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THERMAL STRESS IN CONE ELEMENTS

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THERMAL STRESS IN CONE ELEMENTS

General Model
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ABSTRACT

A basic conical element subjected to axisymmetric linearly varying temperatures on the inner and outer surfaces is solved for internal stress and compatible edge loading and deformation. Uniqueness of the result is proven.

Descriptors

Cones

Thermal Stress

Elasticity

Shells

NOTATION

T	-	Temperature	(degrees Fahrenheit)
ΔT	-	Temperature difference	(" ")
$\bar{T}$	-	Mean temperature	(" ")
s	-	Coordinate distance from apex	(in.)
β	-	Half opening angle of cone	(degrees)
A, B, C, D	-	Thermal constants	(degrees Fahrenheit)
χ	-	Meridian angle change	(dimensionless)
w	-	Normal displacement	(in.)
N_s, N_θ	-	Axial stress resultants in meridional and circumferential directions	(lb/in.)
M_s, M_θ	-	Stress moment resultants in meridional and circumferential directions	(lb.in./in)
α	-	Coefficient of thermal expansion	(in/in/^{o}F)
ν	-	Poisson's ratio	(dimensionless)
E	-	Young's modulus	(lb./in.^{2})
t	-	Thickness of cone	(in.)
$K = \frac{Et^3}{12(1-\nu^2)}$	-	Bending rigidity	(lb.in.)
r_2	-	Principal radius of curvature in circumferential direction	(in.)
ϕ	-	Angle between shell normal and cone axis	(degrees)

NOTATION

$U = \frac{4 s Q_s}{t^2} \tan\beta$ - Stress function (lb./in.)

$(\ldots)^{\cdot} = \frac{d}{ds}(\ldots)$

$(\ldots)' = \frac{d}{dx}(\ldots)$

$x = 2\lambda\sqrt{s}$ - Coordinate transformation

$\lambda^4 = \frac{Et}{K}\cot^2\beta$ - Parameter

F, G, H, I - Constants

$L(\ldots) \equiv s(\ldots)^{\cdot\cdot} + (\ldots)^{\cdot} - \frac{1}{s}(\ldots) \equiv \lambda^2 \bar{L}(\ldots)$

$\bar{L}(\ldots) = (\ldots)'' + \frac{1}{x}(\ldots)' - \frac{4}{x^2}(\ldots)$

C_i - Constant of integration $i = 1, \ldots, 8$

X - Horizontal shear (lb./in.)

SOLUTION SUMMARY

$$
\begin{cases}
\Delta T = T_o - T_i \\
\bar{T} = \dfrac{T_o + T_i}{2} \\
\Delta s = s_2 - s_1
\end{cases}
$$

Thermal Constants

$$
\begin{cases}
A = \dfrac{\bar{T}^2 - \bar{T}^1}{\Delta s} \\
C = \dfrac{\Delta T^2 - \Delta T^1}{t \Delta s}
\end{cases}
\qquad
\begin{cases}
B = \dfrac{s_2 \bar{T}^1 - s_1 \bar{T}^2}{\Delta s} \\
D = \dfrac{s_2 \Delta T^1 - s_1 \Delta T^2}{t \Delta s}
\end{cases}
$$

Edge Loads

$$X = - \frac{\alpha K (1+\nu) C}{\cos \beta} \qquad M_s = \alpha K (1+\nu)(A \tan \beta + Cs + D)$$

Edge deformations

$$\chi = - \alpha A s \tan \beta \qquad w = \frac{\alpha A}{2} s^2 \tan \beta$$

Internal Stress resultants

$$N_s = + N_\theta = - \alpha K (1+\nu) \, C \tan \beta$$

$$M_s = M_\theta = \alpha K (1+\nu)(A \tan \beta + Cs + D)$$

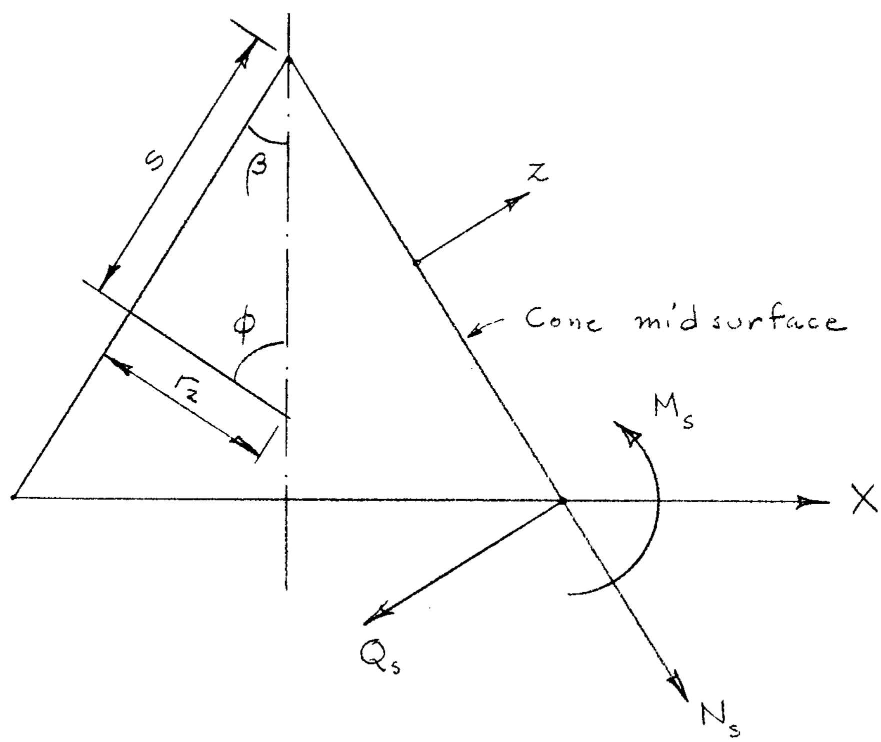
INTRODUCTION

The solution of thermal stress in an arbitrary thin shell of revolution subjected to an axisymmetric temperature distribution specified on the inner and outer surfaces of the shell may be solved by the cone coupling technique developed in SM-38500, "Analysis of a Shell by the Truncated Cone Approximation" by M. B. Harmon, provided that a rigorous solution of an individual conical element is known. It is the purpose of this report to develop such a basic solution for a linear temperature distribution defined at the edges of a truncated conical element.

The governing equations are the well-known Reissner-Meissner shell equations as extended by Melan and Parkus for thermal loading. These consist of a coupled set of second order ordinary linear differential equations. Particular integrals are developed by means of the method of undetermined coefficients in a polynomial for each dependent variable. It is shown that the particular integrals are unique and represent the desired basic solution provided that compatible edge shears and moments are provided at the edges of the conical frustrum.

GEOMETRY

The notation employed will be that of Fluegge [1] to be consistent with previous developments by the author [3], [5].



Cone Section

TEMPERATURE DESCRIPTION

In terms of the normal coordinate z to the shell mid-surface, measured positive outward, a Taylor expansion of the temperature, T, above some arbitrary stress-free datum would be,

$$T = T_0(s) + z\,T_1(s) + \frac{z^2}{2!}\,T_2(s) + \cdots$$

Where $-\frac{t}{2} \leq z \leq \frac{t}{2}$, t = shell thickness,

Now since $t \ll s$, one may ignore the higher order terms and write:

$$T \doteq T_0(s) + z\,T_1(s) \qquad (1)$$

This linearization is consistent with the approximations inherent in the Love-Kirchhoff formulation of thin shell theory.

SHELL EQUATIONS

The Reissner-Meissner equations in terms of T_o and T_1 are [1] :

$$s^2 \ddot{\chi} + s\dot{\chi} - \chi = -t^2 \frac{\cot\beta}{4K} sU + \alpha(1+\nu) s^2 \dot{T}_1 \quad - (2)$$

$$s^2 \ddot{U} + s\dot{U} - U = \frac{4E}{t}\left(s\cot\beta\,\chi + \alpha s^2 \dot{T}_o\right) \quad - (3)$$

Where:

$\chi = -\frac{dw}{ds}$ = Meridian angle change

$K = \frac{Et^3}{12(1-\nu^2)}$ = Bending rigidity

α = Coefficient of thermal expansion

E = Young's modulus

ν = Poisson's ratio

$U = \frac{4 s Q_s}{t^2}\tan\beta$ = Stress function

$(\ldots)^{\cdot} = \frac{d}{ds}(\ldots)$

Note that this definition of U differs from that of Fluegge by a factor of $\frac{4}{t^2}$. This unessential modification is due to Melan and Parkus and involves the particular integral only.

TEMPERATURE CONSTANTS

Let the specified temperature dependence upon s be given by the following linear expressions:

$$\left.\begin{aligned} T_o &= As + B \\ T_i &= Cs + D \end{aligned}\right\} \quad \text{——} \quad (4)$$

The unknown constants, A B C D, may be evaluated from temperatures defined at the outer and inner surfaces at points $s = s_1$ and $s = s_2$.

$$\underline{s = s_1} \begin{cases} T_o^1 = As_1 + B + \frac{t}{2}(Cs_1 + D) & \text{outer temperature} \\ T_i^1 = As_1 + B - \frac{t}{2}(Cs_1 + D) & \text{inner temperature} \end{cases}$$

$$\underline{s = s_2} \begin{cases} T_o^2 = As_2 + B + \frac{t}{2}(Cs_2 + D) & \text{outer temperature} \\ T_i^2 = As_2 + B - \frac{t}{2}(Cs_2 + D) & \text{inner temperature} \end{cases}$$

These are four equations in four unknowns,

$$\begin{bmatrix} s_1 & 1 & s_1 & 1 \\ s_1 & 1 & -s_1 & -1 \\ s_2 & 1 & s_2 & 1 \\ s_2 & 1 & -s_2 & -1 \end{bmatrix} \begin{bmatrix} A \\ B \\ \frac{t}{2} C \\ \frac{t}{2} D \end{bmatrix} = \begin{bmatrix} T_o^1 \\ T_i^1 \\ T_o^2 \\ T_i^2 \end{bmatrix}$$

with solutions given by:

$$A = \frac{\overline{T}^2 - \overline{T}^1}{\Delta s} \qquad B = \frac{s_2 \overline{T}^1 - s_1 \overline{T}^2}{\Delta s}$$

$$C = \frac{\Delta T^2 - \Delta T^1}{t \Delta s} \qquad D = \frac{s_2 \Delta T^1 - s_1 \Delta T^2}{t \Delta s}$$

where

$\Delta T = T_o - T_i$ = Temperature difference

$\overline{T} = \dfrac{T_o + T_i}{2}$ = Average temperature

$\Delta s = s_2 - s_1$ = Slant height

GOVERNING EQUATIONS

Differentiating equations (4),

$$\dot{T}_0 = A \,, \qquad \dot{T}_1 = C$$

and substituting these values into the Reissner-Meissner equations (2), (3) one obtains,

$$s^2 \ddot{\varkappa} + s\dot{\varkappa} - \varkappa = -t^2 \frac{\cot\beta}{4K} sU + \alpha(1+\nu)s^2 C$$

$$s^2 \ddot{U} + s\dot{U} - U = \frac{4E}{t}\left(s\varkappa\cot\beta + \alpha s^2 A\right)$$

For convenience of manipulation, define the constants.

$$\left.\begin{aligned} F &= \frac{t^2\cot\beta}{4K} & G &= \alpha(1+\nu)C \\ H &= \frac{4E}{t}\cot\beta & I &= \frac{4E}{t}\alpha A \end{aligned}\right\} \quad (6)$$

The governing equations become:

$$s^2\ddot{x} + s\dot{x} - x = -FsU + Gs^2 \qquad \text{———} \qquad (7)$$

$$s^2\ddot{U} + s\dot{U} - U = Hsx + Is^2 \qquad \text{———} \qquad (8)$$

This system is a set of two ordinary coupled second-order linear non-homogeneous differential equations whose complete primitive consists of a fundamental set of solutions associated with the homogeneous part and a particular integral of the non-homogeneous equations. [6] p. 69.

HOMOGENEOUS SOLUTION

The homogeneous equations are:

$$s^2\ddot{x} + s\dot{x} - x = -FsU \qquad \text{———} \qquad (9)$$

$$s^2\ddot{U} + s\dot{U} - U = Hsx \qquad \text{———} \qquad (10)$$

Define the cone operator [2] p. 196,

$$L(\ldots) \equiv s(\ldots)'' + (\ldots)' - \frac{1}{s}(\ldots)$$, then

$$L(x) = -FU \qquad \text{———} \qquad (11)$$

$$L(U) = Hx \qquad \text{———} \qquad (12)$$

Successive operations yield,

$$LL(\chi) = -FL(U) = -FH\chi$$

$$LL(U) = HL(\chi) = -FH\cancel{\chi}U$$

i.e.

$$\left.\begin{aligned} LL(\chi) + FH\chi &= 0 \\ LL(U) + FHU &= 0 \end{aligned}\right\} \quad \text{— (13)}$$

Now $U = \dfrac{4sQ_s}{t^2}\tan\beta$, so that the second equation may be written:

$$LL(sQ_s) + FH\,sQ_s = 0 \qquad \text{— (14)}$$

In order to solve this equation, one may factorize the operator and obtain the two equations:

$$L(sQ_s) \pm i\lambda^2 sQ_s = 0 \qquad \text{— (15)}$$

where $\lambda^4 = FH = \dfrac{Et}{K}\cot^2\beta$ — (16)

The solution of equations (15) may be obtained in terms of the Kelvin function [2*] , ber x, bei x, ker x, kei x and their derivatives employing the coordinate transformation,

$$x = 2\lambda\sqrt{s} \qquad \text{——— (17)}$$

[* The Thompson functions quoted by Fluegge are the Kelvin function since Thompson was also known as Lord Kelvin.]

to give,

$$\left.\begin{aligned} sQ_s = \; & C_1\left(\operatorname{ber}x - \frac{2}{x}\operatorname{bei}'x\right) + C_2\left(\operatorname{bei}x + \frac{2}{x}\operatorname{ber}'x\right) \\ & C_3\left(\operatorname{ker}x - \frac{2}{x}\operatorname{kei}'x\right) + C_4\left(\operatorname{kei}x + \frac{2}{x}\operatorname{ker}'x\right) \end{aligned}\right\} \text{(18)}$$

Where $C_1 \ldots C_4$ are constants of integration.

An identical solution for χ may be written down in terms of new constants of integration $C_5 \ldots C_8$; however, the two sets of constants are related by equations (11) and (12) and it is easier to obtain the solution of χ directly from these relations.

Define the fundamental set of solutions by equation (18),

$$u_1 = \operatorname{ber}x - \frac{2}{x}\operatorname{bei}'x \,, \qquad u_2 = \operatorname{bei}x + \frac{2}{x}\operatorname{ber}'x$$

$$u_3 = \operatorname{ker}x - \frac{2}{x}\operatorname{kei}'x \,, \qquad u_4 = \operatorname{kei}x + \frac{2}{x}\operatorname{ker}'x$$

Then the homogeneous solution becomes briefly,

$$sQ_s = C_1 u_1 + C_2 u_2 + C_3 u_3 + C_4 u_4 \qquad \text{— (19)}$$

Now from equation (12),

$$x = \frac{1}{H} L(U) = \frac{t}{4E\cot\beta} \cdot \frac{4\tan\beta}{t^2} L(sQ_s)$$

$$x = \frac{\tan^2\beta}{Et} L(sQ_s)$$

In order to operate on $u_1 \cdots u_4$ we must define L in terms of x.

$$s = \frac{x^2}{4\lambda^2} \qquad \frac{d}{ds} = \frac{2\lambda^2}{x}\frac{d}{dx}$$

$$\frac{d^2}{ds^2} = \frac{2\lambda^2}{x}\frac{d}{dx}\left(\frac{2\lambda^2}{x}\frac{d}{dx}\right) = \frac{4\lambda^4}{x}\left(-\frac{1}{x^2}\frac{d}{dx} + \frac{1}{x}\frac{d^2}{dx^2}\right)$$

One obtains the new operator $\bar{L}(\ldots)$ from,

$$L(\ldots) = s(\ldots)^{\cdot\cdot} + (\ldots)^{\cdot} - \frac{1}{s}(\ldots)$$

$$L(\ldots) = \lambda^2\left[(\ldots)'' + \frac{1}{x}(\ldots)' - \frac{4}{x^2}(\ldots)\right] = \lambda^2 \bar{L}(\ldots)$$

where $(\ldots)' = \frac{d}{dx}(\ldots)$

The expression for x becomes:

$$x = \frac{\lambda^2\tan^2\beta}{Et}\bar{L}(C_1 u_1 + C_2 u_2 + C_3 u_3 + C_4 u_4)$$

Due to the linear property of the operator, one writes:

$$\bar{L}(u_1) = \bar{L}(\operatorname{ber} x - \tfrac{2}{x}\operatorname{bei}' x) = -\operatorname{bei} x - \tfrac{2}{x}\operatorname{ber}' x = u_5 = -u_2$$

$$\bar{L}(u_2) = \bar{L}(\operatorname{bei} x + \tfrac{2}{x}\operatorname{ber}' x) = \operatorname{ber} x - \tfrac{2}{x}\operatorname{bei}' x = u_6 \quad (20) \quad = u_1$$

$$\bar{L}(u_3) = \bar{L}(\operatorname{Ker} x - \tfrac{2}{x}\operatorname{Kei}' x) = -\operatorname{Kei} x - \tfrac{2}{x}\operatorname{Ker}' x = u_7 = -u_4$$

$$\bar{L}(u_4) = \bar{L}(\operatorname{Kei} x + \tfrac{2}{x}\operatorname{Ker}' x) = \operatorname{Ker} x - \tfrac{2}{x}\operatorname{Kei}' x = u_8 = u_3$$

$$x = \frac{\lambda^2 \tan^2\beta}{Et}\left(C_1 u_5 + C_2 u_6 + C_3 u_7 + C_4 u_8\right) \quad \text{—} \quad (21)$$

Where one has used the relations [2] p. 171

$$\operatorname{ber}'' x = -\operatorname{bei} x - \tfrac{1}{x}\operatorname{ber}' x, \qquad \operatorname{Ker}'' x = -\operatorname{Kei} x - \tfrac{1}{x}\operatorname{Ker}' x$$

$$\operatorname{bei}'' x = \operatorname{ber} x - \tfrac{1}{x}\operatorname{bei}' x, \qquad \operatorname{Kei}'' x = \operatorname{Ker} x - \tfrac{1}{x}\operatorname{Kei}' x$$

PARTICULAR INTEGRAL

Turning now to the particular integral of (7) (8) and guided by the polynomial form of the coefficients, a solution by the method of undetermined coefficients is tried.

$$\left.\begin{aligned} x &= C_0 + C_1 s + C_2 s^2 \\ U &= D_0 + D_1 s + D_2 s^2 \end{aligned}\right\} \quad \text{— (22)}$$

$C_0\ C_1\ C_2$ and $D_0\ D_1\ D_2$ are not related to previously defined constants.

Substituting equations (22) into equation (7), one obtains,

$$s^2(2C_2) + s(C_1 + 2C_2 s) - (C_0 + C_1 s + C_2 s^2) + Fs\,(D_0 + D_1 s + D_2 s^2) - G s^2 \equiv 0$$

Equating coefficients of s,

(0)	$-C_0 = 0$	$C_0 = 0$
(s)	$C_1 - C_1 + F D_0 = 0$	$D_0 = 0$
(s^2)	$2C_2 + 2C_2 - C_2 + F D_1 - G = 0$	$3C_2 + F D_1 - G = 0$
(s^3)	$F D_2 = 0$	$D_2 = 0$

Substituting equations (22) into equation (8)

$$s^2(2D_2) + s(D_1 + 2D_2 s) - (D_0 + D_1 s + D_2 s^2) - Hs(C_0 + C_1 s + C_2 s^2) - Is^2 \equiv 0$$

Equating coefficients of x,

(0)	$-D_0 = 0$	$D_0 = 0$
(s)	$D_1 - D_1 - HC_0 = 0$	$C_0 = 0$
(s^2)	$2D_2 + 2D_2 - D_2 - HC_1 - I = 0$	$3D_2 - HC_1 - I = 0$
(s^3)	$-HC_2 = 0$	$C_2 = 0$

The final solution is:

$$C_0 = C_2 = D_0 = D_2 = 0$$

$$D_1 = \frac{G}{F} = \frac{4K\alpha(1+\nu)C}{t^2 \cot\beta} \qquad C_1 = -\frac{I}{H} = -\alpha A \tan\beta$$

$$\varkappa = C_1 s = -\alpha A s \tan\beta \qquad \text{——— (23)}$$

$$U = \frac{4sQ_s}{t^2}\tan\beta = D_1 s = \frac{4K\alpha(1+\nu)C}{t^2\cot\beta}\, s$$

$$Q_s = \alpha K(1+\nu)C \qquad \text{——— (24)}$$

One may now write the complete primitives of (7) and (8)

$$\chi = \frac{\lambda^2 \tan^2\beta}{Et} \left(C_1 u_5 + C_2 u_6 + C_3 u_7 + C_4 u_8 \right) - \alpha A s \tan\beta \qquad \text{— (25)}$$

$$s Q_s = C_1 u_1 + C_2 u_2 + C_3 u_3 + C_4 u_4 + \alpha K s (1+\nu) C \qquad \text{— (26)}$$

Consider now a cone closed at the apex and extending to infinity at the open end.

When the shell is closed at the apex, one obtains the boundary conditions $C_3 = C_4 = 0$, [2] p. 198, while C_1 and C_2 are associated with stresses which are non-zero only in a neighborhood of the bottom edge, i.e. in a neighborhood of infinity.

One concludes that for values of s defined in the open interval

$$0 < s < \infty$$

The solutions are:

$$\left.\begin{aligned} \chi &= -\alpha A s \tan\beta \\ s Q_s &= \alpha K s (1+\nu) C \end{aligned}\right\} \qquad \text{— (27)}$$

UNIQUENESS

It follows, moreover, that these particular integrals are unique. The proof is comparatively simple.

I. The complete primitives are unique since the coefficients in the differential equations (7) and (8) are analytic [6] p.48 and 69.

II.
The form of two primitives with different particular integrals ϕ_1 and ϕ_2 would be:

$$(sQ_s)_1 = \Phi_1(s) + \phi_1(s)$$

$$(sQ_s)_2 = \Phi_2(s) + \phi_2(s)$$

Where Φ_1 and Φ_2 are the homogeneous solutions,

Now since $\Phi_1 \to 0$, $\Phi_2 \to 0$ while

$(sQ)_1 \equiv (sQ)_2$, in the interval $0 < s < \infty$

one obtains necessarily,

$$\phi_1 \equiv \phi_2$$

STRESSES AND DISPLACEMENTS

From (1) one may now compute the remaining forces and displacements.

$$N_s = -Q_s \tan\beta = -\alpha K(1+\nu)C\tan\beta \qquad \text{— (28)}$$

$$N_\theta = -(sQ_s)'\tan\beta = -\alpha K(1+\nu)C\tan\beta = +N_s \qquad \text{— (29)}$$

$$M_s = -K\left[\dot{\varkappa} + \frac{\nu}{s}\varkappa - \alpha(1+\nu)T_1\right]$$

$$= -K\left[-(1+\nu)\alpha A\tan\beta - \alpha(1+\nu)(Cs+D)\right]$$

$$M_s = \alpha K(1+\nu)\left[A\tan\beta + Cs + D\right] \qquad \text{— (30)}$$

$$M_\theta = -K\left[\frac{\varkappa}{s} + \nu\dot{\varkappa} - \alpha(1+\nu)T_1\right]$$

$$= -K\left[-(1+\nu)\alpha A\tan\beta - \alpha(1+\nu)(Cs+D)\right]$$

$$M_\theta = \alpha K(1+\nu)\left[A\tan\beta + Cs + D\right] = M_s \qquad \text{— (31)}$$

$$w = -\int_0^s \varkappa\, ds = -\int_0^s -\alpha A\tan\beta \cdot s\, ds$$

$$w = \frac{\alpha A}{2} s^2 \tan\beta \qquad \text{— (32)}$$

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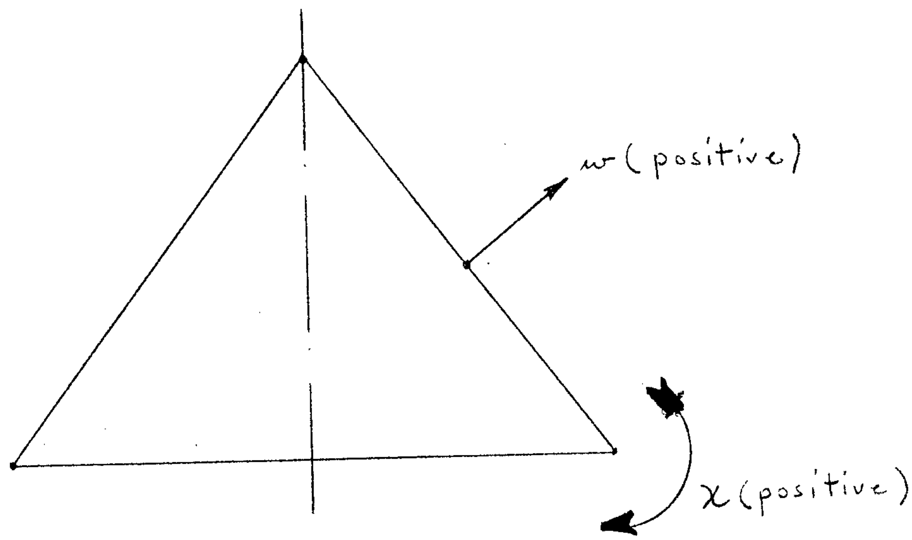
w is positive in the direction of z,

χ is positive when the rotation is as shown in the sketch.



CONICAL FRUSTRUM

If a conical frustrum "free body" is isolated from the infinitely long cone and loaded by reactive edge loads of magnitude,

$$N_s = -\alpha K (1+\nu) C \tan\beta$$

$$Q_s = \alpha K (1+\nu) C$$

$$M_s = \alpha K (1+\nu)(A \tan\beta + Cs + D)$$

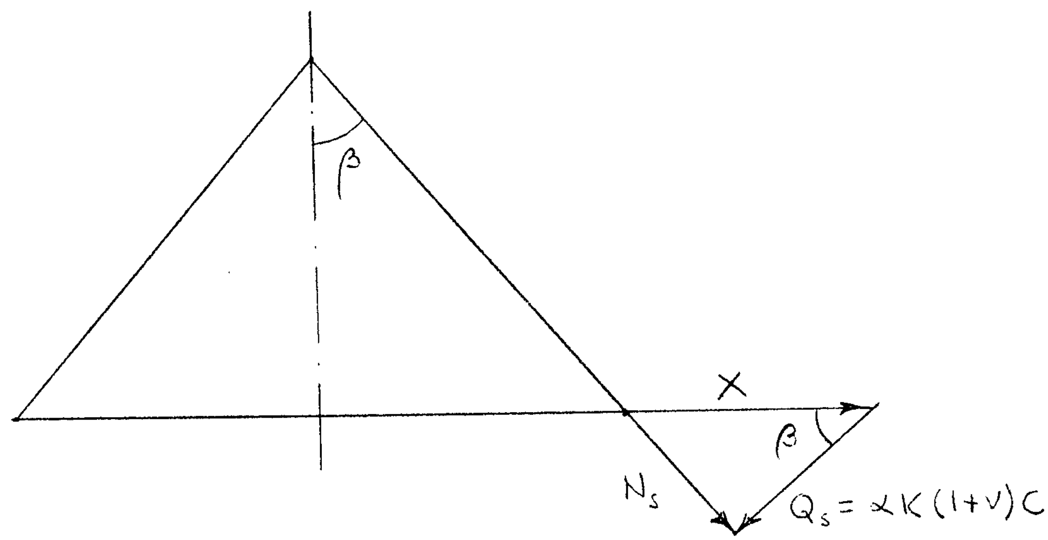
at the top and bottom edges, and having edge deformations given by,

$$\chi = -\alpha A s \tan\beta$$

$$w = \frac{\alpha A}{2} s^2 \tan\beta$$

it is clear that the solution presented in this report will satisfy these loading and edge conditions and will consequently represent the required basic conical element required for incorporation in an extended version of SM-38500.

One small detail is to be observed in such a formulation. Edge loads in SM-38500 are input as a line load perpendicular to the axis of rotation. The relation between such an edge load and N_s, Q_s is shown below.



$$X = -\frac{\alpha K(1+\nu)C}{\cos\beta} \qquad \text{—— (33)}$$

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