

OTS: 60-11, 913

JPRS: 5022

14 July 1960

**DISTRIBUTION STATEMENT A**  
**Approved for Public Release**  
**Distribution Unlimited**

SHOCK WAVES IN SOLIDS

- USSR -

by L. P. Orlenko and K. P. Stanyukovich

RETURN TO MAIN FILE

**Reproduced From**  
**Best Available Copy**

20000707 168

Distributed by:

OFFICE OF TECHNICAL SERVICES  
U. S. DEPARTMENT OF COMMERCE  
WASHINGTON 25, D. C.

---

U. S. JOINT PUBLICATIONS RESEARCH SERVICE  
205 EAST 42nd STREET, SUITE 300  
NEW YORK 17, N. Y.

**DTIC QUALITY INSPECTED 4**

JPRS: 5022

CSO: 3712-N

## SHOCK WAVES IN SOLIDS

[This is a translation of an article written by L.P. Orlenko and K.P. Stanyukovich in Izvestiya Vysshikh Uchebnykh Zavedeniy, Fizika (News of the Higher Institutions of Learning, Physics), No. 6, 1958, pages 14-24.]

(Moscow Higher Technical School imeni N.Ye. Bauman; submitted to editors 16 April 1958)

The propagation of shock waves in solids under high pressures considering the compressibility of the material is examined in this article. The motion of the material is studied in the zone of an incident shock wave and the reflected wave zone, where the body has finite dimensions. The solution is obtained in the form of finite formulas.

The motion of the incident shock wave is investigated by means of a special solution of differential equations of motion. The wave reflected from a free surface is investigated by a general solution of differential equations of motion. The results obtained make it possible to determine all of the parameters of both incident and reflected waves.

By utilizing the theoretical results obtained, a dynamic diagram of deformation stress can be derived with the appropriate experiments.

To solve the dynamic problems arising when detonation or shock charges are applied to a solid

it is necessary to know the dynamic deformation-stress curve. An experimental determination of such diagrams presents considerable difficulties. It is known that when an instantaneous pressure is applied to a solid, which then falls in the course of time, elastic, plastic or shock waves can be propagated in the solid, depending on the magnitude of the applied pressure and the properties of the solid (see work (1)).

In the present work the solution of the dynamic problem at high detonation charges is analyzed, where shock waves are propagated in a solid medium. In this area of great dynamic pressures, where the solid medium behaves like a compressible fluid (1), the dynamic deformation-stress volume diagram is only qualitatively known, i.e. it can be given, for example, by the equation having the following form:  $\sigma - \sigma_0 = -A(\xi - \xi_0)^{-3}$ , where  $A$ ,  $\sigma_0$ ,  $\xi_0$  are unknown constants. In such an approximation of the  $\sigma - \xi$  curve, one can obtain in the form of finite formulas the laws of velocity and pressure change in both the incident shock wave and the reflected wave (were the body, for instance, to be a plate of finite thickness). It is known that if impact loading is applied at one side of a finite body (a plate), at the rear a part of the material will be split off with great velocity. Using this phenomenon, it is possible by means of the solution obtained in this work to find the unknown constants  $A$ ,  $\sigma_0$  and  $\xi_0$ , the dynamic deformation-stress volume curve with compression will be determined by this.

We will examine the propagation of a shock wave in a solid, where a great instantaneous pressure which subsequently decreases with time is applied to the latter.

It is known that the "deformation-stress" compression curve for solids have the form shown in Fig. 1 (see work (1)).

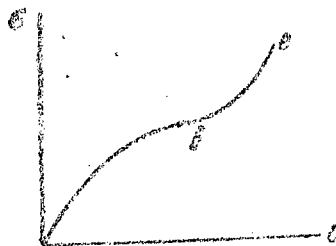


Fig. 1

If the pressure in the wave exceeds the pressure

corresponding to the discontinuity b of the  $\sigma - \epsilon$  curve, shock waves will be propagated in the solid. It is apparent that under these pressures the solid may be regarded as a quasifluid (1).

It is known that the basic equations of one-dimensional motion have the form

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \quad \frac{\partial u}{\partial h} = \frac{\partial v}{\partial t} \quad (1)$$

where  $u$  - is the particle velocity;  $v$  - is specific volume;  $t$  - is time;  $p$  - pressure;  $h$  - is the Lagrangian coordinate.

If one introduces the terms  $\sigma = -p$ ,  $\epsilon = \frac{v-v_0}{v_0} =$   
 $= \frac{p_0}{p} - 1,$  (2)

then we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial \sigma}{\partial h} = \frac{d\sigma}{d\epsilon} \frac{\partial \epsilon}{\partial h}, \quad \frac{\partial u}{\partial h} = v_0 \frac{\partial \epsilon}{\partial t} \quad (3)$$

The equation of state (curve b - e in Fig. 1) is approximated by the relation having the following form:

$$\sigma - \sigma_0 = -A(\epsilon - \epsilon_0)^{-3}, \quad (4)$$

where  $\sigma_0$ ,  $A$  and  $\epsilon_0$  are constants.

We then obtain

$$\frac{d\sigma}{d\epsilon} = \frac{3A}{(\epsilon - \epsilon_0)^4} \quad (5)$$

The velocity of sound in this case is determined by the expression

$$c = \sqrt{\frac{dp}{d\rho}} = (\epsilon + 1) \sqrt{\frac{d\sigma}{\rho_0 d\epsilon}} = \frac{\epsilon + 1}{(\epsilon - \epsilon_0)^2} \sqrt{\frac{3A}{\rho_0}} =$$

$$= \frac{\sqrt{3A\rho_0}}{\rho \left[ \frac{\rho_0}{\rho} - (1 + \epsilon_0) \right]^2} \quad (6)$$

The velocity of propagation of disturbances from particle to particle will be equal to

$$\begin{aligned}
 w &= \sqrt{\frac{d\sigma}{\rho_0 d\epsilon}} = \frac{c}{\epsilon + 1} = \frac{\rho}{\rho_0} \quad c = \frac{1}{(\epsilon - \epsilon_0)^2} \sqrt{\frac{3A}{\rho_0}} \\
 &= \sqrt{\frac{3A}{\rho_0} \frac{1}{\left[\frac{\rho_0}{\rho} - (1 + \epsilon_0)\right]^2}} \quad (7)
 \end{aligned}$$

We write equation (5) in the form

$$\begin{aligned}
 \frac{\partial u}{\partial t} &= \frac{d\sigma}{d\epsilon} \frac{\partial \epsilon}{\partial h} = \frac{3A}{(\epsilon - \epsilon_0)^4} \frac{\partial(\epsilon - \epsilon_0)}{\partial h} \\
 \frac{\partial u}{\partial h} &= v_0 \frac{\partial(\epsilon - \epsilon_0)}{\partial t} \quad (8)
 \end{aligned}$$

We will find particular solutions for this system of equations. We will assume for this purpose, as usual, that  $\epsilon = \epsilon(u)$ , then we will arrive at the equations

$$\frac{\partial u}{\partial t} = \frac{d\sigma}{du} \frac{\partial u}{\partial h}, \quad \frac{\partial u}{\partial h} = v_0 \frac{d\epsilon}{du} \frac{\partial u}{\partial t}$$

whence we obtain

$$\frac{du}{d\sigma} = v_0 \frac{d\epsilon}{du} \quad \text{or}$$

$$du = \sqrt{v_0} d\epsilon d\sigma = \sqrt{v_0 \frac{d\sigma}{d\epsilon}} d\epsilon = w d\epsilon \quad (9)$$

$$\frac{\partial u}{\partial t} = \rho_0 \frac{\partial u}{\partial h} \frac{du}{d\epsilon} = \frac{\partial u}{\partial h} \sqrt{\rho_0 \frac{d\sigma}{d\epsilon}} \quad (10)$$

The solution of equation (10) has the form

$$h = \sqrt{\rho_0 \frac{d\sigma}{ds}} t + F(u) = \rho_0 \omega t + F(u). \quad (11)$$

In the equation of state (4) the specific solution may be written in the form

$$\left. \begin{aligned} u &= \pm \sqrt{\frac{3A}{\rho_0} \frac{1}{s - \epsilon_0}} + \text{const} \\ h &= \pm \sqrt{\frac{3A}{\rho_0} \frac{\rho_0 t}{(s - \epsilon_0)^2}} + F(u) \end{aligned} \right\} \quad (12)$$

We will now seek the general solutions of the system of equations (8). Having inverted the dependent and independent variables, we arrive at the equations:

$$v_0 \frac{\partial h}{\partial s} = \omega^2 \frac{\partial t}{\partial u}, \quad \frac{\partial t}{\partial s} = v_0 \frac{\partial h}{\partial u}. \quad (13)$$

We find from this that

$$v_0 \frac{\partial^2 h}{\partial s \partial u} = \frac{\partial^2 t}{\partial u^2} = \omega^2 \frac{\partial^2 t}{\partial u^2}. \quad (14)$$

In the equation of state (4) we will have

$$\frac{\partial^2 t}{\partial (s - \epsilon_0)^2} = \frac{3A}{\rho_0 (s - \epsilon_0)^4} \frac{\partial^2 t}{\partial u^2}. \quad (15)$$

We will introduce the new variable  $z = \frac{\alpha}{s - \epsilon_0}$ , then  $s - \epsilon_0 = -\lambda \left(\frac{z}{\alpha}\right)^3$ , and equation (15) assumes the form

$$\frac{\partial^2 t}{\partial z^2} + \frac{2}{z} \frac{\partial t}{\partial z} = \frac{3A}{\rho_0 \alpha^2} \frac{\partial^2 t}{\partial u^2}. \quad (16)$$

Assuming that  $\alpha^2 = \frac{3A}{\rho_0}$ , then

$$z = \sqrt{\frac{3A}{\rho_0} \frac{1}{\epsilon - \epsilon_0}} = w(\epsilon - \epsilon_0) = \frac{\epsilon - \epsilon_0}{\epsilon + 1} C = \frac{aw}{z}$$

$$w = \frac{z^2}{a}$$

equation (16) assumes the form

$$\frac{\partial^2(zf)}{\partial z^2} = \frac{\partial^2(zf)}{\partial u^2} \quad (17)$$

The solution of this equation:

$$t = \frac{\Phi_1(u-z) + \Phi_2(u+z)}{z} = \frac{F'_1(u-z) + F'_2(u+z)}{z} \quad (18)$$

where  $\Phi_1, \Phi_2, F_1$  and  $F_2$  are arbitrary functions (the prime designates differentiation according to the arguments).

We find from the 2nd equation (13) that

$$\frac{\partial t}{\partial \epsilon} = -\frac{z^2}{a} \frac{\partial t}{\partial z} = + \left[ \frac{F'_1 - F'_2}{z} + \frac{F_1 + F_2}{z^2} \right] \frac{z^2}{a} = \frac{1}{\rho_0} \frac{\partial h}{\partial u}$$

whence it follows that

$$h = \frac{\rho_0}{a} \left[ z(F'_1 - F'_2) + (F_1 + F_2) \right] \quad (19)$$

If the function  $\varphi = \varphi(\epsilon, u)$  is introduced which satisfies the second equation (13), then

$$t = \frac{\partial \varphi}{\partial u} \quad h = \rho_0 \frac{\partial \varphi}{\partial \epsilon} = -\frac{\rho_0 z^2}{a} \frac{\partial \varphi}{\partial z} \quad (20)$$

Whereupon the solution of (18) assumes the form

$$\psi = \frac{f_1(u-z) + f_2(u+z)}{z} \quad (21)$$

where  $f_1$  and  $f_2$  are new arbitrary functions,

$$l = \frac{f_1 + f_2}{z} \quad h = \frac{p_0}{a} \left[ (f_1' - f_2') z + f_1 + f_2 \right] \quad (22)$$

The special solution of (11) assumes the form

$$\frac{p_0 z^2}{a} \frac{\partial \psi}{\partial z} = z \sqrt{-\frac{p_0}{a} \frac{dc}{dz}} \frac{\partial \psi}{\partial u} + F(u) \quad (23)$$

Whereupon

$$du = -\frac{1}{z} \sqrt{-\frac{dc}{p_0 dz}} dz \quad (24)$$

We multiply (23) by  $du$ , then

$$z \frac{\partial \psi}{\partial z} \sqrt{-\frac{p_0}{a} \frac{dc}{dz}} + z \sqrt{-\frac{p_0}{a} \frac{dc}{dz}} \frac{\partial \psi}{\partial u} du = F(u) du.$$

Whence

$$\begin{aligned} z \sqrt{-\frac{p_0}{a} \frac{dc}{dz}} d\psi &= F(u) du \text{ or } d\psi = \\ &= \tilde{\Phi}(u) du, \quad (25) \end{aligned}$$

there

$$\Phi(u) = \frac{F(u)}{\sqrt{\frac{\rho_0}{a} \frac{d\sigma}{dz}}} = \frac{F(u)}{\rho_0 \frac{z - \varepsilon_0}{\alpha} z \sqrt{\frac{1}{\rho_0} \frac{d\sigma}{d\varepsilon}}}$$

$$= \frac{F(u)}{\rho_0 \sqrt{\frac{1}{\rho_0} \frac{d\sigma}{dz}}}$$

or

$$\Phi(u) = \frac{F(u)}{\rho_0 \alpha}$$

Hence, along the characteristics in plane  $(u, z)$  which are determined by the special solution

$$\psi = \psi(u) = \psi(z). \quad (26)$$

If  $F(u) = 0$ , then  $\psi = \text{const.}$ , since without limiting the general representation it may always be assumed that  $\text{const.} = 0$ , hence if  $F(u) = 0$ , along the characteristic  $\psi = 0$  (27). In the equation of state (4), we have

$$u = \pm z + \text{const.}$$

$$h = \pm \frac{\rho_0 + \rho_0 z^2}{a} + F(z) = \rho_0 \alpha t + F(z). \quad (28)$$

If, for example, we determine  $\psi$  along the characteristic  $u + z = \text{const.}$ , then in this individual case, where  $F(z) = 0$ ,  $f_2 = 0$ .

If  $\psi$  is determined along the characteristic  $u - z = \text{const.}$ , then when  $F(z) = 0$ ,  $f_1 = 0$ .

We will look at the solution of a concrete example. Let us assume at the moment of time  $t = 0$  in cross-section  $h = 0$  there is a pressure  $p = p_n$  applied, where

$p_H$  is greater than  $\sigma_0$ , where  $\sigma_0$  is the pressure corresponding to the discontinuity point in curve  $\sigma - \hat{\epsilon}$  (Fig.1) which then is found according to the law

$$\frac{p - p_0}{p_H - p_0} = \left( \frac{\tau}{t + \tau} \right)^{3n}, \quad (29)$$

where  $p_0, \tau$  and  $n$  are constants.

It is known from detonation theory that it is possible to assume  $p_0 = 0$ ,  $n = 1$  and  $\tau = \frac{l}{D}$ , where  $l$  is the height of the loading and  $D$  is the velocity of the detonation.

We find the law of motion of the medium in the passing wave. The motion of the medium in the wave will be subject to the specific solution (28) where it is necessary to determine  $F(z)$ :

$$\left. \begin{aligned} u - z &= \text{const} \\ h &= \frac{\rho_0 l z^2}{a} + F(z) \end{aligned} \right\} \quad (A)$$

We will determine the constant in the first equation of the system (A).

When  $u = u_H$ ,  $z = z_H$ , inasmuch as at the wave front

$$\sigma_H = \frac{u_H^2 \rho_0}{\epsilon_H}, \quad \text{then const.} = -z_0 = \sqrt{\frac{\sigma_H \cdot \epsilon_H}{\rho_H} \cdot \frac{a}{\epsilon_H - \epsilon_0}}$$

Using equation (4), (29) can be written in the form

$$\frac{z}{z_H} = \left( \frac{\tau}{t + \tau} \right)^n. \quad (30)$$

The first equation (A) now assumes the form:  $u = z - z_0$ .

$$(31)$$

For the initial moment

$$u_H = z_H - z_0. \quad (32)$$

If it is assumed in equation (A) that  $h = 0$ , and the time  $t$  is expressed from (30), then we determine  $(z)$ :

$$F(z) = - \frac{\rho_0 z^2 \tau}{a} \left[ \left( \frac{z_H}{z} \right)^{\frac{1}{n}} - 1 \right]. \quad (33)$$

System (A) is finally written in the form

$$u = z - z_0,$$

$$h = \frac{\rho_0 z^2}{a} \left\{ t - \tau \left[ \left( \frac{z_H}{z} \right)^{\frac{1}{n}} - 1 \right] \right\}. \quad (34)$$

If  $n = 1$ , we derive the quadratic equation relative to  $z$ :

$$z^2 - \frac{z_H \tau}{t + \tau} z - \frac{a h}{\rho_0 (t + \tau)} = 0.$$

The solution of this equation will be

$$z = u + z_0 = \frac{z_H \tau \pm \sqrt{z_H^2 \tau^2 + 4 \rho_0 a h (t + \tau)}}{2(t + \tau)}. \quad (35)$$

We find the law of motion of the shock front  $= h(t)$ :

$$D_y = \frac{dh}{\rho_0 dt} = \frac{u + c + c_0}{2} = \frac{z - z_0 + c + c_0}{2}, \quad (36)$$

where

$D_y$  is the velocity of the shock wave,  
 $c_0$  is the velocity of sound in metal.

We express the local velocity of sound  $c$  by  $z$ :

$$c = \frac{\varepsilon + 1}{\varepsilon - \varepsilon_0} z = \frac{(\varepsilon_0 + 1)z^2 + az}{a} = z + \frac{\varepsilon_0 + 1}{a} z^2. \quad (37)$$

Thus, the velocity of wave  $D_y$  will be equal to

$$D_y = z + \frac{c_0 - z_0}{2} + \frac{\varepsilon_0 + 1}{2a} z^2. \quad (38)$$

By means of equation (35) we eliminate  $z$ :

$$\frac{dh}{\rho_0 dt} = \frac{c_0 - z_0}{2} + \frac{(\varepsilon_0 + 1)h}{2\rho_0(t + \tau)} + \left[ 1 + \frac{\varepsilon_0 + 1}{2a} \cdot \frac{z_H \tau}{t + \tau} \right] \frac{z_H \tau + \sqrt{z_H^2 \tau^2 + 4\rho_0 a h(t + \tau)}}{2(t + \tau)}. \quad (39)$$

Integrating this equation which when  $h = 0$ ,  $t = 0$ , we find the law of propagation of the shock wave:

$$h = h(t). \quad (40)$$

If the body has finite thickness  $d$ , then when the wave reaches the back side of the solid it will be reflected from the free surface as a rarefaction (expansion [Russian: discharge]) wave, moving toward the shock wave. The motion of the medium in the rarefaction wave is determined by the general solution (22). The arbitrary functions in the general solution are determined primarily from the condition of the connection between the reflected and incident waves; secondarily, by the fact that when  $t \geq T$ ;  $h = \rho_0 d$ ,  $\tau = 0$ , where  $d$  is the thickness of the plate. The first condition implies that on the line  $u = z = z_0$  takes place according to (25) and (33):

$$\begin{aligned} \psi &= \int \Phi(u) du = \frac{u}{\rho_0} \int \frac{F(z)}{z^2} dz = \tau \int \left[ \left( \frac{z_H}{z} \right)^{\frac{1}{n}} - 1 \right] dz = \\ &= \tau z \left[ -1 + \frac{n}{n-1} \left( \frac{z_H}{z} \right)^{\frac{1}{n}} \right]. \end{aligned}$$

$$\text{If } n = 1, \text{ then } \phi = \tau z \left[ -1 + \frac{z_H}{z} \ln \frac{z}{z_H^*} \right] = \frac{f_1 + f_2}{z}$$

or

$$-\tau z^2 + \tau z_H z \ln \frac{z}{z_H^*} = f_1 + f_2, \quad (41)$$

where  $f_1 = f_1(2z - z_0)$  and  $f_2 = f_2(-z_0)$ ,

$z_H^*$  is the integration constant.

The second condition provides that when  $h = \rho_0 d$  the condition

$$ad = f_1 + f_2 + (f_2 - f_1) z_d, \quad (42)$$

is fulfilled, where

$$z_d = a \sqrt{\frac{c_0}{A}}$$

Thus, by means of (41) and (42) the parameters of the reflected wave can be determined. The found solution is complicated for investigation, hence we will find a less precise, although considerably simpler solution of the problem. It is known from detonation theory that the pressure on the surface of the plate falls according to the law

$$\rho = \rho_H \left( \frac{l}{Dt} \right)^2, \quad (43)$$

where

$$\rho_H = \frac{16}{27} \rho_0 D^2 \quad (44)$$

The velocity of the shock wave  $D_y$  is determined by the formula

$$D_y = \frac{u + c + c_a}{2} = \frac{dx}{dt} \quad (45)$$

We take the equation of state in the form

$$p = A_1 \rho^k - B_1 \quad (46)$$

where  $A_1$ ,  $B_1$  and  $k$  are constants.

The motion of the shock wave will be determined by special solutions of the system of equations (1) written in Euler's form:

$$c = c_a + \frac{k-1}{2} u \quad (47)$$

$$x = (u + c)t + F(u) \quad (48)$$

The velocity of the shock wave  $D_y$  is determined by the expression

$$D_y = \frac{dx}{dt} = c_a + \frac{k+1}{2} u \quad (49)$$

Function  $F(u)$  is determined by the condition that when  $x = 0$  the pressure varies according to the law (43):

$$p = p_H \left( \frac{l}{Dt} \right)^3$$

whence follows that

$$\frac{Dt}{l} = \left( \frac{p_H}{p} \right)^{\frac{1}{3}} = \frac{\bar{c}_H}{c} \quad (50)$$

where  $\bar{c}_H$  is the initial speed of sound in the plate where  $p = p_H$ ,

$$\bar{c}_H = \sqrt{\frac{dp}{d\rho}} = \sqrt{k A_1 \left( \frac{p_H + B_1}{A_1} \right)^{\frac{k-1}{2k}}} \quad (51)$$

By means of (47) equation (50) assumes the form

$$t = \frac{l}{D} \left( \frac{c_a + \frac{k-1}{2} u}{\bar{c}_H} \right)^{-1} \quad (52)$$

When  $x = 0$  we determine  $F(u)$  according to (48) and (52):

$$F(u) = - \left( c_a + \frac{k+1}{2} u \right) \left( c_a + \frac{k-1}{2} u \right)^{-1} \frac{l}{D \bar{c}_H}$$

Equation (48) now has the form

$$x = \left( c_a + \frac{k+1}{2} u \right) \left[ t - \left( \frac{c_a + \frac{k-1}{2} u}{\bar{c}_H} \right)^{-1} \frac{l}{D} \right] \quad (53)$$

When  $k = 3$  this expression is written in the form

$$x = (c_a + 2u) \left( t - \frac{\bar{c}_H}{u + c_a} \frac{l}{D} \right) \quad (54)$$

We convert this expression to the form

$$x(u + c_a) \frac{D}{l} = (c_a + 2u) \left[ \frac{Dt}{l} (u + c_a) - \bar{c}_H \right] \quad (55)$$

or

$$2 \frac{Dt}{l} u^2 + u \left[ \frac{Dt}{l} 3c_a - x \frac{D}{l} - 2\bar{c}_H \right] + \frac{Dt}{l} c_a^2 - \bar{c}_H c_a - x c_a \frac{D}{l} = 0.$$

Inasmuch as  $u \ll c_0$ , neglecting the term with  $u^2$ , we find that

$$u = \frac{\bar{c}_K + x \frac{D}{l} - \frac{Dt}{l} c_a}{\frac{Dt}{l} 3 - \frac{x}{c_a} \frac{D}{l} - 2 \frac{c_H}{c_a}} \quad (56)$$

When  $k = 3$  equation (49) assumes the form

$$D_y = \frac{dx}{dt} = u + c_a = \frac{c_a \left[ 2 \frac{Dt}{l} - \frac{c_K}{c_a} \right]}{\frac{Dt}{l} 3 - \frac{x}{c_a} \frac{D}{l} - 2 \frac{c_H}{c_a}} \quad (57)$$

We assume  $\frac{Dt}{l} = \tau + \frac{c_H}{2c_a}$ ,  $\xi = \frac{c_H}{2D} + \frac{x}{l}$ ,

we then get

$$\frac{d\xi}{d\tau} = \frac{c_a}{D} \frac{2\tau}{3\tau - \frac{D}{c_a} \xi} \quad (58)$$

We will designate the relation  $\frac{D\xi}{c_a} = z$ , then

$$\frac{dz}{d\tau} = \frac{c_a}{D} \frac{2}{3 - \frac{D}{c_a} z}$$

or

$$d \ln \tau = \frac{Dz}{c_a \frac{2}{3 - \frac{D}{c_a} z}} = \frac{\left( 3 - \frac{D}{c_a} z \right) dz}{\frac{2c_a}{D} 3z + \frac{D}{c_a} z^2}$$

After integration we obtain

$$\tau = A \left| \frac{z - 2 \frac{c_a}{D}}{\left(z - \frac{c_a}{D}\right)^2} \right|, \quad (60)$$

where A is the integration constant.

The constant A is determined from the condition that when  $x = 0$ ,

$$t = \frac{l}{D} \quad \text{or when } \tau = 1 \quad \tau = 1 - \frac{\bar{c}_K}{2c_a}$$

$$t = \frac{\bar{c}_K}{2D}, \quad z = \frac{\bar{c}_K c_a}{D(2c_a - \bar{c}_K)}$$

The constant A is equal to

$$A = -\frac{2\bar{c}_K}{D} \frac{\left(1 - \frac{c_a}{c_K}\right)^2}{4 \frac{c_a}{c_K} + 1}. \quad (60)$$

Equation (59) can now be written in the form

$$\tau = -\frac{2\bar{c}_K}{D} \frac{\left(1 - \frac{c_a}{c_K}\right)^2}{4 \frac{c_a}{c_K} + 1} \frac{z - 2 \frac{c_a}{D}}{\left(z - \frac{c_a}{D}\right)^2}. \quad (61)$$

Let us turn to the new variables according to equation (58)

$$x^2 + x \left( 2 \frac{\bar{c}_K}{D} - \frac{2tc_a}{l} - A \right) + \left( \frac{c_K}{D} - \frac{tc_a}{l} \right)^2 l^2 +$$

$$+ A \left( \frac{3}{2} \frac{\bar{c}_K}{D} - 2 \frac{tc_a}{l} \right) l^2 = 0. \quad (62)$$

From this we determine  $x = x(t)$  - the law of motion of the shock wave:

$$x = -\frac{l}{2} \left( 2 \frac{\bar{c}_n}{D} - \frac{2\bar{c}_a}{l} - A \right) \pm \sqrt{\frac{l^2}{4} \left( \frac{2\bar{c}_n}{D} - \frac{2\bar{c}_a}{l} - A \right)^2 - B} \quad (63)$$

where

$$B = \left( \frac{\bar{c}_n}{D} - \frac{\bar{c}_a}{l} \right)^2 l^2 + A \left( \frac{3}{2} \frac{\bar{c}_n}{D} - 2 \frac{\bar{c}_a}{l} \right) l^2. \quad (64)$$

The obtained relations can be used to determine pressure in the passing waves.

To determine the pressure in the passing wave at a given distance  $x = h$  one must determine  $t_h$  according to formula (63) when  $x = h$ , then according to (56) determine  $u_h$ , in accordance with which  $c_h$  is determined with (47). The pressure is determined by means of the equation of state (46).

#### BIBLIOGRAPHY

- (1) Stanyukovich, K.P.: Irregular Motion in a Uniform Medium. M.-L., 1956.

END