NACA TN 2201

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2201

MEASUREMENT OF THE MOMENTS OF INERTIA OF AN AIRPLANE BY A SIMPLIFIED METHOD

By Howard L. Turner

Ames Aeronautical Laboratory
Moffett Field, Calif.

Washington
October 1950

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2201

MEASUREMENT OF THE MOMENTS OF INERTIA OF AN AIRPLANE BY A SIMPLIFIED METHOD

By Howard L. Turner

SUMMARY

A simplified method for the experimental determination of the moments of inertia, product of inertia, and inclination of the principal axes; the associated equipment and techniques; and the application of this method to a conventional 13,000-pound airplane are described. Measurements were made with the landing gear retracted for full and empty fuel conditions. The equipment, which consisted primarily of knife-edge supports and restraining springs for the pitch and roll axes and a single-shaft torsional pendulum for the yaw axis, was designed for increased accuracy as well as for simplicity of operation and ease of handling as compared with previous methods. At no time was it necessary to hoist or jack the airplane in an abnormal fashion.

Analysis showed the maximum possible error of the inertia measurements to be ±1.7, ±1.2, and ±0.6 percent of the true moments of inertia about the X, Y, and Z axes, respectively. For each suspension system, measured moments of inertia of known masses of simple form agreed within 0.5 percent with the calculated values.

The results of brief tests have indicated that suitable application of the torsional pendulum would permit evaluation of the inclination of the principal axes to within less than $\pm 0.1^{\circ}$, which corresponds to an error of less than ±35 slug-feet squared in the product of inertia of the test airplane.

INTRODUCTION

The dynamic-stability problems accompanying the unusual configurations and the increases in the relative density of modern aircraft, and the application of rational design procedures to servomechanism installations necessitate an accurate knowledge of the dynamic response characteristics of the airplane. These response characteristics, in turn, are dependent upon the accurate evaluation of product of inertia (or inclination of the principal axes) and moments of inertia. The effect of product of inertia was usually neglected in earlier dynamic-stability work, but in recent years this effect has become more important and can no longer be ignored (references 1 and 2).

The practical problems involved in the experimental determination of the moments of inertia and product of inertia have become critical with modern aircraft. It has been the practice to suspend and swing the airplane as a compound and as a bifilar pendulum, and to correct the resulting data for the displacement of the axes of oscillation from the body axes through the center of gravity of the airplane (references 3 to 6). It is difficult to find a structure from which airplanes weighing over 10,000 pounds can be suspended for swinging. Usually a building with sufficient strength and space to permit swinging has such a high overhead structure that the hoisting and handling problems become unreasonable as accuracy considerations for a compound pendulum require the shortest possible pendulum lengths. Even with short pendulum lengths, which might be obtained by hoisting and swinging the airplane high above the hangar floor, the corrections required for the transfer of axes alone (as shown by the data in references 4 to 7) would be as high as 200 to 700 percent of the final results. Hence, it can be seen that the accuracy of the results using such a swinging system would be dependent upon small differences in large numbers.

In view of the structural, hoisting, handling, and accuracy problems involved, it appeared impractical to extend these swinging methods to the larger and heavier aircraft of the present and future. These practical difficulties led the Cornell Aeronautical Laboratory to employ a system of pivots and springs to measure the moment of inertia about the pitch axis of a B-25J airplane (reference 7).

When the problem of measuring the moments of inertia and product of inertia of a 13,000-pound airplane arose, it was decided to design and install equipment that could be used to measure the moments of inertia of aircraft weighing up to 20,000 pounds. This equipment was to be so designed that the axes of oscillation would be on or as near as possible to the body axes of the airplane. The necessity of hoisting and swinging the airplane high above the hangar floor was to be eliminated. Handling problems were to be reduced to the point where only the handling and jacking techniques such as normally used for checking retractable landing gear would be employed. The equipment was to be flexible in principle to allow its use on any modern aircraft with a minimum amount of special fittings.

A description of this moment-of-inertia gear and its application to the measurement of the moments of inertia and product of inertia of a 13,000-pound airplane are given in this report. During activities not associated with this program, the torsional pendulum was damaged prior to the completion of an accurate determination of the airplane product of inertia. Rather than delay a flight program scheduled for the test airplane, the inertia measurements were discontinued. However, sufficient product-of-inertia data were obtained to warrant discussion at this time.

SYMBOLS

Refer to figures 1 and 2 for clarification of the definition of certain symbols. The notation of reference 8 was used as a basis for the symbols used in this report.

A aspect ratio of the surface $\left(\frac{b^2}{S}\right)$

C_X, C_Y static spring constants of the restraining springs for the X- and Y-axes oscillations, respectively, pounds per foot

C_Z equivalent spring constant of torsional pendulum, supporting roof truss and airplane cradle, foot-pounds per radian

D_Γ dihedral-angle correction factor

D_λ plan-form taper-ratio correction factor

I_X, I_Y, I_Z moments of inertia about the roll, pitch, and yaw axes, respectively (the axes are further defined by subscripts ref, prin, etc.), slug-feet squared

I_{XZ} product of inertia, slug-feet squared

I_{X_θ} moment of inertia about an axis in the XZ plane, parallel to the axis of oscillation, inclined from the X-body reference axis by an angle θ, and passing through the airplane center of gravity, slug-feet squared

I_{Z_G} moment of inertia of the Z-axis torsional-pendulum gear (includes the pendulum shaft and the airplane support cradle)

L_f fuselage length, feet

l_X perpendicular distance from center line of the restraining spring to the X axis of oscillation, feet

L_y perpendicular distance from the center line of the restraining spring to the Y axis of oscillation, feet

M.A.C. mean aerodynamic chord of wing $\left(\frac{\int c^2 \, dy}{\int c \, dy}\right)$, feet

P period of oscillation, seconds

S area of the surface denoted by the subscript, square feet

V total volume of airplane, cubic feet

W airplane weight, pounds

b span of the surface denoted by the subscript, feet

c local chord, feet

$\bar{c}$ mean chord of the surface $\left(\frac{S}{b}\right)$, feet

d geometric average depth of the fuselage, feet

g acceleration due to gravity, 32.2 feet per second per second

h_x, h_y vertical component of the distance from the X and Y axes of oscillation, respectively, to the airplane center of gravity, feet

k coefficient of additional mass of an equivalent flat rectangular plate

k' coefficient of additional moment of inertia of an equivalent flat rectangular plate

k_{f_y}, k_{f_z} coefficients of additional mass of an equivalent fuselage ellipsoid for motion along the Y and Z axes, respectively

k'_{f_y}, k'_{f_z} coefficients of additional moment of inertia of equivalent fuselage ellipsoid about the Y and Z axes, respectively

l_{f_x} perpendicular distance in the vertical plane from the X axis of oscillation to the centroid of the side area of the fuselage, feet

l_{f_y} component of distance in the XY plane of the perpendicular distance between the Y axis of oscillation and the centroid of the top area of the fuselage, feet

l_{f_Z}	perpendicular distance in the vertical plane from the Z axis of oscillation to the centroid of the side area of the fuselage, feet
l_{t_Y}	component of distance in the XY plane of the perpendicular distance between the centroid of the horizontal-tail area and the Y axis of oscillation, feet
l_{t_Z}	perpendicular distance from the Z axis of oscillation to the centroid of area of the vertical tail, feet
l_X, l_Y	perpendicular distance from the X and Y axes of oscillation, respectively, to the airplane center of gravity, feet
m	mass $\left(\frac{W}{g}\right)$, slugs
w	geometric average width of the fuselage, feet
ϵ	angle in the XZ plane between the X-body reference axis and the X principal axis, positive when the reference axis is nose up, degrees
θ	angle between the X-body reference axis and an inclined axis in the XZ plane, positive when the reference axis is nose up, degrees
ρ	air density at test altitude, slugs per cubic foot

Subscripts

add mass	additional mass
fuse	fuselage
ht	horizontal tail
knife edge	axis of oscillation
meas	as measured (uncorrected for transfer of axes, additional mass, etc.)
prin	principal axis
ref	body reference axis passing through airplane center of gravity

vt vertical tail

wing wing

θ axis in XZ plane inclined from body reference axis by an angle θ

1 load condition 1

2 load condition 2

APPARATUS, TESTS, AND ANALYSIS METHODS

Moments of Inertia About Roll and Pitch Axes

Of the methods of measuring moments of inertia considered, the most promising from the practical and the accuracy standpoints appears to be a system whereby the airplane is pivoted about an axis of rotation located on the airplane structure and restrained from rotating about this axis by a spring. The moment of inertia about the axis of rotation is then a function of the spring constant, the location of the spring, and the period of the resulting oscillation. The apparatus used in these tests, the manner in which the tests were carried out, and the method of data analysis are described below.

The position of the airplane center of gravity was determined by weighing the airplane in a tail-up and tail-down attitude while holding a known reference point on the airplane at a fixed height. By geometry, the horizontal and vertical positions of the center of gravity with respect to this reference were calculated from the weight and balance data. The positions of the airplane center of gravity for the full and empty fuel conditions (load conditions 1 and 2, respectively) are given in Appendix A. A sketch of the airplane showing the center-of-gravity positions and other pertinent dimensions is given in figure 1.

Roll axis.- The airplane as set up for measuring the moment of inertia about the roll axis (X axis) is shown in figure 3. The two knife edges fixing the axis of oscillation were located in the plane of symmetry below and astride the center of gravity. The restraining springs were attached outboard on the front wing spar. As the knife edges were below the center of gravity, the springs were preloaded to stabilize the airplane in roll.

The hoisting and jacking of the airplane necessary to position it for testing were reduced to a minimum. The airplane was towed into position with the main landing wheels rolling up on low ramps. The restraining springs were secured and the tail was raised. Knife edges mounted on hydraulic jacks were then positioned under the V-block fittings attached to the airplane structure (fig. 4). The hydraulic jacks were used to raise the airplane to permit retraction of the landing gear, after which the jacks were lowered, positioning the airplane for test. This procedure was reversed to remove the airplane from the test position.

A standard NACA position recorder coupled with a 1/10-second timer was connected to the left wing tip. An oscillation was induced manually at the spring and photographic records of the time histories of the resulting oscillations were obtained. A double exposure of the oscillation in roll is shown in figure 5.

The moment of inertia about the axis of oscillation is given by

$$I_{X_{knife\ edge}} = \left(C_X\ L_X^2 - Wh_X\right)\left(\frac{P}{2\pi}\right)^2 \tag{1}$$

where P is the period of oscillation, and C_X is the sum of the static spring constants of the two springs. The moment of inertia at a given test attitude about a roll axis through the airplane center of gravity and parallel to the axis of oscillation as obtained from the measured moment of inertia about the knife-edge axis is given by the equation

$$I_{X_{ref}} = I_{X_{knife\ edge}} - I_{X_{add\ mass}} - \left(\frac{W}{g} + V\rho\right)\ l_X^2 \tag{2}$$

where $I_{X_{add\ mass}}$ is the moment of inertia due to the apparent additional mass effect of oscillation in a fluid medium (air) and the term $[(W/g)+V\rho]l_X^2$ represents the transfer of axes and the buoyancy and entrapped air corrections. Moments of inertia were measured about two axes in the plane of symmetry; one parallel to the body reference axis ($\theta=0^\circ$), and one inclined from the body reference axis ($\theta=7.60^\circ$). Use of these equations in the evaluation of the product of inertia is discussed later.

Pitch axis.- A knife-edge and restraining-spring method similar in principle and handling procedures to that for the roll axis was used to measure the moment of inertia about the pitch axis (Y axis). The airplane as set up for test is shown in figure 6. The V blocks were fastened to the rear wing spar aft of the center-of-gravity position, and the restraining spring was secured to the arrester-hook structure at the tail. The same knife edges on hydraulic jacks as used for the roll-axis measurements were employed. The knife-edge and V-block assembly is shown in detail in figure 7. The same instrumentation that was described for the roll-axis measurements was attached to the tail of the airplane to obtain photographic time histories of the oscillations.

The moment of inertia about the pitch axis passing through the airplane center of gravity is given by the equation

$$I_{Y_{ref}} = \left(C_y\ L_y^2 - Wh_y\right)\left(\frac{P}{2\pi}\right)^2 - I_{Y_{add\ mass}} - \left(\frac{W}{g} + V\rho\right) l_y^2 \qquad (3)$$

Moment of Inertia About Yaw Axis

A long pendulum length, so detrimental in the case of the compound pendulum, has a favorable effect in the case of a bifilar torsional pendulum. It can be shown that the accuracy of the bifilar-torsional-pendulum method is increased as the ratio of the suspension length to the distance between the bifilar supports is increased. It seemed logical to extend this principle to the point where a single-shaft torsional pendulum would be used for the yaw-axis oscillations; the axis of the pendulum shaft then would be the axis of oscillation. The single-shaft torsional pendulum has been used extensively in the past to measure the moments of inertia of small objects such as projectiles, missiles, and dynamic wind-tunnel models.

The torsional pendulum shown in figures 8 and 9 was made from a 4.5-inch-outside-diameter chrome-molybdenum steel tube with solid end fittings. The upper fitting was rigidly secured to a suitable roof truss. The lower fitting was connected to the airplane support cradle by a pin joint in such a manner that the airplane was free in pitch but restrained in roll by the bending of the shaft and in yaw by the twisting of the shaft. The legs of the cradle were bolted securely to primary structure of the airplane. Slots in the cradle beams permitted fore-and-aft adjustment of the legs to allow for various center-of-gravity positions. The restoring force was provided by the twisting of the shaft. The moment of inertia about the axis of the shaft is given by the equation

$$I_{Z_{meas}} = C_z \left(\frac{P}{2\pi}\right)^2 \qquad (4)$$

where C_z is the equivalent spring constant of the system. This torsional-pendulum spring constant in foot-pounds per radian was evaluated by measuring angular deflections resulting from known applied torques.

It should be noted that equation (4) is rigorous only when the axis of oscillation is a principal axis. When the axis of oscillation is not a principal axis, there is a coupling between the rolling and yawing motions and complex equations relating the two degrees of freedom must be considered. Preliminary estimates for the test airplane (verified later by the test results) indicated that the principal axis was displaced less than 4° from the axis of oscillation. Calculations showed that the effects of the rolling on the period of the oscillation in yaw would be negligible, so that equation (4) was a valid approximation in the present tests.

The pendulum was checked with a test frame having a mass and moment of inertia about the vertical axis similar to the airplane to be tested. Blocks of lead were added to the test frame so as to increase the moment of inertia about the vertical axis approximately 30 percent. It was found that the moment of inertia of the lead blocks as measured by the pendulum agreed within 0.40 percent with the calculated moments of inertia. A photograph of the calibration test frame on the pendulum is shown in figure 8. The moment of inertia of the torsional pendulum and cradle about the axis of the shaft was determined experimentally.

The airplane handling procedures were somewhat more complicated than those used for the knife-edge measurements. The cradle legs were bolted to the airplane and the airplane towed into position under the pendulum. The airplane was then lifted in a level attitude to join the legs to the cradle beam. The pendulum length was predetermined such that the distance the airplane was lifted was just sufficient to permit landing-gear retraction. The cradle was then adjusted so that the axis of the shaft was coincident with the Z-body reference axis. The airplane as set up for oscillating about the yaw axis is shown in figure 9.

The photographic recording instruments were attached to the tail of the airplane to measure the yawing oscillation. A torque was applied to the airplane and held until any undesirable motion had been damped out. The torque was then abruptly released and the airplane oscillated about the yaw axis. A double exposure showing the motion of the oscillation in yaw is shown in figure 10. The moment of inertia about the yaw reference axis passing through the airplane center of gravity is given by the equation

$$I_{Z_{ref}} = I_{Z_{meas}} - I_{Z_{add\ mass}} - I_{Z_G} \tag{5}$$

Inclination of and Moments of Inertia About the Principal Axes

Product of inertia and inclination of the principal axes.- It is assumed that the vertical plane passing through the center line of the airplane is a plane of symmetry. Hence the pitch axis is a principal axis, since it is perpendicular to the plane of symmetry, and, consequently, the products of inertia I_{XY} and I_{ZY} will be zero. In figure 2, let X_{ref} and Z_{ref} be the body reference axes, X_θ and Z_θ be a set of axes inclined from the body axes by a known angle θ, and the axes X_{prin} and Z_{prin} be the principal axes, inclined at an angle ϵ to the body reference axes. Then the moment of inertia about the X_θ axis is given by the equation

$$I_{X_\theta} = \int z'^2 \, dm = \int (z \cos \theta - x \sin \theta)^2 \, dm$$

$$= \int z^2 \cos^2 \theta \, dm - 2 \int (xz \sin \theta \cos \theta) \, dm + \int x^2 \sin^2 \theta \, dm$$

$$I_{X_\theta} = I_{X_{ref}} \cos^2 \theta + I_{Z_{ref}} \sin^2 \theta - 2 \, I_{XZ_{ref}} \sin \theta \cos \theta$$

so that the product of inertia referred to the body reference axis is

$$I_{XZ_{ref}} = \frac{I_{X_{ref}} \cos^2 \theta + I_{Z_{ref}} \sin^2 \theta - I_{X_\theta}}{2 \sin \theta \cos \theta} \tag{6}$$

Since, by definition of principal axes, $I_{XZ_{prin}}$ equals zero, from figure 2

$$I_{XZ_{prin}} = \int (x''z'') \, dm = \int (z \cos \epsilon - x \sin \epsilon)(z \sin \epsilon + x \cos \epsilon) \, dm = \cos \epsilon \sin \epsilon \left(\int z^2 \, dm - \int x^2 \, dm\right) + \cos 2\epsilon \int xz \, dm = 0$$

or

$$I_{XZ_{prin}} = \frac{1}{2}\left(I_{X_{ref}} - I_{Z_{ref}}\right) \sin 2\epsilon + I_{XZ_{ref}} (\cos 2\epsilon) = 0$$

hence

$$\tan 2\epsilon = \frac{2 \, I_{XZ_{ref}}}{I_{Z_{ref}} - I_{X_{ref}}}$$

or

$$\epsilon = \frac{1}{2} \tan^{-1} \frac{2 \, I_{XZ_{ref}}}{I_{Z_{ref}} - I_{X_{ref}}} \tag{7}$$

If the moments of inertia $I_{X_{ref}}$ and $I_{Z_{ref}}$ about the body reference axes and a moment of inertia I_{X_θ} about an axis inclined θ° from the X body reference axis are measured, then the product of inertia $I_{XZ_{ref}}$ may be determined from equation (6), and the inclination ϵ of the principal axes with respect to the body reference axes can be determined from equation (7).

The noticeable rolling motions which occurred during the torsional swingings suggested another method of determining ϵ. This method is based on the fact that application of a pure yawing moment to the system produces no rolling when the axis of oscillation corresponds to a principal axis of the suspended body. The angle between the reference axis and the axis of no roll represents the inclination of the principal axis of the airplane and gear combination. It can be shown that the correction which must be applied to yield ϵ for the airplane alone is closely approximated by the expression

$$(\Delta\epsilon)_G \approx \frac{I_{XZ_G}}{I_{Z_{ref}} - I_{X_{ref}}}$$

where I_{XZ_G} is composed of the product of inertia of the gear about its own center of gravity and the terms involved in correcting for the difference in center-of-gravity location of the gear, airplane, and airplane-gear combination.

Brief tests of a preliminary nature were made for load condition 1 with the airplane suspended with the X reference axis at various angles from $+3.7^\circ$ to -2.9° to the horizontal. A position recorder was attached to the left wing tip to measure the amplitude and period of the roll and the same instrumentation as used for the yaw axis swingings was used to measure the corresponding yaw amplitude and period.

Principal moments of inertia.- The moment of inertia about the Y_{ref} axis will be a principal moment of inertia, hence

$$I_{Y_{ref}} = I_{Y_{prin}} \tag{8}$$

From figure 2,

$$\begin{aligned} I_{X_{prin}} &= \int z''^2 \, dm = \int (z \cos \epsilon - x \sin \epsilon)^2 \, dm \\ &= \int z^2 \cos^2 \epsilon \, dm - 2 \int xz \sin \epsilon \cos \epsilon \, dm + \\ &\quad \int x^2 \sin^2 \epsilon \, dm \end{aligned}$$

or

$$I_{X_{prin}} = I_{X_{ref}} \cos^2 \epsilon + I_{Z_{ref}} \sin^2 \epsilon - 2 I_{XZ_{ref}} \sin \epsilon \cos \epsilon \tag{9}$$

and

$$\begin{aligned} I_{Z_{prin}} &= \int x''^2 \, dm = \int (z \sin \epsilon + x \cos \epsilon)^2 \, dm \\ &= \int z^2 \sin^2 \epsilon \, dm + 2 \int (xz \sin \epsilon \cos \epsilon) \, dm + \\ &\quad \int x^2 \cos^2 \epsilon \, dm \end{aligned}$$

or

$$I_{Z_{prin}} = I_{X_{ref}} \sin^2 \epsilon + I_{Z_{ref}} \cos^2 \epsilon + 2 I_{XZ_{ref}} \sin \epsilon \cos \epsilon \tag{10}$$

RESULTS AND DISCUSSION

Moments of Inertia About Body Reference Axes

Basic data.– The dimensions and physical characteristics of the airplane are given in Appendix A. All measurements of length were made several times to at least the nearest 0.01 foot. The airplane was weighed eight times and the average values were used in the determination of the horizontal and vertical positions of the airplane center of gravity. It is believed the positions of the center of gravity were known within ±0.02 foot. The period data as obtained from the knife-edge and pendulum measurements are given in table I. The period values for each run are averages of about 30 cycles for the X and Y axes and 15 cycles for the Z axis. The timing error was less than 0.01 second per minute. A mean value of the period of oscillation for each set of runs was used for the determination of the moments of inertia. The equipment used for the measurements of the moments of inertia was tested by oscillating known masses; the calculated and measured values agreed within 0.50 percent in all cases.

Corrections to basic data.– Additional mass and buoyancy effects were considered. Additional mass corrections were made according to reference 8 and are included in the sample calculations given in Appendix B for load condition 1. The resulting true moments of inertia about the body reference axes are given in table II.

Precision.– The effect on the moment-of-inertia calculations of the possible errors in the various measured and computed quantities is summarized in table III, which shows the percentage error in the true moments of inertia due to individual errors in each variable taken one at a time. The possible errors in the variables were estimated on the basis of the present test techniques and the previous experience of references 4, 6, and 8. The total of the individual percentage errors is a measure of the over-all precision of the method. The values of ±1.7, ±1.2, and ±0.60 percent for $I_{X_{ref}}$, $I_{Y_{ref}}$, and $I_{Z_{ref}}$, respectively, are slightly lower than the values of ±2.5, ±1.3, and ±0.8 estimated in reference 6 for the usual swinging methods. Detailed comparison with the data of references 4 and 6 indicates that, in general, the errors in measured moments of inertia are slightly greater for the new method than for previous methods because of the direct effect of errors in the evaluation of the spring constant C. However, this disadvantage is more than offset by the reduction, due to the shorter suspension lengths, in the magnitude and resultant errors of terms involving transfer from the axes of oscillation to axes through the center of gravity. This is illustrated by the fact that for the present tests the maximum difference between the measured moments of inertia and the true moments of inertia about axes through the center of gravity is less than 16 percent of the latter, compared with the 200- to

700-percent differences inherent with compound-pendulum methods previously used (references 4 to 7).

Inclination of the Principal Axes and Principal Moments of Inertia

Inclination of the principal axes by two-suspension method.- The calculations, from equations (6) and (7), of the inclination of the principal axis from measurements of the moments of inertia about the Z and X reference axes and an inclined axis in the XZ plane are given in Appendix C. With regard to precision, the net effect on $I_{XZ_{ref}}$ of a small error in the directly measured quantity θ is small, as is the effect of an error of ±0.6 percent (see table III) in $I_{Z_{ref}}$. However, the term $I_{X_{ref}} \cos^2 \theta - I_{X_\theta}$ in equation (6) represents the small difference between large numbers, so that $I_{XZ_{ref}}$ is very sensitive to errors in $I_{X_{ref}}$ and I_{X_θ}. Since $\cos^2 \theta$ is nearly equal to 1, the possible error in the difference is approximately equal to the error in $I_{X_{ref}} - I_{X_\theta}$, which arises, in turn, from errors in P, L, and l. Table III indicates that these items can cause an error of about ±0.55 percent in each I_X value, giving a possible error in $I_{X_{ref}} \cos^2 \theta - I_{X_\theta}$ of about 1.10 percent of $I_{X_{ref}}$. Substitution of this error in Appendix C yields a maximum possible error in $I_{XZ_{ref}}$ of ±659 slug-feet squared corresponding to about ±1.84° in terms of ϵ. Computations have shown that in order to obtain reasonable accuracy in the analysis or prediction of the dynamic lateral stability characteristics of high-performance airplanes, it is often necessary to know ϵ to less than ±1.0° (reference 2). The accuracy of the two-suspension method could be increased somewhat by measuring I_{X_θ} at large angles of inclination θ. However, this procedure does not appear promising, in view of the handling difficulties which might be encountered with airplanes of large size or unusual configuration.

Inclination of principal axes by the "Null" method.- In the torsional-pendulum swingings with the airplane X reference axis at various angles to the horizontal, there was a rolling motion at all test attitudes, so that the inclination of the principal axis of the airplane-gear combination was not determined directly. However, as shown in figure 11, this inclination could be established by interpolation from a plot of the value of the dimensionless ratio of maximum rolling-motion amplitude to the corresponding yawing-motion amplitude, where these amplitudes were measured across the envelope of the oscillations. The data indicate an inclination of 2.4°, with a precision of about ±0.1°. The torsional pendulum was damaged prior to measurement of the product of inertia of the gear itself. However, it was estimated that the correction to ϵ due to the gear would be of the order of +0.3°, so that ϵ for the airplane alone would be about 2.7°. It is believed that, with minor modifications to apparatus and technique, ϵ for the test airplane could be evaluated to within ±0.1°, which corresponds to an error of ±35 slug-feet squared in $I_{XZ_{ref}}$. Although the estimated value of ϵ of 2.7° is in excellent agreement with the value of 2.77° determined by the two-suspension method, even this must be considered as fortuitous in view of the possible error of ±1.84° for the latter.

Principal moments of inertia.- The principal moments of inertia were determined from equations (8), (9), and (10). The sample computations (based on the two-suspension $I_{XZ_{ref}}$ data) for load condition 1 are given in Appendix C. The principal moments of inertia, product of inertia, and inclination of the principal axes for the two load conditions are summarized in table IV. Since ϵ is so small, the moments of inertia about the principal axes and the resulting possible errors are nearly the same as the moments of inertia about the reference axes and the corresponding possible errors (tables II and III).

Comments on Apparatus and Procedures

Compared with previous methods, the simplicity of the apparatus and the handling procedures cannot be stressed too highly. Handling of the airplane was reduced to a minimum and at no time was it necessary to hoist or jack the airplane in unnatural positions or to any great height. In view of the apparent simplicity and accuracy of the Null method for determining the inclination of the principal axes, provisions in the Z-axis support cradle to facilitate continuous and accurate changes in airplane attitude would be desirable. Since the amplitude of small rolling motion is of importance in this method, sensitive roll measuring instruments based perhaps on strain gages or an optical lever should be employed.

These methods of inertia measurement can be applied, of course, to other airplanes, even to very heavy airplanes, if adequate provision is made for increasing the weight-carrying capacities of the loaded members. There appears to be no great difficulty in the application of the X-axis and Y-axis equipment to other airplanes; the detail suspension design would be dependent upon the particular airplane configuration and structure. It may be necessary, in some cases, to account for the effect on the spring constant of the flexibility of the structure between the pivots and spring anchors.

For the Z-axis measurements the application of the overhead torsional pendulum is limited by the load-carrying capacity of the available supporting structure (design load of present equipment was 20,000 pounds). This limitation might be overcome by a torsional pendulum which supports the airplane from below. Preliminary estimates indicate the practicability of such a system which would employ a platform flush with the ground as the support cradle. The airplane would be supported from this platform at the axle axis of the extended landing gear.

CONCLUDING REMARKS

The methods employed in the present investigation for measuring the moments of inertia of a 13,000-pound airplane reduced the handling problems and inherent inaccuracies of previous methods and appear suitable for extension to inertia measurements on very heavy airplanes.

The test equipment was checked by measuring moments of inertia of known masses; the calculated and measured values agreed within 0.50 percent. Analysis of the precision of the airplane inertia measurements showed the maximum possible errors to be ±1.7, ±1.2, and ±0.6 percent of the true moments of inertia about the X, Y, and Z axes, respectively. At no time was the maximum difference between the measured moments of inertia before correcting for additional mass, transfer of axes, etc., and the true moments of inertia greater than 16 percent of the true moments of inertia, as compared with the 200- to 700-percent differences inherent in the swinging methods previously employed.

The airplane product of inertia and inclination of the principal axes were determined by two methods. The first method was dependent upon values of moments of inertia about an inclined axis in the XZ plane and about the X and Z reference axes, and was characterized by possible errors of $\pm 1.9^{\circ}$ in the derived value of ϵ. The other method utilized the coupled motion between roll and yaw which occurred when the airplane was yawed about an axis other than a principal axis. Brief tests with this method indicated that ϵ could be evaluated to within $\pm 0.1^{\circ}$, which corresponds to an error of approximately ±35 slug-feet squared in the product of inertia of the test airplane.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., April 5, 1950.

APPENDIX A.- PHYSICAL CHARACTERISTICS OF AIRPLANE AND INERTIA GEAR AS USED FOR MOMENT-OF-INERTIA MEASUREMENTS

General

Type: Single-engine, propeller-driven, two-place dive bomber

Weight and balance

Load condition 1
Basic airplane
Pilot and observer (400 pounds)
Research instrumentation
23 gallons oil
300 gallons gasoline (fuel tanks full)

Weight	13,090 lb
Longitudinal center-of-gravity position Gear up	30.28% M.A.C.
Vertical center-of-gravity position from fuselage reference (thrust) line	−0.130 ft

Load condition 2
(Load condition 1 less fuel)

Weight	11,525 lb
Longitudinal center-of-gravity position Gear up	27.12% M.A.C.
Vertical center-of-gravity position from fuselage reference (thrust) line	0.124 ft

Dimensions for inertia measurements

X axis

Perpendicular distance from the axis of the spring to axis of oscillation, L_X	
$\theta = 7.60^\circ$	10.30 ft
$\theta = 0^\circ$	10.21 ft
Static spring constant of the restraining springs, C_X (total)	5832 lb/ft
Perpendicular distance from the axis of oscillation to the airplane center of gravity, l_X	
Load condition 1	
$\theta = 7.60^\circ$	1.34 ft
$\theta = 0^\circ$	1.93 ft
Load condition 2	
$\theta = 7.60^\circ$	1.63 ft
$\theta = 0^\circ$	2.19 ft
Vertical component of the distance between the X axis of oscillation and the airplane center of gravity h_X	
Load condition 1	1.93 ft
Load condition 2	2.19 ft

Y axis

Perpendicular distance from the axis of spring to the axis of oscillation, L_y	16.49 ft
Perpendicular distance from axis of oscillation to airplane center of gravity, l_y	
Load condition 1	3.064 ft
Load condition 2	3.411 ft
Static spring constant, C_y	5820 lb/ft

Z axis

Equivalent spring constant of the torsional pendulum and airplane support cradle combination, C_z	82,000 ft-lb/radian
Moment of inertia of torsional pendulum and airplane support cradle combination about axis of pendulum shaft, I_{Z_G}	216 slug-ft^2
Vertical component of the distance between the Y axis of oscillation and the airplane center of gravity, h_y	
Load condition 1	0.751 ft
Load condition 2	1.005 ft

Wing

Area, S	422 ft^2
Span, b	49.72 ft
Aspect ratio, A	5.87
Taper ratio, λ	2.32
Mean chord, $\bar{c}$	8.48 ft
M.A.C.	109.3 in
Wing volume	636 ft^3
Dihedral angle (top surface front spar)	6°
Additional moment-of-inertia coefficient for X swinging of a flat rectangular plate (for A=5.87, fig. 4, reference 8), k'	0.88
Taper-ratio correction factor (fig. 6, reference 8), D_λ	0.78
Dihedral correction factor, (fig. 5, reference 8) D_Γ	0.80
Distance aft from leading edge of wing to leading edge M.A.C.	0.03 ft

Fuselage

Fuselage length, L_f	34 ft
Geometric average width, w	3.5 ft
Geometric average depth, d	6.11 ft
Fineness ratio of equivalent fuselage ellipsoid	7.04
Width-depth ratio, w/d	1.745
Perpendicular distance in the vertical plane from the X axis of rotation to the centroid of side area of the fuselage, l_{f_x}	
$\theta = 7.6^\circ$	1.59 ft
$\theta = 0$	2.51 ft
Component of distance, in the XY principal plane, of the perpendicular distance between the Y axis of rotation and the centroid of top area of fuselage, l_{f_y}	0.05 ft

Perpendicular distance in the vertical plane from the Z axis of rotation to the centroid of side area of fuselage, l_{f_Z}	
$l_{f_{Z_1}}$, load condition 1	2.44 ft
$l_{f_{Z_2}}$, load condition 2	2.72 ft
Coefficient of additional mass of equivalent fuselage ellipsoid for motion along the Y and Z axes (fig. 7, reference 8),	
k_{f_Y}	1.54
k_{f_Z}	0.57
Coefficient of additional moment of inertia of equivalent fuselage ellipsoid about the Y and Z axes (fig. 8, reference 8),	
k'_{f_Y}	0.44
k'_{f_Z}	1.25
Fuselage volume	727 ft^3
Horizontal tail	
Area, S	107.4 ft^2
Span, b	19.04 ft
Aspect ratio, A	3.37
Taper ratio, λ	2.30
Mean chord, $\bar{c}$	5.65 ft
Volume horizontal tail	40.3 ft^3
Component of distance in the XY plane of the fuselage of the perpendicular distance between the centroid of the horizontal-tail area and the Y axis of rotation, l_{t_Y}	16.07 ft
Coefficient of additional mass of an equivalent flat rectangular plate of A=3.37 (fig. 3, reference 8), k	0.876
Vertical tail	
Area, S	45.7 ft^2
Span, b	7.78 ft
Aspect ratio, A	1.32
Taper ratio, λ	2.00
Mean chord, $\bar{c}$	5.88 ft
Volume vertical tail	17.94 ft^3
Perpendicular distance from the centroid of area of the vertical tail to the Z axis of rotation,	
$l_{t_{Z_1}}$, load condition 1	19.31 ft
$l_{t_{Z_2}}$, load condition 2	19.60 ft
Additional mass coefficient of an equivalent flat rectangular plate of A=1.32 (fig. 3, reference 8), k	0.65

APPENDIX B.— CALCULATIONS OF MOMENTS OF INERTIA ABOUT BODY REFERENCE AXES FOR LOAD CONDITION 1

Additional Mass Corrections

X axes

(a) $\theta=0^{\circ}$, $l_{f_x}=2.51$ feet

$$\left(I_{add\ mass}\right)_{\theta=0^{\circ}} = \rho\left\{\frac{\pi}{48}\left(k'\ D_{\lambda}D_{\Gamma}S^{2}b\right)_{wing} + \left[k_{f_y}\ L_f\ wd\ \left(l_{f_x}\right)^{2}\right]_{fuse}\right\}$$

$$= (0.002378)\left[\frac{\pi}{48}(0.88)(0.78)(0.80)(422)^{2}(49.72)+(1.54)(34)(3.5)(6.11)(2.51)^{2}\right]$$

$$= (0.002378)(318{,}244 + 7054.34)$$

$$\left(I_{add\ mass}\right)_{\theta=0^{\circ}} = 773.56 \text{ slug-feet}^{2}$$

(b) $\theta=7.60^{\circ}$, $l_{f_x}=1.59$ feet

then

$$\left(I_{add\ mass}\right)_{\theta=7.60^{\circ}} = 763.47 \text{ slug-feet}^{2}$$

Y axis

$$I_{add\ mass} = \rho\left\{\left[\frac{1}{5}k'_{f_y}\ L_f\ wd\left(\frac{L_f^{2}}{4}+\frac{3d^{2}}{2\pi}\right)\right]_{fuse} + \left[k_{f_z}\ L_f\ wd\ \left(l_{f_y}\right)^{2}\right]_{fuse} + \left[\frac{\pi}{4}k\frac{S^{2}}{b}\left(l_{t_y}\right)^{2}\right]_{ht}\right\}$$

$$= 0.002378\left\{\left[\frac{1}{5}(0.44)(34)(3.5)(6.11)\right]\left[\frac{(34)^2}{4} + \frac{3(6.11)^2}{2\pi}\right] + \right.$$

$$[0.57\ (34)(3.5)(6.11)(0.05)^2] +$$

$$\left.\left[\frac{\pi}{4}(0.876)\ \frac{(107.4)^2}{19.04}\ (16.07)^2\right]\right\}$$

$$= 0.002378(19631.85 + 1.04 + 107{,}635.46)$$

$$I_{add\ mass} = 302.64 \text{ slug-feet}^2$$

Z axis

$$I_{add\ mass} = \rho\left\{\left[\frac{1}{5}k'_{f_z}\ L_f\ wd\left(\frac{{L_f}^2}{4} + \frac{3w^2}{2\pi}\right)\right]_{fuse} + \left[k_{f_y}\ L_f\ wd\ \left(l_{f_z}\right)^2\right]_{fuse} + \right.$$

$$\left.\left[\frac{\pi}{4}k\frac{S^2}{b}\left(l_{t_z}\right)^2\right]_{vt}\right\}$$

$$= 0.002378\left\{\left[\frac{1}{5}(1.25)(34)(3.5)(6.11)\right]\left[\frac{(34)^2}{4} + \frac{3(3.5)^2}{2\pi}\right] + \right.$$

$$\left.[1.54\ (34)(3.5)(6.11)(2.44)^2] + \left[\frac{\pi}{4}(0.65)\ \frac{(45.7)^2}{7.78}\ (19.31)^2\right]\right\}$$

$$= 0.002378\ (53608.45 + 6663.88 + 51099.48)$$

$$I_{add\ mass} = 264.85 \text{ slug-feet}^2$$

Moments of Inertia About Body Reference Axes
Through the Airplane Center of Gravity

X axes

(a) $\theta=0^{\circ}$

From equation (2)

$$I_{X_{ref}} = I_{X_{meas}} - I_{add\ mass} - \left[\frac{W}{g} + V(\rho) \right] l_x{}^2$$

$$= C_x L_x{}^2 \left(\frac{P}{2\pi}\right)^2 - Wh_x \left(\frac{P}{2\pi}\right)^2 - I_{add\ mass} - \frac{W}{g} (l_x)^2 - V(\rho)\ (l_x)^2$$

$$= 5832\ (10.21)^2(0.03065) - 13090\ (1.93)(0.03065) - 773.56 - 406.52\ (1.93)^2 - 1421\ (0.002378)(1.93)^2$$

$$= 18633.72 - 774.33 - 773.56 - 1514.25 - 12.59 = 18633.72 - 3074.73$$

$$I_{X_{ref}} = 15559 \text{ slug-feet}^2$$

(b) $\theta=7.60^{\circ}$

From equation (2)

$$I_{X_\theta} = C_x L_x{}^2 \left(\frac{P}{2\pi}\right)^2 - Wh_x \left(\frac{P}{2\pi}\right)^2 - I_{add\ mass} - \frac{W}{g} (l_x)^2 - V(\rho)\ (l_x)^2$$

$$= 5832(10.30)^2(0.02891) - (13090)(1.93)(0.02891) - 763.47 - 406.52\ (1.34)^2 - (1421)(0.002378)(1.34)^2$$

$$= 17887.11 - 730.37 - 763.47 - 729.95 - 6.07 = 17887.11 - 2229.86$$

$$I_{X_\theta} = 15657 \text{ slug-feet}^2$$

Y axis

From equation (3)

$$I_{Y_{ref}} = I_{Y_{meas}} - I_{add\ mass} - \left[\frac{W}{g} + V(\rho) \right] (l_y)^2$$

$$= C_y L_y^2 \left(\frac{P}{2\pi}\right)^2 - Wh_y \left(\frac{P}{2\pi}\right)^2 - 347.44 - \frac{W}{g} (l_y)^2 - V(\rho)\ (l_y)^2$$

$$= 5820\ (16.49)^2\ (0.01906) - (13090)(0.751)(0.01906) - 302.64 - 406.52\ (3.064)^2 - 1421\ (0.002378)(3.064)^2$$

$$= 30163.88 - 187.37 - 302.64 - 3816.41 - 31.72 = 30163.88 - 4338.14$$

$$I_{Y_{ref}} = 25826 \text{ slug-feet}^2$$

Z axis

From equation (5)

$$I_{Z_{ref}} = I_{Z_{meas}} - I_{add\ mass} - I_{Z_G} = C_z \left(\frac{P}{2\pi}\right)^2 - I_{add\ mass} - I_{Z_G}$$

$$= 82000\ (0.44502) - 264.85 - 216 = 36492 - 480.85$$

$$I_{Z_{ref}} = 36011 \text{ slug-feet}^2$$

APPENDIX C.– CALCULATIONS OF THE INCLINATION OF THE PRINCIPAL AXES AND THE PRINCIPAL MOMENTS OF INERTIA FOR LOAD CONDITION 1

Product of Inertia

From equation (6)

$$I_{XZ_{ref}} = \frac{I_{Z_{ref}} \sin^2 \theta + I_{X_{ref}} \cos^2 \theta - I_{X_\theta}}{2 \sin \theta \cos \theta}$$

when

$$\theta = 7.60^\circ$$

then

$$I_{XZ_{ref}} = \frac{(36011)(0.01749) + (15559)(0.98252) - 15657}{0.26219} = \frac{629.83 + 15287.03 - 15657}{0.26219} = \frac{259.86}{0.26219}$$

$$I_{XZ_{ref}} = 991.11 \text{ slug-feet}^2$$

Inclination of the Principal Axis

From equation (7)

$$\epsilon = \frac{1}{2} \tan^{-1} \frac{2 I_{XZ_{ref}}}{I_{Z_{ref}} - I_{X_{ref}}} = \frac{1}{2} \tan^{-1} \frac{2(991.11)}{36011 - 15559} = \frac{1}{2} \tan^{-1} \frac{1982.22}{20452} = \frac{1}{2} \tan^{-1} 0.09692$$

$$= \frac{1}{2}(5.536^\circ)$$

$$\epsilon = 2.768^\circ$$

Principal Moments of Inertia

From equation (8)

$$I_{Y_{prin}} = I_{Y_{ref}} = 25826 \text{ slug-feet}^2$$

From equation (9)

$$I_{X_{prin}} = I_{X_{ref}} \cos^2 \epsilon + I_{Z_{ref}} \sin^2 \epsilon - 2\, I_{XZ_{ref}} \sin \epsilon \cos \epsilon = (15559)(0.99846)^2 + (36011)(0.05551)^2 - 2(991.11)(0.05551)(0.99846) = 15511 + 111 - 110$$

$$I_{X_{prin}} = 15512 \text{ slug-feet}^2$$

From equation (10)

$$I_{Z_{prin}} = I_{X_{ref}} \sin^2 \epsilon + I_{Z_{ref}} \cos^2 \epsilon + 2\, I_{XZ_{ref}} \sin \epsilon \cos \epsilon = (15559)(0.05551)^2 + (36011)(0.99846)^2 + 2(911.11)(0.05551)(0.99846) = 48 + 35900 + 110$$

$$I_{Z_{prin}} = 36058 \text{ slug-feet}^2$$

REFERENCES

1. Sternfield, Leonard: Effect of Product of Inertia on Lateral Stability. NACA TN 1193, 1947.

2. Sternfield, Leonard: Some Considerations of the Lateral Stability of High-Speed Aircraft. NACA TN 1282, 1947.

3. Green, M. W.: Measurement of the Moments of Inertia of Full Scale Airplanes. NACA TN 265, 1927.

4. Soulé, Hartley, and Miller, M. P.: The Experimental Determination of the Moments of Inertia of Airplanes. NACA Rep. 467, 1933.

5. Miller, M. P.: An Accurate Method of Measuring the Moments of Inertia of Airplanes. NACA TN 351, 1930.

6. Gracey, William: The Experimental Determination of the Moments of Inertia of Airplanes by a Simplified Compound-Pendulum Method. NACA TN 1629, 1948.

7. Pauly, U. J., Meyer, R. J., and Infanti, N. L.: The Determination of the Moment of Inertia About the Lateral Axis of a B-25J Airplane. Cornell Aero. Lab. Rep. No. TB-405-F-9, Feb. 17, 1948.

8. Malvestuto, Frank S., Jr., and Gale, Lawrence J.: Formulas for Additional Mass Corrections to the Moments of Inertia of Airplanes. NACA TN 1187, 1947.

TABLE I.– PERIODS OF OSCILLATION, MOMENT-OF-INERTIA MEASUREMENTS

[Load condition 1. Fuel tanks full, crew of two (400 pounds), airplane weight 13,090 pounds]

Run	X axis (roll axis)		Y axis (pitch axis)	Z axis (yaw axis)
	θ=7.60°	θ=0°		
	period (sec)	period (sec)	period (sec)	period (sec)
1	1.0691	1.1016	0.8681	4.1898
2	1.0714	1.1016	.8676	4.1928
3	1.0690	1.0994	.8676	4.1856
4	1.0692	1.1000	.8687	4.1972
5	1.0700	1.1014	.8686	4.1848
6	1.0683	1.0999	.8665	4.1952
7	1.0690	1.0996	.8651	4.1980
8	1.0660	1.0992	.8661	4.1925
9	1.0682	1.1009	.8668	4.1840
10	1.0692	1.1011	.8689	4.1945
11	1.0661	1.1007	.8675	---
12	1.0690	1.0973	.8680	---
13	1.0655	1.1017	.8680	---
14	1.0686	1.1003	.8677	---
15	1.0682	1.0976	.8651	---
16	1.0693	1.1004	---	---
17	1.0676	1.1008	---	---
18	1.0672	1.0975	---	---
19	1.0715	1.1007	---	---
20	1.0681	1.1008	---	---
21	1.0667	1.0972	---	---
22	1.0696	1.1013	---	---
23	1.0680	1.1002	---	---
24	1.0657	1.0983	---	---
Mean period, second	1.0684	1.1000	.8674	4.1914
Maximum variation from mean period, percent	.290	.254	.2653	.1765
Maximum variation from mean moment of inertia, percent	.578	.330	.613	.351

NACA

TABLE I.– CONCLUDED

[Load condition 2. Fuel tanks empty, crew of two (400 pounds), airplane weight 11,525 pounds]

Run	X axis (roll axis)		Y axis (pitch axis)	Z axis (yaw axis)
	$\theta=7.60^{\circ}$	$\theta=0^{\circ}$		
	period (sec)	period (sec)	period (sec)	period (sec)
1	1.0399	1.0585	0.8669	4.1064
2	1.0392	1.0569	.8661	4.1171
3	1.0403	1.0585	.8663	4.1197
4	1.0378	1.0578	.8663	4.1140
5	1.0400	1.0585	.8665	4.1257
6	1.0382	1.0585	.8651	4.1199
7	1.0400	1.0565	.8659	4.1110
8	1.0384	1.0584	.8654	4.1145
9	1.0401	1.0581	.8656	4.1113
10	1.0381	1.0588	.8650	4.1210
11	---	1.0595	---	---
12	---	1.0587	---	---
Mean period, second	1.0392	1.0582	.8659	4.1161
Maximum variation from mean period, percent	.106	.161	.116	.236
Maximum variation from mean moment of inertia, percent	.185	.351	.267	.187

NACA

TABLE II.– MOMENTS OF INERTIA ABOUT BODY AXES
THROUGH AIRPLANE CENTER OF GRAVITY

Item	Load condition 1	Load condition 2
I_{X_θ}, slug-feet^2 (θ=7.60°)	15,657	14,687
$I_{X_{ref}}$, slug-feet^2 (θ=0°)	15,559	14,022
$I_{Y_{ref}}$, slug-feet^2	25,826	25,329
$I_{Z_{ref}}$, slug-feet^2	36,011	34,710

TABLE III.– RESULTS OF PRECISION ANALYSIS

Variable		Possible error, percent of true moment of inertia (±)		
Symbol	Estimated error (±)	$I_{X_{ref}}$	$I_{Y_{ref}}$	$I_{Z_{ref}}$
C	0.5 percent	0.59	0.58	0.50
L	0.01 foot	.24	.14	– –
P	0.0005 second	.11	.13	.02
$I_{add\ mass}$	10 percent	.57	.13	.08
W	5 pounds	<.01	<.01	– –
l	0.02 foot	.20	.20	– –
$V\rho$	10 percent	<.01	.01	– –
θ	0.01°	– –	– –	– –
Total		±1.71	±1.19	±.60

TABLE IV.– PRINCIPAL MOMENTS OF INERTIA, PRODUCT OF INERTIA, AND INCLINATION OF PRINCIPAL AXES

Item	Load condition 1	Load condition 2
$I_{X_{prin}}$, slug-feet^2	15,512	14,215
$I_{Y_{prin}}$, slug-feet^2	25,826	25,329
$I_{Z_{prin}}$, slug-feet^2	36,058	34,517
$I_{XZ_{ref}}$, slug-feet^2	991	−1155
ϵ, angle between principal axis and X reference axis, degrees	2.77	−3.19

NACA

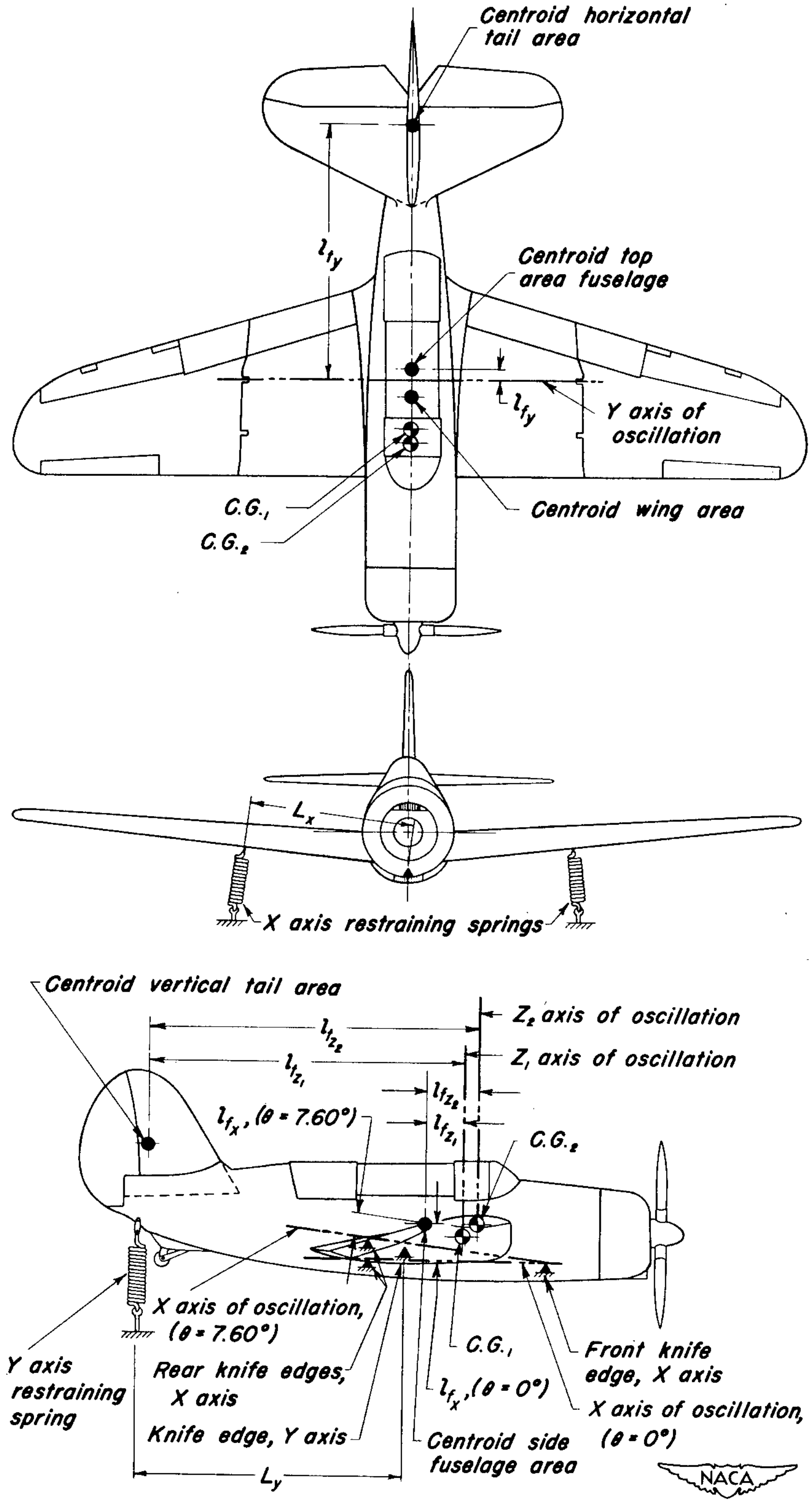


Figure 1.- Sketch of test airplane showing pertinent symbols for moment-of-inertia measurements.

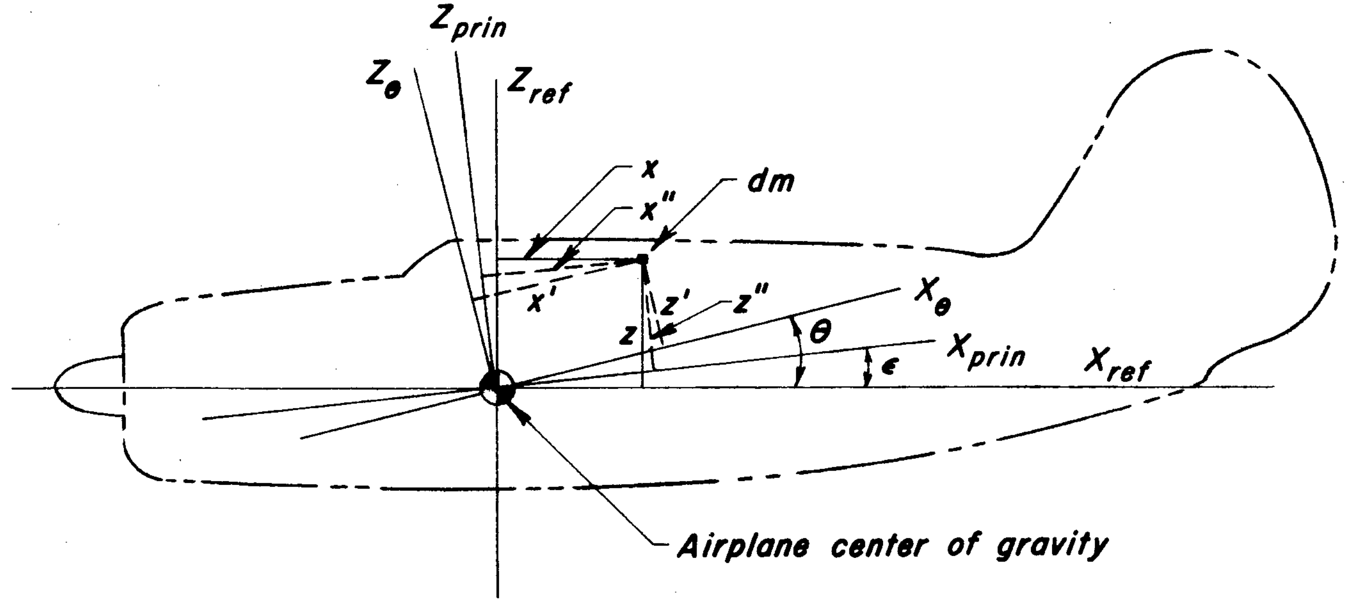


Figure 2.- Location of axes used in the derivation of the equations for product of inertia and principal moments of inertia.

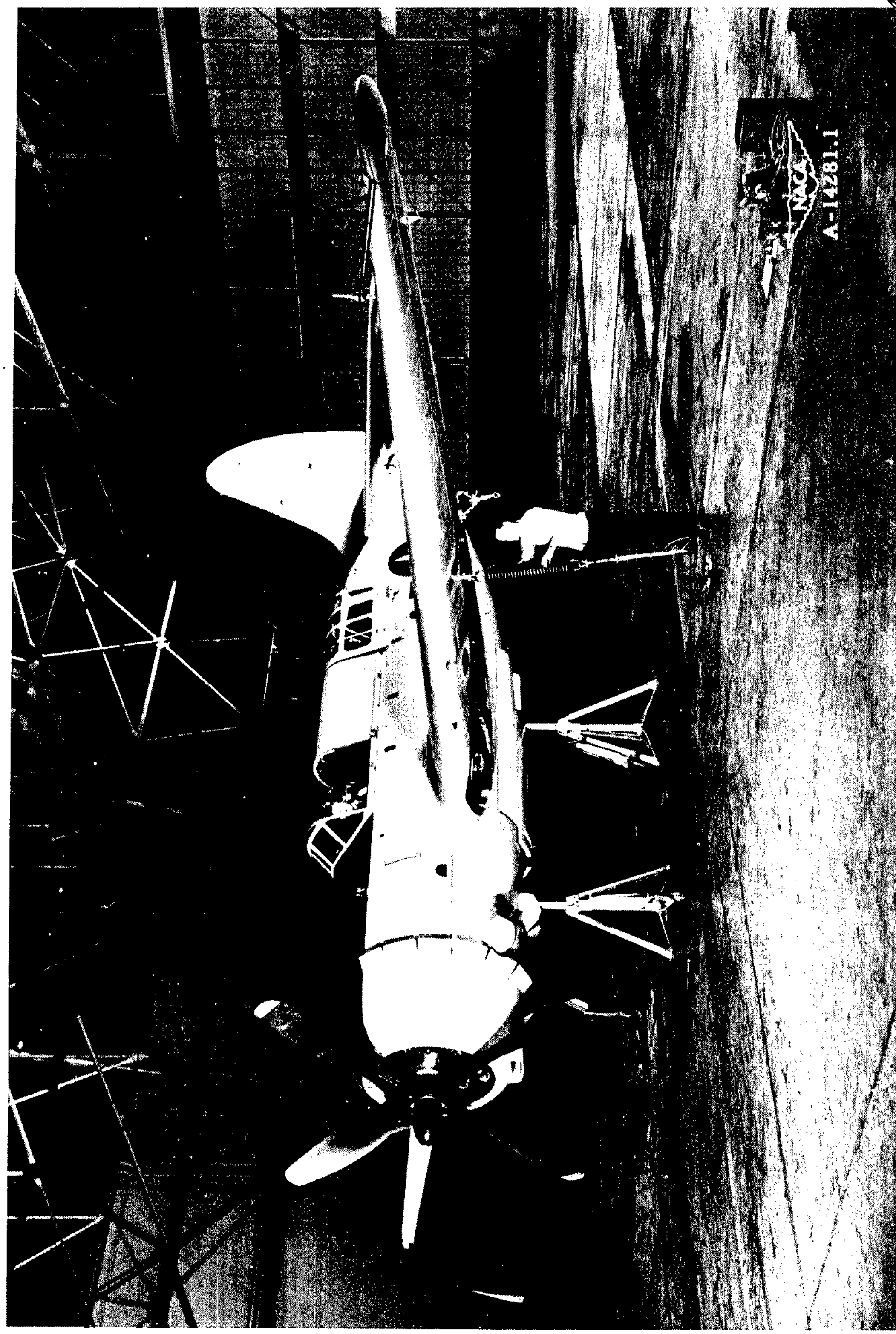
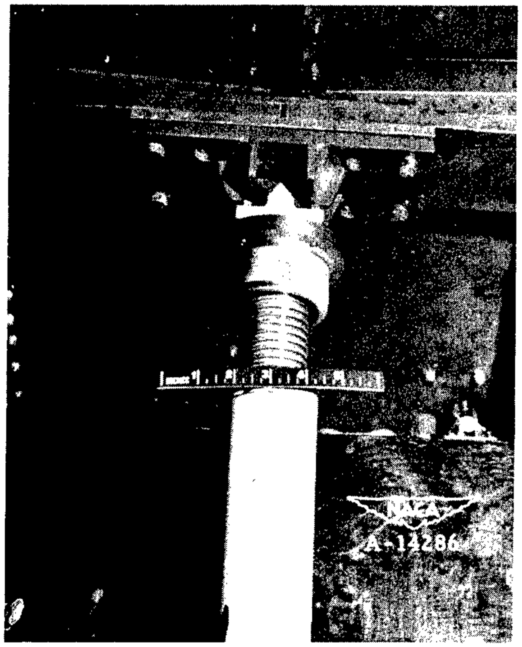
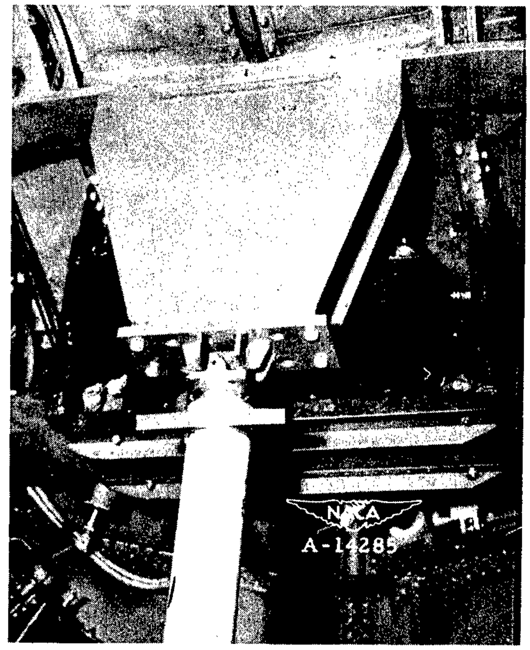


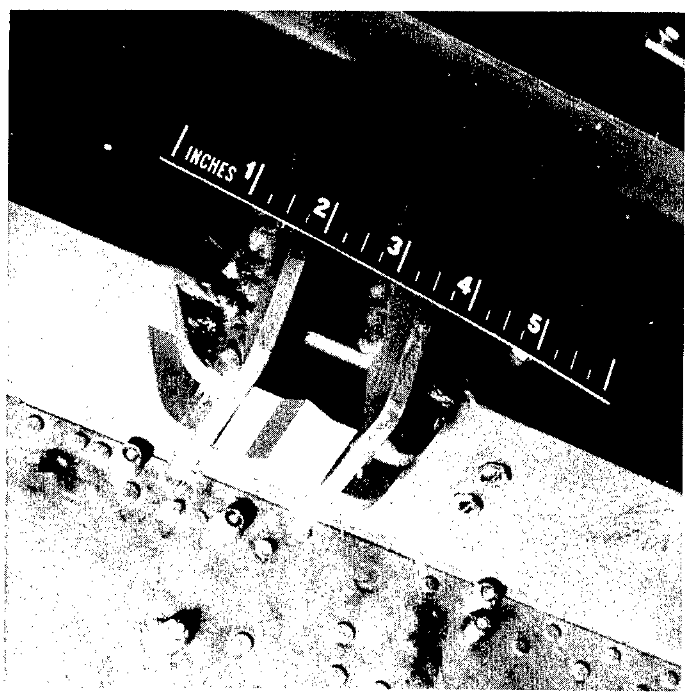
Figure 3.– Moment of inertia gear, X axis (roll), $\theta=0^\circ$.



(a) Rear fitting, θ=7.60°.



(b) Rear fitting, θ=0°.



(c) Front V-block.

Figure 4.- X axis knife-edge fittings.

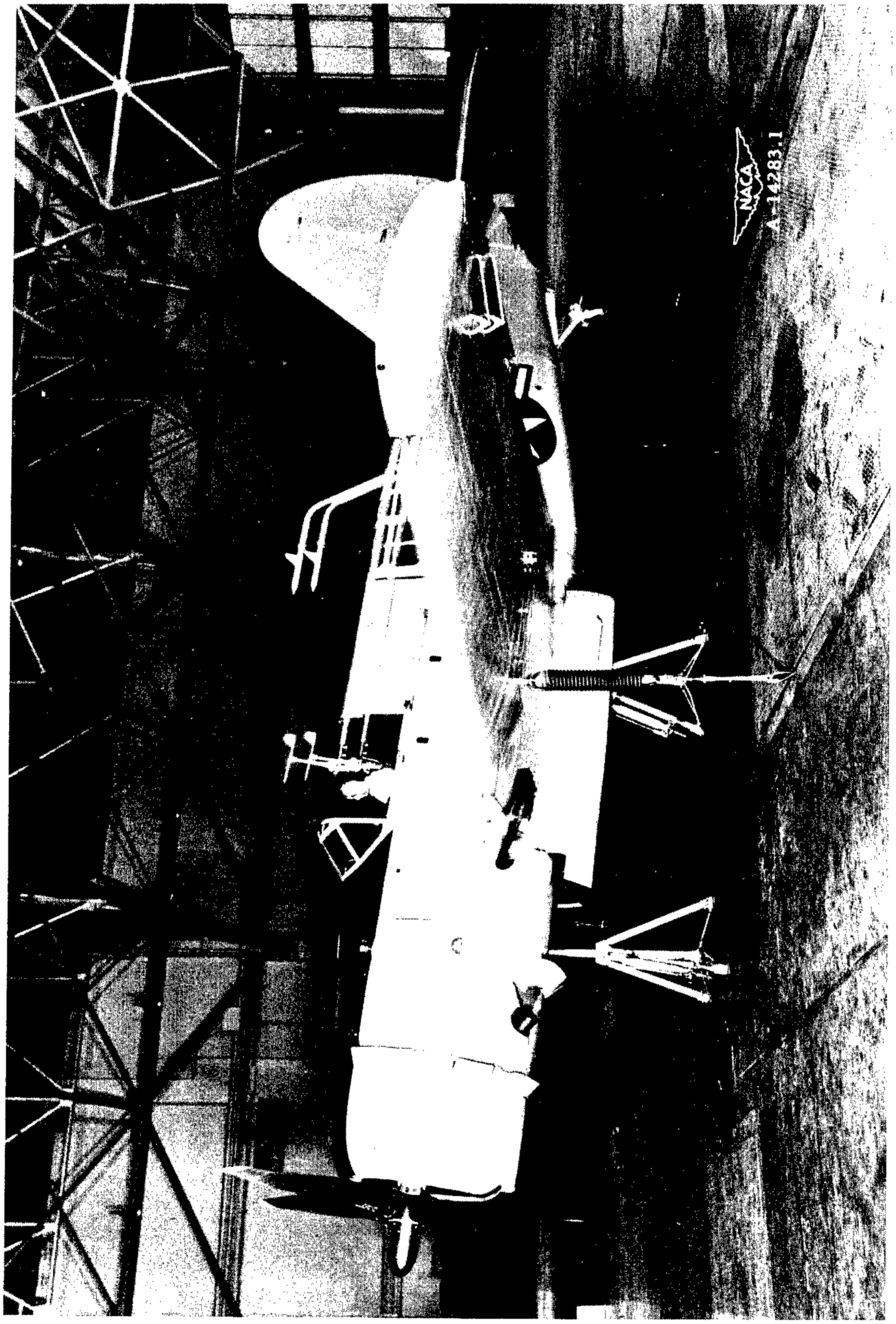


Figure 5.- Double exposure showing oscillations in roll.

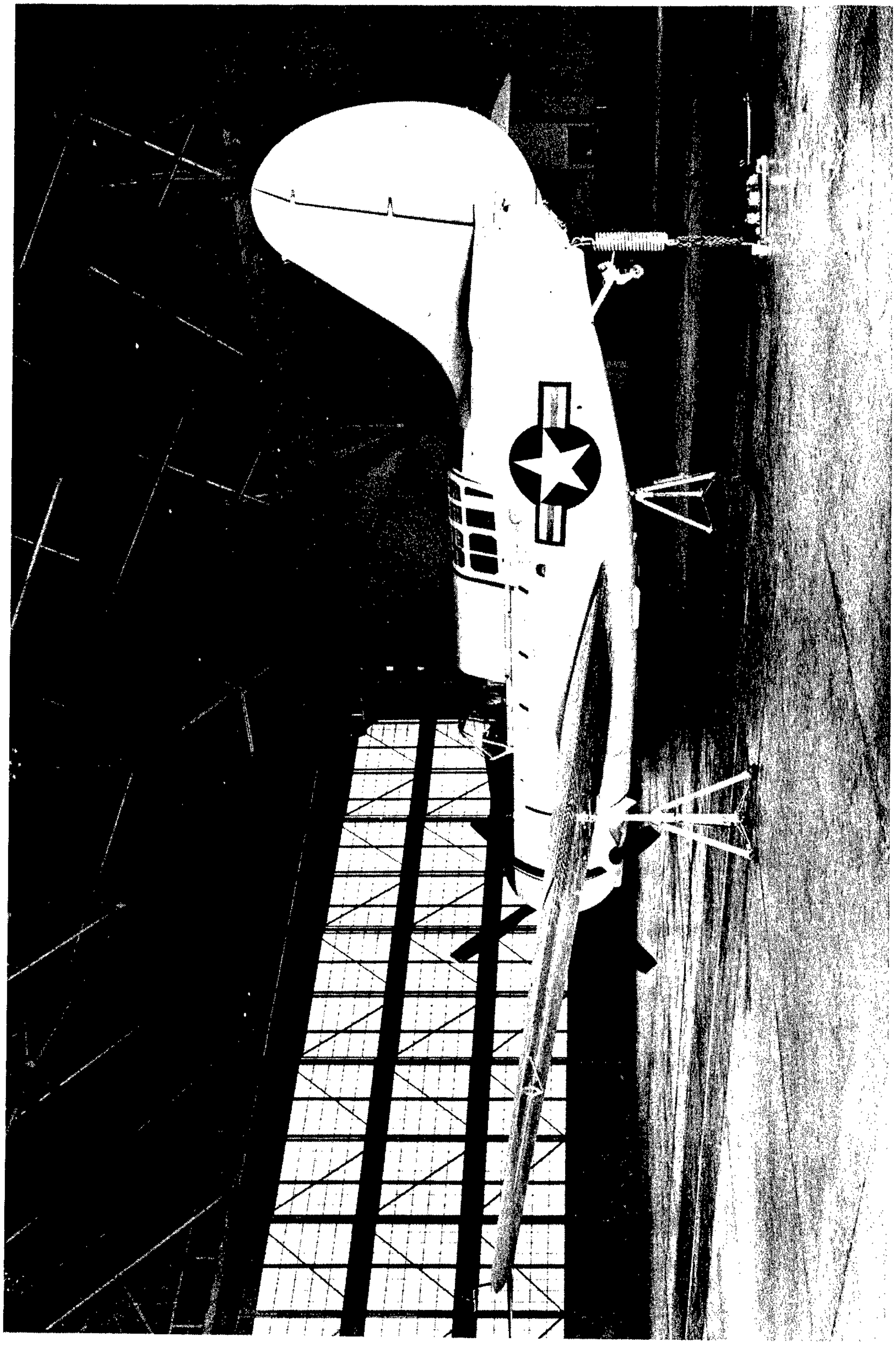


Figure 6.- Moment of inertia gear, Y axis (pitch).

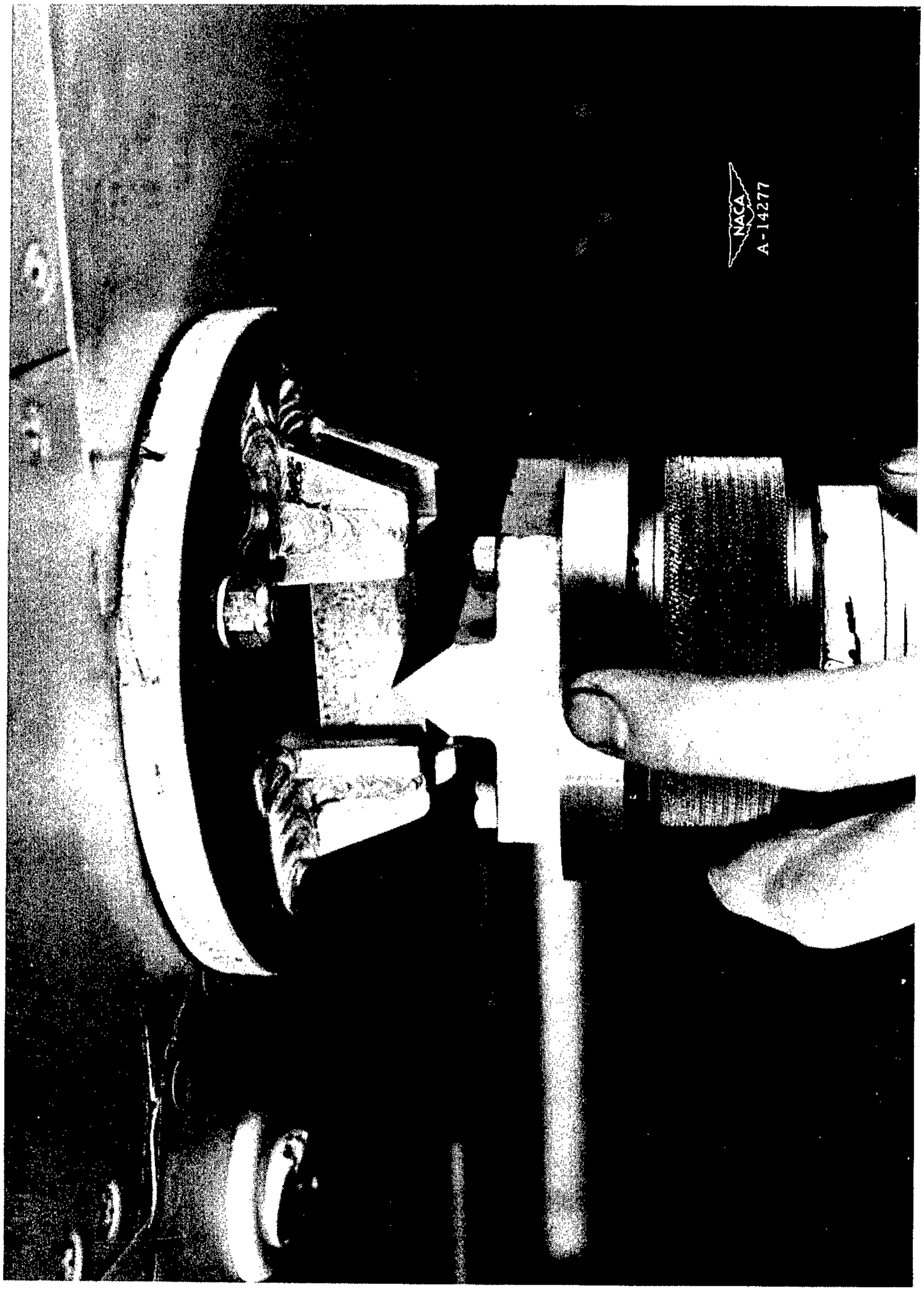


Figure 7.- Knife-edge and V-block assembly.

Figure 8.- Torsional-pendulum calibration test frame.

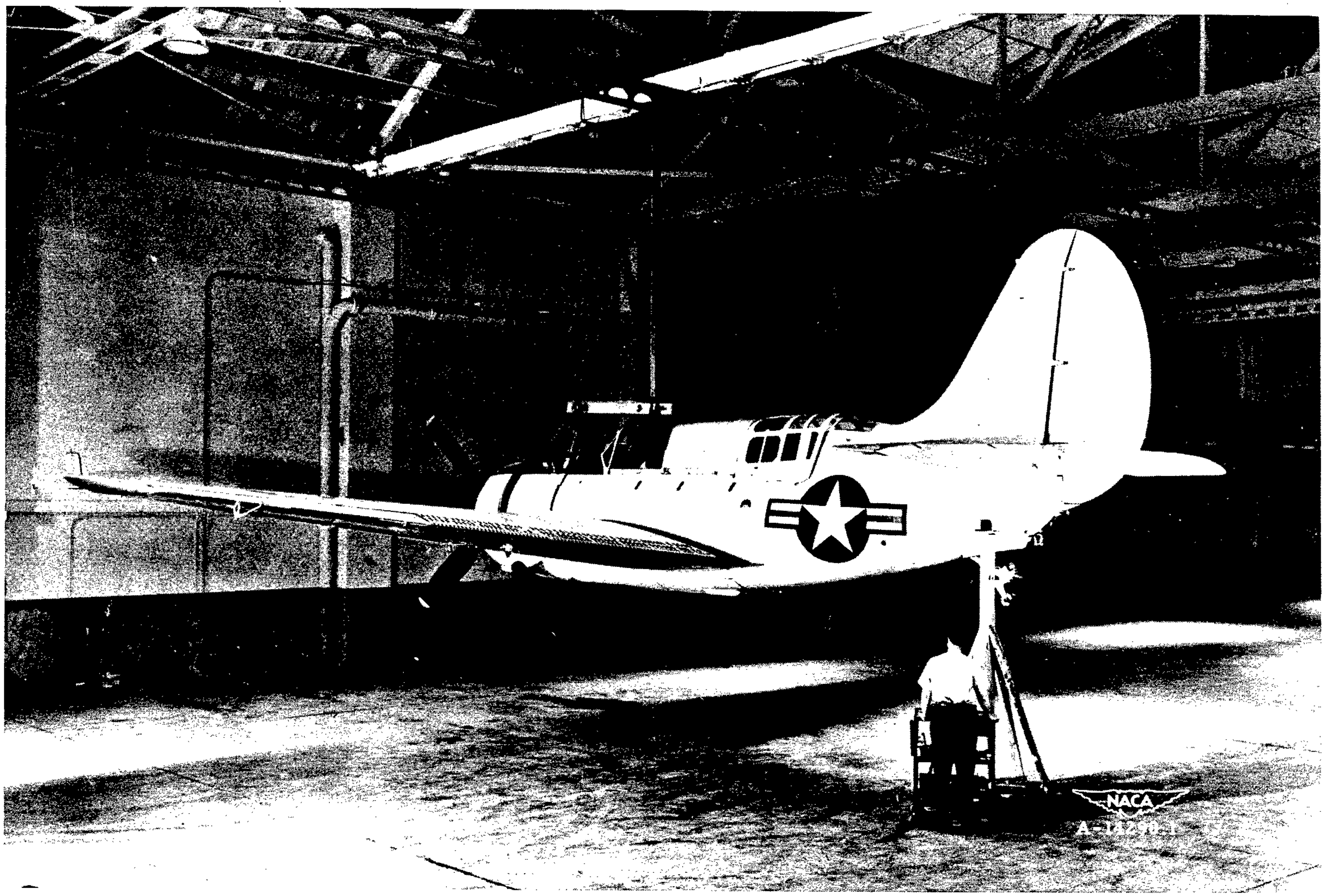


Figure 9.– Moment of inertia gear, Z axis (yaw), $\theta=0^{\circ}$.

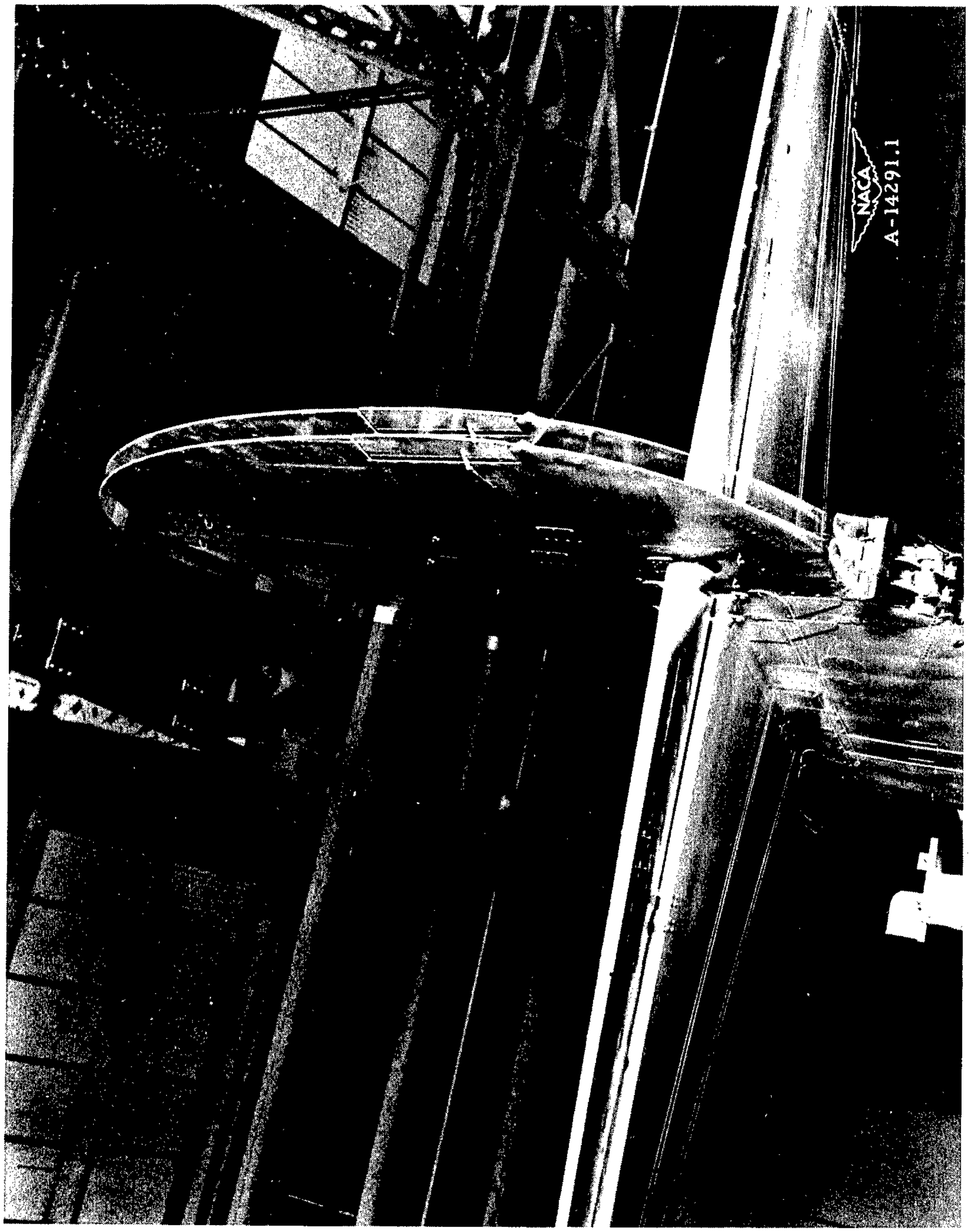


Figure 10.- Double exposure showing oscillation in yaw.

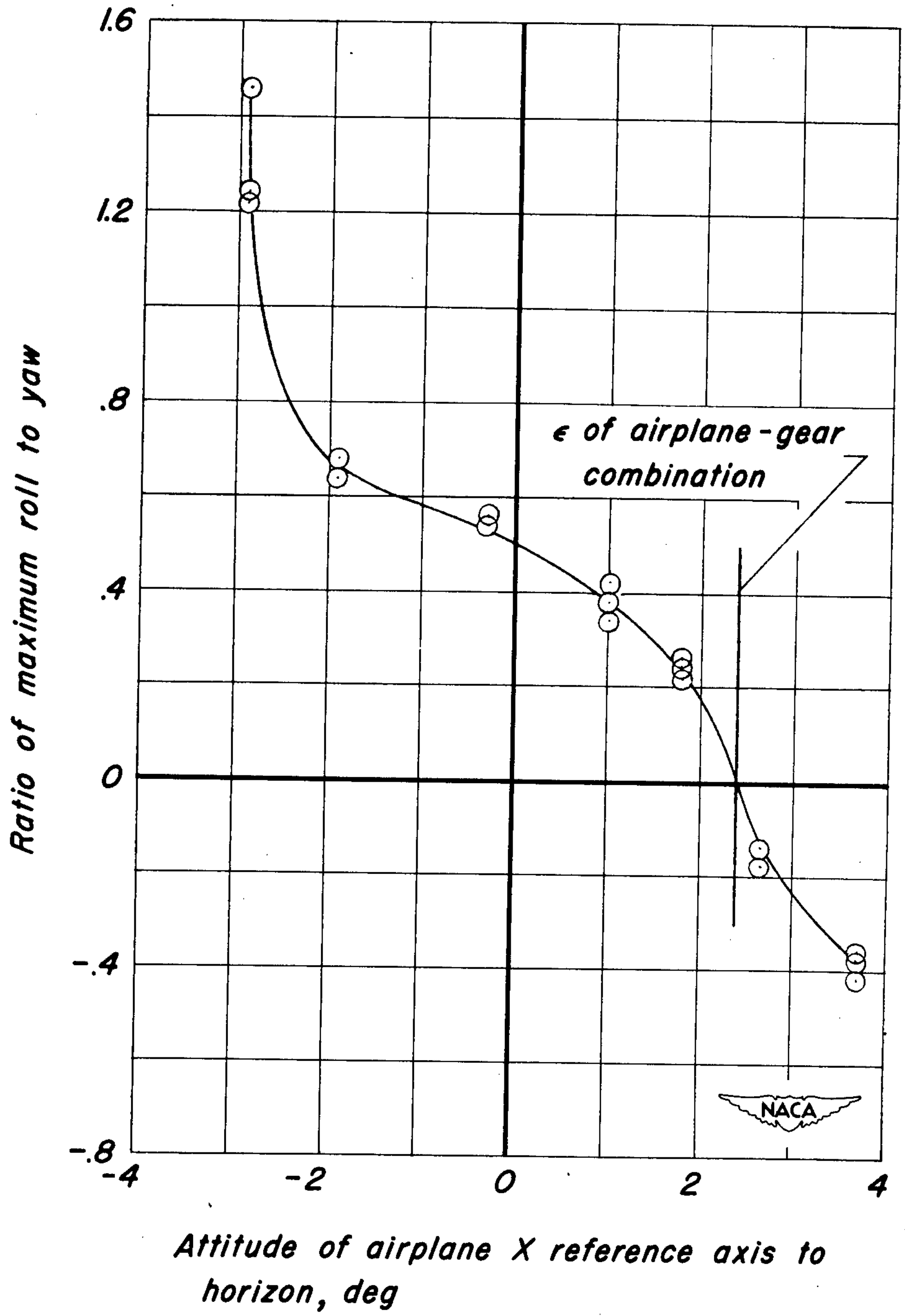


Figure 11.- Ratio of maximum roll to yaw as a function of airplane attitude as measured with the torsional pendulum. Load condition 1.

NACA-Langley - 10-11-50 - 1075