Final Report
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Nonlinear Estimators for Multi-sensor/Multi-target Tracking and Synchronization
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1 Summary of Research Accomplishments

The majority of the research focussed on developing new anti-jam and code synchronization algorithms for GPS receivers (funding from NAWCWPNS). In addition, support for a graduate student (Craig Agate) was provided by BMDO for part of the period 9/30/98 to 12/30/98, during which time he started writing his dissertation. We anticipate that the final dissertation, [1], describing work completed under both this contract, and the previous ONR contract N00014-95-1-1213, will be completed in the Summer of 2000. The following specific algorithms were developed in this reporting period.

1. Modulated Lapped Transform (MLT) for GPS C/A code. The MLT is related to Fourier and wavelet transforms, and generates a time-frequency decomposition of the received signal.
2. Extended Kalman Filter (EKF) for code tracking using the MLT (MLT-EKF). This algorithm jointly estimates the code delay, jammer, and received code amplitude. The jammer estimate is subtracted from the received signal as part of the EKF algorithm.
3. Generalized Likelihood Ratio Test (GLRT) for C/A code acquisition. The EKF requires prior knowledge of the code delay to within half a chip. To provide initial acquisition, a GLRT-based algorithm was developed which provides constant false alarm rate (CFAR.)

The GPS code tracking/anti-jam algorithms based on modulated lapped transforms (MLT) are now summarized.

2 Transform-Domain Model for the GPS Signal/Interferer

In [2] we developed a vector model for the received GPS C/A code signal using the MLT. The discrete-time received signal is given by

$$r(k) = a(kT_s)PN_{lp}(kT_s - \tau(kT_s))\cos(2\pi F_c kT_s + \theta) + j(k) + n(k), \tag{1}$$

where T_s is the intermediate frequency (IF) sampling interval. The jammer $j(k)$ and additive thermal noise are real-valued, bandpass processes, as is the modulated C/A code $PN_{lp}(t)$. Note that the signal must be real-valued to use the MLT. The sequence $PN_{lp}(t)$ is a filtered pseudo-random binary waveform, with chip rate 1.032 mcps.

The overall estimation problem is to (a) acquire and track the code delay $\tau(kT_s)$, while simultaneously (b) rejecting the jammer $j(k)$.

A general form of the jammer was adopted in [2], encompassing both sinusoidal and chirp waveforms. Thus,

$$j(k) = \sqrt{J}\cos(2\pi((F_C + F_J)kT_s + \dot{F}_J(kT_s - mT_{swp})^2)), \tag{2}$$

where F_J is the jammer center frequency, and $2\dot{F}_J$ is the sweep rate in Hz./sec.

In order to apply the MLT, the received samples $r(k)$ are grouped into vectors of length $4N_c$ samples, where N_c is the number of chips per vector, and $1/T_s = 4/T_c$ is the sampling rate. The received vectors are then compactly written as

$$\mathbf{r}(k) = \mathbf{S}(k, \tau(k))\mathbf{a}(k) + \mathbf{j}(k) + \mathbf{n}(k), \tag{3}$$

where

$$\mathbf{r}(k) = [r(kM), r(kM+1), \ldots, r((k+1)M-1)]^T \tag{4}$$

The signal matrix $\mathbf{S}(k, \tau(k)) \in \mathcal{R}^{M\times 2}$ is defined in [2]. Note that the received amplitude has been separated into its inphase and quadrature components, yielding

$$\mathbf{a}(k) = [a(kMT_s)\cos(\theta), a(kMT_s)\sin(\theta)]^T. \tag{5}$$

The MLT is defined by a transformation matrix $\mathbf{T} \in \mathcal{R}^{M\times 2M}$ [2]. The transformed received vector used to develop the code acquisition and tracking algorithms is finally defined by

$$\underline{r}(k) = \mathbf{T}[\mathbf{r}(k-1)^T, \mathbf{r}(k)^T]^T. \tag{6}$$

Note that $\mathbf{T}$ transforms a $2M$ length vector to an M length one, and furthermore, is overlapped, in that both $\underline{r}(k)$ and $\underline{r}(k+1)$ depend on the same time-domain vector $\mathbf{r}(k)$. The transform-domain received signal model is then

$$\underline{r}(k) = \underline{S}(k, \tau(k))\mathbf{a}(k) + \underline{n}'(k), \tag{7}$$

where $\underline{n}'(k)$ denotes the transformed noise plus jammer.

The major advantage of the lapped transform is that the correlated time-domain jammer plus noise, $j(k) + n(k)$ is now converted to an uncorrelated vector sequence $\underline{n}'(k)$. This greatly simplifies development of the EKF algorithm, as described in the next section.

3 Modulated Lapped Transform-domain EKF and GLRT-based Acquisition

In order to apply the EKF, a state vector comprising the delay and amplitude is first defined by

$$\mathbf{x}(k) = [\tau(k), \mathbf{a}(k)^T]^T. \tag{8}$$

An EKF form of the measurement equation is then given by [2]

$$\underline{r}(k) = \mathbf{h}(k, \mathbf{x}(k)) + \underline{n}'(k), \tag{9}$$

where the nonlinear measurement function is

$$\mathbf{h}(k, \mathbf{x}(k)) = \underline{S}(k, \tau(k))\mathbf{a}(k). \tag{10}$$

The EKF can be directly applied to the measurement model (9), and the resulting algorithm is simplified since $\underline{n}'(k)$ is approximately white Gaussian noise. However, the covariance matrix

$$\mathbf{R}(k) = E\{\underline{n}'(k)\underline{n}'(k)^T\}$$

is unknown due to the presence of the jammer. A method based on maximum-likelihood is presented in [2] for estimating this matrix.

```
Initialize ACQ_FLAG = 0
For $l = 0, 1, \ldots$
    Input next received vector $\mathbf{r}(l)$.
    Compute MLT vector.
        $\underline{r}(l) = \mathbf{T}[\mathbf{r}(l-1)^T \mathbf{r}(l)^T]^T$
    Update noise covariance matrix estimates
        $\mathbf{R}_0(l) = \frac{1-\lambda}{1-\lambda^{l+1}} \underline{r}(l) \underline{r}(l)^T + \lambda \frac{1-\lambda^k}{1-\lambda^{k+1}} \mathbf{R}_0(l-1)$

        $\mathbf{R}_1(l) = \frac{1-\lambda}{1-\lambda^{l+1}} [\underline{r}(l) - \underline{S}(l, \hat{\tau}(l|l-1)\hat{\mathbf{a}}(l|l-1)][\underline{r}(l) - \underline{S}(l, \tau(l|\hat{l}-1)\hat{\mathbf{a}}(l|l-1)]^T$
        $+\lambda \frac{1-\lambda^k}{1-\lambda^{k+1}} \mathbf{R}_0(l-1)$
    Update error covariance matrix.
        $\mathbf{P}(l|l)^{-1} = \mathbf{P}(l|l-1)^{-1} + \mathbf{H}(l)^T \mathbf{R}_1(l)^{-1} \mathbf{H}(l)$
    Compute EKF measurement update.
        $\mathbf{x}(l|l) = \mathbf{x}(l|l-1) + \mathbf{P}(l|l)\mathbf{H}(l)^T \mathbf{R}_1(l)^{-1}[\underline{r}(l) - \underline{S}(l, \hat{\tau}(l|l-1)\hat{\mathbf{a}}(l|l-1)]$
    Compute one-step predictions.
        $\mathbf{x}(l+1|l) = \mathbf{F}\mathbf{x}(l|l)$
        $\mathbf{P}(l+1|l) = \mathbf{F}\mathbf{P}(l|l)\mathbf{F}^T + \mathbf{Q}$
    Test for acquisition using GLRT.
        If $|\mathbf{R}_0(l)|/|\mathbf{R}_1(l)| > T_h$ then
          ACQ_FLAG = 1
          $\tau(l+1|l) = \tau(l+1|l) + \tau_{inc}$
        Else
          ACQ_FLAG = 0
Next $l$
```

Table 1: MLT-EKF Algorithm Summary.

The acquisition algorithm is based on the GLRT. Let hypothesis H_0 and H_1 correspond to not acquired and acquired, respectively. Then the measurement models are

$$\begin{aligned} H_0 : \underline{r}(k) &= \underline{n}_0(k) \\ H_1 : \underline{r}(k) &= \underline{S}(k, \hat{\tau}(k|k-1)\hat{\mathbf{a}}(k|k-1) + \underline{n}_1(k). \end{aligned} \tag{11}$$

That is, if acquisition is occurring, the received vector consists of the estimated signal vector, thermal noise, and jammer. However, if the estimate $\hat{\tau}(k|k-1)$ is far from $\tau(k)$, it is assumed that the C/A signal component is effectively a white noise. Hence, under H_0, the received signal is a Gaussian noise vector $\underline{n}_0(k)$.

The GLRT is defined by

$$\frac{\max_{\mathbf{R}} \prod_{l=0}^{k} p(\underline{r}(l)|H_1, \mathbf{R})}{\max_{\mathbf{R}} \prod_{l=0}^{k} p(\underline{r}(l)|H_0, \mathbf{R})} \underset{H_0}{\overset{H_1}{\gtrless}} T_h, \tag{12}$$

where T_h is a threshold, and $\mathbf{R}$ corresponds to the unknown covariance matrices of $\underline{n}_0(k)$ or $\underline{n}_1(k)$.

Acquisition is performed as follows: If the GLRT is greater than the threshold, acquisition is declared, and the MLT-EKF updates normally. Otherwise, the delay estimate is updated by a fixed valued τ_{inc}, to step through the code delay uncertainty.

The final MLT-EKF algorithm, with GLRT-based acquisition is summarized in the following table from [2].

4 Example Results

An example result from [2] is shown in Figures 1 and 2. In Fig. 1, a swept tone jammer, with $2\dot{F}_J = 2000$ mHz./sec. interferes with the C/A code waveform, received with an SNR of -5 dB. Again, without AJ,

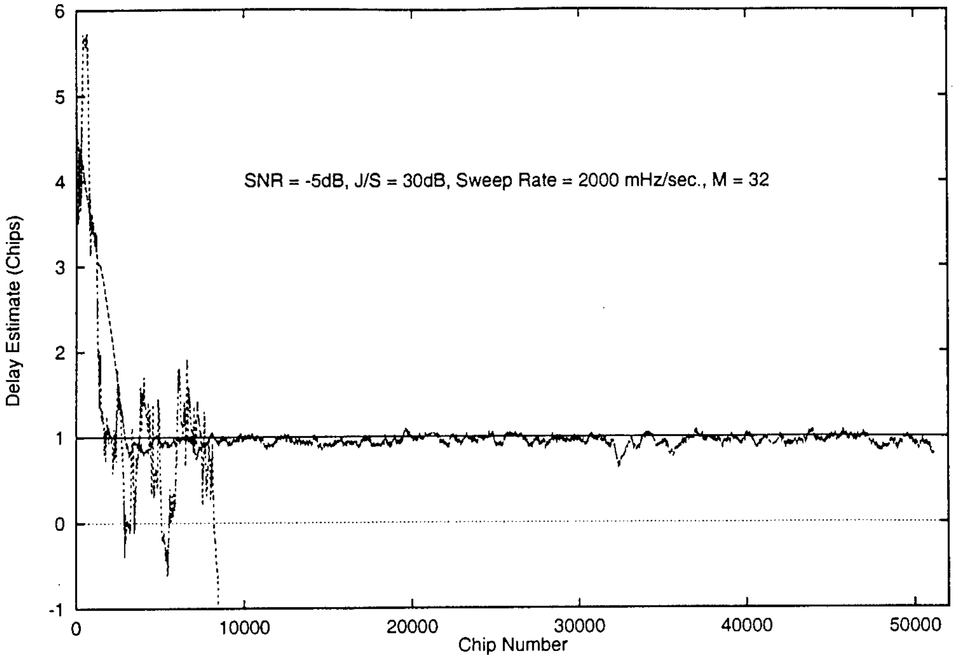

Figure 1: C/A code acquisition and tracking, with and without interference rejection for a swept tone jammer, 2000 mHz. sweep rate .

the MLT-EKF algorithm fails to acquire. The MLT bin powers (diagonal elements of $\mathbf{R}(k)$) are shown in Figure 2 for this swept tone jammer.

5 Conclusions

New algorithms for jammer suppression and joint C/A code acquisition/tracking were developed for GPS receivers. The modulated lapped transform was employed to decorrelate the jammer, and provide better isolation of time/frequency content. As a result, the overall algorithm was able to reject both tone and chirp jammers with high sweep rates.

References

[1] C. Agate, "Tracking of point and extended targets using density function approximation." Dissertation in Preparation.

[2] R. Iltis, "An EKF-based algorithm for C/A code tracking using the Modulated Lapped Transform," tech. rep., Department of Electrical and Computer Engineering, University of California, Santa Barbara, Dec. 1998. Report submitted to NAWCWPNS.

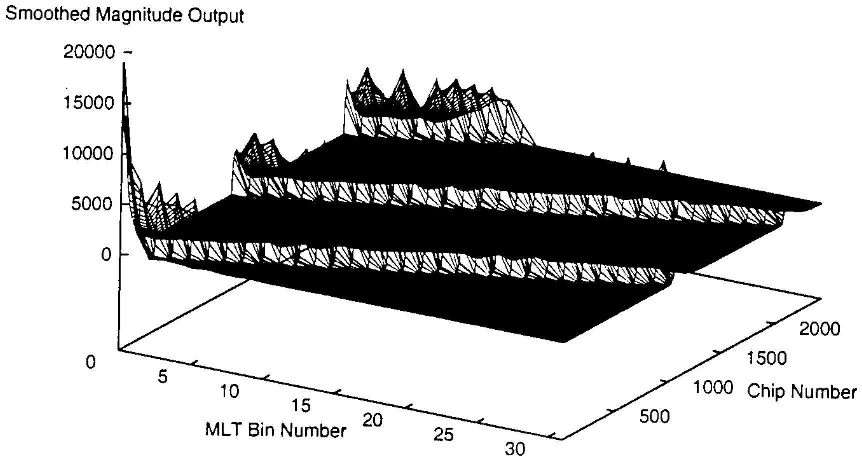

Figure 2: Averaged output power of MLT bins for the swept tone jammer.

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13. ABSTRACT *(Maximum 200 words)*

Research was conducted on code synchronization and jammer suppression for GPS receivers. The modulated lapped transform (MLT) was employed to decorrelate the jammer in time, and better isolate jammer components from the signal in the time-frequency plane. The extended Kalman filter (MLT-EKF) and generalized likelihood ratio test (GLRT) were used to both track and acquire the unknown code delay. Simulation studies for various stationary and swept tone jammers were conducted. It was found that the MLT-EKF could acquire and track the GPS C/A code in the presence of a chirp jammer with a sweep rate of up to 2000 mHz per second..

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