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15. SUBJECT TERMS

Atmospheric Turbulence, Stratified and Rotating Fluid Flows, Phase Turbulence, Ageostrophic Turbulence

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FINAL REPORT

FUNDAMENTAL THEORETICAL AND NUMERICAL ISSUES OF PHASE TURBULENCE IN GEOPHYSICAL FLOWS

AFOSR GRANT F49620-96-1-0165

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1. Synopsis

This project focuses on turbulence in stratified fluids, with applications to laser-beam propagation in the stratosphere and upper troposphere. Such middle atmospheric layers are characterized by the presence of high shear (e.g. jet stream and thermal winds) and stable density stratification (due to the thermal structure of the atmosphere). Asymptotic regimes of atmospheric flow dynamics are described for different Burger number limits. Rotating 3D Euler-Boussinesq equations are analyzed in the asymptotic limit of strong stratification in the Burger number of order one situation as well as in the asymptotic regime of strong stratification and weak rotation. Substantial progress has been achieved in unraveling the physics and mathematics of strong nonlinear interactions involving three-wave resonances between inertio-gravity waves and vortical (potential vorticity) modes. Applications to optical (scintillation) turbulence of relevance to the Air Force Air Borne Laser (ABL) Program are investigated.

2. Accomplishments

This project falls within the realm of the USAF mission of developing air defense systems, a component of which is the ongoing Airborne Laser (ABL) Program. The recent congressional mandate to develop an airborne missile defense system for the US will bolster the ABL program, since laser technology is a viable tool for such an effort. One of the major challenges of the ABL program is the development of laser-beam propagation codes. Such codes must account for the beam propagation through an extended turbulent medium, consisting of the bulk of the troposphere and stratosphere. From the operational perspective (Tactical Decision Aid support), rigorous modeling of the refractive index structure function for long horizontal or nearly horizontal paths under high scintillation conditions characteristic of the atmosphere is imperative. In spite of demonstrated importance of stratification on atmospheric optical turbulence, currently available ABL phase screen theories are hinged upon isotropic Kolmogorov spectrum (Rytov's theory), and hence they cannot represent large amplitude fluctuations of atmospheric stratified turbulence. In the context of such turbulence the collusion between the stratification and shear leads to many intriguing phenomena such as the formation of thin, elongated turbulent layers (pancakes) and instabilities (such as Kelvin-Helmholtz (K-H) billowing) that ultimately break down into turbulence. The turbulence so generated is often patchy and temporally intermittent, characterized by strong anisotropy. It produces strong optical scintillation due to refractive index fluctuations, which needs to be quantified accurately in developing advanced beam control concepts for atmospheric laser-beam propagation.

From the underlying fluid dynamics perspective, we have described asymptotic regimes of geophysical dynamics for different Burger number limits. Rotating Boussinesq equations are analyzed in the asymptotic limit of strong stratification in the Burger number of order one situation as well as in the asymptotic regime of strong stratification and weak rotation. It is shown that in both regimes the horizontally averaged buoyancy variable is an adiabatic invariant (approximate conservation law) for the full Boussinesq system. Spectral phase shift corrections to the buoyancy time scale associated with vertical shearing of this invariant are deduced. Statistical dephasing effects induced by turbulent processes on inertial-gravity waves are evidenced. The "split" of the energy transfer of the vortical and the wave components is established in the Craya-Herring cyclic basis. As the Burger number increases from zero to infinity, we demonstrate gradual unfreezing of energy cascades for ageostrophic dynamics. This property is related to the nonlinear geostrophic adjustment mechanism which is the capacity of ageostrophic dynamics to transfer energy to small scales. The energy spectrum and the anisotropic spectral eddy viscosity are deduced with an explicit dependence on the anisotropic rotation/stratification time scale which depends on the vertical aspect ratio parameter. Intermediate asymptotic regime corresponding to strong stratification and weak rotation is analyzed where the effects of weak rotation are accounted for by an asymptotic expansion with full control (saturation) of vertical shearing. The regularizing effect of weak rotation differs from regularizations based on vertical viscosity. Two scalar prognostic equations for ageostrophic components (divergent velocity potential and geostrophic departure) are obtained.

Regimes of geophysical dynamics presenting the global picture for small Froude (Fr) or Rossby (Ro) numbers are shown in Figure 1 which summarizes the physical implications of our mathematical analysis. The rotation-dominated case corresponding to $\mathrm{Ro} \to 0, \mathrm{Bu} \to 0$, and Fr finite was considered in Babin, Mahalov, Nicolaenko (henceforth abbreviated as BMN) (1995, 1997a,b,c), Zhou (1995), and Mahalov and Zhou (1996) (Figure 1, vertical axis). In this case we proved the generalized Taylor-Proudman theorem establishing splitting between vertical averages of the velocity $\mathbf{U}$, density ρ_1 (two-dimensional–four-component (2D-4C) barotropic fields, and reduced ageostrophic field. Following Reynolds and Kassinos (1996) 2D-4C refers to fields with four components depending on two variables x_1 and x_2; 3D-2C refers to fields with two components depending on three variables x_1, x_2 and x_3, etc. By splitting we mean that the barotropic field decouples from the ageostrophic one, without feedback from the latter onto the former, to the lowest order. In the limit of the Brunt-Vaisälä frequency of gravity waves $N \to 0$, the usual quasi-geostrophic field reduces to vertically averaged fields; this is what is meant by "geostrophic" in this limiting context. In this limit vertical shearing is fully controlled (bounded) which is reflected in adiabatic invariants associated with vertical shearing (exact conservation laws in reduced equations).

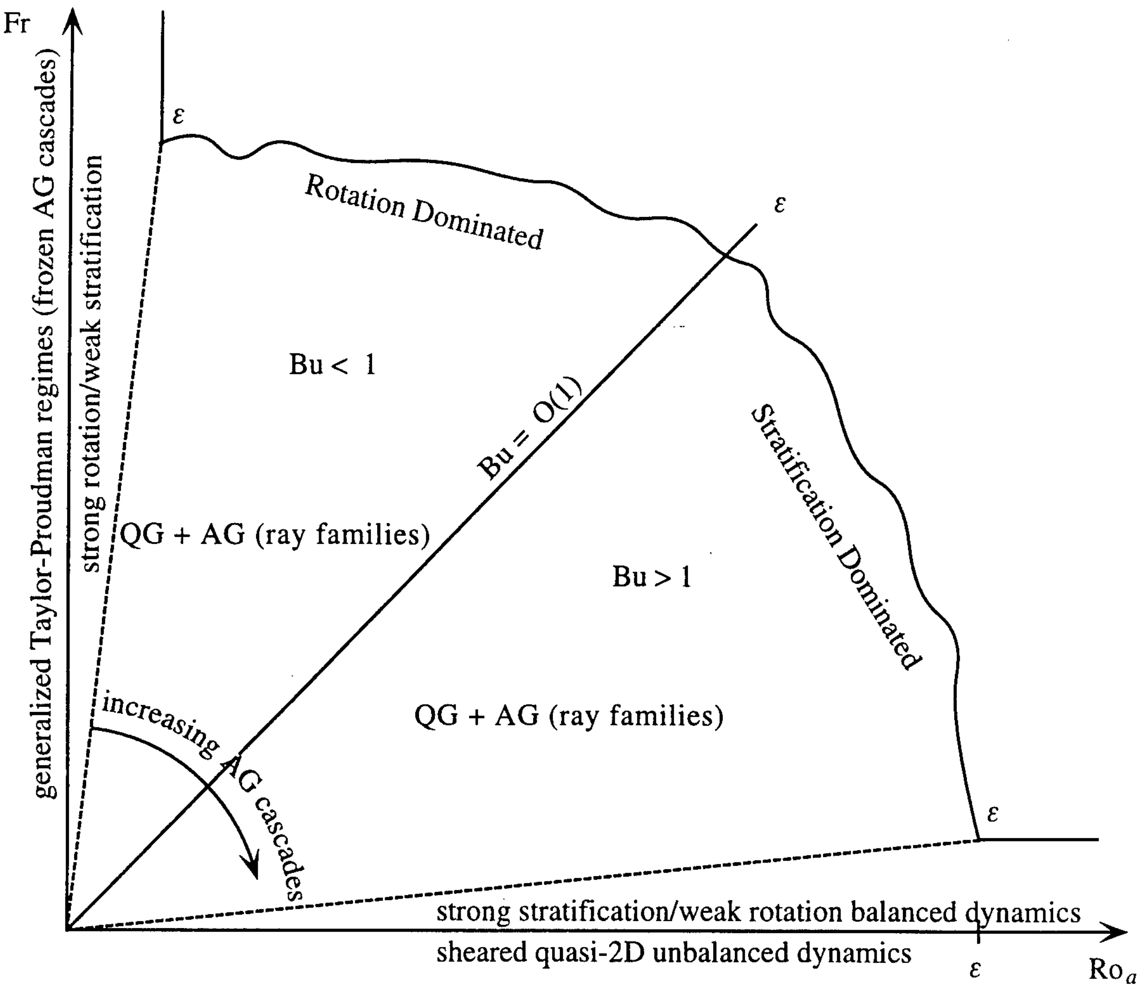

Fig. 1. Geophysical Dynamics: the global picture for small Froude or small Rossby regimes.

In the limit $N \to 0$ the temperature decouples from the dynamics and behaves as a passive scalar. The dynamics of vertically averaged velocity fields reduce to classic 2D-3C Euler equations and are subject to inverse energy cascades in U^1, U^2 as in 2D turbulence. For Bu $\to$ 0 we have shown rigorously that energy cascades for the ageostrophic (AG) field are completely frozen in the vertical direction x_3 and the AG dynamics in pure *phase turbulence* (BMN, 1996a, 1997a,b,c); freezing of energy cascades in x_3 for the "baroclinic" component follows. In pure phase turbulence, the amplitudes of the AG modes remain constant in absolute values; turbulent dynamics are restricted to the phases of the AG modes. The AG field is phase locked to phases associated with vertically averaged vertical vorticity and vertical velocity which are advected by 2D turbulence of vertically averaged fields (BMN 1996a, 1997b). There is no slaving of the amplitudes of AG modes by the 2D turbulence, only phase locking. We calculated Doppler phase shifts induced by turbulence of vortical (vertically averaged) fields on inertio-gravity waves in this limit. In the case of 3D rotating Euler in the small anisotropic Rossby number situation we described regimes with no vertical energy flux in the AG component and formation of KAM-type regimes with frozen in x_3 AG cascades (pure phase turbulence, frozen turbulence). Similar freezing of energy cascades was observed by Farge (1998) in the context of rotating shallow-water equations and in Pushkarev and Zakharov (1996) in numerical experiments describing turbulence of capillary waves.

Next is the regime of strong rotation and weak stratification as shown in Figure 1. Besides the operation of vertical averaging there is a piece of 3DQG (three-dimensional quasi-geostrophic) (Pedlosky, 1987) which plays an important role in the dynamics. It is formally obtained by expanding 3DQG equations in a small parameter N/f, N, Brunt-Vaisälä frequency of gravity waves, f rotation frequency. This procedure is similar to the one for the regime of strong stratification and weak rotation. The corresponding reduced equations, higher-order corrections, and mathematical convergence results for the case of balanced and unbalanced initial data are presented in Avrin, Babin, Mahalov, and Nicolaenko (henceforth ABMN) (1997).

As the effects of stratification are increased further (see Figure 1) AG cascades become possible. In the limit of strong rotation and strong stratification corresponding to $\mathrm{Ro} \to 0, \mathrm{Fr} \to 0$, but $\mathrm{Bu} = O(1)$ we established splitting between 3DQG and the reduced AG field (BMN, 1996b) confirming the similar splitting for rotating shallow-water equations which we obtained in Mahalov and Marcus (1995) and for the rotation-dominated case in BMN (1995, 1996a). Again, by splitting we mean that the QG field decouples from the AG one. Dynamics of AG further splits along uncoupled resonant quadruplets of Fourier rays with AG energy conserved on each resonant quadruplet of rays. Energy cascades are now allowed (unfrozen) for the AG field but they are restricted to families of four *rays* in Fourier space. Direct energy cascades of the AG field provide the mechanism for *nonlinear geostrophic adjustment*. This is fundamentally different from the rotation-dominated regimes where AG cascades are frozen. The nonlinear geostrophic adjustment mechanism is indeed the capacity of the AG dynamics to transfer to smaller scales and eventually dissipate its inertio-gravitational energy (Sadourny, 1975). As shown by Farge and Sadourny (1989) in the context of rotating shallow-water equations, rotation inhibits nonlinear transfers and confines the inertio-gravity waves to scales larger than the Rossby deformation radius; therefore, geostrophic adjustment is possible only for scales smaller than the Rossby deformation radius.

Spectral differential molecular viscosities for QG and AG fields with explicit dependence on the rotation/stratification parameters behave differently. Let ν_1 and ν_2 be the kinematic viscosity and the heat conductivity, respectively; the ratio ν_1/ν_2 is known as the Prandtl number. Through a simple computation of two-wave resonances in the Craya-Herring basis (Lesieur, 1987) the effective differential spectral molecular viscosities ν_{QG} and ν_{AG} are obtained. It shows that dissipation affects QG and AG fields differently. This impacts on direct numerical simulations of QG fields in the context of numerical simulations of atmospheric flows.

Partial control of vertical shearing is obtained allowing us to prove long time existence of solutions of inviscid Boussinesq equations (BMN (1996b)). Also, a flow which is initially wave dominated remains wave dominated even through decay (confirming Métais *et al.* (1996)). We show that the horizontally averaged buoyancy variable $\bar{\rho}_1$ is an adiabatic invariant (approximate conservation law) for the full Boussinesq system. This result is true for all resonances including three-wave resonances. We calculate Doppler phase corrections associated with this invariant to a linear profile (constant N_0). This confirms and generalizes the work of Legras (1980) on phase shifts who showed the existence of statistical dephasing effects induced by turbulent processes on Rossby waves. Frequency shifts induced by turbulent processes on inertial waves were caluclated in BMN (1996a); the case of frequency shifts induced on waves in rotating shallow-water equations was considered in BMN (1997a,c).

As the effects of stratification are increased (see Figure 1) vertical shearing dynamics in the AG field increase and are conveniently characterized using the divergent velocity potential which is coupled to geostrophic departure. The geostrophic departure characterizes imbalance in the vertical motion or omega equation. Up to normalization, the divergent velocity potential χ, the geostrophic departure and 3DQG mode form the Craya-Herring cyclic basis which is used in our work to represent physical fields.

In the inviscid regime $\mathrm{Fr} \to 0, \mathrm{Ro} = O(1), \mathrm{Bu} \to +\infty$ (Figure 1, horizontal axis) there is no bound on vertical shearing associated with the dynamics of 3D-2C decoupled pancakes (parametrized in x_3) with different pressures at every level; this leads to unbalanced dynamics at the lowest order. There is no saturation of the exponential build-up of vertical enstrophy (in small vertical scales) for AG dynamics as the latter is coupled to the quasi-2D field. The major problem is lack of boundedness of vertical shearing in quasi-2D (Q2D) equations (Lilly, 1983). We show that horizontally averaged $\bar{U}^1(x_3), \bar{U}^2(x_3), \bar{\rho}_1(x_3)$ are adiabatic invariants providing a feedback onto AG turbulence. $\bar{U}^1, \bar{U}^2$ are adiabatic invariants only if $\Omega = 0$; otherwise, they undergo rigid Ω-rotation (this result holds for all resonances including the three-wave resonances). However, these adiabatic invariants are not enough to saturate vertical shearing. Worse the lack of boundedness of $\partial U^1_{\mathrm{Q2D}}/\partial x_3, \partial U^2_{\mathrm{Q2D}}/\partial x_3$ leads to explosive exponential growth of the AG dynamics. Of course, control of vertical shearing can be achieved trivially by introducing vertical viscosity; however, this corresponds to a nonphysical laboratory set-up rather than the real atmosphere.

In the intermediate asymptotic regime corresponding to strong stratification and weak rotation ($\mathrm{Bu} \to +\infty, f/N$ small) the effects of weak rotation on the dynamics are accounted for by an asymptotic expansion. Full saturation of vertical shearing is obtained for all times. Equations describing balanced dynamics are intermediate between 3DQG equations valid in the regime $\mathrm{Bu} = O(1)$ and quasi-2D decoupled pancakes without any control of vertical shearing (Lilly, 1983). In our work we show how weak rotation regularizes vertical shearing and calculate its effects on AG dynamics. Our reduced equations have a conservation law associated with vertical shearing which allows us to control AG vertical scales for all times. There is no need to resort to vertical viscosity as the principal stabilizing mechanism (Reynolds number $\mathrm{Re} \sim 10^{12}$ in atmospheric flows). Two scalar prognostic equations for AG components (divergence velocity potential and geostrophic departure) are obtrained. These equations have coefficients depending on regularized quasi-2D fields and can be used for more accurate and robust numerical simulations of geophysical flows in the regime of strong stratification and weak rotation. AG dynamics is driven by regularized vertical shearing of the pancakes.

In our work we emphasize physical predictions which follow from rigorous mathematical analysis of Boussinesq equations in the strongly rotating/stratified Bu $= O(1)$ regime as well as in the asymptotic regime of strong stratification and weak rotation. The mathematical theory is based on rigorous small divisor estimates and KAM-type theoretical considerations to control wave resonances rigorously, especially the three waves of the fast-fast-fast resonances.

On the physical side, for the Bu $= O(1)$ regimes analyzed we establish statistical dephasing effects induced by turbulent processes on inertio-gravity waves with 3DQG turbulence acting to renormalize both frequency and viscosity of the waves. We generalize the work of Legras (1980) and Carnevale and Martin (1982). In particular, we calculate Doppler phase corrections associated with the fact that forizontally averaged buoyancy variable $\bar{\rho}_1$ is an adiabatic invariant. Namely, $\bar{\rho}_1(x_3)$ has an $O(\epsilon)$ variation on large times when Ro $\sim$ Fr $\sim \epsilon$. Rigorous mathematical analysis based on small divisor estimates shows that three-wave fast-fast-fast resonances are rare in the Bu $= O(1)$ regimes (BMN (1996b)) as well as in Bu $\ll 1$ regimes (BMN, 1996a). In fact, just switching on even weak rotation destroys the three-wave resonances found in the pure stratified case $f = 0$. One of the hardest parts of our analysis is to estimate the total probabilities of quasi-resonances, that is the width of Arnold tongues coming out of points (set of measure zero) where three-wave resonances are possible. These resonances are not neglected but rather weights are assigned to them according to their importance (BMN 1996b, 1997b)). Even three-wave resonances do not alter the global picture: they correspond to higher-order corrections ("Arnold drift").

In BMN (1996b) we established splitting between 3DQG and reduced AG field; here we show that the structure of reduced AG equations (via Craya-Herring cyclic basis) implies upscale (inverse) energy transfer of rotational (vortical) energy via QG mode versus downscale cascade of wave energy via slow-fast-fast resonant catalytic interactions (following Bartello, (1995) notaitons). In cases Bu $= O(1)$ and Bu $\ll 1$, the AG field satisfies uncoupled families of equations on four rays in Fourier space. As the effects of stratification are increased direct cascade of AG energy (along these rays) toward small scales is enhanced. In our work we introduce the *anisotropic phase coherence tensor* and model anisotropy in Bu $= O(1)$ regimes of geophysical turbulence. There is a spectral gap (i.e., a power law scaling break) between the QG and AG spectrum with the AG spectrum being shallower than the typical k^{-3} QG spectrum; it varies smoothly between k^{-2} and $k^{-5/3}$ which is in agreement with numerical simulations of Métais *et al.* (1996).

A possible emerging picture of Burger $O(1)$ turbulence is that 3DQG turbulence being a *guiding center* is corrupted by phase turbulence and Doppler phase shifts; with the dynamics of the AG field being constrained to uncoupled four-ray families, with direct AG cascade restricted to the latter. The feedback of the AG field on QG dynamics can be computed at next order in Fr or Ro (ABMN, 1997).

Pancakes Front Dynamics

Among the most important factors impacting atmospheric dynamics are rotation, stable stratification, and mean velocity shear. A manifestation of these influences is that flows become anisotropic both in length scales (horizontal and vertical) and the velocity vector field itself (McWilliams *et al.*, 1994). The combined effects of these influences lead to the formation of coherent structures. In buoyancy-driven low-level planetary boundary layers, the dominant coherent structures are buoyant plumes, which are seen as cumulus clouds in the atmosphere and, more indirectly, as convective remnants, called chimneys, in the ocean. The tropopause acts as a rigid lid to these buoyant plumes which start organizing in "pancakes" with a readjustment of their horizontal and vertical scales. Chimneys, plumes, and pancakes are appreciably influenced by Earth's rotation, so they end up, through a process called geostrophic adjustment, with a geostrophically balanced, horizontally circulating flow. In the stably stratified environment of the tropopause and lower stratosphere, pancakes evolve by adjusting their scales to reach a balanced state of Burger number $O(1)$. This is paradoxical as airborne measurements of local Richardson numbers at the tropopause lid and the troposphere demonstrate values centered at $\mathrm{Ri} \sim 0.15$ and below the critical $\mathrm{Ri} = 0.25$ (Beland, 1996). Shrong vertical shearing instabilities generate strong Kelvin-Helmholtz waves in the tropopause and lower stratosphere. The nonlinear adjustment of pancake scales through even weak rotation balances such waves and ultimately yields near-balanced horizontally circulating systems. It is the goal of our work to shed some light on the exact genesis of these mechanisms, through strongly nonlinear wave/vortex interaction theory. The dynamics of upper-level frontal systems are known to comprise the interactions between the primary (geostrophic) and secondary (ageostrophic) circulations (Keyser and Shapiro, 1986).

The importance of even weak rotation on mesoscale flows and its fundamental role in the scale adjustment process is emphasized in Newley, Pearson, and Hunt (henceforth NPH) (1991) and Rotunno (1983). This is a singular perturbation problem where weak Coriolis accelerations may have large effects on long horizontal scales. The latter are coupled to vertical scales by rotation. Such singular perturbation effects cannot be treated by conventional power series expansions in small Froude/Rossby numbers as emphasized in NPH (1991).

The main mathematical problem that we have investigated is the nonlinear adjustment of strongly stratified turbulence to geostropic turbulence (asymptotic regime of strong stratification and weak rotation). For intermediate scales of motion rotation is present but not dominant, so that the Rossby number is neither very large nor very small. Such systems have *not* been investigated by turbulence closure models. In the asymptotic regime of strong stratification and weak rotation (no hydrostatic assumption) we show how switching on weak rotation triggers frontogenesis. Vertical slanting of these fronts is proportional to $\sqrt{\mu}$ where μ is the ratio of Coriolis rotation and Brunt-Väisälä (frequency of gravity waves) parameters. These slow fronts have frequency proportional to $\sqrt{\mu}$, select the slowest baroclinic waves through adjustment of horizontal scale to vertical scale through rotation, and are the envelope of inertio-gravity waves. The fronts effectively balance the frequencies of baroclinic waves uniformly to $O(\sqrt{\mu})$. This frontogenesis yields vertical "glueing" of pancake dynamics by weak rotation. *The mechanism of its formation is contraction in horizontal dimension balanced by vertical stretching.* From the perspective of Lilly's pancake dynamics (Lilly 1983), the fronts appear as vertically curved sharp pancake edges that "slow-down" the unrestrained horizontal propagation: this is effectively vertical glueing of the pancakes by rotation. The vertical and horizontal scales of the pancakes readjust, leading to an effective decrease of the Burger number from $\mathrm{Bu} \gg 1$ to $\mathrm{Bu} = O(1)$. We predict a definite change in the nature of ageostrophic dynamics under the impact of weak rotation in this problem. The wave fronts that we construct saturate local Kelvin-Helmholtz instabilities (for the local fluctuating ageostrophic field) with small Richardson numbers (strong shearing) through adjustment of vertical and horizontal scales on the slanted fronts.

This agrees with the conclusions of Hoskins and Bretherton (1972) and Hoskins (1982) that the vertical deformation field is crucial in the dynamics of frontal systems, as it balances strong horizontal density gradients. In their study of atmospheric frontogenesis models smaller-scale ageostrophic motions embedded in the baroclinic flow lead to the rapid formation of a front. It has been recognized that the evolution of baroclinic waves provides the dynamical environment for upper-level frontogenesis at the tropopause and lower stratosphere (Keyser and Shapiro, 1986).

In conclusion, in the asymptotic regime of strong stratification and weak rotation (no hydrostatic assumption) switching on weak rotation triggers frontogenesis. Mathematically, this is generated by asymptotic hyperbolic systems describing the strong nonlinear interactions between waves and potential vorticity dynamics. These slow hyperbolic wave fronts are nearly standing waves with frequency proportional to $\sqrt{\mu}$; they select the slowest baroclinic waves through nonlinear adjustment of the horizontal scale to the vertical scale through rotation. The mechanism of its formation is contraction in the horizontal dimension balanced by vertical stretching. From the perspective of Lilly's pancake dynamics (Lilly, 1983), the fronts appear as vertically curved sharp pancake edges that "slow down" the unrestrained horizontal propagation: this is effectively vertical gluing of the pancakes by rotation. The vertical and horizontal scales of the pancakes readjust, leading to an effective decrease of the Burger number from $\mathrm{Bu} \gg 1$ to $\mathrm{Bu} = O(1)$. We predict a definite change in the nature of AG dynamics under the impact of weak rotation in this problem. The wave fronts that we construct saturate local Kelvin-Helmholtz instabilities (for the local fluctuating AG field) with small Richardson numbers (strong shearing) through adjustment of the vertical and horizontal scales on the slanted fronts.

Our asymptotic theory quantitatively describes anisotropy in the AG wave turbulence with strong energy cascades along the front direction: this can be checked against experimental measurements provided that the latter distinguish between wave turbulence and the ambient potential vorticity turbulence; it could lead to effective anisotropic corrections for modeling atmospheric turbulence in the tropopause and the lower stratosphere.

That the impact of rotation triggers mechanisms which allow an internal adjustment of horizontal scales has already been demonstrated in Rotunno (1983) and NPH (1991), in a linear theory. Internal radii of deformation determine the horizontal extent of motion and circulation; they are not resolved accurately by the usual numerical models with coarse gridding in the vertical direction. Current numerical models smear out sharp vertical gradients especially with *ad hoc* vertical eddy viscosities. Paradoxically, they should be benchmarked against our exact asymptotic dynamics to gauge for resolution of vertical stiffness. With realistic potential vorticity configurations, the direct numerical simulations of our *nonstiff* nonlinear asymptotic limit equations for the wave fronts should be compared with actual experiments on pancake dynamics (the latter unconstrained by boundaries and beyond small Reynolds numbers).

Phenomenological Turbulence Modeling at the Asymptotic Limit of Strong Rotation/Stratification in the Burger Number of Order One Situation

At the asymptotic limit of strong rotation/stratification, the existence of two disparate time scales indicates that a phenomenological analysis similar to that of rotation (Zhou, 1995; Mahalov and Zhou, 1996) may be appropriate. The aim of this approach is to estimate the averaged effect of rotation/stratification on trubulent energy transfer. The introduction of the anisotropic time scale based on the aspect ratio parameter in the energy spectrum is an improvement over our previous phenomenological analysis of rotating turbulence. In the context of the QG equations for a Boussinesq fluid in a uniformly rotating and stably stratified environment, McWilliams *et al.* (1994) showed that thier solutions exhibit significant anisotropy associated with the emergency of many long-lived coherent vortices that control the flow evolution. Anisotropy of the QG field impacts on the AG gravity wave field.

Among more mundane immediate consequences, exact operator splitting of our reduced asymptotic equations reach the very roots of the mechanisms of wave-vortex interactions. We designate by $\langle\ \rangle$ the ensemble averaging for any field $\mathbf{U}^\dagger$, and by $\mathbf{U}^\dagger_{\mathrm{F}} = \mathbf{U}^\dagger - \langle \mathbf{U}^\dagger \rangle$ the fluctuations. Then the Reynolds stress operator becomes

$$\langle \mathbf{W}_{\mathrm{QG,F}}, \mathbf{W}_{\mathrm{QG,F}} \rangle + \langle \mathbf{W}_{\mathrm{AG,F}}, \mathbf{W}_{\mathrm{AG,F}} \rangle + 2 \langle \mathbf{W}_{\mathrm{QG,F}}, \mathbf{W}_{\mathrm{AG,F}} \rangle.$$

The last tensor we designate as the *anisotropic phase coherence tensor.* It correlates the fluctuations of the AG $\mathbf{W}_{\mathrm{AG}}$-field with the QG $\mathbf{W}_{\mathrm{QG}}$ field. The anisotropic phase coherence tensor is the key player in the control of rapid 3D pressure fluctuations. The dependence of the full Reynolds stress tensor on the intrinsic mean vorticity does not vanish in the limit of strongly rotating/stratified turbulence, as neither the field $\mathbf{W}_{\mathrm{AG}}$, nor the "anosotropic phase coherence tensor" vanish. Long-lived phase coherence is an important part of turbulence (Bartello and Holloway, 1991; Herring and McWilliams, 1985).

In order to infer the form of the inertial-range spectrum, it is necessary to estimate the magnitude for the triple correlations. In general, τ_3, the time scale for decay of triple correlations which is responsible for inducing turbulent spectral transfer, may depend on any relevant turbulence parameter. Because energy is conserved by the nonlinear interaction and a local cascade has been assumed, ϵ is independent of k. Local cascade also implies that ϵ is explicitly proportional to τ_3 and depends on the wave number and on the power of the omnidirectional energy spectrum. A simple dimensional analysis leads to

$$\epsilon = A^2 \tau_3(k) k^4 E^2(k),$$

where $E(k)$ is the spectral density of turbulent energy, ϵ the energy dissipation, and A is a constant.

In the strongly rotating/stratified case when both effects are of the same order (Burger number of order one situation), the time scale for $\tau_3(k)$, the decorrelation of the triple velocity product, is the controlling parameter to influence the energy transfer process. In a regime of high Reynolds numbers and low Rossby and Froude numbers, turbulence is characterized by two disparate time scales: a short anisotropic time scale associated with the rotation/stratification frequency $\tau_{\Omega N}$ and a nonlinear time scale. We find that a direct application of $\tau_3 = \tau_{\Omega N}$ resulted in the energy spectrum for turbulence subject to strong rotation/stratification:

$$E(k) = C_{\Omega N}(\tau_{\Omega N}^{-1}\epsilon)^{1/2}k^{-2}.$$

We now consider the number of the nondimensional parameters needed. For the turbulence in equilibrium the Rossby/Froude numbers are the only relevant parameter controlling the effects of rotation/stratification on the flow. However, for the nonequilibrium situation, a new nondimensional parameter such as $(\tau_{\Omega N}^{-1}t)$ is required. Here the dimensionality is considered by introducing the aspect ratio. Our analysis suggests that the energy transfer process in the limit of $(\tau_{\Omega N}^{-1}t) \to \infty$ and small spectral Rossby/Froude number (strong rotation/stratification) is as follows. There is inverse energy transfer by the 3DQG component (McWilliams *et al.*, 1994). In the meantime, there is also a direct energy cascade governed by the equation for the inertio-gravity wave component.

Following the usual assumption of EDQNM, we consider that the lifetime of triple correlations in rotating/stratified turbulence might be more accurately treated by taking into account the possibility that these correlations decay because of the influence of both wave propagation and nonlinear interactions. The simple choice

$$\frac{1}{\tau_3(k)} = \frac{1}{\tau_{\rm nl}(k)} + \frac{1}{\tau_{\Omega N}(k)}.$$

satisfies the appropriate limiting case: $\tau_3(k) \to \tau_{\rm nl}$ without rotation/stratification and $\tau_3(k) \to \tau_{\Omega N}$ with strong rotation/stratification. The introduction of the anisotropic time scale based on the aspect ratio parameter in the energy spectrum is an improvement over our previous phenomenological analysis (Zhou, 1995; Mahalov and Zhou, 1996) since now the model can distinguish the anisotropic nature of rotating/stratified flows.

3. Collaboration with Air Force Laboratories and the ABL Program

We are consulting with the AFRL/VSCB at Hanscom AFB staff on ABL programs. The focus is on the EGRETT measurement campaigns by Dr. Owen Cote in 1998 and 1999. The EGRETT plane with his large glider-like wing span and modest velocity can still reach altitudes of 15 km without disturbing the atmospheric field unlike fast jets and AWAC Boeings. Though very accurate calibrated measurements of both static and dynamic pressure fields as well as temperature field, the EGRETT is the only facility which can accurately measure both horizontal and vertical FLUXES. Unlike balloons, the measurement apparatus is shielded by the plane and not subject to many spurious effects. Jointly with Dr. O. Cote, we are analyzing EGRETT data on both horizontal and vertical velocity shears, horizontal temperature gradients, correlation between the temperature field and velocity fields. Our goal is to extract various length scales of turbulence and calibrate them against D.N.S. of similar atmospheric situations.

4. Personnel Supported

- Prof. Alex Mahalov, Department of Mathematics, Arizona State University

- Prof. Basil Nicolaenko, Department of Mathematics, Arizona State University
- Prof. H.J.S. Fernando, Center for Environmental Fluid Dynamics, Arizona State University
- Dr. Frank Tse, Post Doctoral Fellow, Arizona State University
- Prof. Anatoli Babin, Collaborator, University of California, Irvine
- Graduate Students:
  - Patrick Mulhall
  - Le Dung
  - Bongsik Kim
  - Tae-Chang Jo
  - Angela Sdric

5. Doctoral Theses Supervised

- Le Dung (May 1997): "Exponential Attractors of Dissipative Evolution Problems in Banach Spaces."
- Bong Sik Kim (in progress): "Asymptotics of Mathematical Models of Atmospheric Flows."
- Tae-Chang Jo (in progress): "Operator Splitting for Mathematical Models of Mesoscale Atmospheric Dynamics."

6. Publications and Bibliography

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2. D. A. Jones, A. Mahalov and B. Nicolaenko, A numerical study of a new operating splitting method on rotating fluid equations with large ageostrophic initial data, J. Theor. and Comp. Fluid Dyn., 13:143-159 (1999).
3. Le Dung and B. Nicolaenko, Exponential attractors for evolution problems in Banach spaces, accepted, J. Dynamics and Differential Equations.
4. J. Avrin, A. Babin, A. Mahalov and B. Nicolaenko, On the regularity of three-dimensional rotating Navier-Stokes equations, Applicable Analysis, 7, 197-214 (1999).
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6. B. Nicolaenko and Weijie Qian, Inertial manifolds for nonlinear viscoelasticity equations, Nonlinearity, 11: 1075-1093 (1998).
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14. P. Fabrie and B. Nicolaenko, Exponential attractors for non-dissipative systems modeling convection in porous media, Asymptotic Analysis, Vo. 12, 295-327 (1996).

7. Interactions/Transitions

A) Conferences and Workshops (selected)

1. Workshop on Euler and Navier-Stokes Equations, Ecole Normale Suprieure, Paris, France (May 1996).
2. European Dynamics Days, Lyon, France (Plenary Lecture) (July 1996).
3. Conference on Rotating and Stratified Turbulence, NCAR, Boulder (July 1996).
4. International Workshop on Low Order Models and Balanced Dynamics in Atmospheric and Ocean Geophysics, Newton Institute, Cambridge, U.K. (October 1996).
5. International Conference on Dynamical Systems Methods for Coherent Structures in Turbulence, U.C.S.B. (February 1997).
6. International Workshop on Mathematical Developments in Atmospheric and Ocean Dynamics, Newton Institute, Cambridge, U.K. (December 1997).
7. International Symposium on Mathematics and Computation in Geophysics, Warwick University, U.K. (September 1998).
8. Conference in Vortex Dynamics in Geophysical Flows, Castro Marina (Otrante, Italy), June 22-27, 1998: "Nonlinear Baroclinic Wave Dynamics and Pancake Dynamics in Atmospheric Flows."
9. Computational Mathematics Workshop on "Fundamental Issues in Simulating Atmospheric-Oceanic Flows," Univ. of Warwick (Coventry, U.K.), Sept. 20-27, 1998: "On the Asymptotic Regimes of Rotating Boussinesq Equations in Geophysics."
10. Special Session on Dynamical Systems, Regional A.M.S. meeting, Tucson, Nov. 13-15,1998: "Exponential Attractors in Banach Spaces" (invited presentation).
11. Atmospheric Physics and Laser Propagation Workshop, AFRL-Kirtland AFB, Feb. 2-

3, 1999: "Theoretical and Numerical Issues of Pancake Dynamics and Turbulence in the Upper Atmosphere."

12. SIAM Dynamics Conference, Snowbird (Utah), May 24-27, 1999: "Regularity of Strongly Stratified and Rotating 3D Boussinesq-Navier-Stokes Equations."
13. European Geophysics Conference, The Hague (Netherlands), April 19-24, 1999: "On the Asymptotic Regimes and Pancake Dynamics for the Primitive Equations of Atmospheric Dynamics."

B) Consultative Functions of Air Force Research Laboratories

See Section 3 for direct involvement with ABL Programs.