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TECHNICAL NOTE 2916

EFFECT OF THERMAL PROPERTIES ON LAMINAR-BOUNDARY-LAYER CHARACTERISTICS

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SUMMARY

An iteration method is presented for solving the laminar-boundary-layer equations for compressible flow in the absence of a pressure gradient wherein the temperature variation of all the fluid thermal properties is considered. Friction and heat-transfer characteristics have been calculated for a stream temperature of -67° F for Mach numbers from 1 to 10 with the use of values of the heat capacity, conductivity, and viscosity determined from experiment. Consideration of the temperature variation of all the fluid thermal properties causes the recovery factor to decrease substantially with increasing Mach number. Moreover, the heat-transfer rate is found to be proportional to the difference between an effective enthalpy, which is a function of both the surface temperature and stream Mach number, and the surface enthalpy. In contrast, the heat-transfer rate is approximately proportional to the difference between the recovery enthalpy and the surface enthalpy for solutions which employ a constant Prandtl number. The calculated skin friction and heat-transfer rates based upon the use of the Sutherland equation for viscosity and a Prandtl number of 0.75, however, are in excellent agreement with the results of the present analysis.

INTRODUCTION

Most compressible-flow boundary-layer analyses employ approximate relations for the temperature variation of the thermal properties. A simple analytic relation is usually specified for the variation of viscosity with temperature. Moreover, the heat capacity and the Prandtl number are generally considered to be independent of temperature and as a consequence the conductivity and viscosity are required to have the same temperature variation. Experiment shows that these conditions are not satisfied closely over the wide temperature range which occurs in a boundary layer in a high-speed flow. The effect of these approximations on the calculated boundary-layer characteristics is most easily studied by considering the laminar-boundary-layer flow on a flat plate.

Calculations of boundary-layer characteristics which employ approximate relations for the thermal properties are generally satisfactory as long as the temperature variation in the boundary layer is not excessive; the range for which the solutions are satisfactory depends upon the value of the Prandtl number used in the analysis and upon the viscosity-temperature relation employed. Various viscosity-temperature relations and values of the Prandtl number have been used in the analysis of the laminar boundary layer on a flat plate (refs. 1 to 5, for example). The choice of a Prandtl number of unity (ref. 1, for example) or of a linear viscosity-temperature relation (ref. 2, for example) leads to an essential simplification, since in each case only one differential equation need be solved. Better solutions are obtained by taking the Prandtl number as approximately 0.73 (the value for air at normal temperatures) and by utilizing a more realistic viscosity-temperature relationship. The viscosity-temperature relationship is frequently taken as a simple power law (refs. 1 and 3, for example) or as given by the Sutherland equation (refs. 4 and 5, for example). The Sutherland equation for viscosity shows especially good agreement with experiment over a wide temperature range.

At high supersonic Mach numbers, where the temperature variation within the boundary layer is large, the assumption of a constant Prandtl number and the use of an approximate viscosity-temperature relation is not jusitifed a priori. A few calculations of the laminar-boundary-layer characteristics on a flat plate have been made which have not imposed these restrictions. Experimental values of the thermal properties were utilized in some calculations of the laminar-boundary-layer characteristics by an iteration method at Mach numbers from 1 to 10 in reference 6 and the thermal properties employed in the differential-analyzer solutions of references 7 and 8 were taken in part from experiment. For the latter two analyses, the stream conditions were determined from the solution. Consequently, a graphical or trial-and-error procedure was required in the course of the solution to obtain values corresponding approximately to the desired stream conditions. In the iteration methods used in reference 6 and in the present analysis, the solutions are determined for specified stream conditions.

The present investigation is a continuation of the work reported in reference 6. The purpose is to determine accurately some characteristics of the boundary layer at high supersonic speeds and to show the effect of various approximations to the thermal data of the fluid on the accuracy of the calculated friction and heat-transfer characteristics. Tabulated values of the thermal properties, found from experiment, have been employed in the calculations in order to obtain reliable data. Friction and heat-transfer characteristics, calculated for a stream temperature of -67° F for Mach numbers from 1 to 10, are presented in the form of graphs. Tabulated values of the corresponding velocity and temperature profiles and their derivatives are available upon request from the

National Advisory Committee for Aeronautics. The iteration method described herein for the solution of the equations is similar to that presented in reference 6. The computations are facilitated however through a different formulation of the equations; the convergence of the iteration procedure is improved considerably and a troublesome singularity is avoided.

SYMBOLS

x,y	coordinates parallel and normal to stream direction
u,v	components of velocity along x- and y-axes
ρ	density
μ	coefficient of viscosity
i	enthalpy per unit mass
k	thermal conductivity
T	absolute temperature
c_p	heat capacity at constant pressure
$\eta = y\sqrt{\frac{U_1}{\nu_1 x}}$	
ν	kinematic viscosity
U_1	stream velocity
Pr	Prandtl number
ϑ	enthalpy function, $\frac{i - i_1}{U_1^2/2}$
α,β	integrating factors
$V(\eta) = \int_0^\eta \frac{Pr^*}{\mu^*} \beta \, d\eta$	

τ	shear stress at surface
R	Reynolds number
c_f	average skin-friction coefficient
q	local heat-transfer rate per unit area at surface
γ	ratio of heat capacities at constant pressure and constant volume
M	Mach number
r	temperature recovery factor
c	constant

Subscripts:

1	free stream or at edge of boundary layer
0	stagnation condition of stream
e	effective
r	recovery
s	surface

Superscript:

*	dimensionless quantity based on stream conditions

ANALYSIS

Basic equations.- With the x-axis taken in the free-stream direction, the two-dimensional compressible-flow boundary-layer equations for steady motion in the absence of a pressure gradient may be written as follows:

Momentum equation:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{1}$$

Continuity equation:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{2}$$

Energy equation:

$$\rho u \frac{\partial i}{\partial x} + \rho v \frac{\partial i}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are the velocity components in the direction of the x- and y-axes, respectively, ρ is the density, T is the absolute temperature, i is the enthalpy per unit mass, μ is the coefficient of viscosity, and k is the thermal conductivity. The heat-conduction term of equation (3) may be expressed in terms of the enthalpy. With the use of the relation $c_p = \frac{\partial i}{\partial T}$, where c_p is the heat capacity at constant pressure, the energy equation becomes

$$\rho u \frac{\partial i}{\partial x} + \rho v \frac{\partial i}{\partial y} = \frac{\partial}{\partial y}\left(\frac{k}{c_p} \frac{\partial i}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

For a flow with heat transfer (specified constant surface temperature) or for a flow with zero heat transfer, the three partial-differential equations which describe the motion in the boundary layer may be reduced to a set of two ordinary differential equations. For this purpose the laminar-boundary-layer parameter $\eta = y\sqrt{\frac{U_1}{\nu_1 x}}$, where U_1 is the stream velocity and ν_1 is the kinematic viscosity based on the stream temperature, is taken as the independent variable. Transforming equation (2) and solving for ρv results in

$$\rho v = \frac{1}{2}\sqrt{\frac{\nu_1}{U_1 x}} \int_0^{\eta} \eta \frac{d}{d\eta}(\rho u)\, d\eta$$

and after an integration by parts

$$\rho v = \frac{1}{2}\sqrt{\frac{\nu_1}{U_1 x}}\left(\rho u \eta - \int_0^\eta \rho u \, d\eta\right) \tag{5}$$

Transforming equations (1) and (4) and making use of equation (5) gives

$$\left(\frac{1}{2}\int_0^\eta \rho u \, d\eta\right)\frac{du}{d\eta} + \frac{U_1}{\nu_1}\frac{d}{d\eta}\left(\mu \frac{du}{d\eta}\right) = 0 \tag{6}$$

$$\left(\frac{1}{2}\int_0^\eta \rho u \, d\eta\right)\frac{di}{d\eta} + \frac{U_1}{\nu_1}\left[\frac{d}{d\eta}\left(\frac{k}{c_p}\frac{di}{d\eta}\right) + \mu\left(\frac{du}{d\eta}\right)^2\right] = 0 \tag{7}$$

With the introduction of the Prandtl number Pr, the nondimensional function

$$\vartheta = \frac{i - i_1}{i_0 - i_1} = \frac{i - i_1}{U_1^2/2}$$

and the dimensionless quantities

$$u^* = \frac{u}{U_1} \qquad \rho^* = \frac{\rho}{\rho_1} \qquad \mu^* = \frac{\mu}{\mu_1} \qquad Pr^* = \frac{Pr}{Pr_1}$$

equations (6) and (7) become

$$\frac{d}{d\eta}\left(\mu^* \frac{du^*}{d\eta}\right) + \frac{f}{\mu^*}\left(\mu^* \frac{du^*}{d\eta}\right) = 0 \tag{8}$$

and

$$\frac{d}{d\eta}\left(\frac{\mu^*}{Pr^*}\,\frac{d\vartheta}{d\eta}\right) + Pr_1 f\,\frac{Pr^*}{\mu^*}\left(\frac{\mu^*}{Pr^*}\,\frac{d\vartheta}{d\eta}\right) = -2Pr_1\mu^*\left(\frac{du^*}{d\eta}\right)^2 \tag{9}$$

where

$$f(\eta) = \frac{1}{2}\int_0^\eta \rho^* u^*\, d\eta$$

The subscript 1 refers to values at the edge of the boundary layer or values in the free stream and i_0 is the stagnation enthalpy of the stream.

The transformation of the boundary-layer equations employed herein follows essentially that of Schuh (ref. 9). This formulation of the boundary-layer equations for the flat plate has certain advantages over those in which the velocity (refs. 4 and 5, for example) or a function related to the stream function (ref. 6, for example) is used as the independent variable. A singularity arises in the solution of the equations in the latter two forms which complicates the numerical solution of the equations. In contrast, no singularity occurs in the solution of equations (8) and (9). Moreover, the iteration method used to solve these equations converges satisfactorily.

Equations (8) and (9) can be solved numerically by a method of successive approximations. The thermal quantities are functions only of the temperature and consequently functions of the independent variable η. Therefore, the coefficients of the derivatives in equations (8) and (9) may be considered as known functions of η if some initial solution is given. With the coefficients considered as known functions of the independent variable and with the integrating factors

$$\alpha = e^{-\int_0^\eta \frac{f}{\mu^*}\, d\eta}$$

and

$$\beta = e^{-Pr_1 \int_0^\eta \frac{f}{\mu^*} Pr^*\, d\eta}$$

the equation of motion (8) and the energy equation (9) may be written as

$$\frac{d}{d\eta}\left(\frac{1}{\alpha}\mu^* \frac{du^*}{d\eta}\right) = 0 \tag{10}$$

$$\frac{d}{d\eta}\left(\frac{1}{\beta}\frac{\mu^*}{Pr^*}\frac{d\vartheta}{d\eta}\right) = -2Pr_1 \frac{\mu^*}{\beta}\left(\frac{du^*}{d\eta}\right)^2 \tag{11}$$

Solution of boundary-value problem.- The solution of equations (10) and (11) gives the distribution of velocity and enthalpy throughout the boundary layer subject to certain conditions on the boundaries. Solutions which have zero heat transfer at the surface and solutions with heat transfer (specified constant surface temperature) are considered herein. The boundary conditions to be satisfied are:

For an insulated surface:

At the surface $(\eta = 0)$, $u^* = 0$ and

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = \text{Constant}\left(\frac{d\vartheta}{d\eta}\right)_{\eta=0} = 0$$

At infinity $(\eta = \infty)$, $u^* = 1$ and $\vartheta = 0$

For a specified constant surface temperature:

At the surface $(\eta = 0)$, $u^* = 0$ and $\vartheta = \vartheta_s = \text{Constant}$

At infinity $(\eta = \infty)$, $u^* = 1$ and $\vartheta = 0$

The subscript s denotes the values at the surface.

The first integral of equation (10) is

$$\mu^* \frac{du^*}{d\eta} = C_1\alpha(\eta)$$

and a second integration gives the velocity distribution

$$u^* = C_1 J(\eta)$$

where

$$J(\eta) = \int_0^\eta \frac{\alpha}{\mu^*} d\eta$$

Since $u^* = 1$ for $\eta = \infty$, the function $C_1 = \frac{1}{J(\infty)}$ and the distribution of the shear parameter is

$$\mu^* \frac{du^*}{d\eta} = \frac{\alpha(\eta)}{J(\infty)} \tag{12}$$

and the velocity distribution is

$$u^* = \frac{J(\eta)}{J(\infty)} \tag{13}$$

Since the boundary conditions on the velocity are the same for both the boundary conditions of specified constant surface temperature and zero heat transfer at the surface, equations (12) and (13) are valid for both surface conditions.

The temperature or enthalpy distribution throughout the boundary layer is obtained by integrating equation (11). The first integral of equation (11) satisfying the condition of zero heat transfer is

$$\frac{\mu^*}{Pr^*} \frac{d\vartheta}{d\eta} = -\beta(\eta)\varphi(\eta) \tag{14}$$

where

$$\varphi(\eta) = 2\Pr_1 \int_0^{\eta} \frac{\mu^*}{\beta}\left(\frac{du^*}{d\eta}\right)^2 d\eta$$

and after a second integration the enthalpy function ϑ is found as

$$\vartheta = \int_{\eta}^{\infty} \frac{\Pr^*}{\mu^*} \beta\varphi \, d\eta \tag{15}$$

At the surface ϑ becomes the enthalpy recovery factor

$$\vartheta_r = \int_0^{\infty} \frac{\Pr^*}{\mu^*} \beta\varphi \, d\eta$$

The first integral of equation (11) for flow with heat transfer (specified constant surface temperature) is

$$\frac{\mu^*}{\Pr^*} \frac{d\vartheta}{d\eta} = C_2\beta - \beta\varphi$$

where

$$\varphi = 2\Pr_1 \int_0^{\eta} \frac{\mu^*}{\beta}\left(\frac{du^*}{d\eta}\right)^2 d\eta$$

and the function C_2 is dependent on the surface temperature, stream temperature, and Mach number which is to be determined from the boundary conditions. A second integration gives

$$\vartheta = \vartheta_s + C_2V - U$$

where

$$U(\eta) = \int_0^\eta \frac{Pr^*}{\mu^*} \beta\varphi \, d\eta$$

$$V(\eta) = \int_0^\eta \frac{Pr^*}{\mu^*} \beta \, d\eta$$

From the boundary condition at infinity, C_2 is determined as

$$C_2 = \frac{\vartheta_e - \vartheta_s}{V(\infty)}$$

where

$$\vartheta_e = U(\infty)$$

With this value of C_2

$$\frac{\mu^*}{Pr^*} \frac{d\vartheta}{d\eta} = \frac{\vartheta_e - \vartheta_s}{V(\infty)} \beta - \beta\varphi \tag{16}$$

and the enthalpy function is

$$\vartheta(\eta) = \vartheta_s - U(\eta) + \frac{V(\eta)}{V(\infty)}\left(\vartheta_e - \vartheta_s\right) \tag{17}$$

Equations (13) and (15) give the distribution of velocity and enthalpy (and consequently the temperature) throughout the boundary layer for the insulated surface whereas equations (13) and (17) give the distribution of velocity and temperature corresponding to a specified constant surface temperature. The distributions of shear and heat transfer are determined from equations (12) and (16).

Method of numerical solution.- The solution of the equations for the velocity or temperature distributions is one of successive approximations; consequently, some approximate solution for the distribution of velocity and temperature is required. The Blasius solution may be taken for the distribution of velocity and the parabolic relation between temperature and velocity for a Prandtl number of unity (ref. 10) may be used to determine the corresponding temperature distribution. Fewer iteration steps are required, however, if a more refined solution, references 2 and 5, for example, is taken for the first approximation. With the initial values determined by the method of reference 2, four or five iterations were required to obtain differences of less than 1/2 percent between the last two iterations (for both the shear stress and the heat-transfer rate at the surface).

The following procedure was found to be most satisfactory in the numerical solution. With the initial values of velocity and temperature given for a specified surface temperature and Mach number as a function of the independent variable η and with the variation of viscosity and Prandtl number with temperature prescribed, the functions $f(\eta)$, $\alpha(\eta)$, and $J(\eta)$ are determined by numerical integration (the trapezoidal rule was found to be very satisfactory). The velocity distribution is then obtained from equation (13). By making use of the shear function given by equation (12) and the previously calculated values of $f(\eta)$, the functions $\beta(\eta)$, $\varphi(\eta)$, $U(\eta)$, and $V(\eta)$ are determined through numerical integration and the enthalpy function $\vartheta(\eta)$ is determined from equation (17). The corresponding temperature distribution may be found from the enthalpy distribution with the use of enthalpy tables. With these new values of velocity and temperature, the steps outlined above are repeated and a second approximation to the velocity and temperature distribution is determined. This process is continued until the desired accuracy is obtained. The procedure for calculating the velocity and temperature distribution for the insulated surface is similar to that described for the condition of constant surface temperature. The enthalpy function in this case, however, is found from equation (15).

Experimental values of the thermal properties or values found from analytical expressions may be used with equal facility in the computations. The equations may be simplified somewhat for the particular case in which the Prandtl number is taken as a constant; then, $Pr^* = 1$ and the integrating factor β reduces to α^{Pr}. For solutions involving a constant Prandtl number, however, the method of reference 5 is preferable since the solution can be obtained with less calculation.

RESULTS AND DISCUSSION

Some friction and heat-transfer characteristics at supersonic Mach numbers have been computed by the method described in the preceding section. Faired experimental values of the viscosity and Prandtl number, taken primarily from reference 11, have been used in the computations. (The calculations were limited to the temperature range in which experimental data were available.) The free-stream temperature was taken as 392.7° R (-67° F), the value at the isothermal region of the NACA standard atmosphere, and the corresponding values of the thermal properties were taken as:

$$c_{p_1} = 7.718 \quad \text{Btu/(slug)(deg)}$$

$$\mu_1 = 3.058 \times 10^{-7} \quad \text{slugs/(ft)(sec)}$$

$$k_1 = 3.227 \times 10^{-6} \quad \text{Btu/(ft)(sec)(deg)}$$

$$Pr_1 = 0.73$$

The boundary-layer characteristics presented herein include those cases which were calculated in reference 6; the same thermal data were used in both analyses. Some small differences were found between the results of reference 6 and those calculated by the method presented herein. These differences are due primarily to the poor convergence of the solutions in reference 6. Additional calculations have been made for those cases by means of the method of this report in order to make the results consistent.

The calculated velocity and temperature profiles for several supersonic stream Mach numbers for the insulated surface are presented in figure 1(a), and profiles for $T_s^* = 1$, 2, and 4 are presented in figures 1(b), 1(c), and 1(d), respectively. (Tabulated values of the velocity and temperature distributions and their first derivatives, to four decimal places, together with a chart of the thermal properties used herein, can be obtained upon request from the National Advisory Committee for Aeronautics.) The corresponding friction and heat-transfer characteristics are presented in figures 2 to 6 together with the results of several other analyses which make use of various approximations for the viscosity and Prandtl number.

Skin friction.- The shear stress at the surface τ is given by

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\rho_1 U_1^2}{\sqrt{R}} \mu^*\left(\frac{du^*}{d\eta}\right)_{\eta=0}$$

where R is the Reynolds number U_1x/ν_1. Substituting from equation (13) results in the following equation for shear stress:

$$\tau = \frac{\rho_1 U_1^2}{\sqrt{R}} \frac{1}{J(\infty)}$$

since $\alpha(0) = 1$. The average skin-friction coefficient c_f is found by integration as

$$c_f = \frac{1}{\frac{1}{2}\rho_1 U_1^2 x}\int_0^x \tau \, dx = \frac{4}{\sqrt{R}} \frac{1}{J(\infty)} \tag{18}$$

The skin-friction coefficients found from equation (18) for the insulated surface and for the condition of heat transfer are presented in figures 2(a) and 2(b), respectively, together with the results from other analyses for a stream temperature of -67° F. All the analyses show substantially the same variation of skin friction with Mach number for the insulated surface except for the solution based upon the linear viscosity relation $\mu^* = T^*$ for Pr = 1.0; for these conditions the skin friction is independent of the Mach number. The present results and those of reference 5, which are based upon a constant Prandtl number of 0.75 and the use of the Sutherland equation for viscosity, are in excellent agreement for both the insulated surface and for the specified constant surface temperatures. The differences between the results of this paper, those of Moore (ref. 8), and those of Young and Janssen (ref. 7)[1] may possibly be attributed in part to some inaccuracies in the differential-analyzer calculations, differences in the thermal data, and a small difference in stream condition - the calculations from references 7 and 8 are based upon a nominal stream temperature of 400° R.

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[1]In reference 8, experimental values of viscosity are used for temperatures up to 3,240° R and for Prandtl number, up to 2,000° R; for temperatures between 3,240° R and the dissociation temperature the viscosity was calculated with the use of the power law $\frac{\mu}{\mu_{T=3240}} = \left(\frac{T}{3240}\right)^{0.6}$ and for temperatures over 2,000° R the Prandtl number was determined from calculated values of the thermal properties. Values of the Prandtl number from various sources and the Sutherland equation for viscosity were employed in the boundary-layer calculations reported in reference 7.

The agreement of the results based on a power law and a linear viscosity-temperature relation with the results of this analysis is not so satisfactory as that of the results which make use of the Sutherland equation for viscosity.

It should be noted, however, that the variation of skin-friction coefficient with Mach number is dependent on the stream temperature and thus the differences between the results based on a power law and a linear viscosity-temperature relation and the results of this analysis would depend on the stream temperature at which a comparison was made.

With a linear viscosity-temperature relation of the form $\mu^* = \text{Constant } T^*$, the skin friction is independent of Mach number; taking the constant as unity results in $c_f\sqrt{R} = 1.328$ for all surface temperatures and stream Mach numbers irrespective of the value of the Prandtl number. The linear viscosity relation $\mu^* = \text{Constant } T^*$ was utilized in the calculations of reference 2 for a Prandtl number of 0.72 and the constant was adjusted to give the correct value of the viscosity at the surface. This procedure yields a variation of skin friction with Mach number for the insulated surface since the recovery temperature varies with Mach number; for the condition of constant surface temperature, the skin friction is independent of the Mach number and depends only upon the surface temperature.

Heat-transfer characteristics.- The heat-transfer rate per unit area at the surface q is given by the expression

$$q = -\left(k \frac{\partial T}{\partial y}\right)_{y=0} = -\frac{\mu_1}{Pr_1}\sqrt{\frac{U_1}{\nu_1 x}}\,\frac{{U_1}^2}{2}\left(\frac{\mu^*}{Pr^*}\frac{d\vartheta}{d\eta}\right)_{\eta=0}$$

and from equation (16), since $\beta = 1$ and $\varphi = 0$ for $\eta = 0$,

$$q = -\frac{c_{p_1} T_1}{Pr_1}\sqrt{\frac{\rho_1 \mu_1 U_1}{x}}\,\frac{\gamma - 1}{2} M^2 \frac{\vartheta_e - \vartheta_s}{V(\infty)} = -\frac{1}{Pr_1}\sqrt{\frac{\rho_1 \mu_1 U_1}{x}}\,\frac{i_e - i_s}{V(\infty)} \tag{19}$$

where γ is the ratio of the heat capacities at constant pressure and constant volume.

As can be seen from equation (19), the heat-transfer rate is proportional to the difference between an effective enthalpy function ϑ_e and the surface enthalpy function ϑ_s (or the difference between the effective enthalpy i_e and the surface enthalpy i_s). In contrast, the heat-transfer rate is approximately proportional to the difference between the recovery temperature (or enthalpy) and the surface temperature (or enthalpy) when the Prandtl number is taken as a constant. The effective enthalpy function and the recovery enthalpy function (the value of ϑ at the surface for an insulated plate) are presented in figure 3 as a function of the stream Mach number. Both ϑ_r and ϑ_e decrease with increasing Mach number and ϑ_e decreases with increasing values of the surface temperature; the functions ϑ_r and ϑ_e become identical for the condition of zero heat transfer.

The temperature recovery factor r may be determined from ϑ_r with the use of enthalpy tables. The recovery factor is presented in figure 4 as a function of Mach number together with the values found from other analyses. With the assumption that the Prandtl number is independent of temperature, the recovery factor is approximately equal to the square root of the Prandtl number for laminar flow for all Mach numbers (refs. 2 and 5, for example) irrespective of the viscosity-temperature relation; thus the recovery factor is unity for a Prandtl number of 1.0 and the recovery temperature equals the stagnation temperature of the stream. When the variation of Prandtl number with temperature is taken into account, however, the temperature recovery factor decreases substantially with increasing Mach number. The recovery factors of reference 7, based upon the use of the Sutherland equation for viscosity and experimental values of Prandtl number, are in substantial agreement with the results of the present analysis; the recovery factor is approximately 0.85 at low Mach numbers and decreases to approximately 0.76 and 0.65 at Mach numbers of 5 and 10, respectively.

Equation (19) for the heat-transfer rate is considerably more complicated than those determined from analyses based upon the use of a constant Prandtl number. The function ϑ_e is dependent upon both the surface temperature and the stream Mach number for a given stream temperature; whereas the recovery factor (a constant for all Mach numbers) takes the place of ϑ_e when the Prandtl number is taken as a constant.

The empirical relation

$$\frac{\vartheta_e - \vartheta_s}{V(\infty)(\vartheta_r - \vartheta_s)} = 0.30532 - 0.001289\left|2.25 - M_1\right|^{1.35} - 0.006062\ T_s^* \tag{20}$$

represents the variation of ϑ_e with ϑ_r within 1 percent for Mach numbers up to 5 for a stream temperature of -67° F. Values from equation (20) are compared with the calculated values of the quantities in figure 5. Eliminating $\frac{\vartheta_e - \vartheta_s}{V(\infty)}$ between equations (19) and (20) leads to the following expression for the heat-transfer rate:

$$q = -\frac{c_{p_1} T_1}{Pr_1} \sqrt{\frac{\rho_1 \mu_1 U_1}{x}} \frac{\gamma - 1}{2} M^2 \left(0.30532 - 0.001289 \left|2.25 - M\right|^{1.35} - 0.006062\ T_s^*\right)\left(\vartheta_r - \vartheta_s\right) \tag{21}$$

The values of ϑ_r presented in figure 3 and the values of the temperature recovery factor presented in figure 4 for Mach numbers between 5 and 10 were determined from equation (20) with values of ϑ_e and $V(\infty)$ calculated for $T_s^* = 2$. With these values of ϑ_r the heat-transfer rate calculated from equation (21) agrees with those computed from equation (19) for $T_s^* = 1$, 2, and 4 to within $1\frac{1}{2}$ percent for Mach numbers from 1 to 10.

Comparison of heat-transfer coefficients or Nusselt numbers found from different analyses is difficult because of the different temperature functions which arise in the expressions for the heat-transfer rate. A direct comparison of the heat-transfer rates more clearly shows the effects of the various approximations on the calculated heat-transfer characteristics. The heat-transfer rate determined by the method of this analysis is presented in figures 6(a), 6(b), and 6(c) for values of $T_s^* = 1$, 2, and 4, respectively, together with the results of other analyses.

Figure 6 shows that the local heat-transfer rates calculated with the use of the Sutherland equation for viscosity and a constant Prandtl number of 0.75 are in very good agreement with the results of this analysis for all Mach numbers and surface temperatures considered. Likewise the results of reference 8, which employ values of the thermal properties taken in part from experiment, are in good agreement with the results of this analysis for $T_s^* = 1$, the only value of surface temperature where a comparison could be made. The calculated heat-transfer rates which employ more approximate viscosity-temperature relations are considerably higher than the values determined by the method of this report at very high supersonic Mach numbers; below a

Mach number of 4 or 5, however, the agreement is generally satisfactory for a stream temperature of -67° F.

CONCLUDING REMARKS

Friction and heat-transfer characteristics of the laminar boundary layer on a flat plate have been calculated for Mach numbers from 1 to 10 for a stream temperature of -67° F with the use of experimental values for the heat capacity, conductivity, and viscosity. The consequent variability of the Prandtl number throughout the boundary layer causes the recovery factor to decrease from a value of approximately 0.85 at low Mach numbers to approximately 0.76 and 0.65 at Mach numbers of 5 and 10, respectively; whereas the recovery factor is approximately equal to the square root of the Prandtl number for all Mach numbers when the Prandtl number is independent of temperature. Moreover, the heat-transfer rate is proportional to the difference between an effective enthalpy, which is a function of both the surface temperature and stream Mach number, and the surface enthalpy when the temperature variation of all the thermal properties is considered; whereas the heat-transfer rate is approximately proportional to the difference between the recovery enthalpy and surface enthalpy for calculations based upon a constant Prandtl number. Because of the excellent agreement between the results of the present analysis and calculations based upon the use of the Sutherland equation for viscosity and a Prandtl number of 0.75, it appears that the exact variation of the thermal properties of the fluid need not be considered in the calculation of skin friction or heat transfer. More approximate viscosity-temperature relations and the use of a Prandtl number of unity introduce significant errors in the calculated friction and heat-transfer characteristics at Mach numbers above 4 or 5 for a stream temperature of -67° F.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 23, 1953.

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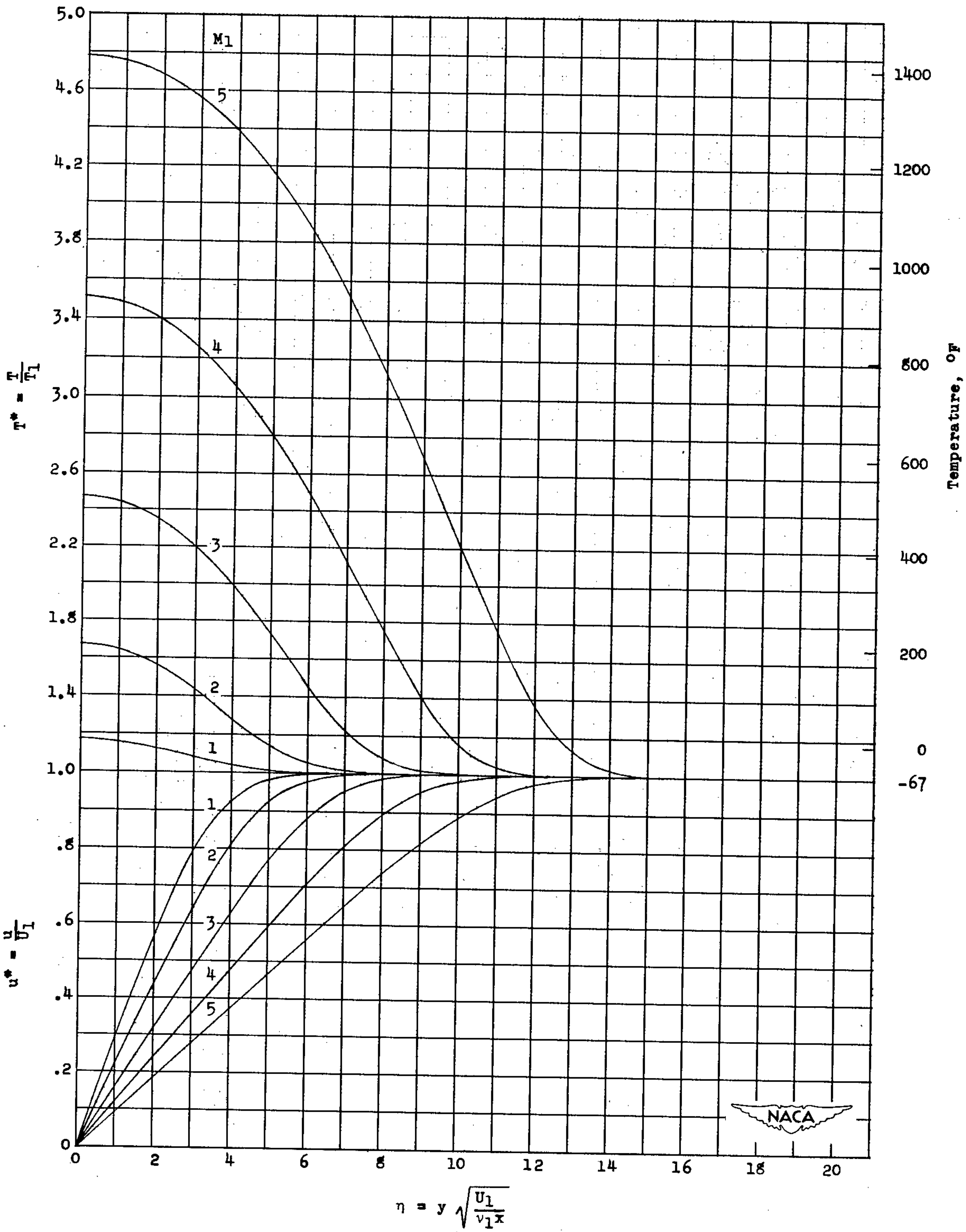
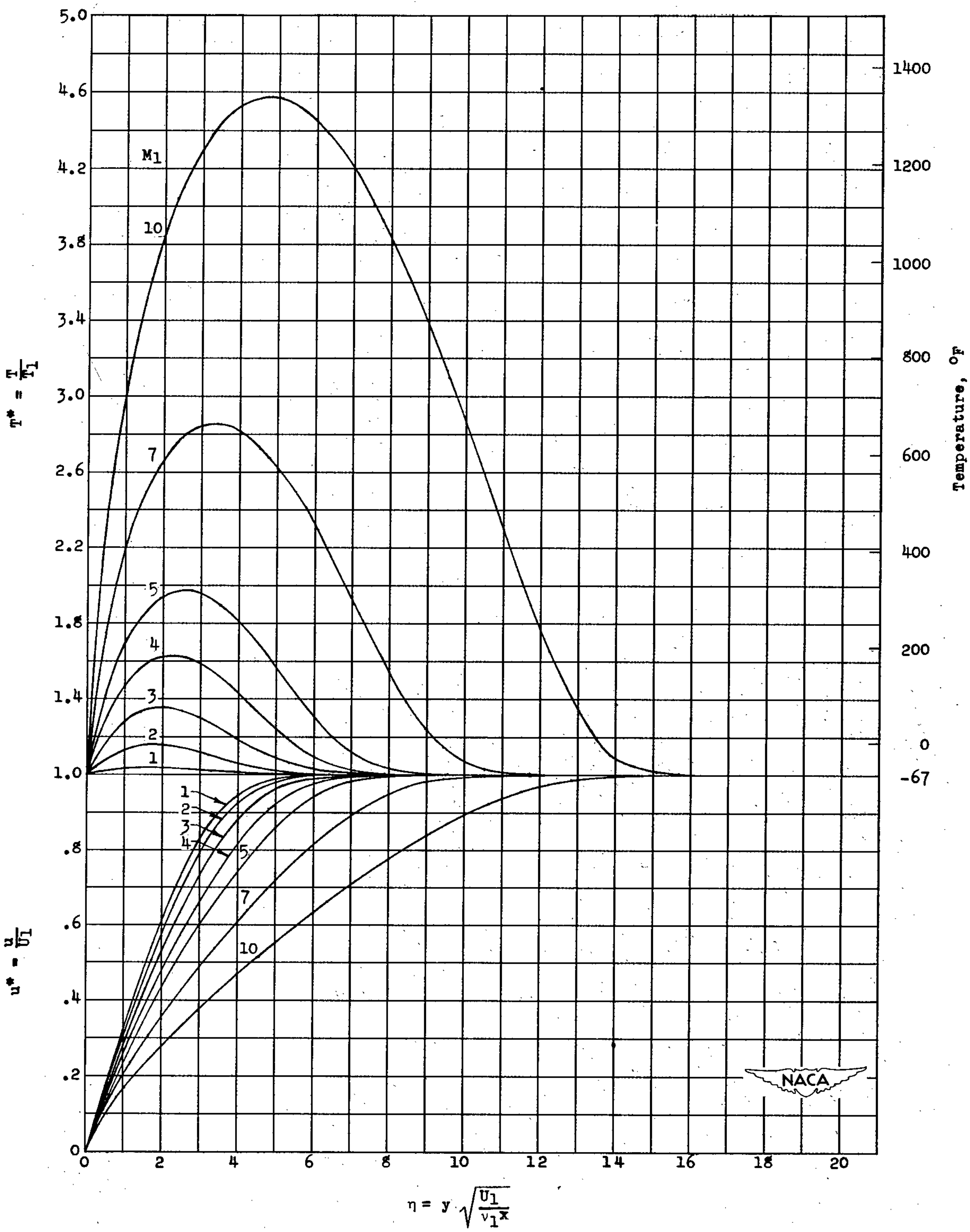


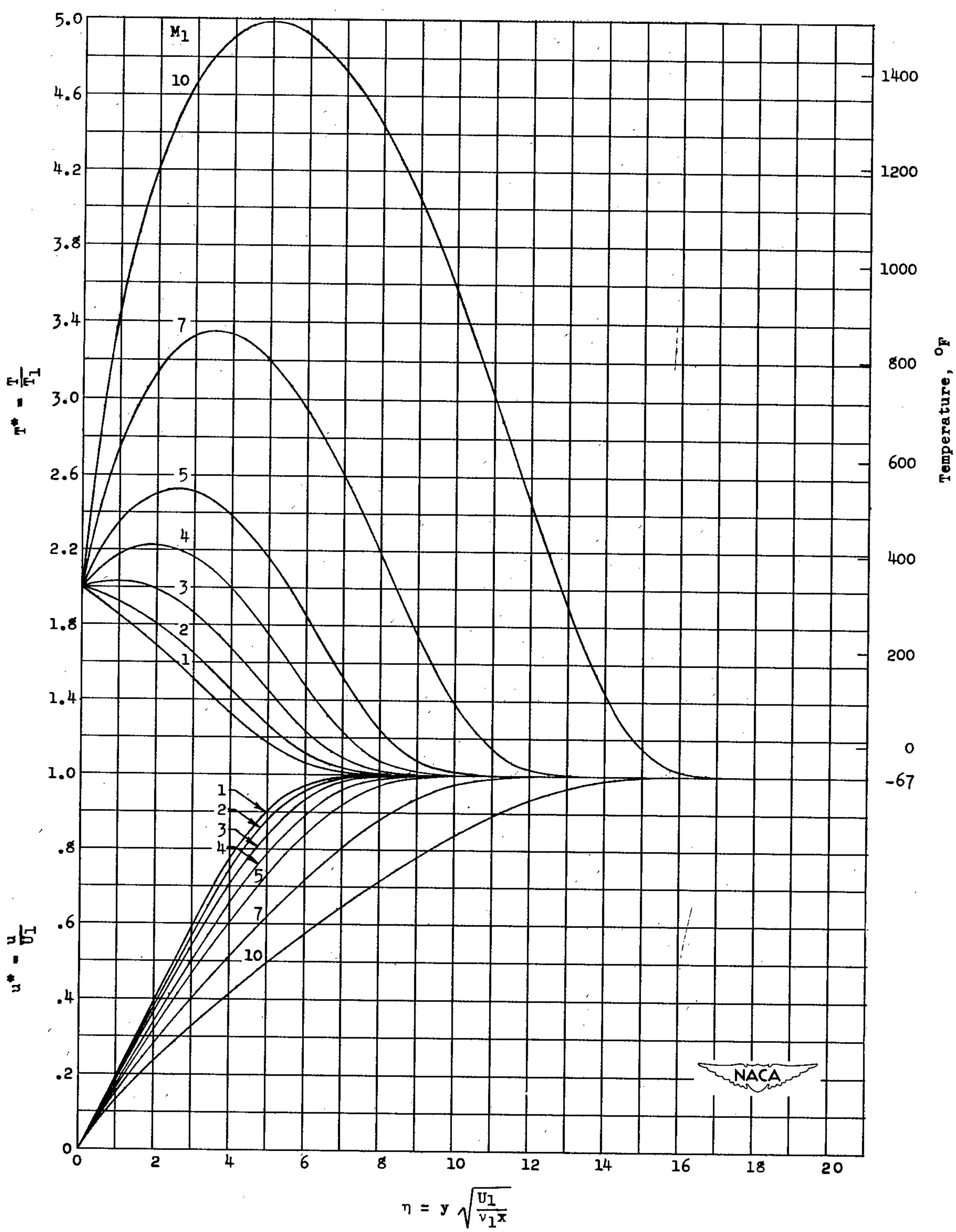
(a) Insulated surface.

Figure 1.- Velocity and temperature profiles.



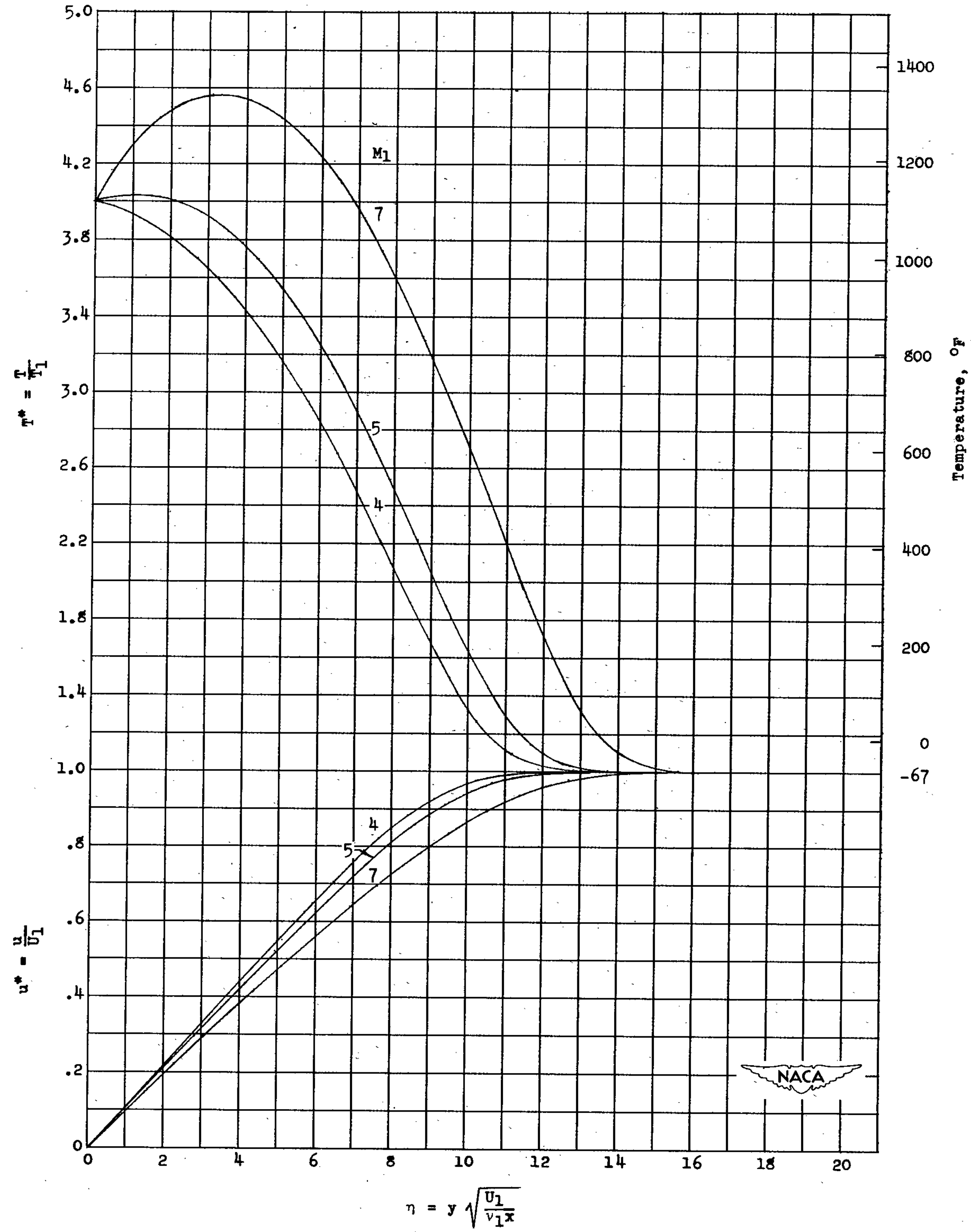
(b) $T_s{}^* = 1$.

Figure 1.- Continued.



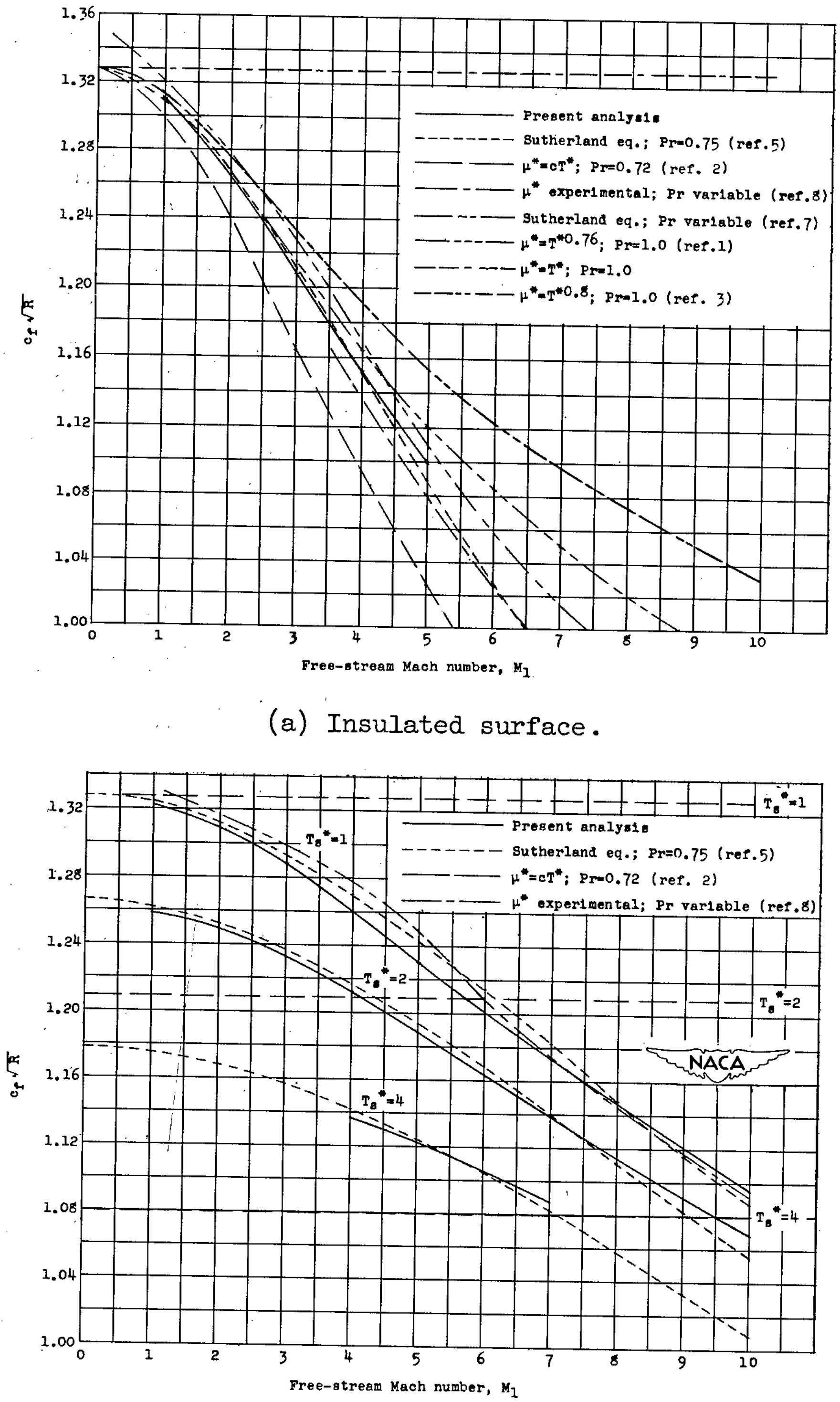
(c) $T_s^* = 2$.

Figure 1.- Continued.



(d) $T_s^* = 4$.

Figure 1.- Concluded.



(a) Insulated surface.

(b) Constant surface temperature.

Figure 2.- Variation of average skin-friction coefficient with stream Mach number.

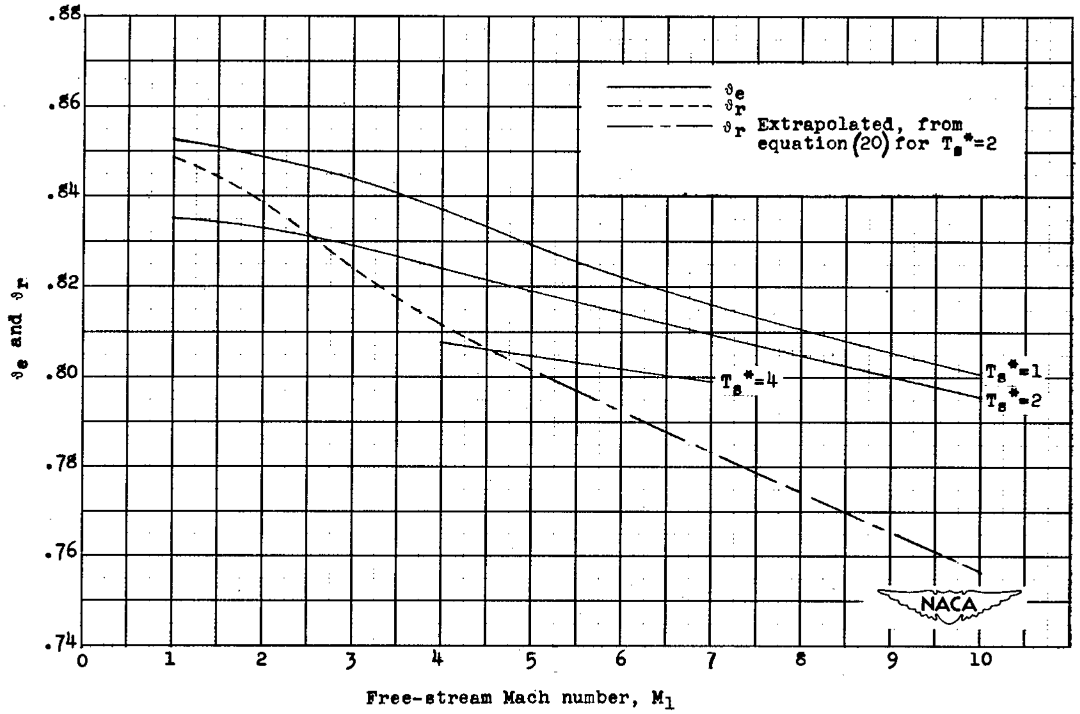


Figure 3.- Variation of effective enthalpy function and enthalpy recovery factor with stream Mach number.

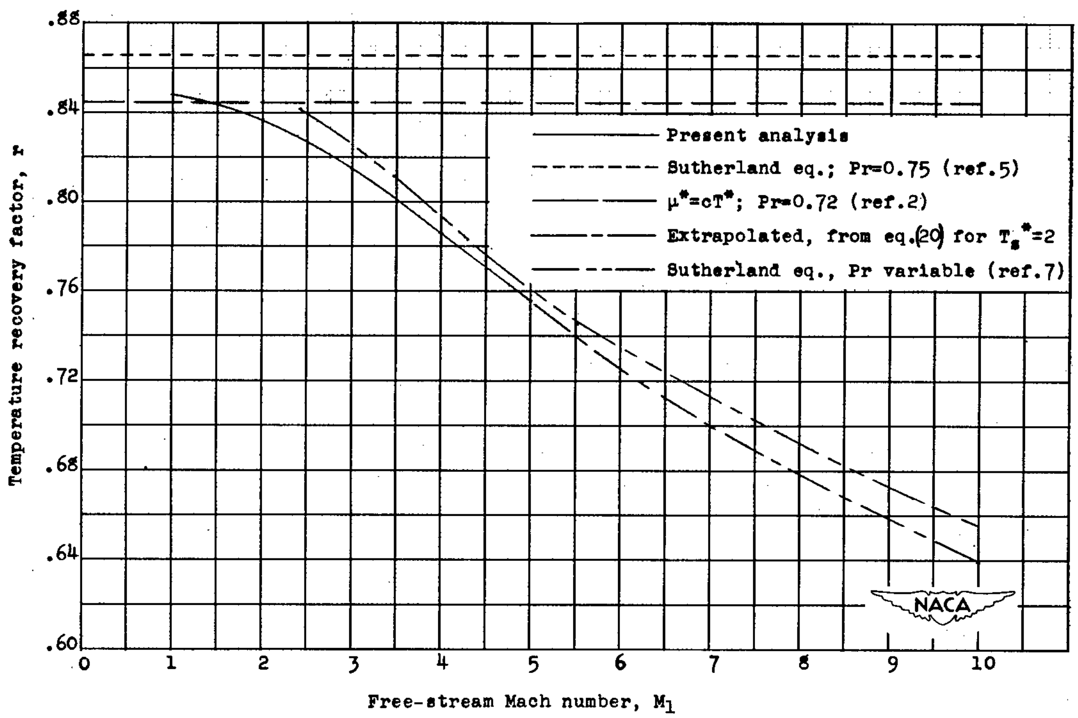


Figure 4.- Variation of temperature recovery factor with stream Mach number.

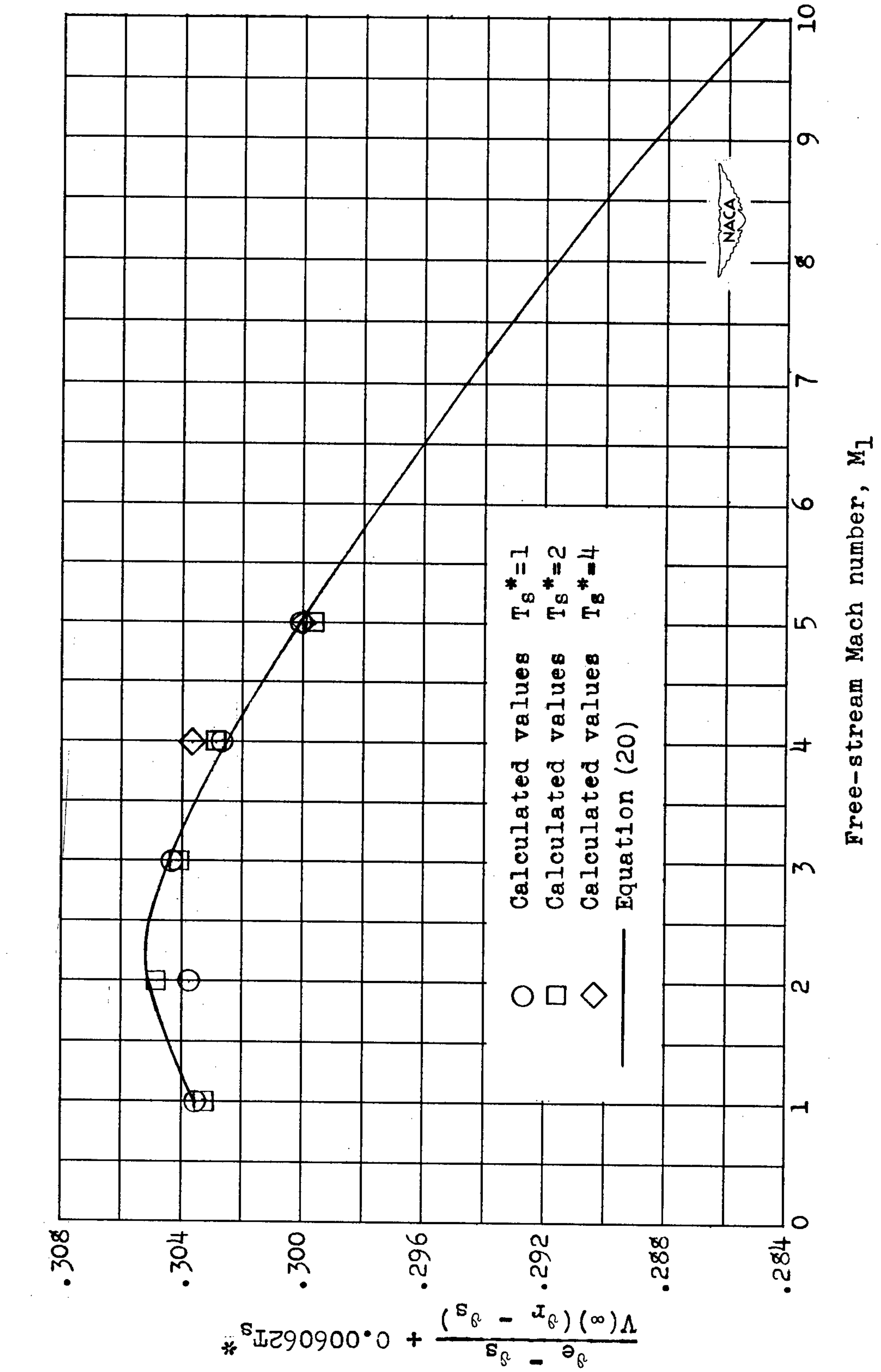
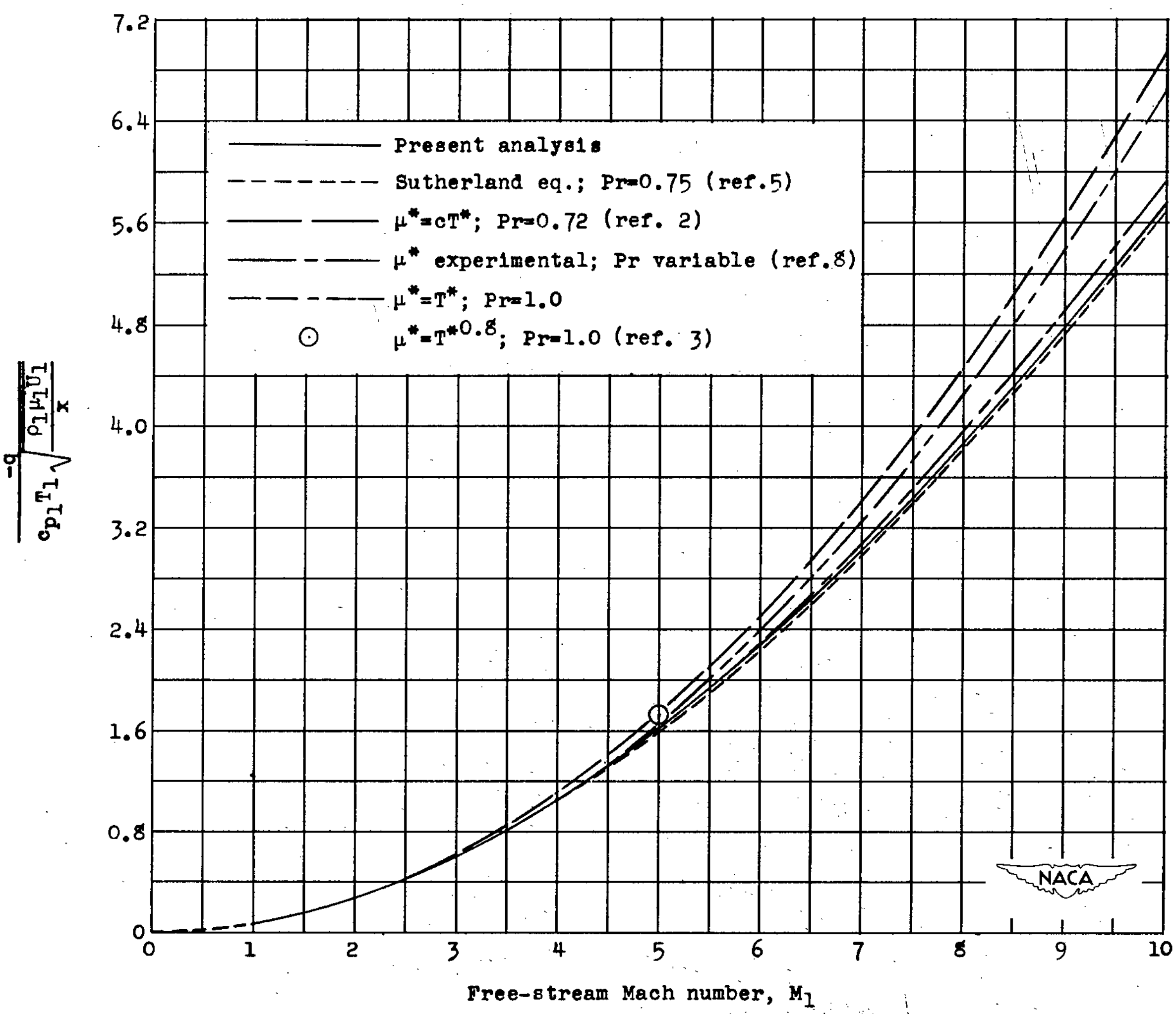
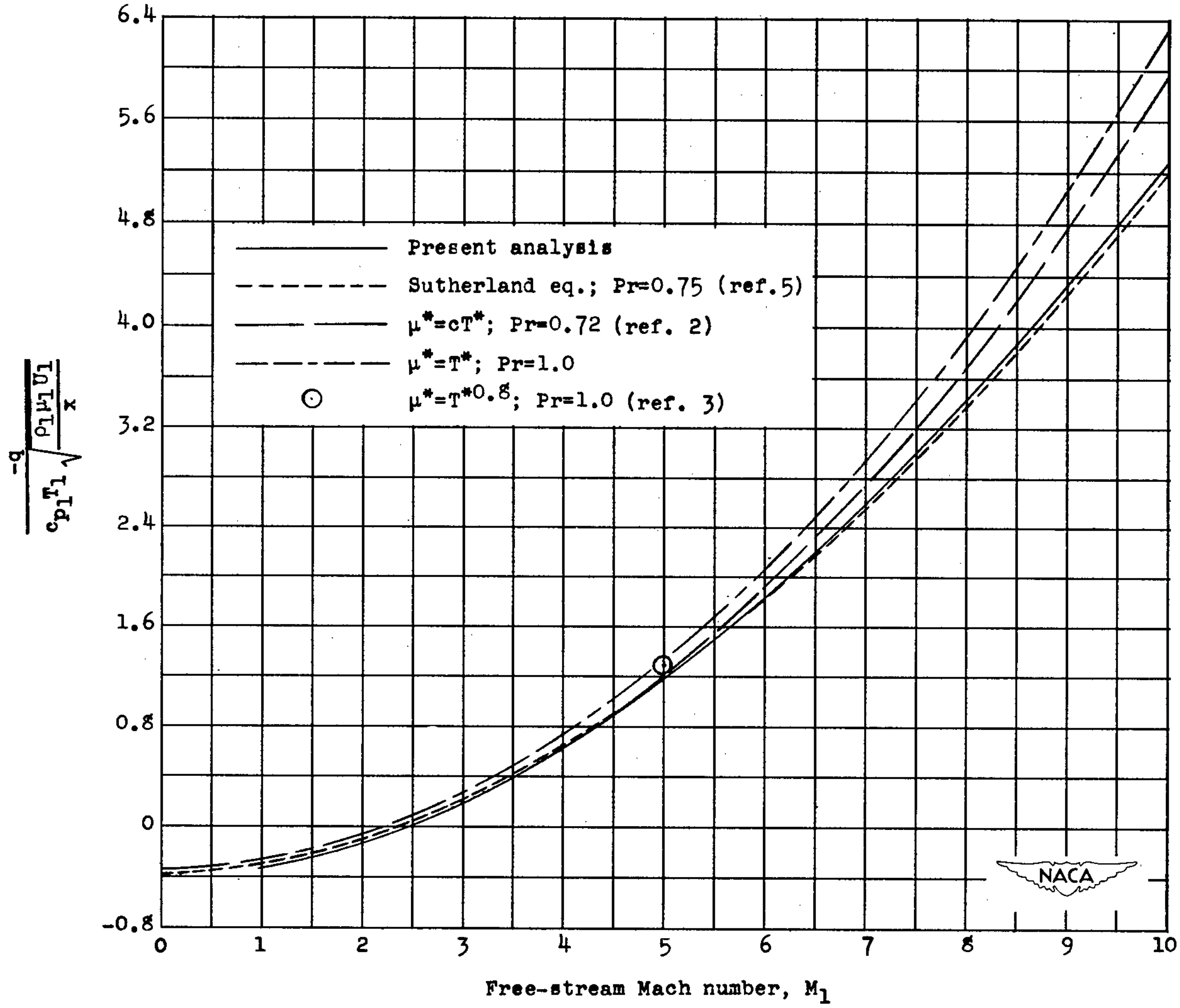


Figure 5.- Relation between effective enthalpy function ϑ_e and enthalpy recovery factor ϑ_r.



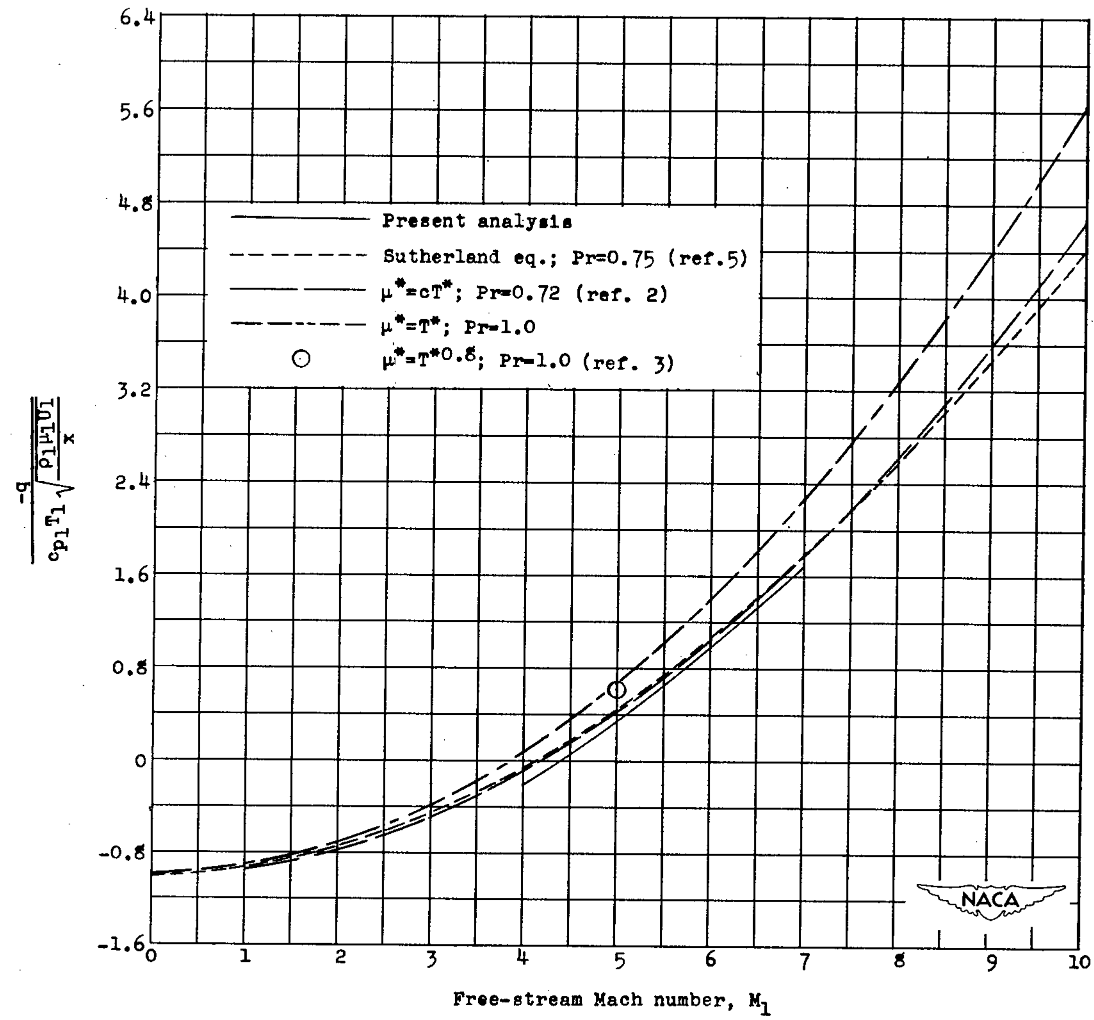
(a) $T_s^* = 1$.

Figure 6.- Variation of local heat-transfer rate with stream Mach number.



(b) $T_s^* = 2$.

Figure 6.- Continued.



(c) $T_s^* = 4$.

Figure 6.- Concluded.

W

NACA TN 2916
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EFFECT OF THERMAL PROPERTIES ON LAMINAR-BOUNDARY-LAYER CHARACTERISTICS.
E. B. Klunker and F. Edward McLean. March 1953.
29p. diagrs. (NACA TN 2916)

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