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A STUDY OF SIMPLE WAVES

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J. Hardy

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ABSTRACT

A method for construction of simple waves is presented. Attention is focused on forward-facing compression waves, specifically at the points at which shocks develop, and what to expect concerning shock strength, etc. Discussion is limited to γ-law gases, although the technique is general.

INTRODUCTION

Section 1 gives the construction of simple wave solutions for γ-law gases. The method is general; restriction to γ-law gases is for convenience. Velocity, density, ..., are constant along a family characteristics presented. The region multiply covered by the family of characteristics is discussed along with the envelope of the family.

Section 2 presents the solution (complete with graphs) to several piston motions, illustrating the properties of the most general solutions.

Section 3 shows that shocks develop only on the envelope of the characteristics. The local behavior of the envelope is given, depending on the sign of a certain differential inequality. The behavior is examined specifically for a neighborhood of zero time. Means are presented for analysis of a general flow.

SECTION 1

The hydrodynamic equations for one-dimensional flow are

$$\begin{aligned} &\rho_t + u\rho_x + \rho u_x = 0, \\ &u_t + uu_x + \frac{1}{\rho} p_x = 0, \\ &\epsilon_t + u\epsilon_x + \frac{1}{\rho}(pu)_x = 0, \end{aligned} \tag{1}$$

where $\rho(x, t)$ is the density, $u(x, t)$ is the material speed, $\epsilon(x, t)$ is the specific energy density, and $p(x, t)$ is the pressure.

In the case of isentropic flow, the system yields $p = p(\rho)$; and for a γ-law gas, $p = p_0\left(\frac{\rho}{\rho_0}\right)^{\gamma}$. Under the enumerated circumstances

$$\begin{aligned} &\rho_t + u\rho_x + \rho u_x = 0, \\ &u_t + uu_x + \frac{c^2}{\rho}\, \rho_x = 0, \end{aligned} \tag{2}$$

where $c^2 = \gamma p/\rho$.

The interest is in solutions to this latter set of equations. The flow considered will be that of a piston pushing into a gas at rest, the piston speed $g(t)$ being such that the flow is everywhere continuous.

The general character of these solutions is well known. They are simple waves, flows whose image in the hodograph plane (ρ, u) is a curve. If $g(t)$ is such that the piston starts from zero speed (necessary) and moves to the right, the following relations among u, c, p, ρ, u_0, c_0, p_0, ρ_0 are known to hold (u_0, ρ_0, c_0, p_0 are the rest values):

$$p = p_0\left(1 + \frac{\gamma - 1}{2}\,\frac{u - u_0}{c_0}\right)^{2\gamma/(\gamma-1)},$$

$$\rho = \rho_0\left(1 + \frac{\gamma - 1}{2}\,\frac{u - u_0}{c_0}\right)^{2/(\gamma-1)}, \tag{3}$$

$$c = c_0 + \frac{\gamma - 1}{2}\,(u - u_0),$$

relations which hold for a foreward-facing simple wave.

Since the value of u_0 makes no difference in any arguments, we shall assume $u_0 = 0$. Substituting the expression for ρ as a function of u into the continuity equation

$$\rho_t + u\rho_x + \rho u_x = 0,$$

we obtain

$$u_t + \left(c_0 + \frac{\gamma + 1}{2}\,u\right)u_x = 0, \tag{4}$$

a quasi-linear partial differential equation whose solution $u(x, t)$ is constant along the characteristic curves

$$\frac{dt}{1} = \frac{dx}{c_0 + \frac{\gamma + 1}{2}\,u}\,.$$

These are straight lines, since u is constant along them.

Consider now the initial value problem (see Fig. 1). At $t = 0$, $u = 0$ along the x axis. Along $x(t)$, $u = g(t)$, the piston speed. Along $t = 0$, the characteristics are the lines $x = x_0 + c_0 t$, and along these lines u, the solution to equation (4) is zero.

At time t_0, the piston is at $x(t_0) = \int_0^{t_0} g(\xi)\,d\xi$ with speed $g(t_0)$. The characteristic passing through this point is

$$x = x(t_0) + \left[c_0 + \frac{\gamma + 1}{2}\,g(t_0)\right](t - t_0), \tag{5}$$

and along it $u = g(t_0)$. Thus, a solution has been constructed, a ruled surface, giving $u(x, t)$ for those points in the (x, t) plane for which the simple wave is a valid solution. Since near $t = 0$, $g(t) \geq 0$, the one-parameter family of characteristics constructed does

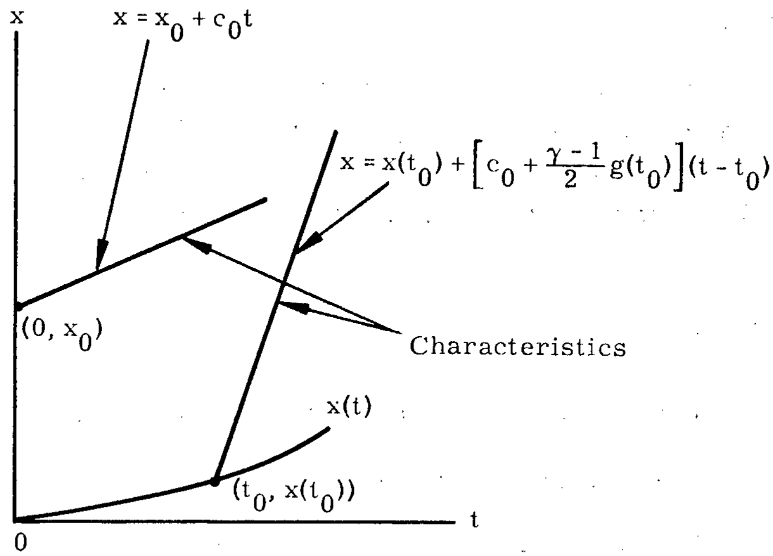

Fig. 1. Construction of characteristics.

not cover the plane simply. Some points in the (x, t) plane must be covered at least twice. It is the implication of this we wish to investigate. Note that if g is discontinuous, double-valued points (x, t) exist beyond this time. The double-valued region for $u(x, t)$ gives physically unacceptable solutions. At some points of the boundary of this region, a shock will develop. The simple wave ceases to be a valid solution to the system (1) in the whole (x, t) plane.

Let the piston be located at $x_1(t)$ at the time t, $x_1(t) = \int_0^t g(\xi)\, d\xi$, $g(\xi)$ is monotone increasing. The characteristic passing through $(t, x_1(t))$ intersects the characteristic through $(0, 0)$, the wave front traveling with velocity c_0, at a time

$$\tau_1(t) = \frac{2}{(\gamma +)g(t)}\left[t(c_0 + \frac{\gamma + 1}{2} g(t) - x(t)\right]. \tag{6}$$

Let t tend to zero. If $g'(0) \neq 0$, then

$$\tau_1(0) = \frac{2c_0}{(\gamma + 1)g'(0)} . \tag{7}$$

Otherwise, the demand that $g(t) > 0$, $t > 0$, requires that

$$\lim_{t \to 0} \tau_1(t) = +\infty .$$

The characteristics passing through $(t, x_1(t))$ and $(t', x_1(t'))$ intersect at a time

$$\tau_2(t, t') = \frac{2(t - t')}{(\gamma + 1)[g(t) - g(t')]}\left[c_0 - \frac{1}{t - t'}\int_{t'}^{t} g(\xi)\, d\xi + \frac{\gamma + 1}{2}\, \frac{g(t)t - g(t')t'}{t - t'}\right]. \tag{8}$$

The envelope (τ_2, x_2) of such lines is given by the limit as t' approaches t,

$$\tau_2(t) = \frac{2}{(\gamma + 1)g'(t)}\left[c_0 + \frac{\gamma - 1}{2} g(t) + \frac{\gamma + 1}{2} t g'(t)\right] \tag{9}$$

where $g'(t) \neq 0$, or, if $g'(t) = 0$,

$$\lim_{t' \to t} \bar{\tau}_2(t) = +\infty . \tag{10}$$

The coordinate x for the envelope is obtained from substituting $\tau_2(t)$ in equation (5).

For the class of monotone increasing functions g(t) (giving rise to forward-facing compression waves) the time at which a shock appears in the flow, a time corresponding to $\inf_{(t)}(t, x)$ for all (t, x) in the region doubly covered by the characteristics, is given by the time

$$\tau_2 = \inf_{t \geqq 0} \tau_2(t). \tag{11}$$

$x_2 = \inf_{(\tau_2)} x(\tau_2)$ located by equation (5). For purposes of examining these functions, the following easily developed formulae are useful:

$$\frac{d}{dt}\tau_1(t) = \frac{\gamma - 1}{\gamma + 1} + \frac{2c_0}{(\gamma + 1)\, g(t)} - \frac{g'(t)}{g(t)^2}\,(c_0 - x_0), \tag{12}$$

$$\frac{d\tau_2(t)}{dt} - \frac{2}{\gamma + 1}\left\{\gamma - \frac{g''(t)}{[g'(t)]^2}\left[c_0 + \frac{\gamma - 1}{2}\, g(t)\right]\right\}, \tag{13}$$

$$\frac{d}{dt}\,[x(\tau_2) = c_0\tau_2] = g(t)\left\{\gamma - \frac{g''(t)}{[g'(t)]^2}\left[c_0 + \frac{\gamma - 1}{2}\, g(t)\right]\right\}, \tag{14}$$

$$\frac{d}{dt}\left\{x(\tau_2) - \left[c_0 + \frac{\gamma + 1}{2} g(t_1)\right]\tau_2\right\} = [g(t_0) - g(t_1)]\left\{\gamma - \frac{g''(t)}{[g'(t)]^2}\left[c_0 + \frac{\gamma - 1}{2} g(t)\right]\right\}, \tag{15}$$

where $t_1 \neq t$ in equation (14).

SECTION 2

Prior to considering general classes of problems, we will consider several examples.

Example 1: $c_0 = 1$, $g(t) = 2t$, $\gamma = 2$.

The solution to the problem is given by

$$u(x, t) = \begin{cases} 0, & x = x_0 + t, \ x_0 \geq 0, \\ 2t_0, & x = t_0^2 + (1 + 3t_0)(t - t_0), \ t_0 > 0, \end{cases}$$

ρ and p being given by equation (3), and it solves the piston problem in the region of single valuedness. This solution is plotted in Fig. 2. The piston path, some characteristics, and the point where the shock originates are indicated. Suppose $t' > t > 0$. Formula (8) gives the time of intersection for the characteristics passing through the piston at these times as

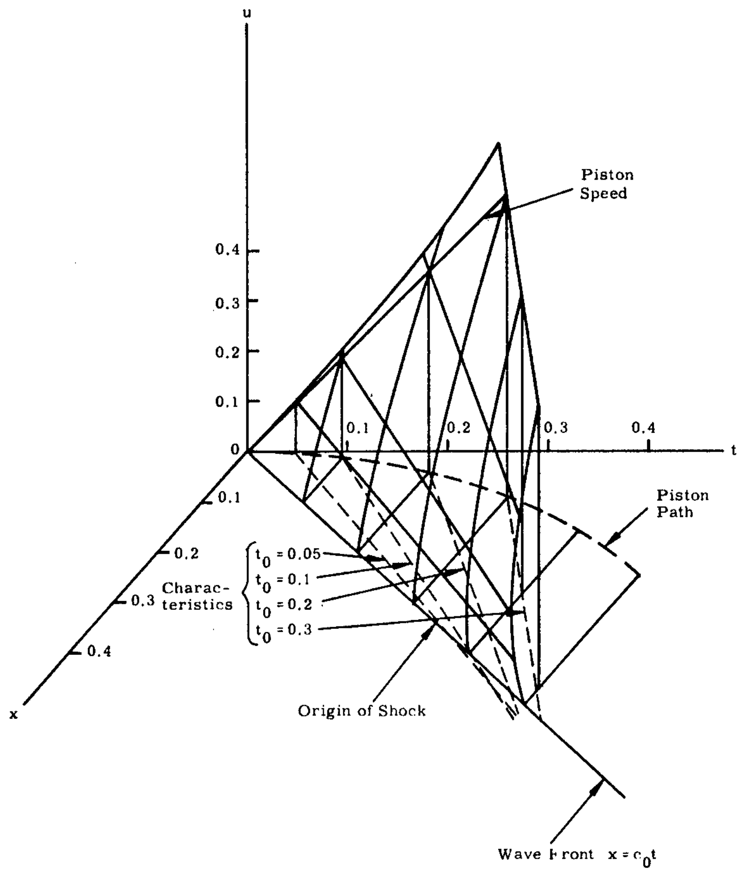

Fig. 2. Example 1.

$$\overline{\tau}_2(t, t') = \frac{2}{3}(t + t') + \frac{1}{3} \geq \frac{1}{3},$$

with equality occurring for $t = t' = 0$. Thus, any characteristic passing through a point near the origin intersects the wave front at a time $\tau \approx 1/3$ and at a point $x(\tau) \approx 1/3$, these values being the limiting values. For $x > 1/3$, $t > 1/3$, $u(x, t)$ is double-valued on the wave front. At $t = 1/3$, a shock starts at $x = 1/3$, with jump in velocity growing continuously from zero.

Example 1 is a special case of $g(t) = 2at$, $a > 0$, where always the infimum of the times of intersection of pairs of characteristics is $\tau = \frac{1}{3}a$, the shock always originating on the wave front. Note that this point recedes to infinity as a tends to zero.

Example 2: $c_0 = 1$, $\gamma = 3$, $g(t) = (1 - 2t)^{-1/2} - 1$. The characteristics all pass through the point $(1/2, 1/2)$. Also, $\lim_{t \to 1/2 - 0} g(t) = +\infty$. This solution is graphed in Fig. 3. The example ceases to be of interest at $t = 1/2$.

From it, an interesting example can be constructed. Let

$$g^*(t) = \begin{cases} (1 - 2t)^{-1/2} - 1, & 0 \leq t \leq 1/4, \\ \sqrt{2} - 1, & t > 1/4. \end{cases}$$

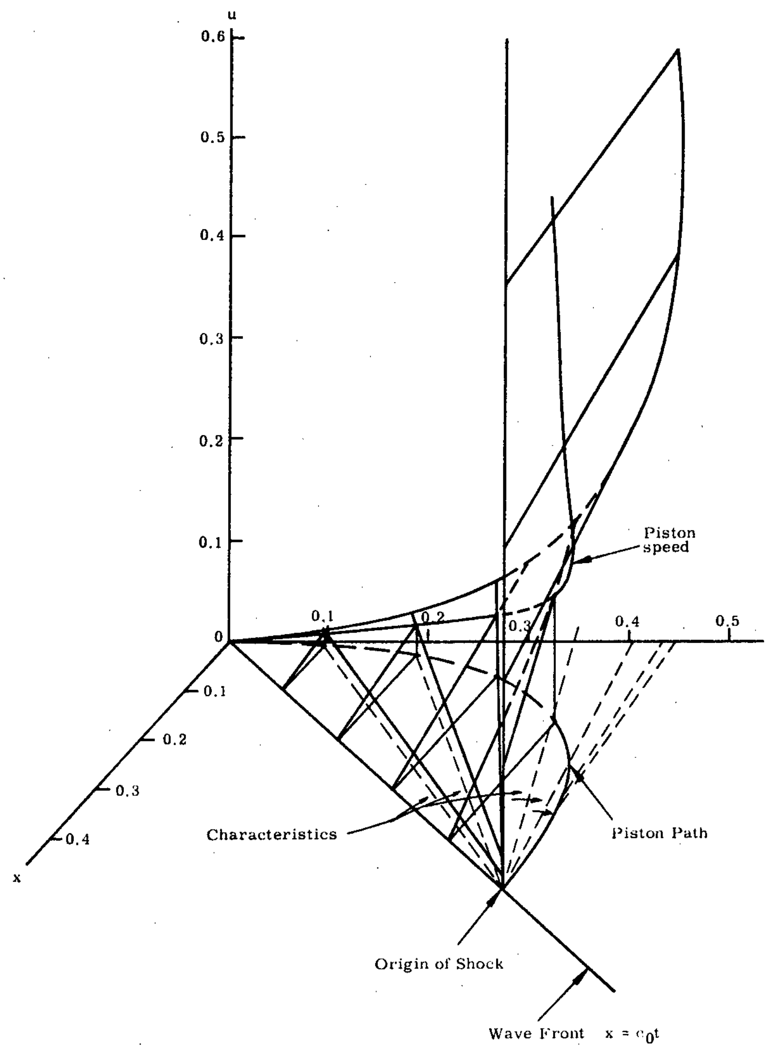

Fig. 3. Example 2.

For $t > 1/4$, the piston follows a straight line in the (x, t) plane, and the characteristics are all parallel straight lines, intersecting $x = c_0 t$ at times at least $1/4$. Along these $u(x, t)$ has the value $\sqrt{2} - 1$. The solution thus consists of this plane parallel to the (x, t) plane, plus that section of example 2 corresponding to characteristics for time $t \leq 1/4$.

At $t = 1/2$, a pair of shocks appear at $x = 1/2$, one with jump in velocity smaller than $\sqrt{2} - 1$ but still positive. This shock proceeds into the undisturbed medium with this initial shock strength. Characteristically, a weaker shock proceeds from this point back into the disturbed medium – toward the piston. A contact discontinuity is present always at the point of origination of these two shocks. Until the weak shock reaches the piston, the shock velocities are constant, the states behind each being constant.

Example 2 is an example of a piston velocity with an important separation property to be discussed directly.

Example 3: $c_0 = 1$, $\gamma = 3$, and $g(t)$ defined by

$$g(t) = \begin{cases} \frac{1}{2}t, & 0 \le t \le 0.3, \\ 4t - 1.05, & t > 0.3, \end{cases}$$

a continuous function with jump in $g'(t)$ at $t = 0.3$. The solution is graphed in Fig. 4 complete with characteristics and piston path. Examination of the characteristics through the formulae (8), (12), and (13) shows that the double-valued region in the range of u has a point on its boundary at $t = 0.44375\ldots$ and $x = 0.20937\ldots$, all other points on its boundary (and interior) corresponding to larger times. This point is on the characteristic passing through the piston location at time $t = 0.3$; $u(x, t)$ has a discontinuous derivative at the piston at $t = 0.3$ and this discontinuity propagates along this characteristic.

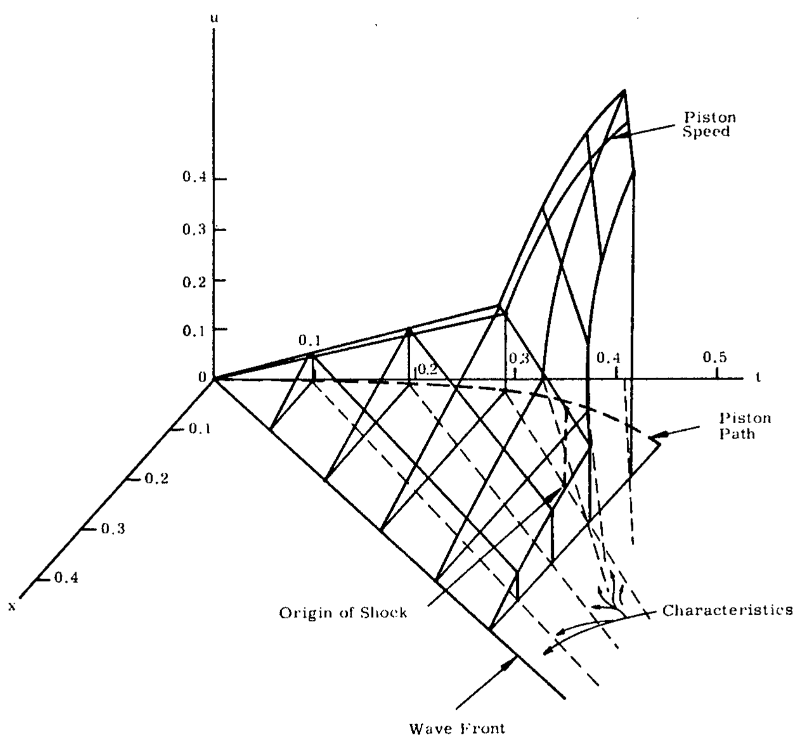

Fig. 4. Example 3.

A shock originates at this boundary point with shock strength (jump in velocity, etc.) growing continuously from zero. Observe that this shock originates in the region of continuous flow well behind the wave front.

From subsequent discussion it will be apparent that it would be possible to modify $g(t)$, $t > 0.3$, in such a fashion that a shock (see Example 2) appears at this point $t = 0.44375$, $x = 0.20937$, with a nonzero jump in velocity.

Another important observation is that the shock can originate in the region of continuous nonconstant flow (behind the wave front) without a discontinuity in any derivative of $g(t)$ occurring. In example 3, take

$$g(t) = t^{4/3}, \quad t > 0.$$

The boundary point of the region of double values with minimum time occurs at $t = 0.97847 \ldots$, $x = 0.9642 \ldots$, which is a point in the region of continuous nonconstant flow.

These examples exhibit the important properties of the solutions to piston problems which persist for a time as simple waves, with shocks then forming somewhere. Because of the arbitrariness of $g(t)$ and the possibility of patching functions of different characters together, flows with quite varied behavior are possible, as for example, simultaneous appearance at $t = t_0$ of n shocks of varying initial shock strengths, etc. (sum of the jumps of velocity at the shocks is strictly less than $g(t)$).

Before discussing local behavior of the envelope, let us show that all candidates for infima are provided by the envelope of characteristics.

SECTION 3

Consider Fig. 5, and piecewise smooth $g(t)$, $g'(t) \neq 0$. Let C and C' be a pair of characteristics corresponding to times t and t'. The slope of the characteristics is a monotone increasing function of this time, the time they pass through the piston location. Thus, for the time $t_1 = (t + t')/2$ the slope is intermediate. It must intersect both C and C' at times that may be equal but can be no later than the time corresponding to the intersection of C and C'. Choose the earlier time. Pick a time t_2 equidistant from t_1 and the time corresponding to the characteristic giving the earliest time. The characteristic C_2 corresponding to t_2 then intersects C_1 and either C or C' (whichever gave the least time) at a time no later than the time of intersection of this later pair. Thus, continuing, we construct a convergent sequence $t_1, t_2, \ldots, t_n \ldots$, the sequence of whose pairs of characteristics is converging, and the sequence of whose points of intersection is converging to a point on the envelope of the characteristics.

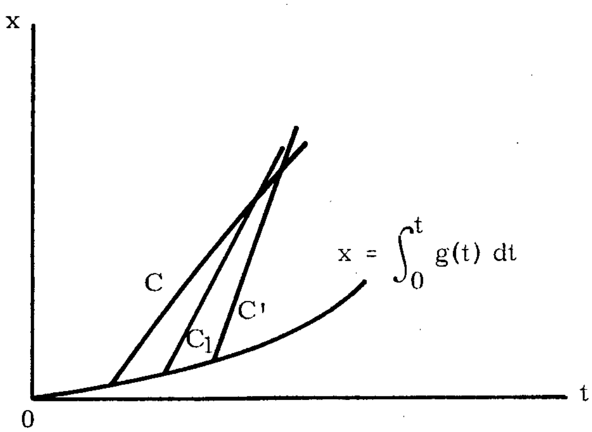

Fig. 5.

Thus, for any such pair C, C' of characteristics, no matter how remote, there is a point on the envelope of the characteristics whose time is not larger than the time of intersection of the pair.

The time $\tau_1(t)$ is the time of intersection of the characteristic corresponding to t with C_0 (Fig. 6); hence, for each time, by the above, there is a value of $\tau_2(t)$ which is no larger. Thus, the infimum sought over the whole field of intersection characteristics will be furnished by $\inf_{t \geq 0} \tau_2(t)$.

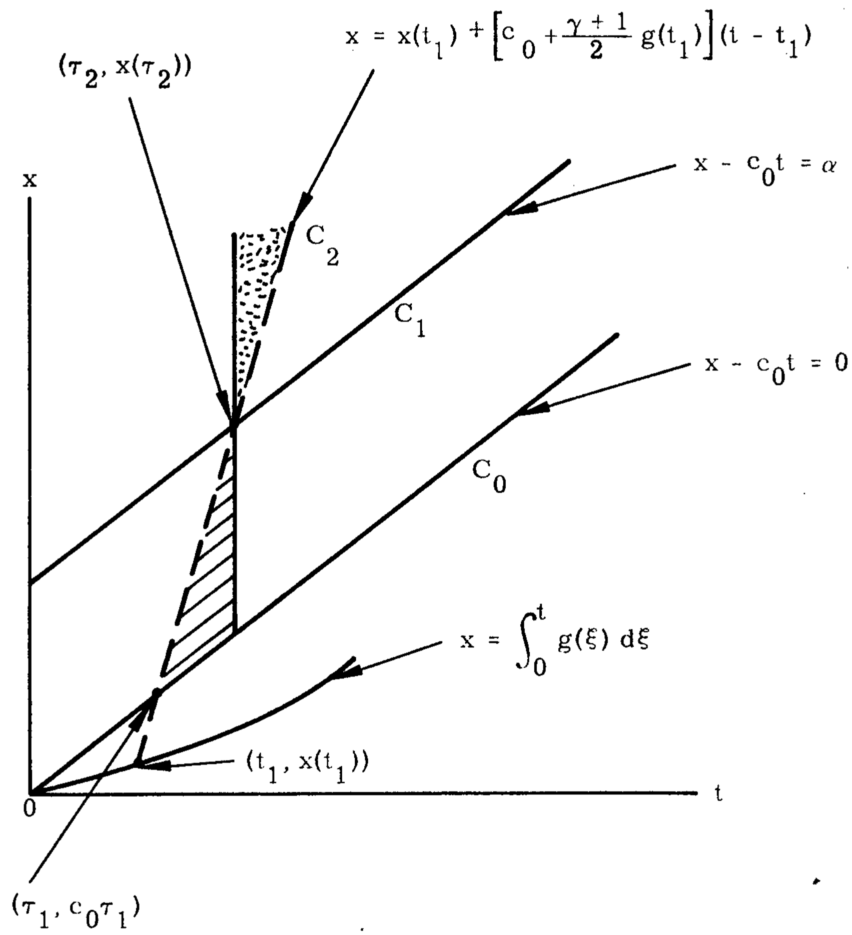

Fig. 6.

Turn now to a discussion of local behavior. Suppose all characteristics corresponding to a time $t_0 > 0$ pass through the same point $(\tau, x(\tau))$. The function g(t) from equation (9) must satisfy the differential equation

$$\frac{2c_0}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} g(t) + (t - \tau)g'(t) = 0$$

which has a solution for $t_0 \leq t < \tau$,

$$g_1(t) = \left[\frac{2}{\gamma - 1} c_0 + g(t_0)\right] \left(\frac{\tau - t_0}{\tau - t}\right)^{\frac{\gamma - 1}{\gamma + 1}} - \frac{2c_0}{\gamma - 1}. \tag{16}$$

This solution satisfies the equation

$$\gamma - \frac{g''}{g'^2}\left[c_0 + \frac{\gamma - 1}{2} g(t)\right] = 0. \tag{17}$$

As it happens, much can be learned concerning the local behavior of intersecting characteristics (a neighborhood of characteristics about that corresponding to t_0) according as

$$\gamma - \frac{g''}{g'^2}\left[c_0 + \frac{\gamma - 1}{2} g(t_0)\right] \tag{18}$$

is positive, negative, or zero. If it is positive, in every sufficiently small neighborhood of t_0 to the left of t_0, $g(t) < g_1(t)$. If it is negative, the inequality is reversed. This property enables a discussion of the local behavior of the envelope of characteristics to be given.

Consider Fig. 6. Suppose that in a neighborhood of t_1, $g(t)$ is twice continuously differentiable and the expression (18) is positive. Then, from formulae (13), (14) and (15) it is clear that all of

$$\frac{d\tau_2(t)}{dt}, \frac{dx(\tau_2)}{dt}, \frac{d}{dt}\left[x(\tau_2) - c_0\tau_2\right], \text{and } \frac{d}{dt}\left\{x(\tau_2) - \left[c_0 + \frac{\gamma + 1}{2} g(t_1)\right]\tau_2\right\}$$

are positive. Thus, as t increases from t_1, $x(\tau_2)$ and τ_2 increase monotonically. Since $\frac{d}{dt}\left\{x(\tau_2) - \left[c_0 + \frac{\gamma + 1}{2} g(t_1)\right]\tau_2\right\}$ is increasing, the point $(\tau_2, x(\tau_2))$ is moving to the left of C_2, that is, into the dotted region. Thus $\tau_2(t_1)$ furnishes a minimum for $\tau_2(t_1)$, those values of t in the neighborhood of t_1 which are larger than t_1.

If the expression (18) is negative, by the same reasoning, the point is moving into the crosshatched region (to the right of C_2 with both τ_2 and $x(\tau_2)$ decreasing monotonically as t increases from t_1).

In either case, the curve τ_2, $x(\tau_2)$ is tangent to C_2 at t_1, since it is the envelope of these characteristics (it is also easily verified from equations (13) and (14) that $\frac{dx(\tau_2)}{d\tau_2} = c_0 + \frac{\gamma + 1}{2} g(t)$).

Observe that a local maximum or minimum in $\tau_2(t)$ corresponds to a cusp in the envelope.

We will now consider a neighborhood of the origin in which $g(t)$ is twice continuously differentiable (permitting $g''(t) \to \infty$ as $t \to 0$) and g' can be zero only at the origin. Observe that if $g'(0) \neq 0$, $\tau_1(0) = \tau_2(0) = 2c_0/(\gamma + 1)g'(0)$. If $g'(0) = 0$, $\tau_1 \to +\infty$ as $t \to 0$ but remains finite for all subsequent values of $g(t)$. $\tau_2(t)$ tends to infinity as t tends to any value of for which $g'(t)$ is zero.

If $g'(0) > 0$, then, according as the quantity (18) is positive or negative, $\tau_2(t)$, $x(\tau_2)$ moves to the left or right of C_0, starting from the point $2c_0/(\gamma + 1) g'(0)$, $2c_0^2/(\gamma + 1)g'(0)$. In the first case both $\tau_2(t)$ and $\tau_1(t)$ increase and a minimun is furnished by $\tau_2(0)$. In the second case, $\tau_2(t)$ is monotone decreasing, so the infimum is given by $\inf \tau_2(t)$ for t approaching $\sup t$, t in the neighborhood.

If $g'(0) = 0$, diverse pathological situations can occur. It is expeditious to consider the case where $g(t) = t^\alpha \phi(t)$, $\phi(0) \neq 0$, ϕ twice continuously differentiable in a neighborhood of $t = 0$, and omit discussion of any pathological situations. (For $\alpha < 1$, $g'(0) = +\infty$, examination shows that a shock appears at the piston at $t = 0$, growing continuously from zero strength.) The interest is in those cases where $\alpha > 1$. In this case,

$$\tau_1(t) - \tau_2(t) = \frac{2}{\gamma + 1} \left\{ \frac{c_0}{t^{\alpha-1}} \left[\frac{1}{\phi(t)} - \frac{1}{\alpha\phi(t) + t\phi'(t)} \right] \right.$$

$$\left. - \frac{1}{t^\alpha \phi(t)} \int_0^t \xi^\alpha \phi(\xi)\, d\xi - \frac{\gamma - 1}{2} \frac{t\phi(t)}{\alpha\phi(t) + t\phi'(t)} \right\} .$$

Since

$$\frac{1}{t^\alpha \phi(t)} \int_0^t \xi^\alpha \phi(\xi)\, d\xi = O(t), \quad \frac{t\phi(t)}{\alpha\phi(t) + t\phi'(t)} = O(t),$$

and

$$\lim_{t\to 0} \left[\frac{1}{\phi(t)} - \frac{1}{\alpha\phi(t) + t\phi'(t)} \right] = \frac{1}{\phi(0)} \left(1 - \frac{1}{\alpha} \right) > 0,$$

there exists a neighborhood of $t = 0$ such that

$$\tau_1(t) - \tau_2(t) > 0;$$

that is, in this neighborhood $x_2(\tau_2)$, τ_2 is to the left of C_0. Both $\tau_1(t)$ and $\tau_2(t)$ tend to $+\infty$ as t tends to zero. Examination of $x(\tau_2)$, τ_2 shows that it is asymptotic to C_0.

Three final observations give sufficient information, coupled with the previous information, to find the infima and the character of the shock which will occur at these various points.

If, in a neighborhood, $g(t)$ satisfies equation (17), the portion of the envelope corresponding to this neighborhood is a point. If this point is outside the region of influence of any other shock which forms in the flow, an outgoing shock appears at this point with jump in velocity less than the jump of $g(t)$ for the set of intersecting characteristics. A weaker ingoing shock also forms.

If $g'(t)$ goes to zero at time t_1, $\tau_2(t)$ goes to $+\infty$, and the envlope goes to infinity asymptotic to the characteristic corresponding to t_1, $x = x_1(t_1) + \left[c_0 + \frac{\gamma+1}{2} g(t_1)\right](t - t_1)$.

If, while $g(t)$ remains continuous at t_1, $g'(t)$ is discontinuous at t_1 but limits of $g'(t)$ exist to the right and left of t_1, the value of $\tau_2(t)$ is discontinuous at t_1 (see equation (9)), as is that of $x(\tau_2)$. Note, however, that the points on the envelope as t approaches t_1 from the left and right are both on the characteristic corresponding to t_1.

The discontinuity in $g'(t)$ can cause the envelope to shift anywhere along the corresponding characteristic, even to the piston front (or behind).

The spectrum of possibilities arising from simple waves should now be apparent. We construct now a $g(t)$ with two infima, both occurring at time τ_2. Let $g(t)$ be linear,

$g(0) = 0$, up to the time t_1. Pick $g'(t_1 + 0)$ so that the corresponding point on the envelope τ_2, $x(\tau_2)$ is τ_2, $x(t_1) + \left[c_0 + \frac{\gamma+1}{2} g(t_1)\right](\tau_2 - t_1)$ and continue $g(t)$ according to equation (16) up to a time $t_2 < \tau_2$. Set $g(t) = g(t_2)$, $t > t_2$. The envelope is as shown in Fig. 7, plus the point at infinity along the characteristic corresponding to t_2.

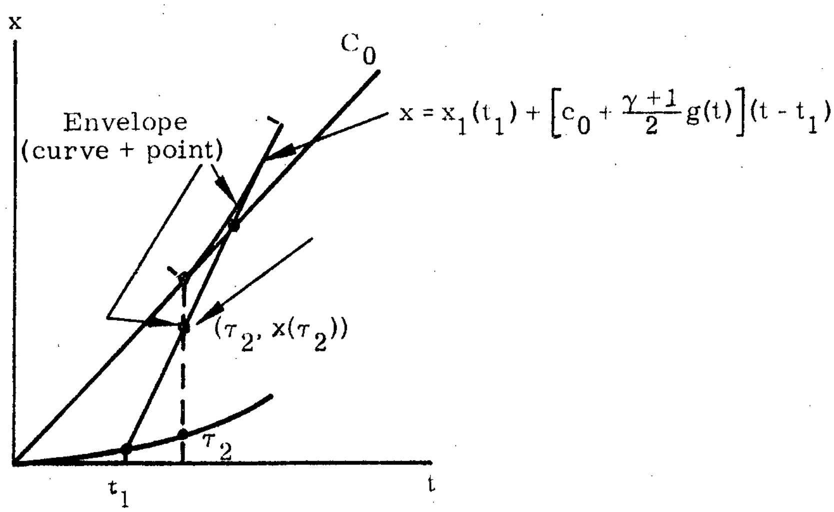

Fig. 7.

At time τ_2, a shock forms on C_0, growing from zero shock strength. Simultaneously at τ_2, $x(t_1) + c_0 + \frac{\gamma+1}{2} g(t_1)(\tau_2 - t_1)$, a pair of shocks with positive jump in velocity appears. Note that a contact discontinuity appears with the latter pair of shocks.

Outside the range of influence of the shocks, the simple waves remain the solution to equations (1).

SUMMARY

The method of construction which is presented is valid for all simple waves. Attention is focused on forward-facing compression waves. Means are developed for the examination of the infima of the time for points in the region multiply covered by the characteristics.

The author has made cursory examination of the existence theory for solutions of equations (1) for times later than that of the appearance of the shock. Adding the equation

$$X = X(\tau_2) + \int_{\tau_2}^{t} U\,dt, \quad U \text{ the shock velocity,} \tag{19}$$

to the Volterra integral equations for continuous flow (requiring the Hugoniot conditions hold across the path, velocity and pressure continuous behind the shock), existence of a solution with a shock proceeds readily for times in a neighborhood of the time the shock appears. Moreover, this is all that is to be expected. Return to the simple wave whose envelope appears in Fig. 7. Had $g'(t, +0)$ been such that its point on the envelope corresponds to a time slightly later than τ_2, the corresponding shocks develop at this slightly later time.

Only by adding (for a forward-facing compression wave with "weak" piston velocity) equation (19) with U defined by

$$U(X, t) = \lim_{\epsilon \to 0} \frac{\rho(X + \epsilon, t)u(X + \epsilon, t) - \rho(X - \epsilon, t)u(x - \epsilon, t)}{\rho(X + \epsilon, t) - \rho(X - \epsilon, t)}$$

(with the additional requirements above on density, velocity, . . .) at every point of the fluid does it seem possible to discuss existence theory in any global sense (all time). Even this assumes something about piston strength. That is, no double shocks, as in example 2, occur. In this later case, it would be necessary to define two shock velocities, U^- and U^+, at each point, and accommodate a more complex algebraic problem.

Estimates for growth of shock strength in terms of $g(t)$ are possible, and will appear in a subsequent report.