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The work completed under this project builds on our previous work sponsored by AFOSR on developing a universal spectral basis for formulating algorithms for high-order accurate solutions of arbitrary nonlinear systes of PDEs in complex-geometry domains. To this end, several classes of classical numerical methods (e.g. finite elements and finite volumes) as well as modern methods (e.g. spectral and spectral elements) are simply subcases of the developed new numerical methodology. One of the unique features of the new methods is that its accuracy is insensitive to the mesh quality, which allows simulation in moving and very distorted domains (without re-gridding) at extremely high-efficiencies, not possible with any other method.

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FINAL REPORT
High-Order Algorithms for 3D Plasma Simulations on Unstructured and Hybrid Meshes
AFOSR number: F49620-97-1-0185

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Objectives

• Develop high-order discretizations on unstructured and hybrid meshes for the viscous unsteady MHD equations in two- and three-dimensions. No such capability exists in the AFOSR labs or elsewhere in the world.

• Advance the state-of-the-art in multi-resolution algorithms and parallel software.

• Apply the new capability to parallel simulations of isothermal and MHD turbulence in complex-geometry domains.

Accomplishments/New Findings

The work completed under this project builds on our previous work sponsored by AFOSR on developing a *universal* spectral basis for formulating algorithms for high-order accurate solutions of arbitrary nonlinear systes of PDEs in complex-geometry domains. *To this end, several classes of classical numerical methods (e.g. finite elements and finite volumes) as well as modern methods (e.g. spectral and spectral elements) are simply subcases of the developed new numerical methodology.* One of the unique features of the new method is that its accuracy is insensitive to the mesh quality, which allows simulation in moving and very distorted domains (without re-gridding) at extremely high-efficiencies, not possible with any other method.

The specific tasks completed include:

• Development of Discontinuous Galerkin projections. This is a *breakthrough* in variational numerical methods, which will have great impact in many AFOSR applications. It allows general high-order differential operators to be discretized with finite jumps (L^2 continuity) across subdomains. This, in turn, implies that "sliding" domains (of great interest to weapons simulation technology) can now be discretized with no extra computational effort. In addition, intelligent trial bases can be used in different subdomains to deal with singularities, discontinuities and sharp gradients while optimizing their efficiency locally without global inter-dependence. The reduced continuity requirements make the method *inherently parallel* and naturally suited to modern parallel platforms at AFOSR labs.

• Development of Discontinuous Galerkin methods for the viscous MHD equations for compressible and incompressible turbulence, and demonstration of the high-order accuracy of the method. The parallel code we developed is the only MHD code with high-order capability in complex-geometry domains. The new formulation is extension to high-order and unsteady flows of the work of Peterkin et al. (Kirkland AFB), Shumlak (University of Washington) and Powell et al. (University of Michigan), who are the others PIs in the AFOSR Computational Math program. In fact, their algorithms are a subset of the new formulation in the degenerate case of setting the spectral order equal to zero, which is the finite volume limit of the discontinuous Galerkin formulation. Therefore, this makes for an easy porting of our numerical algorithms to the current MACH2 and MACH3 (AFOSR codes), which are only limited to second-order accuracy at best and only cartesian grids. Our algorithm is suitable for cartesian grids, unstructured grids, as well as hybrid grids consisting of polymorhic domains.

Personnel

- Faculty: G.E. Karniadakis (US), Professor of Applied Mathematics
- PhD Students: T.C. Warburton and I. Lomtev

Publications

1. T.C. Warburton, "Spectral/hp Methods on Polymorphic Multi-Domains: Algorithms and Applications", PhD thesis, Brown University, 1998 (supervised by the PI).

2. T.C. Warburton and G.E. Karniadakis, "A discontinuous Galerkin method for the viscous MHD equations", J. Comp. Phys., in press, 1999.

3. T.C. Warburton, S.J. Sherwin and G.E. Karniadakis, "Spectral basis functions for 2D hybrid hp elements", SIAM J. Scientific Computing, in press.

4. S.J. Spencer, T.C. Warburton, and G.E. Karniadakis, "Spectral/hp methods for elliptic problems on hybrid grids", Contemporary Mathematics, vol. 218, p. 191-215, 1998.

5. T.C. Warburton, I. Lomtev, M. Kirby, and G.E. Karniadakis, "A discontinuous Galerkin method for the compressible Navier-Stokes equations on hybrid grids", Proc. Tenth Internationa Conference on Finite Elements in Fluids, January 5-8, 1998, Tucson, Arizona, p. 604, Eds. M. Hafez and J.C. Heirich.

6. G.E. Karniadakis and S.J. Sherwin, "Spectral/hp Element Methods for CFD", monograph, Oxford University Press, 1999.

7. H. Marmanis, Y. Du, C.H. Crawford, and G.E. Karniadakis, "Turbulence control via geometry modifications and electromagnetic fields", Proc. ECCOMAS 98, Athens, Greece, 1998.

Interactions/Transitions

The PI will organize the first International conference on Discontinuous Galerkin Methods on May 24-26, in Newport, RI USA. Participants will include several academic and national lab researchers from US.

The PI was invited in the past year to present the AFOSR-sponsored research at:

- Cornell Workshop on POD-Galerkin Models for the Dynamics and Control of Complex Flows
- University of Michigan, Department of Aerospace Engineering
- AFOSR/Princeton Workshop on Plasma-Assisted Drag Reduction
- DARPA/NUWC Workshop on Electromagnetic Turbulence Control
- DOE/Oakridge Workshop on Discontinuous Galerkin Methods for Materials
- NSF Woskshop on New Computational Challenges
- AIAA Fluid Dynamics Conference on LES
- SIAM Annual Meeting/Symposium on MHD
- ICOSAHOM'98 Symposium on Corner Singularities
- Japanese Society of Fluid Mechanics 30th Anniversary Symposium
- University of Tokyo, Department of Mechanical Engineering
- Turkey Workshop on Industrial and Environmental Applications of DNS/LES

Software Distribution: The code NEKTAR has been disributed to more than one dozen Universities and Laboratories. Some of them include MIT, Clatech, Cornell University, Penn State University, University of Wisconsin, Imperial College, North Carolina University, Florida State University, OAK Ridge Labs, Nielsen, Inc., Boeing, etc. There is limited documentation of the code, which has made this distribution somewhat difficult. However, certain researchers, e.g. Hussaini & Erlebacher at FSU have used NEKTAR extensively in LES of compressible turbulence and have developed new extensions for their applications. The porting of the new developments to the AFOSR codes MACH2 and MACH3 has not been done yet and it will require additional resources.