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6. AUTHOR(S)

Mark J. Ablowitz

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Department of Applied Mathematics
University of Colorado
Campus Box 526
Boulder, CO 80309-0526

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Arje Nachman
Program Director, Physical Mathematics
AFOSR/NM
801 N. Randolph St. Room 732
Arlington, VA 22203-1977

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13. ABSTRACT *(Maximum 200 Words)*

This research effort involves the study of the nonlinear wave propagation arising in physical problems. There were a number of significant accomplishments during the period of the grant. Since January 1, 1997, 15 papers were published or accepted, 10 book chapters and papers in conference proceedings were published or accepted, 2 preprints were written and 21 invited lectures were given. Research in quadratically polarized nonlinear optical materials was carried out with novel systems of equations and new classes of stable, localized pulses obtained. Studies of discrete optical waveguides led to a new class of coupled discrete nonlinear systems and the general solution to the associated initial value problem with decaying data was obtained. Research in optical communications has yielded a number of important results including novel equations governing dispersion managed optical systems and an analytical theory governing the timing jitter and four wave mixing due to soliton collisions in WDM systems. The chaotic dynamics of a class of fluid dynamical waves, theoretically predicted by the PI based on earlier work involving numerical chaos, has been observed experimentally. A new class of reflectionless potentials of the nonstationary Schrödinger scattering problem and localized solutions of the Kadomtsev-Petviashvili equation have been obtained.

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Nonlinear Wave Propagation
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Mark J. Ablowitz
Department of Applied Mathematics
University of Colorado
Boulder, CO 80309-0526
Phone: 303-492-5502
Fax:303-492-4066
email address: markjab@newton.colorado.edu

OBJECTIVES

To carry out fundamental and wide ranging research investigations involving the nonlinear wave propagation that arises in physically significant systems. Application areas include studies in nonlinear optics, solutions of multidimensional nonlinear equations, inverse scattering, nonlinear wave dynamics in magnetics films, computationally induced chaos and chaotic dynamics in physical systems.

STATUS OF EFFORT

The research program of the PI in the field of nonlinear wave propagation continues to be very active. There have been a number of important research contributions during the past three years. During the period January 1, 1997 – December 31, 1999, 15 papers were published or accepted for publication in refereed journals, 5 book chapters were published or accepted, 5 conference proceedings were published or accepted, 2 preprints were written and 21 invited lectures were given.

Research investigations in nonlinear optics were varied. In quadratically nonlinear optical media (so called $\chi^{(2)}$ materials) we have derived multidimensional equations governing the asymptotic evolution of quasi-monochromatic optical pulses. The equations obtained are of nonlinear Schrödinger type with coupling to a mean field; we find either scalar or vector systems depending on the polarization. In the scalar case numerical simulations indicate that stable localized waves can result. Soliton interactions in fiber optics, including the implications regarding four wave mixing and timing jitter on communications systems were studied. Novel nonlocal nonlinear Schrödinger (NLS) type equations governing dispersion managed soliton systems in fiber optics were obtained.

The PIs studies of vector discrete NLS systems have been motivated by recent experimental research on discrete optical waveguides. New classes of vector discrete soliton systems, some of which are integrable by the inverse scattering transform have been obtained. The scattering problem has intrinsic interest in its own right.

A new class of lump type solutions of the multidimensional Kadomtsev-Petviashvili (KP) equation and associated eigenfunctions of the nonstationary Schrödinger scattering problem were found. These lump type solutions decay in all directions and exhibit unusual scattering behavior unlike previously known cases. These solutions of the KP equation, which are also "reflectionless potentials" of the nonstationary Schrödinger equation, are characterized by an underlying integer which plays the role of an index or winding number. Given the importance of the KP and the nonstationary Schrödinger equations, this result has important implications in physics and inverse scattering.

Research on nonlinear waves in thin magnetic films was carried out. For those one dimensional wave propagation problems which model laboratory experiments, the governing NLS equations have been derived, and the coefficients have been deduced from first principles. Extensions to multidimensions along the lines of $\chi^{(2)}$ nonlinear optics is being studied. There is considerable experimental interest in the dynamics of multidimensional pulses in these magnetic systems.

Research involving the study of computationally induced chaos in discrete systems related to the NLS and the sine-Gordon equation continues. As part of this research, comparisons between symplectic integrators and "off the shelf" adaptive Runge-Kutta (RK) algorithms were made. Unlike low dimensional dynamical systems our numerical studies of high dimensional Hamiltonian wave systems indicate little error improvement when symplectic algorithms were employed instead of adaptive RK algorithms.

Research studies of computational chaos have indicated novel ways in which high dimensional chaotic dynamics can be induced in physical systems governed by perturbed NLS equations. Recently, water wave experiments and associated theory involving slow modulations of the classical periodic Stokes water wave have delineated, for the first time, parameter regimes where chaotic dynamics is predicted and observed. Similar results can be expected for modulated nonlinear periodic waves in nonlinear optical media. This is a topic for future study.

ACCOMPLISHMENTS/NEW FINDINGS

Nonlinear Optics: Multi-dimensional pulse propagation in $\chi^{(2)}$ optical materials

Unlike fiber optics where the leading order nonlinear polarization effect is cubically nonlinear, in many important optical applications the underlying leading order nonlinearity is quadratic (these are called "$\chi^{(2)}$" materials). In our work [J-6] (this can be found in the section on publications of the PI) we found that in multidimensional nonresonant $\chi^{(2)}$ materials, the nonlinear equations governing the slowly varying envelope of quasi-monochromatic wave trains is not the NLS equation but rather a coupled nonlinear system involving both the optical field and mean terms. We call these equations NLSM systems (M stands for the mean contribution). In water waves analogous scalar systems were derived years ago by Benney and Roskes. A few years later, a special case of this system was found to be integrable. The latter system is often referred to as the Davey-Stewartson (DS) system. The results from these investigations have been useful in our studies.

In the optical application, we derived both scalar and vector NLSM systems directly from Maxwell's equations. The vector NLSM systems generalize to multidimensions the well-known 1+1 vector NLS equations. To our knowledge, the vector multidimensional systems have no analog in water waves. For the scalar NLSM system we have been able to find localized optical pulse solutions. These localized pulses are induced by their interaction with mean terms that have nontrivial boundary values. Hence the localized pulses are boundary induced. This situation is similar to the situation that is known to occur for the Davey-Stewartson system, only in this case the system is unlikely to be integrable. In recent computations we have been able to find improved approximations to the underlying localized pulse with negligible radiation present.

These findings suggest that stable localized multidimensional pulses are a generic feature of the NLSM type equations studied here. We believe that the above described dynamical configuration can be designed experimentally. This possibility is particularly interesting because such experiments could allow the production of stable localized multidimensional optical pulses whose dynamics can be electrically controlled by modification of the relevant d.c. fields.

Potential applications include beam steering, pulse shaping terrahertz imaging and optical switching.

Nonlinear Optics: Discrete optical wave systems and discrete inverse scattering

Discrete soliton systems have been a long standing topic of interest in our research program. Years ago the PI obtained a discrete NLS equation which possesses solitons and can be solved by the inverse scattering transform (IST). This system, called the integrable discrete NLS equation (IDNLS) has been found to apply in a number of physical contexts and is also often used to approximate the so called diagonally discrete NLS system (DDNLS). It is interesting that recent experimental research has confirmed theoretical predictions of the existence of discrete optical solitons in coupled optical waveguides. The experimental situation appears to be closely described by the DDNLS eq. Our recent research investigations have shown that a class of vector discrete nonlinear systems have exact soliton solutions and possess many of the features found in their continuous analog [J-10]. These systems also exhibit strong vector-polarization interaction effects. The equations we have been investigating are of the form:

$$iu_z + (u(n+1) + u(n-1) - 2u(n))/h^2 + (|u(n)|^2 + a|v(n)|^2)(L(u(n)) = 0$$

$$iv_z + (v(n+1) + v(n-1) - 2v(n))/h^2 + (a|u(n)|^2 + |v(n)|^2)(L(v(n)) = 0$$

where a) $L(u(n)) = 2u(n)$ and b) $L(u(n)) = u(n+1) + u(n-1)$. In the latter case (b) we have developed detailed analytical and numerical results and have obtained the complete solution by IST. For example we have obtained soliton solutions, conservation laws and we find wide regions of parameter space where large internal energy is transferred between the u and v components of a two soliton system. In the future we intend to analyze and compare both discrete systems in detail. The experimental work on scalar discrete systems suggests that the novel vector discrete systems of the types we are studying are physically relevant and may be potentially important for device applications such as optical switching.

The scattering problem which applies to the IST of the scalar integrable system (the 2x2 scattering problem) is of the form:

$$\psi_{1_{n+1}} = \lambda\psi_{1_n} + q_n\psi_{2_n}$$

$$\psi_{2_{n+1}} = \tfrac{1}{\lambda}\psi_{2_n} + r_n\psi_{1_n},$$

where q_n, r_n are the potentials and λ is the eigenvalue. Recently we have analyzed the inverse scattering problem for matrix generalizations of arbitrary order of the above 2x2 system (the 2x2 system applies to the scalar discrete NLS equation). In its own right the discrete inverse scattering system is interesting and is likely to have many applications.

We note that the complex function methodology used in our studies of IST in continuous systems can be applied to discrete systems. We find that the scattering theory involves the formulation and solution of a RH problem on a unit circle. The spectral singularities (poles) of the scattering data correspond to solitons, and as a consequence, we have shown that the integrable vector discrete NLS equation has soliton solutions.

Based on our studies of NLS and its discretizations we have also shown that the discrete coupled NLS system provides a useful discretization as a numerical scheme to solve the continuous vector NLS equation. These difference equations, as in the scalar problem, can be expected to provide reliable and effective numerical schemes.

In the future we will study the interaction effects associated with integrable and nonintegrable, vector discrete systems. We wish to develop a detailed understanding of the internal energy transfer between transverse modes.

Discrete solitons in optical waveguides have only recently been observed in experiment. The work described here is the vector extension of the equations governing discrete solitons with one polarization mode. Potential applications involve optical switching and specialized optical communications.

Nonlinear Optics: Soliton communications in fiber optics: Wavelength Division Multiplexing

The development of technologically feasible optical soliton communication systems capable of high speed data transmission has been a major achievement in the field of nonlinear fiber optics. It is well-known that thin optical fibers can support localized pulse/soliton propagation in the anomalous-dispersion regime. Mathematically speaking, the nonlinear Schrödinger (NLS) equation with the addition of suitable perturbative terms, including damping, amplifiers, frequency filters and dispersion management, governs the propagation of such waves. The NLS equation supports multi-soliton solutions and it is these multi-soliton waves which offer significant technological enhancement.

Single channel systems supporting single soliton waves are communication systems in which the solitons are widely separated and there is no possibility of mutual soliton interactions. Laboratory demonstrations have shown that single channel systems are capable of transmitting information at relatively high data rates. However, in order to significantly increase data rates as desired for future communication systems, multi-soliton based systems are being considered. Such systems are usually referred to as wavelength division multiplexed (WDM) systems.

Our research in WDM systems was focused on the study of multisoliton propagation, their interaction properties and associated timing shifts. Our aim is to develop a comprehensive and effective analytical theory describing soliton interactions in both physical and frequency space.

In our work we analyzed multisoliton interactions in ideal and realistic fibers, i.e. soliton interactions in the NLS equation with and without perturbative terms. In general, the formulae are complicated. However, in the limit of large frequency separation, there is a significant simplification, namely our analysis shows that solitons always remain widely separated in frequency space, even when they interact strongly in physical space.

We discovered that: i) significant perturbations can be generated in a different frequency channel from the individual solitons; ii) the perturbations in the new frequency channel are located in specific frequency regimes; more precisely they are excited in the frequency regimes associated with four wave mixing interactions (FWM) contributions; iv) in an ideal fiber we show that as solitons interact, the FWM contributions grow from a zero background and then decay back to zero; v) the results demonstrate, however, that this is not the situation in realistic systems which include damping and amplification. The result is that FWM signals are greatly magnified; they grow and then saturate to become a nontrivial state.

From a communications standpoint, such FWM contributions are undesirable. An important research problem involves how to control/eliminate this phenomena. Our work indicates that optical fibers which employ suitable dispersion parameters (often referred to as dispersion managed fibers) can significantly reduce these undesirable FWM effects.

Another serious problem in WDM soliton technologies is the effect of anomalous timing displacements which are due to multi-soliton interactions. Timing displacements occur when perturbative terms such as damping, amplifiers and frequency filters are present. In recent work we have developed a theory capable of analyzing a wide range of perturbative contributions including damping, amplification and frequency filters. We have shown that in WDM systems the resulting natural collisions of solitons leads to variance in arrival times and hence multisoliton jitter. In our original work we only took into account damping and amplification [J-5]. Subsequently, we added filters and dispersion variation following the loss profile [J-7]. Both filters as well as dispersion following the loss profile significantly reduce timing jitter. A statistical analysis also allows us to consider an arbitrary number of communication channels. Formulae for the mean square timing shift are derived. We have compared the mean square timing shift, with filters included, to the result without filters. It is found that filters decrease the mean square timing shift by about an order of magnitude over a distance of 10,000 km. The effect of dispersion management, following the loss profile, further decreases the timing jitter, and it allows for a significant increase in the number of error free communication channels. We also developed a notion of optimal dispersion managed systems which depend on certain suitable ratios of fibers with different dispersion characteristics.

We recently considered the effect of using lumped type filters and compared it with using the well known distributed filter approximation [cf. J-12]. In this work we show that the distributed approximation, obtained as a continuous limit of many lumped filters, is a good approximation for a wide range of parameters. Most researchers employ distributed approximations. To our knowledge, this work is the first detailed comparison of lumped vs. distributed models.

In the future we shall consider large scale dispersion management by employing dispersion managed fibers with (asymptotically) large average positive and negative dispersion. It is believed that this will be even more effective at reducing timing jitter than dispersion following the loss profile.

In recent work [J-8] we have derived a novel nonlocal nonlinear Schrödinger equation which governs the asymptotic dynamics of strong dispersion managed soliton communication systems. Unlike the derivation of the usual NLS equation with moderate dispersion management, the leading order contribution has strong phase dependence. We find that in frequency space the leading order contribution separates into a product of a slowly varying amplitude and a contribution with a rapidly varying phase—the latter can be found exactly. In physical space the corresponding representation is a convolution integral of a slowly varying amplitude and a term with a rapidly varying phase.

We show that the amplitude satisfies a nonlocal NLS type equation where the linear terms are the same as the usual NLS equation, but the nonlinear terms are nonlocal. Using a novel iterative computational scheme, we have been able to solve these nonlocal equations and obtain their underlying solitary wave solutions. The solution is comprised of a Gaussian-like core with a decaying tail that vanishes at the same exponential rate as the usual soliton of NLS. Comparison with well-known direct numerical simulations demonstrates remarkable agreement. Conserved quantities for the mass, momentum and the Hamiltonian of the system are obtained. We are currently extending this study to numerically solve the time dependent nonlocal NLS equation in order to study the interaction effects of these dispersion managed solitons.

In the future we shall use the above mentioned nonlocal NLS type systems to understand and estimate potential reductions of four wave interaction effects as well as reductions in timing jitter.

This work is important in the field of fiber optic communication since there is considerable interest in ultra fast data transmission. A major research direction in the communications field is wavelength division multiplexing, either via soliton or other technologies.

New classes of lump type solutions in Multidimensional Nonlinear Wave Equations

In our earlier work on 2+1 multidimensional nonlinear wave equations we found an important special class of solutions, namely two dimensional lumps which are solitons/coherent structures which decay in all directions.

Recently we have found a new class of lump type solutions of the Kadomtsev-Petviashvili (KP) equation, which we call multipole lumps. Associated with the KP equation is a linear scattering problem which in this case is the nonstationary Schrödinger equation. Lump type solutions of the KP equations correspond to reflectionless potentials of the the nonstationary Schrödinger problem. We have also found solutions of the nonstationary Schrödinger equation corresponding to these potentials. Given the importance of the nonstationary Schrödinger equation, this work has two equally important themes: solutions of the KP equation and solutions of the nonstationary Schrödinger equation.

Spectrally speaking, these new coherent structure solutions correspond to multiple poles associated with certain eigenfunctions of the nonstationary Schrödinger problem. We have found that these solutions are characterized by an integer which is related to a winding number, or index. We call this number the charge: Q.

The simplest example of a multipole lump is the following. In the the usual spectral description of, say, a one lump solution, the eigenfunction has one pair of poles symmetrically located in the upper/lower half planes. The charge associated with a simple lump is unity. Next let's consider a standard two lump solution. In this case, the eigenfunction has two pairs of poles symmetrically located in the upper/lower half planes. A two lump solution has an overall index of two obtained by simply adding the individual indices of each lump. We have shown, both by taking coalescing limits of (two) lump solutions and by direct analysis of the scattering problem, that the spectral configuration has a double pole in one of the half planes and a simple pole in the other. This new state has index two, which is consistent with the fact that in the limit process one cannot lose "charge".

This process carries on to higher order multipole lumps. We have obtained a number of surprising results which we summarize below [see J-1, J-11, J-13, B-3].

i) The multipole lump solutions are associated with an integer which is related to a winding number. Thus we have found a new underlying index associated with the nonstationary Schrödinger problem. Simple lumps have charge = 1. We have found that higher order lump type solutions can have any integer charge.

ii) The solutions of the nonstationary Schrödinger equation have multiple poles. The poles can have different orders in the upper/lower half planes. We call the order in the UHP/LHP as $m/\bar{m}$ resp. Previously known solutions had only simple poles.

iii) The solution manifold is characterized by the order of the poles of the nonstationary Schrödinger equation and the charge; i.e. $m, \bar{m}, Q$.

iv) The solutions associated with the Kadomtsev-Petviashvili equation have more complicated interaction properties than the previously known lump solutions.

We are currently investigating whether all the the multipole lumps can be succinctly written in terms of the second logarithmic derivative of determinants of polynomials. We are also considering the lump solutions of other physically significant 2+1 dimensional equations such as the Davey Stewartson equation.

This work is important for anyone studying scattering theory in multidimensions as well as nonlinear wave equations possessing multidimensional solitons. The underlying wave equations arise frequently in application as does the direct and inverse scattering problems.

Nonlinear Waves in Ferromagnetic Films

We have studied a class of nonlinear waves in ferromagnetic media. Motivation for these studies comes from experiments by Professor Carl Patton and his group in the Physics Department at Colorado State University. Patton's group has been investigating the generation and evolution of soliton wave pulses in thin film ferromagnets. In these experiments, an yttrium iron garnet (YIG) film is magnetized to saturation causing the dipoles of the ferromagnet to align. An external microwave signal is applied to the film. If the power is large enough, solitons are observed to form and propagate through the film. Our goal is to develop a comprehensive and effective theory governing the propagation of waves in such ferromagnetic media. We are motivated by the strong analogy that exists with nonlinear optics.

In optics the nonlinearity arises from the fact that the polarization is a nonlinear function of the electromagnetic field. In the magnetic systems we have been studying the role of the polarization is played by the magnetization (M) and the electric field is replaced by the magnetic field (H). The nonlinear terms of the magnetic system are generated by a torque equation which describes the precession of dipoles in the magnetic media. This difference is important since the way the nonlinearity arises usually has a major effect on the amplitude equations (i.e. NLS type equations). The fact that we are modeling films means that we must consider three regions: two outside the film and the film itself. Outside the film we take the magnetostatic approximation of the vacuum Maxwell's equations; inside the film is where the torque equation is applied. This means that the way the amplitude equations are derived in a perturbation analysis is different from infinite media. In infinite media, or what is sometimes referred to as bulk media, the amplitude equations result from secularity conditions which require no unbounded growth at infinity. In the case of films the amplitude equations result from Fredholm solvability conditions on the underlying linear system which is found by matching the three wave regimes outside/inside the film.

The calculations are extremely lengthy, but we obtained the nonlinear Schrödinger equation that governs so called forward volume waves—where the applied magnetic field is applied both perpendicular to the film and in the direction of propagation. Previously the NLS equation had only been derived in a heuristic manner.

Computational and Effective Chaotic Dynamics

a) Computational Chaos

We are continuing our investigations involving the computational simulations of a class of nonlinear equations which are perturbations of equations that can be analyzed via the inverse scattering transform (IST). To date we have studied a class of discrete equations which in the continuous limit are also approximations to the nonlinear Schrödinger, modified KdV and sine-Gordon equations (cf. J-2, J-4, J-15). The underlying IST based equations are used as prototypes since they are physically interesting systems, about whose solutions and properties we have concrete analytical understanding. Computationally speaking the discrete equations we are studying provide a vehicle by which: i) computational schemes can be compared and ii) errors in the schemes can be detected. We have found that in certain circumstances computational temporally irregular/chaotic dynamics result. Since these are long time numerical integrations of nonlinear systems, there is no existing theory of error analysis which describes the phenomena.

In our earlier work, we studied the computational chaos associated with the NLS equation with periodic boundary values. We found that the chaos could be excited by both truncation as well as roundoff errors. It will be noted below that the chaotic dynamics we have observed is also a physical phenomena recently observed in laboratory experiments. Thus, for example, the NLS equation is known to govern the modulation of water waves in moderate-deep water and modulational instability in nonlinear optics. When the waves are excited in a periodic manner with small modulation, then the NLS equation with periodic boundary values is the relevant leading order equation governing the physical problem.

With appropriate parameters, the simplest periodically generated waves of the NLS equation are modulationally unstable with M unstable modes of the linearized version of the NLS equation. The NLS with small perturbations, due to the discretization, governs the long time evolution. Thus the problem amounts to understanding the long time dynamics of the NLS equation under small perturbations, be they computationally induced, in this case, or physically induced, as discussed below.

Our computational results, based on extensive numerical and analytical results, indicate that depending on the parameters of the system, there is a significant difference in the long time dynamics depending on whether the perturbations induce evolution close to homoclinic manifolds. Evolution near to homoclinic manifolds indicates that the NLS equation is itself highly unstable in much the same way as are coupled pendula nearly in the "up-position". Thus small perturbations due to numerical errors—or physical perturbations—are capable of causing serious temporal irregularities/dynamical chaotic dynamics in the evolution.

Our computational results, and associated analytical results, show that, for typical parameter regimes, there can significant differences in the long time dynamics depending on whether one excites a small number of unstable modes M (e.g. M=1) or a larger number (e.g. M=3–5). In the former case (M=1), the dynamical evolution should be nonchaotic, explainable and repeatable, in the context of pure NLS theory. In the latter situation (M=3–5), the small perturbations can induce chaotic dynamics. It turns out that the number of unstable modes, M, is the same as the order of the homoclinic manifold. The larger M is, the more likely we are to evolve nearby a homoclinic manifold and execute a homoclinic transition.

Depending on values of the parameters, we have shown that:

a) Computational chaos can result from truncation errors.

b) For suitably large values of M we have demonstrated that the numerical chaos can even be induced by roundoff errors.

c) For even initial data the phase space is foliated and the chaos is explained by continual and temporally irregular crossings of unperturbed homoclinic manifolds (i.e. crossing of the NLS homoclinic manifolds).

d) When the initial data is not even, the phase space is no longer foliated and the solution to the perturbed NLS system can evolve from one "side" of the homoclinic manifold to another without crossing an unperturbed homoclinic manifold. We refer to this situation as a homoclinic transition. The dynamics we have observed in the latter case is depicted by irregular and continual changes of the velocity of the underlying periodic waves. The case of even initial data is typified by the periodic waves being essentially standing waves (no left/right velocity).

e) The situation when the initial data is not even, or the perturbation is not even, is the generic case. Our recent work on water waves has shown that the homoclinic transitions observed in part d) actually occur, but during the time scales we consider, homoclinic crossings like those in part c) above do not occur.

We have also studied the sine-Gordon equation with periodic initial values. Again we find that numerical discretizations of the equation can lead to chaotic dynamics. We have compared a range of numerical schemes based upon their ability to preserve the underlying spectrum of the associated scattering problem which is used to solve the sine-Gordon equation. We found that the spatial discretization plays a more important role than the temporal scheme with different spatial discretizations yielding significantly different results. We have found that pseudo-spectral discretizations are far superior to standard finite difference simulations.

Surprisingly we find "off the shelf" adaptive Runge-Kutta (RK) type algorithms (from the NAG software routines) perform as well as symplectic integrators. The symplectic integrators performed better when they were higher order, with fourth order symplectic algorithms performing about as well as RK algorithms. This calls into question whether symplectic integrators are as useful for the integration of Hamiltonian PDEs as many of its supporters had hoped.

We are continuing our studies of computational chaos related to physically significant equations and comparisons of symplectic integrator's with standard algorithms, e.g. Runge-Kutta algorithms in wider parameter regimes. We believe continuing these studies is important in order for researchers to understand whether these relatively new simplectic based numerical methods are as useful for Hamiltonian PDEs as they are for finite (low) dimensional dynamical systems.

b) Chaotic dynamics in nonlinear wave systems

As mentioned above our research has suggested that the underlying dynamics leading to computational chaos could also be a manifestation of a physical effect potentially observable in laboratory experiments. For example, the NLS equation is known to govern the slow modulation of water waves in moderate-deep water and modulational instability in nonlinear optics. We began working with water wave experimentalists J. Hammack and D. Henderson at the Pennsylvania State University.

Remarkably, in their experiments, Hammack and Henderson have recently observed temporally irregular and chaotic dynamics of water waves in a modulational unstable regime. In their laboratory investigations carefully controlled modulated waves (using state of the art equipment) are excited by a paddle in a periodic manner at the entrance of the tank. Measurements are taken at downstream locations. The data received are then compared with identical experiments conducted at subsequent times. It is found that there are serious discrepancies between the data sets at downstream positions; the discrepancies are magnified as one proceeds downstream. We have worked closely with Hammack and Henderson in order to develop an analytical/numerical framework in order to explain the phenomena.

We have found that the NLS equation with suitable small higher order corrections and periodic boundary data is the relevant equation. We call this the PNLS equation (P stands for perturbed).

As indicated above, the NLS equation has a simple periodic solution which is modulationally unstable. This instability corresponds to the well-known Benjamin-Feir modulational instability of deep water waves. Based on initial conditions there will be a number (which we call M) of unstable modes of the unperturbed NLS equation. The PNLS equation is used as the model governing the long time evolution. Hammack and Henderson have observed that, when there are M=3 unstable modes, the experiment is nonrepeatable. But, when they use solitons as initial data, the experiments are repeatable. Earlier experiments have primarily dealt with solitons. Our results based on direct numerical simulations of the PNLS equation and associated numerical integration of the spectrum of the scattering problem associated with the NLS equation confirm the observations of Hammack and Henderson. We used their experimental data as initial conditions in our computations of PNLS and confirmed that they were in the M=3 regime. When we repeated the numerics with noise corresponding to the same order of laboratory noise, we find similar discrepancies as do Hammack and Henderson. The waveform was not repeatable. We found serious temporal irregularities and numerous homoclinic transitions in the spectral data. When we did the numerics for solitons, we do not find any difficulties—no temporal irregularities, and no spectral homoclinic transitions or homoclinic crossings.

Since the NLS equation is a universal asymptotic equation, this research is important for many other applications. We are continuing this work, and we will use it as a base to develop a related theory of chaotic dynamics associated with modulational instability in optics and magnetics.

PERSONNEL SUPPORTED

- Faculty: Mark J. Ablowitz
- Post-Doctoral Associate: G. Biondini (1997–1999)
- Graduate Student: R. Horne (1997–1998)
- Other (please list role) None

PUBLICATIONS

- ACCEPTED
  - Books/Book Chapters

B-1. Painlevé Test and Painlevé Type Equations, M.J. Ablowitz, in *Encyclopaedia of Mathematics Supplement,* Volume I, Editor-in-Chief, Prof.dr. M. Hazewinkel, Kluwer Acad. Publ., 1997, 397–398.

B-2. Soliton Communications and Wavelength-division Multiplexing, M.J. Ablowitz, G. Biondini, and S. Chakravarty, in *Mathematical and Numerical Aspects of Wave Propagation,* Ed. John A. DeSanto, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1998, 145–149.

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J-6. Multi-dimensional Pulse Propagation in Non-resonant $\chi(2)$ Materials, M.J. Ablowitz, G. Biondini and S. Blair, *Phys. Lett. A,* **236** (1997) 520–524.

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J-10. On Discretization of the Vector Nonlinear Schrödinger Equation, by M.J. Ablowitz, Y. Ohta and A.D. Trubatch, *Phys. Lett. A,* **253** (1999) 287–304.

J-11. On the discrete spectrum of the nonstationary Schrödinger equation, and multipole lumps of the Kadomtsev-Petviashvili I equation, J. Villarroel and M.J. Ablowitz, *Commun. Math. Phys.,* **207** (1999) 1–42.

J-12. A Comparison Between Lumped and Distributed Filter Models in Wavelength-division Multiplexed Soliton Systems, M.J. Ablowitz, G. Biondini, S. Chakravarty and R. Horne, *Optics Communications,* **172** (1999) 211–227.

J-13. Novel potentials of the nonstationary Schrödinger equation and solutions of the Kadomtsev-Petviashvili I equation, M.J. Ablowitz, S. Chakravarty, A.D. Trubatch and J. Villarroel, APPM† #389 (July 1998), *Phys. Lett. A,* accepted.

J-14. On the Extension of the Painlevé Property to Difference Equations, M.J. Ablowitz, R. Halburd and B. Herbst, APPM† #403, February 1999, *Nonlinearity,* accepted.

J-15. Modulated Periodic Waves in Deep Water, M. Ablowitz, J. Hammack, D. Henderson, C. Schober, APPM† #410, Aug 1999, *Phys. Rev. Lett.,* accepted.

– **Conference Proceedings**

C-1. Dynamics of Multi-phase Solutions of a Perturbed Nonlinear Schrödinger Equation, M.J. Ablowitz and C.M. Schober, APPM† #318 (May 1997), accepted *Proc. IMACS Conf.*

C-2. Collision Induced Timing Jitter in Dispersion Managed Soliton Systems, M.J. Ablowitz, G. Biondini, S. Chakravarty and R.L. Horne, in *Nonlinear Guided Waves and Their Applications,* Technical Digest Series, Volume 5, Conference Edition, Optical Society of America, Washington, DC, 1998, 181–183.

C-3. Multidimensional Optical Pulses in Non-resonant Quadratic Materials, M.J. Ablowitz, G. Biondini and S. Blair, *Technical Digest, IEEE/LEOS Conference: Nonlinear Optics '98;* Materials, Fundamentals and Applications Topical Meeting, IEEE Catalog #: 98CH36244, pp.162–164.

C-4. On Integrability and Chaos in Discrete Systems, by M.J. Ablowitz, Y. Ohta and A.D. Trubatch, APPM† #350 (March 1998), *Proceedings on Chaos and Integrability*, accepted.

C-5. Navanlinna Theory and Difference Equations of Painlevé Type, M.J. Ablowitz and R. Halburd, APPM† #420, Dec. 1999, *Conference Proceedings, Nonlinearity, Integrability and All That—20 Years after NEEDS '79,* Gallipoli, Italy, July 1999, accepted.

– **SUBMITTED (Preprints)**

* **Books/Book Chapters — (Preprints)** none
* **Journals — (Preprints)**

P-1. Localized Multidimensional Optical Pulses in Non-resonant Quadratic Materials, M.J. Ablowitz, G. Biondini and S. Blair, APPM† #395 (October 1999), to be submitted to *Phys. Lett. A.*

P-2. Solutions of a Burgers-Stefan Problem, M.J. Ablowitz and S. De Lillo, APPM† #419, November 1999, *Physics Letters,* submitted.

APPM†: Department of Applied Mathematics report

* **Conferences — (Preprints)** none

INTERACTIONS/TRANSITIONS

- Participation/Presentations At Meetings, Conferences, Seminars, Etc.

1. Conference on Soliton Theory, PDE's and Nonlinear Analysis, University of New South Wales, Sydney, Australia, "On Solutions of the Nonstationary Schrödinger and Kadomtsev-Petviashvili Equations", Jan 6-9, 1997.

2. Conference on Chaos and Integrability in Discrete Systems, International Solvay Institutes for Physics and Chemistry, Vrije University Brussels, Brussels, Belgium, "Computational and Effective Chaos in Integrable Systems", July 1-5, 1997.
3. AFOSR Workshop on Nonlinear Optics, Tucson, Arizona, "Wavelength Division Multiplexed (WDM) Solutions: Collision Induced Timing Jitter", September 24, 1997.
4. First Pacific Rim Conference on Mathematics, Hong Kong, "On Solutions to the Nonstationary Schrödinger Equation and the Kadomtsev-Petviashvili Equation", January 21, 1998.
5. Mathematical Association of America Meeting, New Orleans, Louisiana, "Solitons: from Water Waves to Telecommunications", February 28, 1998.
6. Meeting on *Novel Solitons: Nonlinear Guided Waves,* Victoria, Canada, "Four Wave Mixing", March 29, 1998
7. Department of Applied Mathematics Colloquium, University of Colorado, Boulder, "Solitons: from Water Waves to Communication and Encryption", April 3, 1998.
8. Pennsylvania State University, State College, PA, "Computational Chaos in the Nonlinear Schrödinger Equation", April 24, 1998.
9. International Conference on *Solitons, Geometry and Topology: On the Crossroads,* Moscow, Russia, "On Darboux-Halphen (DH) Systems", May 25, 1998.
10. International Meeting on *Integrable Systems: Solutions and Transformations,* Guardamar, Spain, "On Darboux-Halphen (DH) Systems", June 15, 1998.
11. Workshop in Nonlinear Optics, Mathematics Department, University of Arizona, Tucson, Arizona, "Multidimensional Optical Pulses in Chi-2 Materials", September 23–26, 1998.
12. Technion-Israel Institute of Science, Department of Mathematics, Haifa, Israel, "Dispersion Managed Soliton Communications", March 29, 1999.
13. Weizmann Institute of Science, Department of Applied Mathematics, Rehovot, Israel, "Soliton Communications", March 30, 1999.
14. Florida State University, Department of Mathematics, Tallahassee, Florida, "Soliton Communications", April 16, 1999.
15. Conference on *Nonlinearity, Integrability and All That—20 Years after NEEDS '79,* Gallipoli (Lecce), Italy, July 1–10, 1999, "On Difference Analogs of Painlevé Equations", July 4, 1999.

16. Conference on *Solitons, Collapses, and Turbulence,* Landau Institute, Chernogolovka, (Moscow) Russia, Aug 3-10, 1999, "Modulated Periodic Waves in Deep Water", Aug. 6, 1999.

17. Conference: *Nonlinear Optics Workshop,* Tucson, Arizona, Sept. 16-18, 1999, "Long Time Dynamics of Modulational Instability", September 17, 1999.

18. University of Maryland, Department of Mathematics, Novikov Seminar, College Park, Md., "Modulated Periodic Waves in Deep Water", Oct. 23, 1999.

19. Conference on *Integrating Integrability into Mathematics and Science,* Tucson, Arizona, Oct. 29-31, 1999, "Long Time Dynamics of Modulational Instability", October 29, 1999.

20. Nagoya University, Nagoya, Japan, "Modulated Periodic Waves in Deep Water", Nov. 7, 1999.

21. Conference on *Symposium on Massive WDM and TDM Soliton Transmission Systems* Kyoto, Japan, Nov. 9-12, 1999, "On the Evolution and Interaction of Dispersion Managed Solitons", November 9, 1999.

- Consultative and Advisory Functions to Other Laboratories and Agencies: none

- Transitions: none

NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES:
Patent application: "Variable Length Nonlinear Feedback Shift Registers with Dynamically Allocated Taps", M.J. Ablowitz and J. Keiser.

HONORS/AWARDS: none