

# FINAL REPORT

October 1, 1996 to September 30, 1999

## Computational Methods for Image Reconstruction and Enhancement

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| <b>13. ABSTRACT (Maximum 200 words)</b><br>This project had two goals: (1) the development of new computational techniques for image restoration; and (2) the application of these techniques to problems of interest to the US Air Force. The primary application was the identification of space objects from image data collected with ground-based optical telescopes. The achievements of the project included: (1) the publication of a number of research papers in applied and computational mathematics and applied optics journals; (2) the development of two software packages, one for image deblurring and the other for object and phase reconstruction in atmospheric optics; and (3) support for a graduate student in the research toward and preparation of the student's PhD thesis. Algorithms developed under this project have recently been implemented on supercomputers at the USAF Maui High Performance Computing Center to process image data from the USAF Maui Optical Station. |   |  |  |  |
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# 1 Introduction

This project dealt with regularization methods for ill-posed inverse problems and the implementation of these methods for applications in atmospheric imaging which are of interest to the US Air Force. To explain the concept of ill-posedness, some mathematical notation and terminology is needed. In the model equation

$$d = Kf + \eta, \quad (1)$$

the  $f$  represents a desired “true” solution, and  $d$  represents measured data. The  $K$  represents a process which transforms the true solution, while  $\eta$  represents “noise”, which is independent of the solution. The problem of retrieving the solution  $f$  from noisy data  $d$  is called *well-posed* if the transformation operator  $K$  is invertible, and the inverse operator  $K^{-1}$  is continuous. This means that if one could somehow make the noise term  $\eta$  “arbitrarily small”, then  $K^{-1}d$  would be guaranteed to be “arbitrarily close” to the desired true solution  $f$ . A problem that is not well-posed is said to be *ill-posed*. To obtain an accurate approximate solution to an ill-posed problem, one must apply *regularization*. This entails the construction of a family of “approximate inverse operators” for the  $K$  in (1) which are “stable” and “convergent”. For a precise mathematical definition, see [6, Ch. 3] or [25, Ch. 2].

Ill-posed inverse problems occur quite commonly in science and engineering. Examples range from biomedical and seismic imaging to groundwater flow characterization. An ill-posed problem of great importance to the Air Force is the reconstruction of images that have been degraded, or “blurred”, by atmospheric turbulence effects. A mathematical model for the blurring process is the Fredholm first kind integral operator of convolution type,

$$(Kf)(x_1, x_2) = \int \int s(x_1 - x'_1, x_2 - x'_2) f(x'_1, x'_2) dx'_1 dx'_2. \quad (2)$$

Figure 1 below shows simulated data obtained with this model. With this application comes some specialized terminology. The  $f$  in (2) is called the *object*, while the  $s$  is called the *point spread function*, abbreviated by PSF. In atmospheric optics, the PSF has a very special form,

$$s = \left| \mathcal{F}^{-1} (Ae^{i\phi}) \right|^2, \quad (3)$$

where  $A$  is the *aperture function*,  $\phi$  is the *phase function*, or *wavefront profile*, and  $\mathcal{F}$  denotes the two dimensional Fourier transform. See [14] for details.

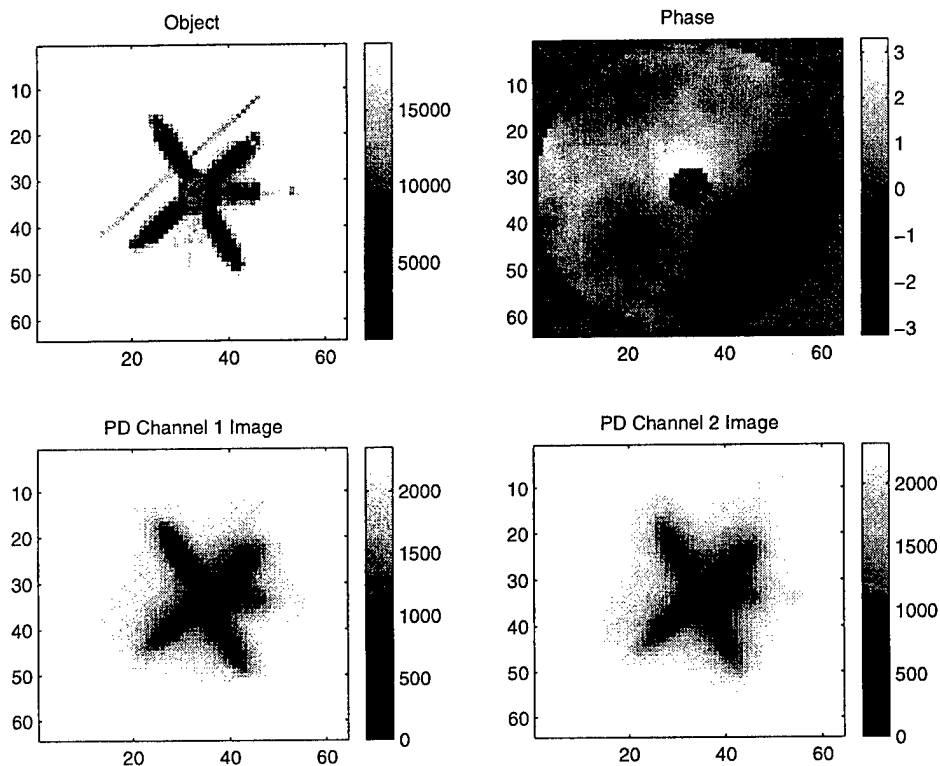


Figure 1: *Simulated Image Data.* The upper left subplot shows the object, a satellite in earth orbit; the upper right subplot shows the atmospheric phase, or wavefront profile; the lower left subplot shows a conventional image (no phase diversity) with blurring resulting from atmospheric turbulence; and the lower right subplot shows a blurred image with phase diversity.

The wave fronts of light emanating from an idealized point source at infinity are planar. As the light propagates through the atmosphere, these wave fronts are distorted due to variations in the index of refraction caused by temperature differences. The phase characterizes this wavefront distortion.

This past decade has seen the development of hardware which uses deformable mirrors to compensate for wavefront distortion to improve the quality of the recorded image. This hardware solution to the atmospheric blurring problem is known as *adaptive optics* [17]. Adaptive optics has several shortcomings: It requires an approximate point source, or *guide star*, to estimate the phase. This may be difficult to obtain, particularly for daylight

imaging. Wavefront compensation is imperfect due to factors like the finite number of actuators in the mirror and the time lag between the detection and compensation. *Postprocessing*, i.e., the application of image reconstruction algorithms implemented with computer software, is needed to further enhance the image. This provides the motivation for this project.

## 2 Objectives

The original goal of this project was the development of fast, robust computational algorithms for image reconstruction. For the US Air Force the most notable application is identification and tracking of objects in earth orbit using ground-based optical telescopes. Numerous other applications occur in biomedical imaging. The relevant mathematical model was given by equations (1)-(2), and the PSF  $s$  in (2) was assumed to be known. Discrete versions of these equations, obtained by pixelization of the data and applying numerical quadrature to the integral operator  $K$ , are notoriously difficult to solve. They are typically quite large, e.g., a  $128 \times 128$  pixel array yields a system with tens of thousands of unknowns. As a consequence of the ill-posedness of the underlying continuous equation, the discrete system is also highly ill-conditioned. In practical terms, a small amount of noise in the recorded image can cause enormous errors in the reconstructed image. This ill-conditioning can be overcome by applying regularization. The PI is an expert in regularization methods and their numerical implementation. His initial contribution to this project was the application of a variety regularization methods to problems in image reconstruction, and the development and implementation of fast algorithms.

During a visit to a US Air Force laboratory (the Starfire Optical Range, Kirtland AFB, New Mexico), the PI became aware that the simple convolution integral equation model (2) was incomplete. A key component of the model, the PSF  $s$ , was typically not available. However, the PSF could be determined from a physical quantity known as the phase, or wavefront profile,  $\phi$  in equation (3). The PI then expanded his goals to include the development of fast, robust computational algorithms for *phase estimation*. Phase estimation is important in its own right, having applications in laser communications and laser weapons systems.

### 3 Major Accomplishments and New Findings

The PI's initial focus was on the development and implementation of new regularization methods. A standard approach to the solution of (1) is Tikhonov regularization [6], or penalized least squares, where one minimizes the functional

$$J(f) = \|Kf - d\|^2 + \alpha \langle Lf, f \rangle. \quad (4)$$

Here  $\langle f, g \rangle = \int \int f(x)g(x) dx$  denotes the  $L^2$  inner product, the regularization parameter  $\alpha$  is a small and positive, and  $L$  is a symmetric positive semidefinite linear diffusion operator, e.g., the negative Laplacian  $Lf = -u_{xx} - u_{yy}$ . A shortcoming of quadratic cost functionals like this is that they produce smooth solutions. Images with sharp features have discontinuities and are not smooth. See the object in Figure 1. A new approach to the removal of noise from discontinuous images had recently been developed by Rudin and Osher [15], based on *total variation*, abbreviated TV,

$$TV(f) = \int \int \sqrt{u_x^2 + u_y^2} dx dy. \quad (5)$$

Since the TV functional is not quadratic, iterative solution methods must be applied to the resulting Euler-Lagrange equations. Rudin and Osher used explicit time marching, or equivalently, the steepest descent algorithm, for TV denoising. Steepest descent is notoriously slow to converge for ill-conditioned systems. Hence, it proved impractical for image deblurring. The PI, together with a graduate student Mary Oman, developed an alternative solution method that was dubbed *lagged diffusivity fixed point iteration* [18]. For the minimization of the Tikhonov functional (4) with  $\langle Lf, f \rangle$  replaced by  $TV(f)$ , this iteration takes the form

$$(K^*K + \alpha L_{TV}(f^\nu)) f^{\nu+1} = K^*d, \quad \nu = 0, 1, \dots$$

In [5], the PI and David Dobson proved that this iteration is globally convergent (i.e., it converges no matter what initial guess is taken). The PI and Oman [23] demonstrated that this method converges rapidly on realistic simulated atmospheric imaging data similar to that shown in Figure 1. On the theoretical side, Robert Acar and the PI have rigorously proved [1] that TV-penalized least squares is indeed a regularization.

Experience of the PI and many others has shown that incorporating non-negativity constraints can dramatically improve the quality of the reconstructed images. Unfortunately, this can also dramatically increase computational cost. In [19] the PI implemented a rapidly convergent projected

Newton algorithm for nonnegatively constrained, regularized image deblurring. Also discussed in this paper was the efficient solution of large, structured linear systems which arise at each iteration of the projected Newton algorithm. In a follow-up paper [8], Martin Hanke, James Nagy, and the PI formulated a general class of efficient quasi-Newton techniques for image deblurring.

The PI and a graduate student, Steve Hamilton, wrote a collection of MATLAB codes which implemented a variety of regularization methods for image deblurring. Included in this package are standard (quadratic) Tikhonov regularization and (nonquadratic) TV regularization. The codes allow for the incorporation of nonnegativity constraints.

Another goal was the development of robust, efficient computational techniques to solve the linear systems arising from the linearization and discretization of regularized inverse problems. As indicated above, these problems typically are quite large. Hence, it is impractical to apply direct matrix decomposition techniques. Instead, iterative methods like the conjugate gradient (CG) method are used. As a consequence of ill-posedness, these systems tend to be somewhat ill-conditioned when the regularization parameter is small. This means that CG convergence may be quite slow, and acceleration techniques called preconditioners are needed. The key to effective preconditioning is to make use of special structure.

The blurring operator  $K$  in (2) gives rise to matrix equations with block Toeplitz-Toeplitz block structure, abbreviated BTTB. Circulant preconditioners have been shown to be very effective for BTTB systems [2, 3]. Unfortunately, other matrices arising in image reconstruction often do not have BTTB structure. This is the case when total variation regularization is used. It is also the case when the blurring process is not spatially translation invariant. The PI and Martin Hanke developed and analyzed a class of *multilevel* preconditioners that require only that  $K$  be a compact linear operator [9, 22]. Compact operators arise not only in image reconstruction, but also in a variety of other important inverse problems. Follow-up work has been carried out by Kyle Riley [12, 11, 13], a PhD student of the PI's.

As indicated above, the model (2) is often incomplete in the sense that the PSF  $s$  is unknown. A variety of approaches have been taken to overcome this difficulty. With *multiframe blind deconvolution* [16], a time-varying sequence of images is captured. It is assumed that the PSF  $s$  varies with time, but the object  $f$  remains constant from frame to frame. An alternative approach is *multichannel phase diversity* [7]. Here the model (3) for the PSF

is assumed, with  $\phi$  representing the phase, or wavefront aberration. Additional information about the phase is obtained by using beam splitters and imposing known phase perturbations and then capturing additional images. Figure 2 illustrates a 2-channel phase diversity setup, while the bottom subplots in Figure 1 show simulated phase diversity image data. The PSF's in the M-channel setting take the form

$$s_m[\phi] = \left| \mathcal{F}^{-1} \left( A e^{i(\phi + \theta_m)} \right) \right|^2, \quad m = 1, \dots, M,$$

where the  $\theta_m$ 's denote the known phase perturbations. Given data  $d_m = s_m[\phi] \star f + \eta_m$ ,  $m = 1, \dots, M$  where  $s \star f$  denotes the convolution integral in equation (2) and  $\eta_m$  denotes error, the goal is to estimate both the object  $f$  and the phase  $\phi$ .

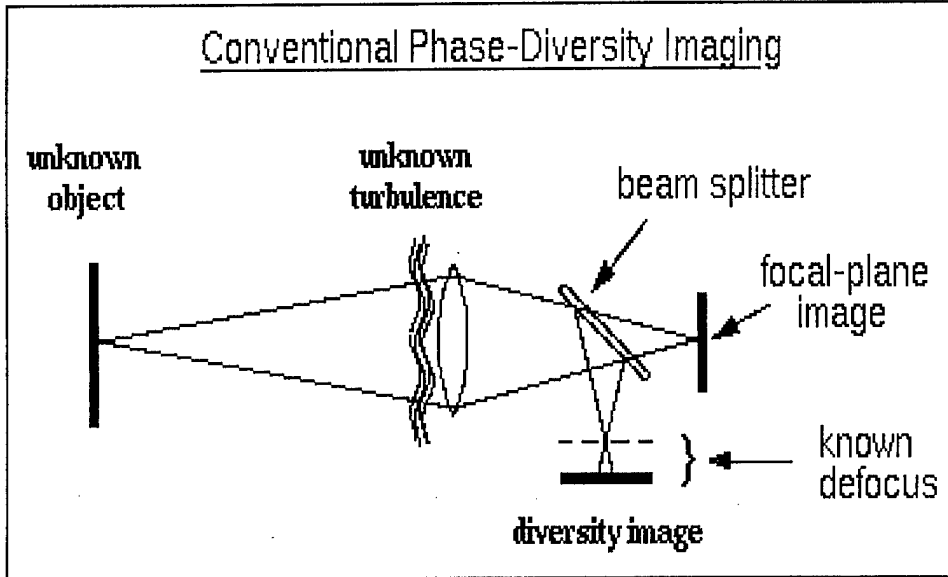


Figure 2: A Simplified Illustration of the Experimental Setup for 2-Channel Phase Diversity Imaging.

An optimization-based scheme for the joint estimation of object and phase was presented by Paxman et al in [10]. The PI and his co-workers (Robert Plemmons of Wake Forest University, Tony Chan of UCLA, and Brent Ellerbroek of the Starfire Optical Range) have made a number of refinements to this scheme. In [20] object and phase regularization terms were incorporated

in a penalized least squares framework. The phase penalty term made use of the covariance structure of the stochastic process used to model atmospheric turbulence. In [24] the PI implemented a limited memory BFGS scheme for the robust, efficient solution of the penalized least squares minimization problem. The PI together with Ellerbroek wrote a package in the MATLAB programming environment for the simulation of noisy, blurred atmospheric image data, and the inversion of this data to obtain the object and atmospheric phase profiles. The inversion codes allow for either single or multiple time frames and either single or multiple phase diversity channels. Robert Plemmons and his colleagues have translated these MATLAB codes into FORTRAN [4], and the codes are running on an IBM SP2 supercomputer at the Air Force's Maui High Performance Computing Center (MHPCC). The algorithms and codes will soon be tested with real atmospheric image data obtained at the Air Force Maui Optical Station (AMOS).

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- [25] C. R. Vogel, *Computational Methods for Inverse Problems*, (in preparation), to be published in the SIAM Book Series *Frontiers in Applied Mathematics*.

## 4 Personnel Supported by Grant

- The PI: Curtis R. Vogel, Professor of Mathematics, Montana State University.
- 3 Graduate Research Assistants: Kyle Riley, Steve Hamilton, and Scott Hyde. Kyle Riley completed his Ph.D. in Mathematics in July 1999 under the direction of the PI. He is now employed as an assistant professor at the South Dakota School of Mines in Rapid City, South Dakota. Steve Hamilton is currently a Ph.D. student in mathematics at Montana State University. Scott Hyde is a Ph.D. student in Statistics, also at Montana State University.

## 5 Publications

1. C. R. Vogel, *Nonsmooth regularization*, in *Inverse Problems in Geophysical Applications*, pp. 1-11, edited by H. W. Engl, A. K. Louis, and W. Rundel, SIAM, 1997.
2. R. C. Smith, K. L. Bowers, and C. R. Vogel, *Numerical recovery of material parameters in Euler-Bernoulli beam models*, *J. Math. Systems, Estimation, and Control*, **7** (1997), pp. 157-195.
3. D. Dobson and C. R. Vogel, *Convergence of an iterative method for total variation denoising*, *SIAM Journal on Numerical Analysis*, **34** (1997), pp. 1779-1791.
4. C. R. Vogel, *Solution of linear systems arising in nonlinear image deblurring*, in *Scientific Computing*, edited by G. H. Golub, S. H. Lui, F. Luk, and R. Plemmons, Springer, Singapore (1997), pp. 148-158.
5. C. R. Vogel and M. E. Oman, *A fast, robust algorithm for total variation based reconstruction of noisy, blurred images*, *IEEE Transactions on Image Processing*, **7** (1998), pp. 813-824.
6. C. R. Vogel, T. Chan, and R. Plemmons, *Fast algorithms for phase diversity based blind deconvolution*, *SPIE Proceedings* **3353** (1998), pp. 994-1006.
7. K. Riley and C. R. Vogel, *Preconditioners for linear systems arising in image reconstruction*, *SPIE Proceedings* **3461-37** (1998), pp. 372-380.
8. C. R. Vogel, *Sparse matrix equations arising in distributed parameter identification*, *SIAM Journal on Matrix Analysis* **20** (1999), pp. 1027-1037.
9. K. Riley, *Two-Level Preconditioners for Regularized Ill-Posed Problems*, Ph.D. Thesis, Department of Mathematical Sciences, Montana State University, July 1999.
10. M. Hanke and C. R. Vogel, *Two-level preconditioners for regularized inverse problems I: Theory*, *Numerische Mathematik*, **83** (1999), pp. 385-402.

11. C. R. Vogel and M. Hanke, *Two-level preconditioners for regularized inverse problems II: Implementation and numerical results*, *SIAM Journal on Scientific Computing*, under review.
12. C. R. Vogel, *A limited memory BFGS method for an inverse problem in atmospheric imaging*, in *Proceeding of IIC98, the Second Interdisciplinary Inversion Conference*, Springer-Verlag, to appear.
13. M. Hanke, J. Nagy, and C. R. Vogel, *A Quasi-Newton approach to nonnegative image restoration*, *Linear Algebra and Its Applications*, to appear.
14. C. R. Vogel, *Negative results for multilevel preconditioners in image deblurring*, in *Proceedings of Scale Space 99*, to appear.
15. K. Riley and C. R. Vogel, *Two-level preconditioners for ill-conditioned linear systems with semidefinite regularization*, *Journal of Computational and Applied Mathematics*, under review.
16. C. R. Vogel, *Computational Methods for Inverse Problems*, Research Monograph under preparation, to be published by SIAM in the book series *Frontiers in Applied Mathematics*.

## 6 Presentations

- Oct. 9-11, 1996, Coeur d'Alene, Idaho. Second SIAM Conference on Sparse Matrices, Attended and gave a Contributed Long Presentation entitled "Sparse matrix equations arising in parameter identification".
- Oct. 16-17, 1996. Phillips Laboratory, Kirtland AFB, New Mexico. Attended AirBorne Laser (ABL) Workshop.
- March 10-12, 1997. Hong Kong, China. Workshop on Scientific Computing '97. Attended and gave Invited Talk entitled "Solution of linear systems arising in nonlinear image deblurring".
- March 14, 1997. Beijing, China. Workshop on Optimization and Numerical Methods. Attended and gave Invited Lecture entitled "Computational Methods for Inverse Problems".

- June 16-19, 1997. Albuquerque, New Mexico. Fourth SIAM Conference on Mathematical and Computational Issues in the Geosciences. Attended.
- Oct. 11, 1997. UCLA Department of Mathematics. Gave colloquium talk entitled "Phase diversity based deconvolution and phase retrieval".
- Oct. 12-17, 1997. Long Beach, California. Annual Meeting of the Optical Society of America. Attended.
- Oct. 29-Nov. 1, 1997. Snowbird, Utah. Sixth SIAM Conference on Applied Linear Algebra. Attended and gave minisymposium presentation entitled "Numerical linear algebra and constrained deconvolution".
- March 23-28, 1998. Kona, Hawaii. SPIE International Symposium on Astronomical Telescopes and Instrumentation. Attended and presented contributed paper entitled "Fast algorithms for phase diversity based blind deconvolution".
- July 5-9, 1998. Mount Holyoke College, South Hadley, Massachusetts. AMS-SIAM-IMS Joint Summer Research Conference on Mathematical Methods in Inverse Problems for Partial Differential Equations. Attended and gave invited talk entitled "Phase diversity based deconvolution and phase retrieval".
- July 20-22, 1998. Wright Patterson AFB, Ohio. AFOSR Grantee's Meeting. Attended.
- July 23-24, 1998. San Diego, CA. SPIE International Symposium on Optical Science, Engineering, and Instrumentation. Attended and presented contributed paper entitled "Preconditioners for linear systems arising in image reconstruction".
- Aug. 12-14, 1998. Copenhagen, Denmark. Keynote Speaker at the Interdisciplinary Inversion Conference. Talk title "Phase diversity based inversion in atmospheric imaging".
- Oct. 4-7, 1998. Baltimore, Maryland. Annual Meeting of the Optical Society of America. Attended and presented contributed paper entitled "Two-level preconditioners for regularized inverse problems".

- Oct. 8-11, 1998. Winnipeg, Canada. International Conference on Operator Theory and Its Applications to Scientific and Industrial Problems. Attended and presented contributed paper entitled “Phase diversity based simultaneous deconvolution and phase retrieval”.
- Jan. 8-9, 1999. Wake Forest, North Carolina. Conference on Linear Algebra: Theory, Applications, and Computations. Attended and presented invited paper entitled “Computational issues arising in atmospheric imaging”.
- Feb. 24-26, 1999. Santa Fe, New Mexico. 1999 IEEE Information Theory Workshop on Detection, Estimation, Classification, and Imaging. Attended and presented invited paper entitled “Nonlinear inverse problems in atmospheric imaging”.
- July 5-9, 1999. Edinburgh, Scotland. 1999 International Conference on Industrial and Applied Mathematics. At the minisymposium on Inverse Problems, presented invited paper entitled “Inverse problems in atmospheric optics”. At the minisymposium on Computational Aspects of Distributed Parameter Estimation in Applications Involving PDE’s, presented invited paper entitled “Multilevel preconditioners for regularized inverse problems”.
- Sept. 26-30, 1999. Santa Clara, California. Annual Meeting of the Optical Society of America. Attended and presented contributed paper entitled “Fast algorithms for nonnegatively constrained image deblurring”.

## 7 Collaborative Research and Transactions at US Air Force Laboratories

During the course of this project, the PI visited the Starfire Optical Range (SOR) at Kirtland AFB, New Mexico, four times. His contacts at the SOR were Dr. Brent Ellerbroek and Dr. Julian Christou. The first visit occurred in June 1997. The PI gave an informal lecture on computational deblurring (i.e., two-dimensional deconvolution) algorithms and demonstrated some deblurring software. At that time, the PI’s goal was to supply the Air Force

with robust, efficient software to improve the resolution of ground-based telescopes used to identify and track objects in earth orbit. In discussions with Ellerbroek and Christou, the PI became aware that the telescope image data available was not enough to apply his algorithms. A key piece of information, the point spread function (PSF)  $s$  in model equation (2), was simply not available.

As a result of this first visit, the PI expanded his research goals to include the estimation of the PSF as well as true image from the recorded (noisy, blurred) image data. An examination of mathematical models for image formation and a search of the literature revealed that a technique known as phase diversity could be used to estimate a quantity known as the phase (which yields the PSF) together with the true image. The PI's next several visits to the SOR dealt with light propagation through the atmosphere and phase diversity. The PI began a collaboration with Dr. Ellerbroek which resulted in a MATLAB software package to simulate image formation in atmospheric optics. The PI also wrote MATLAB codes for the estimation of the phase and the true image from phase diversity image data. One the PI's academic research collaborators, Professor Robert Plemmons of Wake Forest University, has since translated these codes into FORTRAN and implemented them on a supercomputer at the Air Force's Maui High Performance Computing Center.

## 8 Inventions or Patent Disclosures

None. This project dealt with the development of computational algorithms and computer software. These have been made available to the Air Force and to the general public.

## 9 Summary

This project dealt with the development and computer implementation of fast, robust algorithms for atmospheric image deblurring. Specific accomplishments include

- The development of new computational algorithms for total variation and nonnegatively constrained image deblurring.

- The development and analysis of new multilevel preconditioners for the fast solution of linear systems arising in regularized, linearized inverse problems.
- The application of penalized least squares regularization methods for the joint estimation of object and phase (wavefront profile) from multiframe and multichannel phase diversity image data.
- The development of two software packages. One package is a collection of MATLAB routines for image deblurring. The second consists of MATLAB codes for the simulation and inversion of atmospheric phase and image data. The second package has been translated into FORTRAN and implemented on supercomputers at the Air Force's Maui High Performance Computing Center.

Preprints and reprints of papers prepared under this project and computer software prepared under this project can be downloaded directly from the PI's web site at

<http://www.math.montana.edu/~vogel/>

To get the papers, click on [Publications](#). To get the software, click on [Software](#).

A PhD thesis was written by Kyle Riley, a student directed by the PI and supported under this project. This thesis can also be downloaded from the PI's web site under [Publications](#).