Wind Tunnel Branch Technical Note 9

REDUCTION OF FORCES AND MOMENTS TAKEN ON INTERNAL BALANCES AND THE EFFECT OF AXES ORIENTATION

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To present a uniformity in definitions concerning the relationships between forces and between moments for various reference systems and to clarify the effect of rotation of the model on the balance, rotation of the sting, etc.

The notes consider three primary and related topics: (a) a convention of definitions, directions, and angles, (b) a consideration of model body axes, wind axes, and stability axes, and (c) some equations considering movement of the model with respect to the balance, or movements of model and balance with respect to the tunnel.

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REDUCTION OF FORCES AND MOMENTS TAKEN ON INTERNAL BALANCES AND THE EFFECT OF AXES ORIENTATION

The main purpose of this set of notes is to present a uniformity in definitions concerning the relationships between forces and between moments for various reference systems and to clarify the effect of rotation of the model on the balance, rotation of the sting, etc.

The notes will consider three primary and related topics; (a) a convention of definitions, directions, and angles, (b) a consideration of model body axes, wind axes, and stability axes, and (c) some equations considering movement of the model with respect to the balance, or movements of model and balance with respect to the tunnel.

A. Basic Definition of Systems

1. Body Axes - This system, for an internal balance which is directly connected to the model with no type of offset, takes its values as read from the balance instruments.*

2. Wind Axes - For a model-balance set up as above, the only difference between body axes and wind axes is that all body axes forces and moments are rotated back to a coordinate system which corresponds to the body axes at angles of attack and yaw equal to zero.

3. Stability Axes - These are orthogonal axes (as are the others), having at zero yaw the same directions and signs as the wind axes, but differing from the latter in that they rotate with the airplane in yaw (but not in pitch) while the wind axes remain fixed to the relative wind.

Before setting up the coordinate systems that define these three sets it would be well to examine a model in general terms so that a sense of orientation may be established. The wing reference plane is that which passes through the wing tips and is parallel to the fuselage axis. The wind reference plane is that which passes through the relative wind vector and intersects the wing reference plane along a line which is perpendicular to the fuselage axis. The plane of symmetry is that which passes through the fuselage axis and is perpendicular to both the wing and wind reference planes. Angles may be defined in these planes.

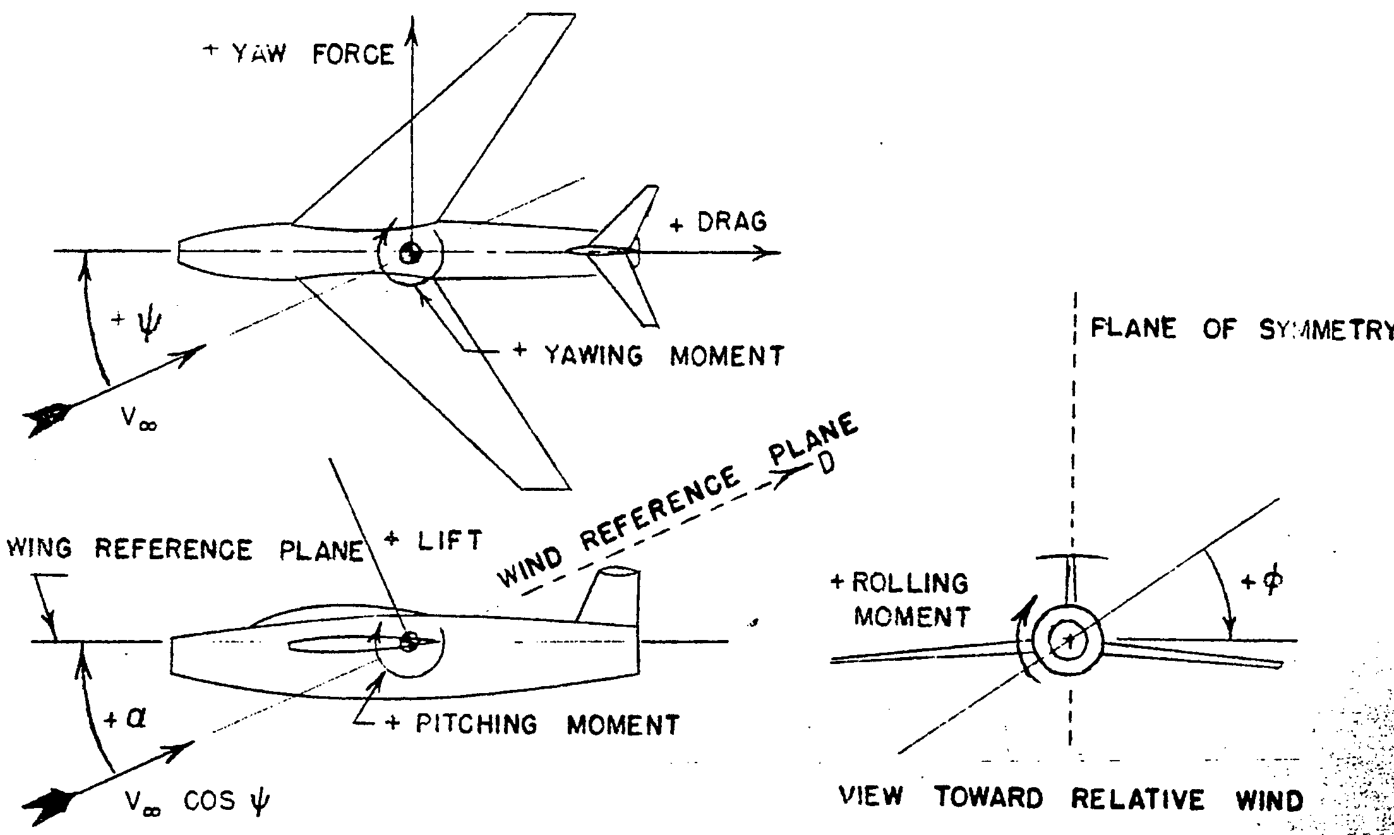
α ~ angle of attack is the angle in the plane of symmetry between the wing reference plane and the wind reference plane.

ψ ~ angle of yaw is the angle between the relative wind vector, V_∞, and the projection of the model reference axis in the relative wind plane.

*The only variation being introduced is that of transferring moments from the balance reference center to the assumed location of center of gravity.

ϕ ~ the angle of roll. In wind tunnel tests, the roll angle, ϕ, with respect to the relative wind, is considered to be zero in all steady state tests. Only in wind tunnel tests with roll oscillations, the instantaneous values of the roll angle deviate from zero while the mean roll angle is still considered to be zero. Of greater interest to the present development, the angle of roll, ϕ, with respect to the support systems in wind tunnels, may be different from zero even in steady state tests.

Consider the following sketch wherein positive directions of forces, moments and angular displacements are indicated:



The following table lists the conventions for designation of forces, moments, and coefficients adopted by the Wind Tunnel Branch. In final reports the subscripts s, w, and B may be dropped once the basic system being used is described. It should be mentioned that no standard convention has been adopted by the aeronautical world, therefore in practically every case the contractor should be informed of these designations.

CONVENTIONS FOR FORCES, MOMENTS, AND COEFFICIENTS

Body Axes		Wind Axes		Stability Axes	
Forces	Coefficients	Forces	Coefficients	Forces	Coefficients
T ~ Tangential	$C_T = T/qA$	D ~ Drag	$C_D = D/qA$	D_s ~ Drag	$C_{D_s} = D_s/qA$
S ~ Side	$C_S = S/qA$	Y ~ Yaw	$C_Y = Y/qA$	S ~ Side	$C_S = S/qA$
N ~ Normal	$C_N = N/qA$	L ~ Lift	$C_L = L/qA$	L ~ Lift	$C_L = L/qA$
Moments	Coefficients	Moments	Coefficients	Moments	Coefficients
M_{ϕ_B} ~ Rolling	$C_{\ell_B} = M_{\phi_B}/qAb$	M_{ϕ_w} ~ Rolling	$C_{\ell_w} = M_{\phi_w}/qAb$	M_{ϕ_s} ~ Rolling	$C_{\ell_s} = M_{\phi_s}/qAb$
M_{α_B} ~ Pitching	$C_{m_B} = M_{\alpha_B}/qAc$	M_{α_w} ~ Pitching	$C_{m_w} = M_{\alpha_w}/qAc$	M_{α_s} ~ Pitching	$C_{m_s} = M_{\alpha_s}/qAc$
M_{ψ_B} ~ Yawing	$C_{n_B} = M_{\psi_B}/qAb$	M_{ψ_w} ~ Yawing	$C_{n_w} = M_{\psi_w}/qAb$	M_{ψ_s} ~ Yawing	$C_{n_s} = M_{\psi_s}/qAb$

In the above Table "c" is the mean aerodynamic chord length and "b" is the span length. In some cases D, the maximum diameter may be used.

B. Transfer Equations

This set of notes will present for 3 and 6 components the following transformations:

1. body axes to wind axes
2. body axes to stability axes
3. wind axes to stability axes
4. stability axes to wind axes
5. stability axes to body axes
6. wind axes to body axes.

Consider the following figure wherein x, y, z define wind axes, x', y', z' define body axes, and x", y", z" define stability axes and capitals designate vectors along these axes. As previously defined ψ refers to a positive yawing angle and is measured in the xy plane; α refers to a positive pitching angle and is measured in the x'z plane.

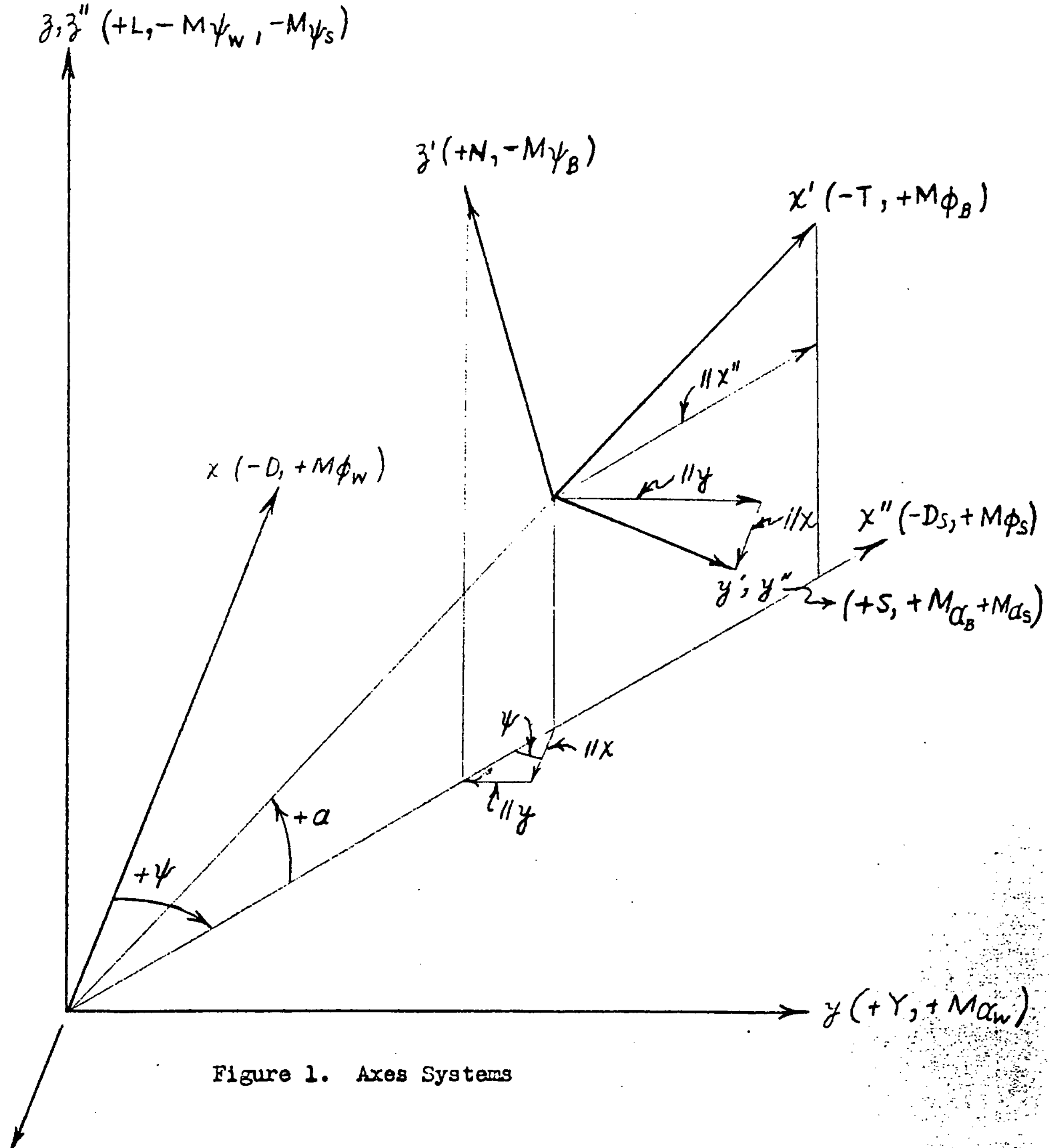


Figure 1. Axes Systems

If the model and balance are fixed to the sting, with no relative motion between the model and the balance, and the sting is capable of movements in yaw and pitch, the model body axes coincide with the balance axes and therefore all forces and moments indicated by the balance are body forces and moments. If the model and balance axes do not coincide additional transformations must be considered.

Case I. Body Axes to Wind Axes

Each vector in the body axes system may be resolved into components in the wind axes system. (bars indicate vectors).

$$\left.\begin{aligned}
\overline{x'} &= \overline{x' \cos\alpha \cos\psi} + \overline{x' \sin\alpha} + \overline{x' \cos\alpha \sin\psi} \\
\overline{y'} &= \overline{y' \sin\psi} + \overline{y' \cos\psi} \\
\overline{z'} &= \overline{z' \sin\alpha \cos\psi} + \overline{z' \cos\alpha} + \overline{z' \sin\alpha \sin\psi}
\end{aligned}\right\} \text{Equation 1}$$

From Equation 1 and Figure 1, the following may be obtained:

$$\left.\begin{aligned}
x &= x' \cos\alpha \cos\psi - y' \sin\psi - z' \sin\alpha \cos\psi \\
y &= x' \cos\alpha \sin\psi + y' \cos\psi - z' \sin\alpha \sin\psi \\
z &= x' \sin\alpha + z' \cos\alpha
\end{aligned}\right\} \text{Equation 2}$$

Consider that

$$\left.\begin{aligned}
x' &= -T & \qquad x' &= M_{\phi_B} \\
y' &= S & \qquad y' &= M_{\alpha_B} \\
z' &= N & \qquad z' &= -M_{\psi_B} \\
x &= -D & \qquad x &= M_{\phi_w} \\
y &= y & \qquad y &= M_{\alpha_w} \\
z &= L & \qquad z &= -M_{\psi_w}
\end{aligned}\right\} \text{Equation 3}$$

Upon substitution of Equation 3, Equation 2 becomes:

$$\left.\begin{aligned}
D &= T \cos\alpha \cos\psi + S \sin\psi + N \sin\alpha \cos\psi \\
Y &= -T \cos\alpha \sin\psi + S \cos\psi - N \sin\alpha \sin\psi \\
L &= -T \sin\alpha + N \cos\alpha \\
M_{\phi_w} &= M_{\phi_B} \cos\alpha \cos\psi - M_{\alpha_B} \sin\psi + M_{\psi_B} \sin\alpha \cos\psi \\
M_{\alpha_w} &= M_{\phi_B} \cos\alpha \sin\psi + M_{\alpha_B} \cos\psi + M_{\psi_B} \sin\alpha \sin\psi \\
M_{\psi_w} &= -M_{\phi_B} \sin\alpha + M_{\psi_B} \cos\alpha .
\end{aligned}\right\} \begin{array}{c}\text{Body to Wind} \\ \text{Equation 4}\end{array}$$

Case 2. Body Axes to Stability Axes

In this case

$$\left.\begin{aligned} x'' &= x' \cos\alpha - z' \sin\alpha \\ y'' &= y' \\ z'' &= x' \sin\alpha + z' \cos\alpha \end{aligned}\right\} \quad \text{Equation 5}$$

Defining

$$\left.\begin{aligned} x'' &= -D_s,\ M\phi_s \\ y'' &= +S,\ M\alpha_s \\ z'' &= +L,\ -M\psi_s \end{aligned}\right\} \quad \text{Equation 6}$$

Then

Body to Stability

$$\left.\begin{aligned} D_s &= T\cos\alpha + N\sin\alpha \\ S &= S \\ L &= -T\sin\alpha + N\cos\alpha \\ M\phi_s &= M\phi_B \cos\alpha + M\psi_B \sin\alpha \\ M\alpha_s &= M\alpha_B \\ M\psi_s &= -M\phi_B \sin\alpha + M\psi_B \cos\alpha \end{aligned}\right\} \quad \text{Equation 7}$$

Case 3. Wind Axes to Stability Axes

$$\left.\begin{aligned} x'' &= x\cos\psi + y\sin\psi \\ y'' &= y\cos\psi - x\sin\psi \\ z'' &= z \end{aligned}\right\} \quad \text{Equation 8}$$

With the previous definitions (Equations 3 and 6)

Wind to Stability

$$\left.\begin{aligned} D_s &= D\cos\psi - y\sin\psi \\ S &= y\cos\psi + D\sin\psi \\ L &= L \\ M\phi_s &= M\phi_w \cos\psi + M\alpha_w \sin\psi \\ M\alpha_s &= M\alpha_w \cos\psi - M\phi_w \sin\psi \\ M\psi_s &= M\psi_w \end{aligned}\right\} \quad \text{Equation 9}$$

Case 4. Stability Axes to Wind Axes

$$\left.\begin{aligned} x &= x'' \cos\psi - y'' \sin\psi \\ y &= x'' \sin\psi + y'' \cos\psi \\ z &= z'' \end{aligned}\right\} \quad \text{Equation 10}$$

Again substituting Equations 3 and 6

Stability to Wind

$$\left.\begin{aligned} D &= D_s \cos\psi + S \sin\psi \\ Y &= -D_s \sin\psi + S \cos\psi \\ L &= L \\ M_{\phi_w} &= M_{\phi_s} \cos\psi - M_{\alpha_s} \sin\psi \\ M_{\alpha_w} &= M_{\phi_s} \sin\psi + M_{\alpha_s} \cos\psi \\ M_{\psi_w} &= M_{\psi_s} \end{aligned}\right\} \quad \text{Equation 11}$$

Case 5. Stability Axes to Body Axes

$$\begin{aligned} x' &= x'' \cos\alpha + z'' \sin\alpha \\ y' &= y'' \\ z' &= z'' \cos\alpha - x'' \sin\alpha \end{aligned}$$

Stability to Body

$$\left.\begin{aligned} T &= D_s \cos\alpha - L \sin\alpha \\ S &= S \\ N &= L \cos\alpha + D_s \sin\alpha \\ M_{\phi_B} &= M_{\phi_s} \cos\alpha - M_{\psi_s} \sin\alpha \\ M_{\alpha_B} &= M_{\alpha_s} \\ M_{\psi_B} &= M_{\psi_s} \cos\alpha + M_{\phi_s} \sin\alpha \end{aligned}\right\} \quad \text{Equation 13}$$

Case 6. Wind Axes to Body Axes

$$\left.\begin{aligned} x' &= x \cos\psi \cos\alpha + y \sin\psi \cos\alpha + z \sin\alpha \\ y' &= y \cos\alpha - x \sin\alpha \\ z' &= z \cos\alpha - x \cos\psi \sin\alpha - y \sin\psi \sin\alpha \end{aligned}\right\} \text{Equation 14}$$

$T = D \cos\psi \cos\alpha - y \sin\psi \cos\alpha - L \sin\alpha$	Wind to Body
$S = Y \cos\alpha + D \sin\alpha$	
$N = L \cos\alpha + D \cos\psi \sin\alpha - y \sin\psi \sin\alpha$	
	Equation 15
$M_{\phi_B} = M_{\phi_W} \cos\psi \cos\alpha + M_{\alpha_W} \sin\psi \cos\alpha - M_{\psi_W} \sin\alpha$	
$M_{\alpha_B} = M_{\alpha_W} \cos\alpha - M_{\phi_W} \sin\alpha$	
$M_{\psi_B} = M_{\psi_W} \cos\alpha + M_{\phi_W} \cos\psi \sin\alpha + M_{\alpha_W} \sin\alpha \sin\psi$	

C. Roll of Model and Balance With Respect to Sting Axis

In this case body axes forces and moments still correspond to balance axes forces and moments and wind-off gravity terms must be entered into the reduction as tare values, as usual. However, the attitude of the balance with regard to angle of attack and yaw must be determined before converting the body axes data to the wind or stability axes systems.

Taking the coordinate systems that appear in Figure 1 and assuming the superscripts that agree with the axes on that figure, then free stream speed, $V\infty$, corresponds to -X with $Y = 0$ and $Z = 0$.

When α_{Ba} and ψ_{Ba} are the setting of the balance at the balance roll angle $\phi_{Ba} = 0°$, then

$$x' = x \cos\psi_{Ba} \cos\alpha_{Ba}$$
$$y' = - x \sin\psi_{Ba}$$
$$z' = - x \cos\psi_{Ba} \sin\alpha_{Ba}$$

Equation 16

Consider that

$$x = -V\infty \qquad x' = -V_x'$$
$$y = + V_y = 0 \qquad y' = + V_y'$$
$$z = + V_z = 0 \qquad z' = + V_z'$$

Equation 17

Introducing Equation 17 into Equation 16 results in the following equations for the velocity components with reference to the balance axis.

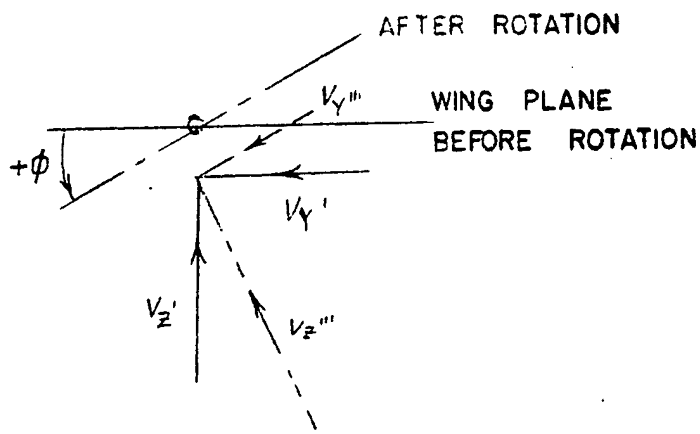
$$V_x' = V\infty \cos\psi_{Ba} \cos\alpha_{Ba}$$
$$V_y' = V\infty \sin\psi_{Ba}$$
$$V_z' = V\infty \cos\psi_{Ba} \sin\alpha_{Ba}$$

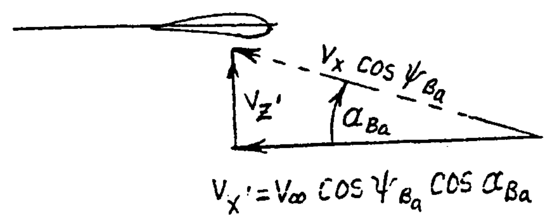
Equation 18

Consider the following sketches:

looking downstream along the balance axis

sideview with flow from right





The velocity components with reference to the rotated balance system are then:

$$\left.\begin{aligned} V_z''' &= V_z' \cos \phi + V_y' \sin \phi \\ V_y''' &= V_y' \cos \phi - V_z' \sin \phi \end{aligned}\right\} \quad \text{Equation 19}$$

corrected α $\qquad \alpha_{corr} = \tan^{-1} \dfrac{V_z'''}{V_\infty \cos \psi_{Ba} \cos \alpha_{Ba}}$

or

$$\alpha_{corr} = \tan^{-1} \left((\tan \alpha_{Ba} \cos \phi) + \left(\frac{\tan \psi_{Ba}}{\cos \alpha_{Ba}} \sin \phi \right) \right)$$

Equation 20

For zero yaw $\alpha_{corr} = \tan^{-1} (\tan \alpha_{Ba} \cos \phi)$

corrected ψ $\qquad \psi_{corr} = \sin^{-1} \dfrac{V_y}{V_\infty}$

$$\psi_{corr} = \sin^{-1} (\sin \psi_{Ba} \cos \phi - \cos \psi_{Ba} \sin \alpha_{Ba} \sin \phi)$$

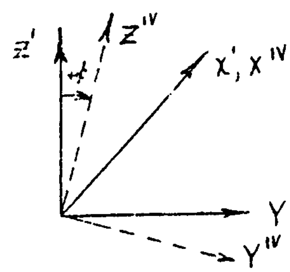
Equation 21

For zero yaw $\psi_{corr} = - \sin^{-1} (\sin \alpha_{Ba} \sin \phi)$

D. Roll of Model With Respect to Balance Axis

Assume that the model is rotated on the balance to some positive angle, and the balance remains in its original attitude. In this case the model axes do not coincide with the balance axes. The anglesof attack and yaw are the same as the corrected values under Part C, however the forces and moments indicated by the balance must first be referred to the model axes before any further transfer may be accomplished.

Consider the following sketch wherein x', y', and z' refer to the balance axes, and x^{IV}, y^{IV}, and z^{IV} are the axes of the model rolled about the balance axis x'.



Then

$$\left.\begin{aligned} x^{IV} &= x' \\ y^{IV} &= y' \cos\phi - z' \sin\phi \\ z^{IV} &= z' \cos\phi + y' \sin\phi \end{aligned}\right\} \quad \text{Equation 22}$$

$$\left.\begin{aligned} x^{IV} &= -T \text{ model}, \; M_{\phi\,model} \\ y^{IV} &= S \text{ model}, \; +M_{\alpha\,model} \\ z^{IV} &= N \text{ model}, \; -M_{\psi\,model} \end{aligned}\right\} \quad \text{Equation 23}$$

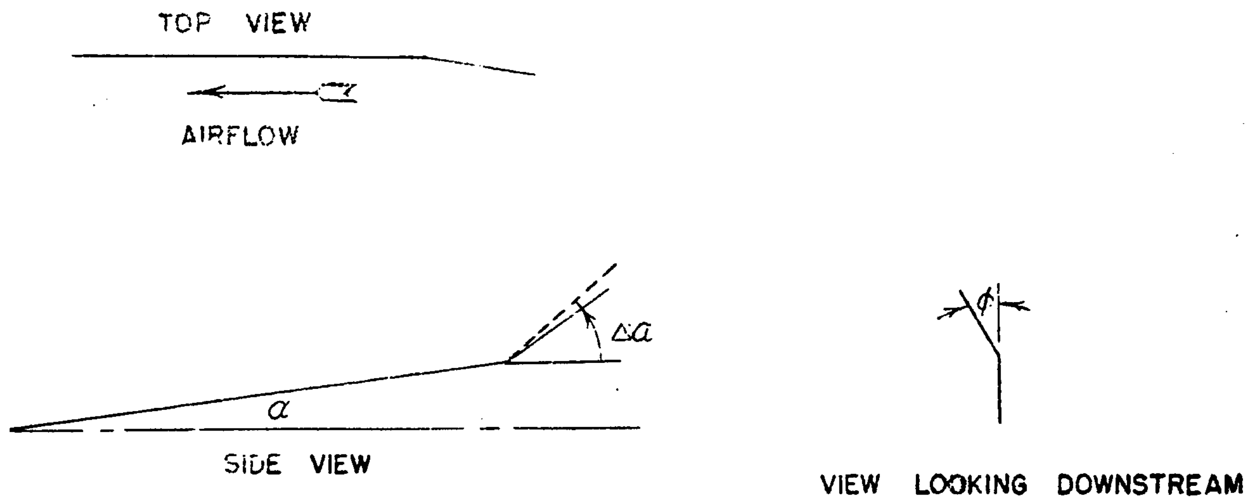
Inserting Equations 23 and 3 into Equation 22,

$$\left.\begin{aligned} T \text{ model} &= T \text{ balance} \\ S \text{ model} &= S \text{ bal} \cos\phi - N \text{ bal} \sin\phi \\ N \text{ model} &= N \text{ bal} \cos\phi + S \text{ bal} \sin\phi \\ M_{\phi\,mod} &= M_{\phi\,bal} \\ M_{\alpha\,mod} &= M_{\alpha\,bal} \cos\phi + M_{\psi\,bal} \sin\phi \\ M_{\psi\,mod} &= M_{\psi\,bal} \cos\phi - M_{\alpha\,bal} \sin\phi \end{aligned}\right\} \quad \text{Equation 24}$$

Case E. Sting Prebent + $\Delta\alpha$, Pitch and Roll of Sting

Body forces and moments coincide with balance values and are transferred to wind and stability axes according to equations reported once the corrected angles of attack and yaw are determined. This case may occur quite frequently with the new sting suspension system, in which the sting swings in the pitching plane.

Consider the following sketch



This is quite similar to Case 3 but with ψ setting = 0°. The $\Delta\alpha$ of prebend is unaffected by rotation. $V_y' = 0$.

Equation 20 becomes:

$$\alpha_{corr} = \tan^{-1} (\tan \alpha_{sting} \cos \phi) + \Delta\alpha_{prebent}$$

Equation 25

$$\psi_{corr} = - \sin^{-1} (\sin \alpha_{sting} \sin \phi)$$

Equation 26

Case F. Rotation of Model in Pitch or Yaw With Respect to Balance

The model may be set off by $\Delta\alpha$ or $\Delta\psi$ or both with respect to the balance pitch or yaw axes. Since this is a very unusual case, it will only be dealt with in general terms. If the model is offset at a negative angle in α and the sting pitches only, then the angle of attack is α sting-$\Delta\alpha$ model and balance axes forces and moment values should be transferred to model axes before using any of the equations. If the sting is rolled to some attitude ϕ, then corrected α and ψ is that as in Case E. Care must be exercised in transferring the moments from the balance reference center to the assumed location of the model c.g. in both the longitudinal and vertical measurements.

G. Control Surface Conventions

The following figure is used to point out the accepted convention for control surfaces. The sketch represents an elevator or aileron in sideview, and a rudder in topview:

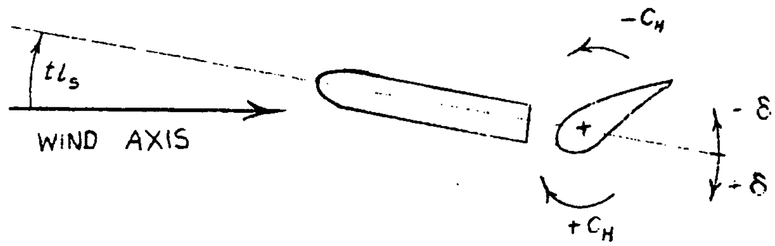


Figure 2. Sign convention for control surfaces

i_s is the symbol for stabilizer incidence

C_H is the hinge moment coefficient

δ is the angular deflection of the control surface.

An important thing to notice is that moment directions indicated are for the moment actions of the air on the control surface.