(a) *Distribution of trapped particles.* To determine the spatial distribution of trapped particles during filtration, we introduced a simple quasi-one-dimensional structure – termed the "bubble" model – upon which it is simple to calculate analytically many interesting aspects of filtration. This system consists of L links in series, in which each link is a parallel bundle of w bonds and each bond represents a pore. An appealing feature of this model in the context of percolation is that it exhibits finite-dimensional behavior when L scales as e^w. We adopt this bubble model as a convenient way to describe the reduced connectivity of the medium as individual bonds are blocked. Due to the gradient nature of the trapping process, the number of blocked bonds in downstream bubbles remains small, even at percolation. This allows us to employ an "unperturbed" approximation in which the initial bond radius distribution is used throughout the clogging process. Within this approximation and assuming a flow rate proportional to r_i^μ, where r_i is the radius of the i^{th} bond, it is a relatively simple exercise to compute the probability that a particle of radius r gets trapped in a single bubble. (Note that $\mu = 4$ corresponds to Poiseuille flow and $\mu = 2$ to plug flow.) By averaging over the particle radius distribution, the final result for the probability that a particle is trapped at downstream distance n is

$$P(n) \propto n^{-\mu}$$

where μ depends on the functional form of the particle and pore radius distributions but invariably lies between 1 and 2.

(b) *Escapee size distribution.* An issue of more practical interest is the properties of the "escapee" particles as a function of distance traveled. Clearly the escapee sizes decrease as they travel further into the medium and it is important to know the rate of this decrease, since the goal of many filtration processes is to remove particles beyond a specified size from a suspension. The radius distribution of particles which reach downstream distance n can be determined by using the observation that particles which are trapped at downstream distance $n' > n$ can be viewed as "escapees" at distance n. The probability that a particle of radius r avoids being trapped by distance n can be again obtained by a simple probabilistic argument within the bubble model and leads to

$$Q_n \propto n^{1/\nu} \exp(-nr^\nu),$$

where $\nu = \mu + 1$. From this distribution, the average radius of an escapee from bubble n decays as $n^{-1/\nu}$. Thus the escapee size decreases relatively slowly with n. As an important consequence, a depth filter with size exclusion as the only trapping mechanism should be relatively long if substantial reduction in the escapee particle size is desired. This suggests that spatially inhomogeneous filters are likely to be much more useful practically.

(c) *Percolation.* This occurs when the filter is completely blocked and no further fluid flow is possible. There are two different behaviors depending on whether the particles are typically of the same size as the pores, or smaller than the pores. The former case is more interesting because clogging is controlled by the spatial gradient in the trapping probability and, in fact, clogging often occurs right at the upstream end of the filter. Within the bubble model, we have used extreme value statistics to compute the number of particle that must be injected to clog the system under the assumption that only the first bubble gets blocked. The basic result of these considerations is that for the bubble model, of the order of $N_c \propto w \ln w$ particles must be injected to clog the system. The logarithm arises from the widest bonds in the first bubble for which many particles need to be injected before blocking occurs. This predicts that the percolation threshold p_c which is very close to 1. This indicates that filter clogging belongs to a new type of gradient-controlled percolation process. On the square lattice, we have found qualitatively similar behavior for $P(n)$ and N_c.

3. Kinetics of Driven Systems

(a) *Lorentz Gas Kinetics.* The dynamics of a point charged particle which is driven by a uniform external field and which moves in a medium of elastic spherical scatters was investigated. We showed that in one dimension the typical speed grows with time as $t^{1/3}$ and that the leading behavior of the velocity distribution is $e^{-|v|^3/t}$. In spatial dimension $d > 1$, we developed an effective medium theory which provides a simple and comprehensive description for the motion of the charged particle. This approach predicts that the typical speed grows as $t^{1/3}$ for all d, while the speed distribution is given by the scaling form $P(u,t) = \langle u\rangle^{-1} f(u/\langle u\rangle)$, where $u = |v|^{3/2}, \langle u\rangle \sim \sqrt{t}$, and $f(z) \propto z^{(d-1)/3} e^{-z^2}$. For a periodic Lorentz gas with an infinite horizon, *e. g.*, for a hypercubic lattice of scatters, a logarithmic correction to the effective medium result was predicted; in particular, the typical speed grows as $(t \ln t)^{1/3}$.

(b) *Polydisperse Sequential Adsorption.* We investigated the process of random sequential adsorption of a polydisperse collection of particles whose size distribution exhibits a power-law dependence in the small size limit, $P(R) \sim R^{\alpha-1}$. We found a simple relation between the adsorption kinetics and the structural properties of the adsorbed particles. We developed a mean-field theory that provided a good description for small α. In the opposite limit $\alpha \to \infty$, highly ordered structures form which are locally identical to an Apollonian packing.

(c) *Coarsening Kinetics and its "Aging".* We investigated the age distribution of domains of a broad class of coarsening processes. As a first step in this general program, we have considered exactly soluble one-dimensional examples. In one dimension, the trajectory of domain interfaces can be monitored and the age distribution can be obtained from the quantity $P(\tau, t)$ which is the probability density that in a time interval $(0, t)$ a given site was last crossed by an interface in the coarsening process at time τ. In contrast to the previous work of aging which has focused on a two-time correlation function, the age distribution function directly probes the history of the process and provides a better understanding of the aging of domains. We have determined $P(\tau, t)$ analytically for two cases, the (deterministic) two-velocity ballistic annihilation process, and the (stochastic) infinite-state Potts model with zero temperature Glauber dynamics. Surprisingly, we find that in the scaling limit, $P(\tau, t)$ is identical for these two models. We also show that the average age, *i. e.*, the average time since a site was last visited by an interface, grows linearly with the observation time t. This latter property is also found in the one-dimensional Ising model with zero temperature Glauber dynamics. Numerically, we examine the age distribution in dimension $d \geq 2$ and find similar qualitative features as in one dimension.

(d) *Deterministic coarsening.* We investigated a 3-phase deterministic one-dimensional phase ordering model in which interfaces move ballistically and annihilate upon colliding.

We analytically determined the autocorrelation function. This was done by first computing the generalized first-passage type probabilities $P_n(t)$ which measures the fraction of space which is traversed by exactly n interfaces during the time interval $(0,t)$, and then expressing the autocorrelation function via the P_n's. We further obtained a rich spatial structure of the system by analyzing the domain size distribution.

4. Kinetics of Islanding in Epitaxial Growth.

(a) *Continuously mobile islands.* We investigated the kinetics of submonolayer epitaxial growth which is driven by a fixed flux of monomers onto a substrate. Adatoms diffuse on the surface, leading to irreversible aggregation of islands. We also account for the effective diffusion of islands, which originates from hopping processes of their constituent adatoms, on the kinetics. When the diffusivity of an island of mass k scales as $k^{-\mu}$, the (mean-field) Smoluchowski rate equations predicts steady behavior for $0 < \mu < 1$, with the concentration c_k of islands of mass k varying as $k^{-(3-\mu)/2}$. For $\mu > 1$, a quasi-static approximation to the rate equations predicts slow continuous evolution in which the island density increases as $(\ln t)^{mu/2}$. A more refined matched asymptotic expansion reveals unusual multiple-scale mass dependence for the island size distribution. Our theory also describes basic features of epitaxial growth in a more faithful model of growing circular islands. For epitaxial growth in an initial population of monomers and no external flux, a scaling approach predicts power-law island growth and a mass distribution with a behavior distinct from that of the non-zero flux system. Finally, we extend our results to one- and two-dimensional substrates. The physically-relevant latter case exhibits only logarithmic corrections compared to the mean-field predictions.

(b) *Mobile Small Islands and Immobile Large Islands.*

The maximal density of stable islands in submonolayer epitaxial growth depends depends on the deposition rate F via $N_{\max} \propto F^{\gamma}$. This exponent γ is known to depend on the substrate dimensionality and the critical size i^* below which islands are mobile and above which they are stable. We studied the dependence of γ on i^* in one–dimensional epitaxial growth. We derived the basic relation $\gamma = i^*/(2i^* + 3)$ for $i^* \geq 2$ and confirmed this result by computer simulations.

5. Evolution of Constrained and Competing Interaction Systems.

(a) *Ising Chain with Competing Interactions.* We investigated the zero-temperature coarsening dynamics of a chain of Ising spins with a nearest-neighbor ferromagnetic and an nth-neighbor antiferromagnetic interactions. For sufficiently large antiferromagnetic interaction, the ground state consists of n consecutive up spins followed by n down spins, etc. We have shown that the asymptotic coarsening into this ground state can be described in terms of a multispecies reactive gas of elementary excitations. The basic elementary excitations are identified and each decays at a different power-law rate in time. The dominant excitations are domains of $n+1$ spins which diffuse freely and disappear through processes which are effectively governed by $n+1$-particle annihilation. This leads to a slow $t^{-1/n}$ temporal approach to the ground state.

(b) *Driven Ising Chain with Conserved Dynamics.* It has been established that systems quenched from a homogeneous high-temperature disordered state to a low-temperature multi-phase state do not order instantaneously; instead, domains of equilibrium ordered phases form and grow with time as the system approaches local equilibrium on larger and larger scales. Little is known about the influence of driving force on kinetics of coarsening. We have studied the low-temperature coarsening of an Ising chain subject to spin-exchange dynamics and a small driving force. We have found that this dynamical system reduces to a domain diffusion process, in which entire domains undergo nearest-neighbor hopping, except for the shortest domains – dimers – which undergo long-range hopping. The system is characterized by two independent length scales: the average domain length $L(t) \sim t^{1/2}$ and the average dimer hopping distance $l(t) \sim t^{1/4}$. As a consequence of these two scales, the density $C_k(t)$ of domains of length k does not obey scaling. This breakdown of scaling also leads to the density of short domains decaying as $t^{-5/4}$, instead of the $t^{-3/2}$ decay that would arise from pure domain diffusion.

(c) *Traffic Flows with Passing.* The role of passing in traffic flows is examined. Specifically, we have considered passing rates that are proportional to the difference between the velocities of the passing car and the passed car. From a Boltzmann equation approach, steady state properties of the flow such as the flux, the average cluster size, and the velocity distributions are found analytically. We have shown that a single dimensionless parameter determines the nature of the flow and helps distinguish between dilute and dense flows. For dilute flows, perturbation expressions are obtained, while for dense flows, a boundary layer analysis is carried out. In the latter case, extremal properties of the initial velocity distribution underly the leading scaling asymptotic behavior. For dense flows, the stationary velocity distribution exhibits a rich "triple deck" boundary layer structure. Furthermore, in this regime fluctuations in the flux may become extremely large.

We have also investigated traffic flows using the Boltzmann equation with a Maxwell collision integral. This approach allows the determination of the transient behavior and the size distributions. The relaxation of the car and cluster velocity distributions towards steady state is characterized by a wide range of velocity-dependent relaxation scales. These relaxation time scales decrease with the velocity, with the smallest scale corresponding to the decay of the overall density. The steady state distribution of clusters of a given number of cars is obtained analytically and it is found to obey an unusual scaling form.

(d) *Citation Distribution.* Numerical data for the distribution of citations are examined for: (i) papers published in 1981 in journals which are catalogued by the Institute for Scientific Information (783,339 papers) and (ii) 20 years of publications in Physical Review D, vols. 11-50 (24,296 papers). A Zipf plot of the number of citations to a given paper versus its citation rank appears to be consistent with a power-law dependence for leading rank papers, with exponent close to $-1/2$. This, in turn, suggests that the number of papers with x citations, $N(x)$, has a large-x power law decay $N(x) \sim x^{\alpha}$, with $\alpha = 3$.

(e) *Wealth Distribution of Economically Active Individuals.* A model for the evolution of the wealth distribution in an economically interacting population is introduced, in which a specified amount of assets are exchanged between two individuals when they interact. The resulting wealth distributions are determined for a variety of exchange rules. For "random" exchange, either individual is equally likely to gain in a trade, while "greedy" exchange, the richer individual gains. When the amount of asset traded is fixed, random exchange leads to a Gaussian wealth distribution, while greedy exchange gives a Fermi-like scaled wealth distribution in the long-time limit. Multiplicative processes are also investigated, where the amount of asset exchanged is a finite fraction of the wealth of one of the traders. For random multiplicative exchange, a steady state occurs, while in greedy multiplicative exchange a continuously evolving power law wealth distribution arises.

(f) *Age-structured Population Dynamics.* We analyzed the dynamics of an age-structured population model in which the life expectancy of an offspring may be mutated with respect to that of the parent. While the total population of the system always reaches a steady state, the fitness and age characteristics exhibit counter-intuitive behavior as a function of the mutational bias. Specifically, we show that if deleterious mutations are favored, the average fitness of the population reaches a steady state, while the average population age is a decreasing function of the overall fitness. When advantageous mutations are favored, the average population fitness grows linearly with time t, while the average age is independent of fitness. For no mutational bias, the average fitness grows as $t^{2/3}$. Our analysis of the rate equations governing the evolution of age-structured populations is a combination of various analytical and numerical approaches.

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13. ABSTRACT (Maximum 200 words)

There were several major results in this project, including topics of ARO relevance, as well as general fundamental studies. We developed a basic understanding of filtration and the attendant clogging process. The role of preferential upstream trapping in governing clogging was elucidated; this should help guide the design of optimal filters. We determined the island size distribution in submonolayer epitaxial growth processes. We made substantial advances in understanding the flow and jamming of fundamental models of traffic with and without passing. We studied anomalously slow coarsening kinetics of spin systems with competing ferromagnetic and antiferromagnetic interaction, as well as the coarsening kinetics of other non-equilibrium processes. In a different area, we provided a comprehensive analysis of the distribution of citations of scientific publications and also introduced and solved a kinetic model of asset exchange to account for the wealth distribution of an economically active population.

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