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# Image Data Compression by Adaptive Thresholding of Wavelet Coefficients

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## ABSTRACT

Image compression is performed to reduce the bit rate for image transmission and the required storage in the case of image archival. Typically, it involves a transformation of the image to reduce the correlation and redundancy between pixels. The wavelet transform has gained wide acceptance as a tool to decorrelate most image sources. The transformed image is decomposed at different scales of details and approximation. Such decomposition facilitates image compression and progressive transmission. In this study, a system and method for image compression is presented based on the wavelet transformation. A threshold is determined adaptively based on the desired compression ratio and only significant coefficients are retained.

**Index Terms:** Wavelet decomposition, image processing, image compression, thresholding, multiresolution analysis, hierarchical pyramid.

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## I. INTRODUCTION

In image processing and compression, image transformation is an important first step since most of the image data is not statistically independent. Various transformation methods such as Karhunen-Loeve transform (K-L) and Discrete Cosine Transform (DCT) among others do exist with different degrees of performance [1]. An excellent review for transformation techniques as applied to the compression of medical images can be found in [2]. The application of the wavelet transform is the fastest growing technique that is gaining wide acceptance among researchers in the field of image compression and coding. The multiresolution analysis [3] is the basis of most of the wavelet-based image processing. An image is processed by a special low pass filter **L** and its complementary high pass filter **H** and then sub-sampled. First, the image is filtered row wise using the **L** filter and then subsampled by a factor of 2. Second, a column wise filtering using the **H** filter is performed. Subsampling by a factor of 2 is then performed on the columns. The process is then repeated with the role of the **L** and **H** filters reversed (**L** for column and **H** for rows). The end result is four images that represent subbands filtered versions of the original image. The **LL<sub>1</sub>** is the approximation image and the other three images are detail images that highlight the horizontal, vertical, and diagonal features of the image. A rigorous mathematical analysis for the performance of wavelet based image compression is in [4].

Figure 1 shows the resultant four images after applying the wavelet transform where each image represents a different subband in the bandwidth of the original image. The designation represents the order of application of the filters. For example, **HL** means that the high pass filter was applied first followed by subsampling and then the low pass filter was applied followed by subsampling. The application of the wavelet transform can be repeated to create higher levels of decomposition and a hierarchical pyramid. Each new decomposition operates on the approximation image of the lower level to produce an approximation and detail images at a coarser scale in the image space. In figure 2, the approximation image **LL<sub>1</sub>** was decomposed to a coarser scale resulting in an approximation image **LL<sub>2</sub>** and detail images **HL<sub>2</sub>**, **LH<sub>2</sub>**, and **HH<sub>2</sub>**. Normally, level zero is known to be the original image, the base of the hierarchical pyramid, while level one is the first level of decomposition, and level two is the second level of decomposition and so on as seen in figure 2.

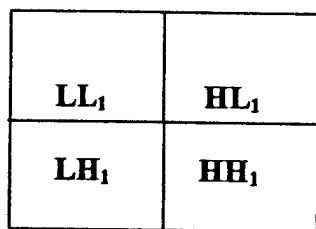


Figure 1, One level of Decomposition

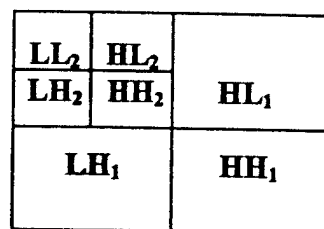


Figure 2, Two levels of Decomposition

The coefficients in the approximation image tend to have larger magnitudes relative to those of the detail images. In figure 3, the histograms of a typical gray level image (Barbara) are presented. It is evident that a great majority of the coefficients in the detail images cluster close to the zero magnitude. Those are the coefficients corresponding to features of insignificant value such as a background in the original image. This fact is normally utilized in image compression and denoising of images [5-6].

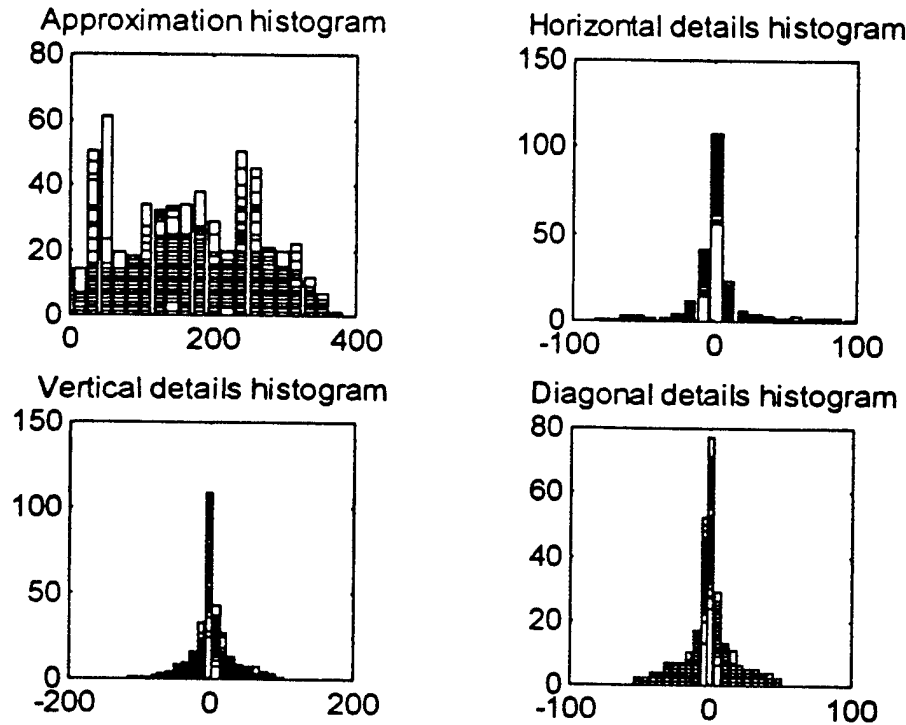


Figure 3, Histograms for approximation and detail images

The objective of this study is to develop a method for image compression for archival purpose. It involves the adaptive selection of significant wavelet coefficients to satisfy a desired compression ratio. The original image is decomposed using the wavelet transform and an image structure similar to that of figure 2 is constructed.

## II. IMAGE COMPRESSION

In an image, the features of significant visual value such as edges and object boundary tend to localize in the decomposed images and have larger wavelet coefficient magnitudes. They have more contribution to the total energy content of an image than the low contrast background. The small valued wavelet coefficients represent fine details and they are the majority of the wavelet coefficients. In a hierarchical pyramidal structure of subband decomposition it is more likely that energy (relevant features) is concentrated at the top most levels of the pyramid and as one goes down the pyramid, the energy content decreases gradually.

In image compression, it is essential that most of the energy contained in the original image be maintained in the compressed image. This energy is synonymous with edges and features of importance in the image. A critical step in the image compression process is the proper selection of a threshold to reduce the number of the wavelet coefficients. Several studies have addressed the issue of threshold selection that ranged from experimental trial and error [7-9] to using models of the Human Visual System (HVS) to take advantage of the spatial clustering of pixels [10-11].

Image coding for network transmission has received a lot of attention. The Embedded Zerotree Wavelet (EZW) algorithm [12] was the first to address the issues of embeddedness, bit plan transmission in decreasing order, and progressive transmission where wavelet coefficients with highest magnitude are transmitted first. The embedded bit stream contains binary decision bits that

allow the decoder to duplicate the encoder's execution path as it sorts the significant coefficients. The encoder can stop the transmission or the decoder can terminate the reception once enough bit budget for a desired compression ratio or distortion rate is achieved. Enhancements to the EZW algorithm have been reported in the literature and other techniques that use block coding of pixels have been developed [13-15]. In all cases, the wavelet coefficients are truncated to a finite precision before coding.

In archival image search, the approximation image at the highest level of decomposition can be sufficient to provide the needed visual information to quickly identify the original image especially if it has enough resolution. If the resolution is insufficient, then the detail coefficients at that highest level can be retrieved and an inverse wavelet transform is performed to synthesize an approximation image at one step lower level. Such approximation image is four times larger than the previous approximation image and thus has higher resolution. This procedure can be repeated until a satisfactory image can be synthesized. The advantage is that at each new synthesize level, only the detail coefficients at that level are needed and they are fast to retrieve because they generally occupy very small storage space due to their small magnitude.

### III. ADAPTIVE THRESHOLDING

The adaptive thresholding of the hierarchical pyramid begins by defining a desired compression ratio. Knowing the original image size, one can determine the desired number of significant coefficients to retain. Those are the wavelet coefficients with the largest magnitude and correspond to highest energy contents. The search for the appropriate threshold level begins with the determination of the largest coefficient magnitude and the standard deviation from the coefficients in the hierarchical pyramid. Two sets are created, the significant coefficients set  $S$ , and the insignificant coefficients  $I$ . Initially,  $S$  is empty and  $I$  is the entire pyramid.

Starting from that largest magnitude value, a threshold is set at one standard deviation ( $\sigma$ ) below it, and starting from the top of the pyramid and down, the coefficients that have absolute value above that threshold (i.e. significant coefficients) are added to the set  $S$  and counted. The remaining coefficients stay in the insignificant coefficient set  $I$ . Each time a coefficient is added to the significant set  $S$ , the value of that coefficient is saved along with a pointer. That pointer indicates the coefficient's position within the hierarchy and its relative position from the previous pixel in a row or from the beginning of the row. A new threshold is determined by subtracting additional standard deviation  $\sigma$ , as shown in figure 4, and the coefficients in the set  $I$  are tested against the new threshold for significance.

Newly determined significant coefficients are moved along with their pointers to set  $S$  and the significant coefficients count is updated. The process of lowering the threshold by  $\sigma$  is repeated until two limiting threshold values are reached. Successive halving of the distance between the two limiting threshold values to achieve the desired significant coefficients count is performed to fine tune the search. The process terminates once the total desired number of coefficients in the set  $S$  is reached. At this point, set  $S$  contains all the significant coefficients and the associated pointers that can be used later to reconstruct the image using the inverse wavelet transform.

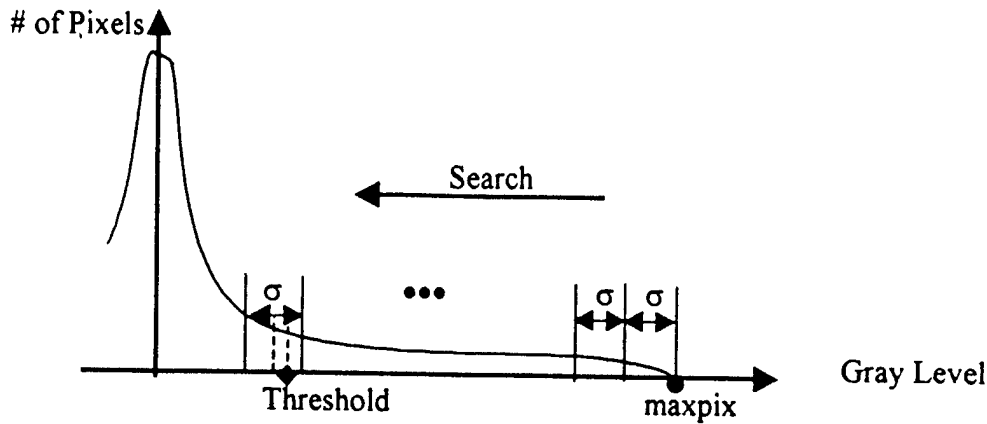


Figure 4, Adaptive threshold search

#### IV. IMAGE FIDELITY

The image fidelity of the reconstructed image is of paramount importance. To quantify the effectiveness of the compression in maintaining the energy content, we define the energy-ratio criterion. If we define  $M(i,j)$  as the original image and  $M_r(i,j)$  as the reconstructed image, then the energy maintained in the reconstructed image  $E_r$  and the original energy  $E$  respectively are,

$$E_r = \sum_{i,j} |M_r(i,j)|^2 \quad \text{and} \quad E = \sum_{i,j} |M(i,j)|^2, \quad i,j = 1..256$$

The Energy-ratio is then,

$$\text{Energy-ratio} = E_r / E = \sum_{i,j} |M_r(i,j)|^2 / \sum_{i,j} |M(i,j)|^2$$

Higher values for that ratio is an indication of a conservative compression and better preservation of image features. To evaluate the distortion in the reconstructed image, the Peak Signal to Noise Ratio is computed (PSNR). The mean-square-error (MSE) for the difference between the original image and the reconstructed image is computed first and then the PSNR. By definition, the mean-square-error is computed as,

$$\text{MSE} = \sum_{i,j} |M(i,j) - M_r(i,j)|^2 / \text{image-size}$$

Thus,  $\text{PSNR} = 10 \log_{10} (255^2 / \text{MSE})$  dB

In addition to those evaluation criteria, visual perception of the constructed image using overlaid graphics will be used to check for different types of distortion.

#### V. EXPERIMENTAL RESULTS

The image of Barbara (256 x 256 pixels) and the biorthogonal 6.8 filters were utilized for this experiment. To eliminate the boundary problems, images were symmetrically extended at the boundary and the wavelet transformation was computed. The Matlab software along with the Wavelet toolbox running on an IBM compatible PC was used. In this experiment, four levels of decomposition were used to generate the hierarchical pyramid. Several compression ratios were tested and the corresponding PSNR was computed.

In figure 5, The original image along with three compressed images at compression ratios of 8, 16 and 32 (1, 0.5, and 0.25 bit/pixel respectively) are presented. The number of significant coefficients retained are 8192, 4096, and 2048 respectively. It is evident that at high compression ratios some blurring effect is introduced especially around the cheeks and the shoulders of Barbara. However, all the significant details are preserved. Higher levels of compressions were also achieved but at the expense of image fidelity. The higher the compression ratio, the more coefficients to drop and thus the loss of fine details and their contribution to the energy content. In figure 6, the PSNR for compression rates of 8-80 (0.1 – 1.0 bit per pixel, bpp) is presented. For all of the compression ratios tested, the energy ratio was computed and found to be better than 99.99%.

Additionally, to evaluate the perceptual fidelity of the constructed image, graphics were added to the original image to evaluate their appearance in the reconstructed image. Graphics in the form of horizontal, vertical, diagonal lines and circles were superimposed on the image as shown in figure 7. The objective is to check for shape distortion. The image plus graphics was decomposed using four levels of decomposition as before and compressed at different compression ratios. The resultant compressed images from this experiment are also shown in figure 7. In each case the graphics were processed as part of the image data. From these images it does not appear that the compression has caused any perceivable distortion in the graphics.

After decomposing the original image at different levels, it is most likely that the wavelet coefficients in the approximation image at the highest level of decomposition are all significant coefficients. Those coefficients require suitable bits per pixel for storage to maintain the information. The remaining coefficients, though significant at the selected threshold but have smaller magnitude, may require fewer bits per pixel to represent. The approximation image at the top of the hierarchical pyramid is a low pass filtered version of the original image and contains many of its features that can be used to identify the original image.

Once an image is compressed and stored, image display can take advantage of the special storage format of the compressed image for progressive construction. In figure 8 the process of progressive construction is demonstrated. Here, The approximation image at the third level of decomposition is shown (image size, 32 x 32). Due to its insufficient information contents, the detail coefficients at that level are utilized and an inverse wavelet transform operation is performed to produce the approximation image at a lower level (second level with image size, 64 x 64). Again, the process is repeated to produce a higher resolution image at a lower level (first level with image size, 128 x 128) and so on. The highest resolution achievable is the resolution of the original image (image size, 256 x 256). The process could have been terminated at any time when enough information can be deciphered from the reconstructed image.

## VI. DISCUSSION AND CONCLUSION

In this work, a method of compressing images based on a desired compression ratio was demonstrated. The appropriate threshold was determined based on the number of significant coefficients that are desired to retain. It is interesting to note that going from a compression ratio of 8 (1 bpp) to a compression ratio of 80 (0.1 bpp), about 11% of the total wavelet coefficients are eliminated. However, the energy ratio drops by less than 0.09%. Certainly there is a significant perceptual difference between the two compressed images produced at those two compression ratios. A total elimination of the insignificant wavelet coefficients may not be the best solution. Due

to their small magnitude, it may be better to keep them using fewer bits per coefficient. In this work the threshold was determined based on the number of significant coefficients. However, a better strategy would base the search for threshold on the number of significant bits instead. The number of significant bits is based on the storage assignment for the significant coefficients at a given threshold. That can include coding the significant coefficients for even smaller storage space. The search for a threshold can terminate once the desired significant bits for a given compression ratio is reached.

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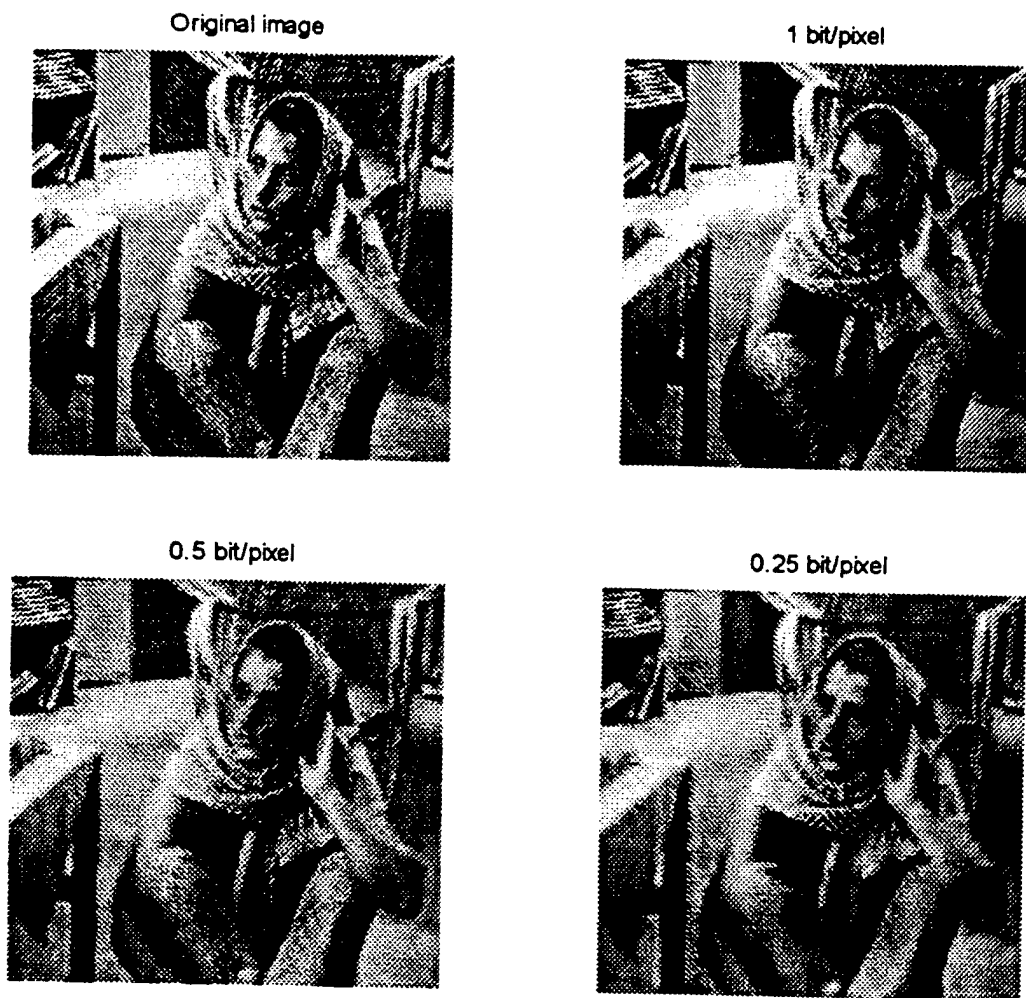


Figure 5, Coding results at selected compression ratios

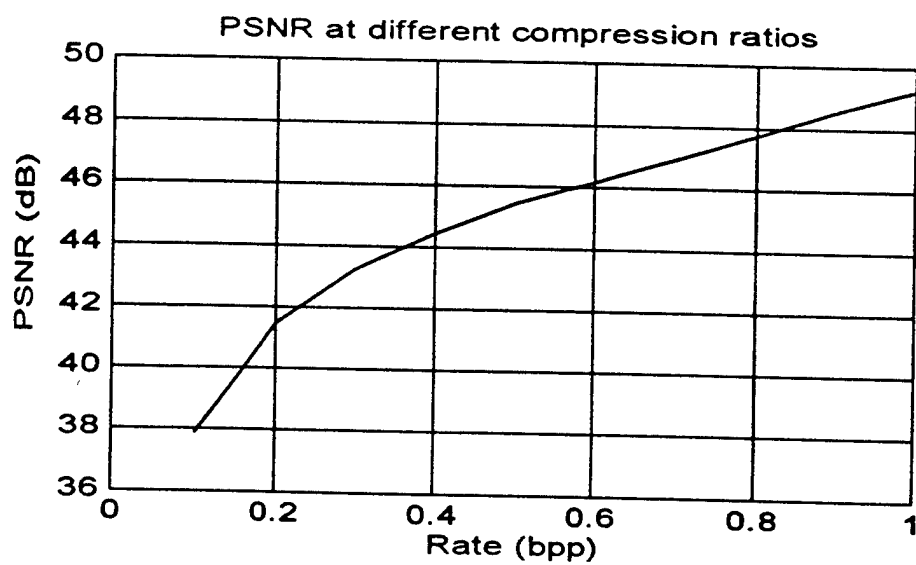


Figure 6, PSNR versus Compression Rate

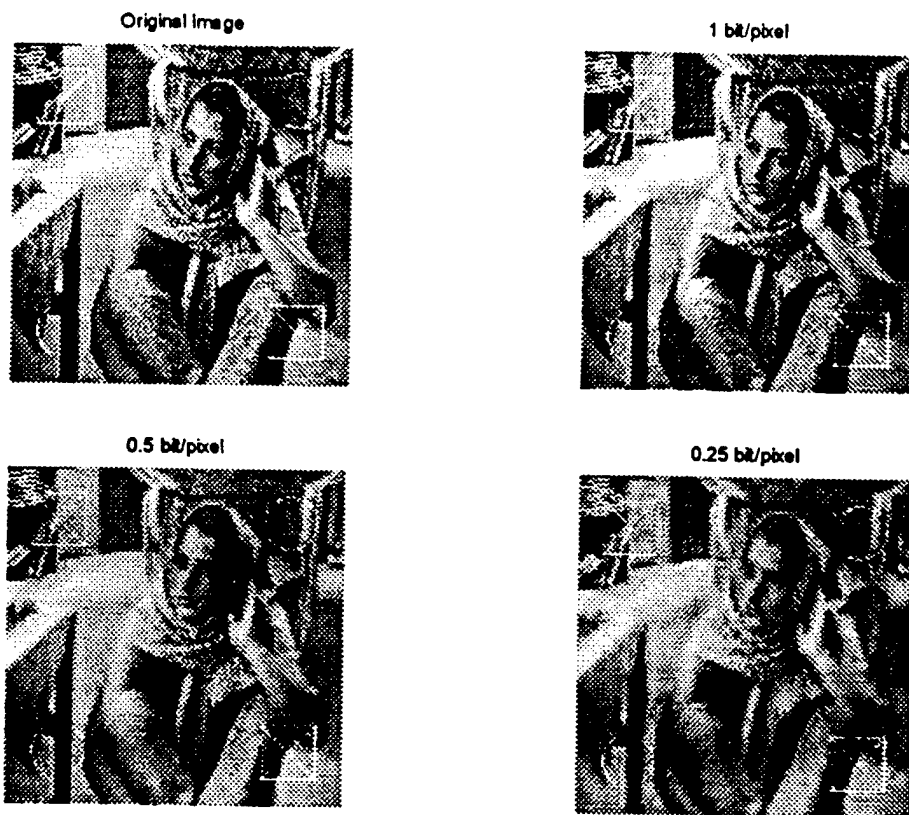


Figure 7, Compression of image with graphics

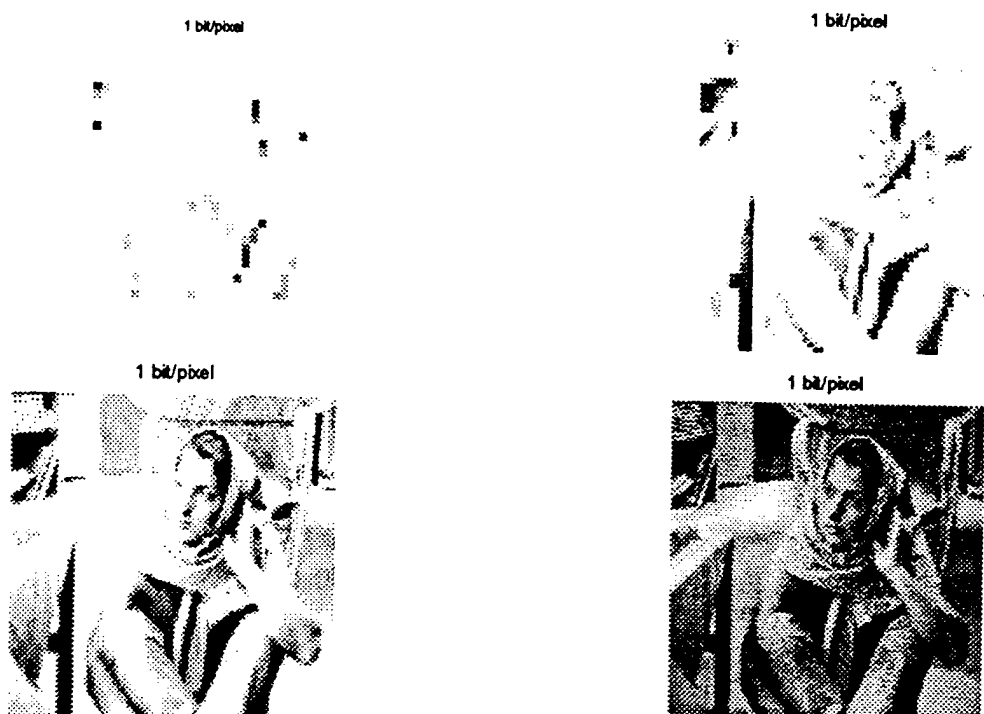


Figure 8, Progressive construction of compressed image