a reduction in PRF from 48,000 to 15,000 pulses per second can reduce the competing clutter by 15 dB (a factor of

32) as long as the target range is an integer multiple of 10 Km.

Note that in the example the PRF was chosen so that the target competed with clutter at the maximum unambiguous range. A better choice would be a PRF where the target was placed in the same range gate as clutter free ranges just less than the altitude of the radar platform. Thus if the radar were at an altitude of h_a, the range to the nearest competing clutter would be:

$$R_o \cong h_a + R_{ua} \cong h_a + c/2\ PRF$$

This increases the range to the nearest competing sidelobe clutter by almost h_a. For a 3 Km radar platform altitude this would increase the signal-to-clutter ratio by 6 dB at 30,000 PRF, and by 3 dB at 15,000 PRF over the values listed in the previous table.

One should check to see if the sidelobe levels from clutter ahead and behind the radar overlap. This would add to the clutter background. The combined level can be estimated by root-sum-squaring the overlaps. An example of an overlap is illustrated in Figure 6.12:

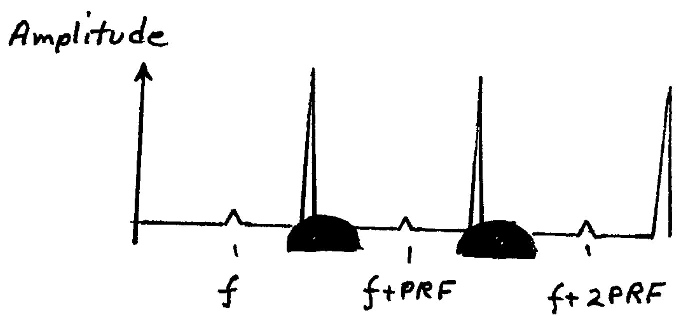


Figure 6.12 Side Lobe Clutter Overlap

For example the extent of the sidelobe clutter for a radar wavelength of 1/30 meter and a 250 m/sec radar surface speed is +/- $2V_c/\lambda$ = +/- 1500 Hz or 3000 Hz. A 15,000 PRF creates a 15,000 Hz spacing between Doppler repeats, so there is no overlap. Thus, the improvements noted in S/C ratios do not need to be modified for clutter overlap in this case.

Summary

Targets can be discriminated from clutter based upon velocity differences between clutter and target motion with respect to the radar. Velocity thresholds are often used to eliminate clutter, but they create Doppler notches when a target flies broadside to a radar.

The PRF and wavelength of a radar determine the maximum unambiguous velocity. Staggered PRFs can be used to raise V_{ua} without decreasing the maximum unambiguous range.

MTI and Doppler processing can separate targets from clutter by differences in closing velocities. High PRFs improve head-on detection for airborne radars, while medium PRFs improve tail-on detection. Equations to estimate S/C in medium PRF modes were presented to illustrate how decreasing the PRF reduces sidelobe clutter. However, adequate clutter suppression alone does not guarantee detection since radar dynamic range requirements and other limiting factors must still be considered.

Chapter 7

Radar Design Considerations

The key radar fundamentals already addressed will be combined in a simple radar design example. This chapter then concludes with a discussion of:

1) Radar frequency bands,
2) Radar PRF modes,
3) Relationship between antenna gain, beamwidth and size, and
4) The blip/scan ratio.

Design Trade-offs

Radar design involves compromises between conflicting goals. Some radar parameters are listed below, along with the implications of an increase and a decrease in the value of each parameter:

Parameter	Increase to	Decrease to
Wavelength	Increase detection range Increase V_{ua}	Improve angle accuracy, Allow smaller antenna
Pulse width	Increase detection range	Improve range resolution, Decrease clutter
PRF	Increase detection range, Increase V_{ua}	Decrease clutter, Increase R_{ua}
Beamwidth	Allow smaller antenna	Decrease clutter and ECM
Antenna height	Increase line of sight	Decrease clutter horizon
Scan rate	Detect fast targets	Increase detection range
False alarm rate	Detect low RCS targets	Decrease clutter false alarms

Radar design is as much an art as a science. It has been said that one can never build the optimum radar, - not even the second best. Perhaps the third best will be a workable design.

Some of the trade-offs listed above can be illustrated by a set of simple calculations. Assume that a radar is desired with the following properties:

1. At maximum radar line of sight range from a 50' antenna height, detect a target of 1 m^2 RCS flying at 40,000' altitude.

$$\text{Radar LOS} = 1.23 \{(h_a)^{1/2} + (h_t)^{1/2}\}$$
$$= 1.23 \{(50)^{1/2} + (40{,}000)^{1/2}\} = 255 \text{ NM}$$

For a 1 m^2 target, the K Value is 255 NM, or 472 KM.

2. No range ambiguities out to 255 NM, or 472 Km.

$$\text{The maximum PRF} = c/2R_{ua} = 3 \times 10^5/2 \times 472 = 318 \text{ pulses/sec.}$$

3. No velocity ambiguities out to 1000 knots (515 m/sec) closing speed, for a 3 GHz (λ = 0.1m) radar.

$$\text{The minimum PRF} = 2V_{ua}/\lambda = 2 \times 515/0.1 = 10{,}300 \text{ pulses/sec.}$$

4. Achieve both of the above goals using staggered PRFs.

Changing the effective PRF from 318 to 10,300 pulses per second requires a 32 fold increase. This could be achieved by a three PRF stagger, with PRI ratios of 31/32/33.

5. Provide a 150 m, or 500', range resolution.

$$\text{Pulse width} = 2 \times \text{Range resolution}/c = 2 \times 150 \;/\; 3 \times 10^8$$
$$= 10^{-6} \text{sec or } 1\ \mu\text{sec.}$$

6. Achieve the needed K Value with a non-coherent scanning radar against a Swerling case 1 target, assuming P_p = 60 dB, $G_t = G_r$ = 40 dB, F_n = 10 dB, L = 10 dB and a 1° azimuth beam.

A. Calculate the single pulse S/N for a K Value of 472 Km.

$S/N = P_p + 2G_t + 2\lambda + \tau + 170 - F_n - L - 4K$

$= 60 + 80 - 20 - 60 + 170 - 10 - 10 - 4(56.7) = - 17$ dB.

B. Determine the S/N_{reqd}. For this Swerling case, using P_d of 0.5 and FAR of 10^{-7}, the S/N_{reqd} is 13 dB.

C. Determine pulse integration and pulse compression needs. A total improvement of 13 + 17 or 30 dB is needed. One of many possible solutions is to provide 10 dB by non-coherent integration of 30 pulses. This leaves 20dB for pulse compression, or a 100 to 1 ratio. Thus, a 100 μsec pulse is needed. (Incidentally, the radar duty cycle will be τ x PRF, or 3.2%).

D. Calculate the dwell time and scan rate required.
The dwell time required is 30 pulses/318 PRF or 0.09 sec.
The scan rate is the beamwidth/dwell time or 1°/0.09 or 11°/sec. or 33 seconds per 360° scan. (This is a very slow scan rate.)

7. Determine the clutter I.F. required to detect a 10 m^2 target 10 NM away from the radar when σ° is -20 dB.

$S/C = \sigma_t - (\sigma^\circ + \tau + R + AZ + 81.8)$ in dB's

$= 10 - (- 20 - 60 + 42.7 - 17.5 + 81.8) = - 17$ dB.

The I.F. = SCV + S/N_{reqd} = 17 + 13 = 30 dB.

If 30 dB (a factor of 1000) clutter improvement can be obtained, this simple design exercise is completed. However, the radar scan rate is very slow and the duty cycle and pulse compression are both high.

Before ending this chapter, the following subjects will be addressed:

1) Radar frequency band categories,
2) Radar PRF mode categories,
3) Some relationships between antenna gain/beamwidth/size, and
4) The term blip/scan ratio.

Radar Frequency Bands

Two sets of radar frequency bands are shown below:

Old Band	Frequency	New Band	Frequency
HF	3 - 30 MHz	A	Up to 250 MHz
VHF	30 - 300 MHz	B	250 - 500 MHz
UHF	300 - 1000 MHz	C	500 - 1000 MHz
L	1 - 2 GHz	D	1 - 2 GHz
S	2 - 4 GHz	E	2 - 3 GHz
C	4 - 8 GHz	F	3 - 4 GHz
X	8 - 12.5 GHz	G	4 - 6 GHz
K_u	12.5 - 18 GHz	H	6 - 8 GHz
		I	8 - 10 GHz
K	18 - 26.5 GHz	J	10 - 20 GHz
K_a	26.5 - 40 GHz	K	20 - 40 GHz
		L	40 - 60 GHz
mm	40 - 300 GHz	M	60 - 100 GHz

The old bands L through K were designated during World War II. The new bands were originally used for electronic warfare, but are now being used also to designate radar frequencies. Note that a radar operating at 3.5 Giga Hertz (GHz) would be in the old S band or the new F band. The initials HF stand for High Frequency, VHF for Very High Frequency, UHF for Ultra High Frequency and mm for millimeter.

PRF Categories

Radar operation is commonly categorized into low, medium and high PRF modes. In a low PRF mode, a radar attempts to operate in an unambiguous range mode. In a high PRF mode, a radar attempts to operate in an unambiguous velocity mode. In a medium PRF mode, both range and velocity ambiguities are present before processing. For a 10 GHz radar, these modes can be roughly categorized as:

Mode	Pulses/sec
Low PRF	100-3,000
Medium PRF	10,000-30,000
High PRF	100,000-300,000

Recall that high PRF Doppler radars can separate a target from sidelobe clutter for head-on closures. Medium PRFs lower the sidelobe clutter for tail-on geometries. Pulse radars depend upon range to detect targets, and thus, use low PRFs.

Antenna Gain and Size

Antenna gain is the ratio between the maximum power per unit solid angle compared to the total power emitted in all directions (over 4π radians). Thus:

$$G = \frac{\text{Max power / unit solid angle}}{\text{Total power / }4\pi}$$

For an antenna with EL and AZ beamwidths (measured in radians) or El and Az beamwidths (measured in degrees), the theoretical antenna gain is:

$$G = \frac{4\pi}{EL\ AZ} = \frac{41{,}000}{El\ Az} \text{ (converting radians to degrees)}$$

The beam shape of a actual antenna limits this gain, changing this equation to:

$$G \sim 32{,}000 \,/\, El\ Az$$

For example, a 1° x 1° antenna would have a gain of about 45 dB, while a 1° x 5° antenna would have about a 38 dB gain.

The length of an antenna is approximately equal to the radar wavelength divided by the beamwidth, or:

$$L_{EL} \sim \lambda/EL \qquad\qquad L_{AZ} \sim \lambda/AZ$$

For example, if Az = 1°, the antenna size would be about 57 times the wavelength. Thus, a 1 cm wavelength (30 GHz frequency) antenna would be about 57 cm or 2 ft long. A 10 cm (3GHz) radar would be about 570 cm or 19 ft long. Note that a higher frequency allows a smaller antenna size for the same beamwidth. If a smaller beamwidth is needed to improve tracking accuracy for a fixed size antenna, a higher radar frequency can be chosen. (Antennas on interceptor aircraft are severely limited in size, forcing most AI radars to operate in the X band or at higher frequencies).

Blip/Scan Ratio

A blip denotes detection of a target on a particular radar scan. The fraction of the scans displaying the target return is called the blip/scan ratio, or probability of detection per scan. This ratio will be very low when targets are far away, but builds rapidly as range decreases. A value of 0.3 may be sufficient for manual target detection by trained radar operators.

Automatic detection schemes are based upon criteria such as: detection requires at least r blips out of n scans. The value of n is usually 5 or more, while r is determined by the blip/scan ratio required. For example, automatic detection for a scanning radar might use a criterion of 3 blips out of 6 scans.

Summary

Choices of many radar parameters are compromises between conflicting goals. A simple search radar design example was used to illustrate the relationships between radar sensitivity, maximum unambiguous range and velocity, range resolution, pulse compression, scan rate and clutter rejection.

Radar frequency bands, radar PRF modes, simple relationships between antenna gain, wavelength and size, and the blip/scan ratio were also discussed.

Chapter 8

Fundamentals of Electro-optics

This chapter describes infrared (IR) and optical detection of airborne targets. First, the electromagnetic spectrum and atmospheric absorptivity are discussed. Then, black body radiance is addressed and simple equations presented to predict IR detection range. The effects of IR losses and background clutter are noted, and a flare countermeasure is described. Optical detection factors are then introduced, with emphasis on detection range, contrast and camouflage paint schemes. The chapter ends with a discussion of two sets of competing requirements: 1) low reflectance camouflage vs low absorptance of thermal radiation from nuclear bursts, and 2) low IR signature vs high penetration speed.

The Electromagnetic Spectrum

A simplified picture of the electromagnetic spectrum is illustrated in Figure 8.1.

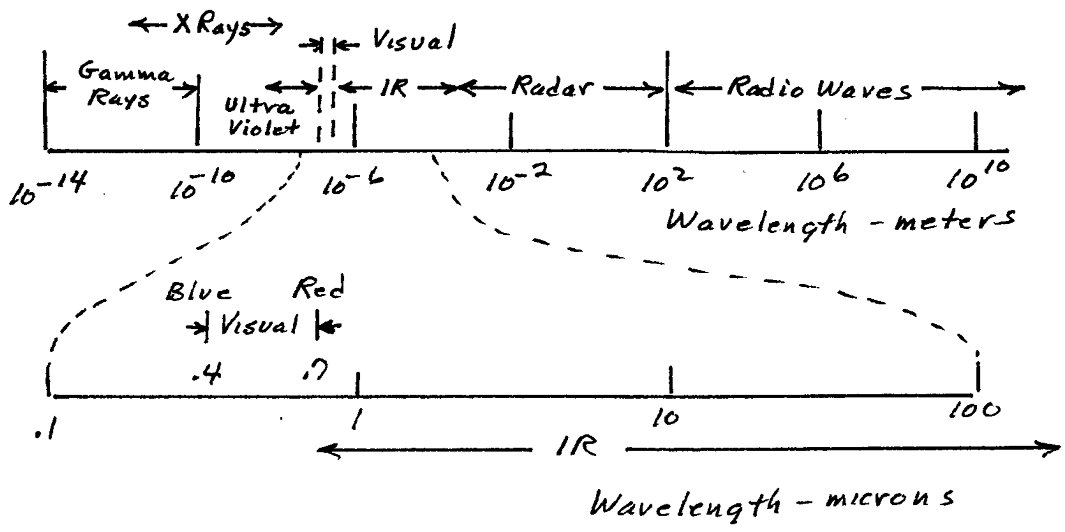


Figure 8.1 Electromagnetic Spectrum

Visual wavelengths vary from 0.4 to 0.7 microns (1 micron or 1μ = 10^{-6} meters), or from blue to red, respectively, on the color spectrum. The infrared or IR (meaning below red in frequency) spectrum extends from 0.7 to above 200 microns. Sometimes the infrared region is divided into short wave IR (0.7 to 3μ), medium wave IR (3 - 5.5μ) and long wave IR (8 - 14μ). The 5.5 - 8 μ band has limited utility due to atmosphere absorption.

Atmosphere Absorption

The absorptivity of the earth's atmosphere severely limits the use of certain wavelengths. Figure 8.2 from Santa Barbara Research Center[6] illustrates absorption by water (H_2O) and carbon dioxide (CO_2) for a 1000' horizontal sea level path with 5.7 mm precipitable water at 79°F.

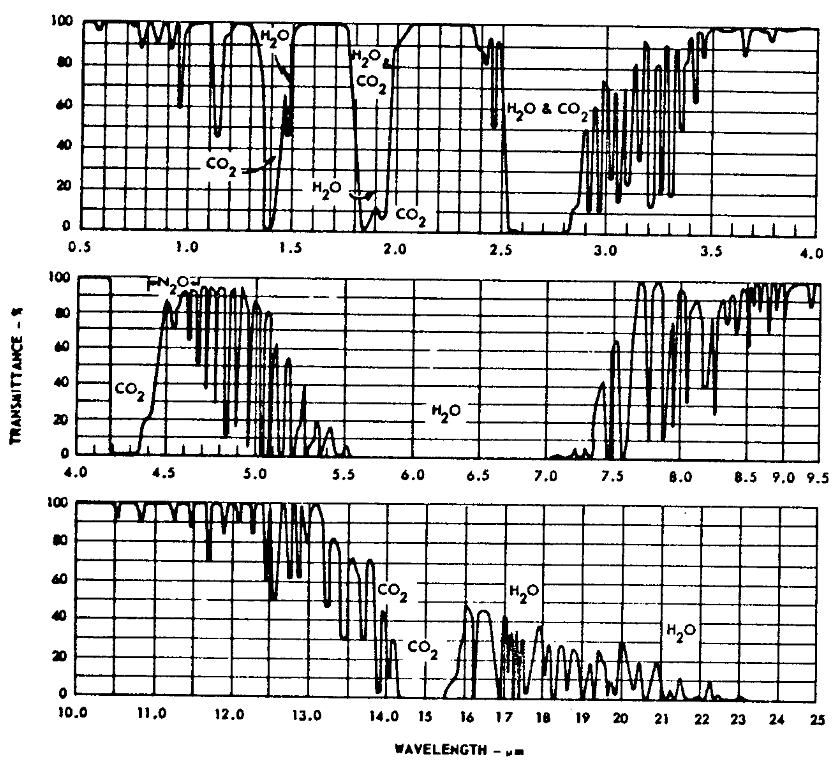


Figure 8.2 Atmospheric Absorptivity

Note the relatively open windows in the visual region and at several wavelengths in the IR region (e.g. 3.5 to 4μ and 8.5 to 13 μ).

Infrared Detection Fundamentals

IR detection fundamentals (radiance, maximum detection range, losses/-clutter and an IR countermeasure will be discussed.

Radiance

The radiance of any object depends upon its black body temperature. A true black body absorbs all incident energy at all wavelengths and radiates energy only due to its temperature. Radiance can be defined as the radiant power (watts) per unit projected area (cm^2) per unit solid angle (steradian). Figure 8.3 illustrates spectral radiance levels (for a 1μ bandwidth) for various black body temperatures.

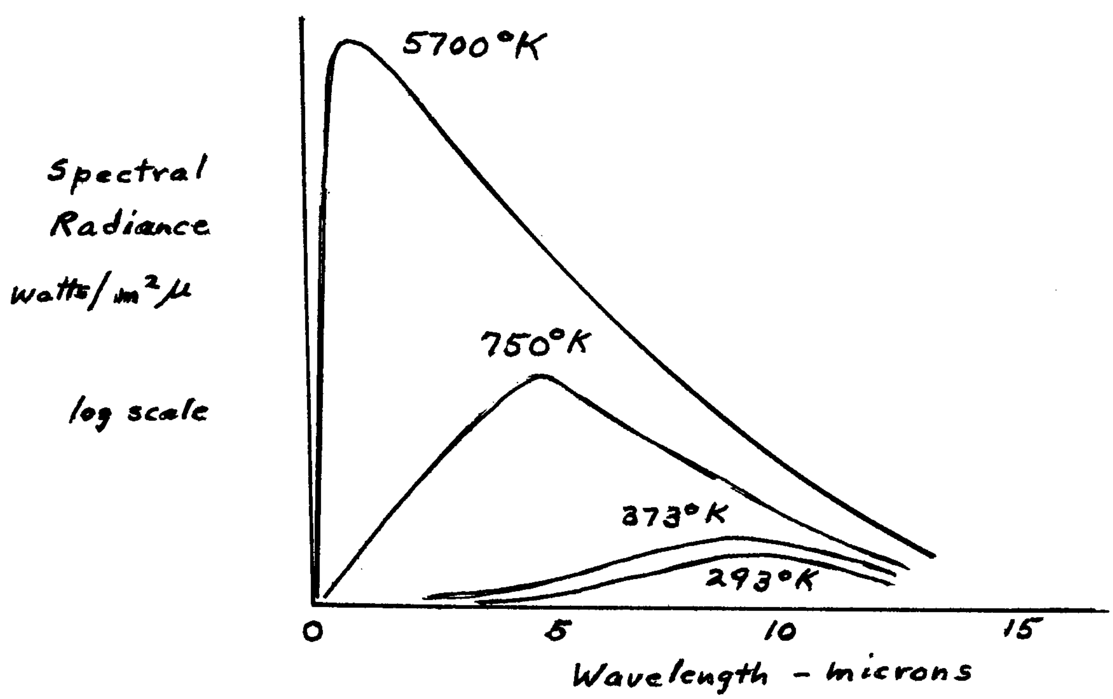


Figure 8.3 Black Body Radiation

Note that a higher black body temperature creates higher radiance levels, with a peak at lower wavelengths. A standard room temperature case is a 293° Kelvin (K) black body temperature, which is 20° Celsius (C) or 68° Fahrenheit (F). The black body radiance for 750°K (477°C or 891°F) - typical of a gas turbine exhaust - is also shown, as well as that for 373°K (boiling water) and 5700°K (the sun).

The infrared intensity (measured in watts per steradian) received by a detector at some distance from an emitter depends upon the radiation from the object in that direction and the atmospheric losses. For a given bandwidth and field of view, the detector measures the intensity of a target (less atmospheric losses). Figure 8.4 illustrates the intensity measured from relatively close to a typical jet engine when the detector is 20° off the tail.

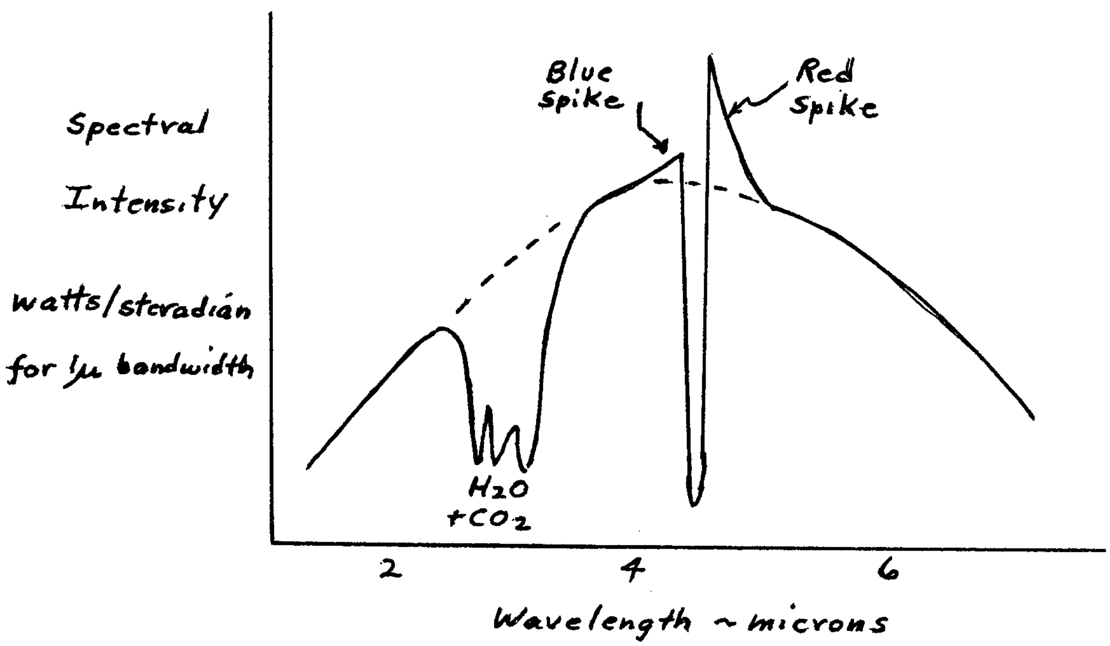


Figure 8.4 Typical Jet Engine Signature

The hot engine produces the characteristic rounded black body curve. Water vapor and carbon dioxide in the atmosphere reduce the intensity near 3 microns. CO_2 causes a major loss near 4.3 microns. The dotted line fills in the black body curve which would have been measured without the atmosphere.

The engine plume creates a spike. This is separated by CO_2 absorption into a blue spike (lower wavelength) and a red spike (higher wavelength). The spike contributes a small portion of the total energy (the integration of this curve over all wavelengths). Most of the energy is emitted by the hot engine. As the distance from the engine increases, the energy received from these spikes is reduced (relative to that from the hot engine parts) due to absorption, and thus, become less important for tail-on IR detection.

Maximum Detection Range

A simple method to estimate the maximum IR detection range of a point target is shown below:

Let I = IR Intensity of the target, within the detector's bandwidth and field of view, in watts/steradian.

NEI = Noise Equivalent Irradiance or sensitivity of the IR detector, in watts/square centimeters.

R = Range between detector and target, in centimeters.

For one way transmission from an IR emitter to the detector, without losses or clutter:

The signal received = $S = I/R^2$ in watts/cm^2

The internal noise in the detector = $N = NEI$ in watts/cm^2

The S/N is then:

Signal to Noise Ratio = $S/N = I/NEI\ (R_{max}{}^2)$

If the S/N required for detection is known, the maximum detection range can be determined as:

$$R_{max}^2 = I/NEI\ (S/N)_{reqd} \quad \text{in } cm^2\text{, or}$$

$$R_{max}^2 = I\ (10^{-10})/NEI\ (S/N)_{reqd} \quad \text{in } Km^2$$

For example, assume that NEI = 10^{-10} watts/cm^2, I = 1000 watts/steradian and the $(S/N)_{reqd}$ is 10. Then the maximum detection range (without losses or clutter) can be determined by:

$$R_{max}^2 = 1000\ (10^{-10})/10^{-10}\ (10) = 100\ Km^2$$

$$\text{or } R_{max} = 10\ Km.$$

Losses and Clutter

Atmospheric attenuation will reduce the signal density at the detector. A loss fraction, L, can be represented by:

$$L = e^{-aR}$$

where a is called the extinction coeficient and depends on the detector wavelength and the atmospheric conditions. These losses are significant in the IR region, but are not usually significant for radars operating below the mm region.

Typical values for extinction coefficients are shown for two IR wavelengths for various sea level visibilities (when R is expressed in kilometers).

Visibility →	40 Km	15 Km	5 Km	3 Km
Called →	Very Clear	Clear	Medium Haze	Haze
$\lambda = 1\ \mu$	.07	.2	.5	.8
$\lambda = 4\ \mu$	.04	.1	.3	.5

Rain can also cause large transmission degrades in the IR and visual regions. The value of a for a rainfall rate r (in mm/hour) can be estimated by:

$$a = 0.24\ r^{2/3}$$

Thus:

Rainfall	Light	Medium	Heavy	Cloudburst
mm/hour	2.5	12.5	25	100
a	0.44	1.3	2.1	5.2

A new equation for the one way transmission range (R) with atmospheric losses added is:

$$R^2 = (I\ 10^{-10}\ e^{-aR})/NEI\ (S/N)_{reqd} \quad \text{in Km}^2$$

This can be written as:

$$R^2 = A\ e^{-aR}$$

where $A = I\ (10^{-10})/NEI\ (S/N)_{reqd} = R_{max}{}^2$

Thus, the value of R can be found from

$$R^2 = R_{max}{}^2\ e^{-aR}$$

Taking the natural logarithm of this expression:

$$2 \ln R = 2 \ln R_{max} - aR$$

Or

$$2(\ln R_{max} - \ln R) = aR$$

This expression is plotted in Figure 8.5 for the example of R_{max} = 10 Km for various values of extinction coefficients.

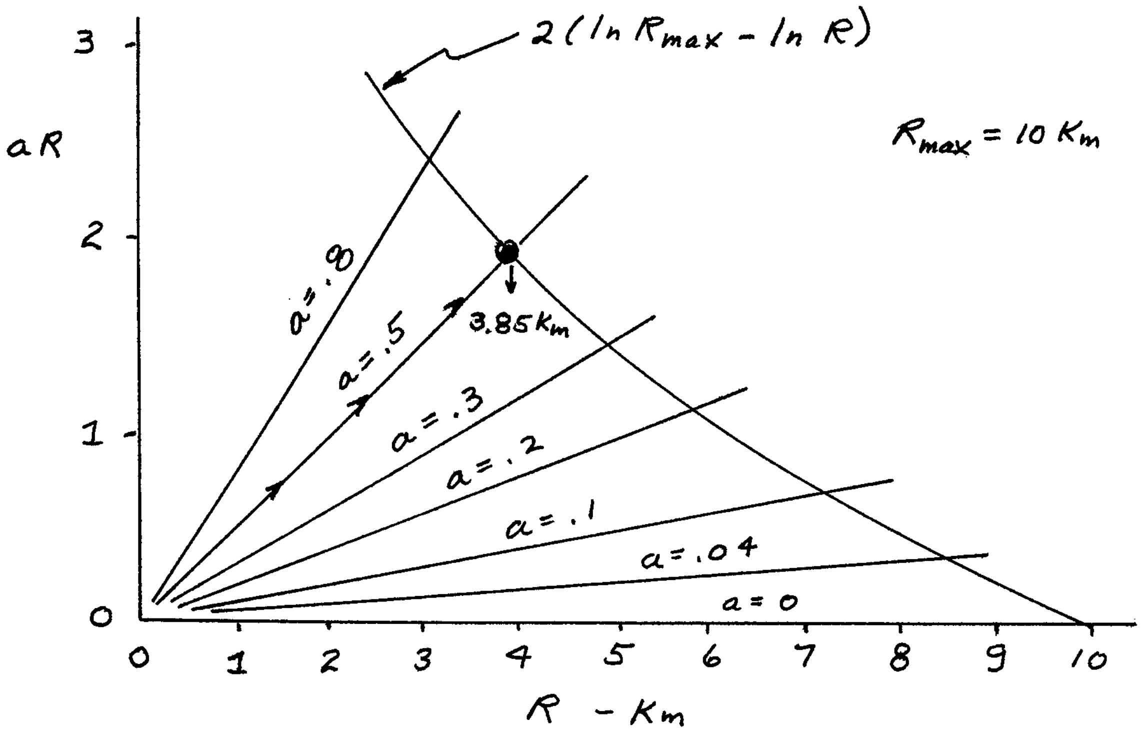


Figure 8.5 Graphical Solution for IR Detection Range

The IR detection range, R, can be found graphically in Figure 8.5 by the intersection of the curve labeled $2(\ln R_{max} - \ln R)$ and the appropriate line aR. For example, for an extinction coefficient of .5, the detection range is found to be 3.85 Km. This is a major reduction in IR detection from the 10 Km maximum range estimated without atmospheric attenuation.

Clutter will dominate detection if the clutter power density from background IR sources exceeds the NEI of the detector. Possible sources of clutter include:

Sun light - either direct radiation, or reflected from clouds, land, or sea surfaces.

Discrete hot spots - from any return which is at a higher temperature than the general background.

Clutter returns will raise the noise threshold from NEI to (NEI + C). The symbol C refers to all clutter returns within the detector's wavelength and field of view, and is measured in watts/cm^2. The modified equation with clutter and atmospheric losses is:

$$R^2 = I\ e^{-aR}/(NEI + C)\ (S/N)_{reqd}$$

Clutter effects can be reduced by spectral, temporal or spatial discrimination. Spectral (or color) discrimination might be achieved by narrowing the bandwidth (to look for a spike) or by use of a two color system (to compare intensities received at two separate wavelengths). The value of a two color system is illustrated in Figure 8.6.

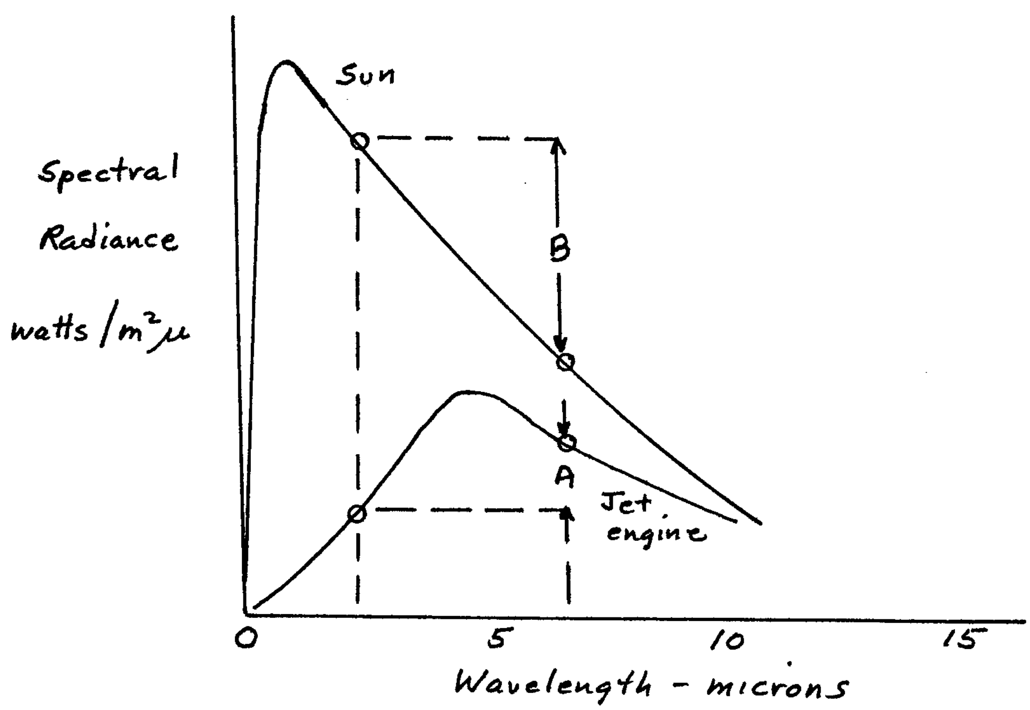


Figure 8.6 Spectral Discrimination

With a two color system, a jet engine target might be distinguished from emitters with a different black body temperature. For example, the sun has a much higher black body temperature than a jet engine, so that its radiance peaks at a lower wavelength. A two color IR system would detect a lower radiance (A) for the jet engine at the lower color wavelength compared to the higher color wavelength. However, the sun would have a higher radiance (B) at the lower wavelength.

Temporal discrimination detects a moving target by frame to frame subtraction, similar to radar MTI processing. Spatial discrimination recognizes a target by its distinct shape and/or size.

Without some discrimination technique, IR sensors faced with background clutter may have very limited capability - due to the high probability of false alarms from clutter.

A Countermeasure

Since IR detectors depend upon receiving a signal or reflection from a hot source, a good countermeasure is to provide false hot sources to decoy the detector away from the air vehicle target. A simple way to provide an alternate source is by dropping flares. The higher intensity of the flare (compared to vehicle emissions) may pull an IR detector off the air vehicle and towards the ground. For example, if missile IR detectors have relatively narrow fields of view and good reacquisition capability, flares must be dispensed with rather precise timing - when both the target and the flare are in the field of view, yet in the last few seconds of missile flight - so that the target can not be reacquired. With a limited inventory of flares, a reliable warning system is required to insure that flares are only dispensed when needed.

Optical Detection Fundamentals

Two topics will be addressed - optical detection range and camouflage.

Detection Range

Factors which affect optical detection of a target can be grouped into three categories - target properties, environmental factors and operational factors. Some examples are shown in the table below:

Target	Environment	Operational
Size, Motion	Masking	Search pattern
Illumination	Atmosphere	Observation time
Compromising signatures	Background contrast	Camouflage

Three factors are of interest here - compromising signatures, background contrast and camouflage. Compromising signatures can be an overriding factor in optical detection of air vehicles, with such tell-tale signs as:

1) Smoke from air vehicle engines,
2) A smoke trail from powered weapons launched,
3) Navigation lights or beacons,
4) Contrails at high altitude,
5) A trailing vortex or rooster tail at very low altitude,
6) Glint due to specular reflections, or
7) A shadow on the earth's surface.

If optical detection is a concern, these compromising signatures should be eliminated. For example, smokeless engines, smokeless rocket exhausts, avoidance of altitudes where contrails are formed, avoidance when flying at low altitude of areas such as loose snow or sand (where rooster tails are formed) and use of flat paint (to avoid glint) can reduce these tell-tale signs. A shadow can not be eliminated on a sunny day. Shadows can assist optical detection, particularly when a low altitude penetrator is flying over smooth, bright surfaces (e.g. water or desert). However, raising the flight altitude will lessen the sharp contrast of the shadow against the background surface.

Assuming that there are no compromising signatures, contrast becomes the most important factor in optical detection.

Contrast is defined as the difference in light intensity between the target and the background, divided by the background light intensity. This is shown below:

$$\text{Contrast} = (I_{target} - I_{background})/I_{background}$$

where I = light intensity in watts per steradian

(Note that contrast values will exceed 1.0, if the target intensity is more than double the background intensity.)

An observer attempting optical detection distinguishes a target by its contrast with the background. The greater the contrast between the target and the background, the greater the detection range. Figure 8.7 illustrates the unaided eye's optical detection range with contrast (in daylight with a clear atmosphere and no compromising signatures).

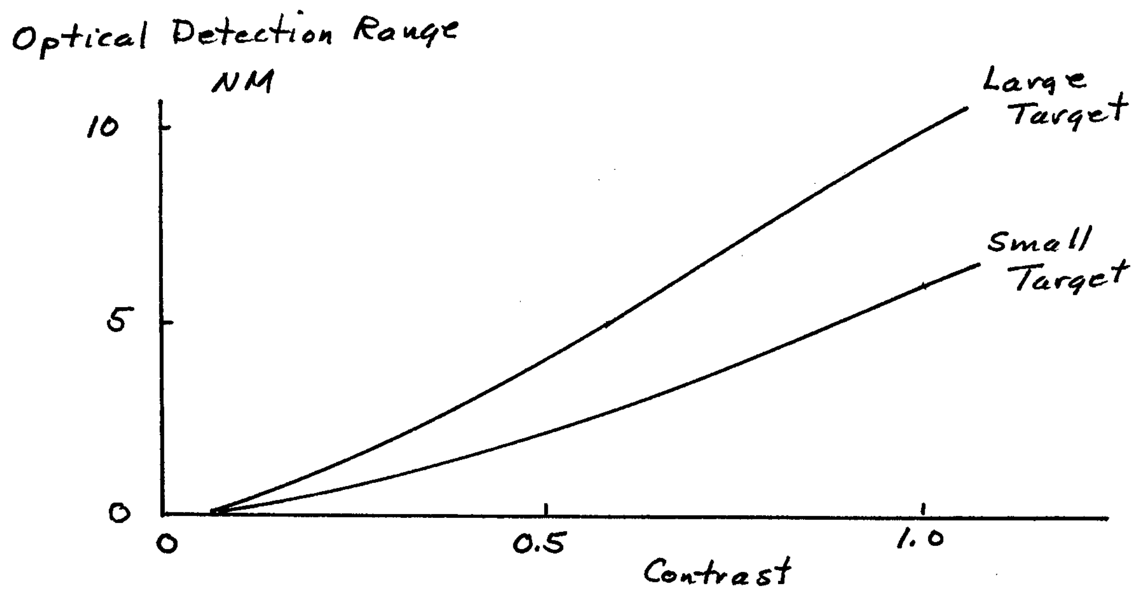


Figure 8.7 Optical Detection Range

Note that detection range varies with target size. A large target (bomber or cargo aircraft) can be detected at about double the range as a small target (fighter or interceptor size). For a small target detection ranges can vary from near 5 NM for high contrasts, to under 1 NM for very low contrast values. For a bomber target detection can range from 10 NM to below 2 NM.

With sufficient visibility and lighting, detection depends upon the contrast between the target and the background. Contrast is proportional to the difference between the reflectance of the target and that of the background. (Note that reflectance is the fraction of the incident radiant flux reflected. Reflectance is equal to (1 - absorbtance). The fraction reflected is the reflectance, the fraction absorbed is the absorbtance.)

Camouflage

Air vehicle surfaces are usually painted. Paint is coded by five numbers. The first digit refers to the spectral property (i.e. 1 = gloss, 2 = semi-gloss and 3 = flat). The second digit refers to the color (e.g. 4 = green, 5 = blue, 6 = grey and 7 = white/black). The last three digits refer to the reflectance (e.g. 100 = 10.0% reflectance). Thus, a flat, grey 10.0% reflectance paint would be labeled 36100 and a glossy, white 80.0% reflectance paint would be labeled 17800.

A paint scheme which minimized the difference in reflectance with the background would minimize optical detection, and so would be said to camouflage the air vehicle.

Two cases will be considered. The first is that of an observer looking <u>down</u> to attempt visual detection against a low altitude penetrator. The second case is looking <u>up</u> into a sky background to detect a penetrator flying above the observer.

Typical reflectance ranges of surface backgrounds are noted below:

Water	3 - 10%	Bare Ground	10 - 20%
Forests	3 - 10%	Desert	20 - 30%
Fields	3 - 20%	Fresh Snow	70 - 86%

Note that paint reflectances in the 5 to 15% region would minimize contrast in look down cases for all but the fresh snow case. Even with snow backgrounds, any surface irregularities, due to trees, cultural features or hills would provide breaks in the snow which could be confused with a target. Since all possible backgrounds can not be matched with one paint scheme, a compromise of 5 - 15% reflectance paint might be chosen.

Paint color can be chosen to match the background color. However, color differences are usually not a factor in increasing detection range, when more than a few miles separate the target and the observer. Thus, the choice of colors is optional, with greys, greens or blues sometimes preferred. (Some colors may not be desired due to higher IR reflectance - such as browns and tans.)

The familiar camouflage patterns using two or more colors, separated by wavy lines, can break up the size/shape contour and delay pattern recognition of the particular air vehicle type (e.g. B-52 vs C-141). One camouflage scheme, called European I, consists of two dark greens (34092 and 34102) and one dark grey (36118).

Flat paints will reduce sun glint. Thus, flat, green/gray colors with 5 - 15% reflectance are good camouflage paint choices for low altitude penetrators against look down observation during daylight.

In the look up case, it is very difficult to seriously degrade daylight detection by camouflage, since the sky is generally much brighter than the vehicle. Simulations in the 1940s showed that lights along the leading edges could deny look up detection, if the intensity of the lights were varied to match the sky shine. For daylight look up conditions, no camouflage scheme can accomplish this, but a flat, grey 20 - 40% reflectance paint scheme can help. Thus, an air vehicle operating at high altitude might benefit from a higher reflectance paint.

At night, camouflage paint is not an issue, except that a shiny black surfaces (such aircraft were called black widows in World War II) can minimize detection by searchlights. Thus, air vehicles on night missions, at low or high altitudes, might be camouflaged with shiny black paint.

Now that IR and optical detection have been discussed, consider some conflicts between reducing IR/optical detection range and other penetrator requirements.

Two Conflicting Requirements

Two conflicts in air vehicle requirements will be addressed. The first is between low reflectance camouflage paint (to reduce optical detection range) and highly reflective surfaces (to survive thermal radiation from nuclear bursts). The second conflict is between low IR signature (to reduce IR detection range) and high speed penetration (to reduce time under defense coverage).

Consider camouflage vs thermal absorption first. Low reflectance paint (5 - 15%) can reduce contrast with surface backgrounds, and thus reduce optical detection range. Since the absorbtance (the fraction absorbed) and the reflectance (the fraction reflected) must equal 1, a low reflectance paint means that a high fraction of the incident thermal energy will be absorbed by the surface, assuming that reflectance does not vary with wavelength in the optical bandwidth.

Air vehicles may be required to survive a certain level of thermal radiation from nuclear bursts. In this case, a higher reflectance paint is a simple way to reduce the thermal energy absorbed, and thus increase vehicle survival. This directly conflicts with low reflectance camouflage requirements. If both can not be satisfied by one paint scheme, a trade off can be performed to determine reflectance values which will maximize some overall measure of effectiveness, such as probability of mission completion. The critical thermal radiation case may be escaping from a air base which is under attack, while the camouflage critical case may be airborne interceptor attacks requiring visual detection.

A second conflict arises in the requirement to reduce IR radiation vs minimizing exposure time to defense threats. Increased speed can satisfy the latter requirement, but this will cause increased skin temperatures, particularly during low altitude flight. Increased skin temperatures, primarily on the leading edges (fuselage, wing, nacelles and tail surfaces), may allow IR detection in the forward hemisphere. Again, a trade off is needed to determine if an optimum speed exists which can balance lower IR signature and higher penetration speed.

Summary

In the electromagnetic spectrum, the optical wavelengths range from blue (0.4 microns) to red (0.7 microns) and the infrared extends from 0.7 to beyond 200 microns.

IR radiance depends on the emitter's black body temperature. A typical jet engine IR signature comes close to the theoretical black body case, with the addition of engine plume spikes. Spikes can be separated into a blue and a red portion by CO_2 absorption.

IR detection range can be estimated from:

$$R^2 = I\ e^{-aR}/(NEI + C)\ (S/N)_{reqd}$$

Atmospheric losses due to rain and haze severely limit the performance of IR systems. IR background clutter can cause many false returns and can dominate detection. Various discrimination techniques (spectral, temporal or spatial) can be used to improve the capability of IR systems in clutter.

One countermeasure to IR missile guidance is use of flares, dispensed at precise times, to decoy the missile into the ground. A reliable warning system is needed to conserve the limited number of flares carried by an air vehicle.

The key factors in optical detection are compromising signatures, background contrast and camouflage. Compromising signatures include smoke, contrails, dust/snow vortices, sun glint and shadows. Without these tell-tale signs, the reflectance contrast between the target and the background becomes the major detection factor. Air vehicle camouflage paint can reduce this contrast, and significantly decrease daylight look down visual detection range in clear weather.

There is a conflict between requirements for low reflective camouflage paint (for low altitude penetrators) and for highly reflective surfaces (to increase survival of thermal radiation - e.g. when escaping from a base under nuclear attack). A second conflict arises in requirements to reduce IR emissions, yet fly at high speeds to reduce exposure time to defense threats. Trade offs are needed to choose a paint scheme and a speed to balance these conflicting requirements.

PART III

OFFENSIVE/DEFENSIVE INTERACTIONS

Chapter 9

Basic Tactics/Countermeasures

Penetration may be considered to be a game - though a deadly serious game - between the offense and the defense. This chapter describes fundamental tactics and countermeasures employed by each side to try to win the game, or at least to end the game to their advantage. The probability of success of various defense actions is defined, and methods of compounding single shot kill probabilities are introduced.

Basic Tactics

Air vehicle penetrators attempt to minimize losses by three basic tactics - avoid, degrade and destroy. The defense attempts to thwart these actions by three basic tactics - detect, destroy and survive (an offensive suppression attack). Key tactics are listed below:

<u>Offense Tactics</u>

<u>Avoid</u>	<u>Degrade</u>	<u>Destroy</u>
Low altitude	Warning	Suppress C^3
High speed	ECM	Suppress SAMs/AAA
Low observables	Mass	Attack air bases
Careful routing	Timing	Develop corridor
Standoff	Surprise	Lethal self-defense

<u>Defense Tactics</u>

<u>Detect</u>	<u>Destroy</u>	<u>Survive</u>
AWACS	Forward CAP	Dispersal
Ground net	Mobility	Deception
Doppler radars	ECCM	Emission control
Look-down AI	Shoot-down missile	Hardening
Forward deploy	Effective weapons	Proliferation

The <u>offense</u> has reacted to the <u>defense</u>. For example, low altitude was a reaction to improved ground nets and high altitude SAM capability. The defense has reacted to the offense. For example, the offensive low altitude tactic hastened defense deployment of AWACS, look-down AIs, shoot-down missiles and improved clutter rejection techniques (MTI and Doppler processing). These interactions (and others) will be outlined in this chapter, and illustrated by applications in later chapters.

During peacetime, each side tries to exploit the other side, by intercepting signals and collecting information. The countermeasure to exploit is to deceive the enemy - by hiding wartime capability and intentions and by planting false information. Since emissions may be a prime source of intelligence, each side should severely limit all active signals (e.g. radar and communications) which could give away information of value to the other side.

Emissions should also be minimized and carefully controlled during conflicts. For example, the offense can give away penetrator location by radar signals (such as ground mapping or active warning systems) and thus lose the advantage of low altitude and low radar cross section. The defense can give away the locations of weapon systems and targets by their unique emission signatures.

Fundamental Defense Actions

Fundamental air defense actions can be divided into: 1) assignment actions, 2) intercept actions by local defenses (i.e. surface-to-air missile and antiaircraft artillery weapon systems) and 3) intercept actions by area defenses (i.e. airborne interceptors). Each of these three categories will be described.

Assignment Actions

Alerted - sensor ready and on-line
Detection - of the penetrator
Tracking - estimating penetrator position and velocity
Identification - as a hostile or an unknown
Command, Control and Communications - leading to an intercept decision
Assignment - of specific weapon systems by the weapons director

Hand off - to other defense networks
Reassignment - if target survives

The above assignment actions are common to any net (or defense network of sensors and C^3 elements) which controls one or more weapon systems. Each action can be represented by a conditional probability of success, which can be combined (if all actions are sequential) as shown below:

P_{assign} = Probability of weapons assignment against an intruder

$$= P_{al}\, P_d\, P_t\, P_i\, P_{c^3}\, P_{as}$$

where P_{al} = Probability sensor is alerted and on-line
P_d = Probability of penetrator detection with sensor
P_t = Probability of track given detection
P_i = Probability of identification given track
P_{c^3} = Probability of C^3 given identification
P_{as} = Probability of assignment given C^3

For example, if all probabilities are .95, then $P_{assign} = .95^6 = .735$.

Detection probability (P_d) has already been examined for radar sensors. Other net assignment probabilities will be illustrated in the attrition analysis (Part IV of this text).

The time from first penetrator entry into the radar net until weapons assignment can be found by adding the following times:

t_{al} = time to alert sensor following entry
t_d = time for sensor detection
t_t = time to track following detection
t_i = time to identify following track
t_{c^3} = time for C^3 following identification
t_{as} = time for assignment following C^3

The total time to assignment is:

$$t_{assignment} = t_{al} + t_d + t_t + t_i + t_{c^3} + t_{as}$$

For example, assume that a EW/GCI net has been fully alerted, that the detection and identification times are each 1 minute, and that all other times take .1 min each. Then:

$$t_{assignment} = 0 + 1 + .1 + 1 + .1 + .1 = 2.3 \text{ minutes}$$

A GCI or AWACS sensor searching for a previously undetected penetrator may need several minutes to initiate an assignment order. A SAM/AAA acquisition radar which has advance warning of the penetrator's track may be able to initiate weapon assignment orders in several seconds. These delay times will be compared to defense coverage times in the next chapter.

SAM and AAA Actions

Alerted - SAM/AAA site ready for intercept action
Detection - by the SAM/AAA Fire Control System (FCS)
Tracking - accurate enough to attack, when target in range
Target Attacked
- Radar Missile - Launch, Guidance and Fuze
- IR Missile - Launch, Guidance and Fuze
- Gun Fire - Fire Rate/Time, Bias/Dispersion and Hits

Weapon Assessment - was target destroyed or damaged?
Re-attack - if target not destroyed and defense system still capable

Each action can be represented by a probability and a time delay. The SAM/AAA conditional probabilities can be combined (if sequential) as shown below, for a single shot at a penetrator from a particular site:

PK = Probability of killing (destroying) the penetrator

$$= P_{al}\ P_d\ P_t\ P_l\ SSPK$$

where P_{al} = Probability that SAM/AAA is available and alerted
P_l = Probability of launch given track
SSPK = Single Shot Probability of Kill given launch

Methods to compound multiple shots will be described later in this chapter, and in later chapters.

SAM time delays can vary from several seconds to more than a minute. For example, less then ten seconds may be required from assignment until missile launch (or commence fire) for an alerted and directed SAM/AAA site. However, if a site had to operate independently (or autonomously), extra time for target search and identification must be added. Autonomous operation may require up to a minute or more from enter cover to launch/fire. SAM time lines between launches will be illustrated in later chapters.

Airborne Interceptor Actions

AI Alerted - receive order to takeoff or leave CAP location
AI Vectored - in the correct direction and to the assigned target
Detection - a sensor on the AI detects the target
Tracking - accurate enough to attack, when target in range
Conversion - AI achieves correct approach geometry
Target Attacked
- Radar Missile - Launch, Guidance and Fuze
- IR Missile - Launch, Guidance and Fuze
- Gun Pass - Fire Rate/Time, Bias/Dispersion and Hits
- Ram Pass - Collision with penetrator

Weapons Assessment - target destroyed or damaged?
Re-attack - if target not destroyed, and AI still capable

Each of these AI actions can be represented by a probability of success and by a time delay. AI conditional probabilities can be combined (if sequential) as shown below, for a single shot at a penetrator:

$$PK = P_{al} P_v P_d P_t P_c P_l \text{ SSPK}$$

where P_{ai} = Probability AI available and alerted
P_v = Probability AI is corrected vectored given alerted
P_d = Probability AI detects given vectoring
P_t = Probability AI tracks given detection
P_c = Probability AI converts given track

The time required from AI alerted to first weapon application (launch/-fire) depends primarily on the alert status of the AI. From combat air patrol this time may be a few minutes. If the AI starts from ground alert a few extra minutes may be required for takeoff and fly out. The time required for non-alert interceptors to reach a penetrator can be quite long, and may exceed the exposure time of that sortie.

Interceptor sensor capability affects both the time required to complete an attack and the probability of success. AIs with effective long range radars (e.g. pulse Doppler or look-down radars) will detect low altitude targets sooner, and will usually have a much higher target kill probability (PK) than older AIs with less capable, pulse radars. PKs for AI radar guided missiles, infrared (IR) guided missiles and gun attacks will be illustrated in later chapters, along with PKs for penetrator lethal self-defense against the AI and these missiles.

It is important to evaluate human factors in all net, SAM/AAA and AI actions. Human factors may be the dominant factor in assessing success probabilities and time delays. C^3 time delays may be quite long - exceeding the sum of all other sensor and weapon system delays.

Fundamental Interactions

The offense may choose from a long list of possible actions to minimize attrition, some of which were described in Chapters 1 and 2. Key interactions between the offense and defense will now be outlined. Many of these interactions will be quantified in the analysis methodology presented in Part IV of this text.

Defense actions and likely offense countermeasures are listed in the next three tables for the three fundamental defense actions: net assignment, SAM/AAA intercept and airborne intercept:

<u>Net Assignment Countermeasures</u>

Alert		Suppression, Surprise
Detect	Speed, Altitude, RCS, ECM	↓ Mass, Turns
Track	↓ ↓ ↓ ↓	Turns
Identify		Turns
Control		Mass
Assign		Mass, Suppression

Note that speed, altitude, radar cross section (RCS) and electronic countermeasures (ECM) can degrade five basic net assignment actions. High penetrator speed reduces the time allowed the defense to complete intercept. Low altitude reduces the distance and time within each individual sensor's coverage, and thus can limit the time for defense actions. (It is sometimes possible to overfly coverage by very high altitude flight). ECM, in its many modes, can prevent or delay detection, degrade tracking, cause misidentification (e.g. by IFF countermeasures) and degrade control and assignment actions. ECM will be considered in Chapter 12.

Suppression attacks on defense elements can open gaps in sensor coverage and destroy weapon systems which might have been assigned. Penetrator turns can cause multiple tracks, errors in track position/velocity and time delays in initiating the new tracks created. Mass - large numbers of penetrators or decoys - will cause delays in the control and assignment functions, as well as creating confusion and dilution of defense resources.

SAM/AAA Countermeasures

Alerted		Suppression
Detect	Speed, Altitude, RCS, ECM,	Mass, Turns, Camouflage
Track		Decoy, Turns
Launch		Decoy, Turns, Lethal self defense, Night, Weather
Guide		Decoy, Turns
Fuze		Decoy, Turns
Gun Fire		Camouflage Lethal self defense

Speed, altitude, RCS and ECM can each degrade six of these SAM/AAA actions, by significantly reducing the time available for intercept. ECM includes electro-optical countermeasures (EOCM) which can degrade defensive EO systems (such as IR search sets, IR guided missiles or optical aids). Decoys include both fly-along objects and expendables (e.g. chaff to provide multiple radar returns and flares to provide a large false IR target).

Turns can degrade SAM/AAA weapon systems by causing time delays and loss of coverage (e.g. if the penetrator turns into a radar Doppler notch or blind speed). Lethal self-defense can kill the SAM/AAA site, or a SAM missile in flight.

Defense weapon systems may use electro-optics. These systems can be degraded by penetrating at night or in weather. Camouflage can degrade optical systems.

Airborne Intercept Countermeasures

Alerted	Speed, Altitude, RCS, ECM,	Mass, Suppression
Vectored		Mass, Turns
Detect		Mass, Turns, Camouflage
Track		Decoy, Turns
Convert		Decoy, Turns
Launch		Decoy, Turns Lethal self-defense
Guide		Decoy, Turns
Fuze		Decoy, Turns
Gun Pass or Ram		Night, Weather, Camouflage Lethal self-defense

Speed, altitude, RCS and ECM can each degrade all these airborne intercept actions. For example, an air vehicle may escape intercept if the AIs do not have sufficient time to reach the penetrator before it leaves coverage, or within several minutes after exiting cover (allowing hot pursuit).

RCS reductions can be accompanied by EO signature reductions to degrade infrared search and track (IRST), IR guided missiles and other EO systems in the defense inventory. ECM is again defined in the broadest sense - to include radar, communications and electro-optical countermeasures. For example, infrared countermeasures (IRCM) would be desired if the AI had an effective IRST set or IR guided missiles.

A large number of penetrators/decoys can provide mass to overload the AIs available, degrade the vectoring process and cause confusion in AI detection of the assigned target. Terminal decoys (fly along or expendables) can be launched as the AI approaches, degrading AI actions from conversion through missile fuze. If the penetrator knows the AI approach geometry, a penetrator turn can degrade the AI attack. Lethal self-defense, using penetrator launched missiles or guns, can kill the AI or its missiles.

Detection as well as AI gun and ram attacks (flying into the intruder) can depend upon visual sighting of the target. Optical counters (such as night penetration, weather or effective optical camouflage) will degrade these actions. IR systems will be degraded by atmospheric moisture (rain) and by backgrounds with hot spots and reflections.

Three Basic AI Attacks

An airborne interceptor attack can include 1) various sensors (e.g. radar, IR and/or optics), 2) various weapons (e.g. radar guided missiles, IR guided missiles and/or guns), 3) various firing doctrines (e.g. one missile, a salvo of two missiles of the same type, or a salvo of two missiles of different type) and 4) various approach geometries (e.g. head-on, tail-on or abeam). The large number of possible combinations of sensors, types and numbers of weapons and intercept geometry will be trimmed to the three basic airborne interceptor attack cases listed on the next page:

Radar Head-on - A head-on approach using the AI radar as the sensor and an attack with a salvo of two radar guided missiles. If unsuccessful, this attack is followed by a radar tail-on attack, if the AI has the proper armament and sufficient fuel.

Radar Tail-on - A tail-on approach using the AI radar as the sensor and an attack with a salvo of two radar guided missiles. If unsuccessful, this attack is followed by a EO tail-on attack, if the AI has the proper armament and sufficient fuel.

IR/Visual - A tail-on approach using EO sensors (IR/visual) and an attack with a salvo of two IR missiles. If unsuccessful, gun equipped AIs with enough fuel initiate a tail-on gun attack.

In order to analyze air vehicle penetration, assume that modern AIs (with Doppler radars) will undertake radar head-on or tail-on attacks against either low or high altitude intruders. The head-on approach will be considered to be the primary attack mode for look-down (LD) interceptors with shoot-down (SD) missiles. Initial tail-on attacks for these LD/SD systems will occur only if an initial head-on attack is not feasible (i.e. due to approach geometry). Also assume that time will not permit an AI to accomplish a head-on attack after tail-on action. Thus, head-on attacks can be followed by tail-on action, but not the reverse.

Older interceptors will be assumed to be effective against low altitude penetrators only in an IR/visual attack with IR missiles and guns. High altitude penetrators will be attacked by these older AIs in the same manner as modern AIs - starting with radar head-on or tail-on actions.

The three AI attack cases are illustrated in Figures 9.1, 9.2 and 9.3. In each figure, defense probabilities are annotated above the arrows. Success in the previous action allows continuation to the right. Failure results in either a miss (termination of the attack) or an attempt to convert to a different type of attack (i.e. from radar head-on to radar tail-on, from radar tail-on to IR tail-on, or IR tail-on to visual tail-on).

The circles represent decision points, and are coded to show where ECM/EOCM degrades are possible (by darkening the lower semicircle) and where low radar cross section degrades are possible (by darkening the upper semicircle). Where both degrades are possible, the decision circle is completely darkened.

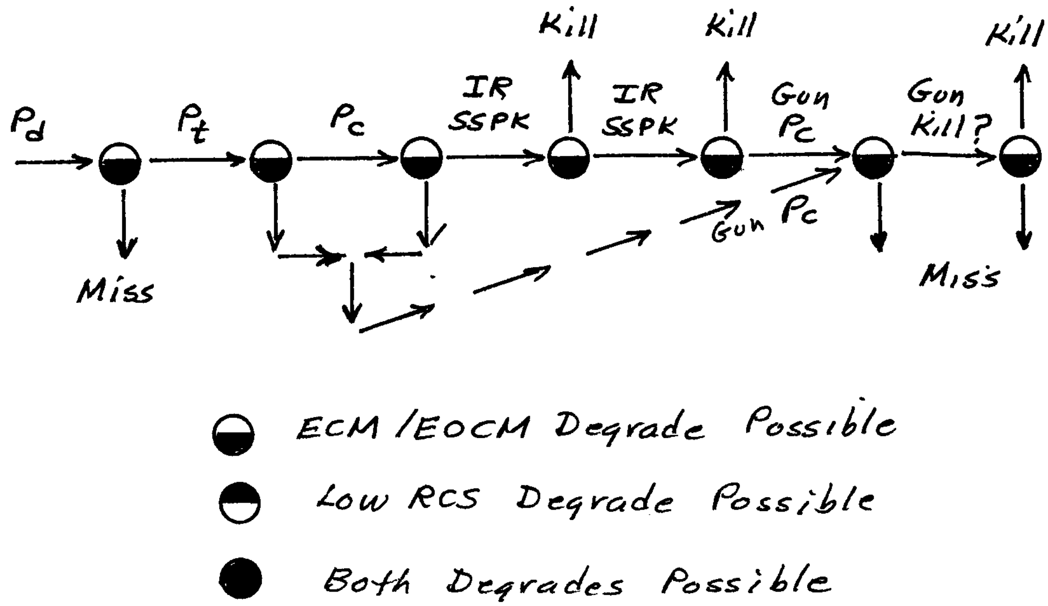


Figure 9.1 IR/Visual Attack

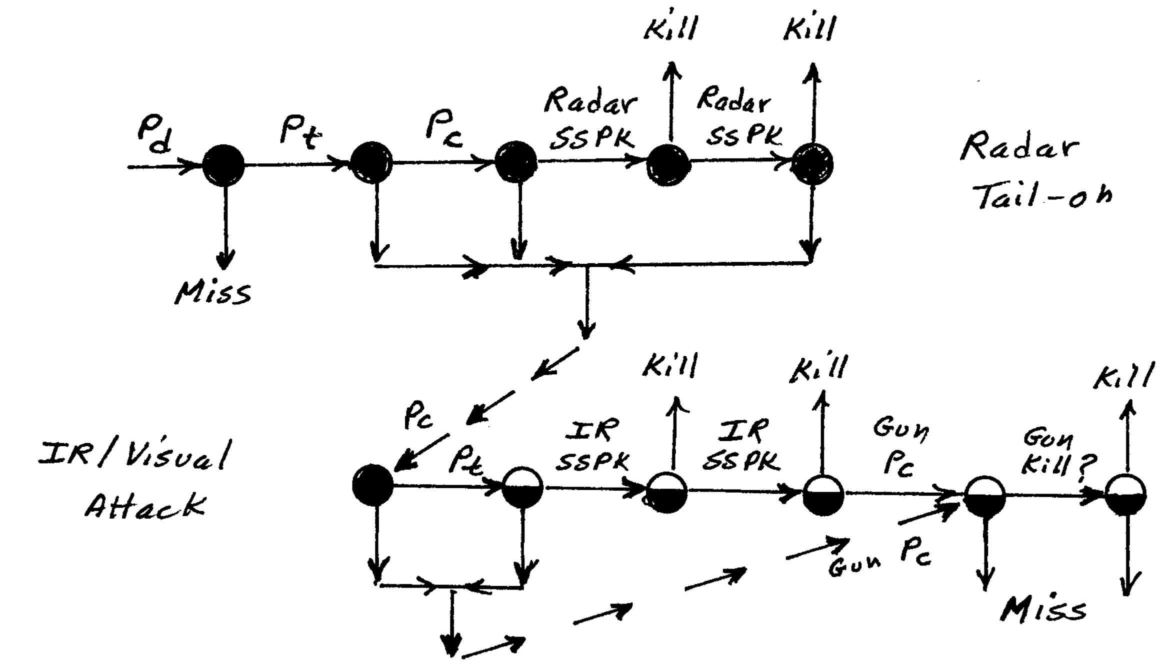


Figure 9.2 Initial Tail-on Radar Attack

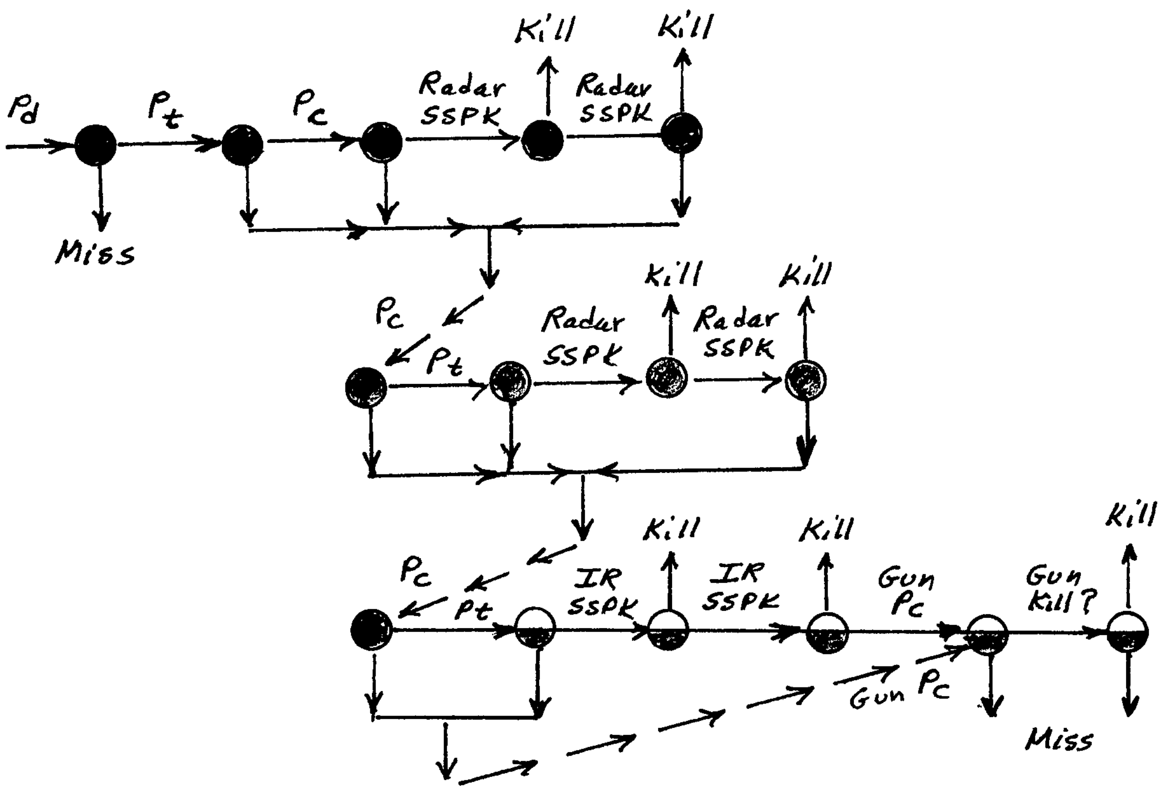


Figure 9.3 Initial Head-on Radar Attack

Note that low RCSs can degrade AI radar detection, conversion and track, as well as radar missile launch and kill. ECM degrades can occur at all decision points requiring radar or C^3 actions. EOCM degrades occur in IRST conversion, against the IR guided missiles and in a visual (or IR aided) gun attack.

Optical camouflage may play an important role in an AI attack. Peace-time rules normally require positive visual identification prior to weapon launch/fire. Combat rules can change this requirement, but visual cues are expected to remain an important factor in detection through launch/fire actions of modern AIs.

Compounding Independent Kill Probabilities

It is important to understand the fundamentals of compounding kill probabilities, since missile salvos and multiple attacks may occur during penetration.

Consider a simple example (a missile salvo aimed at one intruder) to illustrate PK compounding. Assume that adequate (hopefully live missile firing) test data are available for the specific condition to be examined. With this data, there is an excellent estimate of the single shot probability of kill (SSPK).

Next, the issue of independence between launches in a salvo will be addressed. If knowledge of the failure or success of the first missile does not help to predict the failure or success of subsequent missiles, the results of an n missile salvo can be treated as n independent events. If the results of the first launch do help to predict results on subsequent launches in a salvo, these events are dependent on each other.

A tree diagram (shown in Figure 9.4) illustrates a two missile salvo, and shows the correct way to compound two independent SSPKs (single shot probabilities of kill):

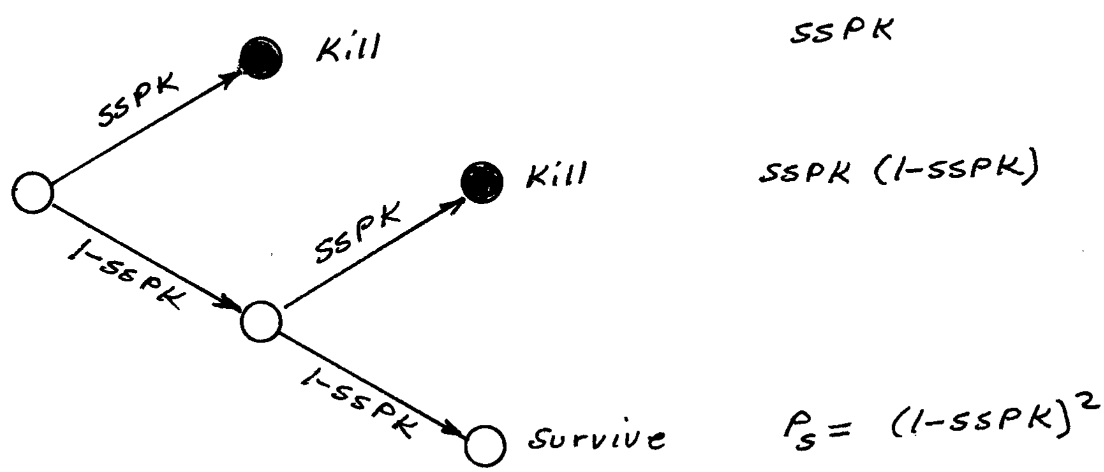


Figure 9.4 Compounding Two Independent SSPKs

The compounded probability of kill (PK) is found by summing up the end points of the tree diagram associated with <u>kill</u>. Figure 8.4 shows that PK is:

$$PK = SSPK + (1\text{-}SSPK)\ SSPK$$

The probability that the penetrator will survive (P_s) the two launches can also be seen from the tree diagram as:

$$\text{Survival probability} = P_s = (1 - SSPK)^2$$

It is far easier to use (and understand) this equation for survival probability then the PK equation, particularly as the number of shots increase beyond two. For example, the probability of surviving each missile is (1-SSPK). The probability of surviving two independent missiles is $(1\text{-}SSPK)^2$. The probability of surviving n independent missiles is $(1\text{-}SSPK)^n$. The general formula for survival probability for independent shots is:

$$P_s = (1 - SSPK_1)\ (1 - SSPK_2)\ (1 - SSPK_3)\ - - - -\ (1 - SSPK_n)$$

for n independent events

For example, if two independent missiles were launched, each with a 0.3 SSPK, the penetrator's P_s and compounded kill probability is:

$$P_s = (1 - SSPK)^2 = (1 - 0.3)^2 = .49$$

$$\text{and } PK = 1 - .49 = \underline{.51}$$

If two independent missiles had different SSPKs, say 0.3 for one and 0.4 for the other, penetrator survival would be:

$$P_s = (1 - SSPK_1)\ (1 - SSPK_2) = (1 - 0.3)\ (1 - 0.4) = .42$$

$$\text{and } PK = 1 - .42 = .58$$

Compounding Dependent Kill Probabilities

Now assume that missiles in a salvo are no longer independent of each other. For example, missiles might be affected by the same launch or flight conditions such as: the same target range, azimuth and elevation, or the same masking, clutter and multipath background, or the same ECM effects. However, missile inflight reliability, fuzing and warhead damage may be independent of these launch/flight conditions. The SSPK of each missile will be split into a dependent ($SSPK_d$) and an independent ($SSPK_i$) part. A tree diagram illustrating this is shown in Figure 9.5.

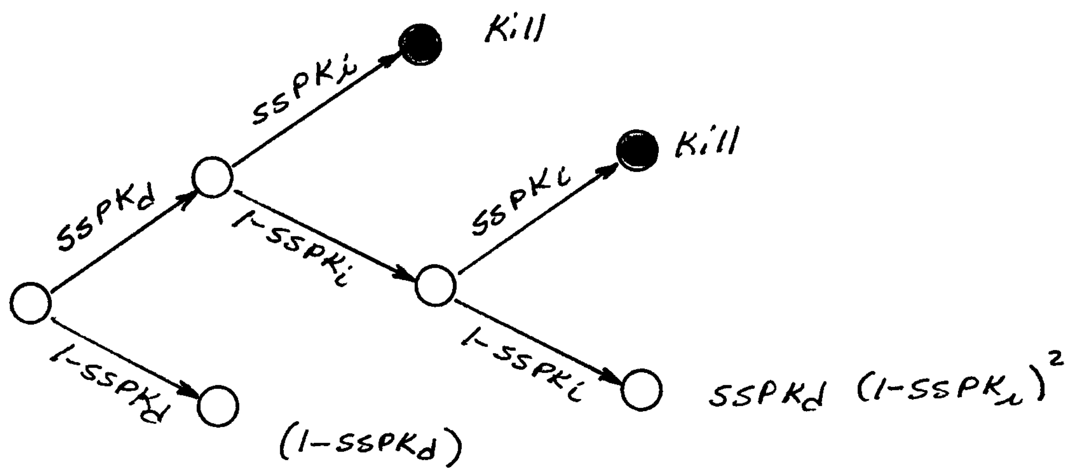


Figure 9.5 Compounding Two Dependent SSPKs

Note that the SSPK has been divided so that:

$$SSPK = SSPK_d \times SSPK_i$$

The compounded PK can be seen to be the dependent part of the SSPK multiplied by (1 - the compounded survival from the independent part). The equation for PK for n shots is now:

$$PK = SSPK_d \{1 - (1 - SSPK_i)^n\}$$

For example, if a missile's SSPK was 0.3 (composed of a dependent part of 0.5 and an independent part of 0.6), the compounded kill for a two missile salvo would be:

$$PK = 0.5 \{1 - (1 - 0.6)^2\} = .42$$

This compares to a compounded PK of .51 for two completely independent missiles.

In order to account for the dependent part of SSPK in a salvo, the assumption $SSPK_d = SSPK^{1/2}$ will be used in all future salvo calculations. The equation for compounded kill probability to be used is:

$$PK = SSPK^{1/2} \{1 - (1 - SSPK^{1/2})^n\}$$

where n = the number of shots in a salvo

This square root compounding is compared to independent compounding in Figure 9.6 below:

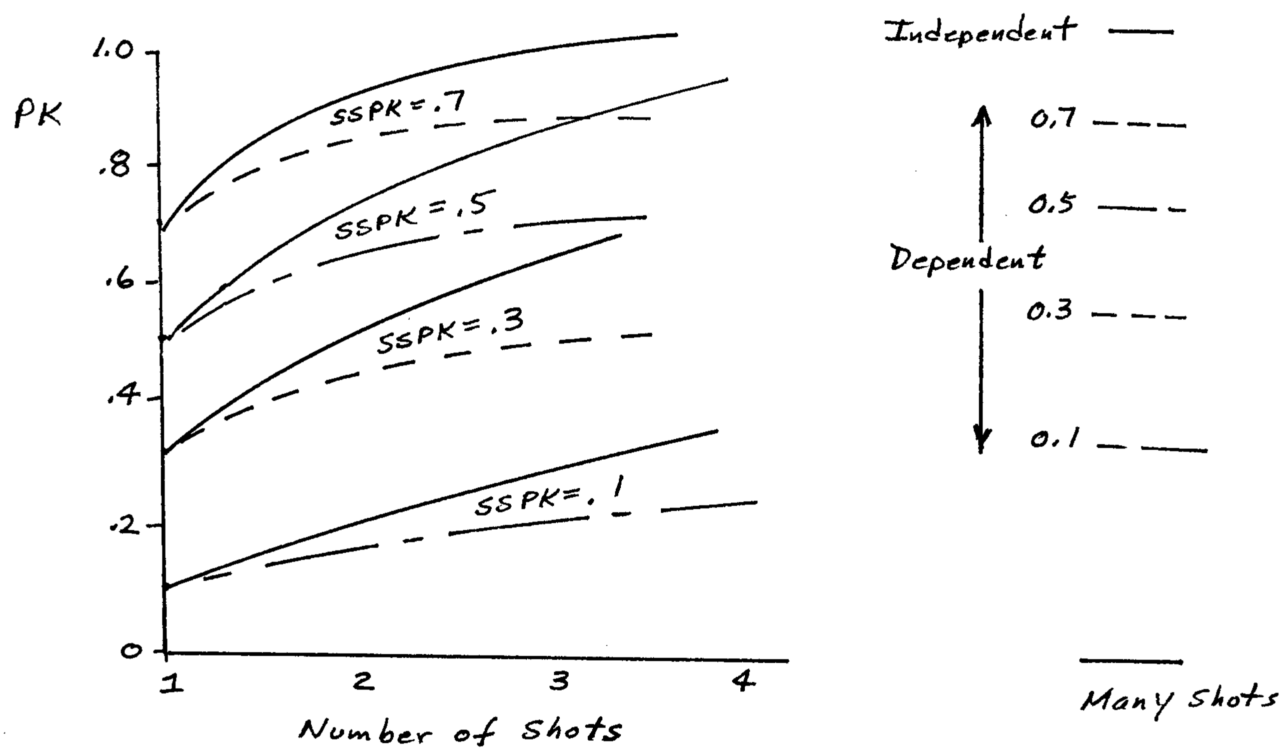


Figure 9.6 Compounding SSPKs

Note that with independence the compounded probability of kill will approach 1.0 as the number of missiles launched in a salvo increases.

With dependence, however, the compounded PK will approach the dependent portion of SSPK as the number of missiles launched increases.

This square root dependence between missiles in a salvo will be used in subsequent examples. However, independence between radar, IR and visual passes by one AI, and independence between different AIs will be assumed (in a later penetration analysis) as well as independence between SAM missiles not fired in salvo, (e.g. in a shoot-look-shoot tactic).

Summary

The basic offense tactics of avoid, degrade and destroy were compared to the basic defense tactics of detect, destroy and survive. Defense actions were divided into assignment, SAM/AAA intercept and airborne interceptor actions, and the fundamental elements of these actions were identified and defined. These elements can be represented by probabilities and time lines, which can be degraded by many different offensive countermeasures.

Three standard AI attacks patterns were presented: a radar head-on, a radar tail-on and an IR/visual tail-on attack. Offensive degrades to these three attacks were described and the transitions between radar missile launches, IR missile launches and gun fire were defined - for later use in the analysis of air vehicle penetration.

Methods to compound independent and dependent shots were described, and a square root SSPK dependence was postulated for missiles launched in salvo. The P_s formula for compounding n independent shots is:

$$P_s = (1 - SSPK_1)\ (1 - SSPK_2)\ - - - -\ (1 - SSPK_n)$$

Compounding dependent SSPKs requires the separation of the dependent part $SSPK_d$ from the independent part $SSPK_i$, as shown:

$$SSPK = SSPK_d \times SSPK_i$$

The formula for compounding n shots having the same dependent and independent SSPK is shown below:

$$\text{Compound PK} = SSPK_d \{1 - (1 - SSPK_i)^n\}$$

Independence will be assumed (in a later penetration analysis) between radar, IR and visual attacks, as well as between weapons not launched in salvo.

Chapter 10

Time in Radar Coverage

This chapter addresses one of the most important factors in air vehicle penetration - the amount of time a penetrator is under radar coverage of defense sensors. A simple model of the mean time under Airborne Warning and Control System (AWACS) and ground system cover will be developed and used to analyze the effects of penetrator speed, altitude and radar cross section on coverage time. A brief discussion of the time required for defense assignment concludes the chapter.

AWACS Detection

A simple model is needed to show how penetrator speed, altitude and radar cross section affects time under AWACS coverage. A model of the mean (average) time in AWACS coverage will be developed in four steps: 1) AWACS detection range, 2) Probability of entering cover, 3) Expected time in cover and 4) Over-the-horizon detection. The results of this simple model has been found to match closely the results of far more complex AWACS simulations.

AWACS Detection Range

Assume that an AWACS is in a racetrack orbit searching for air vehicle penetrators on straight flight paths which enter perpendicular to the longest dimension of the orbit. The orbit end points are separated by a distance d. The line distance assigned to one AWACS is L, as illustrated in Figure 10.1:

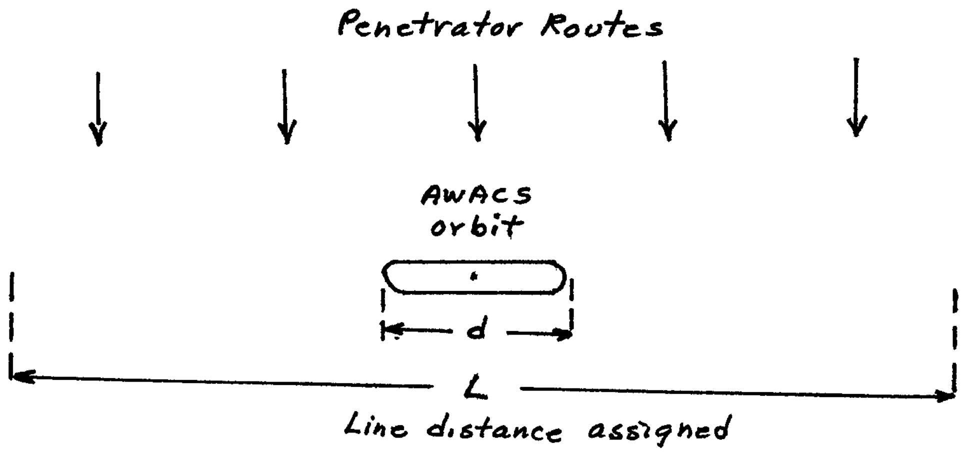


Figure 10.1 AWACS Line Coverage

At any point in the orbit, AWACS's detection range depends upon three factors: the air vehicle's radar cross section (RCS), AWACS radar sensitivity (or K Value) and the maximum radar line of sight. Assume that the penetrating vehicle has a constant RCS, σ_p, in all directions (i.e. a spherical RCS). The AWACS detection range R is the minimum of either the radar LOS distance or the sensitivity limit.

$$R = \min\{\text{radar LOS or } K\,\sigma_p^{1/4}\}$$

For example, if an AWACS is flying at 30,000' altitude and is searching for a penetrator flying at 200' altitude, the maximum radar LOS (for 4/3rds earth radius refraction) between the two vehicles is:

$$\text{Radar LOS} = 1.23\ \{(30{,}000)^{1/2} + (200)^{1/2}\} = 230\ \text{NM.}$$

AWACS detection and tracking of a low altitude penetrator usually requires Doppler processing, so the AWACS radar will likely be in its pulse Doppler mode. To illustrate a possible K Value for this mode, assume that the radar was designed to detect a 10 m^2 target at the radar horizon of 213 NM (when AWACS is at 30,000'). In order to detect a 10 m^2

target at this distance, the K Value must be at least $213/(10)^{1/4} = 120$ NM. Assume that the K Value is 120 NM.

If the penetrator is at 200' altitude, but had a σ_t of only 1 m^2, the minimum range, R, would be determined by K $\sigma_p^{1/4}$, or 120 NM in this case, rather than the 230 NM radar line of sight. Thus, a 1 m^2 spherical target could be detected 120 NM away from AWACS as it moves through its orbit.

AWACS velocity will be annotated by V_R, and the target's velocity by V_p, which is assumed to be perpendicular to the orbit length. The angle between these two velocity vectors is γ, as shown in Figure 10.2.

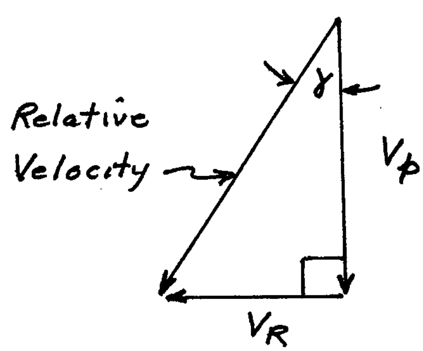


Figure 10.2 Angle between Velocity Vectors

Probability of Entering Cover

Given that the penetrator crosses the line, L, the probability that the penetrator enters AWACS coverage, P_{ec}, is the probability of flying within distance R of AWACS as the penetrator passes this line. P_{ec} is the projection of 2R, the target's vulnerability to cover (measured along the direction of the relative velocity), divided by the total line distance. This is illustrated in Figure 10.3:

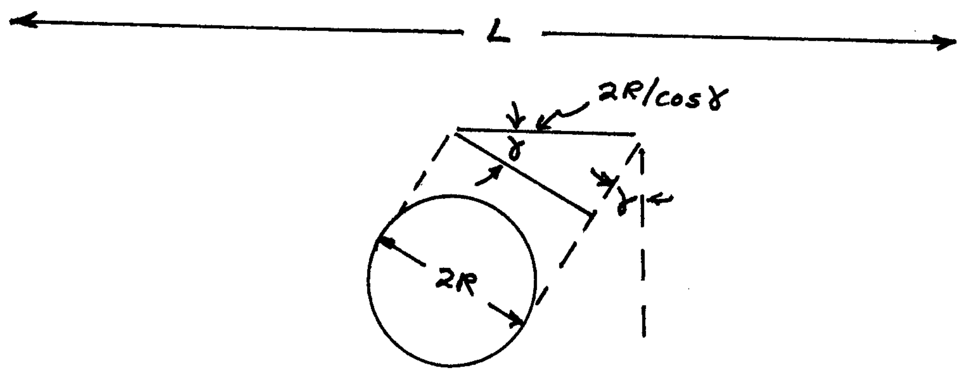


Figure 10.3 Probability of Entering Cover

The equation for this probability of entering coverage is:

$$P_{ec} = \frac{\text{Actual line coverage}}{\text{Total line distance}} = \frac{2R/\cos \gamma}{L}$$

Expected Time in AWACS Coverage

The expected distance in cover, given AWACS coverage is entered, will first be developed. For a random distance from a stationary radar, the expected coverage distance is the average chord of a circle of radius R. The average chord can be determined as the area of the circle divided by the diameter, or:

$$\text{Average chord} = \pi R^2/2R = \pi R/2$$

This average chord is illustrated in Figure 10.4.

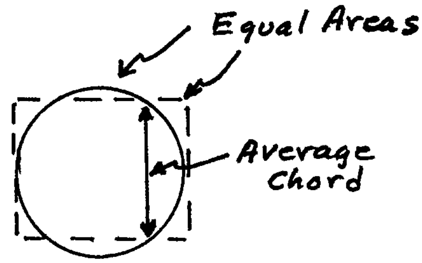


Figure 10.4 Average Chord

Due to the penetrator's relative velocity with respect to the moving AWACS, this average chord must be projected perpendicular to the line coverage. The expected distance under AWACS cover, given coverage is entered, is:

Expected Distance in cover = Average chord x $\cos\gamma$ = $(\pi R/2)\cos\gamma$
given coverage is entered

Multiplying the probability of cover by the expected distance in cover (given entry), yields the expected distance under AWACS cover as:

$$\text{Expected Distance in cover} = \pi R^2/L$$

The expected time in cover, $\bar{t}_A$, is just this distance divided by the penetrator's velocity, V_p:

$$\bar{t}_A = \pi R^2/V_p\ L$$

This equation does not include a dead zone (or no coverage zone) beneath AWACS, caused either by radar interference with the AWACS airframe or by close-in surface clutter exceeding the clutter thresholds. A dead zone will decrease active coverage, although dead reckoning may allow track continuation. The modified equation for active cover with a dead zone of radius r from AWACS is:

$$\bar{t}_A = \pi(R^2 - r^2)/V_p\ L$$

For example, find the mean time in cover for a 560 NM assigned line distance, h_a = 30,000', h_t = 200', V_p = 300 knots, σ_t = 1 m^2 and K = 120 NM in pulse Doppler mode. (Recall that R = 120 NM and radar LOS = 230 NM for this example). Without a dead zone, the expected AWACS coverage distance and time are:

$$\text{Expected distance in cover} = \pi R^2/\ L$$
$$= \pi(120)^2/\ 560 = 81\ \text{NM}.$$

$$\text{Expected time in cover} = \text{Expected distance/target velocity}$$
$$= 81/300 = .27\ \text{hours, or 16 minutes.}$$

With a dead zone, this mean coverage time would decrease as illustrated below for the example AWACS and a 1 square meter spherical target:

Dead Zone	=	0	20	40	60 NM
$\bar{t}_A$	=	16	16	14	12 minutes

Antenna shielding and clutter thresholds, which may create AWACS dead zones in low altitude detection, may not apply to detection of high altitude penetrators.

Over-the-Horizon Detection

A low PRF pulse mode may be used by AWACS operators against penetrators that are over-the-horizon (OTH), or beyond the clutter horizon. A K Value for such a mode can be estimated if radar design requirements are known, or can be approximated. For example, suppose a 10 m^2 target flying at 40,000' altitude must be detected at maximum LOS by an AWACS flying at 30,000'. This sets the minimum OTH K Value at LOS/$(10)^{1/4}$. The radar LOS is 459 NM for the altitudes stated. Thus, the minimum OTH K

Value = $459/(10)^{1/4}$ = 258 NM. Assume this is the OTH K Value.

The example has yielded the following K Values and coverage regions:

AWACS Mode	Pulse Doppler	Pulse OTH
K Value	120 NM	258 NM
Coverage region	0 - 213 NM	213 - 459 NM

Note that a break in coverage could exist at the 213 NM clutter horizon transition range between pulse Doppler (PD) and pulse (OTH) modes, if the penetrator's RCS allowed OTH detection, but not PD detection at this range. In the example, the lowest σ_t that can be detected at the transition range is:

$$\sigma_t < (LOS/K)^4 = (213/120)^4 = 10 \text{ m}^2 \text{ in PD mode}$$

$$= (213/258)^4 = 0.5 \text{ m}^2 \text{ in OTH mode}$$

In the example, targets with spherical RCSs between 0.5 and 10 square meters will have a gap in active radar coverage between these two AWACS modes. Any air vehicle penetrator with a spherical RCS less than 0.5 m^2 will not be detected in the OTH mode. Detection ranges are illustrated in Figure 10.5 as a function of RCS, for the example AWACS.

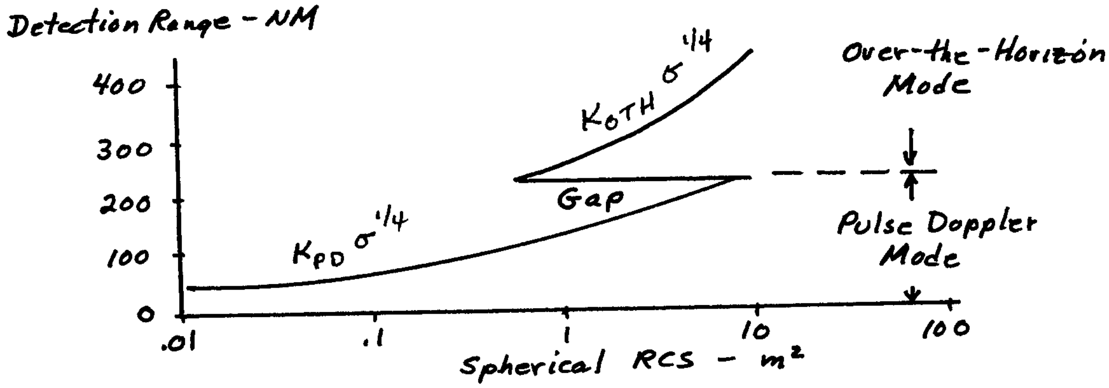


Figure 10.5 Detection Range vs RCS

AWACS Sensitivity Analysis

The expected distance and time in AWACS cover are illustrated in Figure 10.6 as a function of spherical RCS for two penetrator altitudes (200' and 40,000') and two penetrator ground speeds (300 and 600 knots) for straight flights perpendicular to an AWACS orbit line. The AWACS is assumed to be at 30,000' altitude, with a PD K Value of 120 NM and a OTH K Value of 258 NM. A dead zone of 20 NM was assumed for low altitude penetrators, but none for high penetrators.

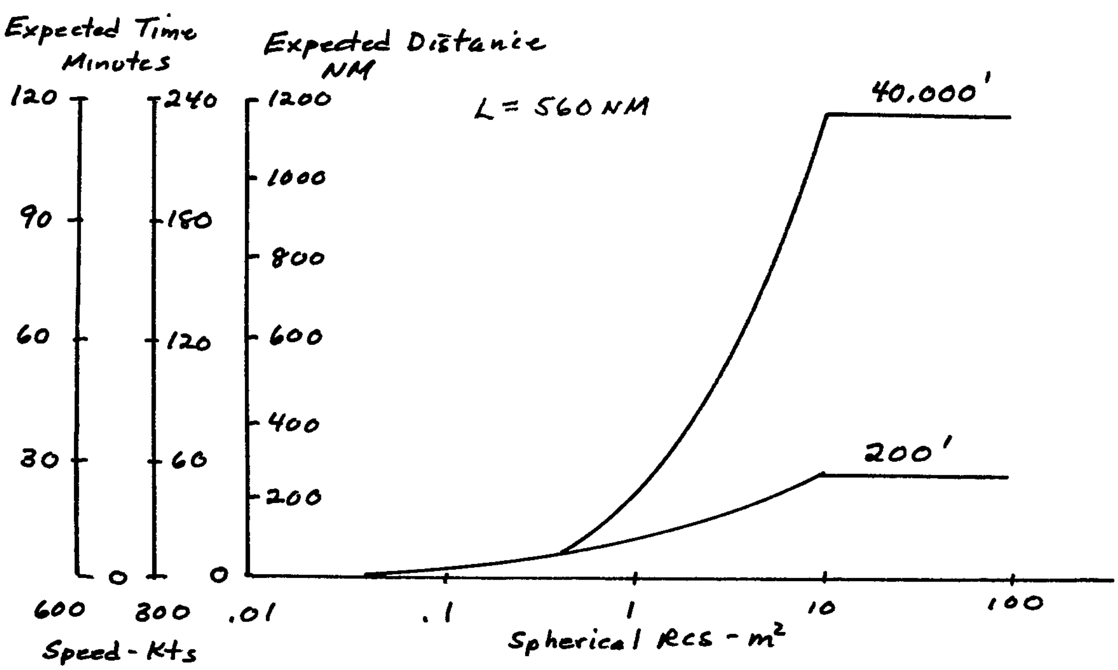


Figure 10.6 Mean Coverage vs RCS, Speed and Altitude

Note that penetrator RCS is a dominant factor in reducing mean coverage times. Low altitude reduces AWACS coverage time dramatically for RCSs above 1 m^2. Time under AWACS cover is inversely proportional to penetrator speed. Thus, doubling the speed cuts coverage time in half. If mean coverage is to be reduced to a few minutes, penetrators spherical RCSs must be reduced to below 0.3 square meters, in the example.

An air vehicle RCS is not spherical, but varies widely in elevation and azimuth as well as fluctuating over time. However, if the mean coverage time of a specific air vehicle can be obtained from test data, this simple model can provide an equivalent spherical RCS for the same mean time under AWACS coverage. For example, suppose test data for a specific air vehicle/AWACS combination showed a 10 minute average coverage time for a 200' penetrator flying at 600 knots. If the conditions were the same as the example in Figure 9.6, then the equivalent spherical RCS would be about 2 m^2. Comparisons with more complex simulations have shown that a simple rule of thumb for estimating spherical RCS of conventional aircraft is to double the median head-on air vehicle RCS (over a +/- 60° azimuth, and +/- 20° elevation range) for the proper frequency and polarization.

Ground Radar Coverage Model

In order to develop a simple ground radar coverage model, assume that a penetrator, on a straight track and at a constant altitude, enters ground radar coverage at a random distance from a site. Assume also that the radar coverage at the target's altitude is a circle of radius R centered at the site. The expected distance in cover for this penetrator is just the average chord of the circular coverage:

$$\text{Expected distance} = \text{average chord} = \pi R/2.$$

However clutter around the site can cause loss in active tracking of the penetrator. Assume that clutter is uniform (a sandpaper earth) out to the clutter horizon, r_c, and that the radar is able to detect the target within clutter at r_f. Figure 10.7 illustrates these definitions:

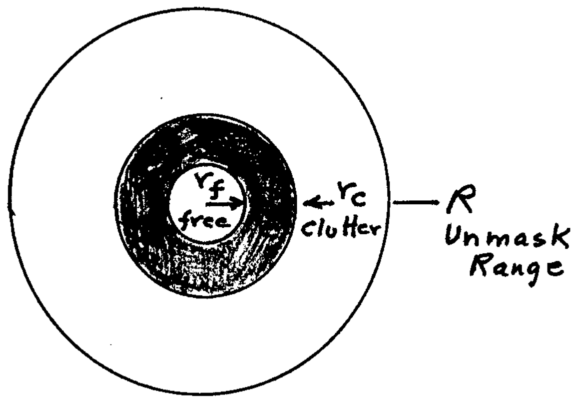


Figure 10.7 Masking and Clutter Circles

The distance R is the minimum of either LOS or K $\sigma_t^{1/4}$. r_c is the radar horizon, while r_f will be determined by the clutter processing model presented in Chapter 6.

For example, assume a 1 m^2 target at 200' altitude flying into coverage of a ground radar with a K Value of 200 NM (for the lowest beam, which will detect the penetrator first) and an antenna height of 50'. The radar LOS of 26 NM is far less than the K $\sigma_t^{1/4}$ value of 200 NM. Thus, the minimum R without masking is 26 NM. r_c is the radar's horizon, or 8.7 NM for a 50' antenna. The calculation of r_f is derived from the following parameters for the example:

$\sigma° = -20$ dB, $\tau = -60$ dB, Az = 1° or -17.5 dB,

$S/N_{reqd} = 13$ dB and a double delay MTI with I.F. = 30 dB.

At the clutter horizon, 8.7 NM, the S/C is:

$$S/C = \sigma_t - (\ \sigma° + \tau + R + AZ + 81.8\ ) \text{ in decibel form}$$

$$= 0 - (\ -20\ -60\ +42\ -17.5\ +81.8\ ) = -26.3 \text{ dB}$$

With a 30 dB improvement factor, the effective S/N = 30 - 26.3 = +3.7 dB. This is well below the 13 dB needed for detection in the example.

The range at which the required S/N is attained will determine r_f. This can be determined in the example by solving for the range (r_f) to obtain a $S/C = S/N_{reqd} - I.F. = 13 - 30 = -17$ dB.

$$r_f = \sigma_t - (\ \sigma° + \tau + S/C + AZ + 81.8\ ) \text{ in decibel form}$$

$$= +0 -(-20\ -60\ -17\ -17.5\ +81.8) = 32.7 \text{ dB, or } 1.9 \text{ Km} = 1 \text{ NM.}$$

Figure 10.8 illustrates the example, with R = 26 NM, r_c = 8.7 NM and r_f = 1 NM:

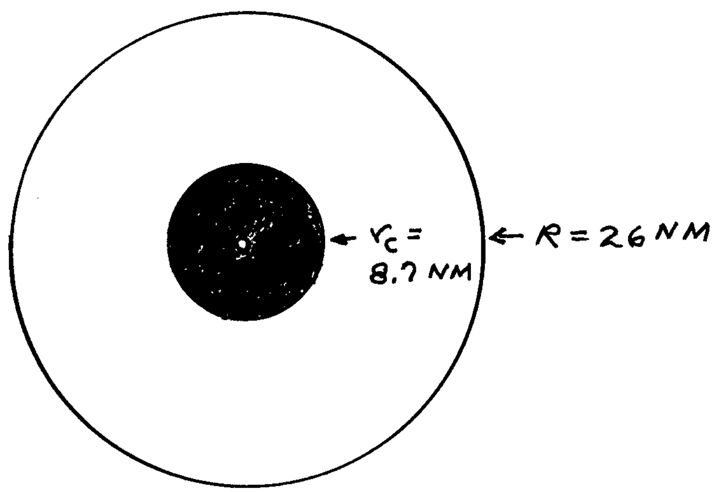


Figure 10.8 Target Detection Regions

The expected distance in cover, for a penetrator which enters ground radar coverage, is:

$$\text{Expected distance in cover} = (\pi/2)\ \{R - (r_c^{\ 2} - r_f^{\ 2})/R\}$$

The expected time in coverage is just the expected distance divided by the penetrator's velocity, V_p.

$$\text{Expected time in cover} = (\pi/2V_p)\ \{R - (r_c^{\ 2} - r_f^{\ 2})/R\}$$

For the example, the expected distance and time in cover (for a 300 knot penetrator at 200') are:

$$\text{Expected distance} = (\pi/2)\ \{26 - (8.7^2 - 1^2)/26\ \} = 36\ \text{NM.}$$

$$\text{Expected time} = 36/300 = .12\ \text{Hours} = 7\ \text{min.}$$

Note that if the radar saw no clutter, the expected distance would be 41 NM, and the time would be 8 minutes. Clutter did not have a major effect on these values since the clutter radius, r_c, is only about 1/3 (8.7/26) of the LOS range, and thus covers only about 1/10 of the radar's potential area of observation. However, this reduction due to clutter for a straight flight path over a sandpaper earth is misleading. The effect of clutter can be far more dramatic for an actual site's masking/clutter conditions, particularly when turns are considered.

Ground Radar Sensitivity Analysis

The mean distance and time under ground radar cover, given enter cover, is shown in Figure 10.9 for the example as a function of spherical RCS and altitude for two speeds with uniform clutter.

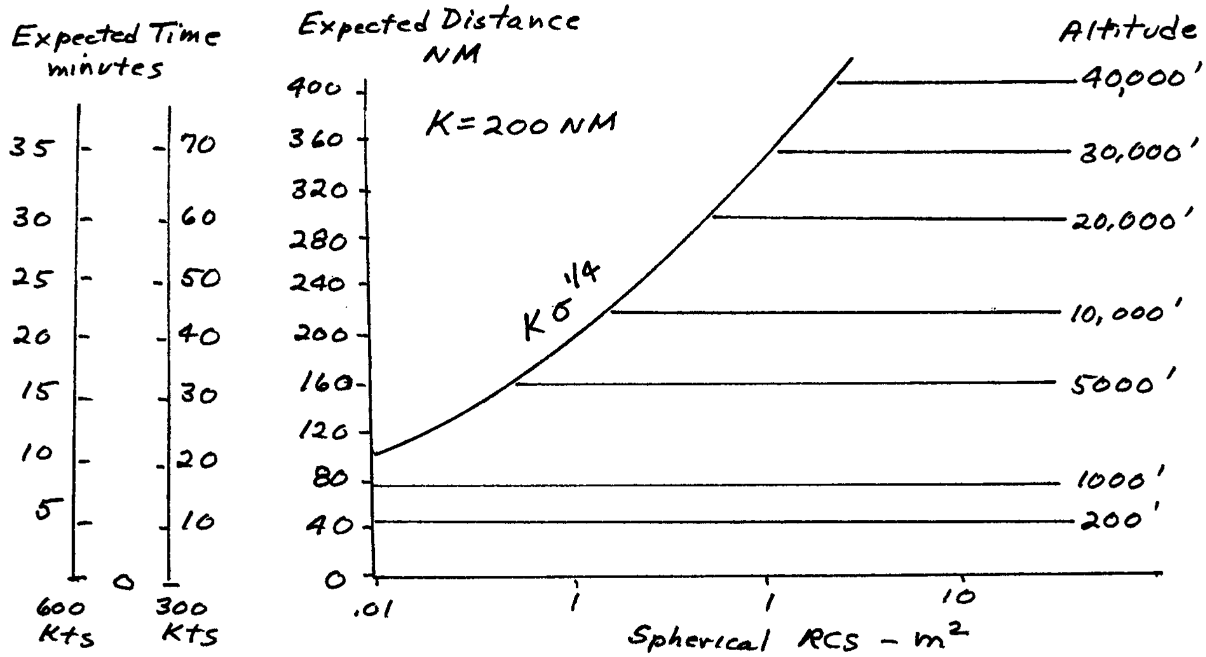


Figure 10.9 Ground Radar Coverage vs Speed, Altitude and RCS

Penetrator altitude is the dominant factor along with penetrator speed. RCS is not a major player in Figure 10.9, due to the large radar K Value and the limited clutter region of a sandpaper earth assumed. If a single ground site's radar coverage is to be reduced to a few minutes, high speed and low altitude appear to be the best tactic.

Using the average unmask values presented in Chapter 4, the unmask range would change from 26 NM to 22 NM in flat terrain, to 17 NM in rolling terrain, and to 13 NM in hilly terrain. These unmask factors would reduce distance and time in the previous example to:

Terrain	Distance	Time/300knots	600 knots
Cue ball earth	36 NM	7 min.	3 1/2 min.
Flat	29	6	3
Rolling	20	4	2
Hilly	11	2	1

Terrain masking can markedly reduce the detection distance and time under coverage (in this case near 70% in hilly terrain). If the defense reaction time (to detect, initiate track, identify as an unknown and assign weapons) exceeds a few minutes, terrain masking can be a major factor in low altitude penetration.

Masking and Clutter Around a Ground Site

The masking and clutter around one ground site is shown in Figure 10.10, for a particular radar antenna height, target altitude and clutter level.

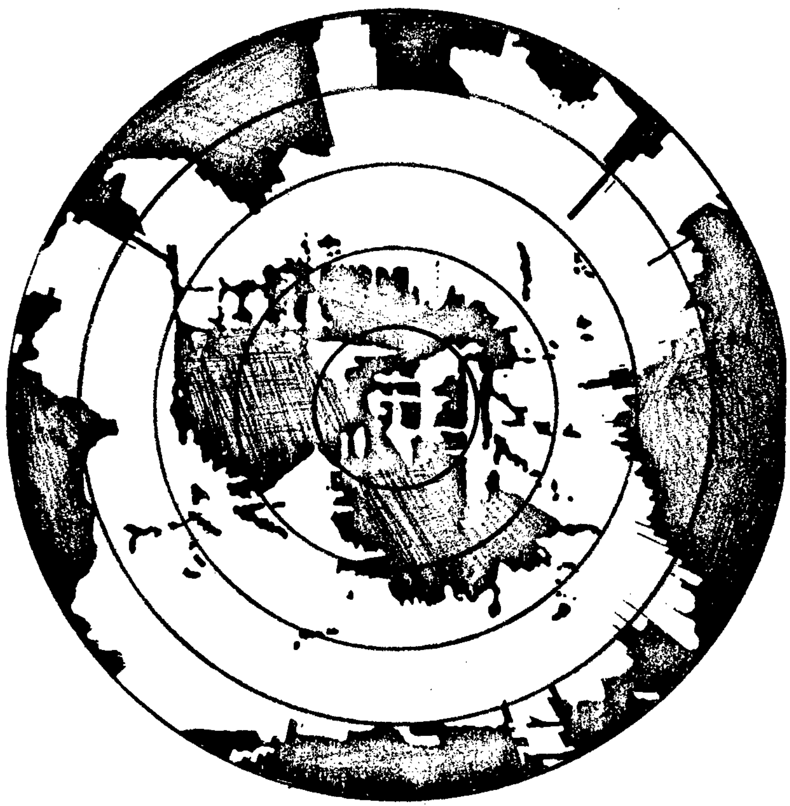


Figure 10.10 Masking and Clutter around a Site

Masking will vary with the antenna height, target altitude and the height of all obstructions (e.g. cultural features and trees) around the site. An increase in either the antenna height or the target altitude will decrease the masked area, and increase the maximum LOS distance. The map shown is for one particular clutter threshold. With clutter processing, radars can detect targets with closing speeds which exceed the

Doppler thresholds, within this clutter area.

Small RCS targets will be difficult to detect when clutter competes with the target return. For a large RCS target and excellent clutter processing these areas can be reduced markedly. The use of MTI or Doppler processing will, however, create blind sectors at the Doppler notches where targets closing with less than the threshold Doppler speeds can not be detected.

If a masking and clutter map of radar cover were available, a penetrator can choose a route to minimize detection time. Such a route with one turn is illustrated in Figure 10.11.

Figure 10.11 Careful Routing

Careful route planning, based on detailed masking and clutter measurements or predictions, can significantly reduce coverage time - and thus reduce attrition. The benefit is greatest for small RCS penetrators flying at low altitude.

Defense Reaction Time

Many factors are involved in determining the reaction time of an air defense net. Some of these factors are listed below:

1. Preconditioning of the defense network.
2. Configuration of the net - sensors, spacing and hand-off.
3. Command, Control and Communications and Information flow within the net.
4. Tactical Warning.
5. Friendly and neutral aircraft traffic, known and unknown.
6. Time delays to detect, identify, alert weapon systems and intercept the penetrator under ideal circumstances.

Depending on the above factors, the time delay from enter coverage to alerting an AI, SAM or AAA may range from tens of seconds to several minutes as mentioned in the previous chapter. The shorter times generally apply to SAM and AAA systems which are fully alerted and under the tight control of a radar network. The longer times generally apply to forward defenses which do not have the benefit of prior warning from a network.

Summary

A simple model was developed to predict the mean time under AWACS coverage. The mean time can be predicted as:

$$\bar{t}_A = \pi(R^2 - r^2)/V_p L$$

where R = AWACS detection range, r = dead zone, V_p = penetrator velocity and L = assigned line distance.

In sensitivity studies of penetrator RCS, speed and altitude, low RCS was found to be the dominant factor in AWACS penetrator, with altitude and speed being the major factors for ground radar threats.

Clutter alone (based on a sandpaper earth) was not a major factor in reducing ground radar coverage. Masking alone can be a major factor in rolling or hilly terrain. The combination of masking and clutter around actual sites can seriously degrade tracking, particularly when penetrators are carefully routed through ground defenses.

Chapter 11

Electronic Warfare

This chapter describes the fundamentals of electronic warfare. The requirements for passive electronic countermeasures (ECM) and for active ECM (including repeater, noise and standoff jammers) will be examined. A discussion of jamming goals and the uncertainties in predicting ECM effectiveness concludes the chapter.

Definitions

Electronic combat include three activities:

1) Suppression of enemy air defenses (SEAD),
2) Command, control and communications countermeasures (C^3CM), and
3) Electronic warfare (EW).

Concepts to suppress enemy air defense elements have been discussed in previous chapters. In later chapters, range requirements for attacks against an AWACS, a ground radar site, an AI and AI/SAM missiles will be analyzed.

Measuring the effectiveness of C^3 countermeasures may be very difficult, since the enemy's critical nodes (central connections or tie-in points), capacities and redundancies are seldom known well enough to take advantage of potential weakness. However, if defense capacities can be estimated, the dilution effects of multiple targets can be quantified. For example, AWACS intercept capability can be estimated by the number of weapons controllers on board multiplied by the number of intercepts each controller can handle simultaneously. AWACS capacity can be stressed by concentrating penetrators, cruise missiles, decoys, expendables and jamming against this threat.

Electronic warfare (EW) is divided into:

1) Electronic support measures (ESM),
2) Electronic counter countermeasures (ECCM), and
3) Electronic countermeasures (ECM).

Electronic support measures include the collection, analysis and exploitation of intelligence data on enemy systems which use the electromagnetic (EM) spectrum. Warning receivers on penetrators can be considered part of ESM, since they provide the most up-to-date information on the defense threat. The sensitivity requirements for radar warning receivers will be examined in this chapter.

Electronic counter countermeasures are actions to insure use of the electromagnetic spectrum, despite enemy ECM. This subject deserves a separate chapter - Chapter 12.

Electronic countermeasures include the development and application of equipment and tactics to deny the enemy the full use of the EM spectrum. The fundamentals of ECM will be presented in this chapter, under four general headings:

1) Passive ECM,
2) Active ECM,
3) ECM goals, and
4) Predicting ECM Effectiveness.

Passive ECM

Passive electronic countermeasures include reduced observables (e.g. lower RCS), threat warning/avoidance and use of reflectors of EM energy. The value of reduced RCS and threat avoidance has been illustrated in prior chapters. Passive reflectors will be discussed now and threat warning next.

Reflectors

Passive reflectors include decoys and chaff (e.g. strips of foil or clusters of fine wire). If the penetrator can not be separated (resolved in location or velocity) from these reflectors, target detection can be delayed or missed completely. However, modern MTI and Doppler radars can discriminate between reflectors with low closure speeds (e.g. chaff drifting in the wind) and higher closure speed penetrators.

If reflectors are scattered randomly throughout a radar display and can not be filtered out by velocity differences, the time required to sort through the returns to find air vehicle targets will delay detection. The presence of multiple returns close to the targets will also degrade tracking (e.g. chaff corridors used to hide penetrators).

If passive reflectors are not randomly distributed they can have the opposite effect - they can assist the defense in detecting and identifying the penetrators. For example, chaff dispensed at regular intervals from a single penetrator on a straight path will form a trail which points directed at the dispensing vehicle, as illustrated in Figure 11.1.

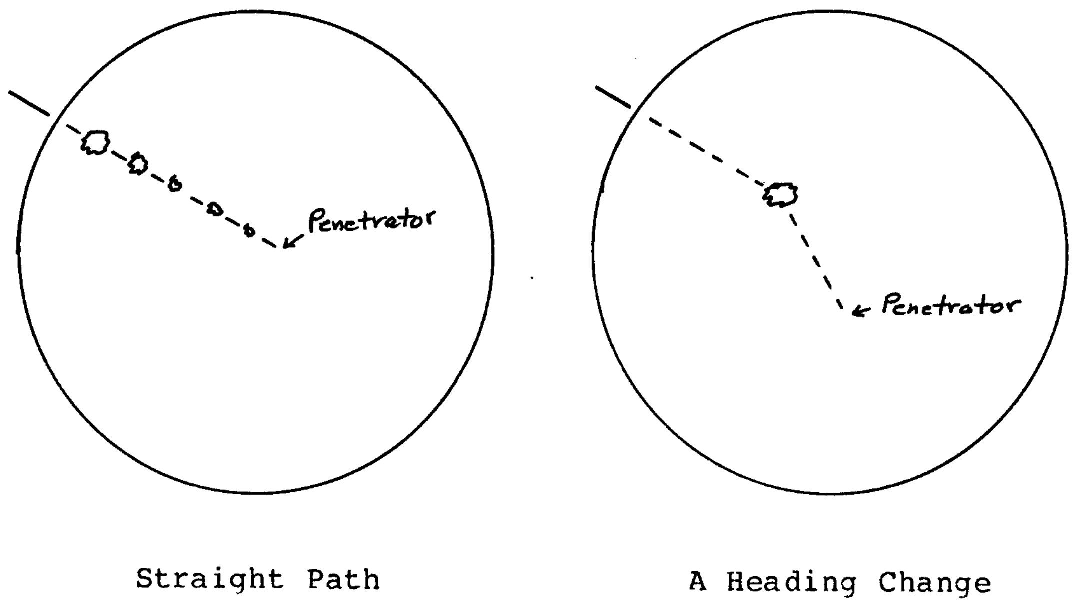


Figure 11.1 Chaff Dispensing

Note that the regular dispense pattern on the left assists the defense in detecting/tracking this solitary penetrator. Irregular dispense patterns can have the opposite effect on the defense, particularly when accompanied by a change in penetrator heading. On the right side of Figure 11.1 is an example of chaff dispensed prior to a turn. Chaff returns highlight the previous track heading, and can cause the defense to miss the turn. A new track may be assigned to the new penetrator heading. This lowers tracking quality, which can lead to missed intercepts.

Warning Receivers

Warning of an active enemy radar is one of the most important functions of a penetrator's ECM suite. The warning system should provide identification of the specific radar type as the basis for possible inflight threat avoidance and ECM actions. Some requirements for a radar warning receiver (RWR) on the penetrator will now be examined.

Assume that the defense radar operates at a frequency covered by the RWR and that the RWR system can recognize the radar signal waveform if sufficient energy is received. The radar signal received by a warning receiver will be labeled S_W. This signal strength can be determined by multiplying the radar signal density at the penetrator location by the RWR antenna aperture. This is shown below:

$$S_W = (P_p G_t / 4\pi R^2)\ (G_W \lambda^2 / 4\pi)$$

Note that the signal received by the RWR varies as the square of the range due to one way transmission of the radar signal to the penetrator. In decibel form this is:

$$S_W = P_p + G_t + G_W + 2\lambda - (22 + 2R)$$

where G_W = antenna gain of the RWR and $(4\pi)^2 \rightarrow 22$ dB

The signal, S, reflected back to the radar from the target varies as the fourth power of the target range due to two way transmission. The equation below (in dB form) was developed in Chapter 3:

$$S = P_p + G_t + G_r + 2\lambda + \sigma_t - (33 + 4R)$$

Assuming for the moment that the radar and the RWR have the same losses, sensitivity and processing gain, the range, $\bar{\bar{R}}$, where the two signals are equal is sought. Setting $S = S_W$, $\bar{\bar{R}}$ is found from:

$$2\bar{\bar{R}} = (G_r - G_W) + \sigma_t - 11$$

If radar losses, sensitivity and processing gain are different from that of the warning receiver, this equation is modified to:

$$2\bar{\bar{R}} = (G_r - G_W) - (L - L_W) + (Proc_r - Proc_W) - (Sens_r - Sens_W) + \sigma_t - 11$$

where Sens = Sensitivity of the radar/warning receiver
Proc = Processing gain of the radar/warning receiver
and L_W = Warning receiver losses

If both the radar and the RWR are limited by their respective receiver internal noise levels, and the radar's sensitivity just allows detection at $\bar{\bar{R}}$, simultaneous radar detection of the target and penetrator detection of the radar will occur at $\bar{\bar{R}}$.

Radars have the advantage of high antenna gain and high processing gain (with complex modulation and matched filtering). RWR requirements for wide angle and broad frequency coverage lead to low antenna gain and low processing gain. For example, assume a 40 dB radar antenna gain and an isotropic or omnidirectional (0 dB) RWR antenna gain. This provides a 40 dB antenna gain advantage to the radar. The difference in processing gain will be assumed to favor the radar by 30 dB.

Sensitivity is normally expressed as a negative value, thus a minus sign is used in comparing sensitivities in the above equation. Assume that the radar has a 10 dB sensitivity advantage over the warning receiver, that losses are equal and that the target RCS is 20 dB (100 m^2). With these values, the range for equal signal strengths is:

$$\bar{\bar{R}} = 1/2\ (40 + 0 + 30 + 10 + 20 - 11) = 44.5 \text{ dB, or } 28 \text{ KM} = 15 \text{ NM}$$

Figure 11.2 illustrates the variation in signal intensity with distance between the radar and the penetrator. Note that the signal reflected from the target varies as R^4 (two way transmission), while the signal received by the RWR varies as R^2 (one way transmission).

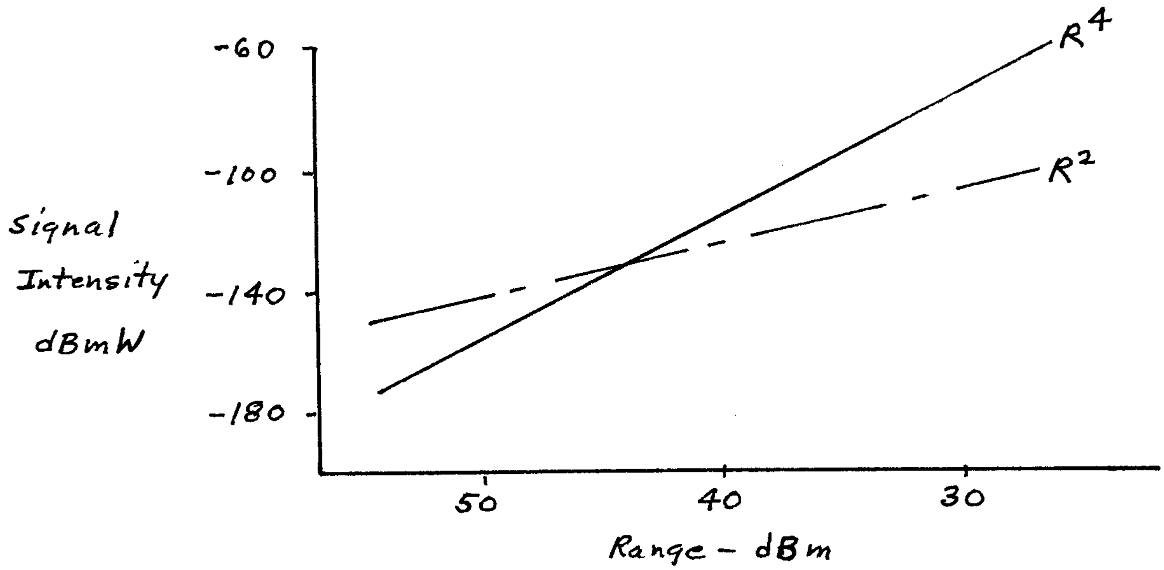


Figure 11.2 Threat Warning Range

In order to detect a 100 square meter target at 15 NM under free space conditions, the radar needs a K Value of at least 4.7 NM. If the radar had exactly this K Value, the RWR could detect the radar at 15 NM - a simultaneous detection of the intruder by the radar and of the radar by the RWR.

Target detection range would increase with a higher K Value, but RWR detection range would increase faster, due to the advantage of the square vs fourth power signal drop off with range illustrated in Figure 11.2. Radar and RWR detection ranges for three K Values are shown below for the example:

K Value	2.4 NM	4.7 NM	9.5 NM
Radar range	7.5 NM	15 NM	30 NM
RWR range	3.75 NM	15 NM	60 NM

If a radar can accomplish its job with a low sensitivity, the radar will be able to detect the penetrator before the penetrator's RWR can detect the radar signal. This is a desirable condition for the defense, particularly if the radar were aboard an airborne interceptor. For example, the AI could hold a position out of range of the RWR and launch missiles without its radar being detected by the penetrator's RWR.

Now reduce the intruder's RCS from 100 m^2 to 1 m^2. The radar/RWR crossover point is then reduced by 20 dB/2 or 10 dB. This factor of 10 reduction brings the crossover range to 1.5 NM. In order to prevent the RWR from picking up the radar beyond 1.5 NM, the radar must limit its detection to 1.5 NM, as illustrated below:

K Value	.75 NM	1.5 NM	3 NM
Radar range	.75 NM	1.5 NM	3 NM
RWR range	.375 NM	1.5 NM	6 NM

The penetrator's RWR can be prevented from detecting the radar if the AI radar had a very low sensitivity and held a position out of range of the penetrator's RWR. However, this limited detection range may compromise the success of the intercept. For example, the AI may not be vectored close enough to allow it to find the intruder, and even if the intruder is found, the AI would be limited to short range missile and gun attacks. Thus, the defense may choose long range detection of the penetrator to increase attrition, in preference to avoiding RWR detection.

With both a 1 m^2 target (a 20 dB RCS reduction) and a 20 dB RWR sensitivity improvement, the crossover point would drop to 0.15 NM, in the example. Since air defense radars must exceed this range to be effective, the radar warning receiver will always have the opportunity for detection of the radar before radar detection of the penetrator. This illustrates the advantage of low RCS and RWR sensitivity in order to detect low power radar signals.

Diffraction

Before ending the discussion of warning receivers, two issues will be considered: diffraction of signals beyond radar line of sight, and high radar densities. Diffraction will be considered first.

When a penetrator is beyond radar line of sight, its RWR can still receive a radar signal due to diffraction of the radar beam by surface irregularities. The normal R^2 loss in signal strength with distance no longer applies, but can approach R^{10} with surface diffraction. Two way transmission losses may place the signal well below radar detectability, but the one way signal received by the penetrator's RWR from a high power radar may still be strong enough to warn of a supposed imminent threat. The penetrator may react by turning on ECM prematurely, which can alert the radar to an approaching hostile, even though it is well beyond line of sight. RWR reception of diffracted signals is a major cause of premature ECM turn-on (particularly at low altitude), which can be a significant aid to the defense.

High Densities

Now consider high radar densities. The goal of a warning system is not just a general warning, but the correct identification of each threat radar signal (e.g. type of AWACS, GCI, SAM or AI). However, receipt of a large number of signals can overload a receiver. The number of radars, beams and pulses that illuminate a penetrator can be quite large, particularly during high altitude flight. Many radars with multiple beams per radar and multiple frequencies and PRIs per beam can cause the number of different signals received to rise into the hundreds. The pulse rate can exceed 100,000 pulses per second, without counting any medium/high PRF Doppler radars. In high threat densities, a RWR system needs a large processing capacity and methods to recognize and prioritize threat signals. The number of signals received can be reduced by making the RWR less sensitive, but this would not allow detection of threat radars with low power.

Active ECM

The signal, S, returned to the radar after reflection from the penetrator's RCS will be compared to the jamming, J, received by a victim radar. Recall that the signal returned to a radar is (in dB form):

$$S = P_p + G_t + G_r + 2\lambda + \sigma_p - (33 + L + 4R)$$

The jamming signal received by the victim radar can be determined by multiplying the jammer signal density in the vicinity of the radar by the radar's effective aperture in receiving this jamming:

The jammer signal density (at a range R) is $P_jG_j/4\pi R^2$

where P_j = Power output of the jammer in the radar's bandwidth
G_j = Antenna gain of the jammer in the radar's direction
P_jG_j = Jammer Effective Radiated Power (ERP)

The radar receiving antenna aperture is $G_r\lambda^2/4\pi$ (from Chapter 3).

The jamming signal received by the victim radar is just the multiplication of these two components. The result is shown below (in decibel form):

$$J = P_j + G_j + G_r + 2\lambda - (22 + L_j + 2R)$$

where L_j is added to allow for jammer losses.
and $P_j + G_j$ is the ERP of the jammer, in dB form.

The jammer-to-signal (J/S) ratio of power levels can be determined from the above equations as:

$$J/S = P_j + G_j + 11 + 2R + (L - L_j) - (P_p + G_t + \sigma_p)$$

For example, assume that P_j = 20 dB, G_j = 10 dB, P_p = 60 dB, G_t = 40 dB and that the losses are equal ($L = L_j$) for a jammer which just repeats the radar signal received. The J/S for a 1 m^2 penetrator at 10 Km (40 dB) range is:

$$J/S = 20 + 10 + 11 + 2(40) + 0 - (60 + 40 + 0) = +21 \text{ dB.}$$

This can be compared to the J/S required for gate pull off. A typical radar operator can hold a target until the jamming rises to about 10 dB above the signal. Automatic systems can hold a target until the J/S exceeds about 3 dB. Less ERP is needed against automatic detecting systems, i.e. operators can cope with jamming by one penetrator better than automatic systems. Thus, the 21 dB J/S in the example is more than adequate to allow gate pull off at 10 Km, when radar and jammer losses are equal.

The J/S required to accomplish other ECM goals varies widely with the task. Sometimes tens of dBs are required to perform more sophisticated ECM techniques.

Range Obscuration

Range obscuration will now be discussed, and the term target burn thru will be introduced in order to illustrate how the term is often misused.

When the J/S is just equal to that required for detection, the target is said to burn thru the jamming at that range. This range can be estimated as shown below (on the dB scale):

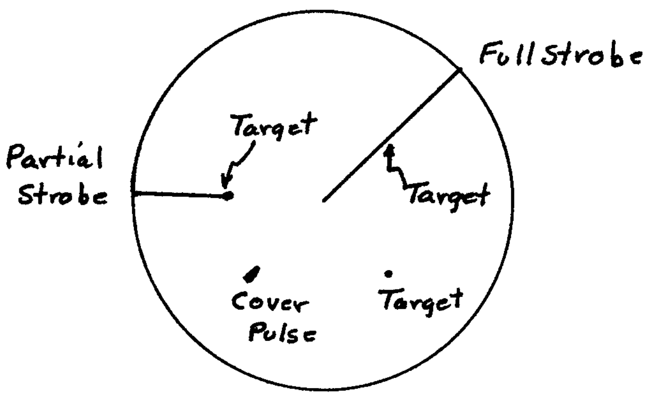
$$\text{Burn thru } R = 1/2\ \{P_p + G_t + \sigma_p + (J/S)_{reqd} - (L - L_j) - (P_j + G_j + 11)\}$$

For the repeater jammer example and a $(J/S)_{reqd}$ of 10 dB, the burn thru range is:

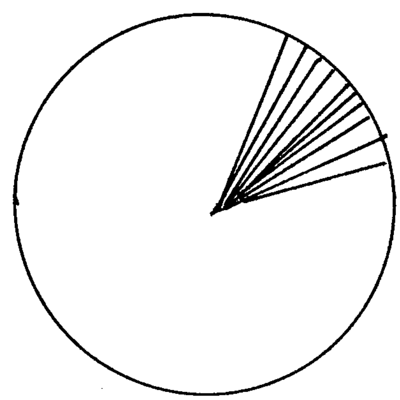
Burn thru R = 1/2 {60 + 40 + 0 + 10 - 0 - (20 + 10 + 11)} = 34.5 dB.

At ranges greater then this 2.8 Km burn thru range the penetrator is said to self screen itself from the radar.

Figure 11.3 illustrates range denial. Note that the jamming can appear as a short line or a narrow strobe (on the left), or as a wide sector of interference (as shown on the right of the figure). The width of the strobe depends on the sidelobes of the victim radar's antenna and the sidelobe suppression techniques (ECCM) used.



Narrow Strobe



Wide Strobe

Figure 11.3 Jamming Strobes

The left side of Figure 11.3 illustrates four cases: a target without jamming, a target with a jamming cover pulse (e.g. for gate pull off) and two cases where the jamming is effective only in the narrow main beam of the victim radar. If a repeater responds for only a portion of the PRI interval, starting just after receipt of the radar signal, the jamming will appear at ranges beyond the target range. This jamming does not really deny target range, since the target is at the closest range that is not jammed. The last case shows jamming for time periods greater than the PRI and provides a full strobe on the radar display. Smart or sophisticated repeater jamming can deny range by anticipating the arrival of pulses (after several are received), and by jamming before later pulses reach the penetrator. However, the victim radar may change PRI, frequency or operate passively to counter this jamming.

Burn thru ranges are not of direct interest unless they can be translated into a delay or denial of some defense action. In fact, even a small burn thru range may not be desired, if the initiation of jamming alerted the defense to the hostile's presence and increased attrition.

The right side of the figure illustrates a case where jamming is effective in the antenna sidelobes. With sufficient power, a jammer can put enough energy into the side/backlobes to completely blank out a scope - if the defense does not employ ECCM to prevent this blanking. In the absence of ECCM, the extra power required for this is just equal to the side/backlobe level (e.g. a 30 dB sidelobe must be matched by a 30dB increase in ERP).

Note that the effective radiated power (ERP) required can be expressed in terms of the J/S required as:

$$\text{Reqd ERP} = P_p + G_t + \sigma_p + (J/S)_{reqd} - (L - L_j) - 11 - 2R$$

ERP requirements vary directly with penetrator RCS. For example, if the RCS is cut in half (a 3 dB reduction), the effective radiated power is cut in half (lowered by 3 dB). This illustrates the complementary nature of low radar cross section and ECM.

Threshold Jamming

Thus far, jamming power requirements (J) have been considered in relation to the penetrators return (S). There is another case where the jamming required is not dependent upon the target return, but is done to exceed a radar threshold level. Recall that the jamming signal received by the radar receiver was:

$$J = P_j + G_j + G_r + 2\lambda - (22 + L_j + 2R)$$

Note that J is proportional to - 2R (in dBs) or dependent upon $1/R^2$. Thus, the closer the jammer is to the radar the stronger will be the signal received - to meet or exceed the radar threshold.

For example, a Doppler radar may eliminate returns at all ranges which exceed a fixed threshold. Jammers that are close to the radar can easily exceed these thresholds, but the distant jammer will be at a disadvantage. This is the opposite effect of jamming to screen a penetrator, where jammers are most effective at greater ranges due to the R^4 relationship of the target return to be obscured. Jamming requirements for the two cases, obscuring a target return and exceeding a fixed threshold, are illustrated in Figure 11.4.

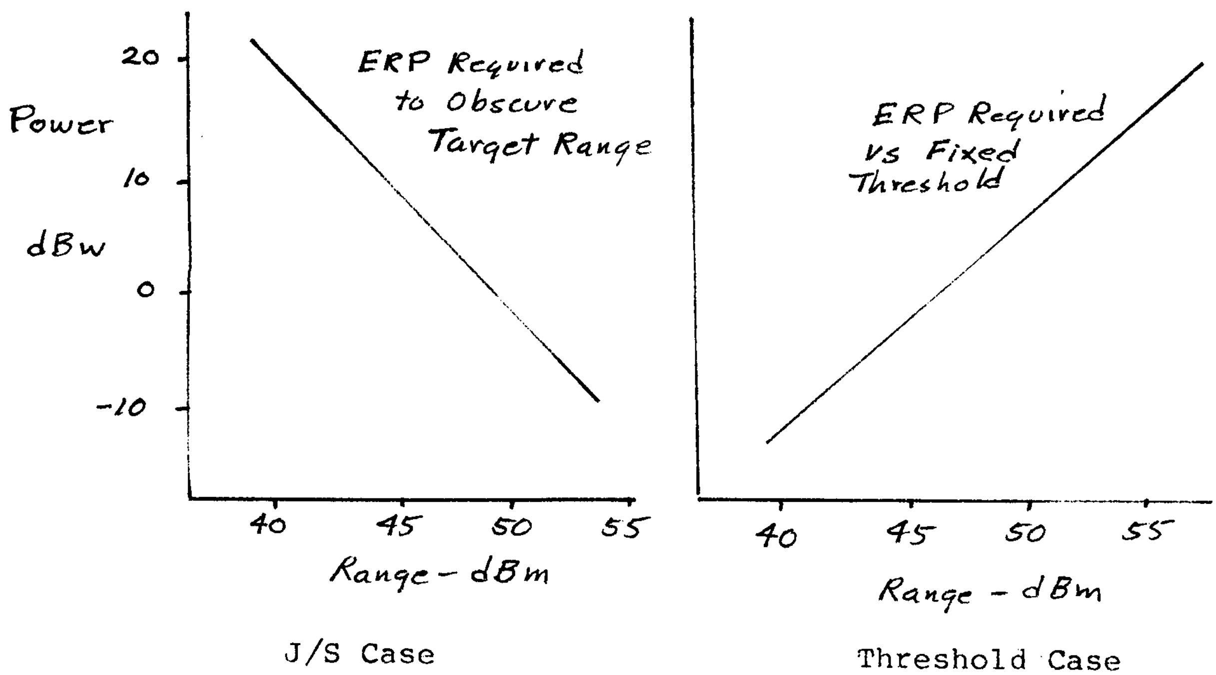


Figure 11.4 ERP Required as a Function of Range

Note that jammer ERP requirements increase markedly at close in ranges with jamming to obscure a penetrator's RCS. However, for the fixed threshold case, jamming requirements decrease as the radar is approached.

Polarization Losses

One problem in jamming is matching the polarization of the victim radar receiver. A penetrator's jamming antenna usually has a fixed polarization. A circular polarization is often chosen, since there is only a 3 dB loss against both vertical or horizontal polarized radar receiving antennas. However, if the radar's receiving antenna was polarized orthogonal to that of the jammer, a 20 dB loss can be expected (e.g. if the receiving antenna had left circular and the jammer had right circular). The polarization of the receiving antenna may not be known. It may be different than the transmitter, and may be at a different location than the transmitter (i.e. a bistatic case).

Repeater/Noise/Standoff Jamming

Active ECM includes repeater, noise and standoff jammers. A repeater receives the victim radar's signal, amplifies it and transmits it back to the radar. Since this jammer just repeats the radar's bandwidth and pulse shape/coding, there are no losses due to mismatch in these two areas. The previous ECM examples assuming $L = L_j$ are based on repeater jamming.

A noise jammer generates its own jamming signal, which is transmitted to degrade a threat radar. The jammer does not repeat the radar's waveform, so three additional loss terms must be considered:

1) Jammer/radar bandwidth ratio,
2) Radar pulse coding/compression, and
3) Coherent integration.

The difference in ERP required for noise jamming compared to repeater jamming is shown next:

$$\Delta\ ERP_{reqd} \text{ for noise jamming} = (B_j - B) + F_{code} + F_{coh}$$

where B_j = Jammer bandwidth, in dB
B = Radar bandwidth, in dB
F_{code} = Additional dBs due to pulse coding
F_{coh} = Additional dBs due to coherent processing

For example, if:

$(B_j - B)$ = 3 dB (the jammer/radar bandwidth ratio is 2)
F_{code} = 20 dB, or a 100 to 1 pulse compression ratio
F_{coh} = 0 dB, or no coherent processing benefits.

The change in ERP required is:

$$\Delta\ ERP = 3 + 20 + 0 = 23 \text{ dB}$$

This is a 200 fold increase in power requirements. This illustrates the advantage of repeater jammers over noise jammers, particularly when the victim radar uses pulse compression.

A standoff jammer is operated aboard a support vehicle, which usually stands off (at a safe distance from the defense threats). The jammer attempts to degrade enemy radars and/or communication systems, thus shielding the penetrator from defense intercept. The standoff vehicle can use repeater and/or noise ECM. A standoff jammer must consider two more potential losses:

1) The standoff jammer is at a range, R_{SO}, from the radar, while the penetrator being supported is at a range, R, from the radar. Usually $R_{SO} > R$.

2) The standoff jammer is attempting to mask the penetrator (in the radar's mainbeam), but the standoff jammer's energy enters the radar's sidelobes (if the standoff vehicle is not in the same beamwidth as the penetrator).

A typical standoff jammer geometry is illustrated in Figure 11.5.

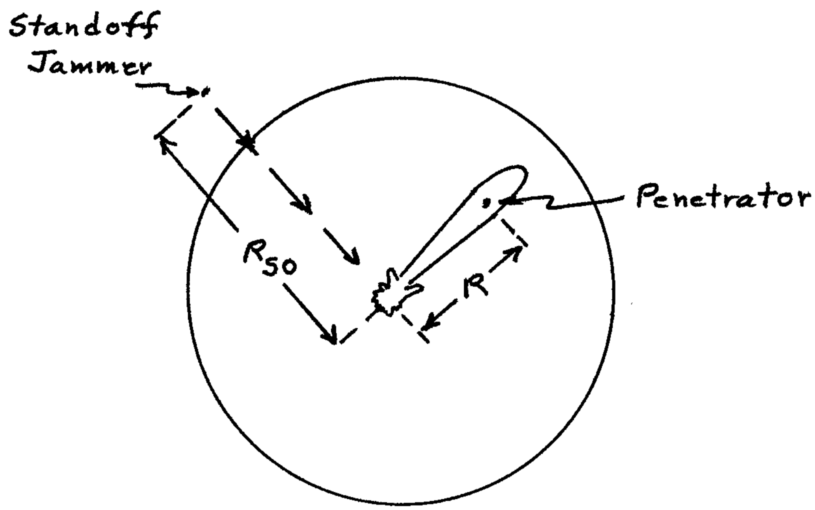


Figure 11.5 Standoff Jammer Geometry

The difference in ERP required for standoff jamming compared to a jammer on the penetrator itself is shown below:

$$\Delta \; ERP_{reqd} \text{ for standoff jamming} = (G_r - G_{sl}) + 2(R_{SO} - R)$$

For example, if the standoff jammer was twice as far away from the radar as the penetrator being protected, and was jamming in a sidelobe which was 30 dB below the mainbeam, the change in ERP would be:

$$\Delta \text{ ERP} = 30 + 2(3) = 36 \text{ dB}$$

A 36 dB increase represents almost a 4000 fold increase in jamming power over the penetrator self screen case. This illustrates the high ERP requirements for standoff jamming.

If the standoff jammer employed noise jamming, the ERP requirement deltas for standoff and noise must be added together (if in dB form). Thus in the example, a 23 + 36 = 59 dB improvement is needed in a standoff noise jammer to match the screening power of a repeater on the penetrator. Power is not the only measure, however, since the standoff jammer may be able to provide special ECM techniques and off angle jamming not possible from the penetrator.

ECM Goals

Electronic countermeasure goals can be categorized into denying (or delaying) detection and creating errors in range, velocity and angle. These four categories, their corresponding measures and ECM technique examples are shown below:

ECM Goal	Measure	Technique Examples
Deny detection	Sensitivity	Raise threshold level False targets
Deny range	Time	Screen target range Range pull off
Deny velocity	Frequency	False targets Velocity pull off
Deny angle	Beamwidth and scan pattern	Angle deception Off board jamming

A prime ECM goal is to deny detection of the penetrator. The measure of a radar's ability to detect a target is its sensitivity. The presence of natural clutter, however, forces the radar to use clutter thresholds to eliminate these sometimes overpowering returns. ECM may also cause an increase in the radar's threshold to the point where a target can no longer be detected. A second example to deny (or delay) detection is the creation of many false targets throughout the radar display. The time required to sort through these false targets may exceed the target time in coverage, or limit the time for defense intercept actions.

Denying or obscuring target range is one of the easiest tasks for ECM. However, a jammer should insure that the radar receives sufficient ERP both before and after the time that the target signal reaches the radar. This can screen the penetrator's range. A different ECM technique, called range gate pull off (RGPO), can be used against range gates of target tracking radars. RGPO starts with a jamming cover pulse over the target return. This cover pulse is then moved in range away from the target return to pull the tracking gate off the target.

Denying target velocity is a prime ECM goal against Doppler radars, which depend upon velocity for target detection and tracking. A jammer can create false targets with different closing velocities (Doppler frequencies). The time delay in sorting through these false targets may delay or prevent intercept. A second example is a velocity gate pull off (VGPO) technique employed against velocity gates of target tracking radars. This technique starts with a velocity cover pulse which is moved in velocity to pull the tracker off the target.

Denying angle may be the most difficult task for an ECM suite. Some angle deception techniques attempt to jam while the antenna sidelobes are pointing to the target, and avoid jamming the mainlobe. This is called Inverse Gain, since the jamming power varies inversely with the antenna gain. This ECM technique creates strobes and false targets at different angles than the true target. As mentioned in standoff jamming requirements, sidelobe jamming requires tens of dB more ERP than main-beam jamming. Another technique is to create false targets at other angles using off board or bounce techniques. Radar susceptibility to sidelobe jamming and angle deception will be discussed in Chapter 12 on ECCM.

Predicting ECM Effectiveness

Predictions of how ECM will degrade an air defense system are made by operating commands, study agencies and avionics manufactures. ECM equipment manufactures tend to claim the greatest benefits. Operating commands tend to place less reliance on ECM, but more reliance on mission planning and tactics.

In past conflicts, ECM has proven to be of great value. However, the value has varied with the shock of encountering new offensive or defensive systems, and the learning curve of offense and defense. Some believe that the value of ECM deceases over time as the enemy learns to counteract it. Others believe that ECM and tactics can improve faster than the defense, and that ECM will become more valuable during a conflict.

In either case, ECM vs ECCM can be considered a game. The game is very dynamic, with no clear winner over time and considerable uncertainty of the effectiveness at any given time. The terms robust and fragile have been used to express the degree of confidence in an ECM or ECCM technique. Robust techniques are expected to do the job without needing any special knowledge of enemy systems and should have been extensively tested. A fragile technique may be one that is relatively easy to counter, or which depends upon a critical assumption about an enemy system which can not be verified.

Summary

This chapter pointed out the advantages of randomly dispersed passive reflectors (e.g. chaff), and the possible disadvantages of regular dispensing by a lone penetrator.

Sensitivity requirements for a radar warning receiver (RWR) to detect radar threats were explored, and shown to be dependent upon the penetrator's RCS. Calculations illustrated how a radar can lower its sensitivity to avoid detection by a RWR. Recognizing radar types in a dense radar environment can be a difficult problem, particularly with the number of different signals received due to improved RWR sensitivity. Diffraction of radar signals can cause early RWR warning and premature ECM turn-on, which can be a significant aid to the defense.

Jamming requirements were shown for repeater, noise and standoff jammers. The effective radiated power (ERP) needed to obscure a target's return was shown to be a function of R^2, and the limitations of burn thru range calculations were pointed out. Jamming to exceed a fixed radar threshold was shown to be a function of $1/R^2$, with the closest jammer being most effective.

ECM goals of denying detection, range, velocity and angle were described, along with some examples of ECM techniques. Predicting ECM effectiveness is quite difficult due to the nature of the ECM/ECCM game. Each side seeks robust techniques which will be effective despite changes in tactics or countermeasures.

Chapter 12

Electronic Counter Countermeasures

This chapter describes electronic counter countermeasures (ECCM) - actions taken to insure use of the electromagnetic spectrum, despite enemy countermeasures. Radar ECCM will be emphasized particularly avoiding saturation and discriminating target range, velocity and angle. A short discussion of penetrator requirements for ECCM and the ECM/ECCM game concludes the chapter.

ECCM Categories

Radar electronic counter countermeasures can be divided into four general categories as noted below:

Category	ECCM Examples
Avoiding saturation	Gain control, Guarding sidelobes.
Range discrimination	Triangulation, Multiple radar signals, Pulse edge tracking.
Velocity discrimination	MTI/Doppler processing, Multiple radar signals, Gate movement logic.
Angle discrimination	Monopulse processing, Track on Jam, Home on jam.

Each category will be examined in turn, discussing the ECCM technique examples listed above.

Avoiding Saturation

Radars must operate within a specified dynamic range in order to process signals in their amplifiers. The jammer power (and clutter returns) received can be thousands of times stronger than target returns, and can saturate or overload an unprotected radar. Two general ECCM techniques to avoid saturation - gain control and guarding the side lobes will be discussed.

Gain Control

A radar operator can prevent saturation by lowering the gain of the radar. However, this would eliminate weaker target returns at all ranges and azimuths. Gain reduction may be compared to turning down the volume on a TV to reduce the sound level, so that only the louder sounds are heard.

Various ECCM techniques are available to automatically control the gain of a radar receiver to avoid saturation, yet retain reasonable target detection capability. Five techniques will now be described.

The first is a countermeasure against sweep jammers, called a Dicke -fix. The Dicke-fix includes a limiter in the receiver to prevent occasional strong signals from saturating the radar.

A second method to control the gain to avoid saturation is by establishing a non-linear relationship between receiver input and amplifier output, for example - using a logarithmic amplifier. Both limiting and non-linearity are illustrated in Figure 12.1.

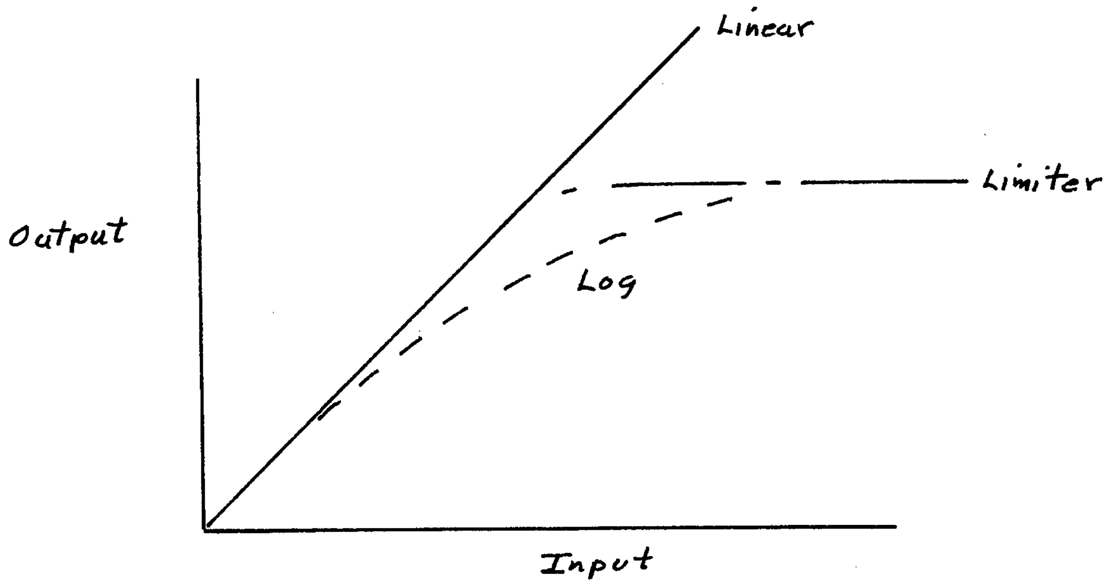


Figure 12.1 Gain Control

Normal radar operation occurs in the linear region. Beyond this linear region, the radar is protected from occasional jamming and clutter signals which can saturate the amplifiers, but at the expense of a loss in radar sensitivity in detecting weaker signals.

A third technique, Sensitivity Time Control (STC), varies radar gain with range, or time from pulse transmission. STC usually adjusts the receiver gain according to $1/R^4$, so that a target will provide approximately the same signal level regardless of the range from the radar. This prevents strong close in returns from saturating the radar, while retaining sufficient sensitivity at maximum range.

Two other gain control techniques, Automatic Gain Control (AGC) and Instantaneous Automatic Gain Control (IAGC), continuously adjust the gain by the strength of the signal received. IAGC has a faster response time, approximately equal to the transmitted pulse width. The faster response limits the time when the radar loses sensitivity.

All five of the above ECCM techniques are designed to avoid saturation by strong returns in the radar's mainbeam, at the expense of a loss in sensitivity. All five techniques can be considered to be Constant False Alarm Rate (CFAR) controls, since they can cause a higher detection threshold in the radar.

Guarding the Sidelobes

A different ECCM technique designed to counter jamming in the sidelobes is called a Guard Channel. This requires a separate guard antenna, as shown in Figure 12.2.

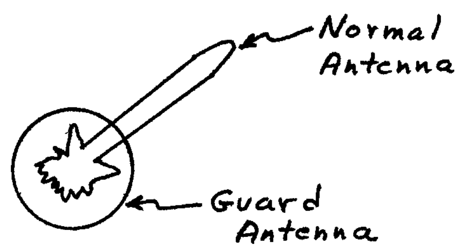


Figure 12.2 Guard Antenna

The guard antenna's gain is set just above that of the highest sidelobe, and is isotropic (equal gain in all directions). The signal received by the normal antenna and the guard antenna are continuously compared. As long as the normal signal exceeds that from the Guard Channel, the radar operates as if there were no guard antenna. However, whenever the Guard Channel has the stronger signal (e.g. due to jamming received in a sidelobe), the normal returns are either blanked or cancelled. Sidelobe Blanking (SLB) blanks or cuts off all returns when the Guard Channel receives stronger signals. Sidelobe Cancellation (SLC) attempts to create an antenna null in the direction of the strong signals in the sidelobe by judicious combination of antenna patterns. This null cancels the strong signals by lowering the effective antenna gain in their direction, without blanking or eliminating target returns in the mainbeam.

Both SLB and SLC work best against a single jammer. Multiple jammers at different azimuths degrade these ECCM techniques. For example, two penetrators at different azimuths from a radar can jam cooperatively, so that each jams only when the other is in the radar's mainbeam. Against SLB, each penetrator can blank the other's return. Against SLC, each penetrator can at least cause a significant reduction in the effective antenna gain used to detect the other's return.

With effective sidelobe and gain control, a radar can avoid saturation and reduce the effects of a single penetrator's simple repeater or noise jamming to no more than a narrow strobe, a beamwidth or so in width. Deceptive techniques may deny detection, but depend upon knowledge of how the radar processes signals.

Range Discrimination

Three general ECCM techniques to discriminate target range - triangulation, multiple radar signals and pulse edge tracking will now be described.

Triangulation

Perhaps the simplest way to obtain the range on one penetrator which is jamming two or more radar sites is by triangulation on the jamming source from multiple radar sites. This is illustrated in Figure 12.3.

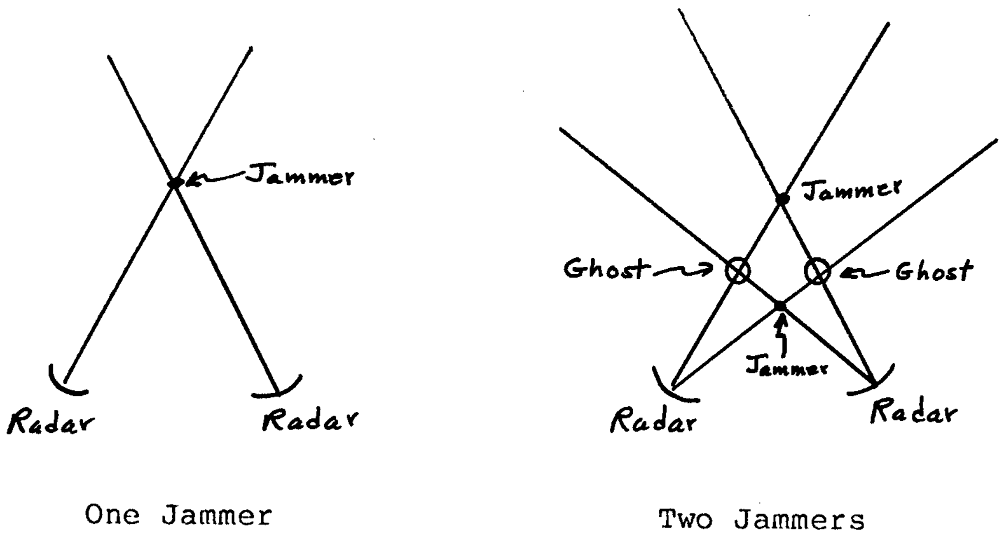


Figure 12.3 Triangulation

Triangulation depends upon finding the intersection of jamming strobes, and so works best with narrow strobes (e.g. when radar side lobes are reduced or guarded). When more than one jammer is in view, ghosts will appear at the intersection of jamming strobes as shown on the right side of Figure 12.3. Ghosts can cause defense delays - due to the time required to observe the movement of all intersection points to find the real targets. The number of ghosts rises dramatically with the number of penetrators in view and the number of radar locations used for triangulation. For example, with two radar locations and two penetrators there are 4 possible intersections and 2 ghosts possible. For 3 penetrators and 2 radars there are 9 possible intersections and 6 ghosts possible. With more than two radars, deghosting is possible by choosing only those intersections common to all radars.

Penetrators at low altitude may not be within radar line of sight of two or more ground radars at all times. Thus, triangulation may not always be possible against low altitude penetrators - unless jamming signals can be picked up over-the-horizon by diffraction.

Multiple Radar Signals

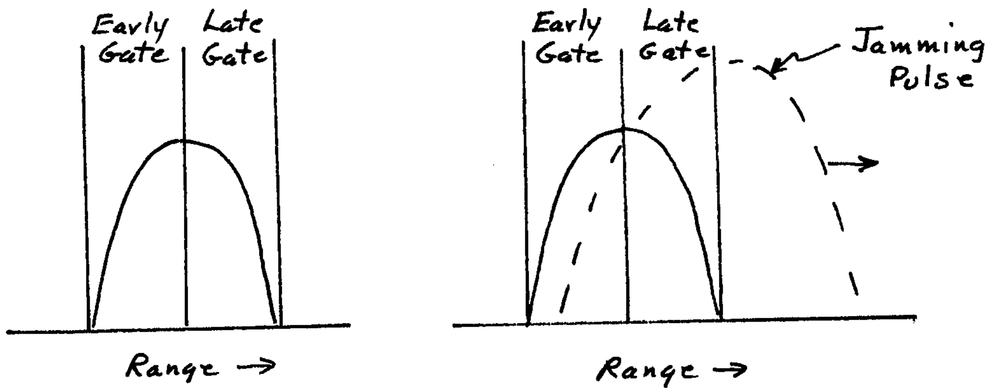
When triangulation is not feasible, a defense net might determine penetrator range by use of ECCM techniques which will be called multiple radar signals. Here, many different radar signals illuminate the penetrator, in an attempt to find some combination which can not be countered effectively by the penetrator's ECM suite. Five of these techniques will be described next.

Three ECCM techniques are frequency diversity, polarization diversity and agilities (e.g. changing frequency and other parameters to minimize jamming). A penetrator that is dependent upon jamming must cover <u>all</u> frequencies, rapidly follow <u>all</u> frequency shifts and have enough effective radiated power to make up for polarization mismatches (which may require up to 20 dB extra power). Agility in PRI can also be effective against repeater jammers which try to predict pulse arrival time to provide range cover pulses. This agility will be discussed later under ECCM against RGPO.

A fourth ECCM technique is provided by an unjammed radar in a passive, listening mode that suddenly turns active to obtain target range before jamming is initiated at its frequency. A fifth technique synchronizes pulses of several radars operating at different frequencies. This can overload ECM suites which can respond to only a limited number of signals at any instant in time.

Pulse Edge Tracking

Once range is obtained, an ECCM technique called pulse edge tracking can be used by tracking radars to avoid some range gate pull off (RGPO) ECM. Pulse edge tracking is illustrated in Figure 12.4.



No Jamming With Jamming

Figure 12.4 Pulse Edge Tracking

The radar can employ Leading Edge Track or Trailing Edge Track, which are also called early or late gates, respectively. If a simple repeater merely responds to the received pulse, jamming would start after a short time delay - due to the time required for a penetrator's ECM suite to recognize the pulse and respond with a cover pulse. The delay, shown on the right side of Figure 12.4, may be large enough so that the early gate stays on the target as the cover pulse moves to greater ranges.

A smart repeater or transponder can anticipate the next pulse and start the cover pulse early to avoid this time delay (and can attempt pull off to shorter or greater ranges). However, the ECCM technique of PRI agility may invalidate the prediction of the arrival time of the next pulse, and prevent this more sophisticated RGPO from decoying a tracking gate.

Gate pull off ECM is generally more effective against automatic systems. Operators who are alerted for a jamming cover pulse may delay moving the gate when jamming is suspected, and thus may not be decoyed into following the pull off cover pulse.

Velocity Discrimination

Three general ECCM techniques to discriminate target velocity - MTI/Doppler processing, multiple radar signals and gate movement logic - will now be discussed.

MTI/Doppler Processing

Moving Target Indication (MTI) and Doppler processing for velocity discrimination has been discussed in Chapter 6. Both are effective ECCM techniques in eliminating returns which do not match the Doppler bandwidths of likely targets.

Multiple Signals

False velocity targets, which match expected target speeds, can be created by ECM. However, the same general multiple radar signal ECCM techniques discussed above under range discrimination are also applicable to velocity discrimination. Recall that the five techniques were frequency diversity, polarization diversity, agilities, passive/active mode and pulse synchronization. If any of these methods succeed, the true target velocity can be obtained.

Gate Movement Logic

The primary purpose of false targets is to delay defense actions against the penetrator. The time delays in sorting through these targets can be speeded by an ECCM technique, which will be called gate movement logic. This logic depends on knowing the aerodynamic velocity and acceleration limits of an air vehicle target, and the physical laws governing range and azimuth movements. If range and velocity gates can be set on each possible target (false and true alike), logic can be applied to eliminate those targets which do not move in accordance with physical laws and aerodynamic limits. This ECCM technique forces the offense to more sophisticated ECM to present false targets with believable range, velocity and acceleration histories. It will also lessen the number of believable targets which can be presented to search radars which observe targets for relatively long periods of time. Effective radar sidelobe control further restricts these false targets to the same direction as the jamming penetrator.

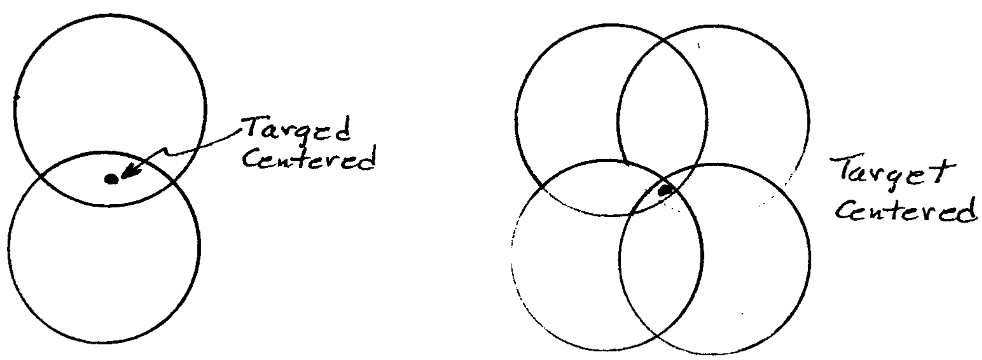
Angle Discrimination

Even if a penetrator is able to obscure its range and velocity, the defense can still use the penetrator's azimuth (and/or elevation) angle to assign and vector interceptors and for guiding missiles. Denying angle is perhaps the most difficult task for ECM. For example, a search radar, with effective sidelobe control, should be able to obtain a good azimuth on a penetrator using ECM to deny range to the radar.

Tracking radars and their ability to measure target azimuth (and elevation) angles will now be discussed. Radar trackers which use a single antenna beam, track a target by keeping the penetrator centered within a scan pattern. Single beam scanning is vulnerable to jamming which is synchronized to the scan rate. If jamming occurs only when the antenna is not centered on the target, but off to one particular side, the center of the next scan will be driven off the target towards the stronger off-centered jamming signal. One ECCM technique to prevent the jammer from synchronizing to the scan is to conceal the scan rate. For example, with ECCM techniques called Lobe on Receive Only (LORO) or Conical Scan on Receive Only (COSRO), the transmitting antenna does not scan, but a separate receiving antenna does the scanning. Without knowing the receiver scan pattern, this jamming (which must be synchronized to the scan rate) can not succeed.

Monopulse Processing

Another way to counter angle deception ECM is for the radar to obtain information on the direction of the target without scanning. This can be accomplished by monopulse processing. Monopulse means one (mono) pulse, and indicates that the radar can obtain range and angle information in one pulse from one antenna position. A monopulse system requires more than one receiving antenna. Examples of two and four receiving antenna patterns are illustrated in Figure 12.5.



2 Beams 4 Beams

Figure 12.5 Monopulse Receiving Antennas

Figure 12.5 shows a target centered in the beams, indicating that the antenna is boresighted on (or pointing directly at) the target.

The special feature of multiple receiving antennas is their relative invulnerability to angle deception by noise or normal repeater jamming. Normal angle deception jamming not only does not degrade monopulse trackers, but the jamming acts as a beacon for the monopulse system to determine angular measurements. ECM techniques have been developed to counter monopulse processing, including special angle pull off deception techniques and use of off board jamming, in the never ending game of measure and countermeasure.

Home on Jam

Home on jam (HOJ) or Track on Jam (TOJ) ECCM techniques allow a tracker to home on the jamming signal in angle, when the normal tracking circuits can not be used. HOJ can be very effective if range data are not required for successful intercept. This might be the case if a missile can be launched within its effective envelope, and a proximity fuze used for weapon detonation against a penetrator.

Offensive ECCM

Thus far, ECCM has been described for defense systems. Now some ECCM requirements for the penetrator will be discussed.

A desire to conceal the penetrator's presence as long as possible dictates low observables. However, the intruder may require an active radar for navigation, particularly if the vehicle is to follow the terrain at low altitude. This radar can be jammed by the defense, or its emissions used to detect and intercept the penetrator. The offense can counter by minimizing use of this radar, lowering the radar power, spreading the frequency, using a high gain antenna with very low side lobes and carefully choosing the frequency bands and polarization. These are all offensive ECCM tactics and techniques.

The intruder may need a warning system to detect AI, SAM and AAA activity around the vehicle. A completely passive system is preferred (for low observable penetration), but an active system might be allowed _after_ threat detection to quickly confirm and provide range/velocity data on the particular threat detected.

A penetrator's ECM should react to defense ECCM. For example, if the enemy depended upon home on jamming, penetrator ECM should include systems which can thwart HOJ, by decoying the defense away from the penetrator. Chaff and expendables jammers could provide such a decoy, as could off board jamming.

The ECM-ECCM Game

ECM and ECCM advocates often propose excellent techniques that could be operational in several years - to counter today's threat. By that time the threat could change markedly, and the proposal may no longer be effective. Systems with rapid software reprogramming capability, however, may allow a significant reduction in the long delays in fielding new ECM and ECCM techniques. This will make the game even more dynamic and more dependent upon fast reaction to counter new threats or techniques.

Another approach to speed up acquisition is to research and develop countermeasures at the same time that friendly systems are being developed (or at least before they are exported). Robust techniques would be sought, although fragile techniques might also be pursued if they are cheap and easily implemented. A danger in this approach is that the system may never be procured (or exported) if decision makers thought it could be countered by ECM or ECCM.

Summary

ECCM techniques to avoid radar saturation by enemy jamming or clutter returns include gain control and guarding the sidelobes. Many techniques provide automatic gain control to prevent overloading of the radar's amplifiers. However, these techniques will reduce radar capability to detect low RCS target returns.

A Guard Channel can blank or cancel jammer energy in a radar's side-lobes, limiting the jamming from one penetrator to no more than a narrow strobe. Multiple jammers may not be handled as easily, particularly if they practice cooperative jamming to protect each other.

ECCM techniques to discriminate target range include triangulation, multiple radar signals and pulse edge tracking. The defense may obtain a penetrator's range by triangulation from several radars which are being jammed. However, multiple penetrators create ghosts due to multiple intersections of jamming strobes. The time required to sort out these ghosts delays defense actions.

Multiple radars signals (multiple frequencies, multiple polarizations, frequency/PRI agilities, passive/active mode and pulse synchronization) complicate a penetrator's task of concealing range (and velocity) from every radar encountered.

Pulse edge tracking is an effective ECCM technique against simple repeaters attempting range gate pull off. Combined with PRI agility, it can be effective in preventing RGPO by smart repeaters.

ECCM techniques to discriminate target velocity include MTI/Doppler processing, multiple radar signals and gate movement logic. MTI/Doppler processing can eliminate returns that are not in the same Doppler bandwidths as likely targets. Gate movement logic can eliminate false targets that do not have the correct velocity and acceleration profiles.

ECCM techniques to discriminate angle includes monopulse processing and home on jam. Monopulse radars use multiple receiving antennas to obtain angle information, and thus are not subject to normal angle deception jamming techniques which are effective against single antenna scanning radars. Several monopulse countermeasures have been developed, however, in the never ending ECM/ECCM game.

A penetrator should practice ECCM by limiting emissions, using passive sensing systems and insuring that its ECM is not vulnerable to track on jam and home on jam.

Robust ECM/ECCM techniques are desired by offense and defense, however the time delay in implementing new techniques may be very long unless software reprogramming capability is included in the design of new systems. This will make the game even more dynamic, and more dependent upon fast reaction to new threats.

Chapter 13

Lethal Self Defense vs SAMs

Penetrator encounters with SAMs are discussed in this chapter. The SAM intercept region will be highlighted, with equations developed for the earliest head-on intercept. Penetrator self defense weapon/sensor range requirements will be analyzed to attack the SAM site before it launches, along with methods to reduce these range requirements. An analysis of weapon and sensor needs to attack the missile itself concludes the chapter.

SAM Encounters

A SAM encounter occurs when a SAM site achieves at least one missile launch against a penetrator. One encounter includes all missiles launched until the penetrator either is killed or survives (exits site coverage).

A penetrator's route may be carefully chosen to avoid en route encounters with SAM sites. However, encounters may still occur. For example:

1) While creating a safe corridor through a known SAM barrier,
2) Penetrating known terminal defenses around a target,
3) Penetrating coverage of moved SAMs, or
4) Penetrating coverage of concealed SAMs.

In the first case, mission plans include an attack against one or more SAM sites. In the other cases, some penetrator action may be required to improve survival, such as:

1) In flight turns (to minimize exposure)
2) ECM - to deny/delay intercept and degrade attrition, or
3) Lethal self defense - vs the site or the missile.

Avoidance may not be possible if the penetrator does not detect a previously unknown SAM in time, or does not have the flexibility to change routes in flight. ECM may be able to delay/degrade SAM actions. However, the effectiveness of ECM may be uncertain - due to lack of robust techniques, or due to changes in defensive tactics or ECCM. (Missions which must be repeated may allow the defense to develop countermeasures over time which may reduce ECM effectiveness significantly.)

If avoidance and/or ECM can not provide sufficient protection, lethal self defense measures may be required. Lethal self defense is usually considered after avoidance and ECM, since carrying defensive weapons usually reduces the offensive payload.

SAM Intercept Range

The intercept region around a SAM site (with no masking, multipath or clutter restrictions) against a non-maneuvering penetrator depends upon:

1) Defense coverage and reaction time,
2) Missile velocity and time of flight, and
3) Penetrator altitude, RCS, velocity and offset to the site.

Figure 13.1 illustrates some of these factors for a SAM's forward hemisphere and first intercept:

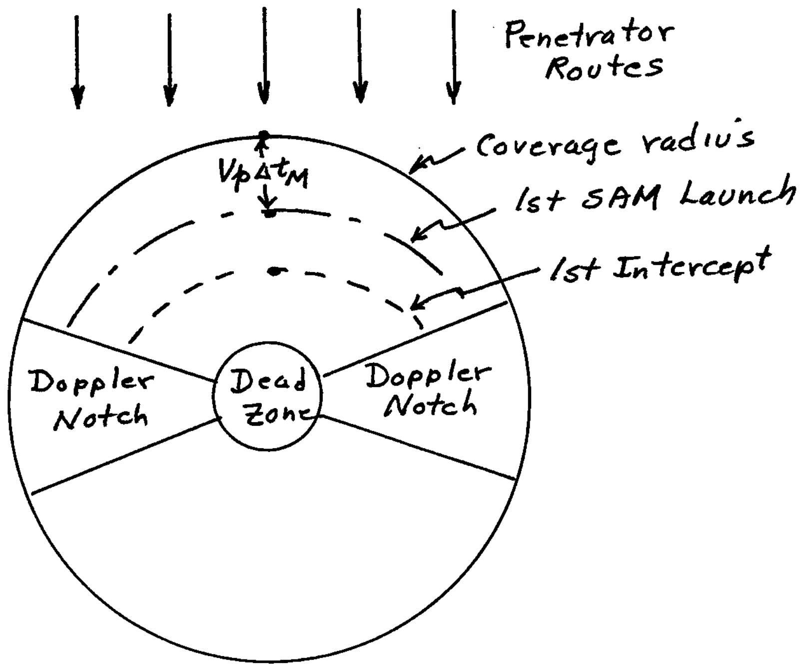


Figure 13.1 1st SAM Intercept

The dead zone is an area close to the site where radar tracking, missile performance or seeker limits severely degrades SAM effectiveness. The Doppler notch prevents active track when a penetrator's radial speed is below the radar velocity threshold.

For a direct overflight of the site (i.e. zero offset distance), the sensor limited launch range R_L of the first missile is

R_L = Entry range - Distance flown during the SAM's reaction time

$$R_L = R_e - V_p \, \Delta t_M$$

But the launch range is also equal to:

R_L = Missile flight time x Closing velocity

$$R_L = TOF_M \, (V_p + V_M)$$

From which $TOF_M = (R_e - V_p\ \Delta t_M)/(V_p + V_M)$.

This equation can be solved for R_M:

$$R_M = TOF_M\ V_M = (R_e - V_p\ \Delta t_M)/(1 + V_p/V_M)$$

where R_L = Penetrator range at the first SAM launch
R_e = Penetrator range at entry (SAM sensor limited)
Δt_M = SAM reaction time (from entry to launch)
V_p = Penetrator ground velocity
TOF_M = Missile time of flight
V_M = Average missile velocity
R_M = Missile range to intercept

Note that TOF_M has both a minimum and maximum limit.

For example, with a 10' SAM radar antenna height, a 200' altitude penetrator flying at 600 knots ground speed, a SAM reaction time of 25 seconds, average missile velocity of 1000 knots, sufficient radar sensitivity and the standard 4/3rds earth refraction model:

$$\text{Sensor limited } R_e = 1.23\ (10^{1/2} + 200^{1/2}) = 21 \text{ NM}$$

$$R_L = R_e - V_p\ \Delta t_M = 21 - 600\ (25/3600) = 17 \text{ NM}$$

$$R_M = (R_e - V_p\ \Delta t_M)/(1 + V_p/V_M) = 17/(1 + 600/1000) = 11 \text{ NM}$$

In the example the penetrator enters cover at 21 NM head-on to the SAM. In the 25 seconds for SAM reaction time, the penetrator flies 4 NM. A missile is launched when the penetrator is 17 NM away. First SAM intercept occurs at 11 NM from the site (with a 1600 knot average closing speed between intruder and missile), if the missile can reach this range. Figure 13.2 shows the sensitivity of entry range, distance flown during the SAM reaction time and penetrator/missile velocity to

SAM launch range and missile range.

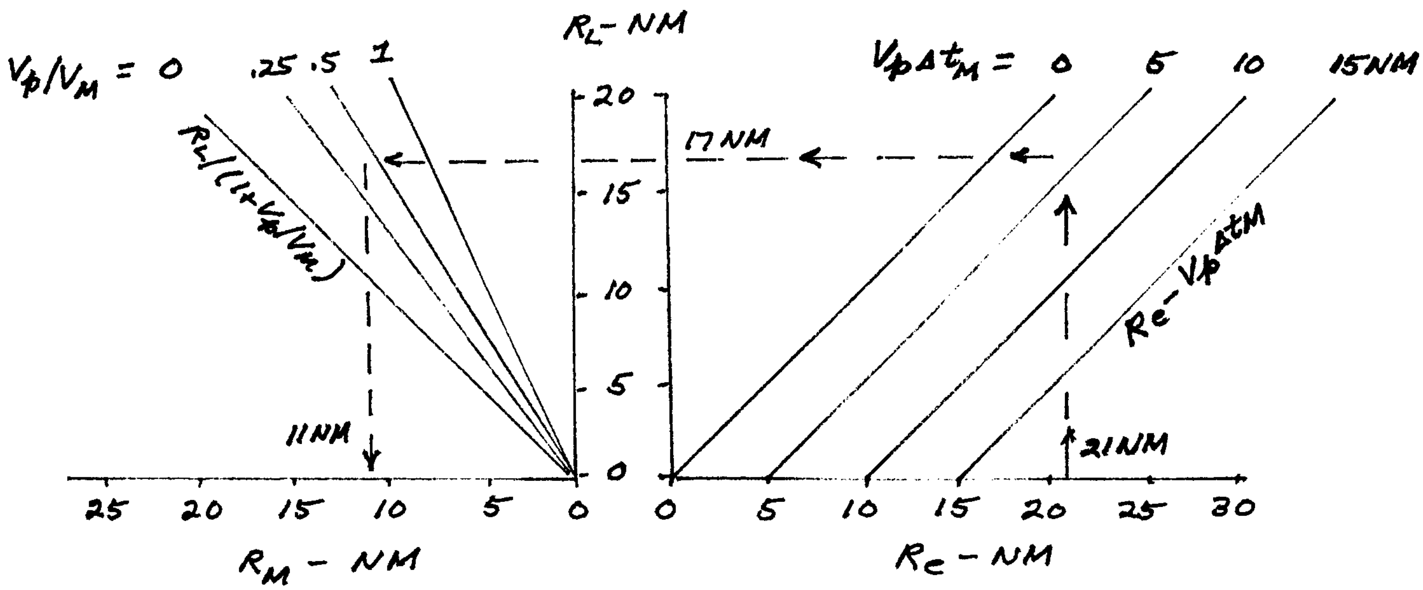


Figure 13.2 SAM Capability

If the missile does not have this range capability, the SAM is missile limited, rather than sensor limited. (SAMs used against low altitude penetrators will most likely be sensor limited. SAMs used against high altitude penetrators will most likely be missile range limited.) When missile limited, the launch range R_L is determined from R_{Max} as:

$$R_L = R_{Max} + V_p \; TOF_{Max} = TOF_{Max} \; (V_M + V_p)$$

The chart on the left of Figure 13.2 can be used - entering R_{Max} to find the SAM launch range R_L.

Penetrator <u>weapon</u> range requirements against a site will now be addressed.

Weapon Range Requirements vs a SAM Site

For a direct overflight of a SAM site (zero offset), the penetrator's standoff weapon range requirements to hit the SAM site prior to SAM missile launch is illustrated in Figure 13.3:

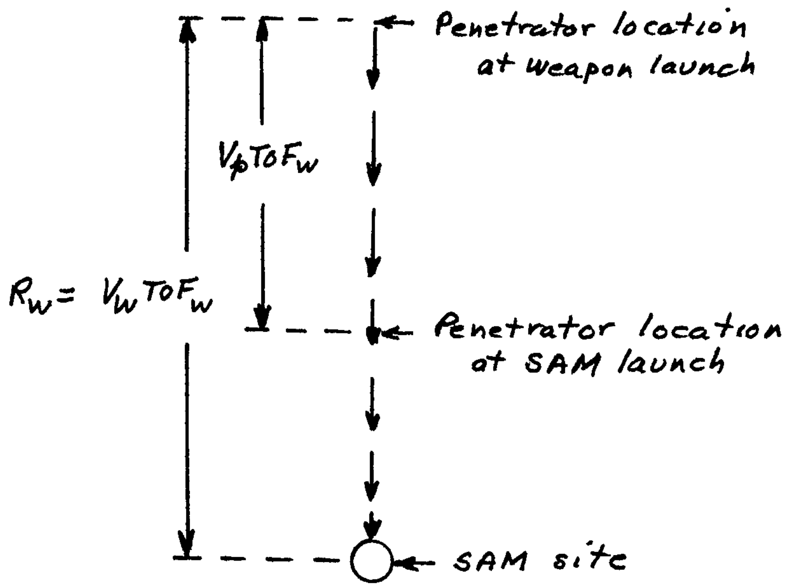


Figure 13.3 Penetrator Weapon Range Requirements

The weapon range R_W is equal to the weapon's TOF multiplied by its <u>average</u> velocity:

$$R_W = TOF_W \ V_W.$$

This range must exceed the SAM launch range plus the distance the penetrator flies during the weapon time of flight, in order that the SAM site be hit before it can launch:

$$R_W > R_L + TOF_W \ V_p$$

If the SAM is sensor limited this inequality becomes:

$$R_W > R_e - V_p \ \Delta t_M + (R_W/V_W) \ V_p$$

This can be solved for R_W as:

$$R_W > (R_e - V_p \Delta t_M)/(1 - V_p/V_W)$$

where R_W = Range of the penetrator's weapon
V_W = Average velocity of the penetrator's weapon

For example, using the same SAM values as before and letting the penetrator's weapon average velocity be 1000 knots:

$$R_W > (21 - 600\{25/3600\})/(1 - 600/1000) = 43 \text{ NM}$$

In order to launch at this range, the penetrator must detect the SAM at a greater distance, identify it and decide to attack. The time for these actions will be labeled Δt_W, and the range at which the penetrator must detect the SAM will be labeled R_d. This must be greater than:

$$R_d > R_W + V_p \Delta t_W$$

In the example, assuming 25 seconds reaction time to detect, identify and decide:

$$R_d > 43 + 600 (25/3600) = 47 \text{ NM}$$

This is well beyond the radar horizon of 17 NM for a 200' penetrator altitude and presents quite a challenge for an airborne sensor. Figure 13.4 illustrates the sensitivity of SAM launch range, penetrator/weapon velocity and reaction time on range requirements.

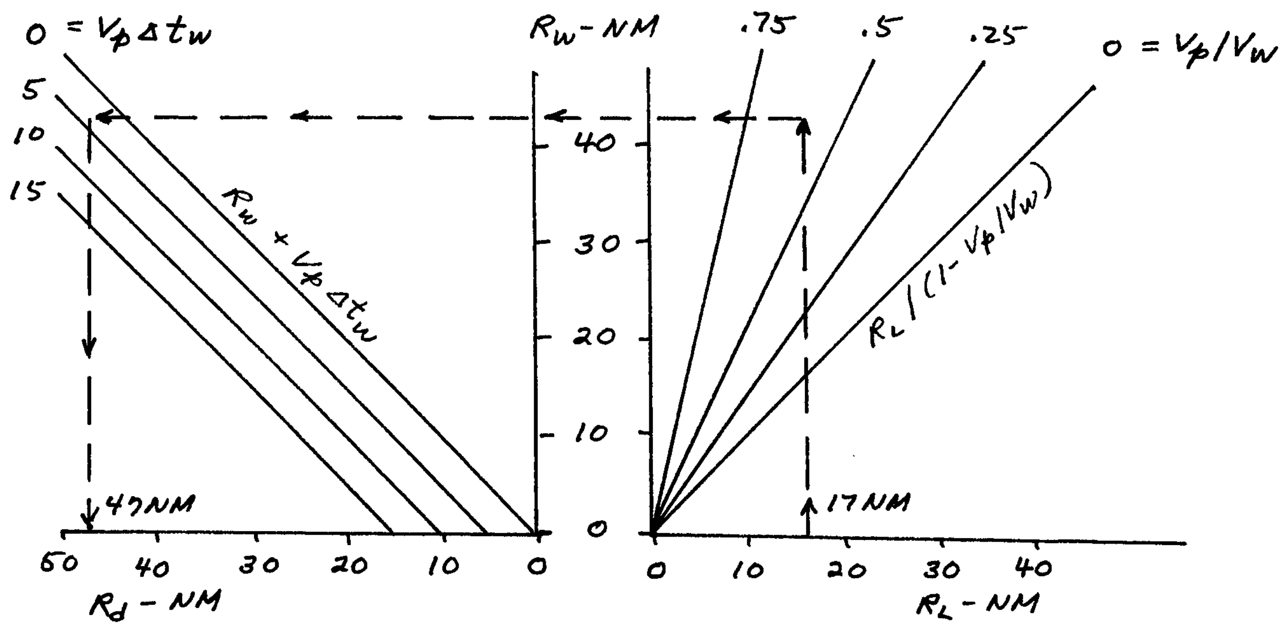


Figure 13.4 Weapon Capability

If the penetrator is sensor limited, Figure 13.4 can be used in reverse - entering the detection range R_d limit and solving for the launch range that can be supported.

A passive sensor on the penetrator (e.g. a radar warning receiver) might be used to detect an active SAM radar signal over-the-horizon, providing an azimuth and a rough measure of range to the site (by the signal intensity received). However, if the SAM maintained radar silence until the penetrator came into SAM detection range (e.g. depended on an acquisition radar for early warning), the passive detection range would be limited to that when the SAM radar first went active.

Some ways to reduce penetrator weapon and sensor range requirements will now be addressed.

Reducing Range Requirements

Penetrator weapon/sensor range can be reduced by:

1) Electronic countermeasures,
2) Penetrator low radar cross section,
3) Lower penetrator speed (or turn) after weapon firing,
4) Reaching the SAM site after launch, but before intercept, or
5) Masking, multipath or clutter around the SAM site.

ECM

ECM can delay SAM launch. The sensitivity to various delay times is shown below, for the 600 knot penetrator example:

Delay	0 min	1/4 min	1/2 min	1 min
R_L	17 NM	15 NM	12 NM	7 NM
R_W	43 NM	36 NM	30 NM	18 NM

ECM delays can show dramatic reductions in lethal defense range requirements. The effects of ECM can be seen as an increase in the term Δt_M in Figure 13.2.

Low RCS

SAM radar detection range is determined by the minimum of the radar LOS and $K\,\sigma_p^{1/4}$. If this detection range could be reduced from the LOS to 1/2 or 1/4 of the LOS, the penetrator weapon requirements would be markedly reduced, as noted below using the previous example:

Detection Range	LOS	1/2 LOS	1/4 LOS
R_e	21 NM	11 NM	5.3 NM
R_M	11 NM	4 NM	0.7 NM
R_W	43 NM	16 NM	3.0 NM

These are dramatic reductions in SAM capability and weapon range requirements. However, the RCS reductions required are also dramatic. For example, if the 21 NM line of sight in the example occurred due to a radar K Value of 21 NM (against a 1 m^2 RCS), then the RCS required to reduce the LOS to 1/2 would be .06 m^2, and to 1/4 → .004 m^2 (assuming linear relationships and no instabilities or dynamic range problems). The effects of reductions in RCS can be seen as reductions in R_e in Figure 13.2.

Reduce Speed or Turn

A reduction in speed, following penetrator weapon firing, will reduce range requirements. This can be shown by the relation for SAM sensor limited capability:

$$R_W > (R_e - V_p \Delta t_M)/(1 - V_p/V_W)$$

In the example, if the penetrator velocity is reduced from 600 knots to 300 knots, the weapon range requirement drops from 43 NM to 27 NM.

A turn to zero radial velocity after weapon firing could place the penetrator in a Doppler notch of the SAM radar, and cause loss of active tracking. If the weapon were fired and the air vehicle could complete its turn beyond the maximum SAM launch range (17 NM in the example), one might have reasonable assurance of avoiding intercept if the penetrator disappeared in the Doppler notch. A more conservative approach might require that the penetrator launch and complete its turn outside coverage range (21 NM in the example). Reductions in penetrator speed can be seen as reductions in the ratio V_p/V_W in Figure 13.4.

Hit Site Before Intercept

A case of hitting the SAM site at a time t before missile interception of the penetrator might be a feasible tactic if the missile required information from the site in the last t seconds of flight. Assume that if this information is not available, the missile will not damage the penetrator. (This might be the case with a command guided missile or with a semi-active radar guided missile which homes on the reflected illumination of the SAM radar on the penetrator.) Figure 13.5 illustrates this case:

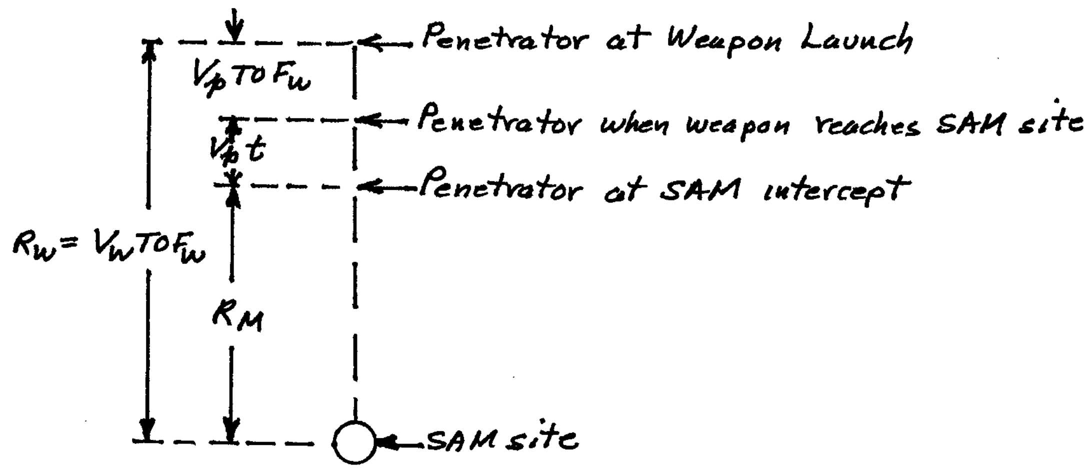


Figure 13.5 Hit SAM Site t Seconds Before Intercept

The relations below follow from Figure 13.5:

$$R_W > R_M + V_p\ TOF_W + V_p\ t$$

And, since $R_W = V_W\ TOF_W$:

$$R_W > (R_M + V_p\ t)/(1 - V_p/V_W)$$

Using the example of a SAM intercept range of 11 NM, a 600 knot penetrator and a 1000 knot average velocity for the weapon, the following table of range requirements can be constructed as a function of t, the time before intercept that the SAM site is hit:

t =	5 sec	10 sec	20 sec	30 sec
R_W >	30 NM	32 NM	36 NM	40 NM

These values are less than the original 43 NM range requirement.

Masking/Multipath/Clutter

Any estimate of SAM degrades due to masking, multipath or clutter should be based on data around a site. However, in order to test the sensitivity of these factors, assume average masks for terrain types as listed in Chapter 4. These masks for flat, rolling and hilly terrain are .85, .65 and .50 of the radar line of sight, respectively for a low altitude penetrator. With the example of a 10' SAM radar antenna height and a 200' penetrator, the average unmask R_e and weapon R_W when flying in flat, rolling or hilly terrain might be:

Mask	None	Flat	Rolling	Hilly
R_e	21 NM	18 NM	14 NM	10.5 NM
R_W	43 NM	35 NM	25 NM	17 NM

The effects of reductions in R_e due to masking can be seen in Figure 13.2.

The severity of multipath effects depends on the radar frequency, antenna height and specular reflection conditions around the site, as noted in Chapter 4. Nulls might be expected into a mile or two of the site, which could markedly degrade tracking and delay SAM missile launch. Weapon range requirements might be as low as 5 NM for severe multipath conditions (e.g. over water or smooth deserts).

Clutter effects around a site were addressed in Chapter 10. Clutter can create a washer shaped area where active coverage is lost. The size of the area is a function of the clutter coefficient, the penetrator RCS and the radar parameters. For a sandpaper earth, clutter effects extend only to the radar horizon (to 4 NM for a 10' antenna height). If SAM intercepts can be limited to these close-in ranges, clutter can be a significant factor in degrading SAM operations.

Recap

Weapon range requirements can be reduced by several methods. The methods, effects and examples of reductions in weapon range are summarized below:

Method	Effect	Example	Weapon Range
None			43 NM
ECM	Delay launch	1 minute	18 NM
Low RCS	Delay detection	1/2 LOS	16 NM
Decrease speed	Buys time	1/2 Speed	27 NM
Turn	Avoids intercept	Broadside	17+ NM
Hit SAM site before intercept	Lose SAM guidance	10 sec before	32 NM
Masking	Delay launch	Rolling terrain	25 NM
Multipath	Delay launch	Severe	~5 NM

These methods can be combined so that, for example, one might depend on a combination of ECM, low RCS and a turn to reduce weapon range. For any combination set, Figures 13.2 and 13.4 or the equations presented can be used to estimate range requirements.

Thus far no operational pad has been included to account for uncertainties - such as range errors between the penetrator and the site, or for different refractions than the standard 4/3rds earth refraction. Such a pad might add 10 - 20% to the range requirements shown in the example.

Now a switch from attacks on the site to intercepts on the SAM missile will be made.

Self Defense vs a Missile

Consider a close-in defense against the missile, such that the missile must be intercepted at a minimum distance, d_{min} from the penetrator. The head-on case is illustrated in Figure 13.6.

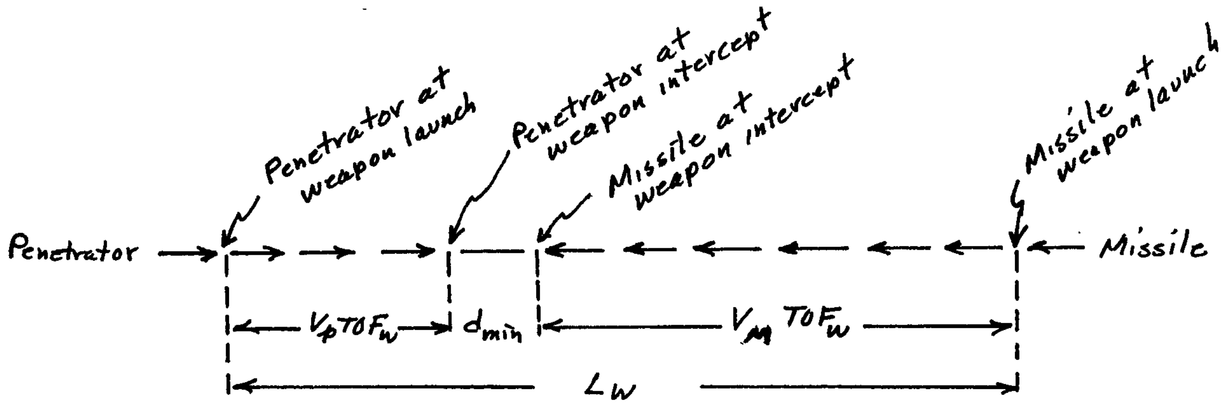


Figure 13.6 Head-on Missile Intercept

It follows from Figure 13.6 that:

$$L_W > d_{min} + TOF_W (V_M + V_p)$$

and:

$$R_W > d_{min} + TOF_W V_p$$

Where L_W = Range between the penetrator and the missile at penetrator weapon launch against the missile.

For example, assume that d_{min} = 1/2 NM and R_W = 2 NM. The penetrator's weapon time of flight, velocity and launch range needed to intercept a 1000 knot SAM missile from a 600 knot penetrator is:

$$R_W = 2 = 1/2 + TOF_W\ (600/3600), \text{ from which } TOF_W < 9 \text{ sec}$$

$$V_W = R_W/TOF_W > 2/(9/3600) = 800 \text{ knots}$$

$$L_W > 1/2 + 9\ (1000 + 600)/3600 = 4.5 \text{ NM}$$

In the example the penetrator's weapon must have less than a 9 second time of flight and greater than 800 knots average velocity.

The penetrator's sensor must detect the missile at a time t_{min} before launch. This is the reaction time for detection and decision to launch. The penetrator's sensor range against the missile must exceed:

$$R_d > L_W + t_{min}\ (V_p + V_M)$$

Which for the example and a 25 second reaction time is:

$$R_d > 4.5 + 25\ (600 + 1000)/3600 = 16 \text{ NM}$$

If the penetrator used a radar as the sensor and the SAM missile RCS were 0.1 m^2, the radar K Value requirement would be:

$$K > R_d/\sigma_p^{1/4} = 16/(.1)^{1/4} = 28 \text{ NM vs } 1 \text{ m}^2$$

This allows missile intercept 1/2 NM from the penetrator. The sensitivity of R_d, TOF_W and t_{min} is shown in Figure 13.7.

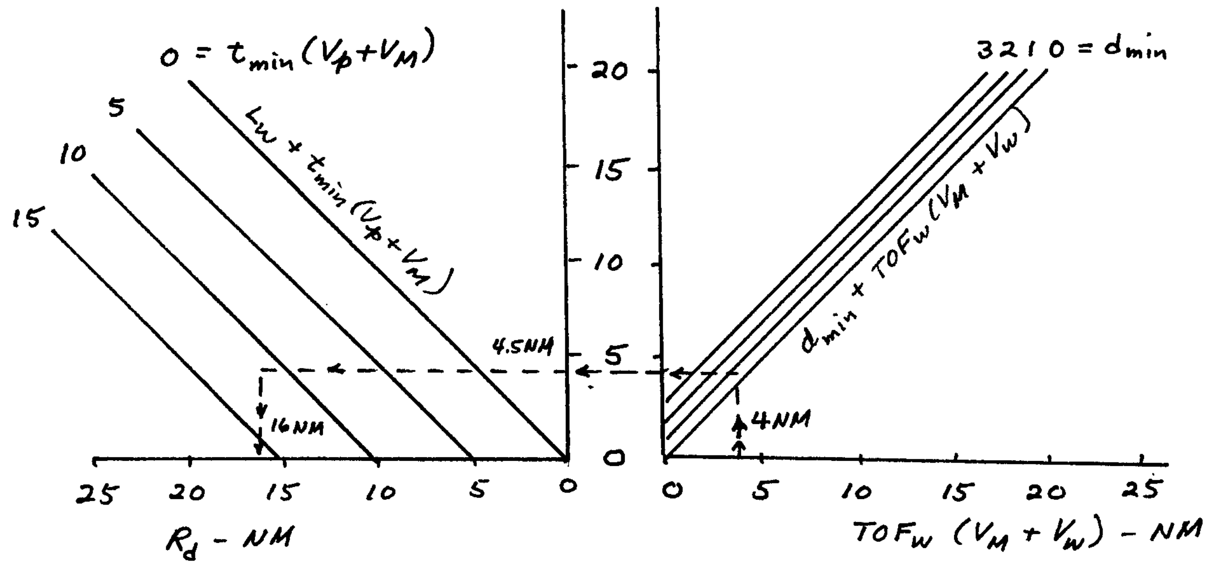


Figure 13.7 Missile Intercept Range Capability

When weapon range limited, enter the chart on the lower right and proceed up and to the left as indicated. When sensor limited, enter the chart on the lower left and proceed up and to the right. In this latter case, a combination of a short weapon time of flight (TOF_W) and short response time (t_{min}) might be needed.

Summary

This chapter has explored lethal self defense against a SAM site and against a missile in flight. SAMs may be encountered, despite a basic policy of avoiding sites, due to:

- Corridor busting
- SAM movements
- Overflying terminal defenses
- Concealed SAM locations

Avoidance and ECM are usually considered first to minimize losses to SAMs, since lethal self defense usually requires downloading offensive weapons in order to carry defensive weapons.

Equations and charts for the initial head-on SAM launch and intercept ranges were presented, along with sensor (and weapon) range limited lethal self defense requirements. Methods to decrease these range requirements include:

Electronic countermeasures	Low radar cross section
Lower speed or turn	Hitting the SAM before intercept
Masking/multipath/clutter	

Combinations of these methods, such as ECM, low RCS and a turn, can be used to reduce range requirements markedly .

Self defense against the SAM missile itself was addressed, with equations and charts presented for analysis of this case. The value of short weapon time of flight and short detection-to-launch response time was stressed in order to minimize penetrator sensor range requirements.

Chapter 14

Lethal Self Defense vs AWACS

This chapter analyzes weapon range requirements for penetrators to attack an Airborne Warning and Control System (AWACS). AWACS detection and reaction time are discussed first, followed by weapon ranges needed for high (and low) altitude entry. Methods to reduce range requirements and examples of possible benefits are then examined. The value of neutralizing AWACS and the residual capability of a Combat Air Patrol (CAP) is the last topic.

AWACS Detection

The line of sight (LOS) limits for a 4/3rds earth refraction model was developed in Chapter 4 as:

$$\text{Radar LOS} = 1.23(h_a^{1/2} + h_p^{1/2})$$

where h_a = Antenna height, or AWACS altitude
h_p = Penetrator altitude

For example, with a 30,000' AWACS and a 1,000' (or 30,000') penetrator:

$$\text{Radar LOS} = 1.23(30{,}000^{1/2} + 1{,}000^{1/2}) = 252 \text{ NM} \quad \text{Low}$$
$$= 1.23(30{,}000^{1/2} + 30{,}000^{1/2}) = 426 \text{ NM} \quad \text{High}$$

Figure 14.1 presents a plot of AWACS radar LOS as a function of penetrator altitude, for a 30,000' AWACS altitude.

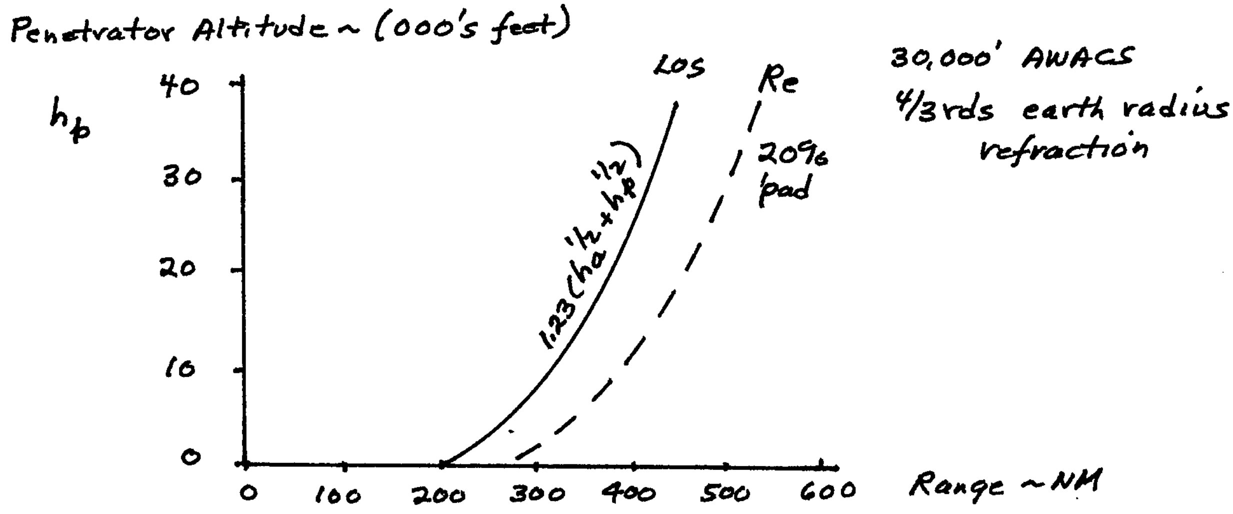


Figure 14.1 AWACS Radar Line of Sight

An approaching penetrator can delay detection if the air vehicle descends to stay below the radar LOS. However, the penetrator may not know: 1) the distance to AWACS, 2) the altitude of AWACS or 3) the existing refraction conditions.

Detection of an active AWACS signal should not be a problem for modern radar warning receivers (RWRs). But a RWR can not measure range accurately from signal amplitude measurements alone. AWACS might be triangulated over time to reduce this uncertainty if the AWACS track is known. Alternate methods include triangulation from multiple penetrators, or from two widely separated antennas on a penetrator.

To improve confidence in avoiding detection, a penetrator might add a pad (to account for the above three uncertainties). A 20% pad is assumed in Figure 14.1. The range with the pad will be labeled R_e, the entry range to possible AWACS coverage. In the example, a 20% pad would raise the 252 NM radar LOS to 302 NM for 1,000' penetration, and raise 426 NM to 511 NM for 30,000'.

Reaction Time

The time needed for AWACS actions from entry through assignment of airborne interceptors (AIs) will be labeled Δt_A. The time from assignment to AI arrival within AI sensor range of the penetrator will be labeled Δt_{AI}. This includes the time required to take off (if the AI is on ground alert) and fly to the penetrator's vicinity. The two terms will be considered together ($\Delta t_A + \Delta t_{AI}$).

This total reaction time may vary from a minute or two (for AIs on CAP) to well over ten minutes (for AIs on the ground hundreds of miles away).

The distance that the penetrator moves during this reaction time is:

$$\Delta R = V_p \ (\Delta t_A + \Delta t_{AI})$$

For example, assuming a 400 knot penetrator speed relative to AWACS and a 5 minute total reaction time, ΔR is:

$$\Delta R = 400 \ (5/60) = 33 \text{ NM}$$

This 33 NM reduction in range in the example is only a 11% difference (302 to 269 NM) for low altitude, and 6% (from 551 to 478 NM) for high altitude. Thus, these reaction times are not expected to be major factors in determining range requirements.

Weapon Range to Reach AWACS Prior to Enter Cover

First assume that a weapon is required to reach AWACS before the penetrator enters AWACS coverage at R_e. Figure 14.2 illustrates weapon range requirements for a penetrator flying perpendicular to an AWACS track.

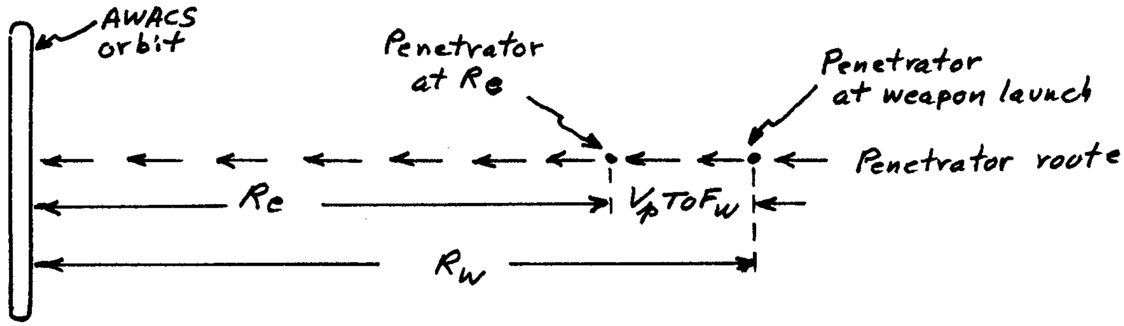


Figure 14.2 Weapon Range Diagram

where R_e = Range at entry to AWACS coverage
V_p = Velocity of penetrator, relative to AWACS
TOF_W = Time of flight of penetrator's weapon
V_W = <u>Average</u> velocity of penetrator's weapon
R_W = Range of penetrator's weapon, $R_W = TOF_W\ V_W$

If the penetrator aligns its heading to point to the current AWACS location, maintains its current heading and velocity after weapon firing, and AWACS velocity is small compared to the average weapon velocity, the required weapon range is:

$$R_W > R_e + TOF_W\ V_p$$

But, since $TOF_W = R_W/V_W$,

$$R_W > R_e/(1 - V_p/V_W)$$

For example, with a 400 knot penetrator and R_e = 302 NM at low altitude (511 NM at high altitude) and a 2000 knot average weapon velocity:

$$R_W > 302/(1 - 400/2000) = 378 \text{ NM} \quad \text{Low}$$
$$> 511/(1 - 400/2000) = 639 \text{ NM} \quad \text{High}$$

Figure 14.3 shows the simple relationship between R_W/R_e and the velocity ratio V_p/V_W.

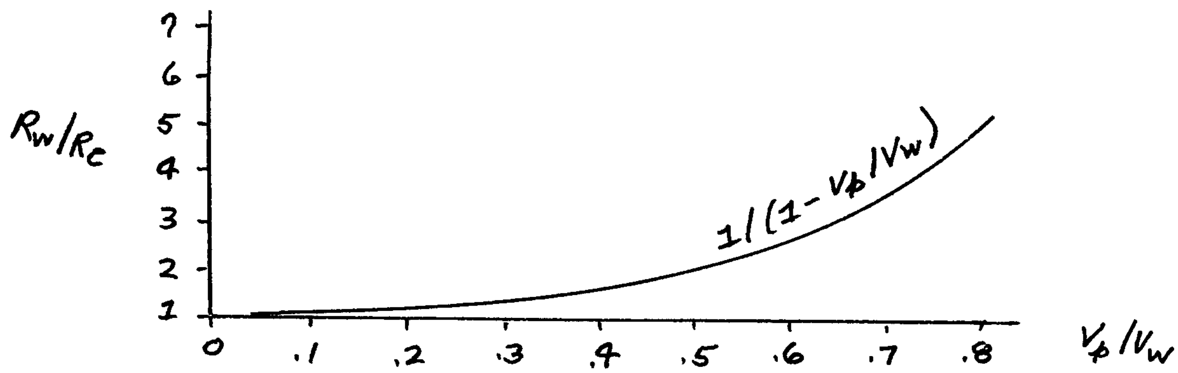


Figure 14.3 Sensitivity to Velocity Ratio

Low velocity ratios reduce weapon range requirements. Sensor requirements can be determined by adding a penetrator reaction time multiplied by the penetrator closing velocity to the weapon ranges found above. For example, with a 25 second reaction time and a 400 knot closing, 3 NM must be added to the weapon ranges to find the required sensor range for the penetrator.

Weapon Range to Reach AWACS Prior to AI Cover

Figure 14.4 illustrates the case of delaying weapon firing until the expiration of the reaction time (for AWACS actions and the AI reaching sensor detection range).

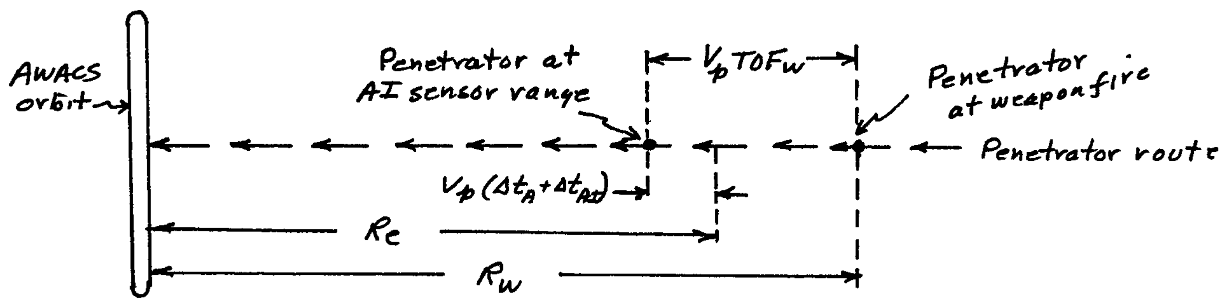


Figure 14.4 Weapon Range Diagram with Reaction Time

The weapon range requirements are now:

$$R_W > R_e - (\Delta t_A + \Delta t_{AI}) V_p + TOF_W V_p$$

With $TOF_W = R_W/V_W$, this relation becomes:

$$R_W > \{R_e - V_p (\Delta t_A + \Delta t_{AI})\}/(1 - V_p/V_W)$$

(This equation is identical to that on Page 13.6 for weapon range against a SAM, if the SAM reaction time replaces the AWACS + AI reaction time.)

For the case of a low (and high) 400 knot penetrator, a 30,000' AWACS (which led to a R_e of 302 NM), a 2000 knot weapon and a 5 minute combined delay:

$$R_W > \{302 - (5/60)\ 400\}/(1 - 400/2000) = 336 \text{ NM}$$
$$> \{511 - (5/60)\ 400\}/(1 - 400/2000) = 597 \text{ NM}$$

This is a reduction of 42 NM due to a AWACS/AI 5 minute delay. These time delay differences are not dominant factors in range requirements against AWACS.

Figure 14.5 illustrates the effect of time delays on weapon range requirements.

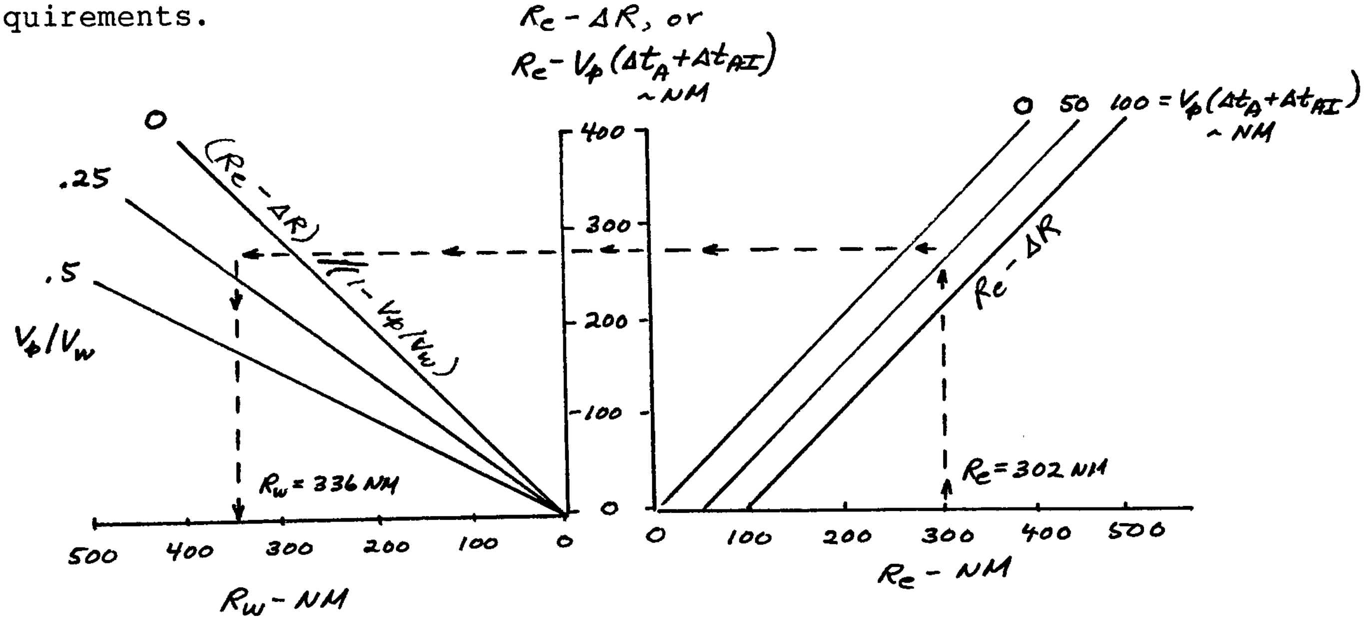


Figure 14.5 Weapon Range Requirements vs AWACS

Reducing Range Requirements

Penetrator weapon range requirements against AWACS can be reduced by:

1) Electronic countermeasures,
2) Penetrator low radar cross section, or
3) Lower penetrator speed, or turn after weapon firing.

ECM

The effect of time delays due to ECM can be seen in Figure 14.5. For example, a 5 minute ECM delay would decrease AWACS detection range by 5 (400)/60 = 33 NM for the case of a 400 knot penetrator. This would have a relatively minor effect on weapon range requirements. For example, with a 2000 knot weapon this results in a reduction of 33/(1 - 400/2000) = 42 NM in weapon range.

Low RCS

AWACS radar detection range is determined by the minimum of the radar LOS and $K\ \sigma_p^{1/4}$. If the detection range could be reduced from the LOS to 1/2 or 1/4 of the LOS, penetrator weapon requirements would be markedly reduced as shown in the table below for the low altitude example:

Detection Range	LOS	1/2 LOS	1/4 LOS
$K\ \sigma_p^{1/4}$	252 NM	126 NM	63 NM
R_e	302 NM	151 NM	76 NM
R_W	378 NM	190 NM	95 NM

These are dramatic reductions in AWACS capability and weapon ranges. The RCS reductions required can be estimated from the previous work on AWACS in Chapter 10. In the example there, a pulse Doppler K Value of 120 NM was used vs low altitude penetrators. In order to reduce the detection range to 126 NM (1/2 LOS) requires a spherical RCS of $(126/120)^4 = 1.2\ m^2$. A reduction to 1/4 LOS requires a RCS of $(63/120)^4 = .08\ m^2$. If these radar cross sections are achievable, major reductions in penetrator weapon ranges are possible. Figure 14.5 can be used to show the effect of reductions in R_e on weapon range requirements.

(Recall that an example in Chapter 10 showed that a vehicle with an RCS less than $.5\ m^2$ will not be detected in the OTH mode. Thus, in the example, penetrators with less than this RCS can approach at high altitude undetected until they are within pulse Doppler mode coverage.)

Reduce Speed or Turn

A reduction in speed, following weapon firing, will reduce range requirements, as shown by the inequality:

$$R_W > R_e/(1 - V_p/V_W)$$

For example, a reduction in penetrator speed from 400 knots to 200 knots would reduce weapon requirements in the example by 11% (the denominator changes from .8 to .9).

If the penetrator turned into the Doppler notch of AWACS immediately after weapon firing, the weapon range can be reduced to that of the entry range for AWACS. This would result in a reduction of 20% in weapon range for the example (the denominator changes from .8 to 1.0). Figure 14.5 shows the general effects of V_p/V_W on range requirements.

Recap

Several methods to reduce weapon range requirements were discussed. These methods, their effects and examples of reductions are summarized below for the low altitude case with a 5 minute reaction time:

Method	Effect	Example	Weapon Range
None			336 NM
ECM	Delay detection or assignment	5 minutes	294 NM
Low RCS	Delay detection	1/2 LOS	190 NM
Decrease speed	Buys time	1/2 Speed	299 NM
Turn	Avoids detection	Broadside	269 NM

These methods can be combined, for example with ECM, low RCS and a broadside turn. Figure 14.5 (or the equations presented) can be used to estimate weapon range requirements for any combination which can be supported.

Consequences of Neutralizing AWACS

Assuming that the penetrator weapon successfully performs its mission of neutralizing AWACS - either killing AWACS or forcing the radar to turn off (to avoid weapon damage), how does this help the penetrators? Although AWACS may be neutralized, AIs on CAP with AWACS can still pose a threat, if they take up a patrol on their own. For example, the AIs may line up abreast to sweep out an area as shown in Figure 14.6.

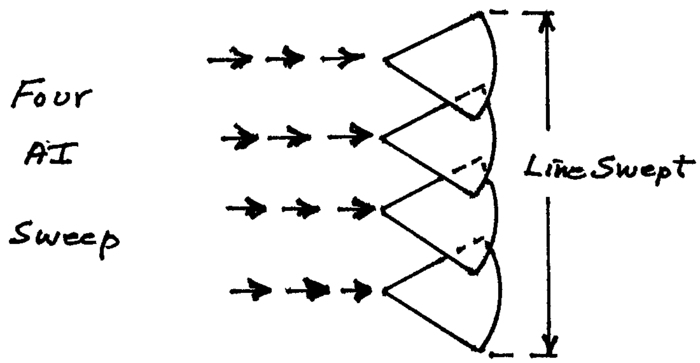


Figure 14.6 AI Patrol Sweep

With an assumed AI K Value of 30 NM, four interceptors can sweep out about a 100 NM wide area (if the penetrator's RCS is 1 m^2). This sweep would provide significant coverage and compares to the 120 NM radius coverage in the example AWACS. However, an AI sweep would have limited capability when side-on or tail-on to the penetrator force (due to the Doppler notch and sidelobe clutter). Also, each AI may be on its own when it sights a target, and will not be able to direct assignments as efficiently as AWACS.

A prime measure of AWACS effectiveness is the overall attrition suffered with and without AWACS direction. This changes with the depth of penetration. For example, air vehicles which had very <u>shallow</u> penetration of defenses may only encounter AWACS/AIs. (This might be the case for cruise missile carriers encountering AWACS prior to last cruise missile launch.) Here the presence or absence of AWACS may be a major factor in attrition of the carrier, and perhaps a lesser factor in the number of cruise missiles launched.

Air vehicles with deep penetration routes may face dense ground sensors and AI/SAM threats which can cause high attrition. In this case, the presence or absence of AWACS may have only a minor effect on overall attrition, but a somewhat greater effect on penetrator mission results (such as target damage) due to early attrition by AWACS defenses.

The effectiveness of lethal defense against AWACS must be weighed in trade off studies against other penetration aids to determine if other options can achieve greater mission results at the same cost. These studies must include the loss of offensive weapons displaced by self defense weapons.

Summary

This chapter discussed AWACS detection ranges against penetrators, and problems in a penetrator's ability to measure the range to AWACS. Since uncertainties will exist in estimating the separation distance, AWACS altitude and propagating conditions, a pad was suggested to be added to the penetrator's estimate of entry into AWACS coverage.

The weapon range to reach AWACS prior to entering its coverage was found to be:

$$R_W > R_e/(1 - V_p/V_W)$$

Adding a reaction time for AWACS and AI actions provides:

$$R_W > \{R_e - V_p\ (\Delta t_A + \Delta t_{AI})\}/(1 - V_p/V_W)$$

These equations and Figure 14.5 can be used to estimate weapon range requirements to reach AWACS prior to entering cover, or prior to AI sensor cover. The reaction time of AWACS (in assigning interceptors, and in AIs arriving within their sensor range), was found to have minor effects on weapon range requirements. Penetrator reaction time (to detect and decide to fire weapons against AWACS) was likewise found to have minor effects upon sensor range requirements.

Range requirements can be reduced by ECM, low RCS or lower penetrator speed (or a turn) after weapon firing. Combinations of these can reduce weapon range needs appreciably.

The effectiveness of lethal self defense against AWACS must be weighed against the residual capability of AIs on combat air patrol, and the loss of offensive weapons (displaced by self defense weapons to attack AWACS). Trade off studies should compare lethal self defense against AWACS with other penetration aid options.

Chapter 15

Lethal Self Defense vs AIs

Penetrator encounters with an airborne interceptor (AI) are discussed in this chapter. AI missile launch ranges and penetrator self defense weapon and sensor range requirements are highlighted. The fundamental relationships between range, velocity and time of flight for head-on and tail-on AI attacks will be developed. Examples of these relationships will point out the long ranges needed for self defense against head-on AI attacks and the short ranges required for tail-on attacks.

Head-on AI Attacks

The maximum AI radar detection range against a penetrator is the minimum of the radar line of sight (LOS) or the AI radar sensitivity times the penetrator radar cross section (RCS):

$$\text{Max AI radar range} = \min \{ \text{Radar LOS, or } K_{AI}\, \sigma_p^{1/4} \}$$

For example, with an AI at 30,000', a 1 m^2 RCS penetrator at 1000' and an AI head-on radar K Value of 30 NM, the maximum detection range is:

$$\text{Max AI radar range} = \min \{ 1.23\ (1{,}000^{1/2} + 30{,}000^{1/2}) = 252 \text{ NM},$$

$$\text{or } 30\ (1^{1/4}) = 30 \text{ NM} \} = 30 \text{ NM}$$

In this example, the maximum detection range is 30 NM - limited by AI radar sensitivity.

The maximum launch range for an AI missile (when AI radar detection of the penetrator is required prior to missile launch) is the minimum of the radar limited range or the missile limited range:

AI launch range = min { Radar limited range, or Missile limited range }

$$L_{AI} = \min \{ \text{Radar } L_{AI}, \text{ or Missile } L_{AI} \}$$

Consider the radar limits first. For a head-on attack, the radar launch limit is the maximum AI radar range less the closure distance between the AI and the penetrator during the AI reaction time. The time delay for AI reaction (including penetrator detection, tracking and decision to launch) will be labeled Δt_{AI}.

$$\text{Radar } L_{AI} = \text{Max AI radar range} - \Delta t_{AI} (V_{AI} + V_p)$$

For example, assume an AI velocity of 700 knots, penetrator velocity of 600 knots, a 30 NM AI radar range limit and a 25 second reaction time. Then the radar limited AI launch range is:

$$\text{Radar } L_{AI} = 30 - (25/3600)(700 + 600) = 30 - 9 = 21 \text{ NM}$$

Now consider the missile limits. The geometry for a head-on AI missile attack as illustrated in Figure 15.1:

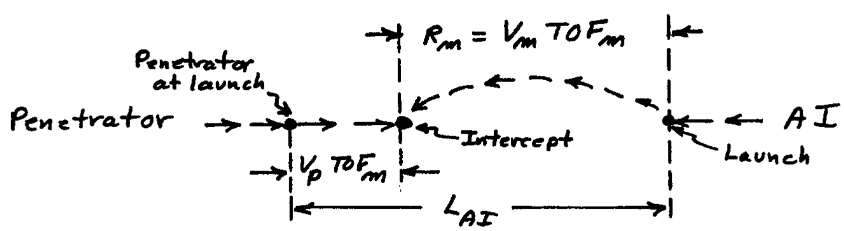


Figure 15.1 Head-on AI Missile Attack Geometry

where L_{AI} = Range between AI and penetrator at AI launch
TOF_m = AI missile time of flight
V_m = AI missile average flight velocity
R_m = AI missile kinematic range = $TOF_m V_m$
V_p = Penetrator velocity
V_{AI} = AI velocity

The missile limited launch range for a head-on geometry is the maximum missile kinematic range plus the distance moved by the penetrator during the missile's time of flight:

$$\text{Missile } L_{AI} = R_{max} + TOF_{max} V_p$$

But, since $R_m = TOF_m V_m$,

$$\text{Missile } L_{AI} = TOF_{max} (V_m + V_p)$$

For example, assume an average missile velocity of 1000 knots and a 36 second maximum time of flight, or a (1000) (36/3600) = 10 NM maximum missile range. The missile limited launch range against a 600 knot penetrator in the example will then be:

$$\text{Missile } L_{AI} = (36/3600)(1000 + 600) = 16 \text{ NM}$$

This 16 NM missile limited launch range is smaller than the 21 NM radar limited launch range in the example, so that L_{AI} is:

$$L_{AI} = \min \{ \text{Radar } L_{AI}, \text{ or Missile } L_{AI} \} = \min \{ 21, \text{ or } 16 \} = 16 \text{ NM}$$

Weapon range requirements will now be analyzed as a function of this AI launch range.

Reach AI Prior to Head-on Missile Launch

The head-on case (illustrated in Figure 15.2) allows the penetrator's weapon to reach the AI prior to AI missile launch.

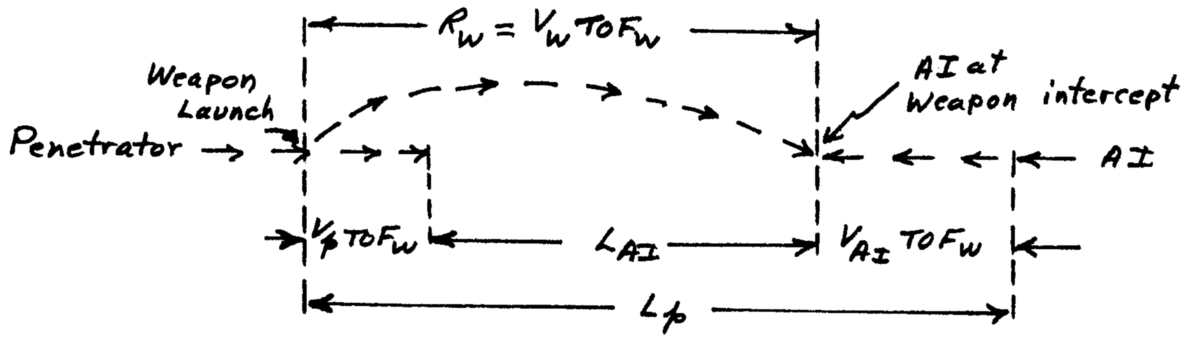


Figure 15.2 Reach AI Before Head-on Launch

where L_p = Launch range of penetrator weapon
TOF_W = Time of flight of penetrator weapon
V_W = Average velocity of penetrator weapon
R_W = Kinematic range of penetrator weapon = $TOF_W\ V_W$

In order that the penetrator weapon reach the AI prior to missile launch, the weapon launch range must exceed the AI launch range plus the closing distance of the AI and the penetrator during the weapon's time of flight:

$$L_p > L_{AI} + TOF_W\ (V_{AI} + V_p)$$

The ratio of the required penetrator launch range to the missile limited AI launch range can be determined by first recalling that:

$$\text{Missile } L_{AI} = TOF_{max}\ (V_m + V_p)$$

If the inequality developed for L_p is divided by the missile L_{AI}:

$$L_p/L_{AI} > 1 + (TOF_W/TOF_{max})(V_{AI} + V_p)/(V_m + V_p),$$

the time of flight ratio can be determined by noting that:

$$L_p = R_W + TOF_W\ V_{AI} = TOF_W\ (V_W + V_{AI})$$

This equation is combined with the earlier inequality for L_p:

$$TOF_W\ (V_W + V_{AI}) > L_{AI} + TOF_W\ (V_{AI} + V_p)$$

For missile limited AI launch ranges, L_{AI} = max TOF_m $(V_m + V_p)$. Thus:

$$TOF_W\ (V_W + V_{AI}) > TOF_{max}\ (V_m + V_p) + TOF_W\ (V_{AI} + V_p)$$

From which the ratio of the (missile limited) times of flight can be obtained:

$$TOF_W/TOF_{max} > (V_m + V_p)/(V_W - V_p)$$

Note that $(V_m + V_p)$ is the average closing velocity of the penetrator with the AI missile, and $(V_W - V_p)$ is the average separating velocity of the penetrator's weapon from the penetrator.

The above expression can be used to develop the required head-on ratios of launch and flight ranges as:

$$L_p/L_{AI} > 1 + (V_{AI} + V_p)/(V_W - V_p)$$

$$R_W/R_m > (1 + V_p/V_m)/(1 - V_p/V_W)$$

For example, with V_m = 1000 knots, V_W = 800 knots (a lower average velocity to allow for shooting up from a low altitude penetrator to a higher altitude AI), a penetrator speed of 600 knots, and a head-on AI at 700 knots, the example weapon/missile limited ratios are:

$$R_W/R_m > (1 + 600/1000)/(1 - 600/800) = 6.4$$

$$L_p/L_{AI} > 1 + (700 + 600)/(800 - 600) = 7.5$$

Penetrator head-on weapon and launch requirements for the example are listed below for various AI kinematic missile ranges, when the launch range of the AI missile is the limiting factor (rather than an AI sensor limit):

R_{max}	=	2 NM	4 NM	8 NM	10 NM	16 NM
R_W	>	13 NM	26 NM	51 NM	64 NM	102 NM
L_{AI}	=	3.2 NM	6.4 NM	13 NM	16 NM	26 NM
L_p	>	24 NM	48 NM	96 NM	120 NM	192 NM

Note the very large penetrator weapon range requirements, which are 6.4 times the AI missile ranges in this example. Even a relatively short 4 NM interceptor missile must be countered with a self defense weapon with more than 26 NM kinematic range, and the penetrator must launch more than 48 NM away from the AI.

Figure 15.3 illustrates the basic kinematic range relationships for a head-on approach as a function of the two velocity ratios: V_p/V_m and V_p/V_W.

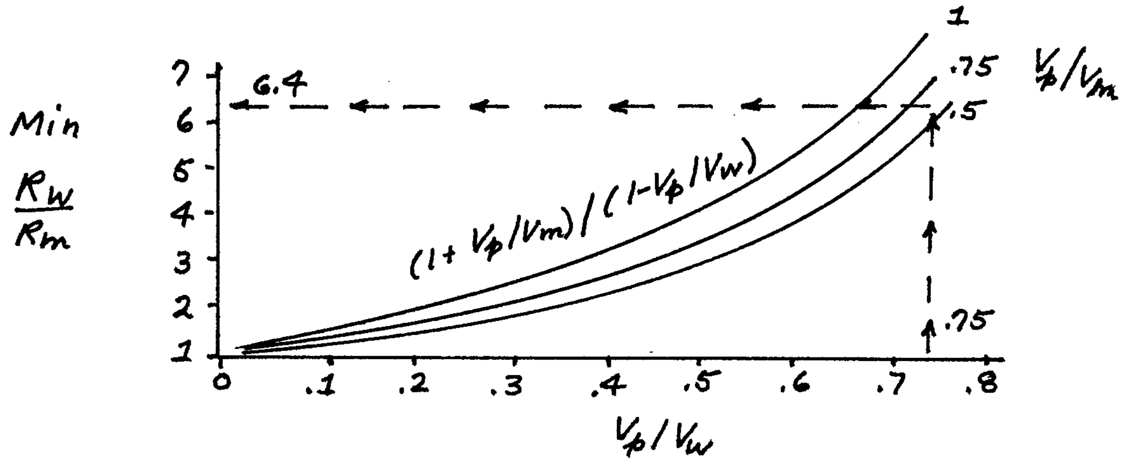


Figure 15.3 Head-on Weapon to Missile Range Ratios

This figure shows the strong dependence of penetrator weapon requirements on penetrator velocity. A reduction in penetrator velocity from 600 to 300 knots in the example will reduce the kinematic weapon range requirements by over 67% (from a ratio of 6.4 to 2.08).

If a laser or directed energy weapon were used by the penetrator, V_W becomes very large (e.g. the speed of light). Now the weapon launch range must just exceed the AI missile launch range (i.e. $L_p > L_{AI}$). On the other hand, if the AI has the laser/directed energy weapon, the penetrator weapon requirements become:

$$R_W/R_m > 1/(1 - V_p/V_W) \quad \text{as } V_m \rightarrow \infty$$

The expression for the ratio of launch ranges does not change, however.

Figure 15.4 illustrates the ratio of penetrator to AI launch ratios as a function of penetrator to AI closing velocity and weapon to penetrator separating velocity.

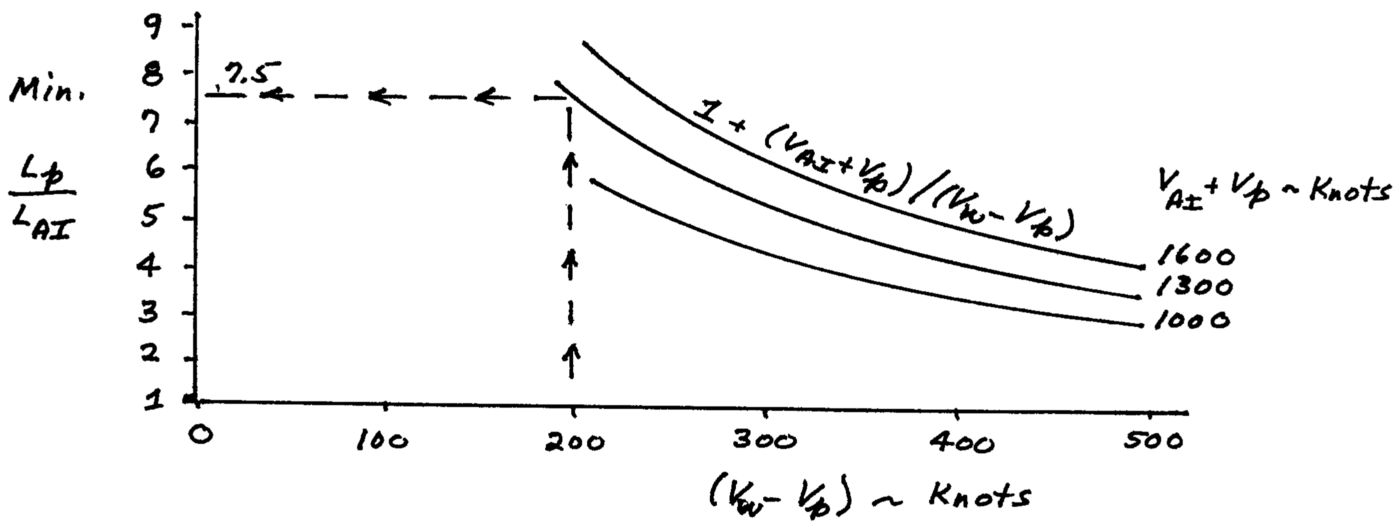


Figure 15.4 Head-on Penetrator to AI Launch Range Ratios

Note that the example 1300 knot closing velocity between the AI and the penetrator and 200 knot separating velocity between weapon and penetrator yields a ratio of 7.5 (when the AI launch is missile, rather than sensor, limited).

Now consider sensor requirements on the penetrator to achieve these weapon launch ranges.

Penetrator Head-on Sensor Requirements

A sensor is required to detect the AI prior to reaching these required weapon launch ranges, so that the penetrator has time to react (i.e. detect the AI, track and decide to launch). The sensor must detect the AI (at detection range R_d) prior to the required reaction time Δt_p:

$$R_d = L_p + \Delta t_p (V_{AI} + V_p) = L_{AI} (L_p/L_{AI}) + \underbrace{\Delta t_p (V_{AI} + V_p)}_{\text{closure distance, } \Delta R}$$

Figure 15.5 illustrates penetrator sensor requirements as a function of

AI launch range and the ratio of penetrator to AI launch range for various closure distances during the penetrator's reaction time.

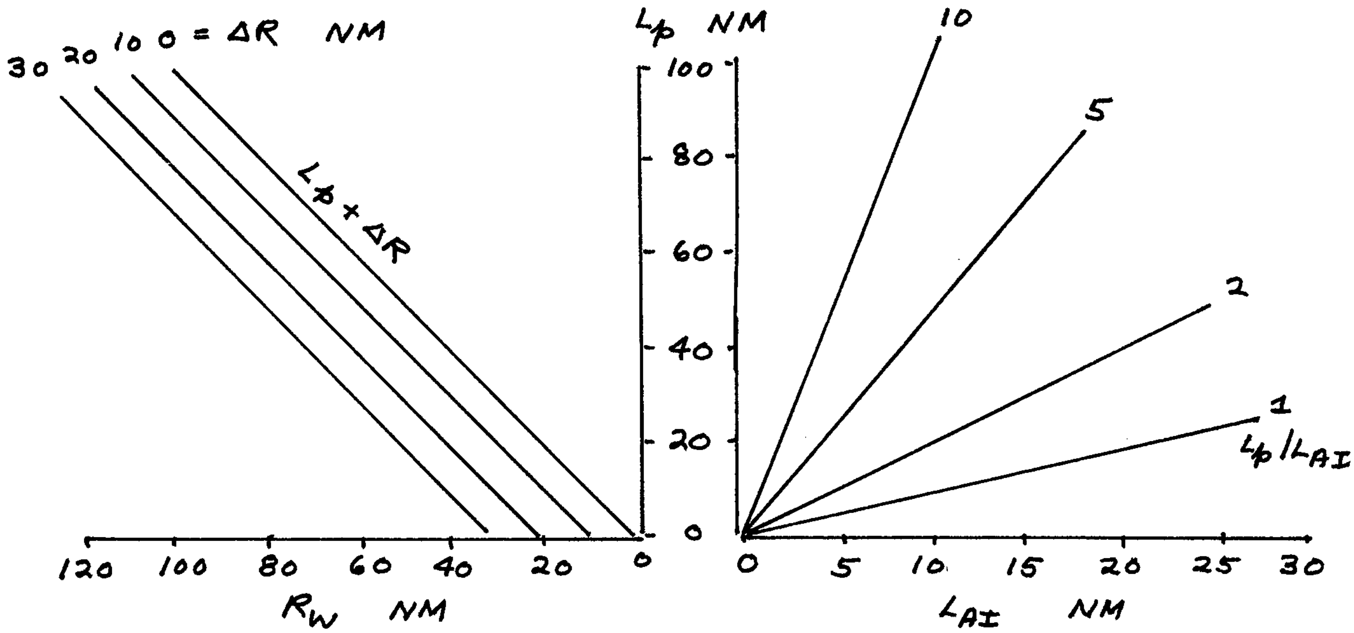


Figure 15.5 Sensor Head-on Requirements

If the penetrator's sensor is an active radar, a head-on radar sensitivity or K Value can be calculated:

$$\text{Head-on K} = (L_p + \Delta R)/\sigma_{AI}^{1/4}$$

For example, using a 25 second reaction time, a closing velocity of 1300 knots, and a head-on AI RCS of 1 m^2:

$$\Delta R = \Delta t_p\ (V_{AI} + V_p) = (25/3600)(700 + 600) = 9 \text{ NM}$$

$$\text{Head-on K} > (L_p + \Delta R)/(1)^{1/4} = L_p + 9 \text{ NM}$$

The prior example showed penetrator launch ranges from 24 to 192 NM, for AI missile ranges of from 2 to 16 NM, respectively. The penetrator reaction time adds 9 NM to these ranges, for a 33 to 201 NM K Value!

Reducing Head-on Range Requirements

Penetrator weapon launch requirements can be reduced by:

1) Electronic countermeasures,
2) Penetrator low radar cross section,
3) Lower penetrator speed (or turn) after weapon firing, or
4) Reaching the AI after launch, but before missile intercept.

ECM

ECM can delay AI launch. The sensitivity to various launch delay times is shown below, for the example of a 10 NM missile limited range and a 1300 knot closing speed between penetrator and AI:

Delay	0 min	1/4 min	1/2 min	1 min
L_{AI}	16 NM	11 NM	5 NM	No Launch
L_p	120 NM	79 NM	39 NM	Not reqd
R_d	129 NM	88 NM	48 NM	Not reqd

High head-on closing speeds translate into a small time window when AI launch is possible. For example, a 16 NM separation is eliminated in 44 seconds at a 1300 knot closure speed. If ECM can delay launch by 1/2 minute or more, dramatic reductions in penetrator weapon launch ranges and sensor ranges are possible. Note that ECM has been assumed to cause a reduction in launch range, not detection range.

Low RCS

AI radar detection range is determined by the minimum of the radar LOS and $K_{AI}\ \sigma_p^{1/4}$. If the detection range can be reduced from the line of sight to 1/2 or 1/4 of the LOS, the penetrator weapon and sensor requirements would be markedly reduced, as noted below for the example:

Detection Range	30 NM	15 NM	7.5 NM
L_{AI} =	16 NM	6 NM	No Launch
L_p >	120 NM	45 NM	Not reqd
R_d >	129 NM	54 NM	Not reqd

These are dramatic reductions in penetrator launch and sensor detection range requirements against the AI. The RCS reductions are likewise dramatic. For example a reduction to 1/2 LOS in radar capability requires that the RCS be reduced by $(1/2)^4$ or from 1 to .06 m^2 in the example. A reduction to 1/4 of the range requires a penetrator head-on RCS of $(1/4)^4 = .004\ m^2$ (assuming linear relationships and no instabilities or dynamic range problems).

Reduce Speed or Turn

A reduction in speed, following weapon firing, will reduce range requirements. This was noted earlier in this chapter and is seen in the ratio for penetrator launch range below:

$$L_p/L_{AI} > 1 + (V_{AI} + V_p)/(V_W - V_p)$$

For example, if the penetrator velocity is reduced from 600 to 300 knots after weapon launch, this ratio changes from 7.5 to 3.0 in the example, a 60% reduction.

A turn to zero radial velocity to the AI would place the penetrator in the Doppler notch of the AI radar, and cause loss of active AI pulse Doppler tracking. Now the penetrator need only fire its weapon outside of AI launch range. This reduces the launch range ratio to just over one, or about a 87% reduction in weapon launch range requirements.

Reach AI Before Missile Intercept

A case of reaching the AI at a time t before missile intercept can be a feasible tactic if the missile requires information from the AI in the last t seconds of flight. Assume that if this information is not available during this time, the missile will not damage the penetrator. An example would be a semi-active radar guided missile which homes on the energy reflected off the penetrator by the AI radar illuminator.

Figure 15.6 illustrates the geometry of the penetrator's weapon reaching the AI t seconds before missile intercept on the penetrator.

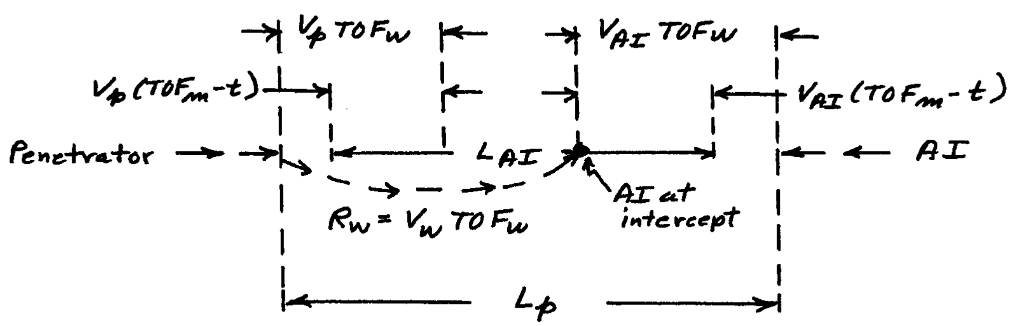


Figure 15.6 Reaching AI t Seconds Before Head-on Intercept

Let t be the minimum time prior to AI missile fuze that AI information is required to insure proper final guidance, arming or fuzing. This allows the penetrator to delay launch by the distance:

$$\text{Launch delay distance} = \text{Time delay} \times \text{Closing velocity}$$

$$\text{Launch delay distance} = (TOF_m - t)\,(V_{AI} + V_p)$$

This delay can be subtracted from the previous inequality for L_p:

$$L_p > L_{AI} + TOF_W\ (V_{AI} + V_p) - (TOF_m - t)(V_{AI} + V_p)$$

This reduces to:

$$L_p > L_{AI} + (V_{AI} + V_p)\ (TOF_W + t - TOF_m)$$

Which, when divided by L_{AI} becomes:

$$L_p/L_{AI} > 1 + \{(V_{AI} + V_p)/(V_m + V_p)\}\ \{(TOF_W/TOF_m) + t/TOF_m - 1\}$$

Note that if $t = TOF_m$, this inequality returns to the original form developed for reaching the AI before its missile launch.

The TOF ratio can be obtained from:

$$TOF_W\ (V_W + V_{AI}) > TOF_m\ (V_m + V_p) + (V_{AI} + V_p)\ \{TOF_W + t - TOF_m\}$$

Which for (t/TOF_m) less than or equal to 1, can be changed to:

$$TOF_W/TOF_m > \{V_m - V_{AI} + (t/TOF_m)(V_{AI} + V_p)\}/(V_W - V_p)$$

Then the kinematic (and launch) range ratios are:

$$R_W/R_m > \{1 - V_{AI}/V_m + (t/TOF_m)(V_{AI} + V_p)/V_m\}/(1 - V_p/V_W)$$

$$\frac{L_p}{L_{AI}} > 1 + \left[\frac{(V_{AI} + V_p)}{(V_m + V_p)}\right] \left[\frac{\{V_m - V_{AI} + (t/TOF_m)(V_W + V_{AI})\}}{(V_W - V_p)} - 1\right]$$

For example, with t = 20 seconds, AI velocity = 700 knots, penetrator at 600 knots (head-on), AI missile at 1000 knots and penetrator weapon at 800 knots, the weapon launch ranges are:

R_m =	2 NM	4 NM	8 NM	10 NM	16 NM
R_W >	13 NM	26 NM	39 NM	41 NM	48 NM
L_{AI} =	3.2 NM	6.4 NM	13 NM	16 NM	26 NM
L_p >	24 NM	48 NM	73 NM	77 NM	92 NM
Original L_p >	24 NM	48 NM	96 NM	120 NM	192 NM

There is no difference in the 2 and 4 NM missile ranges since the time of flight for these cases is less than 20 seconds. (For a 1000 knot missile velocity, a 20 second time of flight is equivalent to almost a 5.6 NM missile range.)

For those cases where the missile time of flight is greater than 20 seconds, the launch ranges have decreased. A 36% reduction in penetrator launch range (from 120 to 77 NM) is noted for the 10 NM AI missile range case.

Now consider tail-on attacks by AI's.

Tail-on AI Attacks

The geometry of an AI missile launch against a penetrator is illustrated in Figure 15.7.

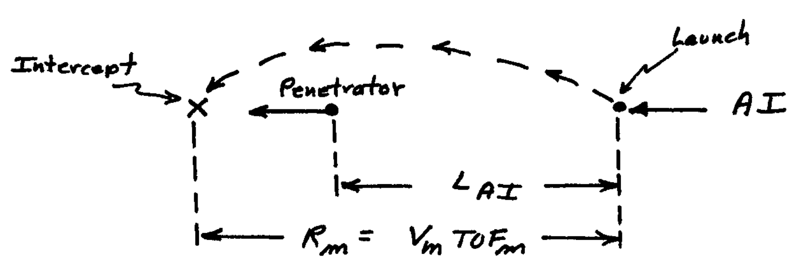


Figure 15.7 Tail-on AI Missile Attack Geometry

The missile limited launch range for a tail-on geometry is the missile kinematic range minus the distance moved by the penetrator during the missile time of flight:

$$\text{Missile } L_{AI} = R_m - TOF_m V_p$$

All relations previously developed for head-on launch are identical, with only the sign of V_p changed. Thus:

$$R_W/R_m > (1 - V_p/V_m)/(1 + V_p/V_W)$$

$$L_p/L_{AI} > 1 + \{(V_{AI} - V_p)/(V_W + V_p)\}$$

Using the same example as the head-on case (V_{AI} = 700 knots, V_p = 600 knots, V_m = 1000 knots and V_W = 800 knots):

$$R_W/R_m > (1 - 600/1000)/(1 + 600/800) = 0.23$$

$$L_p/L_{AI} > 1 + \{(700 - 600)/(800 + 600)\} = 1.07$$

The penetrator weapon requirements are listed for various AI missile ranges below:

R_m =	2 NM	4 NM	8 NM	16 NM
R_W >	0.5 NM	0.9 NM	1.8 NM	3.7 NM
L_{AI} =	0.8 NM	1.6 NM	3.2 NM	6.4 NM
L_p >	0.9 NM	1.7 NM	3.4 NM	6.8 NM

Note that each penetrator weapon launch range (L_p) is just slightly larger than the AI missile launch range (L_{AI}), and is well below the AI missile kinematic range (R_m).

Figure 15.8 illustrates the sensitivity of the kinematic range ratio to the velocity ratios: V_p/V_m and V_p/V_W.

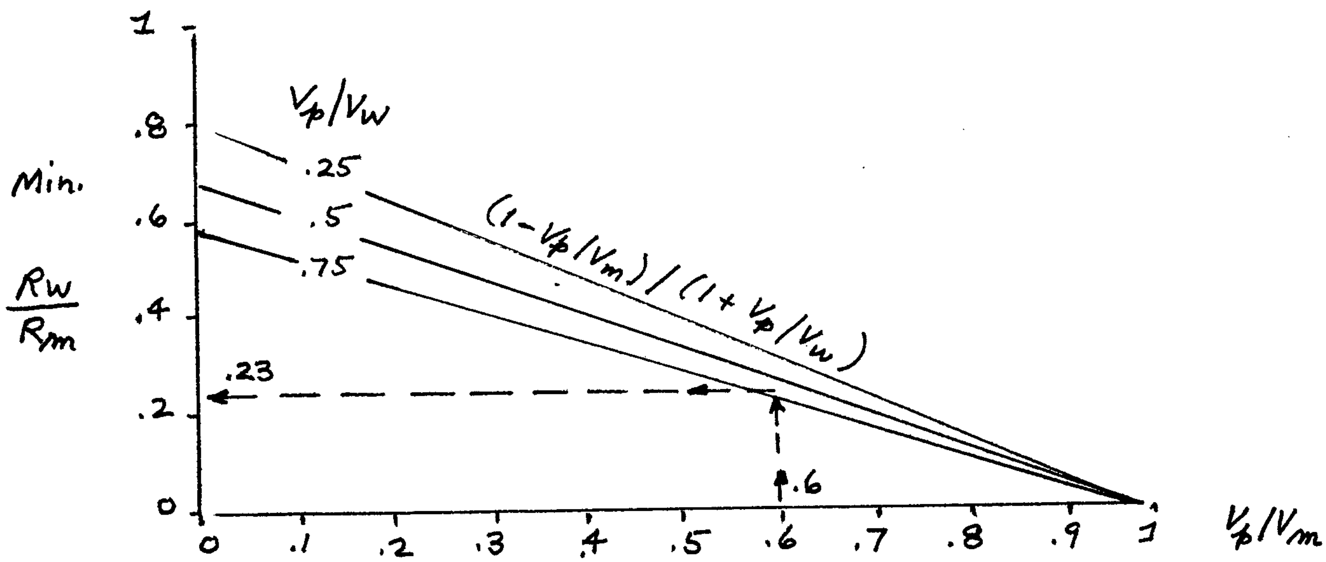


Figure 15.8 Tail-on Weapon to Missile Range Ratios

Note how this ratio drops off as the penetrator velocity increases.

Figure 15.9 illustrates penetrator requirements for weapon launch range as a function of AI missile launch range.

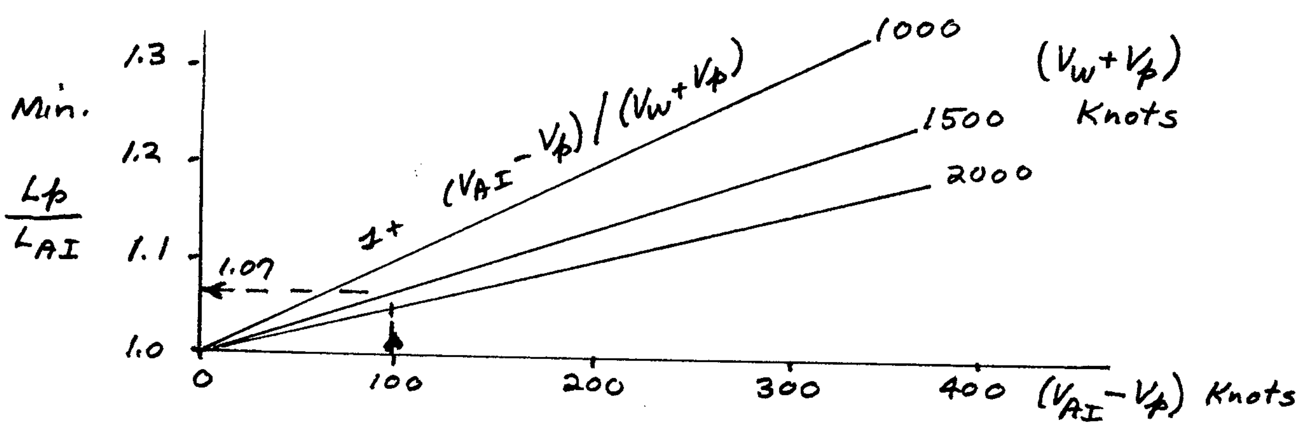


Figure 15.9 Tail-on Penetrator to AI Launch Range Ratios

Penetrator Tail-on Sensor Requirements

With an active tail warning radar, the required radar K Value can be found from:

$$K > \{L_p + \Delta t_p (V_{AI} + V_p)\}/\sigma_{AI}^{1/4}$$

Assuming Δt_p = 25 seconds, a interceptor to penetrator closing velocity of 100 knots and a 1 m^2 RCS for the AI:

$$K > L_p + 0.7 \text{ NM}$$

This adds 0.7 NM to the values of L_p previously calculated, providing a range from 1.6 to 7.5 NM for the minimum required K Values (for AI missile ranges from 2 to 16 NM respectively). Methods to reduce these requirements include increased penetrator (or weapon) speed and decreased reaction time.

So far, only penetrator self defense against the AI itself has been addressed. Now consider self defense range requirements against the AI missile in flight.

Intercept Missile Head-on

The weapon launch range required to intercept a missile at a distance d_{min} from the penetrator is illustrated in Figure 15.10.

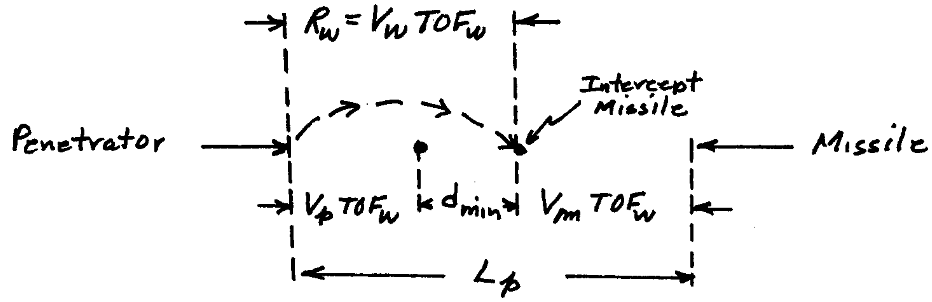


Figure 15.10 Intercept Missile Head-on

Note that:

$$L_p > d_{min} + TOF_W\,(V_p + V_m)$$

and

$$R_W > d_{min} + TOF_W\, V_p$$

For example, assume that d_{min} = 1 NM, V_p = 600 knots, V_m = 1000 knots and R_W is limited to 2 NM. The TOF_W, V_W and L_p to meet the requirement to intercept 1 NM away is to be determined.

$$R_W = 2 > 1 + TOF_W\,(600/3600) \quad \text{Therefore } TOF_W < 6 \text{ sec}$$

$$V_W = R_W/TOF_W > (2/6)\,3600 = 1200 \text{ knots}$$

$$L_p > 1 + 6\,(600 + 1000)/3600 = 3.7 \text{ NM}$$

Adding a 25 second time delay for the penetrator sensor to detect and track and for launch decision, the required coverage range (R_d) is:

$$R_d > L_p + \Delta t_p\,(V_p + V_m)$$

which in the example is:

$$R_d > 3.7 + 25\ (600 + 1000)/3600 = 14.8 \text{ NM}$$

If the AI missile RCS were 0.1 m^2 this would require a radar sensitivity on the penetrator of:

$$K > R_d/\sigma_m{}^{1/4} = 14.8/(.1)^{1/4} = 26 \text{ NM}$$

The detection range is dominated by the reaction time (Δt_p) in the above example. If this reaction time can be reduced from 25 to 2.5 seconds, the detection range will drop to 4.8 NM, and the K Value to 8.5 NM. This illustrates the need for minimum penetrator reaction time when trying to intercept attacking missiles.

Now the tail-on intercept case will be examined.

Intercept Missile Tail-on

The tail-on requirements can be developed by just changing the sign of V_p in the relations developed above for head-on intercept. Thus:

$$L_p > d_{min} + TOF_W\ (V_m - V_p)$$

$$R_W > d_{min} - TOF_W\ V_p$$

For example, using the same values as before (d_{min} = 1 NM, V_m = 1000 knots, V_p = 600 knots and V_W = 1200 knots):

$$R_W = TOF_W\ (1200/3600) > 1 - TOF_W\ (600/3600) \quad \text{Therefore } TOF_W > 2 \text{ sec}$$

$$L_p > 1 + 2(1000 - 600)/3600 = 1.2 \text{ NM}$$

$$R_d > 1.2 + 25\ (1000 - 600)/3600 = 4.0 \text{ NM}$$

$$K > 4.0/(.1)^{1/4} = 7 \text{ NM}$$

Again the reaction time appears to be a critical factor. If the reaction time were reduced to 2.5 seconds, the detection range requirements would change to 1.5 NM and the K Value to 2.7 NM.

With both a 2.5 second reaction time and a reduction in d_{min} to 0.1 NM, the requirements in the example become:

$$TOF_W > 0.2 \text{ sec},\ L_p > 0.12 \text{ NM},\ R_d > 0.4 \text{ NM and } K > 0.7 \text{ NM}$$

This illustrates the importance of reaction time and minimum intercept distance in reducing range requirements for missile intercept.

Summary

This chapter has examined lethal self defense against an interceptor for both head-on and tail-on attacks. Methods to estimate weapon and penetrator launch ranges were developed. Examples showed that head-on AI intercepts before AI launch require very long range weapons and sensors on the penetrator. These requirements can be reduced by:

Electronic countermeasures	Low radar cross section
Lower speeds or turns	Reaching the AI before intercept

Tail-on weapon and launch range requirements against the AI are quite modest, compared to head-on needs.

Defending against the AI missile itself was analyzed from the perspective of intercepting the missile a minimum distance away from the penetrator. Methods to estimate weapon/launch ranges were presented, along with sensor requirements. The importance of reducing penetrator reaction time (to detect, track and decide to intercept the missile) and minimizing keep out range was shown in head-on and tail-on examples.

PART IV

ATTRITION METHODOLOGY

Chapter 16

AI One-on-One Pk

Attrition Methodology (Part IV of this text) starts with an analysis of the probability of kill (Pk) of one airborne interceptor (AI) against one penetrator (one-on-one). AI attacks using radar or electro-optical (EO) sensors will be addressed from both head-on and tail-on aspects to the penetrator.

Simple methods to evaluate the results of these attacks are developed and illustrated for undegraded and several degraded conditions. Degrades considered are electronic countermeasures (ECM), an infrared countermeasure (IRCM), optical camouflage, penetrator turns, low radar cross section and lethal self defense against the AI and its missiles. The chapter ends with a brief discussion of the likelihood of head-on vs tail-on vectoring of the AI by area sensor networks.

Definitions

It will be assumed that an AI attack can begin by an attempt to detect the penetrator in one of four ways:

1) Initial Tail-on Electro-optical (T/E) detection attempt,
2) Initial Head-on Electro-optical (H/E) detection attempt,
3) Initial Tail-on Radar (T/R) detection attempt, or
4) Initial Head-on Radar (H/R) detection attempt.

With electro-optical attempts, the AI's radar is assumed to be ineffective, so that the AI can use only IR or optical means for detection. With radar attempts, the AI's radar is assumed to be effective in detecting the penetrator. The weapons used following successful detection include head-on launched radar guided missiles, tail-on launched radar guided missiles, head-on and tail-on IR guided missiles and a tail-on gun pass (until either the penetrator is destroyed or the AI has completed its defined activity). The definitions to be used for AI one-on-one probability of kill are:

$Pk_{T/E}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Tail-on Electro-optical detection. Weapon attacks planned are one tail-on IR guided missile salvo followed by a tail-on gun pass.

$Pk_{H/E}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Head-on Electro-optical detection. Weapon attacks planned are one head-on IR guided missile salvo and a head-on gun pass, followed by a tail-on IR guided missile salvo and a tail-on gun pass.

$Pk_{T/R}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Tail-on Radar detection. Attacks planned are one tail-on radar guided missile salvo, followed by one tail-on IR guided missile salvo and a tail-on gun pass.

$Pk_{H/R}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Head-on Radar detection. Weapon attacks planned are one head-on radar guided missile salvo, followed by one tail-on radar missile salvo, one tail-on IR guided missile salvo and a tail-on gun pass.

A Simple Pk Model

Assume that the entire sequence of AI weapon attacks is dependent upon the initial detection (and track) probability (Pd). However, assume that the penetrator's survival probability after each AI weapon attack/pass is independent of the success of the previous pass. With these assumptions, the following equations define the P_k equations for the four cases:

$$Pk_{T/E} = Pd_{T/E}\{1 - PS_{T/IR}\ PS_{T/gun}\}$$

$$Pk_{H/E} = Pd_{H/E}\{1 - PS_{H/IR}\ PS_{H/gun}\ PS_{T/IR}\ PS_{T/gun}\}$$

$$Pk_{T/R} = Pd_{T/R}\{1 - PS_{T/R}\ PS_{T/IR}\ PS_{T/gun}\}$$

$$Pk_{H/R} = Pd_{H/R}\{1 - PS_{H/R}\ PS_{T/R}\ PS_{T/IR}\ PS_{T/gun}\}$$

(These equations are lined up on a right hand margin in order to illustrate the common tail-on IR/gun attack in all four cases.)

For radar initial detection attempts, the probabilities of detection will be determined by compounding radar, infrared and visual detection probabilities (assuming independence between these P_d's):

$$Pd_{H/R} = 1 - (1 - Pd_{H/radar})(1 - Pd_{H/IRST})(1 - Pd_{H/visual})$$

$$Pd_{T/R} = 1 - (1 - Pd_{T/radar})(1 - Pd_{T/IRST})(1 - Pd_{T/visual})$$

For EO initiated detection attempts, the infrared and visual detection probabilities will be compounded (assuming independence between Pd's):

$$Pd_{H/E} = 1 - (1 - Pd_{H/IRST})(1 - Pd_{H/visual})$$

$$Pd_{T/E} = 1 - (1 - Pd_{T/IRST})(1 - Pd_{T/visual})$$

where IRST = infrared search and track system

PS is the probability of survival after a specific weapon attack/pass on a Head-on or Tail-on approach, with a salvo of either Radar or IR guided missiles or a gun pass. Each of the six possible weapon attack PS values is determined from:

$$PS = (1 - P_c\, P_l\, PK) \quad \text{for each missile salvo, or}$$

$$PS = (1 - P_c\, P_f\, PK) \quad \text{for a gun pass}$$

where P_c = Probability of conversion, given detection on the initial approach

P_l = Probability of missile salvo launch, given conversion

P_f = Probability of gun fire, given conversion

PK = Probability of kill by a radar or infrared missile salvo given launch (or by a gun pass, given fire)

This PS equation was developed assuming that each probability is conditional on the previous action having been successfully completed. That is, conversion can not occur unless the penetrator is detected by the AI initially, launch/fire can not occur unless conversion is successful, and missile/gun kill can not occur unless launch/fire is successful. In the case of a re-attack (e.g. from radar head-on to radar tail-on, from radar to an IR attack, or from IR to a gun pass), the probability of conversion includes this re-attack probability.

For the missile salvo PK term, assume partial dependence between missiles in a salvo, or:

$$PK = SSPK_{dependent}\{1 - (1 - SSPK_{independent})^n\}$$

Assuming that $SSPK^{1/2}$ represents the dependent part:

$$PK = SSPK^{1/2}\{1 - (1 - SSPK^{1/2})^n\}$$

where SSPK = Single shot probability of kill

n = number of missiles in a salvo

For a gun attack, PK represents the probability of kill for all the rounds which can be fired against the penetrator on that pass.

AIs will be divided into two categories - those with look down (LD) capability and those without look down capability (non LD). A look down AI is assumed to have the capability from high altitude to detect low altitude penetrators against a clutter background (e.g. with a pulse Doppler radar). Shoot down missiles on LD AIs are assumed to have the capability to intercept low altitude penetrators in a clutter environment (unless degraded by low penetrator radar cross section as discussed later in the chapter).

Non look down AIs are assumed to have no significant radar detection or radar guided missile shoot down capability against low altitude penetrators. These non LDs are assumed to descend to detect low altitude penetrators and to employ IR guided missiles and guns as their only effective weaponry.

An Undegraded Pk Example

An undegraded case (no ECM, IRCM, camouflage, turns, low radar cross section or lethal self defense) will be illustrated for an AI against a low altitude penetrator. For example, assume that the AI sensor and weapon capabilities are:

Look down only: $Pd_{H/radar} = .75$, $Pd_{T/radar} = .5$

All AIs: $Pd_{H/IRST} \approx 0$, $Pd_{T/IRST} \approx 0$

$Pd_{H/visual} \approx 0$, $Pd_{T/visual} = .5$

Conversion probability = .75 for all attacks

Launch/fire probability = .75 for all attacks

2 radar missiles in a salvo,	SSPK: head-on = .5, tail-on = .25
2 IR missiles in a salvo,	SSPK: head-on $\approx$ 0, tail-on = .25
Gun pass,	PK: head-on $\approx$ 0, tail-on = .25

These are example values only, and do not represent any particular AI or penetrator. Actual values depend upon many factors including AI sensor, missile and gun capability and penetrator speed, altitude and vulnerable areas. The example values were chosen just to illustrate the simple Pk model being developed.

The one-on-one probability of kill is developed below for the case of an <u>initial head-on</u> LD intercept, with values shown for the example:

$$\text{Head-on radar 2 missile salvo PK} = SSPK^{1/2}\ \{1 - (1 - SSPK^{1/2})^n\}$$

$$= .5^{1/2}\ \{1 - (1 - .5^{1/2})^2\} = .646$$

$$\text{Head-on radar PS} = \{1 - P_c\ P_1\ PK_{H/R}\} = \{1 - (.75 \times .75 \times .646)\} = \underline{.64}$$

$$\text{Tail-on radar 2 missile salvo PK} = .25^{1/2}\ \{1 - (1 - .25^{1/2})^2\} = .375$$

$$\text{Tail-on radar PS} = \{1 - (.75 \times .75 \times .375)\} = \underline{.79}$$

$$\text{Tail-on IR 2 missile salvo PK} = .25^{1/2}\ \{1 - (1 - .25^{1/2})^2\} = .375$$

$$\text{Tail-on IR PS} = \{1 - (.75 \times .75 \times .375)\} = \underline{.79}$$

$$\text{Tail-on gun pass PS} = \{1 - (.75 \times .75 \times .25)\} = \underline{.86}$$

The combined probability of survival (cPS) of all four independent weapon actions is:

$$cPS = PS_{H/R}\ PS_{T/R}\ PS_{T/IR}\ PS_{T/gun}$$

$$= .64 \times .79 \times .79 \times .86 = .34$$

The probability of LD head-on detection is:

$$Pd_{H/R} = 1 - (1 - Pd_{H/radar})(1 - Pd_{H/IRST})(1 - Pd_{H/visual})$$

$$= 1 - (1 - .75)(1 - 0)(1 - 0) = .75$$

The one-on-one probability of penetrator kill for one AI (in the example) is:

$$Pk = Pd\ (1 - cPS) = .75\ (1 - .34) = .50$$

The table below illustrates these results, plus two other cases (an initial radar tail-on attack by a look down AI and a initial tail-on EO attack by a non LD AI). Note that non LDs are not vectored for <u>head-on</u> EO attacks, due to the assumed near zero probability of head-on detection and IR missiles/gun kill.

	Head-on Radar Missiles	Tail-on Radar Missiles	Tail-on IR Missiles	Tail-on gun Pass
SSPK	.5	.25	.25	---
PK	.646	.375	.375	.25
P_c	.75	.75	.75	.75
$P_{1/f}$	.75	.75	.75	.75
PS	.64	.79	.79	.86

T/E: cPS = .68
Pd = .5 → Pk = .16

T/R: cPS = .54
Pd = .75 → Pk = .35

H/R: cPS = .34
Pd = .75 → Pk = .50

In the undegraded example, an initial Tail-on EO non look down attack will result in a 16% probability of one-on-one kill of the penetrator. An initial Tail-on Radar look down attack results in 35%, and an initial Head-on Radar look down attack results in a 50% probability of one-on-one kill. Note how the survival probabilities are combined to add radar weapon attacks to the basic IR/gun attack.

Degrades

Six degrade categories will be considered:

1) ECM
2) IRCM
3) Optical camouflage
4) Turns
5) Low radar cross section
6) Lethal self defense

ECM

Assume that ECM may degrade three AI radar actions - radar detection, launch of radar guided missiles and the independent portion of SSPK for radar guided missiles. These degrades will change the original undegraded values to the degraded (asterisked) values noted below:

$$Pd^* = Pd\ (1 - \text{Degrade to } \underline{\text{radar}} \text{ detection})$$

$$P_1^* = P_1\ (1 - \text{Degrade to launch of radar guided missiles})$$

$$SSPK_{indep}^* = SSPK_{indep}\ (1 - \text{Degrade to each radar guided missile})$$

Note that a Pd degrade applies to all weapon actions, a P_1 degrade applies only to launches of radar guided missiles, and a SSPK degrade applies only to the independent portion of SSPK for each radar guided missile. Thus, a detection degrade has the greatest potential influence, while the SSPK degrade has the least (since compounding will reduce its effect).

The estimate of any degrade value should be based on flight test data which verifies a prior analysis of effectiveness. In order to illustrate the sensitivity of each degrade, assume that a .5 degrade has been determined for radar detection, radar launch and radar guided missile SSPK. For the ECM degrade example, a 50% degrade in each of the three areas would reduce the look down attack capability to:

$$\text{Head-on} \quad Pd^* = .75\ (1 - .5) = .375 \text{ (radar only)}$$

$$\text{Tail-on} \quad Pd^* = 1 - \{1 - .5(.5)\}(1 - .5) = .625 \text{ (radar/visual)}$$

$$P_1^* = .75\ (1 - .5) = .375$$

$$\text{Head-on: } SSPK_{indep}^* = (.5)^{1/2}\ (1 - .5) = .35 \qquad PK^* = .41 \text{ for salvo}$$

$$\text{Tail-on: } SSPK_{indep}^* = (.25)^{1/2}(1 - .5) = .25 \qquad PK^* = .22 \text{ for salvo}$$

The table below illustrates the one-on-one Pks for each of these three ECM degrades, for the example values:

	One-on-one Pk			
Begin \ Degrading	Pd	P_1	SSPK	None
Tail-on EO	.16	.16	.16	.16
Tail-on Radar	.29	.29	.30	.35
Head-on Radar	.25	.38	.41	.50

If all three degrades applied independently, the probabilities of survival for the three approaches would be:

$$cPS^*_{T/E} = .68 \quad \text{(no change)}$$

$$PS^*_{T/R} = \{1 - .75 \times .375 \times .22\} = .94$$

$$PS^*_{H/R} = \{1 - P_c\ P^*_1\ PK^*_{H/R}\} = \{1 - .75 \times .375 \times .41\} = .88$$

The degraded Pk in the example can be determined from:

$$Pk^* = Pd^* (1 - cPS^*)$$

For the tail-on EO case, Pk = .5 (1 - .68) = <u>.16</u> (no change)

For the tail-on radar LD case, Pk = .625 (1 - .94 x .68) = <u>.23</u>

For the head-on radar LD case, Pk = .375 (1 - .88 x .94 x .68) = <u>.16</u>

Note that a tail-on radar LD approach now shows a higher one-on-one probability of kill (.23) than a head-on radar approach (Pk* = .16). This is due to the higher Pd on a tail-on approach with both radar and visual detection chances.

These results are compared to the undegraded example in Figure 16.1.

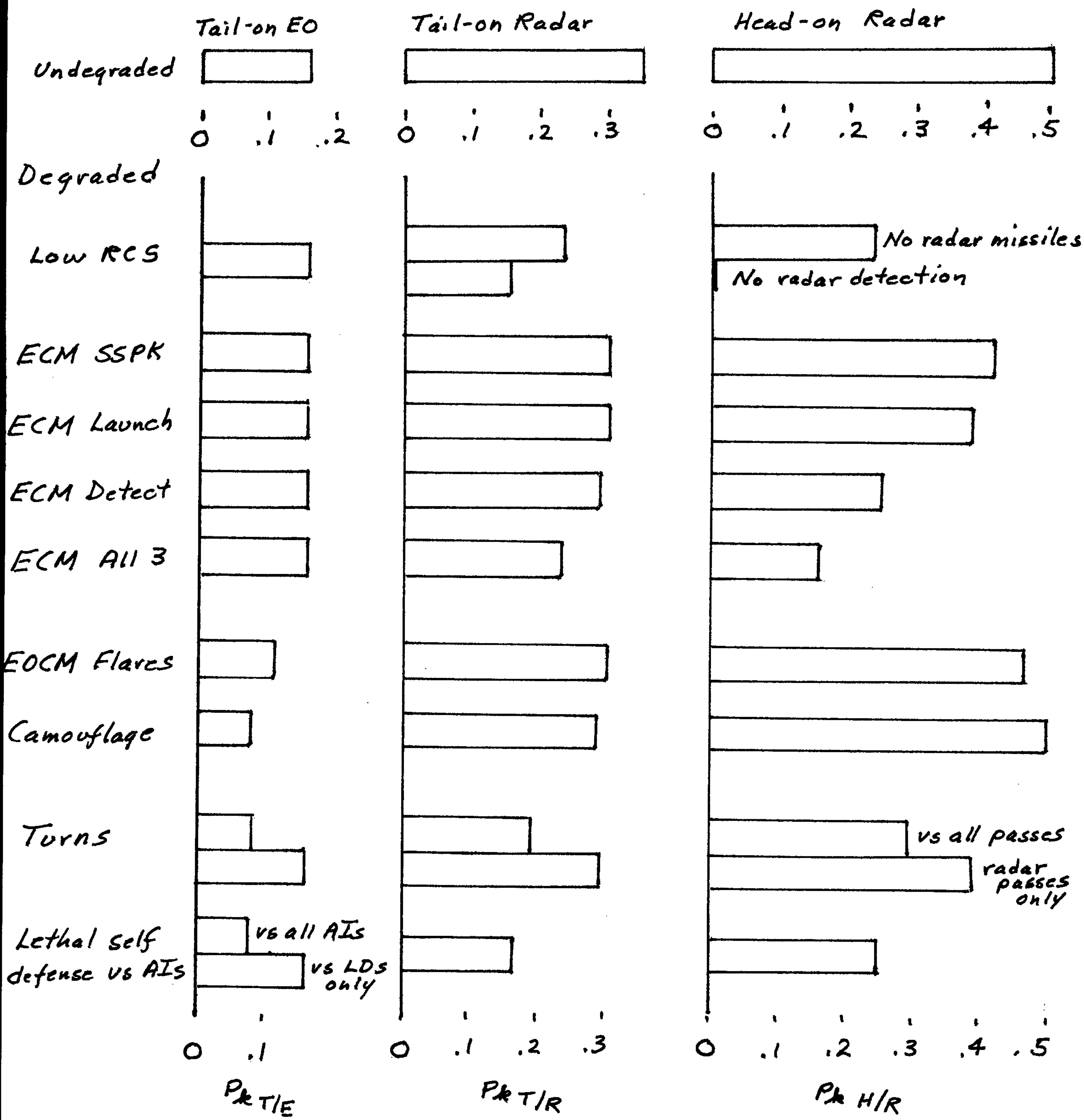


Figure 16.1 Comparison of Pks For Various 50% Degrades

IRCM

For an infrared countermeasure (IRCM), assume a degrade to each IR missile due to flares. If this degrade is .5 against $SSPK_{indep}$ for IR guided missiles, the PK for two missiles changes from .375 to .22. The one-on-one Pk for a tail-on EO attack changes from .16 to .12, a 25% reduction. Reductions for initial radar attacks are smaller as shown in Figure 16.1.

Camouflage

Assume a .5 visual detection degrade due to camouflage, under daylight, clear weather conditions. If the example case had been a dark night or another poor visibility condition, this camouflage degrade would not apply, but visual detection probability under those degraded optical conditions might be very low even without camouflage.

For a tail-on EO attack, the PK is degraded by 50%, from .16 to .08. Reductions for an initial tail-on radar attacks are about 17% (from .35 to .29) as shown in Figure 16.1.

Turns

Penetrator turns might be carefully timed to the AI attack to degrade radar tracking and launch (e.g. by turning into the AI radar's Doppler notch) and to degrade EO launch/firing (e.g. by turning towards the AI attack direction). If the penetrator can turn adequately after detecting the AI attack and its direction, assume for sensitivity purposes that a .5 degrade occurs to the launch of each missile salvo and to the gun pass. This would increase the probability of survival of each weapon attack in the example to:

$$PS^*_{T/gun} = .93, \quad PS^*_{T/IR} = .89, \quad PS^*_{T/R} = .89 \text{ and } PS^*_{H/R} = .82$$

The resulting Pk values are .08 for a tail-on EO approach, .19 for a tail-on radar, and .29 for a head-on radar approach, if all AI weapon actions are independently degraded by penetrator turns. If turns are employed only against radar approaches by LD AIs, the results are .16, .29 and .38 respectively for T/E, T/R and H/R cases, respectively.

Low RCS

Low radar cross section penetrators can stress an airborne interceptor's radar by reducing the time available between AI radar detection of the penetrator and the minimum missile launch distance. Figure 16.2 illustrates how low penetrator RCS can reduce the launch time available.

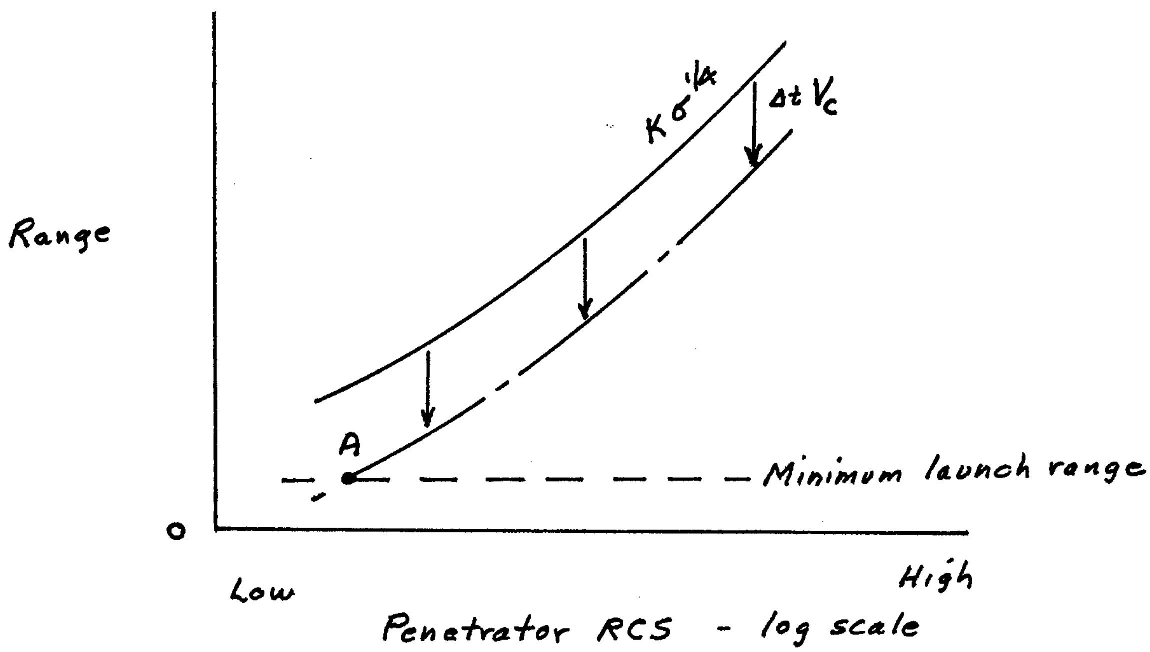


Figure 16.2 AI Radar Capability vs RCS

The range at which the penetrator enters AI coverage is usually determined by $K\,\sigma_p^{1/4}$, where K is the AI's radar K Value or sensitivity, and σ_p is the radar cross section of the penetrator. The launch ready range is determined by deducting a delay distance. This delay distance is equal to the detection, track and decision delay time (or reaction time) multiplied by the closing velocity of the AI to the penetrator.

A minimum launch range is indicated in Figure 16.2. At point A in this figure, the penetrator's RCS is so small that there is just enough time to launch a missile at the penetrator at minimum missile range. Any smaller RCS will deny an effective launch (since a missile can not be launched effectively at less than minimum range).

The RCS at minimum detection range for launch can be found from:

$$K\,\sigma_p{}^{*1/4} - \Delta t\, V_c = \text{Min } R_1$$

Thus:

$$\sigma_p{}^{*} = \{(\text{Min } R_1 + \Delta t\, V_c)/K\}^4$$

where K = K Value or sensitivity of the AI's radar
$\sigma_p{}^*$ = Radar cross section at minimum launch range
Δt = AI reaction time
V_c = Closing velocity between the AI and the penetrator
Min R_1 = Minimum launch range of the AI's missile

For example, assume that an AI's head-on K Value is 30 NM, the reaction time is 25 seconds, the closing velocity is 1300 knots and the minimum launch range is 5 NM. Then the RCS required to stress minimum launch is:

$$\text{Head-on} \quad \sigma^* = \{(5 + (25/3600)1300)/30\}^4 = .47^4 = .05 \text{ m}^2$$

For a tail-on case, a tail-on K Value of 10 NM, a 25 second reaction time, a 100 knot closing velocity and a minimum launch range of 3 NM will be used as an example. The RCS to stress the minimum launch is:

$$\text{Tail-on} \quad \sigma^* = \{3 + (25/3600)100)/10\}^4 = .37^4 = .02 \text{ m}^2$$

If a penetrator's RCS is less than the lower of these values, the AI radar can not detect the penetrator in time to launch radar missiles during radar approaches. Then no radar guided missiles will damage the penetrator, but tail-on EO detection could occur and IR guided missiles and gun passes allowed.

In the example, RCS values below .02 m^2 will reduce the one-on-one Pk tail-on radar approach from the undegraded .35 to .24. For the head-on radar approach, the change is from the undegraded .50 to .24. Figure 16.1 illustrates these results for a low RCS case which denies any radar guided missile launches. Head-on radar degraded values can drop to 0 if the AI can not detect the penetrator (e.g. completely misses the penetrator on the assigned approach), and tail-on radar approach degraded values can drop to the EO case of .16 with visual detection backup.

Lethal Self Defense

Penetrator lethal self defense can kill an AI during head-on or tail-on attacks or can kill a missile approaching the penetrator. Degrades due to lethal self defense against the AI will be denoted as P_{sd}, the probability of killing the AI on the first approach to the penetrator. Assume that this is the only lethal defense action against the AI.

P_{sd} will increase the penetrator's survival by reducing the probability of the first and subsequent missile launches, or the gun pass, if the AI is killed by the lethal defense weapon. This P_{sd} is applied as:

$$Pk^* = P_d \, P_{sd} \, (1 - cPS)$$

For example, with a .5 degrade and a head-on radar approach, the one-on-one Pk is:

$$Pk^* = P_d \; P_{sd} \; (1 - cPS) = .75 \times .5 \; (1 - .34) = .25$$

Similarly, with lethal self defense against tail-on attacks, the Pk for tail-on radar approach is .17, and .08 for tail-on EO approaches.

If the lethal defense is against LD AIs only, the degraded Pks are the same for radar approaches by the LDs. However, non LDs which are not attacked retain their .16 Pk for EO detection and IR/gun weaponry.

For defense against the missile itself, the degrade reduces the SSPK. Recall the following relationship between SSPK and PK:

$$PK = SSPK^{1/2}\{1 - (1 - SSPK^{1/2})^2\} \quad \text{for a 2 missile salvo}$$

With a .5 degrade to the independent portion of SSPK, the head-on and tail-on 2 missile salvo cases become:

$$\text{Head-on} \quad PK^* = .5^{1/2}\{1 - (1 - .25^{1/2})^2\} = .41$$
$$\text{Tail-on} \quad PK^* = .25^{1/2}\{1 - (1 - .125^{1/2})^2\} = .22$$

There is no reduction for a gun pass, since only missiles (not bullets) are assumed to be intercepted.

The degraded Pk for lethal defense against AI missiles then becomes for head-on/radar, tail-on/radar and tail-on/EO approaches:

$$Pk^*_{H/R} = .37 \qquad Pk^*_{T/R} = .25 \qquad Pk^*_{T/E} = .12$$

One might chose from many different combinations of ECM, IRCM, camouflage, turns, low RCS and lethal self defense degrades. The simple Pk model allows sensitivity analyses of any combination desired, once one knows the range of possible degrades, the effect of each degrade on Pk and the independence or dependence of various degrades on each other.

Head-on vs Tail-on Vectoring

Before ending this chapter, the likelihood of AIs being vectored on head-on or tail-on approaches will be discussed. For the non look down AI, the only productive vector (in the example) is a tail-on attack. Thus, one might assume that the area defense will try to vector all non look down AIs on tail-on approaches.

Look down AIs are usually more productive against low altitude penetrators on head-on approaches, due to relative freedom from clutter at high closure speeds. However, the geometry of the intercept may dictate a tail-on approach as the only way to catch a penetrator before it exits area sensor coverage. Thus, some fraction (f_T) of the look down AIs might have to vectored for tail-on approach. The overall look down Pk can be determined by:

$$\text{LD Pk} = (1 - f_T)\ Pk_{H/R} + f_T\ Pk_{T/R}$$

Factors which affect f_T include AI location, distance between the AI and the area sensors, AI/penetrator speeds and penetrator routes.

For example, if f_T = .25 in the undegraded case, the LD Pk would be:

$$\text{Undegraded LD Pk} = (1 - .25) \times .50 + .25 \times .23 = \underline{.43}$$

Interceptors reassigned from a missed head-on approach to a tail-on approach will increase this fraction. A missed tail-on approach might occasionally allow a later head-on approach, but this is restricted by the short combat time (fuel remaining) of an AI.

Degrades against area sensors can change the likelihood of head-on attacks by delaying assignment and thus increasing the likelihood of vectoring on a tail-on approach.

For example, an area sensor degrade may change f_T from .25 to .75. With 75% of the AIs making tail approaches the Pk example changes to:

$$\text{LD Pk} = (1 - .25) \times .50 + .75 \times .23 = \underline{.30}$$

The Pk model developed allows any fraction of tail-on approaches to be chosen, based on the particular assessment of area/AI defense capability and penetrator routes/speeds/altitudes.

Summary

The simple Pk model developed allows head-on and tail-on radar and EO approaches by an AI to a penetrator to be analyzed. The model is based on a building block of combining independent probabilities of survival (PS) as shown below:

$$Pk_{T/E} = Pd_{T/E}\{1 - PS_{T/IR}\ PS_{T/gun}\}$$

$$Pk_{H/E} = Pd_{H/E}\{1 - PS_{H/IR}\ PS_{H/gun}\ PS_{T/IR}\ PS_{T/gun}\}$$

$$Pk_{T/R} = Pd_{T/R}\{1 - PS_{T/R}\ PS_{T/IR}\ PS_{T/gun}\}$$

$$Pk_{H/R} = Pd_{H/R}\{1 - PS_{H/R}\ PS_{T/R}\ PS_{T/IR}\ PS_{T/gun}\}$$

Each PS can be determined from:

$$PS = (1 - P_c\ P_{1/f}\ PK)$$

where P_c, $P_{1/f}$ and PK are all conditional probabilities.

PK can be determined for a salvo of n missiles by the following equation:

$$PK = SSPK_{dependent}\{1 - (1 - SSPK_{independent})^n\}$$

Examples illustrated in this chapter split SSPK into a $SSPK^{1/2}$ dependent and independent part.

The sensitivity of this AI Pk model was illustrated for an undegraded case and for six types of degrades:

ECM	IRCM	Optical camouflage
Turns	Low RCS	Lethal self defense

This model is simple enough to allow the user to understand the impact of various types and levels of degrades. When estimated degrades are verified by flight tests, the model becomes a powerful tool to analyze penetrator attrition.

Against low altitude penetrators, an area defense may attempt to vector non look down AIs for tail-on approaches, and look down AIs for head-on detection approaches. Limitations in area coverage (e.g. due to ECM or low RCS) and in AI combat time, as well as the likelihood of missed head-on detections causing tail-on re-vectors, will increase the fraction of LD AIs vectored for tail-on detection. These factors should lower the Pk of LD AIs due to their lesser capability in clutter on tail-on attacks, illustrating the importance of assessing head-on vs tail-on vectoring by the defense system.

Chapter 17

Surviving AI Attacks

A simple model for penetrator mission survival of AI attacks under close net control is developed in this chapter. AI assignments required to service penetrators will be assessed, at first with no limits on AI resources. Then, defense resources and capabilities will be considered to determine a limit on the number of AIs that can be assigned by a particular defense during a penetration mission.

The ratio of this limit to the number of AIs required will define the expected number of AI assignments per penetrator during an air campaign. The chapter concludes with a discussion of methods to analyze multiple penetrator types, multiple airborne interceptor types and preferential assignments against certain penetrators.

AIs Required

The number of AIs required under close net control to continue assignments against penetrators until they are killed (or escape coverage) will be addressed in two parts. A single penetrator will be discussed first, followed by the case of many penetrators.

A Single Penetrator

The number of assignments against one penetrator can be estimated from 1) the penetrator's time under area defense coverage, 2) the time for the defense to make the first AI assignment, 3) the time for the first AI to reach the penetrator, 4) the time to complete the next assignment (if the last was unsuccessful in killing the penetrator), 5) the probability of kill per assignment and 6) any defense resource limits.

Assume, for the moment, that the number of AIs is unlimited (all penetrators can be serviced when needed) and that the probability of kill per assignment (Pk/A) is 0. These two assumptions lead to the maximum number of assignments against one penetrator, abbreviated as Max A/p. This can be determined by:

$$\text{Max A/p} = \left| \frac{(\text{Time in coverage} - \text{time first AI reaches penetrator})}{\text{Average Time between assignments}} + 1 \right|_{\text{round down}}$$

For example, assume that a penetrator is under coverage for 100 minutes, the average time between entering coverage and the first AI completed assignment is 10 minutes and the average time between completed assignments is 10 minutes. Then,

$$\text{Max A/p} = 1 + (100 - 10)/10 = 10 \text{ assignments per penetrator}$$

If AI resources are still unlimited, but the Pk per assignment (Pk/A) is greater than 0, the expected number of assignments per penetrator (Exp A/p) until the intruder is either killed, or exits defense coverage, can be determined by:

$$\text{Exp A/p} = \sum_{i=1}^{\text{Max A/p}} (1 - \text{Pk/A})^{i-1}$$

This summation was derived by finding the probability that a new assignment is needed, based on the results (kill/no kill) of the previous assignment. For example, the first assignment is always needed: thus the first term is 1.0. A second assignment is needed only if the first is unsuccessful. Failure will occur on any assignment if the penetrator is not killed: a probability of (1 - Pk/A). A third assignment is needed only if the first and second are both failures: a probability of (1 - Pk/A)(1 - Pk/A) or $(1 - \text{Pk/A})^2$. The probability of the nth assignment being required is the probability that the previous n - 1 assignments have been unsuccessful: $(1 - \text{Pk/A})^n$. The expected number of assignments against one penetrator is then:

$$\text{Exp A/p} = 1 + (1 - \text{Pk/A}) + (1 - \text{Pk/A})^2 - - - - (1 - \text{Pk/A})^{\text{Max A/p} - 1}$$

$$\text{Exp A/p} = \sum_{i=1}^{\text{Max A/p}} (1 - \text{Pk/A})^{i-1}$$

Note that with an infinite number of assignments, Exp A/p = 1/(Pk/A).

The expected number of assignments per penetrator is illustrated below for several Pk/A values, when the defense coverage allows a maximum of 10 assignments:

Pk/A	=	0	.1	.2	.4	1.0
Exp A/p	=	10	6.5	4.5	2.5	1.0

The value of Pk/A can be determined, recalling work in Chapter 16, as:

$$\text{Pk/A} = (1 - f_T)\ \text{Pk/A-H} + f_T\ \text{Pk/A-T}$$

Where f_T is the probability of a tail-on assignment, and Pk/A-H (Pk/A-T) is the probability of kill on a head-on (or tail-on) assignment. The head-on assignment Pks can be determined by:

$$\text{Pk/A-H} = \{\ 1 - (1 - \text{Pk}_H)^n\ \}$$

where n is the number of AIs per assignment

Tail-on Pk/A can be determined by:

$$\text{Pk/A-T} = \{\ 1 - (1 - \text{Pk}_T)^n\ \}$$

For example, with a .5 probability of a tail-on assignment, a tail-on Pk per AI of .03, a head-on Pk of .08, and 2 interceptors per assignment, the Pk/A can be determined as:

$$\text{Pk/A-T} = \{\ 1 - (1 - .03)^2\ \} = .06$$

$$Pk/A\text{-}H = \{ 1 - (1 - .08)^2 \} = .15$$

$$Pk/A = (.5)(.06) + (.5)(.15) = .1$$

Thus far only one penetrator has been considered. Now consider many penetrators in an air campaign.

Many Penetrators

The number of penetrators entering coverage usually depends upon the readiness status of the offense. Two levels will be considered: a low (day-to-day or normal) level and a high level (e.g. where all available air vehicles are launched in a maximum effort). For example, the number of penetrators entering defense coverage will be assumed to be 100 with low readiness and 300 with high readiness.

The number of AIs required (AIs Reqd) to be assigned against a penetrator force consisting of P penetrators (all of one type) entering coverage is:

$$\text{AIs Reqd} = (P)(\text{Exp } A/p)(\text{AIs}/A)$$

The number of AIs sent on each assignment (AIs/A) depends upon the defense doctrine. This doctrine can vary from one AI per assignment (for very high quality, scarce interceptors), to many AIs/A (if interceptors have relatively poor capability, but are plentiful).

For example, assume that 2 AIs are assigned per penetrator and that Pk/A = .1 (based on flight test data which validates one-on-one analysis). Exp A/p = 6.5 for this Pk/A and a maximum of 10 assignments per penetrator, so that the expected number of AIs needed against a force of 100 (and 300) penetrators is:

$$\text{AIs Reqd} = (100)(6.5)(2) = 1300 \text{ AIs}$$

$$\text{AIs Reqd} = (300)(6.5)(2) = 3900 \text{ AIs}$$

These requirements will be compared to the defense capability to provide these AIs.

Defense Capability

Defense capability will be addressed in three areas:

1) AIs ready,
2) Multiple use, and
3) Efficiency

AIs Ready

The number of AIs in the defense inventory is often expressed as the Primary Aircraft Authorization (PAA). However, only a certain fraction (f_{ready}) of this number will be ready and within range of the penetrators during a campaign. Thus:

$$\text{AIs Ready} = (f_{ready})(\text{PAA})$$

For example, assume a PAA of 2000 airborne interceptors, all of one type. If results from extensive exercises and analyses indicate that .6 of these AIs will be ready and within range of the intruder force for one particular scenario, then:

$$\text{AIs Ready} = (.6)(2000) = 1200 \text{ AIs}$$

Multiple Use

During one sortie an AI may attack more than one penetrator, if the interceptor has sufficient combat time and weapons capability. The average number of assignments by one AI will be designated (A/sortie).

An AI may also fly more than one sortie (be recycled), if there is time to land, refuel, load missiles/guns and commit the AI before all penetrators exit area coverage. The average number of AI sorties, or cycles (during the entire penetration) depends on the scenario and two factors: 1) AI operating bases - either main operating bases (MOBs) or alternate bases (ABs), and 2) lethal self defense by the penetrators.

The primary reason for operating from alternate bases (where the recycle time is usually longer than on MOBs due to reduced maintenance facilities and personnel) is loss of MOBs due to offensive attacks against these bases. The fraction of all AIs which survive and operate from ABs will be designated f_A. The average number of cycles from ABs will be abbreviated as cycles-AB. The fraction which operate from surviving MOBs will be designated f_M. The average number of cycles from MOBs will be abbreviated as cycles-MOB.

Penetrator lethal self defense against airborne interceptors can reduce AI recycles, if the AI is unable to undertake new assignments due to damage from a penetrator's weapon. Fractional AI capability after this reduction will be designated f_{sd}, measuring the fraction of the AIs available for assignment when lethal self defense is employed by the penetrators compared to that without self defense. (Lethal self defence against AI missile will reduce Pk/A, and will be considered later.)

These factors provide a multiplier to the number of AIs ready of:

$$\text{Multiplier} = (\text{A/sortie})(f_{sd})\ \{(f_M)(\text{cycles-MOB}) + (f_A)(\text{cycles-AB})\}$$

For example, assume that extensive exercises and analyses support the following values, when penetrators have no lethal self defense (f_{sd} = 1):

$$\text{A/sortie} = 1.5, \quad \text{cycles-MOB} = 2, \quad \text{cycles-AB} = 1$$

$$\text{(With no attacks on AI bases) } f_M = 1, \; f_A = 0$$

$$\text{(With the attack on AI bases) } f_M = .1, \; f_A = .4$$

Note that in the example with no defense suppression (i.e. no attacks against AI bases), all AIs survive and operate from their MOBs (f_M = 1). With the base attack, 10% of the total AI force are assumed to operate from surviving MOBs, while 40% of the AI force operate from alternate bases. The other 50% of the AI force are assumed to have been lost due to the base attack which preceded the penetration. Thus, in the example with no suppression:

$$\text{Multiplier} = (1.5)\ 1\ \{(1)(2) + (0)(1)\} = 3$$

With the prior suppression:

$$\text{Multiplier} = (1.5)\ 1\ \{(.1)(2) + (.4)(1)\} = .9$$

With the prior attack on the AIs bases, the defense capability has been dramatically reduced (in the example) from a multiplier of 3 to .9, a 70% reduction. Part of this reduction is due to 50% of the AIs being lost (either destroyed/damaged on the ground, or unable to find a suitable landing airfield - if they flushed to escape an attack on their MOB). The rest is due to the 50% reduction in the number of cycles (or sorties) from alternate bases compared to cycles from the MOBs.

Efficiency

Two assignment efficiency issues will be addressed: 1) missed assignments with no revectors possible, and 2) combat air patrol (CAP).

Missed assignments with no revectors reduce the assignment efficiency, η_{assign}. This will be evaluated as:

$$\eta_{assign} = 1 - (\text{Prob of miss})(\text{Prob no revector})$$

With CAP, AIs dedicated to this patrol will only be available for assignment against a penetrator for a fraction of their total flight time (due to long distances to/from their CAP stations). If the fractional assignment capability of CAP AIs (compared to non CAP AIs) is $f_{A/CAP}$, and the fraction of all AIs assigned to CAP is f_{CAP}, the efficiency with CAP (η_{CAP}) will be:

$$\eta_{CAP} = (f_{CAP})(f_{A/CAP}) + (1 - f_{CAP})$$

If no AIs are assigned to CAP ($f_{CAP} = 0$), the CAP efficiency is 1. If all AIs are assigned to CAP stations, the CAP efficiency is $f_{A/CAP}$.

For example, assume that:

Prob miss = .2, Prob no revector = .4, $f_{CAP} = .2$, and $f_{A/CAP} = .5$

Therefore:

$$\eta_{assign} = 1 - (.2)(.4) = .92$$

$$\eta_{CAP} = (.2)(.5) + (1 - .2) = .9$$

Now these AI capabilities will be combined into a model to determine the limit on the number of AIs expected to be assigned against the penetrating force.

AI Limits

A limit on the number of AIs expected to be assigned will be modeled as:

$$\text{AI Limit} = (\text{AIs Ready})(\text{Multiplier})(\eta_{assign})(\eta_{CAP})$$

Where:

$$\text{AIs Ready} = (\text{PAA})(f_{ready})$$

$$\text{Multiplier} = (\text{A/sortie})(f_{sd})\{(f_M)(\text{cycles-MOB}) + (f_A)(\text{cycles-AB})\}$$

$$\eta_{assign} = 1 - (\text{Prob of miss})(\text{Prob no revector})$$

$$\eta_{CAP} = (f_{CAP})(f_{A/CAP}) + (1 - f_{CAP})$$

Using the example values and no defense suppression against AI bases:

$$\text{AI Limit} = (1200)(3)(.92)(.9) = 2980 \text{ AIs}$$

With AI base suppression:

$$\text{AI Limit} = (1200)(.9)(.92)(.9) = 894 \text{ AIs}$$

Now a simple survival model will be developed based on these defense limited maximums and the required number of AIs.

Survival Model

The required number of AIs (AIs Reqd) and the limited number of AIs that the defense can be expected to provide for all assignments (Max AIs) will now be compared. If the defense can provide all the assignments needed until the penetrators either are killed or escape coverage (i.e. AI Limit > AIs Reqd), the average probability of penetrator survival ($\bar{P}s$) is:

$$\bar{P}s = (1 - Pk/A)^{(Max\ A/p)}$$

For example, for the low penetrator case 1300 AIs were required. Without any attacks on AI bases or penetrator lethal self defense the defense can provide 2980 AIs. Thus, the defense has sufficient resources to provide 2 AIs for assignment every 10 minutes, if the penetrator survives each assignment with probability (1 - Pk/A) = .9.

The penetrator $\bar{P}s$ for this worse offense/best defense case (with Pk/A = .1 and Max A/p = 10) is:

$$\bar{P}s = (1 - .1)^{10} = \underline{.35}$$

This is the survival to the end of the coverage period (100 minutes in this example). Note that the survival to any lesser time (e.g. a penetrator weapon release time) can be determined by reducing the maximum number of assignments on a penetrator based on the reduced time under area coverage.

If the defense can not provide the number of AIs needed to fill the required assignments, the penetrator's probability of survival will increase. The general equation for Ps then becomes:

$$\bar{P}s = (1 - Pk/A)^{(f_{assign})(Max\ A/p)}$$

$$\text{where } f_{assign} = \min \{ \text{(AI Limit)/(AIs Reqd), or } 1 \}$$

When the exponent, $(f_{assign})(Max\ A/p)$, is less than 1, the Ps equation integer solution becomes:

$$\bar{P}s = 1 - (Pk/A)(f_{assign})(Max\ A/p)$$

Note that the evaluation of the fractional assignments (f_{assign}) in the model is based on an average number of AIs per assignment, an average interval between assignments and only one penetrator and AI type. It has been assumed that the AIs can service the penetrators all along their route. If AI basing or combat radius precludes some of these assignments a further reduction in f_{assign} is required. (Different penetrator and AI types will be considered later in this chapter.)

If lethal self defense is employed against missiles launched from an AI, the value of Pk/A must be reduced, to account for those missiles which are successfully intercepted by penetrator weapons.

A summary of the example results for Reqd AIs and Max AIs shows:

AIs Reqd: Low AIs ready = 1300 High AIs ready = 3900
AI Limit: With base suppression = 894 No base suppression = 2980

The penetrator's expected survival for the best offense/worse defense case (high offense readiness with base suppression) is:

$$f_{assign} = \min \{ (894/3900) \text{ or } 1 \} = .23$$

$$\bar{P}s = (1 - .1)^{(.23)(10)} = .9^{2.3} = \underline{.79}$$

(Note that .3 have 3 asignments and .7 have 2 assignments.)

The table below illustrates the four possible results in the example:

Pen	Base attack?	AIs Reqd	AI Limit	f_{assign}	$\bar{P}s$
100	No	1300	2980	1.0	.35
300	No	3900	2980	.76	.45
100	Yes	1300	894	.69	.48
300	Yes	3900	894	.23	.79

Note that either high numbers of penetrators (300 compared to 100) or AI base suppression has a significant impact on penetrator survival. With both high numbers and AI base suppression, there is a dramatic increase in penetrator survival (up to .79 in the example).

The results apply equally to small or large numbers of penetrators as long as the ratio of AIs to penetrators stays the same. For example, the results for 10 penetrators and 200 AIs would be the same as for 100 penetrators and 2000 AIs in this model, so far. The ratio of the number of air vehicles entering coverage to the number of AIs was found to be a dominant factor. However, a large mass of penetrators can degrade assignment efficiency. The defense may then try to discriminate high value penetrators (such as heavy bombers with many weapons) from lower value penetrators (such as fighter bombers or cruise missiles) in order to concentrate their forces. These two factors (mass and discrimination) are considered next.

Discrimination and Mass

Assume that the defense system can discriminate against low value penetrators with probability $P_{discrim}$ and that those penetrators which are perceived to be of lower value will receive no assignments. The required number of AIs against low value penetrators includes only those incorrectly designated as being high value. The AIs required against these low value intruders will be called ΔAIs Reqd and is:

$$\Delta\text{AIs Reqd} = (\text{Lower Value P})(\text{Exp A/p})(\text{AIs/A})(1 - P_{discrim})$$

The Ps equation must be modified, when discrimination is considered to:

$$Ps = (1 - Pk/A)^{(f_{assign})(\text{Max A/p})(1 - P_{discrim})}$$

When the exponent is less than 1, the integer solution is:

$$\overline{Ps} = 1 - (Pk/A)(f_{assign})(Max\ A/p)(1 - P_{discrim})$$

Massing of penetrators (e.g. at entry points or in corridors) can degrade the defense by delaying or denying assignments (due to confusion and to dilution/saturation of defense resources). Assume that exercise data and analyses allow a factor, f_{mass}, to be estimated for a particular campaign and defense capability to account for the remaining assignment effectiveness with mass degrades. (The derivation of mass effects is a separate subject beyond the scope of this text.) If a mass factor can be assessed, the degraded AI Limit is:

$$AI\ Limit^* = (f_{mass})(AI\ Limit)$$

A mass degrade and a mix of low and high value penetrators will be illustrated by an example: assume that both the mass factor and probability of correct discrimination (vs lower value penetrators) are .8, and that 1000 (or 3000) low value air vehicles are added (for the low and high readiness conditions, respectively). If the Pk per assignment against low value penetrators is .2, their time in coverage is 50 minutes and the earlier assumptions hold for all other factors, the AIs Reqd, AI Limit, f_{assign} and Ps for both the high and low value penetrators are developed as:

1. Added AIs required against low value penetrators which were incorrectly designated as high value penetrators:

$$\Delta AIs\ Reqd = (P)(Exp\ A/p)(AIs/A)(1 - P_{discrim})$$

With 50 minutes under coverage and 10 minutes between completed assignments, the Max A/p = 5, and Exp A/p = 3.4 (with a Pk/A of .2):

$$(1000\ Pen')\quad \Delta AIs\ Reqd = (1000)(3.4)(2)(1 - .8) = 1360\ AIs$$

$$(3000 \text{ Pen}') \quad \Delta\text{AIs Reqd} = (3000)(3.4)(2)(1 - .8) = 4080 \text{ AIs}$$

2. The total number of AIs required is the sum of the AIs Reqd against all the correctly designated high value penetrators plus the above ΔAIs Reqd. The assumption is that there are no errors in classifying high value penetrators, so that $P_{discrim} = 1$ for high value penetrators. Thus, 1300 (and 3980) AIs are still required for these low (and high) number of penetrators, respectively. The total AI requirements are:

$$(100 \text{ Pen}) \quad \text{Total AIs Reqd} = 1300 + 1360 = 2660 \text{ AIs}$$

$$(300 \text{ Pen}) \quad \text{Total AIs Reqd} = 3900 + 4080 = 7980 \text{ AIs}$$

3. The mass degraded AI Limit is:

$$\text{AI Limit}^* = (f_{mass})(\text{Max AIs})$$

$$(\text{With suppression}) \quad \text{AI Limit}^* = (.8)(894) = 715 \text{ AIs}$$

$$(\text{No suppression}) \quad \text{AI Limit}^* = (.8)(2980) = 2380 \text{ AIs}$$

4. The f_{assign} values against low and high value penetrators are determined below for one case - with the best defense (2380 AI Limit) and worse offense (2660 AIs Reqd)

$$f_{assign} = \min \{ (\text{AI Limit}^*)(1 - P_{discrim})/(\text{AIs Reqd}), \text{ or } (1 - P_{discrim}) \}$$

For the low value penetrators, $P_{discrim} = .8$ and:

$$f_{assign} = \min \{ (2380)(1 - .8)/(2660), \text{ or } (1 - .8) \} = .18$$

For the high value penetrators, $P_{discrim} = 1$ and:

$$f_{assign} = \min \{ (2380)/(2660), \text{ or } 1 \} = .89$$

5. The $\bar{P}s$ is:

$$\bar{P}s = (1 - Pk/A)^{(f_{assign})(Max\ A/p)}$$

When the exponent is < 1, the integer solution is:

$$\bar{P}s = 1 - (Pk/A)(f_{assign})(Max\ A/p)$$

For the low value penetrators in the best defense/worse offense case, the exponent (f_{assign} x Max A/p) is less than 1 (i.e. .18 x 5 = .9) so that:

$$\bar{P}s = \{ 1 - .2 \times .18 \times 5 \} = \underline{.82}$$

For the high value penetrators, the exponent is .89 x 10 = 8.9. (.9 have 9 assignments and .1 have 8 assignments.)

$$\bar{P}s = (1 - .1)^{(.89)(10)} = \underline{.39}$$

A complete table of survival values for the four possible cases in the example is shown below:

				Higher Value Penetrators		Lower Value Penetrators	
Pen	Suppress?	AIs Reqd	AI Limit	f_{assign}	$\bar{P}s$	f_{assign}	$\bar{P}s$
100	No	2660	2380	.89	.39	.18	.82
300	No	7980	2380	.30	.73	.06	.94
100	Yes	2660	715	.27	.75	.05	.95
300	Yes	7980	715	.09	.91	.02	.98

This again illustrates the value of defense suppression and mass (large numbers of penetrators - high readiness) in raising the probability of survival.

Multiple Penetrator Types

A case of two penetrator types (with discrimination against the lesser value penetrator) has been illustrated. Any number of penetrator types can be handled by summing the AIs required against each, as shown below:

$$\text{AIs Reqd} = (\text{AIs Reqd})_1 + (\text{AIs Reqd})_2 \text{ ----- } (\text{AIs Reqd})_J = \sum_{j=1}^{J} (\text{AIs Reqd})_j$$

This equation can be used to find the required number of airborne interceptors against multiple types of penetrators for one type of AI .

Whenever interceptors can not be aggregated into a single type a different approach is needed.

Multiple AI Types

With multiple AI types but only one penetrator type, the model can be modified by solving for the probability of surviving attacks by all AI types as the multiplication of the survival probabilities against each AI type:

$$\text{Ps vs all AI types} = Ps_1\ Ps_2 \text{ --- } Ps_K = \prod_{k=1}^{K} Ps_k$$

With multiple AI types and multiple penetrator types, the above equation can be used if each penetrator receives the same distribution of AI types during its penetration. For example, there may be four types of AIs - categorized as either look down or non-look down and long range or short range, and four penetrator types: categorized as heavy bombers, medium bombers, fighter-bombers and cruise missiles. As long as each of the four AI types have the same probability of being assigned against each of the four penetrator types, the above equation can be used to estimate the Ps of each penetrator type, independently.

This equal distribution of AIs may not hold if cruise missile carriers are attacked prior to cruise missile launch, or if the defense attempts to preferentially attack one penetrator type. The case of early cruise missile carrier attack can be analyzed by breaking up the penetration route into time periods - before, during and after all cruise missiles are launched. Then the AI activity during each time period can be evaluated separately, with the Ps for the total period found as the multiplication of the Ps values for each successive period. The model could be further complicated by analyzing, for example, different penetrator entry times and different distributions of AI types with time. But the model would be far more complex, and may lose the purpose of the simple model - to help understand penetration cause and effect relationships and dominant factors.

If certain AIs are preferentially assigned against certain penetrators, for example look down AIs against heavy bombers, the defense doctrine for these assignments must be known, or assumed. But success in discrimination can be an important factor in assessing the allocation mix of AIs, as illustrated next.

One possible preferential allocation is: look down AIs against penetrators perceived to be high value bombers, and non LD AIs against penetrators perceived to be low value cruise missiles. A simple table showing the probability of correctly or incorrectly making these designations is shown below:

Penetrator type	Prob no error	Prob of error
Higher Value	$P_{discrim-H}$	$1 - P_{discrim-H}$
Lower Value	$P_{discrim-L}$	$1 - P_{discrim-L}$

where $P_{discrim-H}$ = the probability of correctly discriminating a high value penetrator

$P_{discrim-L}$ = the probability of correctly discriminating a low value penetrator

If a high value penetrator is incorrectly categorized it will not receive a look down AI assignment. If a low value penetrator is incorrectly categorized it will receive a LD assignment, if available.

For example, if 1000 low value and 100 high value penetrators enter, and if the probability of correct assignment is .8 for both vehicles, the number of correct and incorrect designations is illustrated below:

Penetrators	Correct	Incorrect
100 Higher value	$\boxed{80}$	20
1000 Lower Value	800	$\boxed{200}$

With sufficient LD resources, the initial allocation will be 80 against bombers and 200 against cruise missiles. Thus, over 70% of the look down force will be initially misapplied. Even with a 90% probability of successful discrimination, over 50% will be misapplied. This example illustrates the defense problem 1) in discriminating against large numbers of low value penetrators which enter defense coverage and 2) in implementing a preferential assignment doctrine.

The simple model presented can be used with a preferential assignment doctrine to analyze errors in assignment and their consequences.

Just as AIs can be used preferentially against penetrators, penetrator lethal self defense can be employed preferentially by the penetrators against only certain AI types (such as look down AIs). Appropriate reductions must then be made in multiple use of an AI (if self defense is used against these AIs) and in Pk/A (if self defense is used against missiles launched by these AIs), for the particular AI type affected.

Summary

This chapter developed a simple model which can be used to evaluate many penetrators against many AIs. The model compares the number of AIs required for assignment against a penetrator force with the resource limited number of AIs that can be expected to be assigned against that force. The equations used (for each AI type) are:

$$\text{AIs Reqd} = (P)(\text{Exp } A/p)(\text{AIs}/A)(1 - P_{discrim})$$

$$\text{AI Limit*} = (f_{mass})(\text{AIs Ready})(\text{Multiplier})(\eta_{miss})(\eta_{CAP})$$

where:

$$\text{AIs Ready} = (f_{ready})(\text{PAA})$$

$$\text{Multiplier} = (A/\text{sortie})(f_{sd})\{(f_M)(\text{cycles-MOB}) + (f_A)(\text{cycles-non MOB})\}$$

$$\eta_{assign} = 1 - (\text{Prob of miss})(\text{Prob no revector})$$

$$\eta_{CAP} = (f_{CAP})(f_{A/CAP}) + (1 - f_{CAP})$$

The penetration mission probability of survival (Ps) is:

$$Ps = (1 - Pk/A)^{(f_{assign})(\text{Max } A/p)}$$

When the exponent is less than 1, the integer solution is:

$$Ps = \{\ 1 - (Pk/A)(f_{assign})(\text{Max } A/p)\ \}$$

The value of f_{assign} can be determined from:

$$f_{assign} = \min\ \{(\text{AI Limit*})(1 - P_{discrim})/(\text{AIs Reqd}),\ \text{or}\ (1 - P_{discrim})\}$$

The probability of surviving K different AI types is:

$$\bar{P}s \text{ all AI types} = \prod_{k=1}^{K} \bar{P}s_k$$

Preferential allocation of high quality AIs may be significantly degraded by discrimination errors. Preferential allocation of penetrator lethal self defense weapons will affect the Pk/A and the recycle of those AIs attacked.

The value of defense suppression against interceptor bases and of large numbers of penetrators (mass degrades) was illustrated. With extensive test/exercise data to validate analyses, this model can be a powerful tool in evaluating air vehicle penetration and in conducting sensitivity studies of the factors which dominate penetrator survival.

Chapter 18

SAM One-on-One Pk

Air vehicle penetrators usually attempt to avoid en route encounters with surface-to-air missile (SAM) threats. This chapter starts with a discussion of avoidance tactics which might be needed when SAM locations are uncertain. SAM threat fundamentals and SAM actions are described and a simple model developed to examine the effects of penetrator coverage time and SAM capability/degrades.

Three penetration cases are considered: 1) a direct attack on a SAM site, 2) a random encounter and 3) careful route planning (depending upon masking and clutter degrades to pass safely by a SAM site). The chapter concludes with a brief discussion of penetrator warning system reliability, the probability of degrading a SAM and the variables which affect SAM probability of kill.

Avoidance

If the location of all SAM sites are known before a penetration mission <u>and</u> SAMs can not be moved, mission planners may be able to route penetrators around these known locations and so avoid all SAM encounters en route to targets. (A SAM <u>encounter</u> occurs when a SAM launches at least one missile at a penetrator within its coverage.) If SAM locations are uncertain or SAMs move, avoidance is more difficult to achieve. The table on the next page lists some sequential requirements for missions attempting to avoid SAMs:

Plan to Avoid	Initially, SAM Locations are: Known	Uncertain
Fixed SAMs	Careful routing	Reconnaissance, Careful rerouting
Moved SAMs	Predict moves, Careful rerouting, Onboard warning	Reconnaissance, Predict moves, Careful rerouting Onboard warning

The offensive route planner can minimize the effects of SAM location uncertainty and SAM movements, if the following information is available (and there is sufficient replanning time and route flexibility to take advantage of this information):

1. Data on SAM deployment restrictions (e.g. land with slopes greater than a specified angle, swamps, lakes, dense forests)

2. Data on land cover and transportation routes, which can be used to predict likely SAM relocation areas. For example, SAMs which are relocated are more likely to move to areas which have few trees and which are near roads or railroads, than to very heavily forested areas which are far from any roads or railroads.

3. Data on terrain and cultural obstructions which allow penetrators to take advantage of masking, multipath and/or clutter. For example, a penetrator may be able to use terrain masking to minimize exposure to SAMs in certain areas, or to fly down the side of a mountain ridge (being masked from SAMs on the other side of the ridge, and providing a heavy clutter background to SAMs that can see the penetrator).

4. Data on areas damaged by prior strikes. For example, penetrators can be routed over these areas before the defense can recover.

5. Recent information on the locations of mobile SAMs, and a general understanding of movement doctrine. For example, SAMs may accompany a field army on the move. If the location of SAM elements of this army can be determined hours before penetration, predictions of probable army movement patterns might enable the planner to route around the most likely locations of these mobile SAMs.

Despite these avoidance tactics, SAMs may still be encountered by penetrators en route to their targets, as well as in the target area. In order to understand SAM attrition, the fundamentals of the SAM threat will now be examined.

SAM Threat Fundamentals

The fundamental offensive and defensive factors which affect penetrator attrition are listed below, and categorized by SAM coverage, SAM capability (and degrades) or suppression:

Factor	Offense	Defense
SAM Coverage	Altitude, Offset Velocity, RCS Onboard warning and avoidance	Antenna height, Clutter Subclutter visibility Masking, Multipath Sensitivity, Passive
SAM Capability (and Degrades)	Surprise Support EW Onboard EW Turns, Mass	Netting, Alert status Reaction time, ECCM Dead zone, Reliability Missile SSPK
Suppression	Prior damage Current attack	Overlapping coverage SAM relocation

Many of these factors were discussed in Chapter 9. The effects of offset distance and antenna height will be examined here. Coverage distance varies with route offset to a site as illustrated in Figure 18.1 for a coverage radius R. The chord of a circle is $2\ (R^2 - d^2)^{1/2}$, where d is the penetrator's offset distance from the site. (The offset distance is the closest distance to the SAM site from the penetrator's route.)

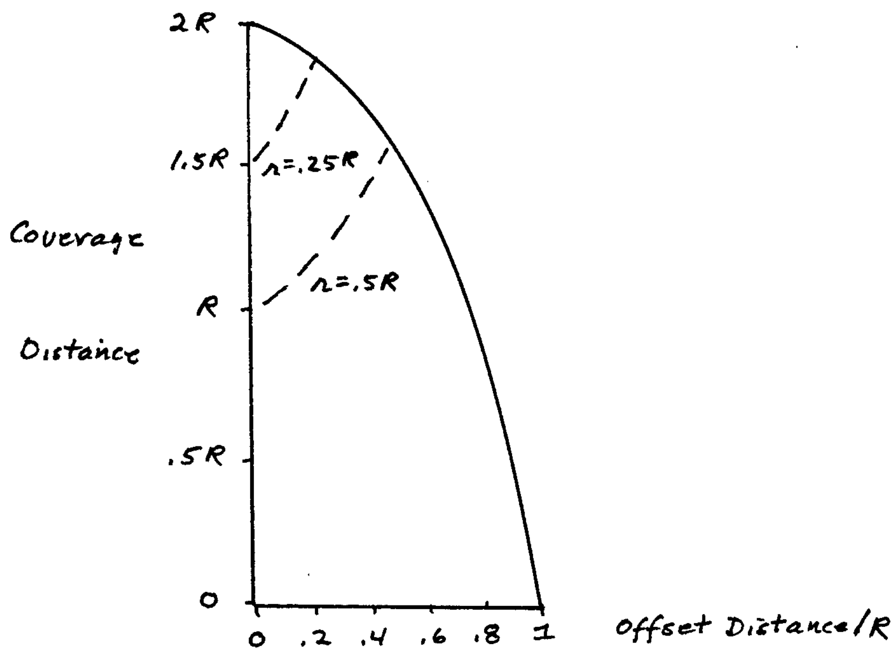


Figure 18.1 Coverage Distance vs Offset Distance

Note that the coverage distance remains quite high until the offset approaches R. If a penetrator can not be tracked within a circular dead zone of radius r, the coverage distance will decrease as shown by the dotted lines in Figure 18.1. Dead zones with radius .25 R (and .5 R) are illustrated. Cusps indicate where penetration routes are tangent to the dead zones.

The effect of antenna height on the maximum radar line of sight (LOS) is shown in Figure 18.2. Radar LOS limits were computed for a smooth 4/3rds earth radius propagation as:

$$\text{Coverage radius} = 1.23\ (h_a^{1/2} + h_p^{1/2}) \quad \text{(in NM)}$$

where h_a = antenna height, and h_p = penetrator altitude (in feet)

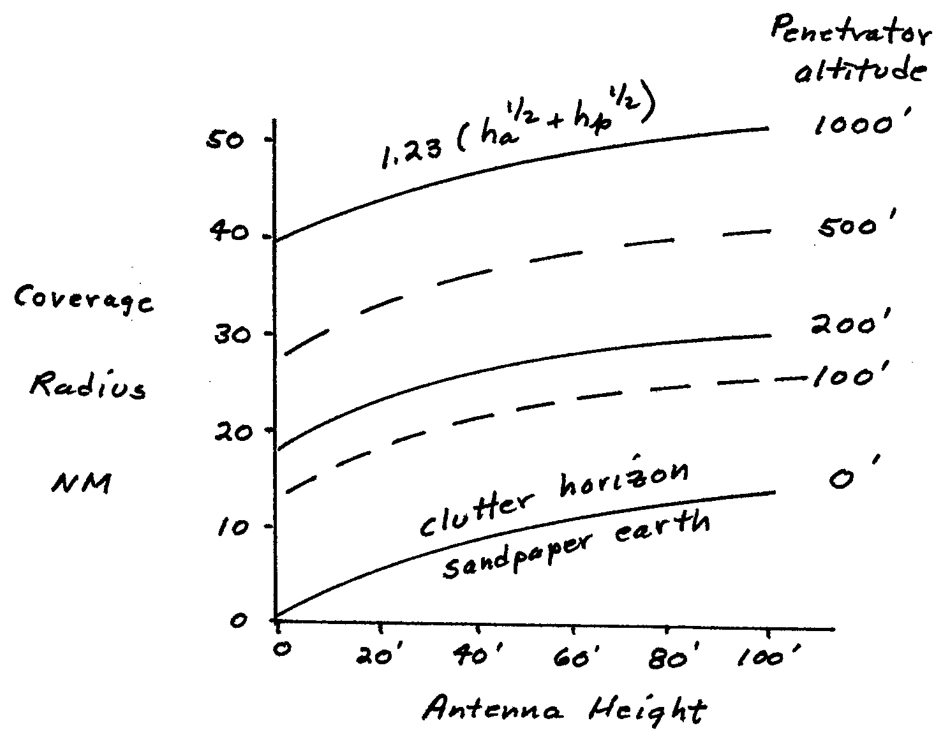


Figure 18.2 Effect of SAM Antenna Height

Both the radar LOS and the clutter horizon increase as the antenna height is raised. Raising the antenna height will increase total coverage distance and may be desired for early detection with SAM acquisition radars. However, raised antennas may be detrimental to radar tracking in clutter at ranges close to a SAM site where intercepts are more likely to occur. Thus, SAM systems which are limited by clutter may have higher acquisition radar antennas and lower tracking antenna heights.

Now consider SAM actions in attempting to down a penetrator.

SAM Actions

The sequence of SAM (and SAM C^3 network) actions against a penetrator was discussed in Chapter 9. A summary is shown on the next page.

<u>Network Actions</u>
Early warning or acquisition
Identification
Net hand-off to Fire Control System (FCS)

<u>SAM Actions</u>
FCS Detection
FCS Tracking
Missiles launched in a salvo
$\left.\begin{array}{l}\text{Missile guidance}\\ \text{Missile fuze}\\ \text{Missile lethality}\end{array}\right\}$ SSPK = Single Shot Probability of Kill
Later missile salvos

If a SAM is not netted to early warning/acquisition sensors, it is said to be operating autonomously. Then it must perform its own acquisition which usually takes more time and has a lower probability of success than net acquisition.

A penetrator's survival against one SAM will be assessed by determining the number of SAM launches (along a penetrator's track), and then compounding missile probability of kill for these launches. Salvo Pk will in general not be the same, but will vary with offset and intercept distance. Figure 18.3 illustrates the definitions used for each SAM reaction time (Δt) and missile time of flight (TOF) for a SAM shoot-look-shoot tactic (which requires that the results of a missile salvo be assessed before another salvo is launched).

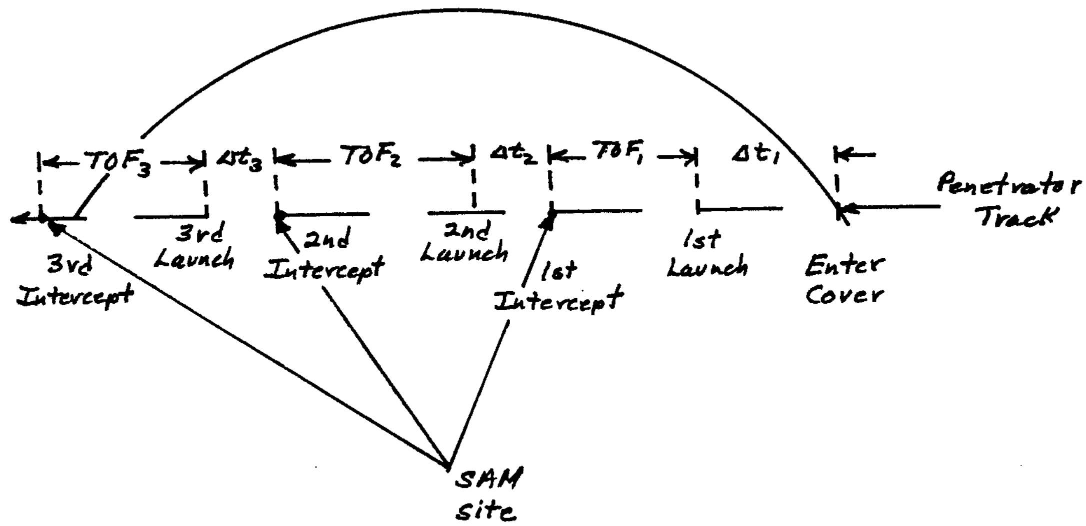


Figure 18.3 SAM Launch Opportunities

The definitions are:

Δt_1 = reaction time from penetrator entry into cover until the first launch opportunity

Δt_2 = reaction time between the first intercept and the second launch opportunity. Later reaction times are indicated by Δt_3, Δt_4, etc.

TOF_1 = missile time of flight from first launch to intercept. Later TOFs are indicated by TOF_2, TOF_3, etc.

Entry into maximum radar line of sight coverage depends upon the penetrator altitude, radar masking and SAM antenna height. Δt_1 is determined by SAM reaction time to detect and track the penetrator and to initiate launch of the first missile salvo. Initial detection range depends upon SAM netting vs autonomous operation, radar sensitivity, multipath, clutter, subclutter visibility and penetrator RCS and ECM.

Later Δt values depend upon the time to assess penetrator kill and to launch again, in a shoot-look-shoot (S-L-S) doctrine. The TOF values depend upon the distance between the SAM site and the penetrator at launch and the penetrator and missile velocity vectors.

Next a simple model will be developed to consider these factors in assessing SAM attrition.

Simple One-on-One Pk Model

The attrition against one penetrator by one SAM (firing N salvos) can be modeled for <u>netted</u> SAM operation by the following equation for the expected probability of kill (Pk):

$$Pk = P_{acq}\, P_{hand\text{-}off}\, P_{track}\, \{ 1 - (1 - P_{launch}\, PK)^{N} \}$$

For <u>autonomous</u> operation, the model is:

$$Pk = P_{acq\ A}\, P_{track\ A}\, \{ 1 - (1 - P_{launch}\, PK)^{N} \}$$

where P_{acq} = Probability of net acquisition of the penetrator

$P_{hand\text{-}off}$ = Probability of successful net hand-off to the SAM, given net acquisition

P_{track} = Probability of SAM FCS track, given hand-off

P_{launch} = Probability of successful missile salvo launch, given SAM FCS tracking

PK = Probability of missile salvo kill, given launch

N = Maximum number of salvos

$P_{acq\ A}$ = Probability of autonomous acquisition of the penetrator

$P_{track\ A}$ = Probability of SAM FCS track, given autonomous acquisition

Note that all probabilities will be evaluated at their average (or expected) values.

The maximum number (N) of missile salvos is determined by the effective time under SAM coverage (t_{eff}) and the time to complete each intercept (Δt + TOF). Assume that the results of each launch are assessed for possible kill before the next launch is ordered (a shoot-look-shoot doctrine). N can be found from:

$$t_{eff} > \sum_{i=1}^{N} (\Delta t_i + TOF_i)$$

Note that intercept zone limits may increase Δt_1 and truncate TOF_N.

The expected salvo probability of kill (with n missiles in each salvo) is determined by the relative dependence of each missile in a salvo. Assuming the square root dependence of Chapter 16,

$$\bar{P}K = SSPK^{1/2} \{ 1 - (1 - SSPK^{1/2})^n \}$$

where SSPK = Average Single Shot Probability of a missile Killing the penetrator, given launch

The single shot probability of kill includes missile guidance, fuze and lethality against the penetrator. (Missile miss distances are designed to be smaller than the lethal radius so that the undegraded SSPK is a function of guidance and fuzing success.)

Three examples will be examined:

1) A direct attack against a SAM site,
2) A random encounter, and
3) Masking and clutter degrades to an actual site.

Direct Attack

For a direct attack on a SAM site, the time from enter cover to the SAM's dead zone will be called the effective time in cover (t_{eff}):

$$t_{eff} = (\text{Enter cover - Dead zone})/\text{Penetrator ground speed}$$

The <u>maximum</u> coverage range is the minimum of the radar line of sight and the sensitivity limit ($K\ \sigma^{1/4}$). This maximum range can be degraded by masking, multipath, clutter, ECM and other offense actions.

For example, with a 200' penetrator, a 10' SAM radar antenna height and 4/3rds earth radius propagation a penetrator will enter maximum radar line of sight coverage at 21 NM. If this LOS limit is less than the sensitivity limited detection range and if there are no degrades, the time in cover for a 600 knot penetrator is 126 seconds until the site is overflown. With a 5 NM dead zone, the undegraded time in cover is 126 - 5/(600/3600) = 96 seconds.

Assume that the undegraded, initial SAM reaction time (Δt_1) is 25 seconds for netted operation, and 60 sec for autonomous operation. Equations to determine missile time of flight were developed in Chapter 13. The equation for the ith salvo TOF for a head-on launch is:

$$TOF_i = L_i/(V_p + V_M)$$

where L_i = SAM launch range for the ith salvo
V_p = Penetrator ground velocity
V_M = Average missile velocity

With a 1000 knot average missile velocity, the flight time for the first launch (if TOF_1 is not missile limited) in the netted SAM example is found after determining the first launch range (L_1).

$$L_1 = R_{entry} - V_p\ \Delta t_1 = 21 - (600/3600)25 = 17\ \text{NM}$$

$$TOF_1 = 17/(600 + 1000)/3600 = 38\ \text{seconds}$$

$$\Delta t_1 + TOF_1 = 25 + 38 = 63\ \text{seconds}$$

The maximum number of missile salvos (or intercepts) can be found by enumerating the times for subsequent shoot-look-shoot launches within the total coverage time. Assuming a 10 second assessment time (Δt) after the first intercept, the launch distance and time required for the second salvo in the example (if TOF_2 is not missile limited) is:

$$L_2 = L_1 - V_p\ (\Delta t_2 + TOF_1) = 17 - (600/3600)(10 + 38) = 9\ \text{NM}$$

$$TOF_2 = 9/(600 + 1000)/\ 3600 = 20\ \text{seconds}$$

The second launch and intercept used 10 + 20 = 30 seconds. Combining this time with the 63 seconds for the first intercept shows that 93 seconds have elapsed. A third launch requires an additional 10 seconds, for a total time of 103 seconds. Since the total coverage time is 96 seconds, a third launch is not possible while the penetrator is within coverage. Thus, the maximum number of shoot-look-shoot (S-L-S) launches is two in the direct attack example for a netted SAM.

The maximum number of launches can also be calculated by the t_{eff} inequality (with no intercept zone limits):

$$t_{eff} = 96 > (25 + 38) + (10 + 20) + (10 + TOF_3) + \text{------}$$

$$> \underbrace{63 + 30 + (10}_{103\ \text{sec} \rightarrow \text{outside cover for 3rd launch}} + TOF_3) + \text{-----}$$

With an average SSPK of .25, the compounded kill from a salvo of two missiles is:

$$\bar{P}K = .25^{1/2} \{ 1 - (1 - .25^{1/2})^2 \} = .38$$

With a separate .75 average probability for SAM track and for launch actions, and a net acquisition and hand-off average probability of .9 each, the undegraded Pk for two missile salvos against the penetrator attacking the SAM site is:

$$\bar{P}k = (.9)(.9)(.75) \{ 1 - (1 - .75 \times .38)^2 \} = \underline{.30} \quad \text{netted}$$

Assume for autonomous operation that the average acquisition probability is reduced to .5 and the initial reaction time (Δt_1) is raised to 60 seconds, with other values unchanged. The first launch range and missile time of flight are:

$$L_1 = 21 - (600/3600)60 = 11 \text{ NM}$$

$$TOF_1 = 11/(600 + 1000)/3600 = 25 \text{ seconds}$$

The first launch is completed in 60 + 25 = 85 seconds. Thus:

$$t_{eff} = 96 \geq \underbrace{(60 + 25) + (10}_{95 \text{ sec}} + TOF_2) + \text{-----}$$

Since a second salvo certainly needs more than a 1 second time of flight, only one salvo is possible with the S-L-S doctrine. Therefore, the autonomous Pk is:

$$\bar{P}k = (.5)(.75) \{ 1 - (1 - .75 \times .38)^1 \} = \underline{.11} \quad \text{autonomous}$$

In the example, autonomous operation reduced the expected Pk over 63% (from .30 to .11) compared to the netted example. The decreased acquisition probability caused a 38% reduction. The remainder is due to the reduction from two salvos to one salvo.

Now consider degrades to SAM operations. Degrades against four defense probabilities (acquisition, track, launch and SSPK) and against the reaction times (Δts), are shown below:

Action	Degrade Possible by
P_{acq}	ECM, RCS, Turns, Suppress, Surprise, Masking
P_{track}	
P_{launch}	Decoys, Self defense
SSPK	
Δt	Speed, Altitude

In order to test the sensitivity of possible degrades, assume that a 50% reduction is possible in each of the four probabilities and a 30 second delay is possible in all Δts.

The table below lists the undegraded and degraded values and the degraded Pks, for a netted SAM for the example:

Action	Undegraded Value	Degraded Value	Degraded Pk
P_{acq}	.75	.375	.15
P_{track}	.75	.375	.15
P_{launch}	.75	.375	.16
SSPK	.25	.125	.17
Δt_1	25 sec	55 sec	.17

The Pk of .17 for a 30 second delay (from a Δt_1 of 25 to 55 seconds) is based on only one S-L-S salvo.

If all of these five degrades were applicable and all operated independently of the others, the Pk would drop to .01 for a direct approach to this SAM in the example.

Now consider the second case - a random encounter.

A Random Encounter

Chapter 10 presented the expected time in cover for a random encounter with a ground site as:

$$\bar{t}_{cover} = (\pi/2V_p) \{ R - (r_c^2 - r_f^2)/R \}$$

For example, assume that a low altitude target allows a SAM a maximum detection range of 15 NM, with no loss of tracking due to clutter. A 600 knot penetrator would have the following time in coverage (if there were no loss of cover in a Doppler notch):

$$\bar{t}_{cover} = \{ \pi/(2)(600)/3600 \} (15) = 141 \text{ seconds}$$

The geometry for the ith launch for this expected time in cover case is shown in Figure 18.4.

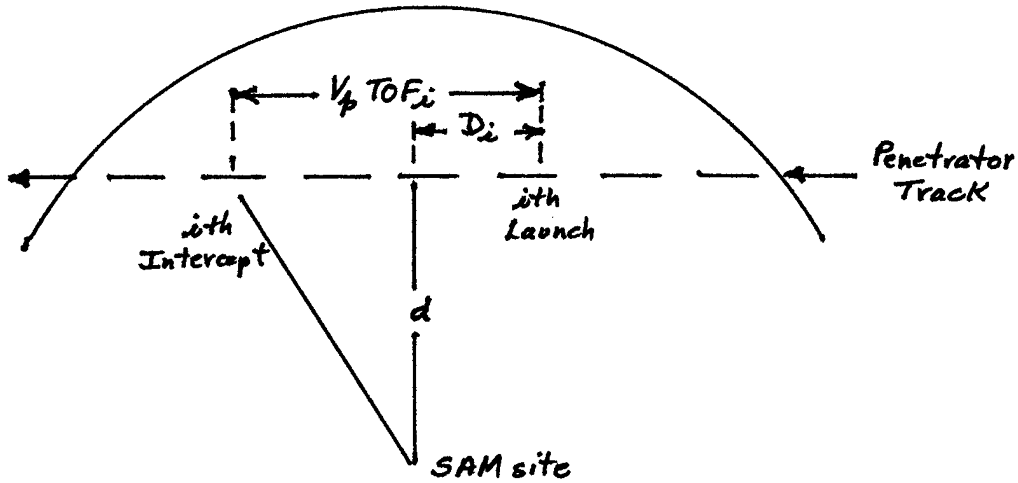


Figure 18.4 Random Encounter - ith Launch

In order to develop a model for intercept at any offset distance, define a point D_i along the penetrator's track where the ith launch occurs. The SAM time of flight can be determined from the following equation based on finding the sides of the right triangle formed by d and (V_p TOF$_i$ - D_i):

$$D_i = V_p\, TOF_i + \{ V_m^2\, TOF_i^2 - d^2 \}^{1/2}$$

where D_i = the distance from the broadside point on the track to the penetrator's position at the ith launch. D_i is positive when the penetrator is approaching the SAM (and negative when receding).

TOF$_i$ = the missile time of flight for the ith launch

d = offset distance to the SAM site

The solution of this quadratic equation for TOF$_i$ is:

$$TOF_i = \{ -b \pm (b^2 - 4ac)^{1/2} \}/2a$$

where $a = V_p^2 - V_m^2$

$b = -2D_i V_p$

$c = D_i^2 + d^2$

The expected chord is $\pi R/2$, or $\pi(15)/2 = 23.6$ NM for the example. The total coverage time is 23.6/(600)/3600 = 141 seconds. The distance from the broadside point at which launch is first possible is:

$$D_1 = (1/2)(\text{Expected chord}) - V_p \Delta t_1$$

With a 25 second initial reaction time:

$$D_1 = (1/2)(23.6) - (600)(25)/3600 = 7.6 \text{ NM}$$

Since the offset distance (for the average chord) is equal to .619 R, the example value of d is 9.3 NM. The first salvo time of flight can be found to be 34 seconds by solving the quadratic equation with $D_1 = 7.6$ NM, $d = 9.3$ NM, $V_p = 600$ knots and $V_m = 1000$ knots. Thus, the 1st intercept is completed in 25 + 34 = 59 seconds.

A second launch can occur 10 seconds later, and requires a 41 sec time of flight. The second launch is completed in 69 + 41 = 110 seconds.

A third S-L-S launch requires an additional 10 seconds. A missile time of flight of 91 seconds would be required to catch the receding penetrator at 23.4 NM beyond the broadside point. But the penetrator will leave cover at 11.8 NM. Thus, in the example a third intercept will not occur within cover. (Maximum missile TOF may limit this intercept.)

The next table summarizes this case (without a Doppler notch):

Event	Distance from Broadside	Time to Next Event	Cumulative Time
Enter Cover	11.8 NM	25 sec	25 sec
1st Launch	7.6	34	59
1st Intercept	1.9	10	69
2nd Launch	0.2	41	110
2nd Intercept	-6.6	10	120
3rd Launch	-8.3	21	141
Leave Cover	-11.8	69	211
3rd Intercept	-23.4		

Thus, only two S-L-S intercepts are possible within coverage. The geometry of this example is shown in Figure 18.5.

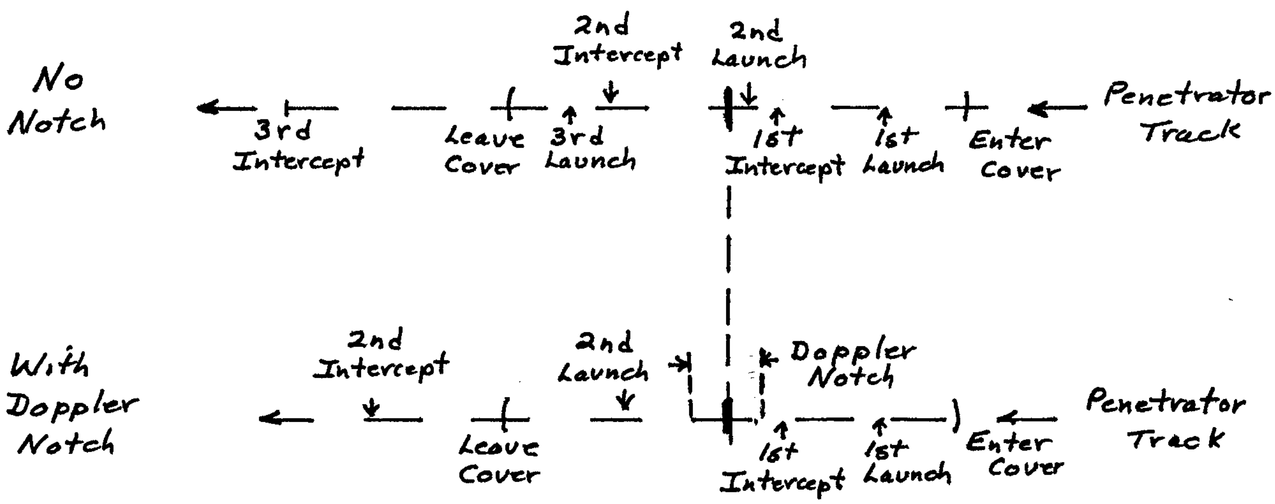


Figure 18.5 SAM Intercept Geometry

With a 100 knot Doppler notch, the results for the 600 knot penetrator example are:

Event	Distance from Broadside	Time to Next Event	Cumulative Time
Enter Cover	11.8 NM	25 sec	25 sec
1st Launch	7.6	34	59
1st Intercept	1.9	2	61
Enter Notch	1.6	19	80
Leave Notch	-1.6	25*	105
2nd Launch	-5.8	36	141
Leave Cover	-11.8	35	177
2nd Intercept	-17.8		

* 25 seconds is required, since tracking was lost in the Doppler notch.

Note that in the example a second intercept will occur outside of cover with a 100 knot Doppler notch. If missile guidance depends on SAM radar site coverage, intercepts out of cover will not damage the penetrator. (Examples of missile guidance which depend upon the SAM radar for intercept are command guided and semi-active guided modes).

This simple approach to a random encounter case allowed either one or two intercepts, depending upon a Doppler notch. The expected Pk depends upon the various defense probabilities and degrades. An undegraded example with netting might show a Pk of .30 for 2 intercepts (with netting) and a Pk of .17 for a single intercept. Degrades will reduce these values markedly, as illustrated in the direct attack case.

Now consider the use of actual masking and clutter around a site to plan a safe route.

Using Masking and Clutter

The third, and last, case involves taking advantage of site specific masking and clutter. Detailed knowledge of the terrain, land cover and clutter data around a particular known SAM location may enable a route planner to reduce exposure time to less than the SAM reaction time. One route is illustrated in Figure 18.6 with a site at the center of the display. Masking depends on the penetrator's altitude and SAM antenna height. Clutter depends on the penetrator's RCS and the SAM antenna height and subclutter visibility.

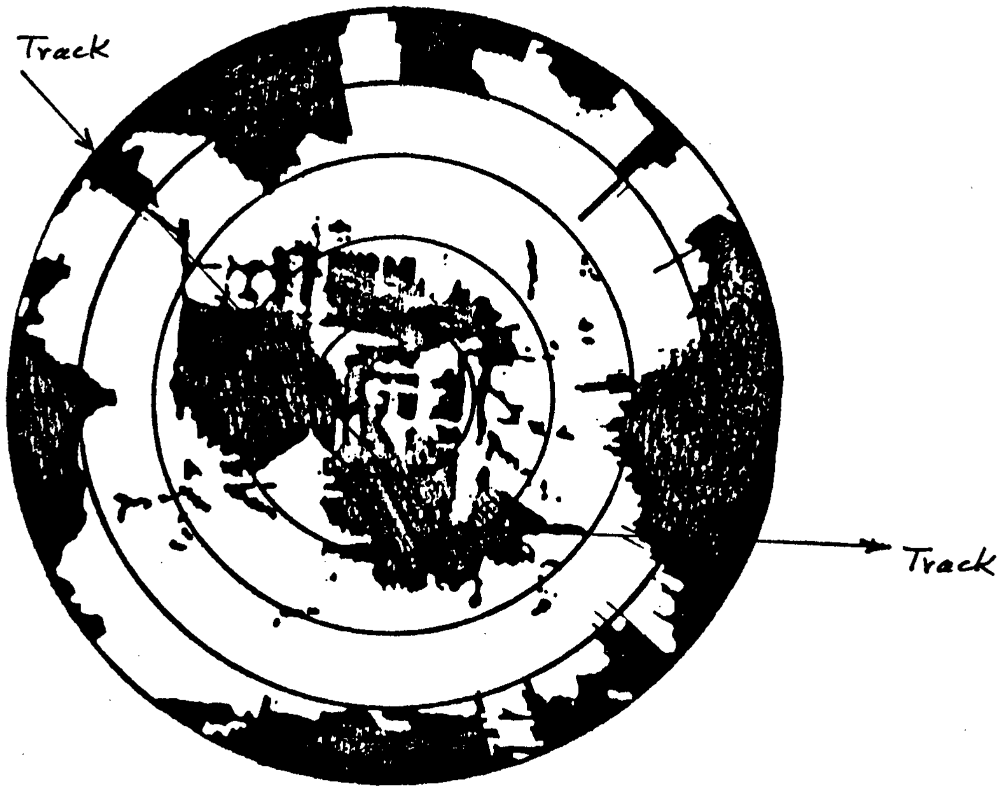


Figure 18.6 Use of Masking and Clutter

Note that the effective time under cover can be reduced markedly by taking advantage of the specific masking and clutter around the site. A turn can degrade any tracking which might occur, so that a SAM launch is unlikely in this case.

This chapter concludes with brief discussions of: 1) onboard warning system reliabilities and degrades, 2) SAM capability charts, and 3) a few added items of interest.

Warning Reliability and Penetration Degrades

If there are I different warning systems operating independently, the probability of <u>at least one</u> providing the proper warning is:

$$\text{Prob}_{\text{warning}} = 1 - (1 - R_1)(1 - R_2)(1 - R_3)----(1 - R_I)$$
$$= 1 - \prod_{i=1}^{I}(1 - R_i)$$

Where R_1, R_2, etc are the respective reliabilities, or probabilities of correctly intercepting and recognizing the threat signal.

If J multiple degrades are possible and each operates independently, the probability of achieving either all J degrades or no degrade is:

$$\text{Prob (all J degrades)} = \prod_{j=1}^{J} P_j$$
$$\text{Prob (no degrade)} = \prod_{j=1}^{J} (1 - P_j)$$

Combining the onboard warning system reliabilities with these degrade probabilities:

$$\text{Prob (warning + all degrades)} = \{ 1 - \prod_{i=1}^{I}(1 - R_i) \} \prod_{j=1}^{J} P_j$$

$$\text{Prob (no degrades)} = \text{P (no warning)} + \text{P (warning + but no degrades)}$$
$$= \prod_{i=1}^{I}(1 - R_i) + \{ 1 - \prod_{i=1}^{I}(1 - R_i) \} \{ \prod_{j=1}^{J}(1 - P_j)$$

These reliabilities and degrade probabilities should be obtained from analysis validated by flight tests.

As an example, assume that there are two independent warning systems and each reliability is .75. Two independent degrades are possible with a .5 probability of achieving each degrade.

$$\text{Prob (both degrades)} = \{ 1 - (1 - .75)^2 \} (.5)^2 = .23$$

$$\text{Prob (no degrades)} = (1 - .75)^2 + \{ 1 - (1 - .75)^2 \} \{ (1 - .5)^2 \} = .30$$

With just one warning system which operates reliably 75% of the time, the probability of no degrade increases almost 50% (from .30 to .44). This illustrates the desirability of multiple, independent onboard warning systems on a penetrator.

SAM Capability Charts

Example SAM capability charts portraying the number of launches and expected Pk against a particular penetrator as a function of offset distance for an undegraded and a degraded case are shown in Figure 18.7.

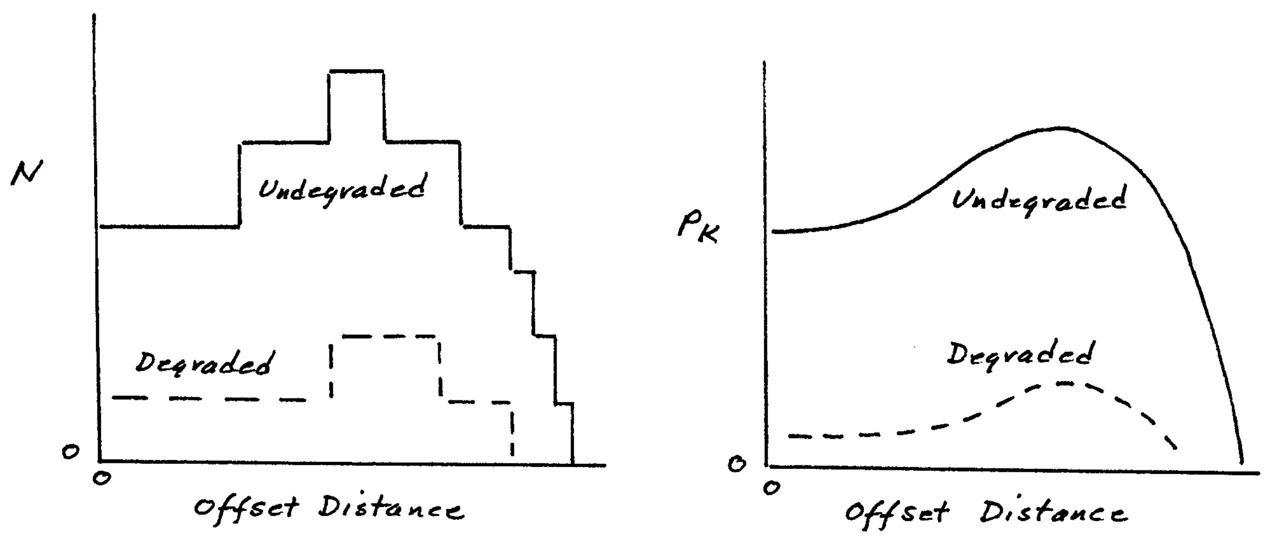


Figure 18.7 SAM Capability Charts

These charts apply to only one:

Penetrator type - altitude, speed and RCS
Site condition - masking, multipath, clutter and SCV
SAM condition - netting, antenna heights, response times, MTI/Doppler processing and notch, ECCM, launch doctrine, reliabilities and SSPKs
Degrade set - Onboard warning capability and reliability, ECM techniques/tactics - their effectiveness, reliability and independence

Many of these parameters affect N. All effect Pk. The dominant factors are usually those that affect N and the SSPK.

Items of Interest

Before ending this chapter, six items of interest will be discussed:

1) Missile guidance modes,
2) Missile fuzing,
3) Approaching vs receding targets,
4) Overlapping coverage,
5) Standoff attack, and
6) Antiaircraft Artillery (AAA).

Missile Guidance Modes

Four basic missile guidance modes - command guided, active guidance, semi-active guidance and passive guidance - are explained below:

Command guidance is provided by uplink signals from the SAM FCS to control the missile's course. The missile may provide a downlink signal to the FCS to assist tracking of the missile, confirm receipt of the last uplink command and provide information (collected by the missile) to the FCS.

Active guidance allows the missile to detect, track and home on energy reflected from the penetrator by the missile's active sensor system.

Semi-active guidance allows the missile to home on the energy reflected from the penetrator by the SAM's FCS illuminator.

Passive guidance allows the missile to home on emissions from the penetrator. For example, infrared emissions are used by IR guided missiles. Electronic emissions are used in a home-on-jam (HOJ) mode by radar guided missiles.

Missile Fuzing

A fuze can be prematurely activated by ground clutter or chaff returns. A high missile approach path can minimize this premature fuzing, and also allow greater missile range.

A very short fuze delay is needed to damage small vehicles (e.g. missiles), while larger delay times increase damage against large vehicles. Fuze delays might be varied to match the perceived size of the target being intercepted, two different fuze settings used (a short delay on one SAM missile and a long delay on the other missile in a salvo of two) or a compromise setting used for all missiles.

Approaching vs Receding Targets

The defense doctrine usually concentrates attacks on approaching penetrators, so that those that survive head-on SAM attacks may be in little jeopardy thereafter. Receding targets require far longer missile intercept times, which reduces the number of launches possible, and thus the threat on the outbound leg for those SAMs which can overcome clutter effects in a tail-on intercept.

Overlapping Coverage

Overlapping coverage between adjacent SAM sites can decrease penetrator survival if more than one SAM site launches against a penetrator. Overlapping coverage can also be used to provide mutual protection between SAMs so that each can defend the other against penetrator attacks. It is often difficult to find a route which underflies, overflies, or masks a penetrator from all closely grouped sites.

Standoff Attack

Standoff weapon launch may allow a penetrator to avoid SAM intercept zones. Penetrator weapon ranges were developed in Chapter 13 for direct attacks against a SAM. This model can also be used to evaluate penetrator avoidance of SAM intercept regions during launch of standoff weapons against targets defended by a SAM site.

Antiaircraft Artillery

Air vehicle attrition due to antiaircraft artillery (AAA) fire depends upon: 1) the number of AAA encounters, 2) the number of hits on the target and 3) the lethality of each round.

The number of encounters depends upon the density of AAA batteries both along the penetrator's route and in target areas and upon the effective firing range and altitude region. A high AAA site density is usually required to insure a high probability of kill. For example, a line density of heavy ($\geq$ 90 mm) AAA guns of about 20 per NM was found to be effective against German V-1 buzz bomb attacks in World War II.

The number of hits by AAA fire depends upon the firing time, the rate of fire, the accuracy of track prediction, and the dispersion pattern. The firing time is a function of the time within gun range, the timeliness of net acquisition, identification, tracking and hand-off, the delays in AAA target detection and track, the maximum and minimum effective firing ranges and the defense firing doctrine. The dispersion pattern includes not only the dispersion about the line of sight, but also the line of sight bias (or angular error) relative to the target. Bias can be a dominant factor in reducing hits on a target. For example if there is a large bias relative to the dispersion all rounds may miss the target.

The lethality of AAA fire should be carefully examined. Results from World War II show that prewar calculations of the number of hits required to down air vehicles were low by a factor of more than 10 (i.e. World War II aircraft were found to be far less vulnerable to AAA hits than expected).

Generally AAA capability increases as firing pattern dispersion decreases, as bias decreases and as fire rates increase, as well as with earlier detection and improved track prediction. AAA sites can pose a threat to aircraft flying within AAA effective altitudes, particularly under good visual conditions (when optical tracking can be employed). AAA fire can be a serious threat to slower speed air vehicles on predictable flight paths (particularly to helicopters during combat landing operations) and to air vehicles attacking AAA sites.

AAA is generally less effective when visibility is restricted, ECM/EOCM is effective, penetrators avoid the AAA's effective altitude region or site density is low. In the past, air vehicles with high subsonic speeds (or greater) on unpredictable flight paths which attempt to minimize exposure to AAA defenses have experienced relatively little attrition from AAA.

Summary

Avoidance of SAMs en route to targets is feasible if all SAM locations are known and no SAMs move after the route is chosen. When locations are uncertain, a reconnaissance mission prior to penetration may remove this uncertainty and allow rerouting of penetrators around the confirmed locations. When SAMs are mobile, recent reconnaissance plus predictions of movement patterns and likely locations (e.g. based on land cover and transportation routes) will reduce encounters. Onboard warning systems can allow penetrators to react to active SAMs ahead.

The fundamentals of attrition from SAMs were discussed, with the effects of offset distances and SAM antenna height emphasized. SAM and network actions in acquiring, tracking and killing a penetrator were described in order to illustrate the probabilities and time delays involved in these actions.

A simple model for one SAM against one penetrator was developed, showing 1) the relationships between expected acquisition, hand-off, track and launch and 2) how to determine the number of launches for a shoot-look-shoot doctrine. For netted operation, the expected probability of kill (Pk) is:

$$Pk = P_{acq}\, P_{hand\text{-}off}\, P_{track}\, \{ 1 - (1 - P_{launch}\, PK)^{N} \}$$

Where PK is the compounded probability of kill for all the missiles in a salvo, given launch.

The number of salvos (N) can be determined by:

$$t_{eff} \geq \sum_{i=1}^{N} (\Delta t_i + TOF_i)$$

Where t_{eff} is the effective time in cover, Δt_i is the SAM reaction time for the ith launch and TOF_i is the missile time of flight for the ith launch.

Three cases were considered - 1) a direct attack against a SAM site, 2) a random encounter and 3) taking advantage of site specific masking and clutter. Methods to analyze the first two cases were developed, with examples shown for the number of intercepts within SAM coverage for a shoot-look-shoot defense doctrine.

This chapter concluded with discussions of degrade probabilities for multiple onboard warning systems, of SAM capability charts and of missile guidance modes, fuzing, receding targets, overlapping coverage, standoff attack and the AAA threat.

Chapter 19

Penetration Mission Survival

This chapter addresses how AI and SAM survival probabilities can be combined to determine mission probability of survival (Ps). Outcome variability is then introduced and illustrated by an example of the probability of various numbers of penetrators surviving.

Next, the assumptions of independence between various defense (and offense) actions are summarized. This is followed by comments on the value of large Monte Carlo models in validating some of these assumptions.

The chapter concludes with a discussion of input uncertainty and an illustration of confidence levels on mission survival when input uncertainties can be quantified.

Expected Mission Survival

The expected (or average) probability of mission survival (Ps) of one penetrator against the AIs assigned and SAMs encountered can be found by multiplying the expected survival probabilities, as shown below:

$$\overline{Ps} = \overline{Ps}_{AI} \ \overline{Ps}_{SAM}$$

where Ps_{AI} = Expected probability of surviving all AIs assigned
Ps_{SAM} = Expected probability of surviving all SAMs encountered

Expected penetrator survival of all AIs assigned can be determined by multiplying the expected survival values against each group of interceptors assigned. Thus:

$$\overline{Ps}_{AI} = \overline{Ps}_{AI_1} \ \overline{Ps}_{AI_2} \text{ --- } \overline{Ps}_{AI_K} = \prod_{k=1}^{K} \overline{Ps}_{AI_k}$$

AIs might be grouped by: 1) assignments, 2) individual AI types, 3) aggregating AIs (e.g. into look downs and non look-down types), 4) AI control types (e.g. AIs under AWACS control and under EW/GCI control) or 5) penetrator event intervals (e.g. AIs assigned from entry to 1st target, from 1st to mid target, from mid to last target, from last target to exit cover).

For example, divide AIs into look down and non-look down groups. The expected survival for all assignments with look down AIs may be .5, with .7 for all assignments with non-look down AIs. Then the expected penetrator survival of the AI threat is:

$$\bar{P}s_{AI} = (.5)(.7) = .35$$

The Ps for SAM encounters depends on the expected number of encounters with (and Pk for) each SAM type. The expected penetrator survival of all SAM types encountered is:

$$\bar{P}s_{SAM} = \prod_{m=1}^{M} (1 - \bar{P}k_m)^{E_m}$$

where $\bar{P}k$ = Expected probability of kill in a SAM encounter
E = Expected number of encounters

When all exponents (E_m) are less than 1, the Ps integer solution is:

$$\bar{P}s_{SAM} = \prod_{m=1}^{M} (1 - E_m \bar{P}k_m)$$

E_m could represent less than one encounter (e.g. when there is site location uncertainty, or when a SAM may receive damage from a prior strike). For example, a .5 probability of prior damage and one planned encounter leads to a .5 probability of an encounter with an undamaged SAM site, or a .5 expected number of encounters.

Assume that there are two different SAM systems - type 1 and type 2. If $\overline{Pk}_1$ is .3, E_1 is .4, $\overline{Pk}_2$ is .05 and E_2 is .4, then the expected penetrator survival against these SAM threats is:

$$\overline{Ps}_{SAM} = \{ 1 - (.4)(.3) \} \{ 1 - (.4)(.05) \} = \{.88\}\{.98\} = .86$$

The combined AI and SAM threats yield an expected mission survival of:

$$\overline{Ps} = \overline{Ps}_{AI}\, \overline{Ps}_{SAM} = (.35)(.86) = \underline{.3}$$

This expected mission survival will now be used to estimate the number of survivors for a penetrator force.

Outcome Variability

If the mission Ps is the same for a group of P penetrators, the expected number of survivors is:

Expected survivors = (P)(Expected mission Ps)

Thus, if each of 10 penetrators had an expected probability of survival of .3, the expected number of survivors would be:

Expected survivors = (10)(.3) = $\underline{3}$

This is the average or expected value. But the number of survivors can vary from 0 to 10 due to chance. For example, imagine that a random sample of ten balls will be drawn from a very large number of red and white balls. If 70% of the balls are red (representing kills) and 30% are white (representing survival), occasionally 10 red balls (10 are killed) or 10 white balls (10 survive) will be chosen.

When there are repeated and independent trials with only two possible outcomes (survival or kill in this case), and the probability of survival is known and constant, the probability of various outcomes is determined by the binomial distribution. This distribution can be expressed as:

$$\text{Prob (p out of P)} = \{ P!/p!(P-p)! \} Ps^{p} (1 - Ps)^{P-p}$$

where p is the number of surviving penetrators out of P which enter

$$\text{and } x! \text{ is x factorial} = (1)(2)(3) \text{ --- } (x) = \prod_{i=1}^{x} i$$

For example, if 10 penetrators have the same Ps (.3), the binomial distribution predicts the chance of exactly 0 to 10 surviving. These probabilities (and the chance that at least x survive out of the 10 that enter). are shown in the table below:

Survivors	Probability of exactly	Probability of at least
0	.0282	1.0
1	.1211	.9718
2	.2335	.8507
3	.2668	.6172
4	.2001	.3504
5	.1029	.1503
6	.0368	.0473
7	.0090	.0106
8	.0014	.0016
9	.0001	.0001
10	.0000	.0000

The probability of exactly 3 out of 10 is almost 27%, and of at least 3 out of 10 is almost 62%. The most likely outcome (the mode) is 3 out of 10, which is also the expected outcome. The pessimist (or prudent planner) may wish to know - what minimum outcome can be depended upon 90% of the time? That answer is 1 out of 10 (which occurs 97.16% of the time). Stated another way, there is a 97.16% probability that the statement - at least one out of ten survive - is a true statement.

This probability is often equated to a statistical confidence level in that outcome.

The table also shows that the outcome of at least 2 out of 10 survivors occurs about 85% of the time (corresponding to a 85% statistical confidence level). An optimist might note that 15% of the time at least 5 out of 10 penetrators survive.

The development of any mission $\bar{P}s$ involves many assumptions about independence/dependence of defense/offense actions. These assumptions will now be summarized, and the value of Monte Carlo models in their validation discussed.

Independence

Independence has been assumed in the model between:

1) Pks of different AIs within one assignment and between assignments,
2) Pks of different SAMs of the same type and of different types,
3) All degrades,
4) Different missile salvos by one SAM site,
5) Different missile salvos (and the AI gun pass) by one AI, and
6) the square root of SSPK between all missiles launched in a salvo.

Other actions within an AI assignment (or a SAM encounter) have been treated as conditional probabilities.

Different assumptions would change the penetration assessment. For example, the assumption of square root SSPK dependence changed the compounded PK for a salvo of two missiles with SSPK = .5 from .75 (if independent) to .65 (with square root dependence), a reduction of over 13%.

A second example is a change from independence to complete dependence for the effectiveness of all ECM degrades. The degrades (in AI Pk covered in Chapter 16) would change from 50% applied three separate times (the independent case) to a single 50% chance of ECM working as planned and a 50% chance of complete ECM failure (a new completely dependent case). The expected Pk would then change (using the head-on radar undegraded Pk of .50 in Chapter 16) from a Pk of .16 (if independent) to .5 (.50) + .5 (0) = .25 (if completely dependent). This is an increase of over 56% in kill probability, due to changing from independence to complete dependence in ECM degrades.

Another aspect of the model will now be considered - the probabilistic approach used with the assumptions of independence/dependence. This approach never evaluates success/failure (or survival/kill), but deals only in probabilities - from defense effectiveness through penetrator survival. There is another approach - using a Monte Carlo procedure.

Monte Carlo Procedure

A Monte Carlo procedure starts with similar input probabilities, but draws random numbers to assess the specific result (success/failure or survival/kill) for each action or event. For example, if the probability of damage to a particular AI base is .5, the selection of a random number from 0 to 49 could represent success in base damage, while a number from 50 to 99 could represent failure. Monte Carlo procedures result in a specific yes/no result for each action or event which is modeled.

Each Monte Carlo run can yield a different numerical result for each measure of effectiveness (such as the number of survivors or penetrator weapons on target). Thus, many computer runs are needed to determine an average result for one case. Sometimes one Monte Carlo result will differ markedly from others - pointing out a dependency (the consequences of success/failure) in some key action or event (e.g. in damaging/not damaging a key AI base or SAM site).

Monte Carlo techniques are almost always used when individual calculations are extremely involved and cumbersome. Other reasons given for use of Monte Carlo techniques are:

1) Interactions between various offensive/defensive actions are too complex for <u>simple</u> modeling (although they must be modeled) or
2) Critical dependencies might dramatically change the measure of effectiveness (these dependencies must also be modeled).

Examples of Monte Carlo air vehicle penetration models are:

ACE - Advanced Campaign Effectiveness
(used by AF/Aeronautical Systems Division and Rockwell Int'l)
APM - Advanced Penetration Model
(used by AF/Studies and Analysis)
STRAT DEFENDER - Strategic Aerospace Defense
(used by AF/Studies and Analysis and NORAD)

Monte Carlo models can be used to investigate known dependencies between various defense/offense actions and events (e.g. area control and net actions in assigning AIs and SAMs). Once understood, simpler models can be modified to include these dependencies.

Probabilistic air vehicle penetration models provide much faster response time and generally higher visibility of inputs. This encourages sensitivity studies that can generate understanding of the dominant factors involved. The larger Monte Carlo models are generally too slow running and have too many input details to allow sensitivity studies in practical cases. However, results from Monte Carlo models can be used to calibrate the simpler probabilistic models, and thus add credibility to the simpler models.

Thus far, all inputs have been represented by a single value - the mean or expected value - for both probabilistic and Monte Carlo models. Now consider input variability.

Input Variability

There may be considerable uncertainty in offense and defense effectiveness (e.g. ECM vs ECCM). The use of a single value for each input masks this uncertainty and precludes any analysis of this uncertainty in mission Ps (or other measures of overall effectiveness). For example, an expected PK may be .5, but all values from 0 to 1.0 may be equally likely, as illustrated in Figure 19.1.

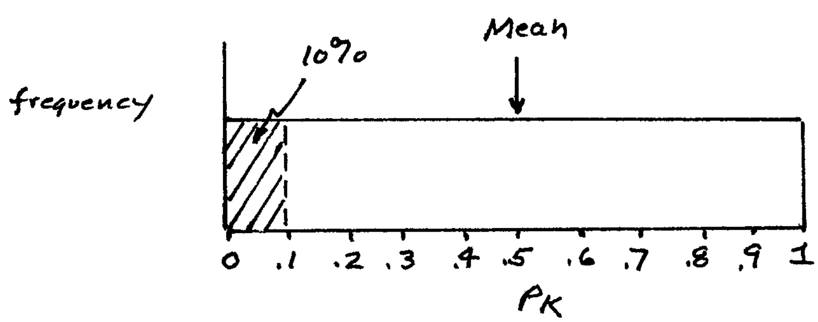


Figure 19.1 Uncertainty in Inputs

In Figure 19.1, the mean (expected or average) PK is .5. But 10% of the time the PK will be no more than .1. One could say that there is 90% statistical confidence that the PK will be at least .1, and only 50% confidence that it will be at least .5.

Input variability might be determined from answers to such questions as - what is the lowest (-3σ) and highest ($+3\sigma$) value that could be justified for each input? Or, what values would be found no more than 10% (and 90%) of the time? Answers to either question will allow a Pk variance to be estimated for the effectiveness of each AI and SAM threat action. The variance in survival of AI and SAM threats can then be determined.

Now uncertainties in various AI and SAM Ps values will be combined to illustrate one example of confidence in mission survival.

Confidence in Mission Survival

In order to illustrate one method of using input uncertainties to predict confidence levels in mission survival, assume that uncertainties in the effectiveness of defense and offense actions have been quantified and that these uncertainties can be expressed as Ps distribution functions for each AI and SAM type. One possible set of four Ps functions is shown in Figure 19.2.

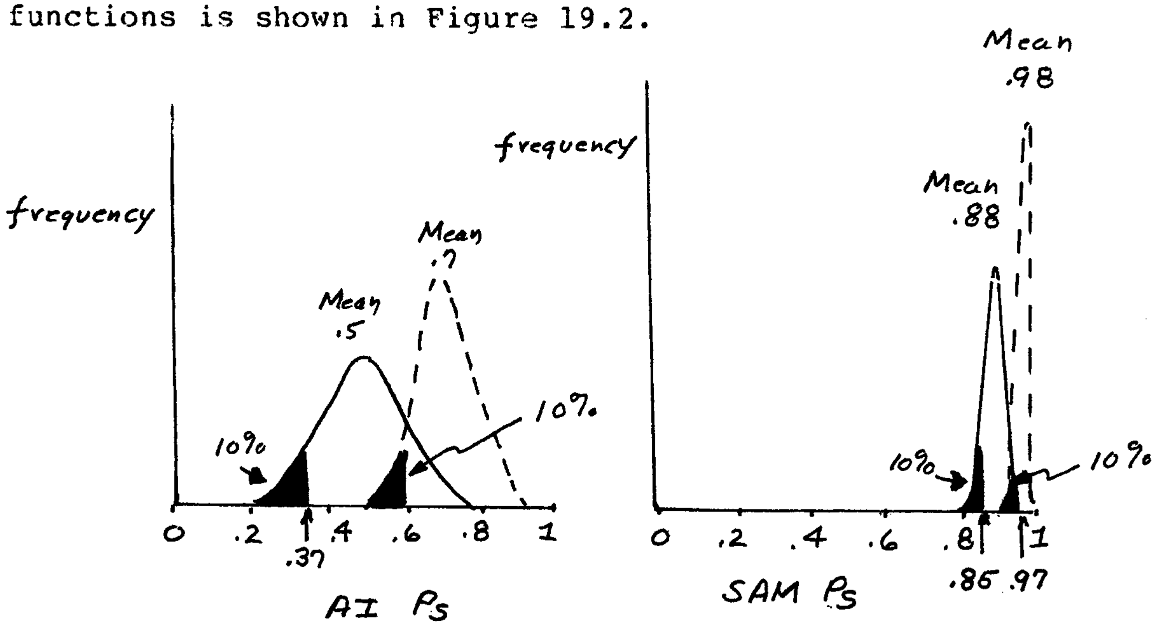


Figure 19.2 AI and SAM Ps Distributions

One method of combining these frequency functions will be illustrated in order to find a mission survival distribution. For the penetration survival case with independence between individual survival terms, the distribution has a mean equal to the expected mission Ps, and a variance that can be approximated by:

$$\sigma^2 \cong (\text{mission Ps})^2 \sum_{i=1} (\sigma_i^{\,2}/\overline{Ps}_i^{\,2})$$

where $\sigma_i^{\,2}$ = variance of the ith Ps distribution

For example, the expected mission Ps was:

$$\overline{Ps} = \overline{Ps}_{\text{LD AI}} \; \overline{Ps}_{\text{non-LD AI}} \; \overline{Ps}_{\text{SAM}_1} \; \overline{Ps}_{\text{SAM}_2}$$

$$= (.5)(.7)(.88)(.98) = .3$$

The four individual standard deviations (the square root of each variance) are noted in the Figure 19.2 example as:

$$\sigma_{\text{LD AI}} = .1, \quad \sigma_{\text{non-LD}} = .08, \quad \sigma_{\text{SAM}_1} = .02, \quad \sigma_{\text{SAM}_2} = .005$$

The mission Ps variance is then:

$$\sigma^2 = (.3)^2 \{ .1^2/.5^2 + .08^2/.7^2 + .02^2/.88^2 + .005^2/.98^2 \}$$

$$= (.09) \{ \underline{.04} + .013 + .0005 + .0000 \} = .0048$$

Note the dominance of the LD AI input uncertainty, representing .04/.0535, or almost 75% of the total variance in this example. Generally, the threat causing the lowest Ps will dominate the mission Ps variance. If the variability in the effectiveness of all defense/-offense actions can not be quantified, the variability of the major penetration threat alone can provide a rough estimate of the overall variance.

The standard deviation (σ) is the square root of this variance. Thus $\sigma = (.0048)^{1/2} = .069$ in the example. (Since the mean is more than 3σ away from both the 0 and 1.0 boundary, a univariate normal is a reasonable approximation.) The lower 90% confidence level for a univariate normal distribution with a mean of .3 and a σ of .069 is:

Lower 90% confidence level = Mean - 1.28 σ = .3 - 1.28 (.069) = .21

With independence assumed between AI (and SAM) threats, the distribution of mission probability of survival in the example can be plotted as shown in Figure 19.3.

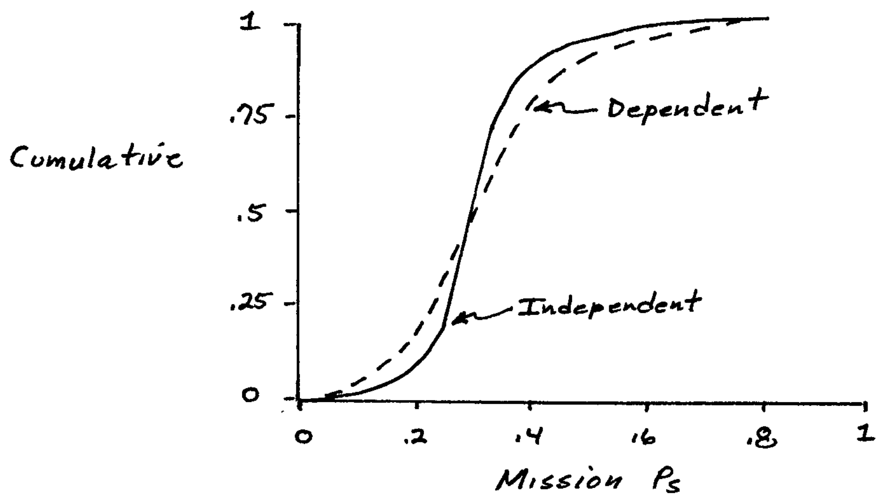


Figure 19.3 Normal Distribution - Mission Ps

If there is complete dependence between penetrator survival of AI and SAM threats (but with the same Ps functions as before), the expected mission Ps is still .3, but now a choice of one Ps (from any one of the four functions) will determine the value of Ps for the other three. Assume that the choice of the lower 90% level for any one, forces the lower 90% level for the other three. In this case, the lower 90% confidence level on mission survivability is determined by multiplying the lower 90% levels from each function. As shown in Figure 19.2, these values are:

Lower 90% confidence level = (.37)(.60)(.85)(.97) = .18 dependent

The lower 90% confidence level has dropped from .21 with independence, to .18 with complete dependence in the example (as shown in Figure

19.3). More dramatic differences could occur if the Ps variance had not been dominated by one threat - the LD AI in the above case.

Summary

The expected mission probability of survival against the AI and SAM threats can be determined by:

$$\overline{Ps} = \overline{Ps}_{AI} \ \overline{Ps}_{SAM}$$

The expected Ps vs individual AI (and SAM) threat types can be determined by the multiplication of individual $\overline{Ps}$ values for each AI assignment (and SAM encounter). AIs and SAMs may be grouped into various types to simplify calculations.

The expected number of survivors out of a force of P penetrators is ($\overline{P}$s)(P). However, outcomes of from 0 to P survivors are possible based on chance. Various outcomes were illustrated for a force of 10 penetrators by evaluating the binomial distribution for various numbers of survivors.

The assumptions of independence between various defense (and offense) actions were reviewed to point out the importance of dependencies in penetration. The value of large Monte Carlo models in investigating some of these dependencies was discussed, as well as the value of probabilistic models for rapid response and better visibility of inputs. Sensitivity studies with simple models can generate an understanding of the dominant penetration factors.

Input variability was discussed, and a simple example of four survival functions was used to develop a distribution of mission probability of survival and confidence levels with independence and complete dependence between the four survival functions.

PART V

EFFECTS ON TARGET DAMAGE

Chapter 20

Target Damage

This chapter addresses the effects of penetration on target damage. The target damage expected from one weapon is discussed first. Methods to compound damage from multiple weapons on one target are then described, followed by an illustration of outcome variability for many targets.

Input variability is then discussed. This chapter concludes with illustrations of statistical confidence in damage, showing the importance of independence/dependence between launch, reliability, penetration and weapon damage.

Expected Target Damage

The Damage Expected (DE), or average damage, to a target can be evaluated by multiplying the expected value of four probabilities (previously defined in Chapter 2) when each is independent:

1) Probability of Launch Survival ($\bar{P}LS$),
2) Weapon System Reliability ($WS\bar{R}$), given safe launch,
3) Probability of survival ($\bar{P}s$) through enemy defenses to weapon release, given a reliable weapon system, and
4) Probability of Damage ($\bar{P}D$) to the target by one weapon, given penetration survival to weapon release.

$$DE = (\bar{P}LS)(WS\bar{R})(\bar{P}s)(\bar{P}D)$$

Or,

$$DE = (\bar{P}A)(\bar{P}D)$$

where $\bar{P}A$ = Probability of Arrival at weapon release

For example, assume that $\overline{PLS}$ = .8, $WS\overline{R}$ = .95, $\overline{P}s$ = .5 and $\overline{P}D$ = .53. Then the average damage is:

$$DE = (.8)(.95)(.5)(.53) = .2$$

If higher damage levels are desired, a more effective weapon can be used or more than one weapon can be assigned to the target and damage from these weapons compounded.

Compounding Damage

Consider damage to a point target at a known location from:

1) many weapons released on one pass from one air vehicle,
2) one weapon released from each of many different air vehicles, and
3) many weapons released on one pass from each of many vehicles.

Note that PD is defined as the probability of damage to one target by one weapon from one air vehicle.

Many Weapons/One Vehicle

If the same air vehicle delivers more than one weapon on a target at one time, the probability of the vehicle arrival (PA) at the weapon release point is a dependent factor for all weapons. Multiple weapons from one vehicle can be compounded if the probability of target damage for each weapon released is independent of the other weapons released. Weapons will be considered to be independent if their impacts are randomly distributed about the aimpoint(s) and if prior damage does not change the hardness of the target.

The average compound Probability of Damage from independent weapons, denoted as $c\overline{P}D$, is:

$$c\bar{P}D = 1 - (1 - \bar{P}D)^n$$

where n = the number of weapons released on one target

For example, the compounding of a $\bar{P}D$ of .53 for one to five weapons is:

n	1	2	3	4	5
$c\bar{P}D$	.53	.78	.90	.95	.98

The compound Damage Expected (cDE) for n independent weapons from a single air vehicle is:

$$cDE = (\bar{P}A)\ \{ 1 - (1 - \bar{P}D)^n \} \quad \text{n weapons/one vehicle}$$

For example:	n =	1	2	3	4	5
	cDE =	.20	.30	.34	.36	.37

However, if weapon impacts are not randomly distributed about the aim point, but about a bias point (due to a common delivery error of the air vehicle), weapons will not be independent. If this bias is large compared to weapon dispersion, multiple weapons from the same air vehicle may not significantly increase damage beyond that of a single weapon. This problem can be avoided if multiple air vehicles (not sharing a common bias) each deliver one weapon on a target.

One Weapon/Many Vehicles

Multiple vehicles can have independent PAs, if each vehicle takes off from a different base, has an independent weapon system reliability and encounters different penetration threats. Then the compound damage expected can be found by:

$$cDE = \{ 1 - (1 - DE)^n \} \quad \text{one weapon/many vehicles}$$

where $(1 - DE)$ = Target survival of one weapon from one vehicle

For example, using expected values of PLS = .8, WSR = .95, Ps = .5 and PD = .53, the table below illustrates the cDE for one to five weapons scheduled against one target by one vehicle and by different vehicles:

Vehicles \ Weapons	1	2	3	4	5
one	.20	.30	.34	.36	.37
different		.36	.49	.59	.67

This illustrates the value of compounding DE (in the many independent vehicle case) as opposed to compounding PD only (in the one vehicle, independent weapon case).

Many Weapons/Vehicles

One might ask - how many weapons and air vehicles need to be assigned to a target in order to achieve some compound damage level (e.g. .90)? If there is independence between weapons, the $c\bar{P}D$ term can be raised from $\bar{P}D$ to $1 - (1 - \bar{P}D)^n$ with n weapons per vehicle. For example, with a maximum of 5 weapons per vehicle and a $\bar{P}D$ of .53, the cDE is:

$$c\bar{P}D = 1 - (1 - .53)^5 = .98$$

If different vehicles have independent PAs (as well as independent weapons) the compound DE is:

$$cDE = 1 - (1 - \{ (\bar{P}A)(c\bar{P}D) \}^N)$$

where N = Number of vehicles scheduled against a target

In the example, with $\overline{PA}$ = (.8)(.95)(.5) = .38 and $c\overline{PD}$ = .98 (for five independent weapons), the cDE is:

$$cDE = 1 - (1 - \{ (.38)(.98) \}^{N})$$

Values for cDE are illustrated below for one to five air vehicles:

N	1	2	3	4	5
cDE	.37	.61	.75	.85	.90

In the example, five air vehicles are needed to achieve an expected damage of .90, assuming independence between vehicles and between weapons delivered.

Variability will now be considered in the number of targets damaged when many targets are attacked.

Outcome Variability

Assume that the number of targets damaged (t) out of the total number of targets attacked (T) by an air vehicle force is to be predicted. With repeated and independent trials, only two possible outcomes (target survival or damage) <u>and</u> with a constant, known probability of damage, the probability of various outcomes can be expressed by the binomial distribution as:

$$\text{Prob (t out of T)} = \{ T!/t!\ (T-t)! \}\ cDE^{\,t}\ (1 - cDE)^{\,T-t}$$

For example, if 10 targets are to be attacked, and each has the same probability of being damaged, the binomial distribution predicts the chance of exactly 0 to 10 (and at least 0 to 10) being damaged. These probabilities are shown in the table on the next page for the example case of cDE = .9:

Targets Damaged	Probability of exactly	Probability of at least
0	~.0000	1.0
1	~.0000	~1.0
2	~.0000	~1.0
3	~.0000	~1.0
4	.0001	~1.0
5	.0015	.9999
6	.0112	.9984
7	.0574	.9872
8	.1937	.9298
9	.3874	.7361
10	.3487	.3487

The expected number of targets damaged is (N)(cDE). In the example this is (10)(.9) = 9 targets. The mode is also 9 out of 10, with a probability of occurrence of 38.74%. The prudent planner would note that 8 out of 10 targets will be damaged over 90% of the time. The optimist would note that all 10 targets will be damaged almost 35% of the time.

These values apply to the example with independent PAs and weapon PDs. Different assumptions of independence/dependence between these parameters will yield different values of cDE, and the probability of various numbers of targets damaged.

Now consider input variability and confidence in damage achieved.

Input Variability and Confidence

Assume that PLS, WSR, Ps and PD are all random variables. An average value and a variance of each distribution will be used to illustrate methods of handling input variability.

For example, the average values in the example for one vehicle carrying 5 independent weapons against one target are:

$$\overline{PLS} = .8 \quad \overline{WSR} = .95 \quad \overline{Ps} = .5 \quad c\overline{PD} = .98$$

The average, or expected, probability of damaging a target is:

$$DE = (\overline{PLS})(\overline{WSR})(\overline{Ps})(c\overline{PD}) = (.8)(.95)(.5)(.98) = .37$$

The standard deviation (σ) of each distribution will be assumed to be:

Probability	$\overline{PLS}$	$\overline{WSR}$	$\overline{Ps}$	$c\overline{PD}$
Standard Deviation	.05	.01	.1	.005

If independent, the overall variance can be estimated as:

$$\sigma^2 \cong DE^2 \sum_{i=1}^{4} (\sigma_i{}^2/\overline{P}_i{}^2)$$

where the $\overline{P}_i$s are the individual PLS, WSR, Ps and cPD average values

In the example, the overall σ^2 is:

$$\sigma^2 = (.37)^2 \{ .05^2/.8^2 + .01^2/.95^2 + .1^2/.5^2 + .005^2/.98^2 \}$$

$$= .14 \{ .004 + .0001 + \underline{.04} + .00003 \} = .0062$$

Note that the variance in probability of survival (penetration) contributes over 90% to the total variance in the example. Generally the term with the lowest probability (in this case Ps) will dominate the overall variance.

The standard deviation of target damage is the square root of the variance. Thus, $\sigma = (.0062)^{1/2} = .079$ in the example. The lower 90% confidence level (L_{90}) for a univariate normal distribution with a mean of .37 and a σ of .079 is:

$$L_{90} = \text{Average} - 1.28\ \sigma = .37 - 1.28\ (.079) = \underline{.27} \quad \text{with independence}$$

The cumulative normal distribution for this independent case is shown in Figure 20.1. (Note that the beta or binomial distribution could be used if the normal distribution did not provide an adequate approximation.)

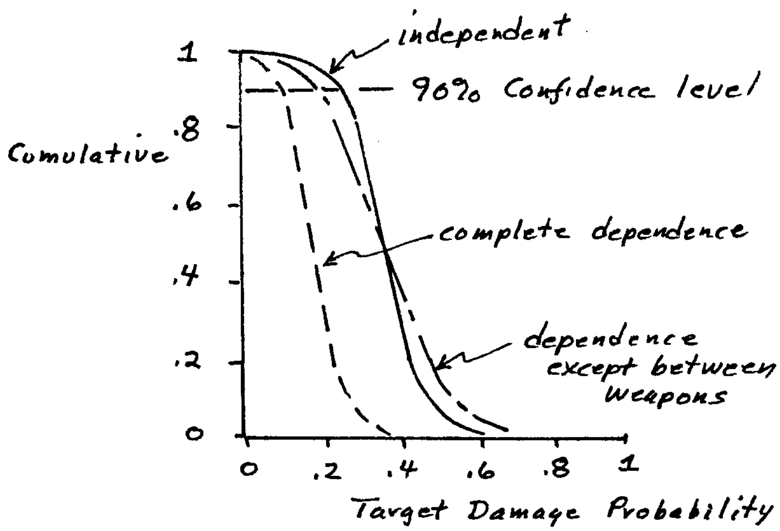


Figure 20.1 Normal Distribution - Target Damage

Assume now that only one vehicle delivers all weapons (PA is dependent), but that weapons released from one vehicle are independent. If the level of each variable is under the control of one chance draw (i.e. a single enemy action will determine a value for PLS, WSR, Ps and cPD), the four lower 90% levels can be multiplied to provide a lower 90% estimate on target damage. This assumption of a common dependent thread is the most pessimistic assumption for high (greater than 50%) confidence calculations.

Each lower 90% value can be estimated as 1.28 σ below the mean (assuming a univariate normal distribution for each variable) as noted on the next page.

Term	PLS	WSR	Ps	cPD
Average	.8	.95	.5	.98
σ	.05	.01	.1	.005
L_{90}	.736	.9372	.372	.9736

The multiplication of these dependent lower 90% confidence levels yields the pessimistic estimate of:

$$L_{90} = (.736)(.9372)(.372)(.9736) = \underline{.25} \quad \text{with dependence}$$

The level for 90% confidence dropped about 7% (from .27 with independence to .25 with dependence between these four variables).

Now assume that the compounding of weapons delivered by one vehicle on the same target is no longer independent but completely dependent, so that only one weapon from each vehicle contributes to target damage. With this dependence, the PD stays at .53. The lower 90% confidence level on PD will be assumed to be .52. The new average and L_{90} level on target damage, with complete dependence between weapons, PLS, WSR and Ps (one draw determines all values - the most pessimistic assumption for high confidence calculations) is:

$$\text{Average} = (.8)(.95)(.5)(.53) = .2$$

$$L_{90} = (.736)(.9372)(.372)(.52) = \underline{.13} \quad \text{with complete dependence}$$

This is quite a change from the mean and L_{90} values (.37 and <u>.25</u>, respectively, in the independent case). The three cases are illustrated in Figure 20.1. This example indicates the importance of the dependence/independence relationships between the variables affecting target damage.

Summary

The damage expected from one weapon scheduled to be delivered on a target (when there is independence between PLS, WSR, Ps and PD) is:

$$DE = (\overline{PLS})(\overline{WSR})(\overline{Ps})(\overline{PD})$$

If weapons are independent, the compound probability of damage (cPD) is:

$$c\overline{PD} = 1 - (1 - \overline{PD})^n \quad \text{for n independent weapons}$$

The compound damage expected (cDE) equation can take one of three forms:

$$cDE = 1 - (1 - DE)^n \quad \text{one weapon each from n vehicles}$$

$$cDE = \overline{PA} \{ 1 - (1 - \overline{PD})^n \} \quad \text{one vehicle with n independent weapons}$$

$$cDE = 1 - (1 - \{ (\overline{PA})(c\overline{PD}) \}^N) \quad \text{N independent vehicles with n weapons each}$$

The probability of any given number of targets being damaged out of a set of targets can be estimated by the binomial distribution, if the the probability of damage is the same for all targets and there is independence in damage between targets. This distribution predicts the outcome variability for known and constant target damage probabilities.

Input variability may exist due to uncertainty in the value of the various PLS, WSR, Ps and PD parameters. When this variability can be quantified, statistical confidence statements may be made on the probability of target damage. Three cases were illustrated with different dependencies to illustrate the importance of dependencies on target damage estimates.

Chapter 21

Four Viewpoints

This text has approached air defense and air vehicle penetration from the viewpoint of an operations analyst - crediting both the enemy and ourselves (friendly forces) with being smart in using military force (at least in the undegraded cases). But an assessment might have allowed being stupid in force application. The four matrices below indicate four different views of smartness and stupidity in the use of friendly (we) and enemy (they) forces:

	Avionics Advocate		Intelligence Analyst	
	Smart	Stupid	Smart	Stupid
We are	✓			✓
They are		✓	✓	

	Operations Analyst		Reality	
	Smart	Stupid	Smart	Stupid
We are	✓			✓
They are	✓			✓

An avionics advocate may believe the enemy will be stupid (in reacting to our new ECM/ECCM capability) and may be correct.

An intelligence analyst may believe friendly forces will be stupid (in not reacting quickly enough to counter postulated enemy capabilities) and may be correct. A prudent planner may also share this tendency to understate our readiness, capability or effectiveness in combat.

A stupid-stupid case has few advocates. However, this combination of an intelligence analyst's view of ourselves with an avionics advocate's view of the enemy might result in estimates closer to some combat (or operational test) results when compared to overly optimistic assumptions of force capability.

An operations analyst may assume that both friendly and enemy forces are smart, and be wrong on both counts! Reality may be that both sides are stupid (i.e. have lower combat readiness/capability and use forces less effectively than planned). Consideration of defensive/offensive degrades and reactive tactics should prevent overly optimistic assumptions in friendly/enemy force capability estimates. It is hoped that this text will encourage the use of:

1. Operational degrades,
2. Better assessment of defensive/offensive tactics, and
3. Uncertainty analysis on the dominant factors

in order to bring attrition estimates closer to combat/operational test results.

1. Swerling, P. Probability of Detection for Fluctuating Targets, March 1954, RAND Corp. Research Memorandum RM-1217
2. Blake, L. A Guide to Basic Pulse-Radar Maximum-Range Calculation Part 1, NRL Report 6930, December 23, 1969
3. Cunningham, J. Parameters Affecting SAM Coverage vs Low Altitude Targets, November 1973, General Research Corp. RM-1740/1
4. Skolnik, M. Radar Handbook, 1970, Mc Graw-Hill
5. Cunningham, J. Unpublished notes and conversations
6. Santa Barbara Research Center, Infrared Components Brochure Edition 17, 1983

Other references

Radar

* Introduction to Airborne Radar by G. Stimson, 1983, Hughes Aircraft

Introduction to Radar Systems by M. Skolnik, 1980, McGraw-Hill

Radar Systems Analysis by D. Barton, 1976, Artech House

Radars, in Seven Volumes by D. Barton, 1974-79, Artech House

Radar Principles for the Non-Specialist by J. Toomay, 1982, Lifelong Learning

Electro-optical

Electro-optics Handbook 1974, RCA Technical Series EOH-11

Infrared System Engineering by R. Hudson, 1969, Wiley-Interscience

Camouflage - A History of Concealment and Deception in War by G. Hartcup, 1980, Scribners

Electronic Warfare

* Applied ECM - Volumes I and II by L. Van Brunt, 1982, EW Engineering

* The History of US Electronic Warfare by A. Price, 1984, The Association of Old Crows (AOC)

* The History of US Electronic Warfare by A. Price, 1984, The Association of Old Crows (AOC)

* Instruments of Darkness - The History of Electronic Warfare by A. Price, 1977, Scribners

Radar Electronic Counter-Countermeasures by S. Johnston et al, 1979 Artech House

Radar Anti-Jamming Techniques by M. Maksimov (translation), 1979 Artech House

Fundamentals of Electronic Warfare by R. Fitts, 1972, US Air Force Academy

An Introduction to Radar Electronic Warfare by A. Golden, 1980 AFIT

Intelligence Warfare by W. Kennedy, 1983, Crescent

The Electronic Battlefield by P. Dickson, 1976, Indiana Univ.

Secrets of Electronic Espionage by J. McCarroll, 1966, Dutton

Models

Military Modeling Edited by W. Hughes, 1984, Military Operations Research Society (MORS)

The Military Applications of Modeling by J. Battilega and J. Grange 1984, AFIT

Lanchester Models of Warfare, Volumes I and II by J. Taylor, 1980 Operations Research Society of America (ORSA)

One-on-One Stochastic Duels by C. Ancker, 1982, ORSA

Force-on-Force Attrition Modeling by J. Taylor, 1980, ORSA

Estimating Bomber Penetration and Weapons Effectiveness by F. Nyland 1974, RAND R-1278/1-PR

The Penetration Integral and Tables: A Unifying Solution for Armament and Encounter-Limited Air Battles by C. Builder, June 1973, RAND P 1257-PR

Weapon Systems

Jane's All the Worlds Aircraft Edited by J. Taylor, Yearly, Jane's

Jane's Weapon Systems Edited by R. Pretty, Yearly, Jane's

Air Warfare

* Air Warfare - the 4th Generation by C. Campbell, 1984, ARCO

* History of the German Night Fighter Force 1917-1945 by G. Aders, 1980 Jane's

German Aircraft Guns - World War I and II by E. Hoffschmidt, 1969 Old Greenwich

Some Cruise Missile History: Performance of the Allied Defenses Against the V-1 by D. Briggs, 12/15/77 M.I.T. Lincoln Laboratory Paper

British Intelligence in the Second World War by F. Hinsley et al Volume I, II and III (1979, 1981 and 1984), Her Majesty's Stationary Office

The United States Air Force in Korea by R. Futrell et al, 1961 Duell, Sloan and Pearce

The Quest for Victory by J. Alger, 1982, Greenwood

Air Combat for the 80's - Red Flag by M. Skinner, 1984, Presidio

Strategy and Tactics of Air Warfare by J. Pimlott, 1979, Chartwell

Strategy and Tactics of Sea Warfare by N. Wilmott, 1979, Chartwell

Bomber Harris by D. Saward, 1985, Doubleday

The Bomber in World War II by A. Price, 1976, Scribners

Luftwaffe Handbook 1935-1945 by A. Price, 1977, Scribners

Night Fighter, A Development and Combat History by W. Gunston, 1976 Scribner

The Cactus Air Force by T. Mitler, 1969, Harper and Row

Air Raid: Pearl Harbor Edited by P. Stillwell, 1981, Naval Institute

The Dauntless Dive Bomber of World War II by B. Tillman, 1976, Naval Institute

The Miracle at Midway by G. Prange et al, 1982, McGraw-Hill

Above and Beyond: 1941-1945 by W. Morrison, 1983, St. Martin's
Air Power - A Concise History by R. Higham, 1972, St. Martin's
Air Assault - The Development of Air Mobile Warfare by J. Gavin, 1969 Hawthorn
Out of the Sky - A History of Airborne Warfare by M. Hickey, 1979 Scribner
Helicopters At War - A Pictorial History by D. Wragg, 1983, St. Martin's
Airpower - The Decisive Force in Korea Edited by J. Stewart, 1957 Van Nostrand

Vietnam Combat

* Air War - Vietnam Introduction by D. Middleton, 1978, Bobbs-Merrill
The Wars in Vietnam by E. O'Ballance, 1975, Hippocrene
The 10,000 Day War by M. Maclear, 1981, St. Martins
The Naval War in Vietnam P. Mersky and N. Polmar, 1981, Nautical and Aviation
Rolling Thunder: Understanding Policy and Program Failure by J. Thompson, 1980, University of North Carolina
Linebacker II - A View from the Rock by J. McCarthy and G. Allison 1979, Volume VI Monograph 8 USAF Southeast Asia Monograph Series Maxwell AFB, Air War College
Vietnam - The War in the Air by G. Guerney, 1985, Crown
The War in Indochina by Air War Study Group, Cornell Univ., 1972 Beacon
The 25 Year War - America Military Role in Vietnam by B. Palmer, 1984 Univ. of Kentucky
The Infantry Brigade in Combat by D. Wolf, 1984, Sunflower

Arab-Israel Wars

* Moscow's Lessons from the 1982 Lebanon Air War by B. Lambeth (translation from Soviet Air Force monthly), 1984, RAND R-3000 AF

The Yom Kippur War by the London Sunday Times Insight Team, 1974 Doubleday

The Yom Kippur War by P. Allen, 1982, Scribners

The Yom Kippur War by A. Barker, 1974, Ballantine

On the Banks of the Suez: an Israel General's Personal Account of the Yom Kippur War by A. Adan, 1980, Presidio

No Victor, No Vanquished - The Yom Kippur War by E. O'Ballance, 1978 Presidio

The War of Atonement by C. Herzog, 1975, Little Brown

Soviet Policy in the October 1973 War by W. Quandt, 1976, RAND R-1864-15A

Arms for the Arabs by J. Glassman, 1975, Johns Hopkins Univ.

Return to Sinai: the Arab Offensive, October 1973 by D. Palit, 1974 Palit and Palit

The Israeli Air Force Story by R. Jackson, 1970, Northumberland

Zanek! A Chronicle of the Israeli Air Force by W. Stevenson, 1971 Viking

Falklands Combat

* Air War South Atlantic by J. Ethell and A. Price, 1983, Macmillan

War in the Falklands - the Full Story by the Insight Team of the London Sunday Times, 1982, Harper and Row

Fight for the Falklands by J. Laffin, 1982, St. Martins

The Battle for the Falklands by M. Hastings and S. Jenkins, 1983 Norton

Military Lessons of the Falkland Island War Edited by B. Watson and P. Dunn, 1984, Westview

Weapons of the Falkland Conflict by B. Perrett, 1982, Blanford

(* - Recommended reading)

AAA = Antiaircraft Artillery
AB = Alternate Base
A_c = Area of clutter
A_e = Antenna effective aperture Area
AGC = Automatic gain control
AI = Airborne Interceptor
A/p = Assignments per penetrator
AWACS = Airborne Warning and Control System

B = Bandwidth
B_d = Doppler Bandwidth

c = Speed of light
C = Clutter
C^3 = Command, Control and Communications
CAP = Combat Air Patrol
cDE = compounded Damage Expected
CFAR = Constant False Alarm Rate
COSRO = Conical Scan on Receive Only
cPD = compounded Probability of Damage
CW = Continuous Wave

d = distance
Δ = (delta) increment

E = Electro-optics AI pass
ECM = Electronic Countermeasures
ECCM = Electronic Counter Countermeasures
EO = Electro-optics
EOCM = Electro-optical Countermeasures
ERP = Effective Radiated Power
ESM = Electronic Support Measures
Exp = expected, or average, value
EW = Electronic Warfare

f = frequency
F = Factor
FAR = False Alarm Rate
FCS = Fire Control System
F_n = noise Factor, or noise Figure

γ = (gamma) clutter coefficient
GCI = Ground Control Intercept
GHz = Giga Hertz (10^9 Hertz)
G_j = jammer transmitter antenna gain
Gr = Grazing angle
G_r = receiving antenna gain
G_t = transmitting antenna gain
G_W = RWR antenna gain

H = Head-on AI pass
h_a = height of antenna
h_t = height of target
HOJ = Home-on-Jam
Hz = Hertz, or cycles per second

I = Light intensity
IAGC = Instantaneous Automatic Gain Control
I.F. = Improvement Factor
IR = Infrared
IRCM = Infrared Countermeasures

J = Jammer signal, or infrared signal
J/S = Jammer-to-Signal ratio

k = Boltzmann's constant
K = K Value (radar free space detection range vs 1 m^2 target)
KHz = KiloHertz (10^3 Hertz)

l = Infrared loss factor

L = Radar loss factor

λ = (lambda) wavelength, in cycles per second or Hertz

LD = Look down

LORO = Lobe on Receive Only

LOS = Line of sight

L_{90} = Lower 90% confidence level

m = meters

μ = (mu) micron (10^{-6} meters)

Max = Maximum

Min = Minimum

mm = millimeters (1/1000 meter)

MHz = MegaHertz (10^{6} Hertz)

MOB = Main Operating Base

MTI = Moving Target Indicator

N = Noise, or maximum number of SAM salvos

NEI = Noise Equivalent Irradiance

Nr = Number

OTH = Over The Horizon

P = Probability, or number of penetrators

$\overline{P}$ = Expected, or average, Probability

π = (pi) 3.14159--

PA = Probability of Arrival

PAA = Primary Aircraft Authorization

P_{aver} = average Power

P_c = Probability of conversion

P_d = Probability of detection

PD = Probability of target Damage

Pen = Number of Penetrators

P_j = jammer power

Pk = Probability of kill

PK = compounded single shot probability of kill

P_l = Prob of launch

PLS = Prelaunch Survival probability

P_p = radar peak Power

PRF = Pulse Repetition Frequency

PRI = Pulse Recurrence Interval

Prob = Probability

Ps = Penetrator survival probability

PS = Probability of Survival after one AI pass

P_t = Probability of track

r = range, or dead zone

R = detection range

RCS = Radar Cross Section

Reqd = Required

RGPO = Range Gate Pull Off

RH = Radar Head-on AI pass

R_{LOS} = Radar Line of Sight

R_{un} = maximum unambiguous range

RT = Radar Tail-on AI pass

RWR = Radar Warning Receiver

S = Signal

σ = (sigma) radar cross section, or standard deviation

σ° = backscatter clutter coefficient

σ_p = penetrator radar cross section

σ_t = target radar cross section

SAM = Surface-to-Air Missile

S/C = Signal-to-Clutter ratio

SCV = Subclutter Visibility

SLB = Side Lobe Blanking

SLC = Side Lobe Cancellation

S-L-S = Shoot-Look-Shoot tactic or doctrine

S/N = Signal-to-Noise ratio

SSPK = Single Shot Probability of Kill

STC = Sensitivity Time Control

t = time

$\bar{t}$ = expected, or average, time

T = Temperature, or Tail-on AI pass

T (as subscript) = Tail-on (attack)

τ = (tau) Pulse width, in seconds

Tgt = Target

TOJ = Track-on-Jam

UHF = Ultra High Frequency

UV = Ultra Violet

V = Velocity

V_c = Closing Velocity

VGPO = Velocity Gate Pull Off

VHF = Very High Frequency

V_{ua} = maximum unambiguous velocity

V_R = Velocity of Radar platform (e.g. AWACS)

WSR = Weapon System Reliability

$\overline{WSR}$ = expected, or average, Weapon System Reliability

ments needed until the penetrators either are killed or escape coverage (i.e. AI Limit > AIs Reqd), the average probability of penetrator survival (Ps) is:

$$Ps = (1 - Pk/A)^{(Max\ A/p)} \quad \text{(when Max A/p is an integer)}$$

For example: without any attacks on AI bases or penetrator lethal self defense, the defense can provide 2980 AIs. For the 100 penetrator case 1300 AIs are required to provide 2 AIs for assignment every 10 minutes, for 100 minutes in coverage, if the penetrator survives each assignment with probability (1 - Pk/A) = .9.

The penetrator Ps for this case (with Pk/A = .1 and Max A/p = 10) is:

$$Ps = (1 - .1)^{10} = \underline{.35}$$

This is the survival to the end of the coverage period (100 minutes in this example). Note that the survival to any lesser time (e.g. an earlier penetrator weapon release time) can be determined by reducing the number of assignments on a penetrator based on a reduced time under coverage.

If the defense can not provide the number of AIs needed to fill the required assignments, the penetrator's probability of survival will increase. The general equation for Ps then becomes a subtraction model:

$$Ps = 1 - \frac{\sum_{i=1}^{Limit} (Pk/A)\ Pen\ (1 - P_k/A)^{i-1}}{Pen}$$

where Pen $(1 - P_k/A)^{i-1}$ is the number of penetrators surviving after cycle "i". The Limit is determined by either the number of cycles penetrators are under coverage, or, a lesser number, the number of cycles that the defense can generate AIs based on AIs available.

It has been assumed that the AIs can service the penetrators all along their route. If AI basing or combat radius precludes some of these assignments a further reduction in assignments is required. (Different penetrator and AI types will be considered later in this chapter.)

If lethal self defense is employed against missiles launched from an AI, the value of Pk/A must be reduced, to account for those missiles which are successfully intercepted by penetrator weapons.

A summary of the example results for AI Limits and AIs Reqd:

AI Limit: With base suppression = 894, No base suppression = 2980
AIs Reqd: Against 100 Pen = 1300, Against 300 Pen = 3900

The penetrator's expected survival for base attack, 300 penetrators, and only 894 AIs available is noted below:

Nr Cycle	Prob Assign	Nr AIs	Exp. Kills	Cum AIs Assigned
1st	1	600	30	600
2nd	.9	294*	14.7	894 = Available
		Total =	44.7	

* The limit of 894 AIs was reached on this cycle.

$$Ps = 1 - 44.7/300 = \underline{.85}$$

The table below illustrates the four possible results in the example:

Base Attack?	AI Limit	Nr Pen	AIs Used	Exp. Kills	Ps
No	2980	100	1300	65.2	.35
No	2980	300	2980	149	.50
Yes	894	100	894	44.7	.55
Yes	894	300	894	44.7	.85

Note that either higher numbers of penetrators (300 compared to 100) or AI base attack has a significant impact on penetrator survival. With both higher numbers and AI base attack, there is a dramatic increase in penetrator survival (from .35 to .85 in the example).

The results apply equally to small or large numbers of penetrators as long as the ratio of AIs to penetrators stays the same. For example, the results for 10 penetrators and 200 AIs would be the same as for 100 penetrators and 2000 AIs in this model, so far. The ratio of the number of air vehicles entering coverage to the number of AIs was found to be a dominant factor. However, a large mass of penetrators can degrade assignment efficiency. The defense may then try to discriminate high value penetrators (such as heavy bombers with many weapons) from lower value penetrators (such as fighter bombers or cruise missiles) in order to concentrate their forces. These two factors (mass and discrimination) are considered next.

Discrimination and Mass

Assume that the defense system can discriminate against low value penetrators with probability $P_{discrim}$ and that those penetrators which are initially labeled as "low value" never receive assignments. The required number of AIs against low value penetrators includes only those incorrectly designated and engaged as being "high value". AIs required against these low value intruders will be called ΔAIs Reqd and can be calculated as:

$$\Delta \text{AIs Reqd} = (\text{Nr of low value pen})(1 - P_{\text{discrim}})(\text{Exp A/p})(\text{AIs/A})$$

The Ps for the low value penetrators is:

$$\text{Ps} = 1 - (\text{Exp. Nr of low value pen killed / Nr of low value pen})$$

Massing of penetrators (e.g. at entry points or in corridors) can degrade the defense by delaying or denying assignments (due to confusion and to dilution/saturation of defense resources). Assume that exercise data and analyses allow a factor, f_{mass}, to be estimated for a particular campaign and defense capability to account for the remaining assignment effectiveness with mass degrades. (The derivation of mass effects is a separate subject beyond the scope of this text.) If a mass factor can be assessed, the degraded AI Limit is:

$$\text{AI Limit*} = (f_{mass})(\text{AI Limit})$$

A mass degrade and a mix of low and high value penetrators will be illustrated by examples: assume that both the mass factor and probability of correct discrimination (vs low value penetrators) are .8, and that either 1000 (or 3000) low value air vehicles are added for the low (and high) readiness conditions respectively. If the Pk per assignment against low value penetrators is .2, their time in coverage is 50 minutes and the earlier assumptions hold for all other factors, the AI Limit, expected kills and Ps for both the high and low value penetrators are developed next. For the AI Limit:

With base attack, AI Limit* = (.8)(894) = 715 AIs

No base attack, AI Limit* = (.8)(2980) = 2380 AIs

Expected kills and Ps values for high and low value penetrators are evaluated below for the case of no base attack (2380 AI Limit*), 100 high value and 1000 low value penetrators, with only 20% of the later penetrators drawing AI assignments. The low value penetrators have a P_k/A of 0.2 and a 50 minute penetration time (five AI cycles).

	High Value Pen			Low Value Pen		
Nr Cycle	Prob Assign	Nr AIs	Exp. Kills	Prob Assign	Nr AIs	Exp. Kills
1st	1	200	10.0	1	400	40
2nd	.9	180	9.0	.8	320	32
3rd	$.9^2$	162	8.1	$.8^2$	256	25.6
4th	$.9^3$	146	7.3	$.8^3$	205	20.5
5th	$.9^4$	131	6.6	$.8^4$	164	16.4
6th	$.9^5$	118	5.9	0	0	0
7th	$.9^6$	98*	4.9	0	0	0
		Total =	51.8		Total =	134.5

* The limit of 2380 AIs was reached on this cycle.

The Ps for the high value penetrators = 1 - 51.8/100 = <u>.48</u>

The Ps for the low value penetrators = 1 - 134.5/1000 = <u>.87</u>

With base attack, 100 high value (and 1000 low value) penetrators, and an AI Limit* of 715:

	High Value Pen			Low Value Pen		
Nr Cycle	Prob Assign	Nr AIs	Exp. Kills	Prob Assign	Nr. AIs	Exp. Kills
1st	1	200	10.0	1	400	40
2nd	.9	41.4*	2.1	.8	73.6*	7.4
		Total =	12.1		Total =	47.4

* The limit of 715 AIs was reached on this cycle.

The Ps for high value penetrators = 1 - 12.1/100 = .88

The Ps for low value penetrators = 1 - 47.4/1000 = .95

Survival values for the four example cases are shown below:

Base Attack?	AI Limit	High Value Pen Nr	Ps	Low Value Pen Nr	Engaged	Ps
No	2380	100	.48	1000	200	.87
No	2380	300	.87	3000	600	.95
Yes	715	100	.88	1000	200	.95
Yes	715	300	.96	3000	600	.98

Note that the addition of low value penetrators which draw some AIs increases high value penetrator survival markedly. The Ps increased for the case with no base attack from .35 to .48 for 100 penetrators, and from .50 to .87 for 300 penetrators. With base attack, the Ps values increased from .55 to .88 with 100 penetrators, and from .85 to .96 with 300 penetrators.

About the Author . . .

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Frank T. Heilenday is a consultant to the aerospace industry in the fields of air defense, air vehicle penetration and operations research and electronic countermeasures. He is a graduate of Massachusetts Institute of Technology and holds bachelor and masters degrees in science. He has held positions in the defense industry as a senior dynamics engineer with General Dynamics and was a flight research engineer with Calspan. In 1955, he was employed by USAF/SAC as a military operations research engineer. He eventually served as chief of the SAC applied research division with responsibility for the entire research and development program of the air arm. His work in this area was acknowledged by the Air Force which awarded him the Meritorious Civilian Service Medal in 1981. He also received the Exceptional Civilian Service Medal in 1984 primarily for his work in air vehicle penetration studies and analysis.