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LAMINAR-BOUNDARY-LAYER HEAT-TRANSFER CHARACTERISTICS OF A BODY OF REVOLUTION WITH A PRESSURE GRADIENT AT SUPERSONIC SPEEDS

By William R. Wimbrow and Richard Scherrer

Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

Local rates of heat transfer have been measured over the negative pressure-gradient region of a pointed body of revolution with a fineness ratio of 6:1, a vertex angle of 37°, and a contour generated by rotating a segment of a parabola. Data have been obtained with a laminar boundary layer at three Reynolds numbers at a Mach number of 1.49 and two Reynolds numbers at a Mach number of 2.18. The results of this investigation indicate that the heat-transfer characteristics of the parabolic body can be predicted by employing the theoretical relationship between Nusselt number and Reynolds number for conical bodies.

INTRODUCTION

The literature pertaining to convective heat transfer as it applies to high-speed aerodynamics is restricted primarily to the consideration of laminar flow over flat plates. (See reference 1.) Along a flat plate in supersonic flow the pressure is constant; consequently, the static temperature at the outer edge of the boundary layer is also constant. In contrast, the contour of the forward portion of a typical supersonic aircraft body, often a segment of a circle or a parabola, gives rise to a pressure field in which the pressure decreases with length along the body. The static temperature is subject to a corresponding variation. The differential equation defining laminar flow over such a surface can be written readily but cannot be solved by ordinary means due to the variable pressure and temperature terms. Approximate methods for determining the heat-transfer characteristics of bodies of revolution with pressure gradients are presented in references 2 and 3. However, the results obtained are questionable due to the simplifying assumptions employed and both methods are so tedious to apply that they are not considered practical for design purposes.

In reference 4, Mangler presents relations whereby the characteristics of a laminar boundary layer for an axially symmetric body of given contour and pressure distribution can be evaluated by calculating the characteristics of a boundary layer on a two-dimensional surface with an equivalent pressure distribution and then correcting for the effect of the varying circumference of the axially symmetric body. Since two-dimensional solutions are restricted by mathematical limitations to the constant pressure case, the resulting solutions for axially symmetric flow are also limited to the constant pressure case and hence to conical bodies. From Mangler's method, however, one can deduce that the boundary layer, and hence the rate of heat transfer, for a body of revolution with a negative pressure gradient is affected by two factors not affecting the boundary layer on a flat plate. One factor is the variation of the circumference of the body of revolution with length and the other is the effect of the pressure gradient.

The boundary layer on a flat plate grows only in thickness with distance along the surface, while the boundary layer on a pointed body of revolution must spread circumferentially as it grows in thickness. The effect of this circumferential spreading on the boundary-layer thickness, and hence on the heat-transfer characteristics, varies with the rate of change of circumference with respect to length and can be evaluated for any fair contour by relations given in reference 4. For example, it can be shown that on a cone, for which the pressure and the rate of change of circumference are constant with length, the boundary-layer thickness is less by the factor $1/\sqrt{3}$ and the rate of heat transfer is greater by the factor $\sqrt{3}$ than on a flat plate. On a body with a curved contour for which the pressure and the rate of change of circumference decrease with length, these factors defining the boundary-layer thickness and rate of heat transfer relative to a two-dimensional surface with an equivalent pressure distribution vary from the conical values ($1/\sqrt{3}$ and $\sqrt{3}$, respectively) at the vertex, to unity at the point where the rate of change of circumference becomes zero. Therefore, the rate of growth of the boundary layer and the rate of heat transfer on such a body are the same as on a cone at the apex, but the decreasing rate of change of circumference along the body tends to increase the rate of growth of the boundary layer and decrease the rate of heat transfer relative to that for a conical body.

Although the effect of a pressure gradient on the characteristics of a boundary layer cannot be predicted exactly, it is known that a negative pressure gradient causes the boundary layer to thicken less rapidly with length than is the case if the pressure were constant, regardless of the body shape. Thus, on a body for which the pressure and the rate of change of circumference decrease with length, the effect of the pressure gradient tends to counteract the effect of the variation of circumference on the boundary-layer characteristics when comparing boundary-layer development with that of a cone. If these two effects can be assumed to counteract each other exactly over that portion of a body of revolution where the pressure gradient is negative, the heat-transfer characteristics can be readily calculated by employing the known theoretical relations for conical bodies. The method would be valid only for laminar boundary layers but, since negative pressure gradients are favorable to the maintenance of laminar flow, the method would have wide application. The present investigation was undertaken to gain some insight of the degree to which these two effects counteract each other by measuring the local rates of heat transfer on a typical body shape and comparing the results with the theoretical heat-transfer characteristics of cones.

SYMBOLS

c_f	local skin-friction coefficient, dimensionless
c_p	specific heat at constant pressure, Btu per pound, °F
g	gravitational constant, 32.2 feet per second squared
H	total pressure, pounds per square inch absolute
h	local heat-transfer coefficient $\left(\frac{q}{T_s - T_R}\right)$, Btu per hour, square foot, °F
k	thermal conductivity, Btu per hour, square foot, °F per foot
l	length of test body, feet
L	length of basic body (pointed at both ends), feet
M	Mach number, dimensionless
Nu	local Nusselt number (hs/k), dimensionless
Pr	Prandtl number $\left(\frac{c_p \mu}{k} \times 3600g\right)$, dimensionless
q	local rate of heat transfer, Btu per hour, square foot
Re	Reynolds number $(\rho Vs/\mu)$, dimensionless
r	radius of the body at any longitudinal station, feet
s	distance from the vertex along the surface of the body, feet
T	temperature, °F
T_R	recovery temperature (temperature attained by the body surface at the zero heat-flow condition), °F
V	fluid velocity just outside the boundary layer, feet per second

x axial distance from the vertex of the body to any longitudinal station, feet

μ absolute viscosity, pound-second per square foot

ρ air density, slugs per cubic foot

In addition, the following subscripts have been used in combination with the foregoing symbols:

o conditions at total temperature and pressure in the free stream

s local conditions at the surface of the body

v local conditions just outside the boundary layer on the body

APPARATUS AND PROCEDURE

Wind Tunnel

The tests were performed in the Ames 1- by 3-foot supersonic wind tunnel No. 1. This closed-circuit, continuous-operation wind tunnel is equipped with a flexible-plate nozzle that can be adjusted to give test-section Mach numbers from 1.2 to 2.4. Reynolds number variation is accomplished by changing the absolute pressure in the tunnel from one-fifth of an atmosphere to approximately three atmospheres depending on the Mach number and ambient temperature. The water content of the air in the wind tunnel is maintained at less than 0.0001 pound of water per pound of dry air in order to eliminate humidity effects in the nozzle.

Test Body and Instrumentation

The body employed in this investigation is a body of revolution generated by rotating a segment of a parabola in such a manner that the radius at any longitudinal station is given by the relation

$$\frac{r}{L} = \frac{1}{3}\left[\left(\frac{x}{L}\right) - \left(\frac{x}{L}\right)^2\right] \tag{1}$$

This body has a fineness ratio of 6:1 and a vertex angle of 37° which is more blunt than is generally considered desirable for supersonic aircraft, and hence has a more severe pressure gradient than would normally be encountered. Equation (1) defines a body which is pointed at each end. Since the forward portion of this basic body was adequate for the purposes of this investigation, the length L in equation (1) was assigned a value of 18 inches, but only the first 8-1/2 inches of the total shape was employed. (See figs. 1 and 2.) The exterior shell of the body was machined from stainless steel and was polished to provide an estimated 20-microinch root-mean-square (rms) surface. All other parts were made of copper. Heating was provided by passing a high-amperage, low-voltage, alternating electrical current longitudinally through the body shell. The shell thickness was designed to provide a constant surface temperature at a free-stream Mach number of 1.5.[1] Since it was impossible to design the body so that current would flow through the extreme forward portion, the forward 16.5 percent of the 8-1/2 inch body was, in effect, unheated.

Nine thermocouples were installed at equal length increments along the body to determine the surface-temperature distribution. The thermocouples were made from 30-gage iron-constantan duplex wire and were soldered in holes drilled through the shell. Ten leads of 20-gage copper wire were also installed in the shell in a similar manner to measure incremental voltage drops along the body. The locations of the thermocouples and the voltage leads are indicated in figure 2. The instrumentation and wiring of the experimental installation were identical with that for previous tests of an electrically heated cone described in detail in reference 5.

In addition to the heated body, another body identical in contour was employed to determine the pressure distribution and, consequently, the Mach number distribution just outside the boundary layer along the body. Pressure orifices were spaced uniformly along its length and were connected to manometer tubes containing dibutyl phthalate.

Procedure

Data were obtained at nominal total pressures of 6, 12, and 18 pounds per square inch absolute at a free-stream Mach number of 1.49 and at nominal total pressures of 8 and 15 psia at a free-stream Mach number of 2.18. The tunnel was first brought to the desired pressure and allowed to run until the tunnel shell, the air stream, and the test body attained thermal equilibrium. When this condition was reached, the surface-temperature distribution was measured. The temperatures measured under these conditions of zero heat flow are called the recovery temperatures T_R. (See reference 6.) The heating circuit was then closed and the body heated to a nominal surface temperature as measured at the second thermocouple from the vertex. Temperatures of 120°, 160°, and 200° F were arbitrarily chosen as values at which to obtain data. With the body at the

[1]The calculations to determine thickness distribution were based on the assumption that the heat-transfer theories for conical bodies could be applied to the test body.

desired temperature, the following data were recorded: the total pressure and total temperature of the air stream, the current input to the body, the incremental voltage drops, and the local surface temperatures of the body.

Measurements were also made with the pressure-distribution body at the same Mach numbers and total pressures as for the heat-transfer measurements. The data so obtained were used to calculate the Mach number and static-temperature distributions just outside the boundary layer on the body.

Schlieren observations, liquid-film tests, and the absence of discontinuities in the temperature distributions that would denote transition indicated that the boundary layer remained laminar for all test conditions.

ACCURACY OF RESULTS

Since the instrumentation and procedure for the present investigation were identical with those for previous tests of a heated cone, the accuracy of the results is also identical. A detailed discussion of the determination of the accuracy is given in reference 5. The over-all accuracy of the final parameters is given below:

Surface temperature	T_s	$\pm 0.5^{\circ}$ F
Nusselt Number	Nu	± 4.4 to ± 6.6 percent
Reynolds Number	Re	± 1.8 to ± 1.9 percent
Heat-transfer parameter . . .	$Nu/Re^{1/2}$	± 4.5 to ± 6.7 percent

As discussed in reference 5, the effect of radiation from the heated cone to the tunnel walls was evaluated and found to be negligible. Since the same conditions existed, the effect of radiation was assumed to be negligible in the present investigation.

RESULTS AND DISCUSSION

The pressure-distribution measurements indicated that the pressure gradient was essentially linear over the entire length of the body. These measurements were reduced to the more convenient form of local Mach number distributions (shown in fig. 3) which were used in the reduction of the heat-transfer data to determine the local temperature at the outer edge of the boundary layer and the local Reynolds number.

The heat-transfer data obtained at a free-stream Mach number of 1.49 are shown in figure 4 and those obtained at a Mach number of 2.18 are shown in figure 5. The measurements of local power input were converted to local rates of heat transfer q by converting the electrical units to heat units and dividing by the incremental areas. The dimensionless heat-transfer parameter $Nu/Re^{1/2}$ shown in figures 4 and 5 was then determined from the equation

$$Nu/Re^{1/2} = \frac{qs}{k(T_s - T_R)} \left(\frac{\mu}{\rho V s}\right)^{1/2} \tag{2}$$

in which all the fluid properties were evaluated at the local static temperature at the outer edge of the boundary layer. Usually, whenever there is a pressure variation along a body, some boundary-layer dimension such as the displacement thickness is used instead of the length of travel along the surface in the definition of Reynolds number and Nusselt number. Since boundary-layer measurements were not required for any other purpose and would not normally be available to the design engineer, the usual definitions for flat plates and cones were employed in the present investigation.

Inspection of the surface-temperature curves in figures 4 and 5 reveals that the temperature at the first measuring station was considerably higher than the temperature at subsequent stations. After the tests were completed it was found that this local hot region was partially caused by poor electrical contact between the copper sting and the stainless-steel shell. This connection was improved and the body was tested again at a Mach number of 1.49 to determine the effects of the resulting change in temperature distribution. When the body was reinstalled in the wind tunnel, some of the measuring instruments were not the same as those originally employed, although they were of the same type and precision. The leads to the thermocouples nearest the base, which were broken when the body was originally installed, were also repaired at this time. The results of the measurements made with these changes are shown in figure 6.

The size of the symbols on the heat-transfer-parameter plots of figures 4, 5, and 6 is approximately equal to the limits of the experimental accuracy. It should be noted that the variation of the individual measurements from a fair curve is consistent throughout and is apparently caused by small deviations in the thickness distribution of the body shell. A comparison of figures 4 and 5 shows that, although there were some variations of the values of the heat-transfer parameter with total pressure, temperature level, and Mach number, the variations were within the experimental accuracy. A comparison of the data in figures 4 and 5 with the data in figure 6 reveals that improving the connection between the shell and the sting reduced the difference between the temperatures measured at the first thermocouple and the average temperature aft of this point by approximately 60 percent. The resulting reduction of the negative temperature gradient at the beginning of the heated region of the body was accompanied by an increase in the rate of heat transfer in this region as would be expected, and the value of the heat-transfer parameter became essentially constant along the test body.

Since the rate of heat transfer is affected by the surface-temperature distribution, it would be logical to compare the experimental results for the parabolic body with the theoretical heat-transfer characteristics of a conical body with the same surface-temperature distribution as that measured on the parabolic body. An attempt was made to carry out this comparison utilizing the method of reference 7. The first step of this method is to apply Mangler's transformation (reference 4) to determine the relation between the dimensionless length along the axially symmetric body and the corresponding length along a two-dimensional body to produce the same boundary-layer flow conditions. The method then requires that the temperature distribution be expressed by a power-series polynomial as a function of the equivalent two-dimensional length. The dimensionless length s/l to the beginning of the heated section on the parabolic body of the present investigation is 0.1658. The transformation gives 0.0045 for the equivalent two-dimensional length. The temperature varies from approximately the recovery temperature in the tip region to the selected surface temperature at the beginning of the heated section and is essentially constant aft of this point, at least for the data shown in figure 6. It was found to be impractical to express such a distribution by a polynomial because of the excessive number of terms required. However, it can be shown by the theory that the local rate of heat transfer along a surface with a temperature gradient followed by a constant-temperature region differs from the rate of heat transfer on a surface with a constant temperature principally in the region of the gradient. Aft of the gradient the rate of heat transfer approaches asymptotically the value it would have had if the gradient had not been present. This effect has been observed in the investigations of the heat-transfer characteristics of conical bodies reported in references 5 and 8. It is also evident in the data shown in figures 4 and 5, but not in the data obtained with the modified temperature distribution shown in figure 6. Therefore, it appears reasonable to assume that the positive temperature gradient at the vertex is confined to such a small region of the equivalent two-dimensional length that its effect is negligible.

Further comparison of figures 4, 5, and 6 reveals that there was an apparent reduction in the rate of heat transfer over the aft portion of the body when the negative temperature gradient at the beginning of the heated region was reduced. Although this reduction in the rate of heat transfer appears to result from the change in temperature distribution, such is not believed to be the case. The maximum difference in the average values of the heat-transfer parameter measured over the aft portion of the body is approximately 13 percent and occurs between the data presented in figures 5(a) and 6(b). These two sets of data were obtained at different free-stream Mach numbers and total pressures and with different instruments. The average values of the heat-transfer parameter over the aft portion of the body shown in figures 4(a) and 6(a), which were measured at the same Mach number and total pressure, differ by approximately 6.5

percent. If the exact value of the heat-transfer parameter were assumed to lie somewhere between the two sets of data, the measured values would be well within the limits of the estimated experimental accuracy. Therefore, it is believed that the differences in the measured rate of heat transfer over the aft portion of the test body were due principally to the summation of the effects of the differences in Mach number, total pressure, and instrumentation. A comparison of the experimental data measured over this region of the test body with the theoretical heat-transfer characteristics of a cone with a constant surface temperature is also believed to be justified.

The existing analytical studies of heat transfer for laminar flow over flat plates are reviewed in reference 1. It is shown that, in general, the results can be expressed by the equation

$$Nu/Re^{1/2} = \left(\frac{c_f \, Re^{1/2}}{2} \right) Pr^{1/3} \tag{3}$$

in which all the fluid properties are based on the same temperature. The quantity $c_f \, Re^{1/2}$ is a function of Mach number, Prandtl number, and the exponent for the variation of thermal conductivity and viscosity with temperature. However, for the Mach number and temperature range of the present investigation, reference 1 shows that very little error is introduced if $c_f \, Re^{1/2}$ is assumed to be a constant and equal to 0.664 for flat plates.

Mangler shows in reference 4 that the value of $c_f \, Re^{1/2}$ for a cone is greater than that for a flat plate by a factor equal to $\sqrt{3}$. The Prandtl number for air in the temperature range of the experiments is approximately 0.715 and hence equation (3) for a conical body reduces to

$$Nu/Re^{1/2} = 0.514 \tag{4}$$

This value is shown as a dashed line on each of the heat-transfer-parameter plots in figures 4, 5, and 6. The average deviation between the theoretical value for cones and the experimental values shown in figures 4 and 5 is approximately 13 percent. The data presented in figure 6 were obtained with a surface-temperature distribution that more closely approximates a constant temperature condition, and, in this case, the theoretical and experimental values differ by less than the uncertainty in the experimental measurements. As discussed previously, the exact value probably lies somewhere between the limits of the experimental values.

It has been shown that the theoretical value of the parameter $Nu/Re^{1/2}$ is 0.514 for conical bodies and 0.297 for flat plates and cylinders. This difference is due solely to the increase of the circumference with length along a cone since a corresponding variation of geometry does not occur on a flat plate or cylinder, and the pressure distribution is constant in both cases. As discussed previously, the local values of $Nu/Re^{1/2}$ along the test body would be expected to vary from 0.514 at the vertex to 0.297 at the base if the pressure gradient that also results from the body contour had no effect. Since the experimental values of $Nu/Re^{1/2}$ remain constant at approximately 0.514 along the test body, the effect of the pressure gradient is apparently opposite and approximately equal to the effect of the decreasing rate of change of the circumference on the growth of the boundary layer. However, the effect of the pressure gradient may be considered to be partially concealed by the manner in which the data are reduced. The pressure gradient is accompanied by corresponding Mach number and static temperature gradients at the outer edge of the boundary layer and the local values of these parameters were utilized to reduce the measured data to the dimensionless heat-transfer parameter $Nu/Re^{1/2}$. Thus the present investigation may be considered inconclusive as to the exact effects of the pressure gradient. However, it has been shown that the local rate of heat transfer along the test body can be calculated within 13 percent or less of the true value by employing equation (4) and assuming that the theoretical local values of the parameter $Nu/Re^{1/2}$ for cones also hold for the test body. It appears reasonable to assume that this result would be applicable to other pointed body shapes that give rise to negative pressure gradients. On a body that is more slender than the one tested the effects of the changing circumference and the pressure gradient would be less and would still tend to counteract each other. Conversely, on a more blunt body both effects would be larger. However, additional experiments with other body shapes covering a wider range of Mach numbers are necessary before the results of the present investigation can be proven to be applicable to all fair bodies of revolution with negative pressure gradients.

CONCLUDING REMARKS

The investigation indicates that, for the parabolic body shape tested, the effect of the decreasing rate of change of the circumference on the growth of the laminar boundary layer as compared to that of a cone is largely compensated by the difference in the pressure gradients. The net result is that the heat-transfer characteristics of the parabolic test body can be predicted by employing the theoretical relationship between Nusselt number and Reynolds number for conical bodies.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Apr. 21, 1950.

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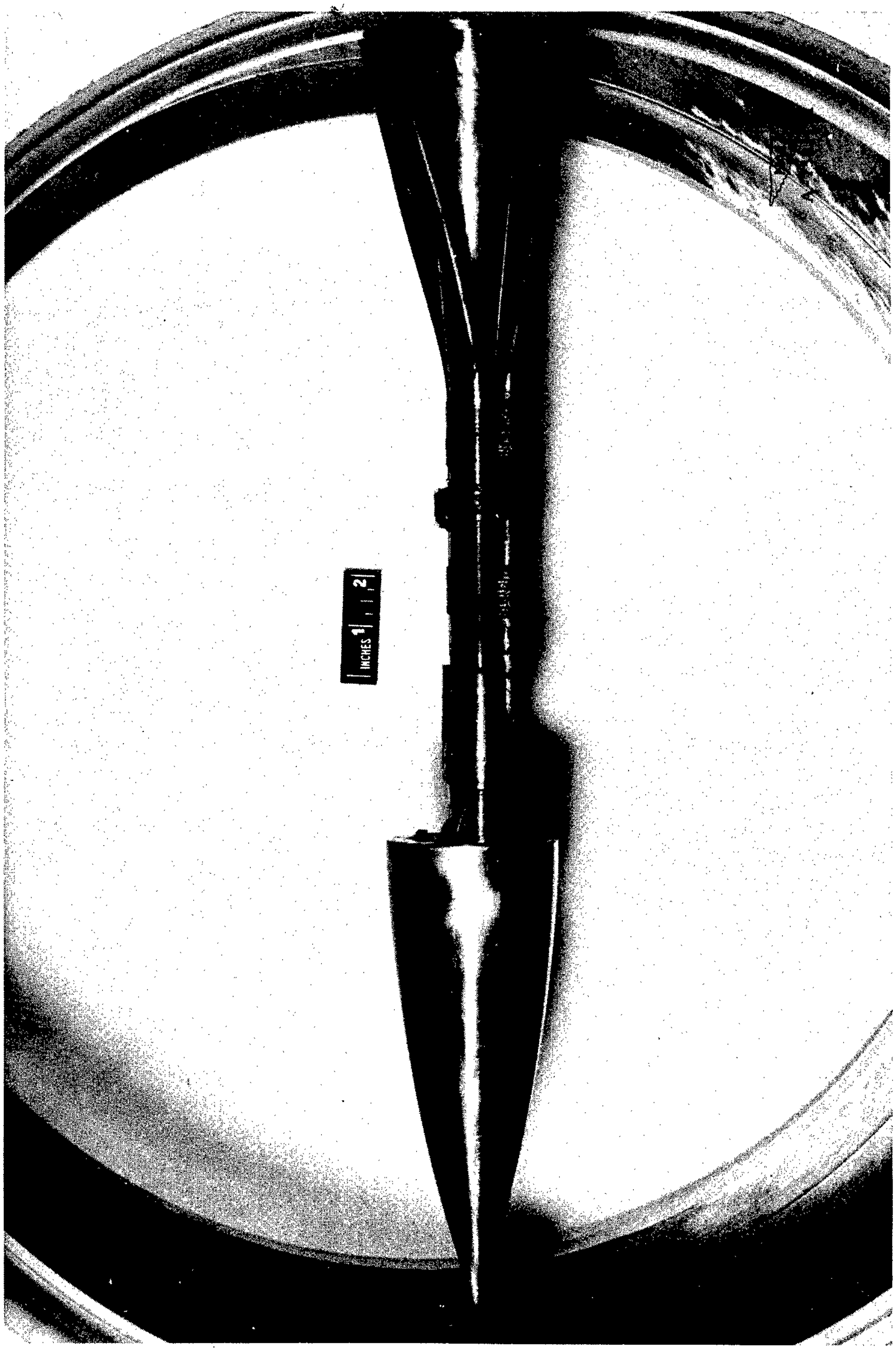


Figure 1.– The test body installed in the Ames 1- by 3-foot supersonic wind tunnel No. 1.

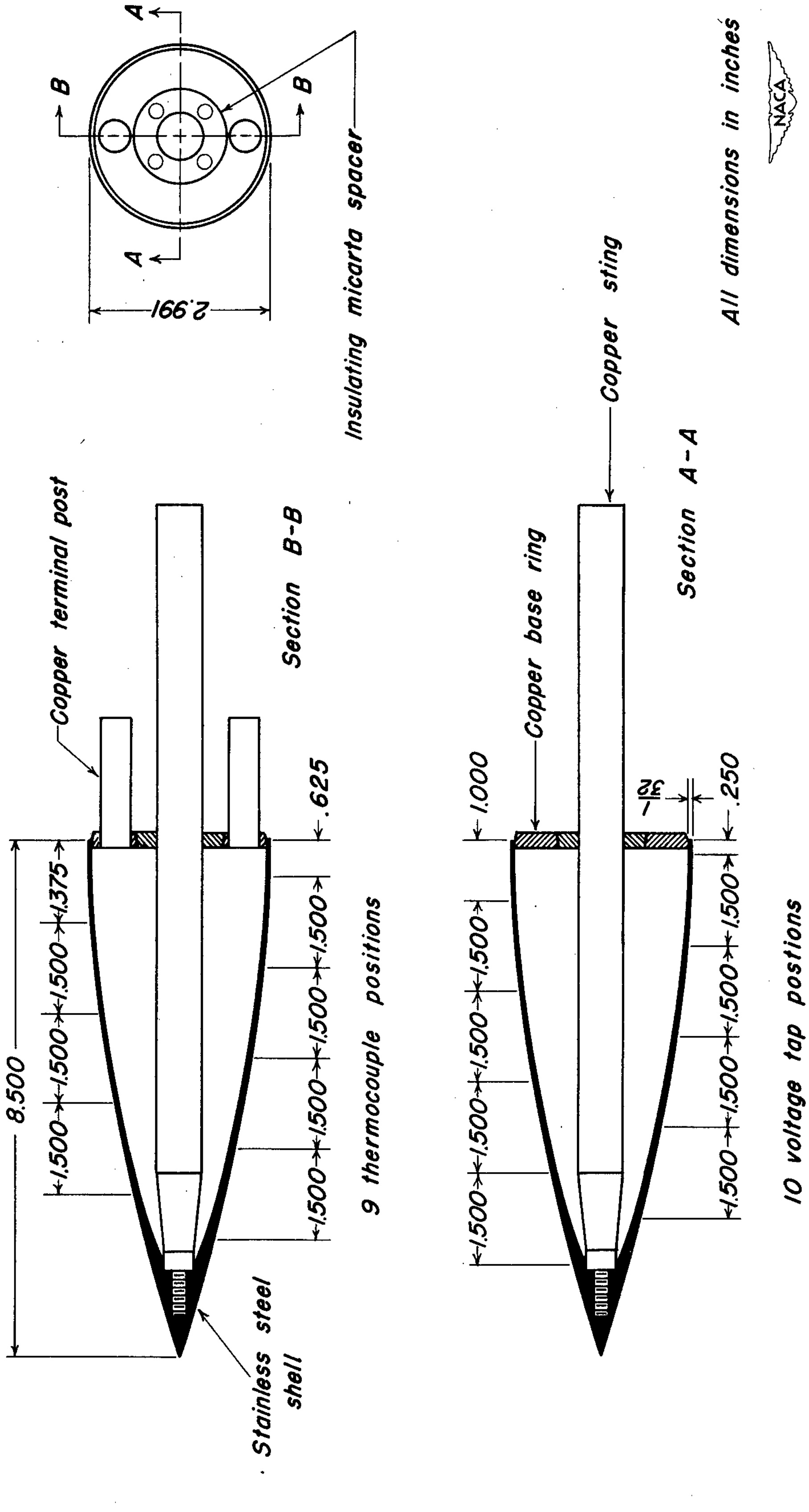


Figure 2.- Details of the test body.

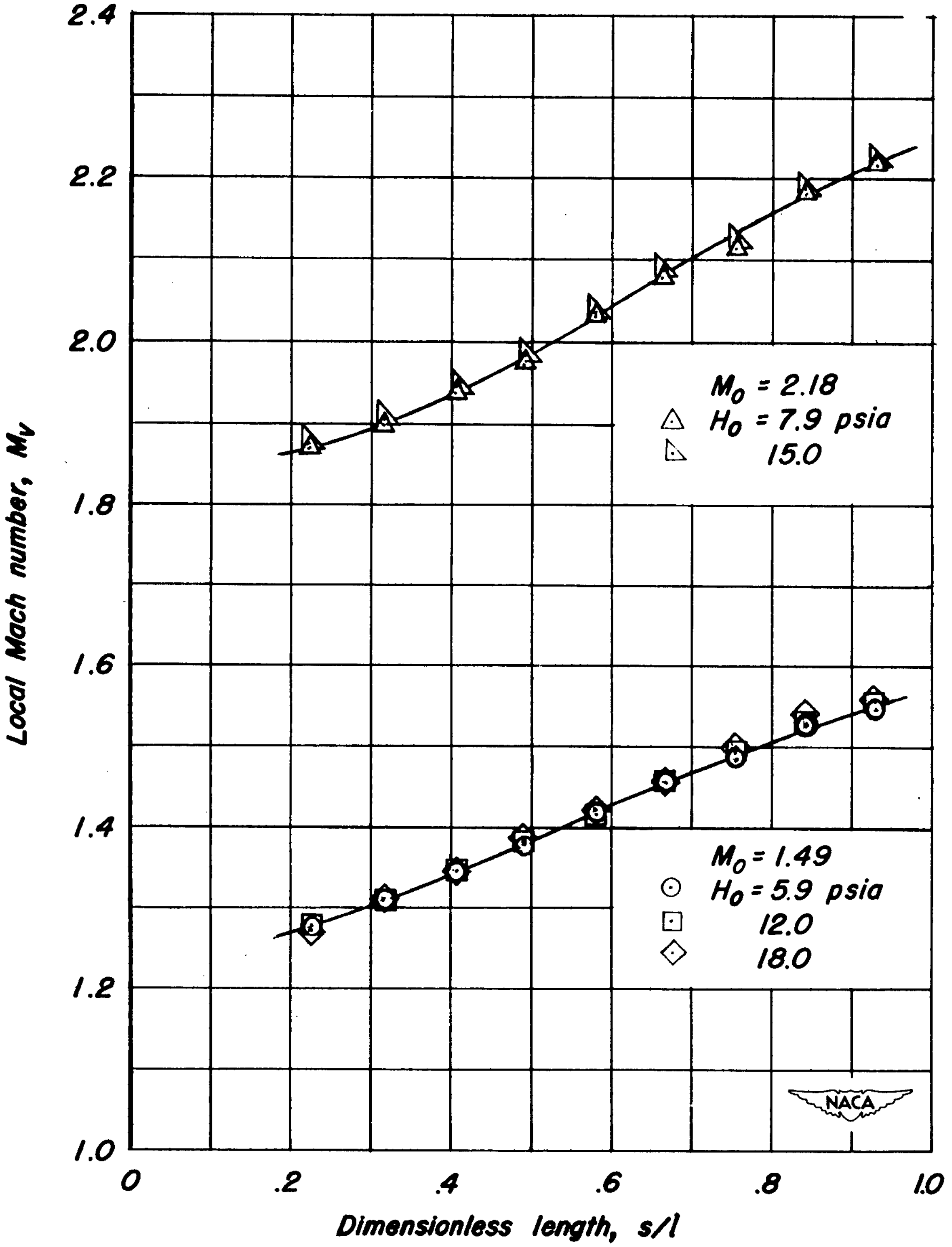
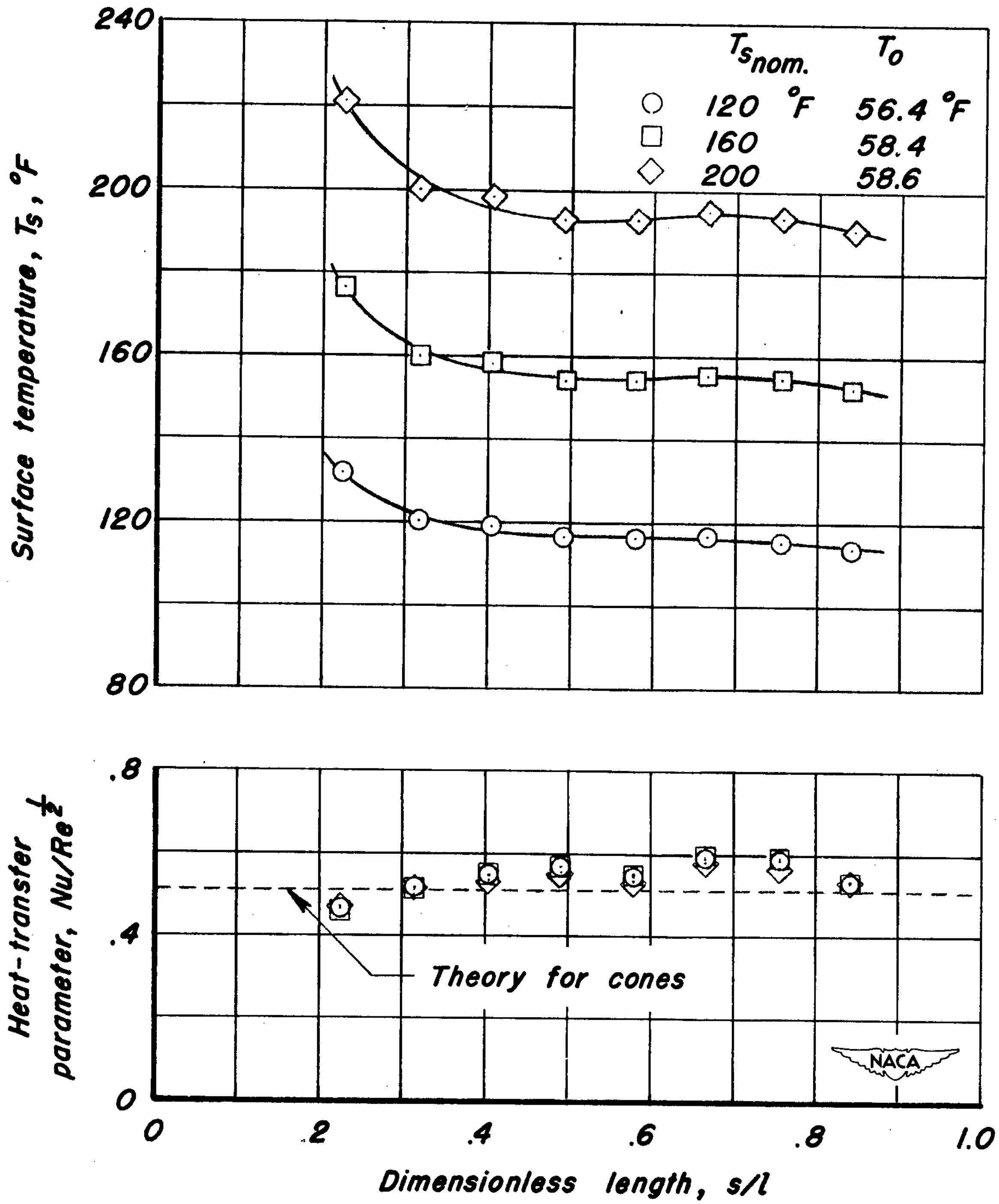
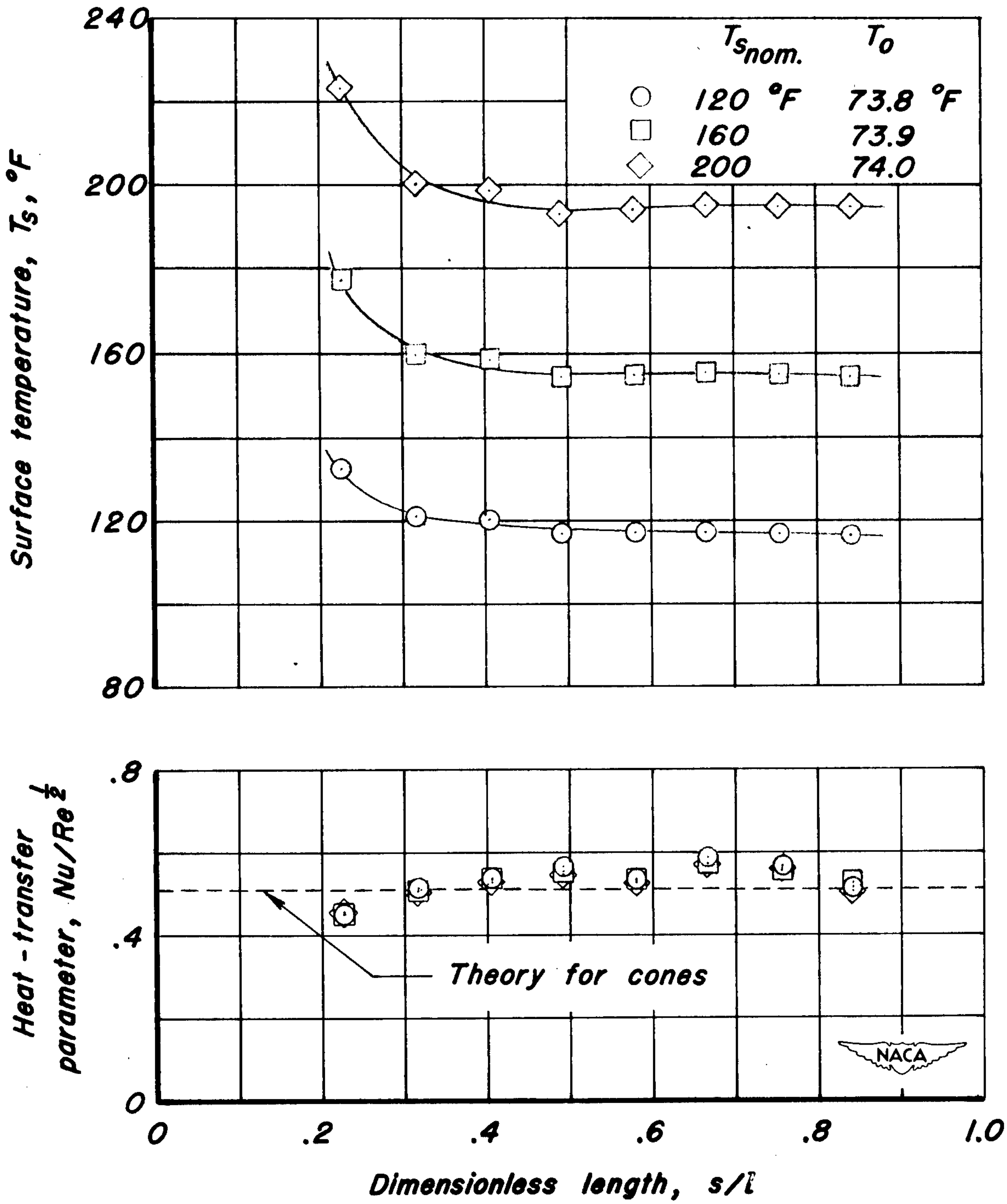


Figure 3. – Variation of local Mach number with length as determined by pressure measurements.



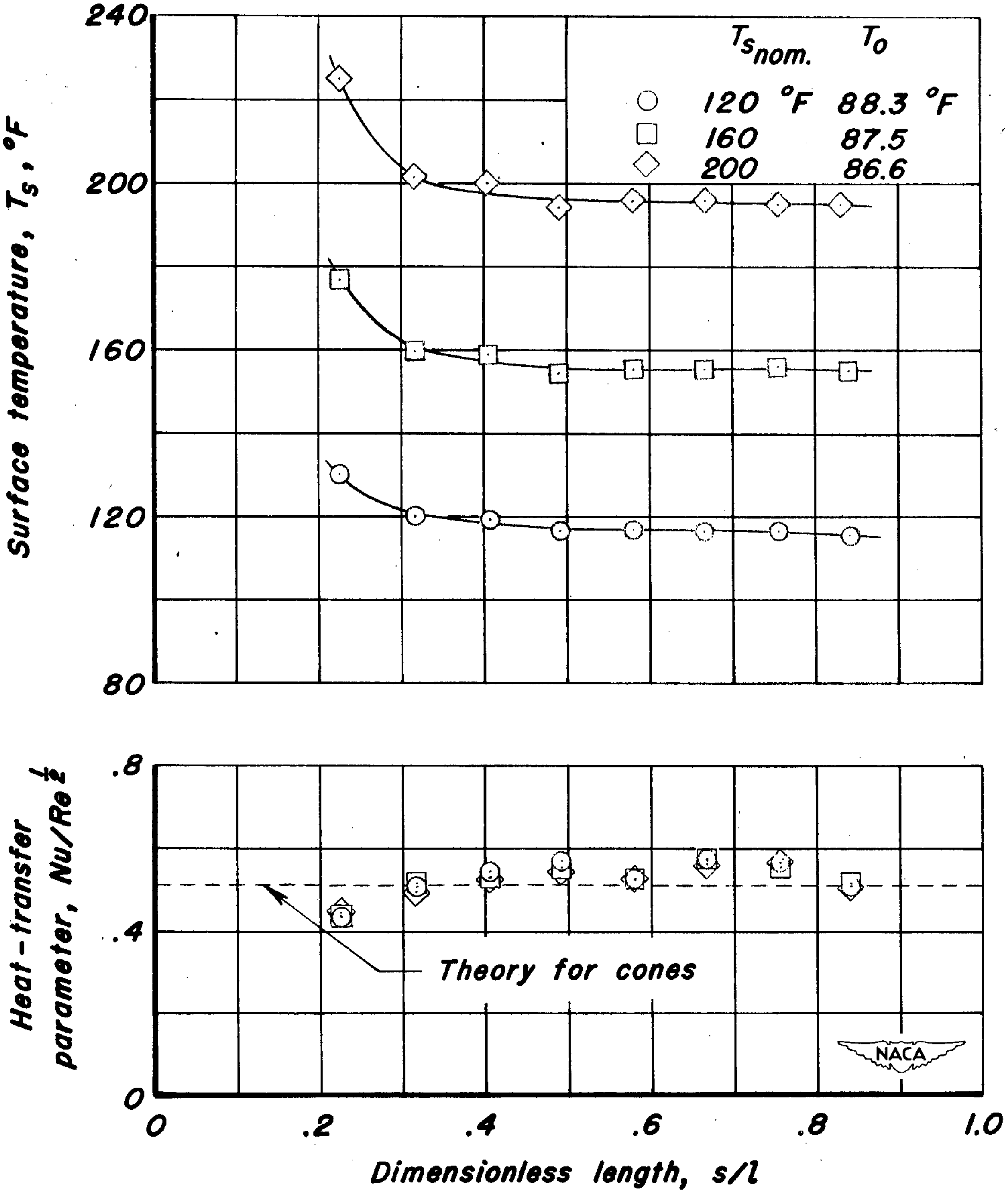
(a) H_0=5.9 psia.

Figure 4.— Heat-transfer characteristics of the parabolic body at M_0= 1.49.



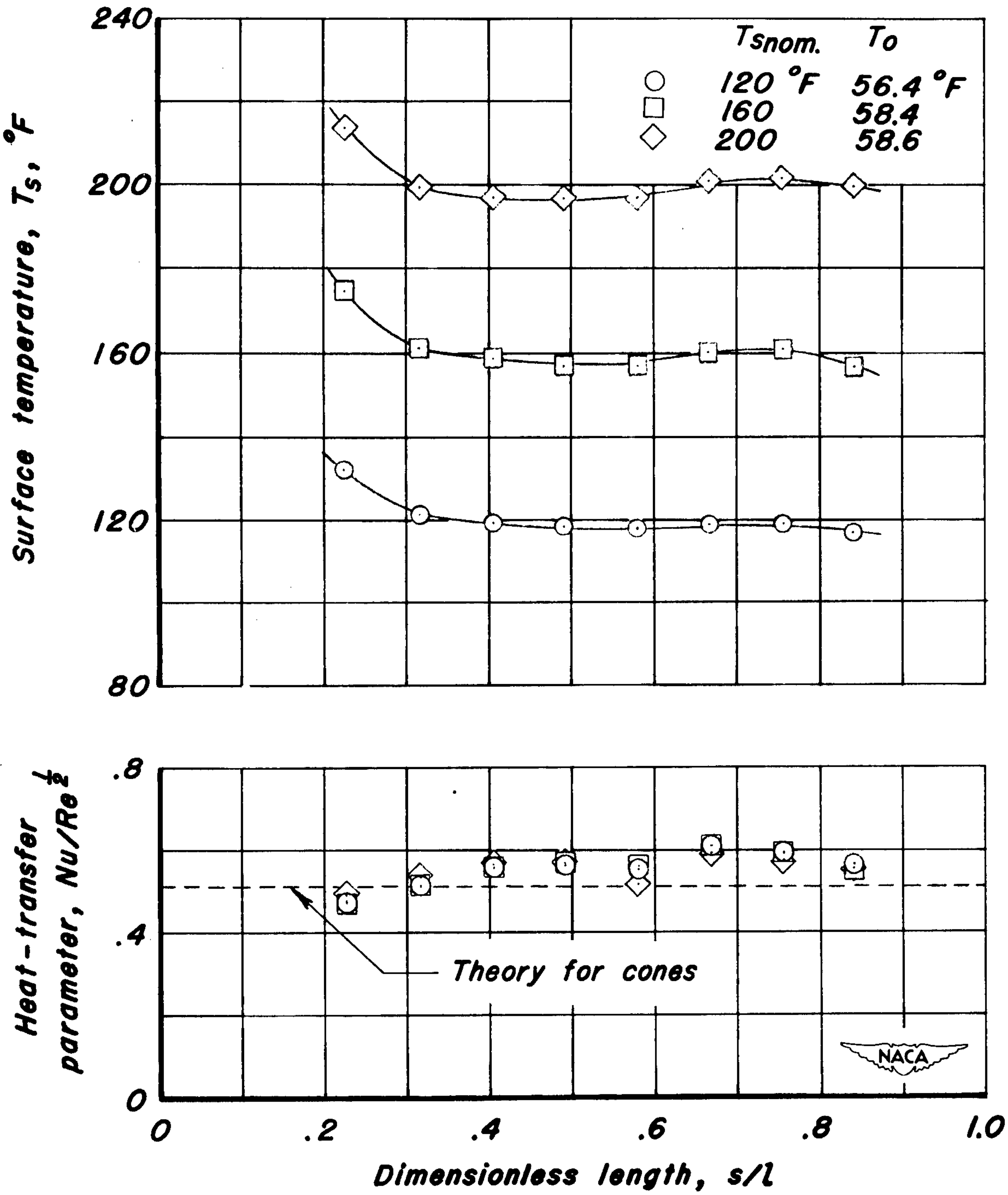
(b) $H_o = 12.0$ psia.

Figure 4.- Continued.



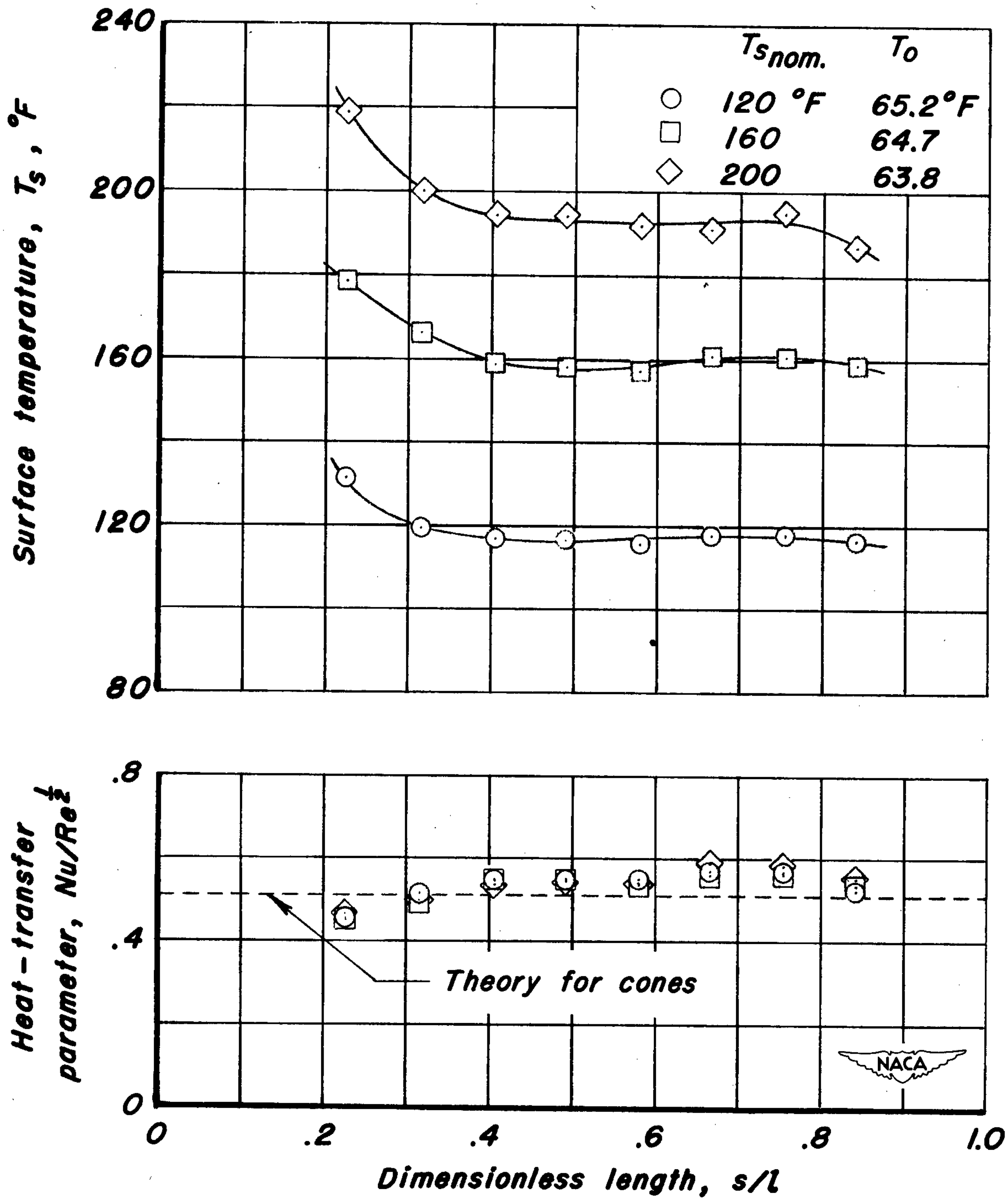
(c) H_o = 18.0 psia.

Figure 4.- Concluded.



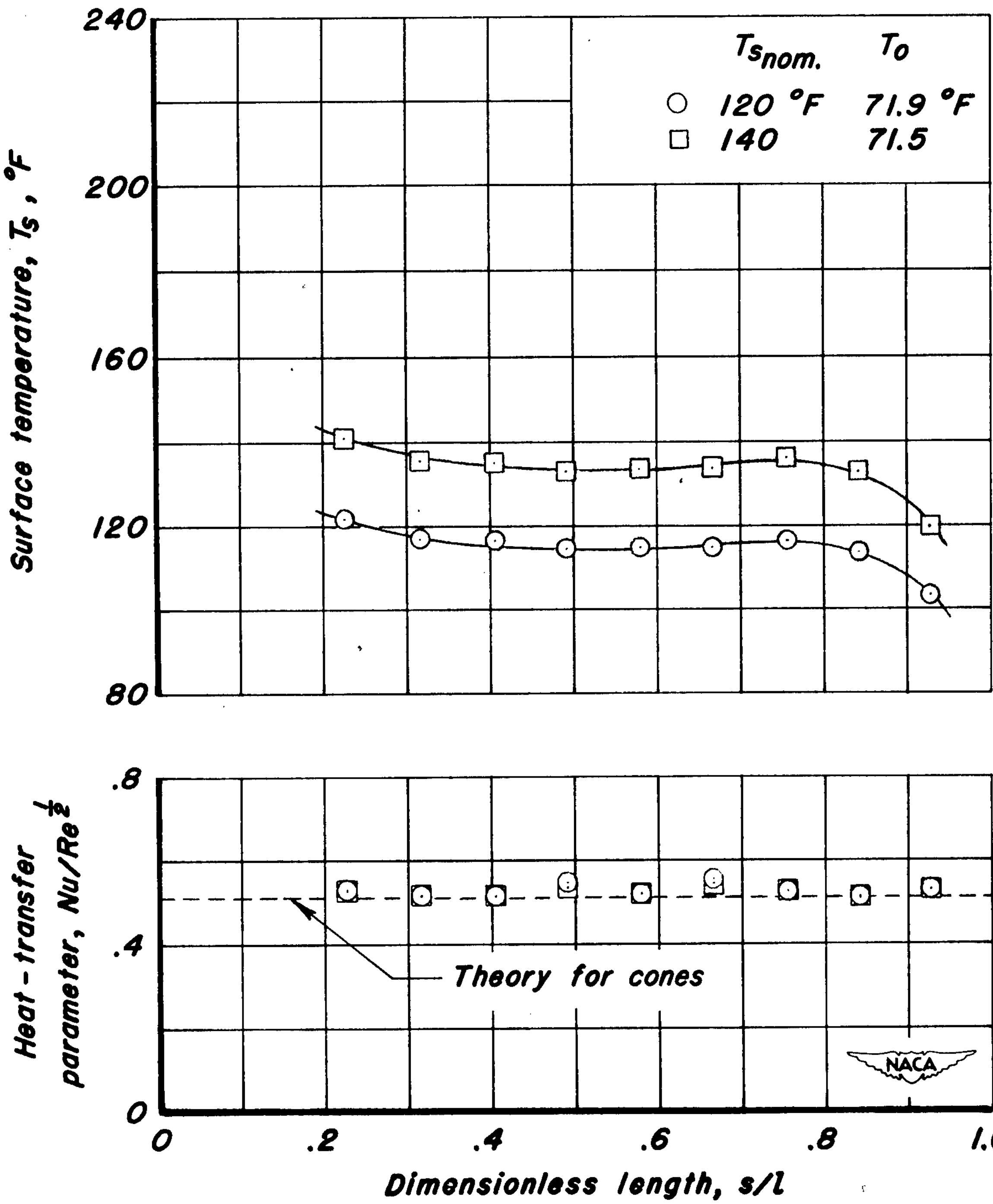
(a) H_o = 7.9 psia.

Figure 5.- Heat-transfer characteristics of the parabolic body at M_o = 2.18.



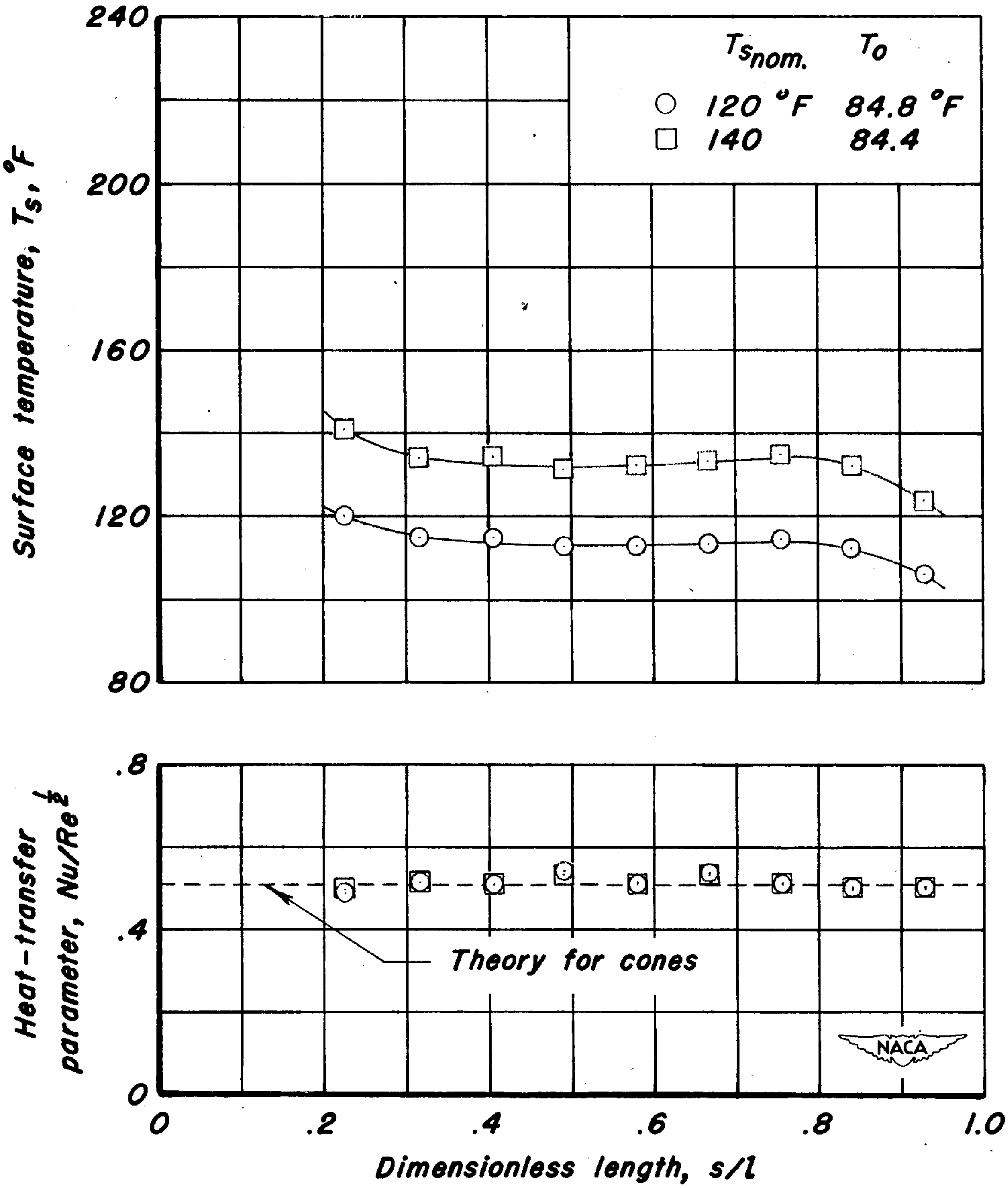
(b) H_0 = 15.0 psia.

Figure 5. - Concluded.



(a) $H_0 = 5.8$ psia.

Figure 6.- Heat-transfer characteristics of the parabolic body with the modified temperature distribution at $M_0 = 1.49$.



(b) H_o= 11.9 psia.

Figure 6. – Concluded.

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