

DESIGN OF HIGH PERFORMANCE CONTROLLERS

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FINAL REPORT

1 Accomplishments

The various lines of progress were:

1.1 Nonlinear Plants

Over the last decade, the extension of H^∞ control to a nonlinear system has been converted to solving two particular PDE's. One partial differential inequality, P.D. I., *the state feedback HJBI*, corresponds to the $CTRL^\infty$ problem where the controller can access the full state of the plant. Indeed solving this PDE gives an optimal solution to the state feedback $CTRL^\infty$ problem and can be solved off line. The second PDE, the *information state PDE*, gives the dynamics of the controller and so it must be solved ON LINE.

Unfortunately, these are P.D.E's on the state space of the original plant (often a high dimensional space) so a numerical solution faces what is called the curse of dimensionality. Getting around this is the main challenge to the field and it requires basic theory as well as numerical analysis.

1.1.1 State Estimation

Collaborators and I have done considerable work on both of these equations. Most of the effort goes to **the information state PDE**, since at first glance, computation (on line) seems impossible in general situations with state-space higher than two. However, recent mathematical work with M. James and partly with Bill McEneaney and Peter Dower discovers that in various important cases the controller dynamics in practice might NOT suffer prohibitively from the curse of dimensionality. One point is that the information state is often supported on a 0,1, or 2 dimensional manifold \mathcal{M} and then the computation required to propagate it is manageable. Thus pinpointing such cases is important though mathematically challenging. Cases under current investigation are

(a) (with James) [HJ99book,prep]

Mixed sensitivity= 2-block, with $\dim \mathcal{M}$ is the antistable manifold of the plant.

(b) (with James, McEneaney) [cdc98]

Measure all but k states directly (cheap sensor case) Example:

Contents

1	Nonlinear Plants	3
1.1	State Estimation	3
1.2	State Feedback	4
2	Symbolic Algebra for Systems Research	4
3	H^∞ Design and Matrix Inequalities	6
3.1	H^∞ Design	6
3.2	Bi H^∞ control or Frequency Response Scheduling	7
3.3	Matrix Inequalities	7
4	Technology and Interactions 1999	8
4.1	Meetings	8
5	Personnel Supported	10
5.1	Consultative and advisory	10
5.2	Transitions	10
6	Publications	11
7	Patents	13

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1. Missile with no air probes $k =$ number of missing airprobes. $\dim \mathcal{M} = k$
2. Compressor $\dim \mathcal{M} = 1$

If # "noise free sensors" $<$ # disturbances, then $\dim \mathcal{M} = 0$.

This work has spread over several years with the main additions from the last year giving reassuring estimates demonstrating how small amounts of noise do not radically change results.

- (c) (with Dower, James) [DHJacc99] Linear systems with non-linear actuators- We examine an obvious architecture for controllers: just feed the actuated control signal into the state estimator (rather than the control signal). This removes the direct effect called "windup". If the noise level is low, then the information state equation reduces to an ODE, so it is solvable. This is a $\dim \mathcal{M} = 0$ situation. Even if there is substantial noise, our theory of nonlinear control is broad enough to prove in many cases that this construction is the best that can be done.

The practical interpretation of the constructions above in (a) and (b) are that they tell us where to put grid points in numerical computation. Ideally all grid points go on the (small) manifold \mathcal{M} . This is risky. In practice one should place some points near \mathcal{M} and a few far from \mathcal{M} . The appropriate mathematics for determining placement of grid points off of \mathcal{M} appears to be ϵ perturbations of solutions to the information state equation. This falls into the framework of singular perturbation theory, and is the next generation of work along these lines.

1.1.2 State Feedback

Our work on the **state feedback HJB equations** focuses on solving them numerically. Work is with Michael Hardt-Ken Kreutz Delgado, ECE Dept. These are old equations and our approach is to take the most refined software publically available (which originally was developed for path planning) and convert it for our purposes. There was gratifying success with the biped walking problem below. To set it in context, we mention the compressor problem completed within the last year though focused on the year before.

- (a) *Moore Greitzer Compressor Stall Model*: This has a 3 dimensional statespace with saturation constraints, and our technique seems to work fine. Ultimately, it produces an explicit 26 parameter (not so big) neural net control law. The technique should extend to 5 dimensions with little modification.
- (b) *Classical Biped Walking Problem- full 5 link model*: This is a horrendous path planning as well as modeling problem. The state space is 14 dimensional with two "phases". One foot on ground imposes 2-algebraic constraints, two feet on the ground imposes 4-algebraic constraints. There is saturation of actuators and states. Finding an energy minimizing motion amounts to solving a HJB at one point in state space. Work in the literature is with vastly compromised models, so we think our solution to this is a considerable advance.

Item (b) gives further evidence of the soft underbelly of the Bellman Equation. Given a single point, even in a 10 dimensional space, one can compute the value function at that

point. (Note this is in gross distinction to elliptic equations). The main difficulty is less in solving the Bellman equation, than it is in determining where you want to solve it and in simplifying data on an irregular grid in high dimensional space.

1.2 Symbolic Algebra for Systems Research

We are the providers to Mathematica of general Noncommuting algebra capability. We have a major computer package, NCAgebra, which is a collection of "functions" for Mathematica designed to facilitate manipulation and reduction of noncommutative algebraic expressions in operator theory and engineering systems. Noncommutative algebra is a cornerstone of matrix theory and, consequently, of research in linear engineering systems. It also occurs in many other fields.

Our work has various phases which run from bread and butter to ambitious research. **NCAIgebra:** The basic commands of Mathematica, Maple etc do NOT apply to noncommuting expressions. Our package NCAgebra does basics of noncommutative algebra in Mathematica. **Simplifying expressions. Eliminating unknowns from collections of equations:** This, the focus of current efforts, is the strict noncommuting generalization of the commutative Solve and Eliminate commands of Mathematica or Maple. It uses our NCGB code plus algorithms of our own construction for eliminating redundant equations.

A major focus of research is using and extending these algorithms to help an engineer "discover" formulas which are critical to his particular problem. This is ambitious but the stakes are high. We do many experiments. We are steadily building a collection of classic theorems and formulas which can be "discovered semi-automatically"; this is how we judge our methods. Also we make new discoveries.

Our work indicates that our software is useful in the area of singular perturbations. There one gets truly long and unpleasant formulas which a human manipulates slowly and with uncertain accuracy. Our computer methods are accurate and our NCGB plus cleaning plus sorting algorithms could well save the user considerable amounts of time. In classic cases our software produces the classical equations. In other problems being studied with McEneaney (see above) we find this symbolic software a valuable tool.

My student D. Kronewitter just determined the solution structure of a class of 3 x 3 matrix completion problems. There were 31,000 cases modulo about 500 permutations which he solved automatically.

Our NCGB software is a long complicated C++, Mathematica, TeX code and we only support it on Solaris platforms. Within the last few months, the C++ GB engine has been heavily modified (modularized) and runs under Linux and Windows. Thus a stand alone C++ version on these platforms is in sight. Testing is in progress as is linking to Mathematica.

Recently we (our group and Bakshee at WRI) found a way to give Mathematica's control package (by Bakshee) noncommuting capability. We can load Bakshee's Mathematica toolbox, NCAgebra, and our new Mathematica file file.m, then the system connection laws portion of the toolbox works with noncommuting A,B,C,D 's. Currently we are extending the file.m to make those parts of the toolbox (where meaningful) work noncommutatively.

This should lead to a good front end for use by engineers who wish to conveniently manipulate many noncommutative linear system formulas on the computer, much as they do by hand.

1.3 H^∞ Design and Matrix Inequalities

1.3.1 H^∞ Design

This concerns a basic question of worst case frequency domain design where stability of the system is the key constraint. This is the H^∞ optimization problem which is crucial in several branches of engineering.

The fundamental H^∞ problem of control. (with O. Merino and T. Walker)

First we state the core mathematics problem graphically. At each frequency ω we are given a set $S_\omega(c) \subset C^N$. The objective is to find a function T with no poles in the R.H.P. so that each $T(j\omega)$ belongs to $S_\omega(c)$. In fact there is a simple picture to think of in connection with a design

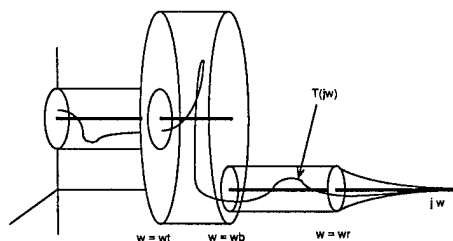


Figure 2

Typically there is a nested family of target sets $S_\omega(c)$ parameterized by a performance level c ; the smaller the sets the better the performance. For the *optimal* c a solution T exists but no solution exists for tighter specs.

The Horowitz templates of control can be transformed into this type of picture. When each $S_\omega(c)$ is a "disk" this problem is solved by traditional H^∞ control. This graphical problem can be formulated analytically in terms of a performance function Γ as

(OPT) Given a positive valued function Γ on $R \times C^N$ (which is a performance measure),
find $\gamma^* \geq 0$ and f^* in A_N which solve

$$\gamma^* = \inf_{f \in A_N} \sup_{\omega} \Gamma(\omega, f(j\omega)).$$

and this of course is what one puts in a computer. Here A_N is the set of N -tuples (f_1, \dots, f_N) of RHP-stable functions.

Collaborators and I have a very broad based attack on the problem and address most aspects of it.

From qualitative theory to numerical algorithms and diagnostics.

Work under the contract was

- (1) to link the theory of this problem with semidefinite programming. (In particular to improve our earlier primal dual algorithms using some ideas from matrix semidefinite programming)
- (2) to begin extension of our OPT theory to the situation where the performance Γ contains an uncertain parameter. (This gives algorithms and diagnostics.)
- (3) (with G. Balas) we have worked out in detail the μ synthesis case of our diagnostics. We ran a few cases with crude coding. Results were intriguing, but there were too few cases to know if they were at all representative. Conversion of the simplest diagnostics to μ -tools format is near completion. Then tests will be easy to run. Conventional wisdom about optimization is that the quality of software and the users enjoyment are considerably aided by having good diagnostics.

Currently, with Harry Dym we are working on interior point methods for MIMO H^∞ .

1.3.2 Bi H^∞ control or Frequency Response Scheduling

This is a frequency domain type of gain scheduling. This (rather new) idea, where one schedules on frequency response functions, leads directly to canonical problems in several complex variables for which a fair amount of theory exists.

An example is H^∞ control for a plant where part of it is known and a subsystem δ is not known, that is, the response of the plant at "frequency" s is $P(s, \delta(s))$. We assume that once our control (closed loop) system is running, we can identify the subsystem δ on line. Thus the problem is to design a function K offline that uses this information to produce a H^∞ controller via the formula $K(s, \delta(s))$. The challenge is to pick K so that the controller yields a closed loop system with H^∞ gain at most γ , no matter which δ occurs. While this is entirely a frequency domain problem, it has the flavor of gain scheduling and one might think of it as H^∞ gain scheduling. Note it is a strict analog to Linear Parameter Varying control LPV or LFT based control, approaches currently meeting with great success.

In article [Hprep] , we show that LMV control problems are equivalent to certain problems of interpolation by analytic functions in several complex variables. These precisely generalize the classical (one complex variable) interpolation (AAK -commutant lifting) problems which lay at the core of H^∞ control. These problems are hard, but the last decade has seen substantial success on them in the operator theory community, resulting from the focus of efforts made by the generation of mathematicians who followed AAK-Nagy-Foias-Sarason.

1.3.3 Matrix Inequalities

Also, we have started to port some of our H^∞ optimization to another nonsmooth problem, namely matrix inequalities. One such result concerns coordinate iteration methods of optimization. We consider optimization of the largest eigenvalue of a smooth selfadjoint matrix

valued function $\Gamma(X, Y)$ of two vector or matrix variables X and Y . A typical problem one faces in control design are matrix versions of minimizing in Y and maximizing in X . Also, minimizing in X and Y is an important problem. We shall assume that Γ is concave or convex in Y and separately in X . As a consequence, assume we have a computer package which will optimize γ over X with Y frozen, and optimize γ over Y with X frozen. This is realistic when the joint behavior in X and Y is bad, since to use existing commercial software, one must apply it to each coordinate separately, and so can be used only to give a "coordinate iteration" algorithm.

Result *On "well behaved Γ ", coordinate iteration always gives a local optimum for the $\min_Y \max_X$ problem.*

Result *We give strong evidence (via optimality conditions) that coordinate iteration almost never gives a local solution to the $\min_Y \min_X$ problem. We give a practical test to tell if you have hit a local optimum.*

2 Technology and Interactions 1999

2.1 Meetings

Feb 97 School of Engineering Gave several Seminars UCSB Kokotivic

May 97 AFOSR Contractors Conf Wright Patterson AFB attended

Sept 97

Conference "Operators Systems and Linear Algebra" Kaiserslauten
Univ of Kaiserslauten

- computer algebra for engineering systems

Oct 97

Pedagogical Inst of Odessa

- computer algebra for engineering systems

Dec 97 Conf on Decision and Control, SanDiego

Workshop on Nonlinear H^∞ control - opening speaker

- Numerical nonlinear (jet engine application)

- Matrix Optimization

1998

February 98 UC Berkeley Math Dept

Analysis seminar - Nonlinear systems

Operator Theory seminar - computer algebra

March 98 UCSanta Barbara - Math Dept Colloquium - Nonlinear Systems

March 98 - Regional AMS meeting Kansas Special Session in Operator Theory
Non-linear systems and operators

AFOSR Contractors LA. AFB attended May 98FO

June 98
International Workshop on Operator Theory and Appl.

July 1998 MTNS
- Nonlinear systems
- Computer algebra and systems
- Optimization

Special Semester on Symbolic Computation at
Math Sci Research Inst, Berkeley
"month long member" -I actually stayed 2 weeks over 3 visits,
and gave one talk on symbolic calculation for engineering problems.

Dec 98 Conf on Decision and Control, Orlando
- Nonlinear Control
- Nonlinear Control
- Numerical nonlinear (jet engine application)
- Matrix Optimization
-

1999

June 1999 American Conference on Control, San Diego
-- Nonlinear control
-- Bi Hinfinity control

AFOSR Contractors LA. AFB attended May 98

Dec 99 Conference on Decision and Control, Pheonix

- Nonlinear Control
- Nonlinear Control
- Nonlinear Control (Robot)
- Computer Algebra in Control
- H^∞ Optimization

3 Personnel Supported

SENIOR PERSONNEL: Helton, Bill Peter Dower PGR Mark Kennel PGR IV Mikhail Shushick VI Trent Walker PGR 1

Independent Contractors: Keller Stankus Merino

GRADUATE STUDENT RESEARCHERS: Dell Kronewitter GSR1 Juan Camino GSR IV Mike Hardt GSR IV Joshua Griffin GSR 1 David Glickenstien GSR 1 Eric Rowell GSR 1 Bob Slobojidan GSR 1 Jeremy Martin GSR 1 Maihong Shi GSR 3 Eric Rowell GSR 1

UNDERGRAD STUDENT STAFF: Marianna Rauk Campbell Mike Torre Phillipe Bergman

3.1 Consultative and advisory

The main US Tokamak is in San Diego at General Atomics Corp. and my former student Mike Walker is in charge of installing a modern control system on it. When the control side of things is moving we have regular discussions about what to do next.

3.2 Transitions

We maintain and expand two computer programs which run under Mathematica which are publicly available.

NCAAlgebra, has potential applications in many fields (get it from <http://math.ucsd.edu/~ncalg>). This is the established package which gives Mathematica noncommuting capability.

OPTDesign, our classical control program is available from <http://math.ucsd.edu/~helton>. Our book [HMer98], while independent of the software, illustrates its use.

Earlier progress reports mentioned discussions on antenna design with NOSC engineers in San Diego. These have led to a Navy initiative to apply H^∞ techniques developed in control problems to antenna matching problems. The problem is challenging because characteristics (scattering parameters) of an antenna (given as data on the $j\omega$ axis) are complicated, due to the interaction of the antenna with much of the metal on the ship. We are using techniques worked out by Merino and I under previous AFOSR funding, and to some extent μ - tools, to find matching circuits. We now have good methods to determine if existing designs are far from the theoretical optimum. Now we are looking at low order models. Contact at Spawar: Dr. Dave Schwartz (619) 553 2021, Jeff Allen (619) 553-6566.

Ford Motor Co. in June 1999 gave a modest gift to support my research. This is the second contribution Ford has made to my work. The possibility of applying the methodology we developed for compressor stall control to engines intrigues them. At Ford: Davor Hrovat (313) 322-1492.

Honors and Awards

Guggenheim Fellow 1985, Outstanding Paper IEEE Control Society 1986

4 Publications

All articles which appeared in 1996.

[HWee96] J. W. Helton and F. Weening: "Some systems theorems arising from the Bieberbach conjecture," *Journal Nonlinear and Robust Control*, vol 6, (1996). (misrecorded as 94 in some lists) p 65-82

[BH 1994, 1996] J. W. Helton and J. Ball: "Viscosity solutions of Hamilton-Jacobi equations arising in nonlinear H_∞ -control." *J. Math Systems Estimation and Control* vol 6 num 1 1996

(JMSEC published an announcement 1994, put the paper on the net, in 1996 they published the full paper)

[HWZ 96] J. W. Helton M. Walker and W. Zhan, " H^∞ Control of systems with compensators which read the command signal" *International Journal of Control* 10 May 96, vol 64, (no.1), pp 9-27

[BHW 96] "A. Ben-Artzi, J.W. Helton and H. Woerdeman: "Nonminimal factorization of nonlinear systems: the observable case" *International Journal of Control*, April 96, vol 63, no.6, pp. 1069-1104

[HSS96] J. W. Helton, M. Stankus, K. Schneider. *Computer Assistance in Discovering Formulas and Theorems in System Engineering II*, CDC96, p. 4412.

[HM96] J. W. Helton, O. Merino. *Semi-Definite Programming Tailored to H-Infinity Optimization Arising from Plant Uncertainty Problems*, CDC96, p. 1333.

[YJH96] S. Yuliar, M. R. James, J. W. Helton. *State Feedback Dissipative Control Systems Synthesis*, CDC96, p. 2862.

[HJ96] J. W. Helton, M. R. James. *J-Inner/Outer Factorization for Bilinear Systems*, CDC96, p. 3788.

[HJ96] J.W. Helton and M. R. James, "On the Stability of the Information State System," *Sys Control Letters* vol 29 (1996) p61-72

All articles which appeared in 1996.

[BaHM97] F. Bailey, J. W. Helton, and O. Merino, "Performance functions and sublevel sets for frequency domain design of single loop feedback control systems with plant uncertainty", *Int. J. Robust and Nonlinear Control*, Wiley, March 1997, vol 7, no 3, pp227-63.

[BaHM97] F. Bailey, O. Merino, and J. W. Helton "Alternative approaches in frequency domain design of single loop feedback systems with plant uncertainty", Int. J. Robust and Nonlinear Control, Wiley, March 1997, vol 7, no 3, pp265-77.

[HZ97] J. W. Helton and W. Zhan: "Piecewise Riccati Equations, the Bounded Real Lemma and Saturation", International Journal of Robust and Nonlinear Control vol 7, pp. 741-757 (1997)

[HV 97] J. W. Helton and A. Vityaev: "Analytic functions optimizing competing constraints", SIAM Math Analysis, vol 28, no 3, pp749-767, May 1997.

[HRW97] J. W. Helton, J. Rosenthal, and Xiaochang Wang: "Matrix Extensions and Eigenvalue Completions, the Generic Case", The Transactions of the American Mathematical Society, August 1997, vol 349, no 8, pp3401-8.

[HMer97] J.W. Helton and O. Merino, Coordinate optimization for bi-convex matrix inequalities, Proceedings of the 36th IEEE CDC, San Diego, CA, USA, Dec 10-12, 1997, vol 4, p3609-13.

[HHD97] M. Hardt and J.W. Helton K. Kreutz-Delgado, Numerical solution of Nonlinear H^2 and H^∞ Control Problems with Application to Jet Engine Control, Proceedings of the 36th IEEE CDC, San Diego, CA, USA, Dec 10-12, 1997, vol 3, p2317-22.

All articles which appeared in 1996.

[HSW98] J. W. Helton, M. Stankus and J. Wavrik, "Computer Simplification of Formulas in Linear Systems Theory", IEEE Trans. on Automatic Control, Mar98, vol. 43, num. 3, pp302-314.

[HMW98] J. W. Helton, O. Merino and T. Walker, " H^∞ Optimization with Plant Uncertainty and Semidefinite Programming" pp1-41 Int Jour of Nonlinear and Robust Control vol 8, 763-802 (1998)

[AHS98] J. Agler, J. W. Helton, M. Stankus "Hereditary Roots" Linear Algebra and Appl.274:125 -160 (1998)

[HMer98] J. W. Helton and O. Merino: "Classical Control using H^∞ Methods." August 1998 SIAM. Long version subtitled: Theory Optimization and Design 20 Chapters and 9 Appendices pp 1- 292 Short version subtitled: An Introduction to Design 7 Chapters and 7 Appendices pp 1- 171

[HKD98] M. Hardt, K. Kreutz-Delgado and J. W. Helton, "Minimal Energy Control of a Biped Robot with Numerical Methods and a Recursive Symbolic Dynamic Model", pp. 1-4, to appear CDC 98.

[HMer98] J. W. Helton and O. Merino, "Sufficient Conditions for Optimization of Matrix Functions", pp. 1-5, CDC 98.

[HJ98] J. W. Helton and M. R. James, "On Verifying the Certainty Equivalence Assumptions in Nonlinear H_∞ Control", pp.1-5, CDC 98.

[HJM98] J. W. Helton, M. R. James and W. M. McEneaney, "Nonlinear control: The joys of having an extra sensor", pp. 1-7, CDC 98.

All articles which appeared in 1996.

- [HS99] J. W. Helton and Mark Stankus, "Computer assistance for "discovering" formulas in system engineering and operator theory " Jour. of Functional Analysis. 161 (1999) pp 289-363
- [H99] J. W. Helton A Type of Gain Scheduling Which Converts to a "Classical" Problem in Several Complex Variables American Conference on Control 1999
- [HD99] P. Dower and J.W. Helton Using Suboptimal State Feedback Controllers in Certainty Equivalence Measurement Feedback Nonlinear H^∞ Control American Conf. on Control 1999
- [HJ99] J. W. Helton and M. James : *Extending H^∞ control to Nonlinear systems*. SIAM 1999 November, RESEARCH MONOGRAPH pp 1- 330
- [HHK99] Michael Hardt, Kenneth Kreutz-Delgado, J. William Helton Modelling Issues and Optimal Control of Minimal Energy Biped Walking 2nd International Workshop and Conference on Climbing and Walking Robots, University of Portsmouth, Portsmouth, UK, 13-15 September, 1999.
- [DHMprep] H. Dym, J. W. Helton, O. Merino Two-Disk and Multidisk Problems in H^∞ Optimization: a Study of Optimality Conditions, CDC 1999 Phoenix, pp 3156-3161
- [HD99] P. Dower, J.W. Helton Certainty Equivalence Measurement Feedback Power Gain Control of Nonlinear Systems CDC 1999 Phoenix, pp 2479 -2484
- [HDJ99] P. Dower, J.W. Helton and M. James, Measurement Feedback Nonlinear H^∞ Control of Linear Systems with Actuator Nonlinearities, CDC 1999 Phoenix, pp 3758-3763
- [HHKprep] M. Hardt and J.W. Helton K. Kreutz-Delgado, Optimal Biped Walking with a Complete Dynamical Model, CDC 1999 Phoenix, pp 2999-3004
- [HK99] Kronewitter Noncommutative Computer Algebra in the Control of Singular Perturbed Dynamical Systems CDC 1999 Phoenix, pp4086-4091
- [HHK00] Numerical Solution of Nonlinear H-two and H-infinity Control Problems with Application to Jet Engine Control. preprint pp ,1- 24 Control Systems Technology, Jan 2000

5 Patents

A patent was applied for primarily with Navy support, but work funded by AFOSR over the years contributed heavily to it. The patent is for a method to determine the ideal limits of an antennas power consumption. The beginnings of the work were my papers of mine in the late 1970s. Indeed this may be the earliest H^∞ engineering work to receive a patent.

THE REGENTS OF THE UNIVERSITY REPORT OF INVENTIONS AND SUBCONTRACTS

(pursuant to "Patent Rights" Contract Clause) (See Instructions on Reverse Side.)

OMB No. 0704-0297
Expires Jun 30, 1992

Public reporting burden for this collection of information is estimated to average 5 minutes per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0297), Washington, DC 20503

1a. NAME OF CONTRACTOR/SUBCONTRACTOR	2a. NAME OF GOVERNMENT PRIME CONTRACTOR	3. TYPE OF REPORT (X one)
	Air Force	a. INITIALS <input checked="" type="checkbox"/> b. FHAI
b. ADDRESS (Include ZIP Code)	b. ADDRESS (Include ZIP Code)	4. REPORTING PERIOD (YYMMDD)
		a. FROM 97/04/01
		b. TO 99/12/31

SECTION I - SUBJECT INVENTIONS

5. "SUBJECT INVENTIONS" REQUIRED TO BE REPORTED BY CONTRACTOR/SUBCONTRACTOR (If "None," so state)	c. DISCLOSURE NO., PATENT APPLICATION SERIAL NO. OR PATENT NO.	d. ELECTION TO FILE PATENT APPLICATIONS		e. CONFIRMATORY INSTRUMENT OR ASSIGNMENT FORWARDED TO CONTRACTING OFFICER
		(1) United States	(2) Foreign	
a. NAME(S) OF INVENTOR(S) (Last, First, MI)		(a) Yes	(b) No	(1) Yes
Helton, William J.	Predictor for Optimal Broadband Matching			
g. ELECTED FOREIGN COUNTRIES IN WHICH A PATENT APPLICATION WILL BE FILED (2) Foreign Countries of Patent Application				
f. EMPLOYER OF INVENTOR(S) NOT EMPLOYED BY CONTRACTOR/SUBCONTRACTOR				
(1) (a) Name of Inventor (Last, First, MI)	(1) Title of invention			
(b) Name of Employer				
(c) Address of Employer (Include ZIP Code)				

SECTION II - SUBCONTRACTS (Containing a "Patent Rights" clause)

6. SUBCONTRACTS AWARDED BY CONTRACTOR/SUBCONTRACTOR (If "None," so state)	b. ADDRESS (Include ZIP Code)	c. SUBCONTRACT NO.(S)	d. FAR "PATENT RIGHTS"		e. DESCRIPTION OF WORK TO BE PERFORMED UNDER SUBCONTRACT(S)	f. SUBCONTRACT DATES (YYMMDD)	
			(1) Clause Number	(2) Date (YYMM)		(1) Award	(2) Estimated Completion
a. NAME OF SUBCONTRACTOR(S)							

SECTION III - CERTIFICATION

7. CERTIFICATION OF REPORT BY CONTRACTOR/SUBCONTRACTOR	(Not required if Small Business or Non-Profit organization) (X appropriate box)	
a. NAME OF AUTHORIZED CONTRACTOR/SUBCONTRACTOR OFFICIAL (Last, First, MI)	c. I certify that the reporting party has procedures for prompt identification and timely disclosure of "Subject Inventions," that such procedures have been followed and that all "Subject Inventions" have been reported.	
Tiffany, Pamela	d. SIGNATURE	e. DATE SIGNED
b. TITLE	<i>William J. Helton</i>	<i>Mar 27 1997</i>
Contract and Grant Officer	Dr. J. William Helton	

1. GENERAL PURPOSE:

The problem of broadband impedance matching is maximizing the power transfer uniformly over a frequency band from a source to a load.

The purpose of this invention is to compute the optimal performance than can be obtained by a passive or lossless matching network.

2. BACKGROUND:

Engineers utilize a variety of methods for constructing broadband matching networks. Some methods may attempt to maximize power transfer uniformly over a frequency band, but most settle for optimizing at a selection of frequencies within the band.

These methods have two things in common:

First, the ignorance of what the optimal performance can be;

Second, the lack of a systematic approach to optimize power transfer uniformly.

This invention resolves the first of these problems by enabling the engineer to compute the optimal performance for a matching network connecting a transmission line to a load.

The application that motivated this effort was the need to maximize power radiated by HF broadband antennas whose power is provided from a 50 Ohm transmission line. However, this approach is not limited to any frequency range and will apply to microwave and electro-optic networks.

Here are two additional benefits of knowledge of optimum performance attainable by a matching circuit:

First, the performance optimum allows an engineer a benchmark to grade the performance of proposed matching circuits.

Second, by computing the optimum performance an engineer can determine if it is possible to satisfy a design objective for a given load.

3. DESCRIPTION AND OPERATION:

Our invention is an algorithm for computing the performance of the optimum matching circuit connecting a load to a constant impedance transmission line over a continuous frequency band F.

This algorithm is implemented in a computer code called AIM for Analytic Impedance Match.

There are three inputs to AIM:

1. q_{in} -- Samples of the reflectance of the load normalized to the characteristic impedance of the line;
2. f_{in} -- The frequencies associated with the normalized reflectances;
3. N -- A parameter specifying the number of frequencies AIM will compute over.

There is a single output from AIM:

G -- The greatest transducer power gain obtainable by any matching circuit. There are two system parameters computed in terms of G:

1. The power mismatch (pm) is the square root of $1-G$;
2. The voltage standing wave ratio $VSWR = (1 + pm) / (1 - pm)$.

The algorithm is completely developed in Attachment 1.
The ``language'' describing this algorithm is MATLAB version 5.1 from The Math Works.
The main program is the ``function'' AIM with the inputs and outputs described above.
For the simplicity of this implementation N is assumed to be a power of 2.

An example of the operation of AIM follows:
Here qin is 401 samples from a data set labeled AOC21B and depicted in Figure 1.
These samples correspond to frequencies fin equally space from 2 to 8 MHz inclusive.
The parameter N is set to 1024.
The output of AIM is $G = 0.705$ as the largest transducer power gain obtainable by a matching circuit.
The corresponding power mismatch is 0.5431 and VSWR equal to 3.378.

4. ADVANTAGES AND NEW FEATURES:

The principle advantage of this invention is the ability to compute the optimal performance attainable by a lossless matching network connecting a transmission line to a load over a frequency band.
No other method is able to calculate what the optimum performance is.

Here are some of the benefits of knowledge of optimum performance attainable by a matching circuit:
First, the ability to decide whether a design objective for a given load and frequency band can be attained;
Second, a benchmark to grade the performance of proposed matching circuits.

5. ALTERNATIVES:

There is no known method that calculates the optimum performance.

6. CONTRIBUTIONS BY INVENTORS:

The idea for this method was developed by Professor J. William Helton in the paper ``Broadbanding: Gain Equalization Directly from Data'' published in IEEE Transactions on Circuits and Systems in December 1981. The implementation was done by David F. Schwartz and Jeffery C. Allen.

7. EXECUTION OF DISCLOSURE:

J. William Helton

4 Apr 99

David F. Schwartz

2 Apr 99

REPORT DOCUMENTATION PAGE

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14. ABSTRACT The accomplishments were to extend H^∞ control in new directions: <ul style="list-style-type: none"> • non-linear control. Theoretical work gave algorithms for determining in real time key properties (the state) of a system one tries to control. Purely numerical work gave a "turnkey" methodology for designing optimal controllers for the compressor stall problem. Mathematically related was numerical solution of the extremely hard path planning problem, biped motion, a "5-link robot". • computer algebra methods appropriate for systems which are aggregates of smaller systems (commercial computer algebra packages now do not do this). • one of the most common techniques now used for performance optimization (co-ordinate descent applied to matrix inequalities) and show that it gives the wrong answer with probability one. Also results give a test to see how far off (local optimum) this is. 					
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