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"Mathematical Problems in Imaging, Statistical Mechanics and Related Topics" for the period 4/1/96-6/30/99.

PI F. Alberto Grunbaum

During the duration of this grant I did carry out work on a few loosely related areas, including diffuse tomography, the phase problem in X-ray crystallography, some relations between the Darboux process applied to orthogonal polynomials and the electrostatic interpretation of their zeros, and a number of specific problems related to the bispectral problem. This last one is an area that I initiated about a decade ago starting from a concrete problem in medical imaging. This field has made contact with several unrelated fields in mathematics, ranging from wave propagation, commutative rings of differential operators, etc.

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My work continues to be centered at the crossroads between "inverse problems" in areas of biomedical interest including X-ray, Magnetic Resonance and Optical tomography on the one hand and a number of mathematical developments in areas like mathematical physics and signal processing. My research program is a two-pronged approach aimed at identifying important areas in "real-life" imaging that are ripe for improved mathematical treatment. In certain cases this has given rise to new mathematical developments that should eventually find useful applications. This work has proved useful in other inverse problems arising for instance in radar detection, noninvasive evaluation, and could be useful in the general problem of seeing through turbid media.

I give below a list of the most recent material that has resulted from this contract.

List of submitted publications

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F.A.Grunbaum, I.Pácharoni and J.A.Tirao  
Matrix valued spherical  
functions associated to the complex projective plane.  
Submitted for publication

F. A. Grunbaum  
Discrete models of the harmonic oscillator and a discrete analog  
of Gauss' hypergeometric equation  
Submitted for publication

F. A. Grunbaum  
Electrostatic interpretation for the zeros of certain polynomials  
and the Darboux Process.  
Submitted for publication

List of recent publications

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F. A. Grunbaum

Variations on a theme of Heine and Stieltjes: an electrostatic interpretation of the zeros of certain polynomials, *J. of Computational and Applied Math.* 99 (1998) 189-194

F.A.Grunbaum and L.Haine

Associated Polynomials, Spectral matrices and the Bispectral problem. *Methods and applications of Analysis*, vol 6 No.6 (2000), pp 209-224

F.A. Grunbaum and L. Haine

The Wilson bispectral involution: some elementary examples, in P. Clarkson and F. Nijhoff (editors) *Symmetries and Integrability of Difference Equations* (Canterbury 1966) London Math. Soc. Lecture Note Series 255, Cambridge U. Press 1999 353-369

F.A.Grunbaum, L.Haine and E.Horozov

Some functions that generalize the Krall Laguerre polynomials. *J. Computational and Applied Mathematics* 106 (1999) 271-297

F. A. Grunbaum and L. Haine

On a q-analog of the string equation and a generalization of the classical orthogonal polynomials, in L. Vinet and J.F. van Diejen (editors) *Algebraic Methods and q-Special functions*, CRM Proc. Lecture Notes, vol 22, American Math Soc., Providence, 1999, 171-181

Some papers by other authors that are related to my recent work

\*\*\*\*\*

L.Haine and P.Iliev

The bispectral property of a q-deformation of the Schur polynomials and the q-KdV hierarchy. *J. Phys A : Math. Gen.* 30 (1997) 7217-7227

L. Haine and P. Iliev

Commutative rings of difference operators of an adelic flag manifold, to appear in *IMRN* (International Math Research Notes)

G. Wilson

Collisions of Calogero-Moser Particles and an Adelic Grassmanian, with an appendix by I.G. Macdonald) *Inventiones Math.* (1998) vol 133 1-41.

G. Wilson

Bispectral Symmetry, the Weyl algebra, and Differential Operators on Curves, *Proc. of the Steklov Inst. of Mathematics* vol 225, (1999) 141-147.

Y.Berest and G.Wilson

Classification of Rings of Differential Operators on Affine Curves, *IMRN* (1999) No.2 105-109

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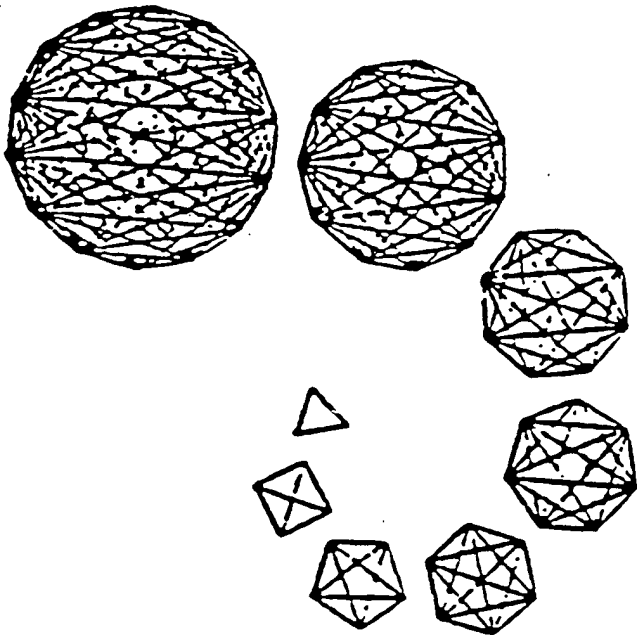
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Your cooperation in the timely submission of this report will be  
appreciated. If you have any questions please contact me.

Thanks,  
Alice

Alice M. Jeffres  
Grant Technician  
ONR Seattle  
(206) 526-3169  
Fax (206) 526-3210  
e-mail: jeffrea@onr.navy.mil