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CONVERGENT WAVE IN A PLASTIC MEDIUM

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CONVERGENT WAVE IN A PLASTIC

MEDIUM

Following is the translation of an article by E. I. Andriankin entitled "Skhodyashchayasya Volna v Plasticheskoy Srede" (English version above) in Doklady Akademii Nauk SSSR (Proc. Acad. Sci. USSR) Vol. 59, No. 4, 1948, pages 659-662/ 131

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(Presented by Academician N. N. Semenov, 13 November 1959)

The problem of an explosion in a plastic compressible medium has been investigated by A. S. Kompaneyets in /1/. In this work a simplified law of compression was assumed. This made it possible to obtain simple equations for the law of motion of the front of a diverging spherical wave.

Utilizing the analogous law of compression (we will assume that the density of the medium reaches the limit value $\rho_* > \rho_0$ at any pressure different from zero), we will consider the problem of the converging plastic wave, which can be formulated as follows.

At the instant $t = \tau_0$, let there be a pressure at the free surface of a spherical layer with an initial outside radius a_0 . Let the said pressure vary according to a given pattern $P = P_0 F(t/\tau_0)$ (in a more general case $F = F(t/\tau_0, x)$ can be assumed). At the inner surface of the spherical layer of initial radius b_0 the pressure is zero. At the instant the pressure is applied, a shock wave is propagated through the material.

We assume the medium behind the front to be incom-

compressible, $\rho_r = \rho_* > \rho_0$, and the condition of plasticity to be satisfied:

$$\sigma_r - \sigma_\theta = k + m(\sigma_r + 2\sigma_\theta). \quad (1)$$

where k and m are assumed to be known constants, σ_r and $\sigma_\theta = \sigma_\phi$ are the main stresses. The subscript f denotes the values at the wavefront. Further, we assume that the initial density of the material in the spherical layer depends on the radius $\rho_0 = \rho_0(r_0)$.

We shall solve the problem in Lagrangian variables, where the equations of continuity will be:

$$\frac{\partial r}{\partial r_0} = \frac{r_0^2 \rho_0(r_0)}{r^2 \rho}, \quad (2)$$

$$\rho \frac{\partial u}{\partial t} = \frac{\partial r}{\partial r_0} \frac{\partial \sigma_r}{\partial r_0} - \frac{2(\sigma_r - \sigma_\theta)}{r}. \quad (3)$$

Here $u = \partial r / \partial t = \dot{r}$, t is time, r and r_0 are the variable and initial coordinates of the particles.

Using (1) and (2), we can bring Eq. (3) into the form:

$$\frac{\partial}{\partial r_0} \left[r^2 \left(\frac{k}{3m} - P \right) \right] = \rho_0(r_0) r_0^2 r^{\alpha-2} \frac{\partial u}{\partial t}, \quad (4)$$

$$\alpha = -\frac{6m}{2m+1} \leq 0, \quad P = -\sigma_r.$$

The boundary conditions for Eqs. (2) and (4) are the laws of conservation at the wavefront, the equality of pressure at the free surface $r_0 = a_0$, and the condition of continuity of the variable radius

$$\begin{aligned} P_\phi = \rho_0(R) \varepsilon_\phi \dot{R}^2, \quad u_\phi = \varepsilon_\phi \dot{R}, \quad r_\phi = R = r_0, \\ P(a_0, t) = P_0 F(t/\tau_0), \quad \varepsilon_\phi = 1 - \rho_0/\rho_*. \end{aligned} \quad (5)$$

We shall introduce the dimensionless parameters

$$\begin{aligned} a = a_0/a_0, \quad b = b_0/a_0, \quad x = R/a_0, \quad s = r_0/a_0, \quad \bar{r} = r/a_0, \\ \bar{\rho} = \rho/\rho_*, \quad z = k/3mP_0, \quad \bar{k} = k/P_0, \quad \tau = t/\tau_0, \quad \bar{u} = \dot{r}/\dot{a}_0, \quad (6) \\ dx/d\tau = -\sqrt{y}, \quad y = \rho_* \dot{R}^2/P_0, \quad \tau_0 = a_0 \sqrt{\rho_*/P_0}. \end{aligned}$$

where a_* and b_* are the outside and inside radii of the spherical layer at the instant t .

Integrating (2) and using Conditions (5), we shall find

$$\begin{aligned} \bar{r}^3 &= x^3 + 3 \int_x^s \bar{\rho}_0(s) s^2 ds, & a^3 &= x^3 + 3 \int_x^1 \bar{\rho}_0(s) s^2 ds, \\ \bar{u} &= -\frac{\lambda(x)}{r^2}, & \lambda &= \varepsilon_\phi x^2 \sqrt{y(x)}, & \varepsilon_\phi &= 1 - \bar{\rho}_0(x). \end{aligned} \quad (7)$$

Integrating Eq. (4) and using (7), we get

$$\bar{r}^2 \bar{p} = x(\bar{r}^3 - x^3) + x^2 \bar{P}_\phi - \frac{d\lambda}{dx} \sqrt{y} \int_x^s \bar{\rho}_0(s) \bar{r}^{\alpha-4} s^2 ds + 2\lambda^2 \int_x^s \bar{\rho}_0 \bar{r}^{\alpha-7} s^2 ds,$$

$$\bar{P}_\phi = \bar{\rho}_0(x) \varepsilon_\phi y, \quad z \neq 0. \quad (8)$$

If the law of motion of the wavefront is known, the pressure and velocity distribution in the entire region $x \leq s \leq 1$.

An ordinary differential equation for the velocity of the wavefront can be obtained from (8), if we use the condition on the free surface. Assuming also

$\bar{\rho}_0 = \rho_0 s^n$, while calculating all integrals in Eqs. (7) and (8), we shall find:

$$\frac{dz}{dx} = K(x)z + N(x, \tau), \quad \frac{d\tau}{dx} = -\frac{x^2 \varepsilon(x)}{\sqrt{\varepsilon_1 z}}, \quad \tau = 1, \quad x = z = 1. \quad (9)$$

Here

$$z = \frac{yx^4 \varepsilon^2}{\varepsilon_1}, \quad \varepsilon = 1 - \beta x^n, \quad \varepsilon_1 = 1 - \beta, \quad \rho_0(a_0) = \beta \rho_0,$$

$$K = \frac{4(\alpha-1)x^2 \varepsilon (a^{2-\alpha} - x^{2-\alpha})}{(\alpha-4)(a^{2-\alpha} - x^{2-\alpha})} + \frac{2\beta(\alpha-1)x^{\alpha+2-2}}{a^{2-\alpha} - x^{2-\alpha}},$$

$$a = \left[x^3 + \frac{3\beta(1-x^{\alpha+3})}{\alpha+3} \right]^{1/3},$$

$$N = \frac{2\beta(\alpha-1)x^2 \varepsilon |\bar{v}_1 - a^2 F(\tau, x)|}{\varepsilon_1 (a^{2-\alpha} - x^{2-\alpha})}, \quad \bar{v}_1 = x(a^2 - x^2).$$

In order to integrate Eq. (9) it is necessary to know its asymptotic expression at $x \rightarrow 0$ and the value of the derivative dz/dx at point $x = 1$.

After a series of calculations, we shall find

$$3 \left(\frac{dz}{dx} \right)_1 = 4z_1 + 8 - 2 \sqrt{\epsilon_1} \left(\frac{dF}{d\tau} \right)_1 + \frac{2}{s_1^3} \left(\frac{ds_1}{dx} \right)_1 + \delta_2, \quad \delta_2 = 2\alpha\beta(z-1); \quad (10)$$

$$\lim_{x \rightarrow 0} z = A, \quad z = B_1 x^{4(x-1)/(x-1)} + \dots \quad (x \rightarrow 0, n > 0); \quad (11)$$

$$z = B_2 x^\alpha + \dots, \quad \omega = \frac{2(x-1)[2z_1 - \beta(x-4)]}{\alpha-4} \quad (n=0, x \leq 0). \quad (12)$$

However, if the outside pressure drops rapidly, the wave may stop before reaching the center.

Equations (8) and (9) are true in case $\alpha \neq 0$; if $m = 0$, the condition of plasticity becomes $\sigma_r - \sigma_\theta = k$ and the solution is given by the same equations, in which we have to assume

$$x = 0, \quad \delta_1 = 2\bar{k} \ln(x/a), \quad \delta_2 = -4\beta\bar{k}.$$

It should also be noted that Eqs. (9) give a solution to the problem of the collapse of a spherical layer of incompressible plastic material. In that case it is necessary to assume

$$r^3 = s^3 + x^3 - b_0^3, \quad x = b, \quad \bar{P}_\phi = 0, \quad z = yx^4, \quad (13)$$

$$k = \frac{4(x-1)x^2(a^{x-1} - x^{x-1})}{(x-4)(a^{x-1} - x^{x-1})}, \quad N = \frac{2(x-1)x^2[\delta_1 - a^x F]}{a^{x-1} - x^{x-1}}.$$

In these cases an asymptotic solution is given by Eq. (11). At $\tau = 1$ and $x = b_0$,

the boundary conditions for this problem will be $y = 0$; in the case of initial motion in the fluid, the boundary condition will be $y = y_0$. The latter occurs, for instance, when a spherical layer of compressible material has been compressed by the outside pressure to the limit condition $\rho_f = \rho_*$; then, after the disappearance of the shock wave (at the boundary with the empty space) the compressed material continues to collapse as if incompressible.

In Figs. 1 and 2 the results of the numerical integration of Eqs. (8) are shown. Curve 1 corresponds to

the solution of the problem, for parameter values of:

$$F = t^{-1.2}$$

$$\epsilon_* = 0.1$$

$$n = 0$$

$$\chi = -0.1$$

$$\alpha = -1$$

for Curve 2, we have:

$$F = t^{-3}$$

$$\epsilon_* = 0.1$$

$$n = 0$$

$$\bar{F} = -0.1$$

$$\alpha = 0$$

for Curve 3, we have:

$$F = 1$$

$$\epsilon_* = 0.2$$

$$nm = 1$$

$$\beta = 0.8$$

$$\chi = -0.01$$

$$\alpha = -1.$$

(In the case of a constant outside pressure, or $F = F(x)$, Eqs. (8) will be integrated in squares).

The values of A and B in the Asymptotic Expressions (11) and (12) can be determined by ~~xxxxxxx~~ numerical calculation. In case 2, $A \approx 0.44$, $B \approx 1.195$, $\omega \approx 1.9$; in case 3, $A \approx 0.166$, $B \approx 1.464$, and $\omega \approx 1.6$.

The concentration of the energy in the center can be investigated through the behavior of the solution at $x \rightarrow 0$. The energy per unit volume consists of kinetic energy, dissipation energy at the wavefront, and dissipation energy due to the work of the forces of plastic deformation; therefore

$$\int_a^1 a^2 F da = \frac{1}{2} \int_x^1 \bar{\rho}_0(s) \bar{u}^2 s^2 ds + \frac{1}{2} \int_x^1 \varepsilon_\phi P_\phi s^2 ds + 2 \int_1^{\bar{r}} \left[\int_x^1 \frac{(\bar{\sigma}_r - \bar{\sigma}_\theta)(1 - \varepsilon_\phi) \bar{u} s^2 ds}{r} \right] d\tau. \quad (14)$$

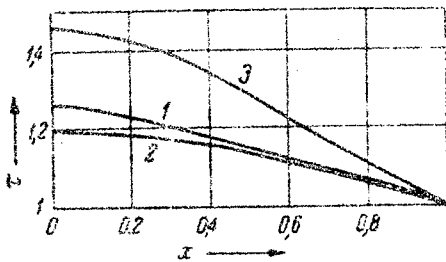


Fig. 1

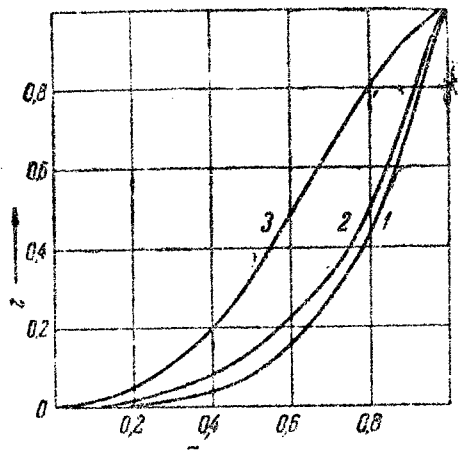


Fig. 2

It can be established, studying the behavior of these integrals at $x \rightarrow 0$ and $s \rightarrow 0$, that the final concentration of energy in the center takes place only in the base of the collapse of a spherical layer of incompressible fluid (the second integral in (14) is absent) under the condition $n = mk = 0$ (\bar{k} is arbitrary). In other cases all integrals in (14) converge, and this means a decrease in the stored energy at the center.

In analogy to the above, the problem can be solved by assuming a variable compression of matter, depending on the amplitude of the pressure at the wavefront /2/. However, as in /3, 4/ it would be necessary here to solve an integral differential equation for the velocity of the front of the shock wave.

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