ARMY RESEARCH LABORATORY

The Effect of Helicopter Main Rotor Blade Damage on the Rotor Disk (Whole Rotor) Motion

by Joseph Fries

ARL-TR-2241

June 2000

Army Research Laboratory
Aberdeen Proving Ground, MD 21005-5068

ARL-TR-2241 **June 2000**

The Effect of Helicopter Main Rotor Blade Damage on the Rotor Disk (Whole Rotor) Motion

Joseph Fries
Survivability/Lethality Analysis Directorate, ARL

Abstract

When a helicopter main rotor blade is ballistically damaged, an imbalance is created in the rotor, causing the rotor disk to execute unwanted motions, which are detrimental to performance. The normally smooth-flying helicopter develops new vibrations that can be physiologically annoying or debilitating to the pilot, can exceed structural fatigue endurance limits, can cause aeromechanical instabilities, and can reduce helicopter performance ability.

This report examines the effect of the loss of the outboard section of one rotating blade of a rotor set of four blades on the fixed-system (nonrotating) rotor disk motion. The report shows, beginning with the rotor blade forcing, how a damaged blade's response changes, and how this change feeds into the rotor's fixed-system disk motion (the disk referring to the blades acting in concert as a whole entity).

With a normally undamaged rotor (referring to all the blades), there exists within the rotor itself the capability of motion canceling of certain frequencies depending on the number of rotor blades in the rotor. This study tracks each individual harmonic (integer multiples of the rotor speed) frequency, one at a time, in order to obtain a first-principles understanding of the phenomena involved with rotor imbalance.

Table of Contents

INTENTIONALLY LEFT BLANK.

List of Figures

List of Tables

INTENTIONALLY LEFT BLANK.

1. Introduction

When a helicopter main rotor blade is ballistically damaged, an imbalance is created in the rotor, causing the rotor disk to execute unwanted motions, which are detrimental to performance. The normally smooth-flying helicopter develops vibrations that can be physiologically annoying or debilitating to the pilot, can exceed structural fatigue endurance limits, can cause aeromechanical instabilities, and can reduce helicopter performance ability.

This report examines the effect of the loss of the outboard section of one rotating blade of a rotor set of four blades on the fixed-system (nonrotating) rotor disk motion. The report shows, beginning with the rotor blade forcing, how a damaged blade's response changes, and how this change feeds into the rotor's fixed-system disk motion (the disk referring to the blades acting in concert as a whole entity).

With a normally undamaged rotor (referring to all the blades), there exists within the rotor the capability of motion and force canceling of certain frequencies depending on the number of blades in the rotor. This study tracks each individual harmonic (integer multiples of the rotor speed) frequency, one at a time, in order to obtain a first-principles understanding of the phenomena involved with rotor imbalance.

It is the purpose of this work to "break" the damaged rotor blade imbalance phenomenon into its most fundamental elements, these elements being the relative individual blade responses between the damaged and undamaged blades, the blade (rotating) forcing frequencies, the fixed-system (nonrotating) rotor disk frequencies and phase angles, the type and location of rotor blade damage, the number of blades that compose the rotor, and the particular blade of the rotor set that is damaged. Among these variables are deterministic coupled relationships that work together to produce fixed-system rotor disk motions as a function of the individual rotor blade response characteristics.

Note: A list of terms used in the equations that follow is included at the end of this report.

Each rotor blade is excited by a common periodic azimuthal harmonic forcing function (i.e., each blade of the rotor experiences the same forcing at the same position in the azimuth). However, for a damaged blade, because of loss of mass, blade length, and/or stiffness, its dynamic response is different from those of the undamaged blades. This difference nullifies the vibration cancellation effect that exists when all the blades are similar or undamaged. As a result, these uncanceled motions at particular frequencies move into the fixed system as new, unwanted forcing of the fixed-system rotor disk motion that ultimately affects the aircraft.

The present analysis uses a Newtonian approach (Meyer 1969) in deriving the blade equations of motion, with 2 degrees of freedom for each blade of the rotor set. A representative four-blade rotor is modeled in the analysis. Each blade is represented as a flapping mass on a blade length with a root effective flap hinge and a root angular spring. In addition, a hub mass is represented at the flap hinge on top of a vertical spring. Each of the blades is forced with harmonic vertical shears at the blade tip.

Since the object of this analysis is to study how and which frequencies are transferred to the rotor's fixed system when a blade is damaged, the magnitudes of the forcing function can be made arbitrary, and only the relative change in magnitude between the damaged and undamaged case is important. Therefore, this analysis normalizes both the blade responses and the rotor disk responses by the undamaged rotor blade response for each individual harmonic. However, it is recognized that there is a natural amplitude relationship among the harmonics for a blade's aerodynamic forcing. The present analysis concentrates on understanding rotor imbalances and vibration propagation fundamentals, rather than attempting to quantify rotor motions and loads.

In this analysis, trigonometric products are involved in the mathematics of transformation from rotating motion into nonrotating motion that results in frequency changes. For instance, trigonometric terms, such as

$$\sin k\psi \cos l\psi$$

occur when a harmonic such as sin k ψ in the rotating system is transformed into the fixed system. Here,

$$\text{k and 1}$$

are different harmonics, and

$$\psi = \Omega t$$

is the rotor blade's azimuth position. Here $\psi = \Omega t$, and when Ω is normalized by itself $\frac{\Omega}{\Omega} = 1$, we call this 1/rev. For the kth harmonic, kΩ normalized again by Ω yields $\frac{k\Omega}{\Omega}$, which we call k/rev. By means of trigonometric identities (Riddle 1974), we know that

$$\sin(k\psi \pm 1\psi) = \sin k\psi \cos 1\psi \pm \cos k\psi \sin 1\psi$$

and

$$\sin k\psi \cos 1\psi = \frac{1}{2}(\sin(k+1)\psi \pm \sin(k-1)\psi).$$

A frequency change occurs going from the rotating blade system into the rotor disk fixed system. We see the rotating sin k ψ is transformed into two different fixed system frequency terms sin (k + 1) ψ and sin (k − 1) ψ. This is one phenomenon; another is the phenomenon of cancellation when the rotor blades are summed to give a total rotor nullifying effect. For instance, we may have a fixed-system mathematical form like

$$\sum_{i=1}^{4} \beta_i \sin k\left(\psi + 2\pi\frac{(i-1)}{4}\right),$$

where

β_i = flap angle of individual blade, and k = fixed-system harmonic.

If all the blades are undamaged and physically equivalent, the flap angle responses will be identical. In this case, it is known that for a four-bladed rotor, the summation will sum to zero (Johnson 1980) for harmonics 1, 2, and 3. This is the cancellation effect. However, if there is blade damage and all the flap angle responses are not equal, then no cancellation occurs, and the 1, 2, and 3/rev frequency vibration effects are felt in the fixed system. These types of considerations are examined in this work.

The physical properties of a generic (16,000-lb gross weight) single main rotor helicopter are represented in this analysis. The blade weight, effective flap hinge offset, hub weight, and the associated springs yield an uncoupled first natural blade flap frequency of 1.05/rev for each of the four undamaged rotor blades.

Damage is assumed to be imposed on blade no. 1 by removing the outboard portion (up to 30%) of the blade, thus resulting in blade mass and length loss. Both of these losses cause the natural frequency of the damaged blade to increase. This reduces the blade's response, especially to the 1/rev forcing from a resonance perspective, which causes the rotor imbalances. There are also differences of response to the other forcing frequencies. These differences nullify the cancellation effect of the normally undamaged rotor.

The ramification of blade damage to the fixed-system rotor disk tilt is that the disk thrust vector is tilted in undesirable directions with undesirable harmonics. The helicopter trim condition when the blades are undamaged is determined by the steady tilt of the rotor disk in the fixed system from 1/rev blade flapping in the rotating system. A damaged blade's 1/rev response is different from the undamaged blades, and thus affects the trim setting. Beside the rotating 1/rev frequency, other frequencies, due to the damaged blade, get into the fixed-system rotor disk motions as harmonics, tilting the disk and the thrust vector back and forth at these frequencies, and shaking the helicopter as vibrations. There will be physiological effects on humans subjected to these vibrations, natural frequencies of the helicopter will change with possible instabilities occurring, structural fatigue life may be exceeded by increased vibrations, and general helicopter performance will be degraded.

Because of these effects, an understanding of the underlying fundamentals of rotor imbalance is important in order to know how to analyze their effects.

2. Equations of Motion

The model used for an individual rotor blade is illustrated in Figure 1.

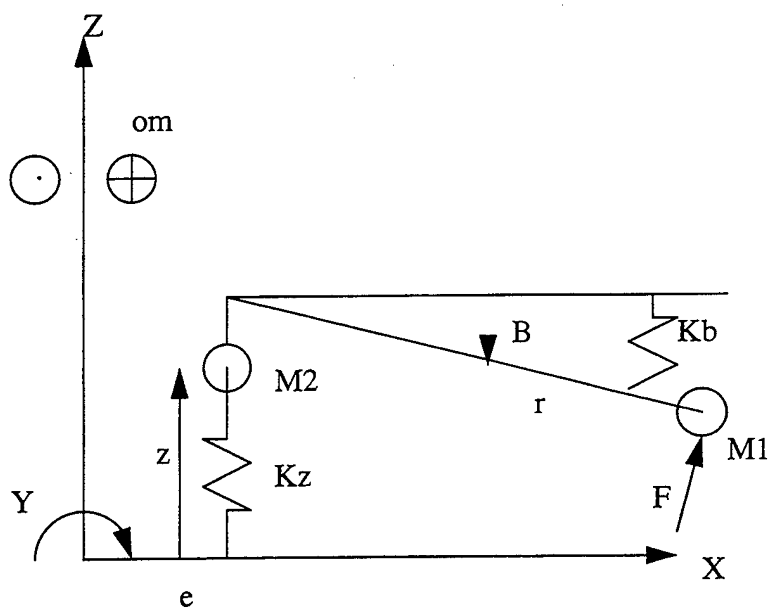

Figure 1. Model of Single Rotor Blade.

From Figure 1, let

Ω = the rotor speed (om),

z = vertical deflection of the hub mass,

β = blade natural angular flap displacement (B),

M_1 = rotor blade mass (M1),

M_2 = hub mass (M2),

K_z = hub spring (Kz),

K_β = blade spring (Kb),

e = blade flap hinge offset,

r = blade radius, and

F = blade forcing.

From Figure 1, om is the rotor speed, denoted by an end of an arrow going into the paper and a dot coming out of the paper. Each of the four rotor blades is represented as per Figure 1. The final equations of motion are

$$\begin{bmatrix} M_1 r^2 & -M_1 r \\ -M_1 r & M_1 + M_2 \end{bmatrix} \begin{bmatrix} \beta'' \\ z'' \end{bmatrix} + \begin{bmatrix} M_1 \Omega^2 r\left((e+r) + K_\beta r^2\right) & 0 \\ 0 & K_z \end{bmatrix} \begin{bmatrix} \beta \\ z \end{bmatrix} = \begin{bmatrix} -Fr \\ F \end{bmatrix},$$

where the primes are the second time derivative of β and z.

Let the forcing function Fi of the ith blade be of the form

$$F_i = A_i \sin k\left(\psi + 2\frac{\pi}{N}(i-1)\right),$$

where

A_i = amplitude,

k = harmonic number,

ψ = azimuth of blade 1, and

N = number of blades in rotor.

Solving for the steady-state solution of the equations of motion, we have for the first element of the solution of the form

$$\beta_i = C_i \sin k\left(\psi + 2\pi\frac{(i-1)}{N}\right).$$

Now, sum the blades together and transform them into the fixed system.

Fixed-System Lateral Disk Plane Tilt:

$$\beta_x = \sum_{i=1}^{4} C_i \sin k\left(\psi + 2\pi\frac{(i-1)}{4}\right)\sin\left(\psi + 2\pi\frac{(i-1)}{4}\right)$$

and

Fixed-System Longitudinal Disk Plane Tilt:

$$\beta_y = \sum_{i=1}^{4} C_i \sin k\left(\psi + 2\pi\frac{(i-1)}{4}\right)\cos\left(\psi + 2\pi\frac{(i-1)}{4}\right)$$

Call the lateral fixed-system disk plane tilt β_x and the longitudinal β_y.

Since $\psi = \Omega t$, where t = time, we need to determine the Fourier coefficients of both the lateral and longitudinal fixed-system tilts to determine the amplitudes and frequencies involved.

3. Computer Software Analysis Implementation

The implementation of the solution of this study is done in integrated steps. The rotor blade dynamic response solution is made with MACSYMA.* This computer-developed code is contained

* MACSYMA is an interactive system for doing mathematical computation. It handles numeric, graphic, and symbolic calculations and incorporates high-level programming language, which allows the user to define his own procedures. MACSYMA Inc., 20 Academy Street, Arlington, MA 02174-6436. The telephone number is 1-800-MACSYMA (1-800-622-7962, toll free in U.S. only); 1-617-646-4550; FAX 1-617-646-3161; and the email address is info@macsyma.com.

in a file named dyn1.sav. The MACSYMA function odematsys is used to solve for the steady-state blade response solution.

These responses are used as inputs into another program in FORTRAN called sum.f, which sums the individual blade responses, makes a coordinate transformation into the fixed system, and performs a Fourier analysis to determine the amplitudes, C_j, and frequencies, k.

4. Results

The individual blade harmonic responses were solved using MACSYMA, and the results given in Table 1 were normalized by the magnitudes of the undamaged blade responses and are presented in the following tabular form. These are the rotor blade flap responses due to an applied sine forcing functions at the blade tips of an arbitrary amplitude. The minus value of −1.0 in column 2 for the first harmonic is due to the 1/rev forcing to be below the blade natural frequency of 1.05/rev (i.e., it is a dynamic resonance phenomenon).

Table 1. Normalized Blade Flap Data (Rotating System)

Sin Harmonic	No Damage	10% Damage[a]	20% Damage	30% Damage
1	−1.0	−0.7929	−0.6151	−0.4636
2	1.0	0.9064	0.8129	0.7182
3	1.0	0.9024	0.8079	0.7103
4	1.0	0.9017	0.8034	0.7052
5	1.0	0.8971	0.8074	0.7112

[a] 10% loss of blade mass and length at tip.

From Table 1, the first column is the harmonic of the sin forcing function that is applied to each blade in the rotating system of the set of four blades from 1 to 5/rev. Column 2 is the blade response for the undamaged condition normalized by the magnitude of the undamaged blade response. The remaining columns are the normalized damaged blade responses for 10%, 20%, and 30% damage. For instance, 30% means a loss of 30% blade mass and 30% of the blade tilt radius.

Next, the total rotor disk fixed-system motion is calculated by summing the contribution of each individual blade and projecting it onto the fixed-system x and y axes. The angular motion projection about the x axis is called BX and about the y axis BY. Figure 2 illustrates this convention.

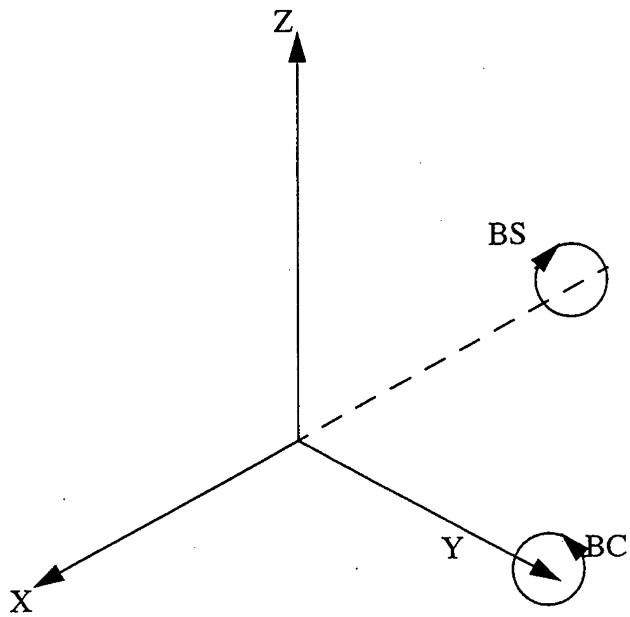

Figure 2. Fixed-System Rotor Disk Tilt Coordinate System.

Figure 2 shows the direction of the disk plane lateral tilt BX and the longitudinal tilt BY. Since BX and BY are time histories, a Fourier series analysis is performed on them to obtain the sine and cosine Fourier coefficients bk and ak, where k denotes the harmonic number.

The computed fixed-system rotor disk tilt information is given in Table 2 (undamaged rotor) and Table 3 (damaged rotor). The results in these tables indicate some interesting phenomena. Table 3 contains more entries than Table 2; this indicates that more frequencies are transmitted into the fixed system when the blades are damaged than when they are undamaged. Thus, in turn, the rotor disk plane is tilting at these additional frequencies. The steady rotor disk tilt of −2.0 in Table 2 has changed to −1.731 in Table 3; this indicates that the damaged blade has caused the helicopter to go

Table 2. Fixed-System Rotor Disk Tilt, No Blade Damage

	1/Rev	2	3	4	5
BXa0	-2.0	—	—	—	—
BYb0	—	—	—	—	—
BXa1	—	—	—	—	—
BYb1	—	—	—	—	—
BXa2	—	—	—	—	—
BYb2	—	—	—	—	—
BXa3	—	—	—	—	—
BYb3	—	—	—	—	—
BXa4	—	—	-2.0	—	2.0
BYb4	—	—	2.0	—	2.0
BXa5	—	—	—	—	—
BYb5	—	—	—	—	—
BXa6	—	—	—	—	—
BYb6	—	—	—	—	—

Table 3. Fixed-System Rotor Disk Tilt, Blade Damage (30% of Tip Removed)

	1/Rev	2	3	4	5
BXa0	-1.731	—	—	—	—
BYa0	—	—	—	—	—
BXa1	—	-0.1409	—	—	—
BYb1	—	-0.1409	—	—	—
BXa2	-0.2672	—	-0.1438	—	—
BYb2	0.2682	—	-0.1448	—	—
BXa3	—	0.1409	—	-0.1474	—
BYb3	—	-0.1409	—	-0.1474	—
BXa4	—	—	-1.854	—	1.855
BYb4	—	—	1.855	—	1.856
BXa5	—	—	—	0.1474	—
BYb5	—	—	—	-0.1474	—
BXa6	—	—	—	—	0.1434
BYb6	—	—	—	—	-0.1444

out of its trimmed condition. For instance, for the helicopter to fly in a straight, level flight condition with undamaged blades, assume the lateral disk plane tilt is normally −2.0. This value (−2.0) is a steady lateral fixed-system disk tilt for the trimmed condition. When the blade is damaged, the tilt changes to −1.731, which causes the helicopter not to fly in a straight line.

The ambient 4/rev rotor disk tilt, as seen in Table 2, for the undamaged blade with a value of 2.0 or −2.0 is a tilting that is normally present in the rotor disk. With the damaged blade, the 4/rev tilt is still present but is reduced to 1.85, as seen in Table 3. This means that the 4/rev vibrations transmitted into the fuselage are reduced, which is a benefit, but at the detrimental cost of other frequencies (1, 2, 3, 5, 6, etc.) appearing.

In Table 2, the numbers across the top are the harmonics in the rotating system, while the numbers in the left-hand column are the harmonics transmitted into the fixed system. For undamaged blades, we see that only the harmonics that are multiples of the number of blades are transmitted (i.e., for the four-bladed rotor, only 0, 4, 8, 12, etc., per revolution will be transmitted).

Another effect to be noted is that all the frequencies are coming through to the fixed system when a blade is damaged (i.e., 1, 2, 3, 4, 5, 6/rev); thus, the rotor thrust vector is tilting at these frequencies and shaking the helicopter. In Table 3, a basic fundamental regular pattern shows up, frequencies are transmitted into the fixed system plus and minus 1/rev of the rotating blade forced response frequency. Also the blade summing process yields a mechanical frequency filtering effect when the blades are undamaged, which is lost when a blade becomes damaged. An explanation of the BXak and the BYbk terms in Tables 2 and 3, where k = 1…6, is given in the Appendix.

5. Pictorial Representation of Rotor Disk Tilt

Graphical representations of the fixed-system BXa2 rotor tilt as a function of time are shown in Figures 3–6 looking in the negative X direction into the Y-Z plane.

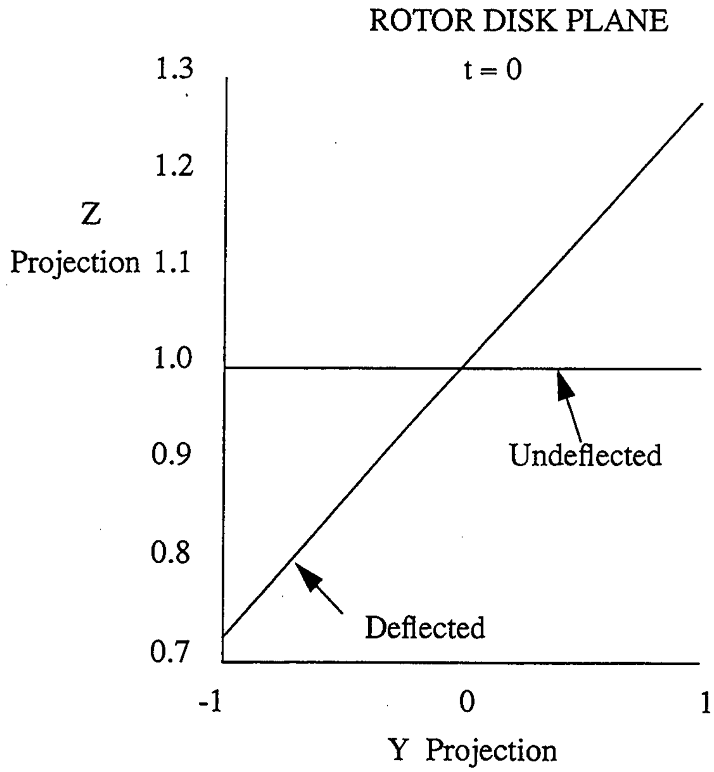

Figure 3. BXa2 Rotor Disk Plane Tilt at t = 0.

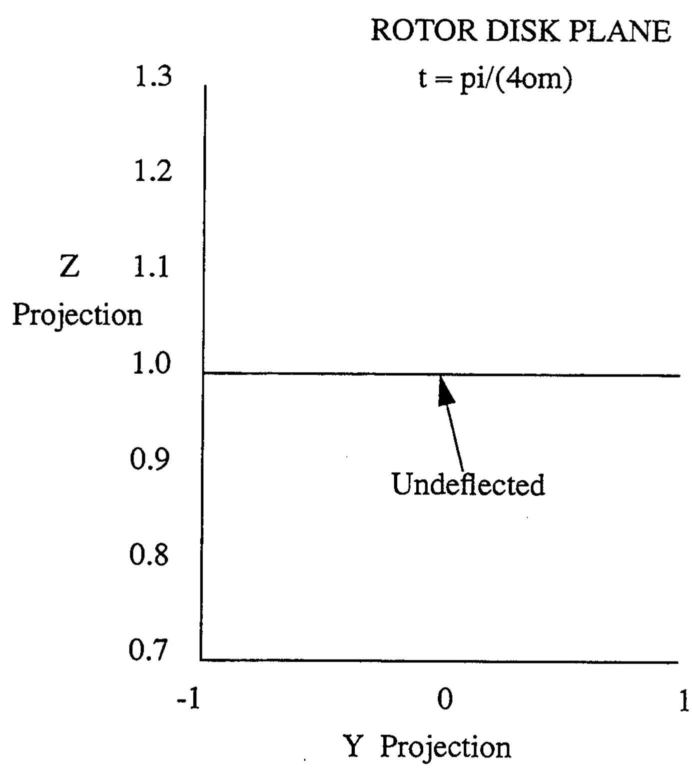

Figure 4. BXa2 Rotor Disk Plane Tilt at t = pi/(4 om).

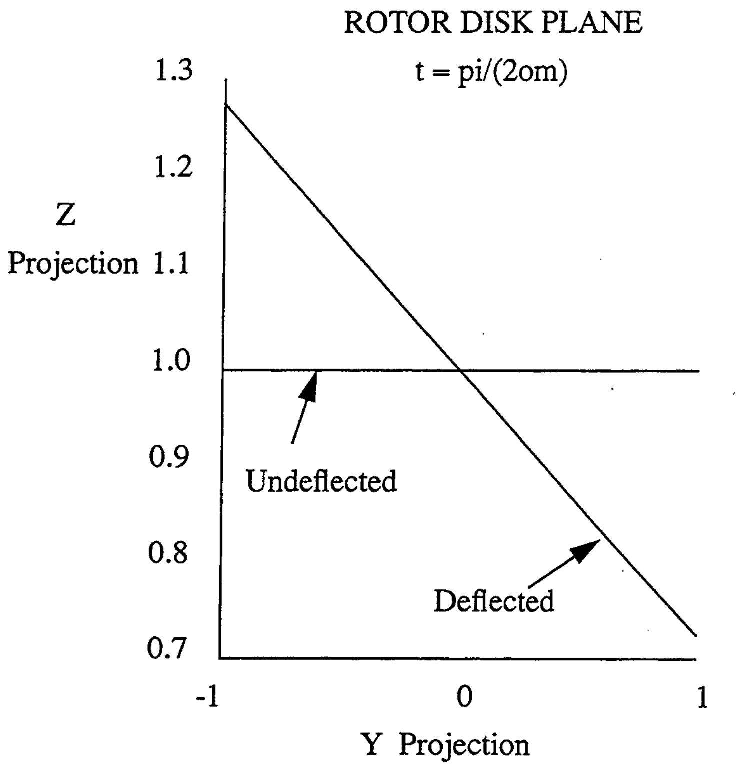

Figure 5. BXa2 Rotor Disk Plane Tilt at t = pi/(2 om).

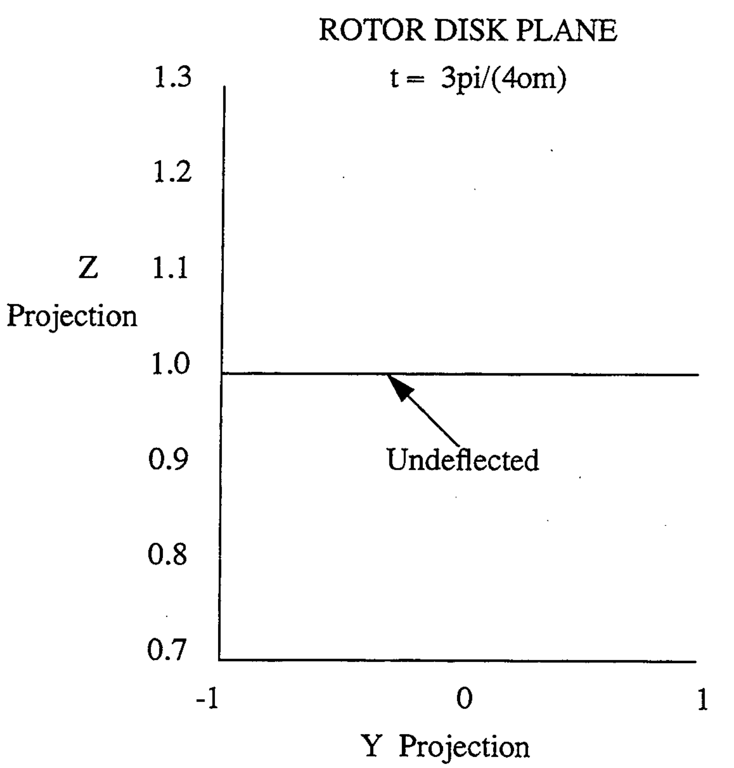

Figure 6. BXa2 Rotor Disk Plane Tilt at t = 3 pi/(4 om).

Figure 3 represents, in a perspective view, a picture of the rotor disk that a viewer would see looking straight on at the disk from the X-axis projected on the Y-Z plane. This shows the undisturbed disk lying horizontally, and also the tilted disk caused by a BSa2 contribution from a 30% damaged blade response to 1/rev forcing. This picture illustrates the magnitude of the 2/rev tilt of the rotor disk in the fixed system at time = 0 due to a damaged blade's response to 1/rev forcing.

In Figure 4, at time = pi/(4 om), the disk has moved back down to the undeflected roll tilt position. Then at time = pi/(2 om), the rotor disk is tilted in the opposite maximum tilted position. Finally, at time = 3 pi/(4 om), it is back to an untilted position again. Collectively, Figures 3–6 show that the rotor disk will go through a complete 2/rev cycle in pi/om seconds, or half a rotor cycle.

From Tables 2 and 3, we see the peculiarity of only BXak and BYbk coefficients present, with no BXbk nor BYak terms. This is caused by blade 1 being damaged; if blade 2 had been assumed damaged, the roles would have been reversed (i.e., BXbk and BYak terms only and no BXak nor BYbk terms). See the Appendix for the mathematical details.

6. Summary

An elementary helicopter rotor blade dynamical system is derived to demonstrate the elemental principles by which blade damage affects rotor system performance and can affect other aspects, such as human tolerance to vibrations, structural fatigue life, and general controllability.

Starting with the rotor's individual blades and their responses to forcing (in this case, harmonic sinusoidal forces), it is shown they have an integrated effect on the fixed-system rotor disk tilt. When the blades are undamaged, the helicopter's trim condition is determined by the blade 1/rev flapping of similarly responding blades. Also, for a four-bladed rotor, there is an inherent natural 4/rev tilting of the rotor disk in the fixed system that is always present.

When a blade is damaged, rotor imbalance exists and all the per rev frequencies are "loosed" to be transmitted into the fixed system. A plus and minus 1/rev frequency conversion is demonstrated on the rotor disk as well as a blade summing and filtering out of frequencies for an undamaged rotor.

One kind of blade damage is studied, that of the loss of the outer portion of one blade of a set of four. Further parametric damages can be imposed on the model to determine transmitted vibration effects as a function of damage type.

Since a helicopter as a whole responds to the entire rotor rather than to a single blade, analysis of the helicopter response due to knowledge of single blade forcing does not give a complete understanding of the damaged blade effect. It is therefore proposed that further work in this area be undertaken.

INTENTIONALLY LEFT BLANK.

7. References

Johnson, W. *Helicopter Theory.* Princeton, NJ: Princeton University Press, pp. 347–348, 1980.

Meyer, A. B. "A Non-Lagrangian Method of Determining the Vector Equations of a Dynamical System Described by Generalized Coordinates." ARA 69-079, Advanced Aeroelastic Group, Boeing Co., Vertol Division, February 1969.

Riddle, D. F. *Calculus and Analytic Geometry.* 2nd Edition, Belmont, CA: Wadsworth Publishing Co., Inc., p. 772, 1974.

INTENTIONALLY LEFT BLANK.

Appendix:

Explanation of BXak and BYbk Coefficients Appearing in Tables 2 and 3

INTENTIONALLY LEFT BLANK.

The type of blade response from this analysis is of the form

$$\sum_{i=1}^{4} C_{ik} \sin k\left(\psi + 2\pi\frac{(i-1)}{4}\right)\sin\left(\psi + 2\pi\frac{(i-1)}{4}\right)$$

and

$$\sum_{i=1}^{4} C_{ik} \sin k\left(\psi + 2\pi\frac{(i-1)}{4}\right)\cos\left(\psi + 2\pi\frac{(i-1)}{4}\right),$$

which are fixed-system rotor disk roll and pitch tilts due to $\sin k\,\psi$ rotor blade forcing.

From trigonometric identifies

$$\sum C_{ik} \sin k\left(\psi + 2\pi\frac{(i-1)}{4}\right)\sin\left(\psi + 2\pi\frac{(i-1)}{4}\right)=$$

$$\frac{1}{2}\sum C_{ik}\left\{\cos(k-1)\left(\psi + 2\pi\frac{(i-1)}{4}\right) - \cos\ (k\ -1)\left(\psi + 2\pi\frac{(i-1)}{4}\right)\right\}=$$

$$\frac{1}{2}\sum C_{ik} \cos(k-1)\,\psi \cos\ (k-1)\, 2\pi\frac{(i-1)}{4} -$$

$$\frac{1}{2}\sum C_{ik} \sin(k-1)\,\psi \sin\ (k-1)\, 2\pi\frac{(i-1)}{4} -$$

$$\frac{1}{2}\sum C_{ik} \cos(k+1)\,\psi \cos\ (k+1)\, 2\pi\frac{(i-1)}{4} +$$

$$\frac{1}{2}\sum C_{ik} \sin(k+1)\,\psi \sin\ (k+1)\, 2\pi\frac{(i-1)}{4}.$$

From this we identify

$$BXa\,(k-1) = \frac{1}{2}\Sigma\ C_{ik} \cos(k-1)\ 2\pi\frac{(i-1)}{4}$$

$$BXb\,(k-1) = -\frac{1}{2}\Sigma\ C_{ik} \sin(k-1)\ 2\pi\frac{(i-1)}{4}$$

$$BXa\,(k+1) = -\frac{1}{2}\Sigma\ C_{ik} \cos(k+1)\ 2\pi\frac{(i-1)}{4}$$

$$BXb\,(k+1) = \frac{1}{2}\Sigma\ C_{ik} \sin(k+1)\ 2\pi\frac{(i-1)}{4}$$

These are the fixed-system rotor disk rolling terms. In a similar manner, the pitching terms are calculated as

$$BYb\,(k+1) = \frac{1}{2}\Sigma\ C_{ik} \cos(k+1)\ 2\pi\frac{(i-1)}{4}$$

$$BYa\,(k+1) = \frac{1}{2}\Sigma\ C_{ik} \sin(k+1)\ 2\pi\frac{(i-1)}{4}$$

$$BYb\,(k-1) = \frac{1}{2}\Sigma\ C_{ik} \cos(k-1)\ 2\pi\frac{(i-1)}{4}$$

$$BYa\,(k-1) = \frac{1}{2}\Sigma\ C_{ik} \sin(k-1)\ 2\pi\frac{(i-1)}{4}.$$

Now, for instance, for k = 2 (2/rev) with blade 1 damaged, let blade 1 have an amplitude of 1.1, and the remaining undamaged blades have an amplitude of 1.0. Then we will have

$$\text{BXa1} = \frac{1}{2}\left\{1.1\ \cos(0) + 1.0\ \cos\left(\frac{\pi}{2}\right) + 1.0\ \cos\ (\pi) + 1.0\ \cos\left(\frac{3\pi}{2}\right)\right\}$$

$$= \frac{1}{2}\ \{1.1 + 0 - 1.0 + 0\} = \frac{1}{2}(0.1)$$

and

$$\text{BXb1} = -\frac{1}{2}\left\{1.1\ \sin(0) + 1.0\ \sin\left(\frac{\pi}{2}\right) + 1.0\ \sin\ (1\pi) + 1.0\ \sin\left(\frac{3\pi}{2}\right)\right\}$$

$$= -\frac{1}{2}\ \{0 + 1.0 + 0 - 1.0\} = 0\,.$$

For blade 2 damage, we have

$$\text{BXa1} = \frac{1}{2}\ \{1.0 + 0 - 1.0 + 0\} = 0$$

$$\text{BXb1} = -\frac{1}{2}\ \{0 + 1.1 + 0 - 1.0\} = -\frac{1}{2}(0.1)\,.$$

We see here how the damaged blade number determines whether a cosine ak or sine bk coefficient is obtained.

INTENTIONALLY LEFT BLANK.

List of Terms

ψ_i	rotor blade azimuthal displacement of ith blade
k_z	rotor blade harmonic number
$B = \beta_i$	ith rotor blade flap displacement
$om = \Omega$	rotor angular speed
z	vertical deflection of the hub mass
$M1 = M_1$	rotor blade mass
$M2 = M_2$	hub mass
$Kz = K_z$	hub spring
$Kb = K_\beta$	rotor blade spring
e	rotor blade flap hinge offset
r	rotor blade radius
F	rotor blade forcing function
A_i	rotor blade forcing amplitude of ith blade
N	number of blades in rotor
t	time
$BX = B_x$	rotor disk lateral tilt in the fixed system
$BY = B_y$	rotor disk longitudinal tilt in the fixed system
$ak = a_k$	rotor disk cosine coefficient in the fixed system
$bk = b_k$	rotor disk sine coefficient in the fixed system
BXak	rotor disk lateral tilt cosine coefficient kth harmonic
BYbk	rotor disk longitudinal tilt sine coefficient kth harmonic
P_i	π
C_i	amplitude of the ith rotor blade flap displacement

INTENTIONALLY LEFT BLANK.

NO. OF COPIES	ORGANIZATION
2	DEFENSE TECHNICAL INFORMATION CENTER DTIC DDA 8725 JOHN J KINGMAN RD STE 0944 FT BELVOIR VA 22060-6218
1	HQDA DAMO FDT 400 ARMY PENTAGON WASHINGTON DC 20310-0460
1	OSD OUSD(A&T)/ODDDR&E(R) R J TREW THE PENTAGON WASHINGTON DC 20301-7100
1	DPTY CG FOR RDA US ARMY MATERIEL CMD AMCRDA 5001 EISENHOWER AVE ALEXANDRIA VA 22333-0001
1	INST FOR ADVNCD TCHNLGY THE UNIV OF TEXAS AT AUSTIN PO BOX 202797 AUSTIN TX 78720-2797
1	DARPA B KASPAR 3701 N FAIRFAX DR ARLINGTON VA 22203-1714
1	NAVAL SURFACE WARFARE CTR CODE B07 J PENNELLA 17320 DAHLGREN RD BLDG 1470 RM 1101 DAHLGREN VA 22448-5100
1	US MILITARY ACADEMY MATH SCI CTR OF EXCELLENCE DEPT OF MATHEMATICAL SCI MADN MATH THAYER HALL WEST POINT NY 10996-1786

NO. OF COPIES	ORGANIZATION
1	DIRECTOR US ARMY RESEARCH LAB AMSRL D D R SMITH 2800 POWDER MILL RD ADELPHI MD 20783-1197
1	DIRECTOR US ARMY RESEARCH LAB AMSRL DD 2800 POWDER MILL RD ADELPHI MD 20783-1197
1	DIRECTOR US ARMY RESEARCH LAB AMSRL CS AS (RECORDS MGMT) 2800 POWDER MILL RD ADELPHI MD 20783-1145
3	DIRECTOR US ARMY RESEARCH LAB AMSRL CI LL 2800 POWDER MILL RD ADELPHI MD 20783-1145
	ABERDEEN PROVING GROUND
4	DIR USARL AMSRL CI LP (BLDG 305)

NO. OF COPIES	ORGANIZATION
1	OUSD AT STRT TAC SYS DR SCHNEITER RM 3E130 3090 DEFENSE PENTAGON WASHINGTON DC 20310-3090
1	OUSD AT S&T AIR WARFARE RM 3E139 R MUTZELBUG 3090 DEFENSE PENTAGON WASHINGTON DC 20310-3090
1	OSD DOT&E LFT RM 1C730 J OBRYON THE DEFENSE PENTAGON WASHINGTON DC 20301-3110
1	OUSD AT S&T LAND WARFARE RM EB 1060 A VIILU 3090 DEFENSE PENTAGON WASHINGTON DC 20310-3090
1	UNDER SEC OF THE ARMY DUSA OR RM 2E660 102 ARMY PENTAGON WASHINGTON DC 20310-0102
1	ASST SECY ARMY ACQUISTION LOGISTICS TCHNLGY SARD ZD ROOM 2E673 103 ARMY PENTAGON WASHINGTON DC 20310-0103
1	ASST SECY ARMY ACQUISITION LOGISTICS TCHNLGY SAAL ZP ROOM 2E661 103 ARMY PENTAGON WASHINGTON DC 20310-0103
1	ASST SECY ARMY ACQUITION LOGISTICS TCHNLGY SAAL ZS ROOM 3E448 103 ARMY PENTAGON WASHINGTON DC 20310-0103

NO. OF COPIES	ORGANIZATION
1	OADCSOPS FORCE DEV DIR DAMO FDZ ROOM 3A522 460 ARMY PENTAGON WASHINGTON DC 20310-0460
1	US ARMY MATERIEL CMD DEP CHF OF STAFF RDA SCIENCE TECH ENG AMCRDA T R PRICE 5001 EISENHOWER AVE ALEXANDRIA VA 22333-0001
1	ARMY TRADOC ATCD B FT MONROE VA 23561-5000
1	ARMY TRADOC ANL CTR ATRC W MR KEINTZ WSMR NM 88002-5502
1	US ARMY RESEARCH LAB AMSRL SL PLANS AND PGM MGR WSMR NM 88002-5513
1	US ARMY RESEARCH LAB AMSRL SL EA R FLORES WSMR NM 88002-5513
1	US ARMY RESEARCH LAB AMSRL EM J PALOMO WSMR NM 88002-5513
1	US ARMY RESEARCH LAB AMSRL SL EI J NOWAK FT MONMOUTH NJ 07703-5602
1	DARPA SPECIAL PROJECTS OFFICE J CARLINI 3701 N. FAIRFAX DR ARLINGTON VA 22203-1714

NO. OF COPIES	ORGANIZATION
1	US MILITARY ACADEMY MATH SCI CENTER OF EXCELLENCE DEPT OF MATHEMATICAL SCI MADN MSCE LTC M PHILLIPS THAYER HALL WEST POINT NY 10996-1786
	ABERDEEN PROVING GROUND
1	US ARMY ATEC AEC CSTE AEC MR HUGHES 4120 SUSEQEHANNA AVE APG MD 21005-3013
1	US ARMY ATEC AEC CSTE AEC SV 4120 SUSQUEHANNA AVE APG MD 21005-3013
2	US ARMY RESEARCH LAB AMSRL DR WADE J BEILFUSS
4	US ARMY RESEARCH LAB AMSRL SL B MS SMITH J FRANZ M VOGEL W WINNER
1	US ARMY RESEARCH LAB AMSRL SL BA M RITONDO
3	US ARMY RESEARCH LAB AMSRL SL BD J MORRISSEY K KUM S POLYAK
2	US ARMY RESEARCH LAB AMSRL SL BE D BELY R SANDMEYER

NO. OF COPIES	ORGANIZATION
1	US ARMY RESEARCH LAB AMSRL SL BG A YOUNG
1	US ARMY RESEARCH LAB AMSRL SL BN D FARENWALD
2	US ARMY RESEARCH LAB AMSRL SL E M STARKS E PANUSKA
1	US ARMY RESEARCH AMSRL SL EA D BAYLOR APG EA MD 21010-5423
1	US ARMY RESEARCH LAB AMSRL SL EM J FEENEY APG EA MD 21010-5423

INTENTIONALLY LEFT BLANK.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	June 2000	Final, Aug - Oct 95

4. TITLE AND SUBTITLE
The Effect of Helicopter Main Rotor Blade Damage on the Rotor Disk (Whole Rotor) Motion

5. FUNDING NUMBERS
6.5 Mission

6. AUTHOR(S)
Joseph Fries

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
ATTN: AMSRL-SL-BA
Aberdeen Proving Ground, MD 21005-5068

8. PERFORMING ORGANIZATION REPORT NUMBER
ARL-TR-2241

9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

When a helicopter main rotor blade is ballistically damaged, an imbalance is created in the rotor, causing the rotor disk to execute unwanted motions, which are detrimental to performance. The normally smooth-flying helicopter develops new vibrations that can be physiologically annoying or debilitating to the pilot, can exceed structural fatigue endurance limits, can cause aeromechanical instabilities, and can reduce helicopter performance ability.

This report examines the effect of the loss of the outboard section of one rotating blade of a rotor set of four blades on the fixed-system (nonrotating) rotor disk motion. The report shows, beginning with the rotor blade forcing, how a damaged blade's response changes, and how this change feeds into the rotor's fixed-system disk motion (the disk referring to the blades acting in concert as a whole entity).

With a normally undamaged rotor (referring to all the blades), there exists within the rotor itself the capability of motion canceling of certain frequencies depending on the number of rotor blades in the rotor. This study tracks each individual harmonic (integer multiples of the rotor speed) frequency, one at a time, in order to obtain a first-principles understanding of the phenomena involved with rotor imbalance.

14. SUBJECT TERMS
helicopter, structural damage, vibration, rotor imbalance

15. NUMBER OF PAGES
30

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18 298-102

INTENTIONALLY LEFT BLANK

USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. ARL Report Number/Author ARL-TR-2241 (Fries) Date of Report June 2000

2. Date Report Received ______

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) ______

4. Specifically, how is the report being used? (Information source, design data, procedure, source of ideas, etc.) ______

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided, or efficiencies achieved, etc? If so, please elaborate. ______

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.) ______

CURRENT ADDRESS

Organization

Name E-mail Name

Street or P.O. Box No.

City, State, Zip Code

7. If indicating a Change of Address or Address Correction, please provide the Current or Correct address above and the Old or Incorrect address below.

OLD ADDRESS

Organization

Name

Street or P.O. Box No.

City, State, Zip Code

(Remove this sheet, fold as indicated, tape closed, and mail.)

(DO NOT STAPLE)