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# Limitations and Strengths of the Fourier Transform Method to Detect Accelerating Targets

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TECHNICAL MEMORANDUM  
DREO TM 2000-078  
November 2000



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DTIC QUALITY INSPECTED 4

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## Abstract

There are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing. In using a Pulse Doppler Radar to detect a non-accelerating target in additive white Gaussian noise and to estimate its radial velocity, the Fourier method provides an output signal-to-noise ratio (SNR) that increases linearly with the number of pulses. When the target is accelerating, the Fourier method may still be used to detect the target and estimate its median velocity, provided the acceleration is small enough in the sense described in the paper. For a given acceleration, when the number of pulses is increased, the output SNR of the Fourier method varies as a concave function, increasing to a maximum and then decreasing, before the method fails catastrophically. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance. Empirical formulae for the dependence of the optimum SNR and the optimum number of pulses on the acceleration are given. The results are shown to be relevant to the design of Generalized Likelihood Ratio Test (GLRT) based detectors that apply a search over a grid.

## Résumé

Ils y a des limites et des problèmes associés avec l'utilisation de la méthode de transformée de Fourier pour la détection des cibles qui accélèrent à cause du phénomène d'élargissement Doppler. Dans l'utilisation des radars Doppler à impulsions pour la détection de cibles non accélérante en présence de bruit blanc et l'estimation de la vitesse radiale, la méthode de transformée de Fourier fournit un rapport de signal au bruit (SNR) qui augmente linéairement avec le nombre de pulses. Quand la cible est en train d'accélérer la méthode de Fourier peut être encore utilisée pour détecter la cible et sa vitesse médiane en autant que l'accélération soit suffisamment petite comme montrée dans ce rapport. Pour une accélération donnée, quand le nombre de pulse augmente, le SNR résultant de la transformée de Fourier varie comme une fonction concave, augmentant à un maximum et après décroissant, avant que la méthode échoue de façon catastrophique. Ainsi le nombre de pulse et l'accélération de la cible doivent être reliés ensemble pour accomplir une performance optimum. Des formules empiriques donnant le SNR optimum en fonction du nombre de pulse et l'accélération sont données. Les résultats sont relevant au développement de détecteurs utilisant un test de rapport de probabilité généralisé qui applique une recherche à travers une grille.

## Executive Summary

The Fourier Transform is at the heart of a wide range of techniques that are used in HF radar data analysis and processing. Mapping the data into the temporal frequency domain is an effective way of recording the data such that their global characteristics can be assessed. However, the change of frequency content with time is one of the main features we observe in HF radar data. Because of this change of frequency content with time, radar signals belong to the class of non-stationary signals.

One of the central problems in High Frequency (HF) radar data is the analysis of a time series. The Fourier transform method, or Doppler processing method, has been generally used in HF radar to detect targets that are moving with constant radial acceleration. Examples of accelerating targets are manoeuvring aircrafts and missiles. In this report we show that there are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing. We show that when the target is constantly accelerating, the Fourier method may still be used to detect target and estimate its median velocity, provided the acceleration is small enough in the sense to be described in this report. It is shown that for a given acceleration, the number of pulses cannot be increased indefinitely without resulting in catastrophic failure of the method. Conversely, for a given number of pulses, the acceleration cannot be arbitrarily large without resulting in catastrophic failure of the method. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance.

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Author, Thayanathan, Thayaparan, Limitations and strengths of the Fourier transform method to detect accelerating targets, Defence Research Establishment Ottawa, DREO TM 2000-078, November 2000.

## Sommaire

La transformée de Fourier est au cœur d'un large éventail de techniques qui sont utilisées dans l'analyse et le traitement des données des radars à hautes fréquences. Le mappage des données dans le domaine temporel de fréquence est une façon effective d'enregistrer les données tel que leurs caractéristiques globales peuvent être évaluées. Cependant, le changement des composantes spectrales avec le temps est une des caractéristiques que nous observons dans les données radar à haute fréquence. Ces signaux radars appartiennent à la classe des signaux non stationnaires à cause de ce changement de composantes spectrales avec le temps.

Un des problèmes centraux dans les données des radars à haute fréquence est l'analyse des séries temporelles. La méthode de transformée de Fourier ou la méthode de traitement Doppler a été généralement utilisée dans les radars à haute fréquence pour détecter des cibles qui se déplacent avec une accélération radiale constante. Exemples de cibles qui accélèrent sont des avions et des missiles qui manœuvrent. Dans ce rapport, nous montrons qu'il y a des limites et des problèmes associés avec l'utilisation de la méthode de transformée de Fourier pour la détection des cibles qui accélèrent à cause du phénomène d'élargissement Doppler. Nous montrons que quand la cible est en train d'accélérer la méthode de Fourier peut être encore utilisée pour détecter la cible et sa vitesse médiane en autant que l'accélération soit suffisamment petite comme montrée dans ce rapport. Ce rapport montre que pour une accélération donnée, le nombre de pulse ne peut être augmenté indéfiniment sans que la méthode échoue de façon catastrophique. D'un autre côté, pour un nombre donné de pulses, l'accélération ne peut être infiniment grande sans que la méthode échoue de façon catastrophique. Ainsi le nombre de pulse et l'accélération doivent être reliés ensemble pour accomplir une performance optimale.

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# 1 Introduction

The Fourier Transform is at the heart of a wide range of techniques that are used in HF radar data analysis and processing. Mapping the data into the temporal frequency domain is an effective way of recording the data such that their global characteristics can be assessed. However, the change of frequency content with time is one of the main features we observe in HF radar data. Because of this change of frequency content with time, radar signals belong to the class of non-stationary signals. One of the central problems in High Frequency (HF) radar data is the analysis of a time series. The Fourier transform method, or Doppler processing method, has been generally used in HF radar to detect targets that are moving with constant radial acceleration. Examples of accelerating targets are manoeuvring aircrafts and missiles.

In this paper, the radar is assumed to be of the Pulse-Doppler type, that is, it sends out a uniform train of radio frequency pulses and phase-coherently receives their returns. It is also assumed that the receiver has pulse compression capability so that a pulse return from an isolated target can be represented by a single sample of the compressed pulse.

A moving target with low per-pulse signal-to-noise ratio (SNR) may be efficiently detected by processing the sequence of Pulse-Doppler radar returns by coherent integration [Whalen, 1971; Meyer and Mayer, 1973; Wehner, 1987; Scheer, 1993]. Under the simplifying assumptions of zero range-walk and constant radial velocity, coherent integration is achieved by taking the Fourier power spectrum of the sequence of complex amplitudes of the received pulses and locating its maximum. If the maximum value exceeds a threshold then the presence of a target is declared, and the location of the maximum is taken to be an estimate of the radial velocity. In an additive white noise background, the Fourier method gives an output SNR that is proportional to the number of pulses [Meyer and Mayer, 1973].

In this paper, we relax the assumption of constant radial velocity and consider using the Fourier method to detect targets that are moving with constant radial acceleration. Thus the aim of the paper is to study the phenomenon usually referred to as *Doppler smearing* and quantify the performance of the Fourier method. In this paper, the term *acceleration* may be interpreted as the *magnitude of the acceleration*, as the conclusions depend only on the magnitude of the acceleration. It is empirically shown that when the target is constantly accelerating, the Fourier method may still be used to detect the target and estimate its

median velocity, provided the acceleration is small enough in the sense to be described in this paper.

It is shown that for a given acceleration, the number of pulses cannot be increased indefinitely without resulting in catastrophic failure of the method. Conversely, for a given number of pulses, the acceleration cannot be arbitrarily large without resulting in catastrophic failure of the method. Moreover, for a given acceleration, when the number of pulses is increased, the output SNR varies as a concave function of the number of pulses, achieving a maximum well before the failure occurs. For a given number of pulses, the output SNR monotonically decreases with the acceleration, until the failure occurs. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance.

The results are presented in terms of a normalized value of the acceleration that takes into account the radar carrier frequency and the pulse repetition frequency. This makes the results widely useful. The results may be used to determine whether, in a given scenario of an accelerating target, the Fourier method can be used or a more sophisticated method such as the Generalized Likelihood Ratio Test (GLRT) is required. In scenarios where the Fourier method is not desirable, and where the GLRT method is considered, it is shown how the results may be used to determine the critical spacing of the grid over which the search is applied.

## 2 Signal Model

Suppose the Pulse-Doppler radar sends out  $N$  pulses, one every  $T$  seconds, and there is a target moving with constant radial acceleration. Assume that the change in range during the observation period of  $NT$  seconds, known as range walk, is negligible compared to the radar range resolution as determined by the width of the compressed pulse. Then the complex amplitudes of the  $N$  range-compressed pulses taken at the range of the target have the form

$$r(n) = s(n) + v(n) \quad (1)$$

where

$$s(n) = ae^{j(b_0 + b_1n + \frac{1}{2}b_2n^2)} \quad (2)$$

for  $n = 0, 1, 2, 3, \dots, (N - 1)$ , is the noise-free signal, and  $v(n)$  is a sequence of independent samples of complex Gaussian noise with mean zero and variance  $\sigma^2$ .

The signal parameters  $a$  and  $b_0$  are the target signal amplitude and phase respectively. The signal parameters  $b_1$  and  $b_2$  are the normalized initial radial velocity and the normalized radial acceleration respectively.

### 2.1 Normalized Initial Radial Velocity and Radial Acceleration

The normalized initial radial velocity  $b_1$  is defined as

$$b_1 = u \left( \frac{4\pi T}{\lambda} \right). \quad (3)$$

where  $u$  is the initial radial velocity in meters/sec towards the radar,  $T$  is pulse repetition interval in secs, and  $\lambda$  is the carrier wavelength in meters. Similarly, the normalized radial acceleration  $b_2$  is defined as

$$b_2 = f \left( \frac{4\pi T^2}{\lambda} \right). \quad (4)$$

where  $f$  is the radial acceleration in meters/sec/sec towards the radar. It can be seen that both of the above normalized quantities are non-dimensional.

To obtain these relations note that the decrement in range  $d$ , measured from the beginning of the observation interval, as a function of continuous-time  $t$ , is  $d(t) = ut + \frac{1}{2}ft^2$ , and therefore the phase increment of a pulse as a function of discrete-time, or pulse index, is given by

$$2\pi \left( \frac{2d(nT)}{\lambda} \right) = u \left( \frac{4\pi T}{\lambda} \right) n + \frac{1}{2}f \left( \frac{4\pi T^2}{\lambda} \right) n^2. \quad (5)$$

In the rest of the discussion, the terms *velocity* and *acceleration* refer to *normalized radial velocity* and *normalized radial acceleration* respectively.

## 2.2 Per-Pulse Signal-to-Noise Ratio

The Per-Pulse Signal-to-Noise Ratio is defined as

$$\text{SNR}_{\text{pulse}} = \left( \frac{a}{\sigma} \right)^2. \quad (6)$$

### 3 The performance of the Fourier method in detecting a non-accelerating target

Although a given signal can be represented in many different ways, the most important are the time and frequency representations. The majority of signals encountered in our everyday life are directly related to time. The frequency representations, on the other hand, were not popular until the early 19th century when Fourier first proposed the harmonic trigonometric series. Since then, the frequency representation has become one of the most powerful and standard tools for studying signals. By using frequency representations, we could better understand many physical phenomenon and accomplish many things that cannot achieved based on time representations. While the time domain function indicates how a signal's amplitude changes with time, the frequency domain function tells how often such changes take place. The bridge between time and frequency is the *Fourier Transform*.

The signal is expanded in terms of sinusoids of different frequencies [Brigham, 1974; Bloomfield, 1976; Papoulis 1977; Bracewell, 1978]

$$s(t) = \frac{1}{\sqrt{2\pi}} \int S(\omega) e^{j\omega t} d\omega \quad (7)$$

The waveform is made up of the addition (linear superposition) of the simple waveforms,  $e^{j\omega t}$ , each characterized by the frequency,  $\omega$ , and contributing a relative amount indicated by the coefficient,  $S(\omega)$ .  $S(\omega)$  is obtained from the signal by

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int s(t) e^{-j\omega t} dt \quad (8)$$

and is called the spectrum or the Fourier Transform. The square of the Fourier Transform  $|S(\omega)|^2$  is called the *Fourier power spectrum*, which indicates how the signal energy is distributed in the frequency domain. While the Fourier Transform  $S(\omega)$  is a linear function of the analyzed signal, the Fourier power spectrum  $|S(\omega)|^2$  is quadratic to the signal  $s(t)$ . The Fourier Transform  $S(\omega)$  in general is complex, whereas the power spectrum  $|S(\omega)|^2$  is always real. The Fourier Transform and the Fourier power spectrum are the two most important tools for frequency analysis.

Before investigating the case of an accelerating target, it is helpful to review the case of a

non-accelerating target.

When the acceleration is zero, and the noise is absent, the Fourier power spectrum

$$S(\omega) = \frac{1}{N} \left| \sum_{n=0}^{N-1} s(n) e^{-j\omega n} \right|^2 \quad (9)$$

$$= \frac{a^2}{N} \left| \sum_{n=0}^{N-1} e^{j(b_1 - \omega)n} \right|^2, \quad (10)$$

has a unique maximum which is attained when  $\omega = b_1 \bmod 2\pi$ . Assuming  $-\pi < b_1 < \pi$ , the maximizing  $\omega$  is an estimate of the normalized radial velocity.

Therefore, the target can be detected and the velocity parameter  $b_1$  can be estimated by using peak detection on the noisy version

$$\Gamma(\omega) = \frac{1}{N} \left| \sum_{n=0}^{N-1} r(n) e^{-j\omega n} \right|^2 \quad (11)$$

of  $S(\omega)$ , that is, maximizing  $\Gamma(\omega)$  with respect to  $\omega$ , and declaring the presence of a target if the maximum value exceeds a threshold; the maximizing  $\omega$ , which is now a random variable due to the presence of noise, is taken as the estimate of  $b_1$ . This method is equivalent to the GLRT.

The notion of output SNR of the method comes from the observation that under the 'noise-only' hypothesis  $\frac{2}{\sigma^2} \Gamma(\omega)$  is a central Chi-Squared random variable of two degrees of freedom, for all  $\omega$ , whereas under the 'signal-and-noise' hypothesis  $\frac{2}{\sigma^2} \Gamma(b_1)$  is a non-central Chi-Squared random variable of two degrees of freedom and non-centrality parameter  $2N \left(\frac{a}{\sigma}\right)^2$  [Whalen, 1971; Meyer and Mayer, 1973].

We, therefore, consider

$$\text{SNR}_{\text{out}} = N \left(\frac{a}{\sigma}\right)^2, \quad (12)$$

as this can be used to approximately compute the probability of detection, assuming that the maximum of  $\Gamma(\omega)$  occurs at  $\omega = b_1$ . Note that  $\text{SNR}_{\text{out}}$  is a monotonically increasing

function of  $N$ . More specifically, it is a linearly increasing function of  $N$ . Normalizing this with respect to  $\text{SNR}_{\text{pulse}}$  gives

$$\text{SNR}_{\text{gain}} = \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{pulse}}}, \quad (13)$$

$$= N. \quad (14)$$

## 4 The performance of the Fourier method in detecting an accelerating target

When the acceleration is non-zero, and the noise is absent, the Fourier power spectrum is

$$S(\omega) = \frac{1}{N} \left| \sum_{n=0}^{N-1} s(n) e^{-j\omega n} \right|^2, \quad (15)$$

$$= \frac{a^2}{N} \left| \sum_{n=0}^{N-1} e^{j((b_1 - \omega)n + \frac{1}{2}b_2 n^2)} \right|^2. \quad (16)$$

Some insight into this function can be gained by rearranging it as

$$S(\omega) = \frac{a^2}{N} \left| \sum_{n=0}^{N-1} e^{j((b_m - \omega)(n - \frac{N-1}{2}) + \frac{1}{2}b_2(n - \frac{N-1}{2})^2)} \right|^2, \quad (17)$$

where

$$b_m = b_1 + \left( \frac{N-1}{2} \right) b_2 \quad (18)$$

is the normalized median velocity over the time interval 0 to  $(N-1)T$  seconds. For simplicity of discussion assume that  $-\pi < b_m < \pi$ . The rearrangement shows that  $S(\omega)$  is symmetric with respect to  $\omega = b_m$ , that is,  $S(b_m + \nu) = S(b_m - \nu)$ , and the spread of the function about  $\omega = b_m$  depends only on  $b_2$  and  $N$ . More specifically, when considered as a function of  $b_2$ ,  $S(\omega)$  depends only on the magnitude of  $b_2$ .

A closed-form expression for the above sum seems difficult to obtain. But numerical evaluations suggest that, for a given acceleration  $b_2$ , there is a threshold value  $N_t$  of  $N$ , below which  $S(\omega)$  has a unique maximum, and above which  $S(\omega)$  has at least two equal maxima. The threshold value  $N_t$  depends on the acceleration  $b_2$  alone. Moreover,  $N_t$  is a decreasing function of  $b_2$ . Thus  $N_t$  implicitly defines the maximum allowable acceleration for a given  $N$ .

## 4.1 The Region $N < N_t$

For  $N < N_t$ , by symmetry considerations, the unique maximum of  $S(\omega)$  is attained when  $\omega = b_m$ . Thus

$$\max_{\omega} S(\omega) = S(b_m), \quad (19)$$

$$= \frac{a^2}{N} \left| \sum_{n=0}^{N-1} e^{j\frac{1}{2}b_2(n-\frac{N-1}{2})^2} \right|^2. \quad (20)$$

Therefore, in this region, we have

$$\text{SNR}_{\text{out}} = \frac{S(b_m)}{\sigma^2}, \quad (21)$$

and

$$\text{SNR}_{\text{gain}} = \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{pulse}}}, \quad (22)$$

$$= \frac{S(b_m)}{a^2}, \quad (23)$$

$$= \frac{1}{N} \left| \sum_{n=0}^{N-1} e^{j\frac{1}{2}b_2(n-\frac{N-1}{2})^2} \right|^2. \quad (24)$$

Provided that  $\text{SNR}_{\text{out}}$  is adequate, peak detection still provides a means of detecting an accelerating target and estimating its median velocity. However, as expected,  $\text{SNR}_{\text{out}}$  is not a linearly increasing function of  $N$ . It is not even a monotonically increasing function of  $N$ . Rather it is a concave function of  $N$  and attains a maximum value in the region  $N < N_t$ . The optimum value  $N_{\text{opt}}$  of  $N$ , where the  $\text{SNR}_{\text{out}}$  is maximum, depends on the parameter  $b_2$  alone. Moreover,  $N_{\text{opt}}$  is a decreasing function of  $b_2$ .

For  $N < N_{\text{opt}}$ , the concentration of the Fourier power spectrum  $S(\omega)$  about  $\omega = b_m$  increases as  $N$  increases towards  $N_{\text{opt}}$ . For  $N > N_{\text{opt}}$ , the concentration of the Fourier power spectrum  $S(\omega)$  about  $\omega = b_m$  decreases as  $N$  increases towards  $N_t$ .

## 4.2 The Region $N > N_t$

For  $N > N_t$ , the Fourier power spectrum  $S(\omega)$  has at least two maxima located symmetrically about  $\omega = b_m$ . Moreover, it is spread symmetrically about  $\omega = b_m$ , and the amount of spread

increases rapidly with  $N$ , soon covering the entire interval of length  $2\pi$ . The combination of the significantly low maxima and the spectral spreading makes it practically impossible to discern the presence of the target or estimate its spectral location. Therefore, the use of peak detection as a method of detecting an accelerating target and estimating its median velocity is precluded in this region.

## 5 Numerical Examples

Figure 1 shows how the Fourier spectrum changes when  $N$  (referred to as the length) is increased while the acceleration is held constant. Here we can see the spectrum concentration first increasing, then slowly decreasing, and eventually rapidly decreasing with multiple peaks appearing.

Figure 2 shows how the Fourier spectrum changes when the acceleration is increased while  $N$  is held constant. Here we can see the spectrum concentration first slowly decreasing and then rapidly decreasing with multiple peaks appearing.

In all cases the median velocity was held at zero for the purpose of compact representation, but the conclusions are independent of this assumption.

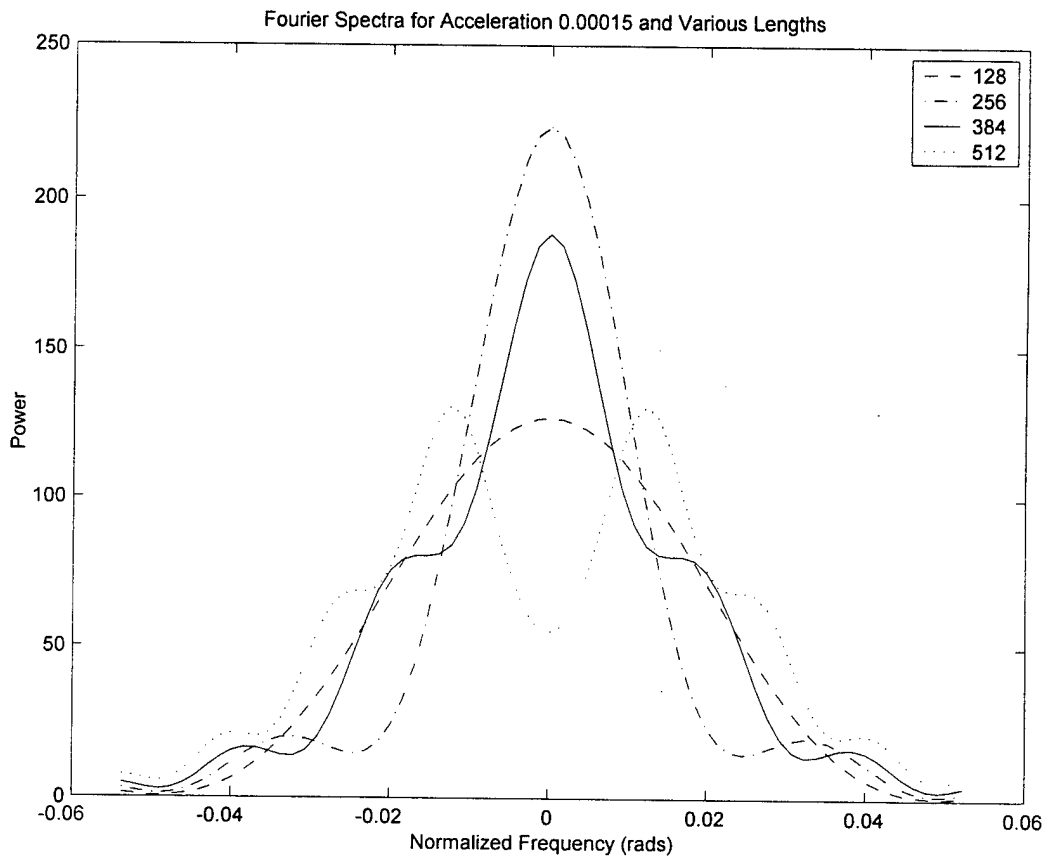


Figure 1: Effect of increasing length.

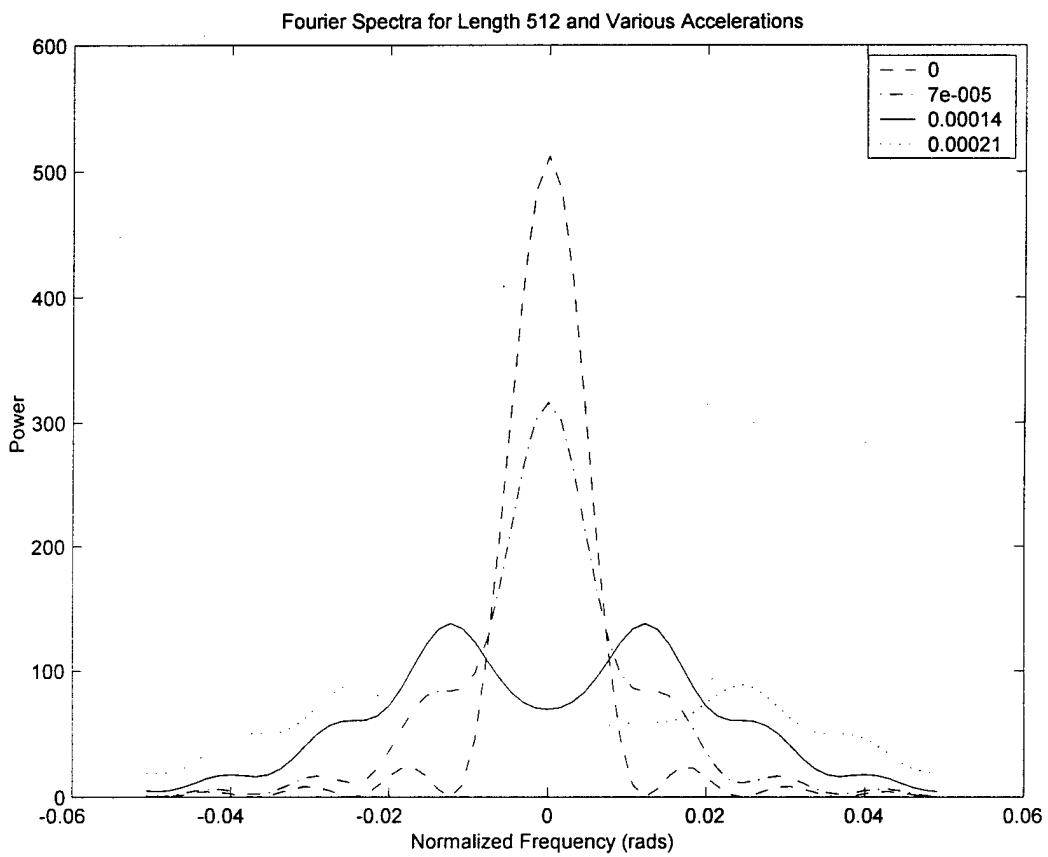


Figure 2: Effect of increasing acceleration.

## 6 Performance Graphs

Graphs quantifying the performance of the Fourier method in detecting an accelerating target are given in this section. The graphs were obtained by empirically studying the Fourier power spectrum for values of  $N$  ranging from 64 to 512 and acceleration  $b_2$  ranging from 0.00005 to 0.0004.

Figures 3 and 4 show the dependence of the  $\text{SNR}_{\text{gain}}$  on Length  $N$  of the signal for various values of the acceleration  $b_2$ . We can see that for  $b_2 \geq 0.00012$ , the failure point has been achieved, that is,  $N_t < 512$ , and for  $b_2 \geq 0.00006$ , the optimum point has been achieved, that is  $N_{\text{opt}} < 512$ .

Figure 5 shows the dependence of the Optimum  $\text{SNR}_{\text{gain}}$  on acceleration. Figure 6 shows the dependence of the Optimum Length  $N_{\text{opt}}$  on acceleration.

Figure 7 shows the dependence of the log of Optimum  $\text{SNR}_{\text{gain}}$  on the log of acceleration. From the linear least squares fit we can infer that

$$10 \log_{10} \text{SNR}_{\text{gain,opt}} \approx -5 \log_{10} b_2 + 4.537. \quad (25)$$

or

$$\text{SNR}_{\text{gain,opt}} \approx 10^{0.4537} b_2^{-0.5}. \quad (26)$$

Figure 8 shows the dependence of the log of optimum length on the log of acceleration. From the linear least squares fit we can infer that

$$\log_{10} N_{\text{opt}} \approx -0.5 \log_{10} b_2 + 0.5606, \quad (27)$$

or

$$N_{\text{opt}} \approx 10^{0.5606} b_2^{-0.5}. \quad (28)$$

Combining these,

$$\text{SNR}_{\text{gain,opt}} \approx 10^{-0.112} N_{\text{opt}}. \quad (29)$$

Thus when operating at the optimum point  $N = N_{\text{opt}}$ , the loss with respect to the non-accelerating case is 1.12dB. This is also the loss with respect to optimum coherent integration.

Every effort was made to ensure the accuracy of the above results. However, caution must be exercised in applying them outside the studied range, especially for larger accelerations.

Figure 9 shows the dependence of the log of threshold  $\text{SNR}_{\text{gain}}$  on the log of acceleration. From the linear least squares fit we can infer that

$$10 \log_{10} \text{SNR}_{\text{gain,opt}} \approx -4.677 \log_{10} b_2 + 3.746, \quad (30)$$

or

$$\text{SNR}_{\text{gain,opt}} \approx 10^{0.3746} b_2^{-0.4677}. \quad (31)$$

Figure 10 shows the dependence of the log of threshold length on the log of acceleration. From the linear least squares fit we can infer that

$$\log_{10} N_t \approx -0.5078 \log_{10} b_2 + 0.6859, \quad (32)$$

or

$$N_t \approx 10^{0.6859} b_2^{-0.5078}. \quad (33)$$

Combining these,

$$\text{SNR}_{\text{gain,opt}} \approx 10^{-0.3745} N_t^{0.9210}. \quad (34)$$

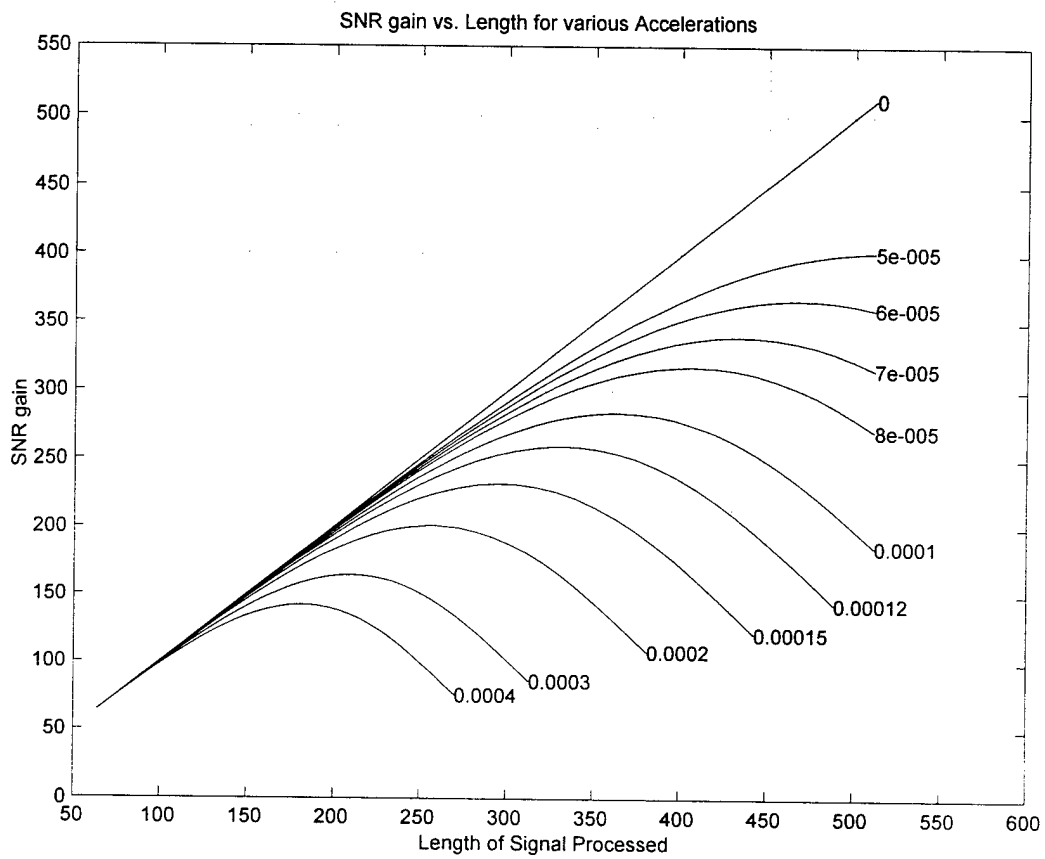


Figure 3: The dependence of SNR on length for various accelerations.

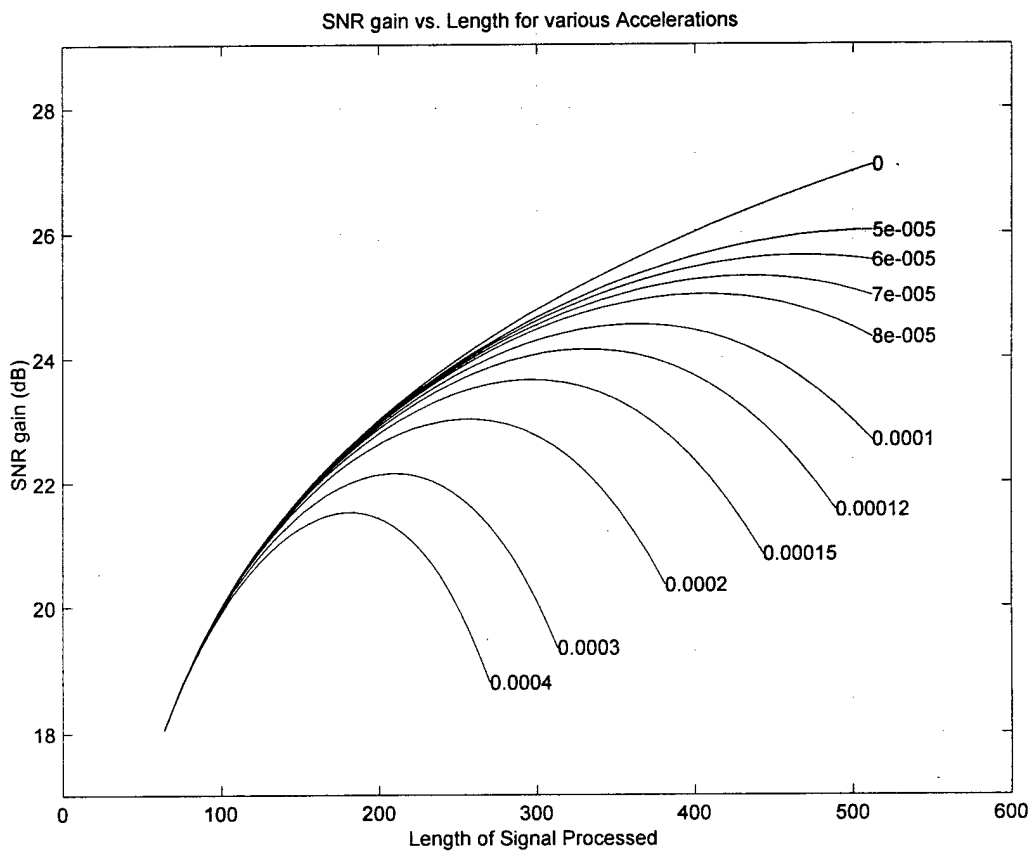


Figure 4: The dependence of SNR on length for various accelerations.

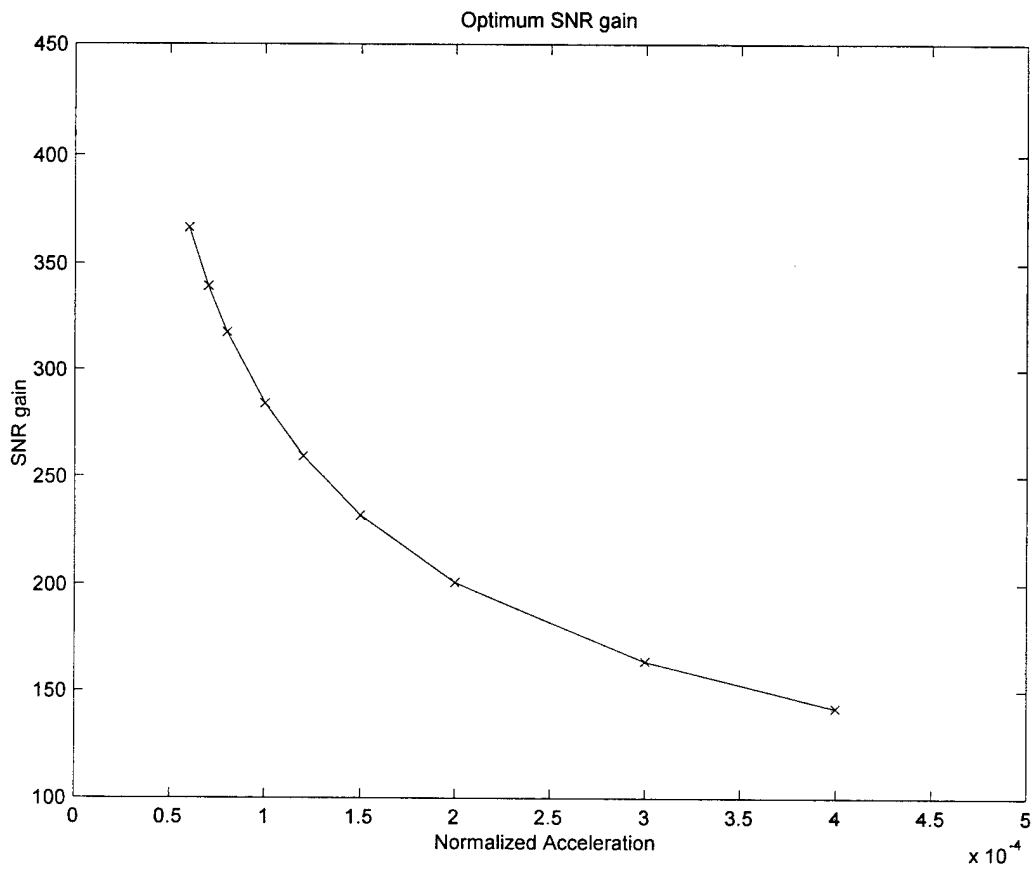


Figure 5: The dependence of the optimum SNR on the accelerations.

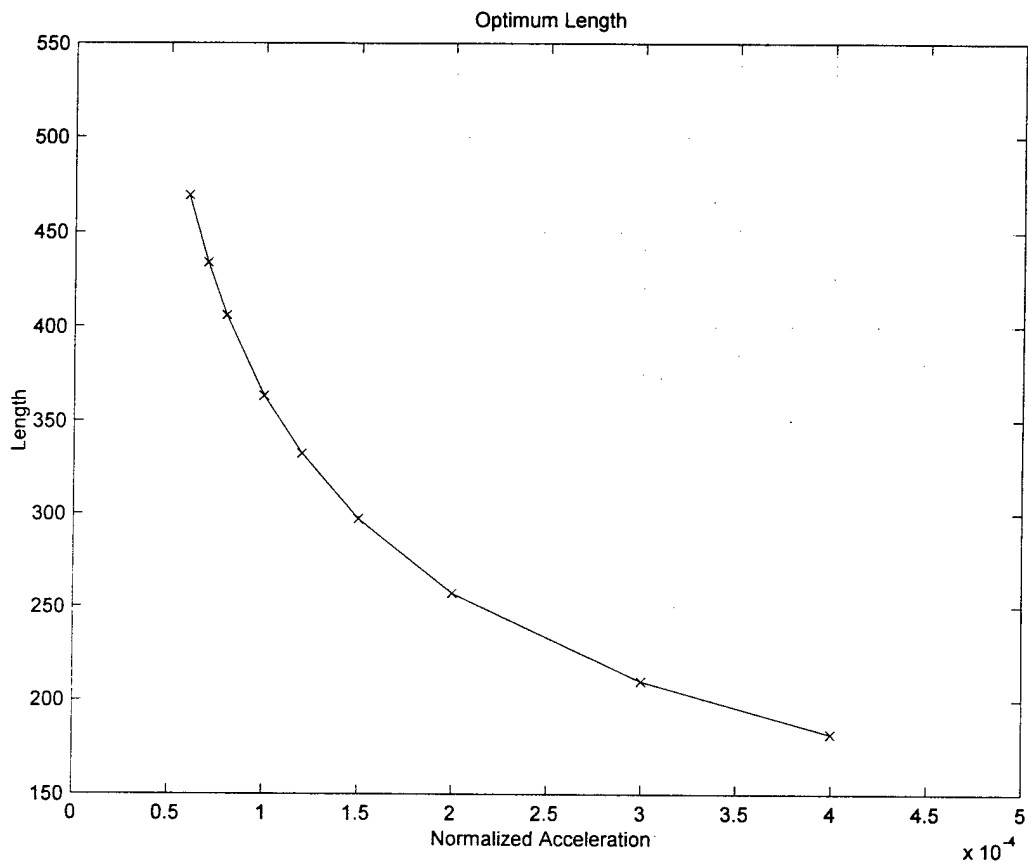


Figure 6: The dependence of the optimum length on the accelerations.

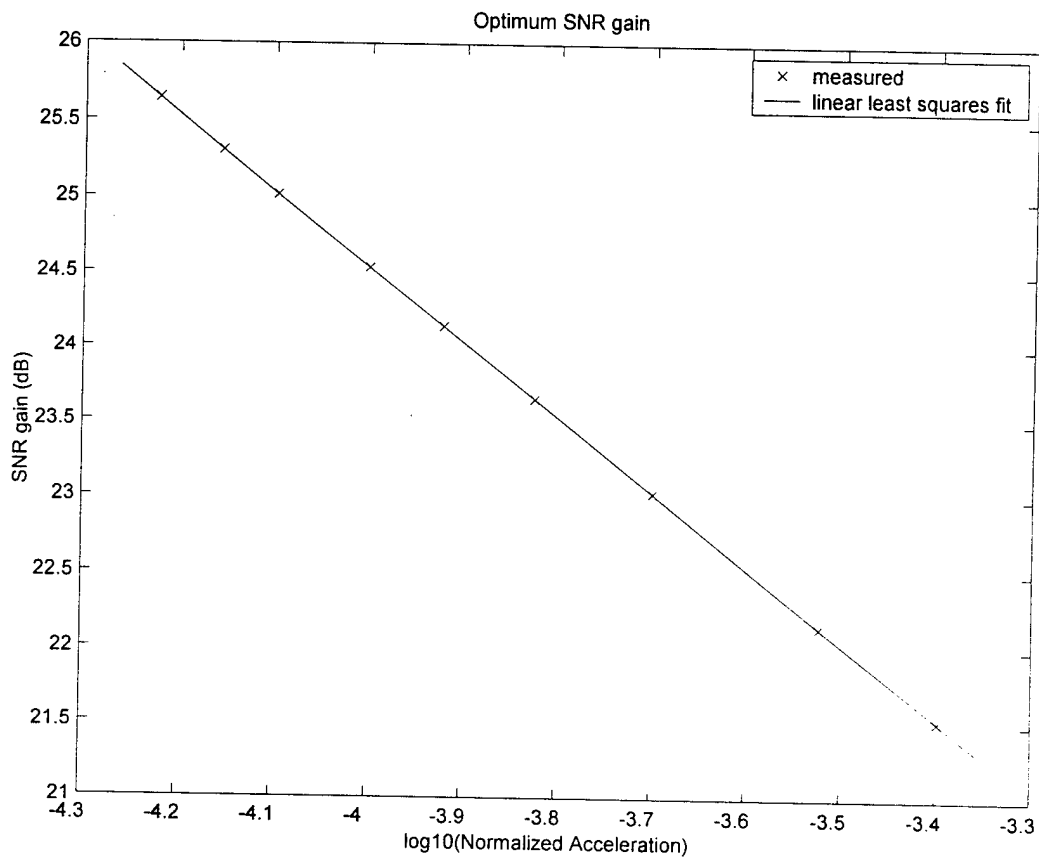


Figure 7: The dependence of the  $\log_{10}(\text{optimum SNR})$  on the  $\log_{10}(\text{acceleration})$ .

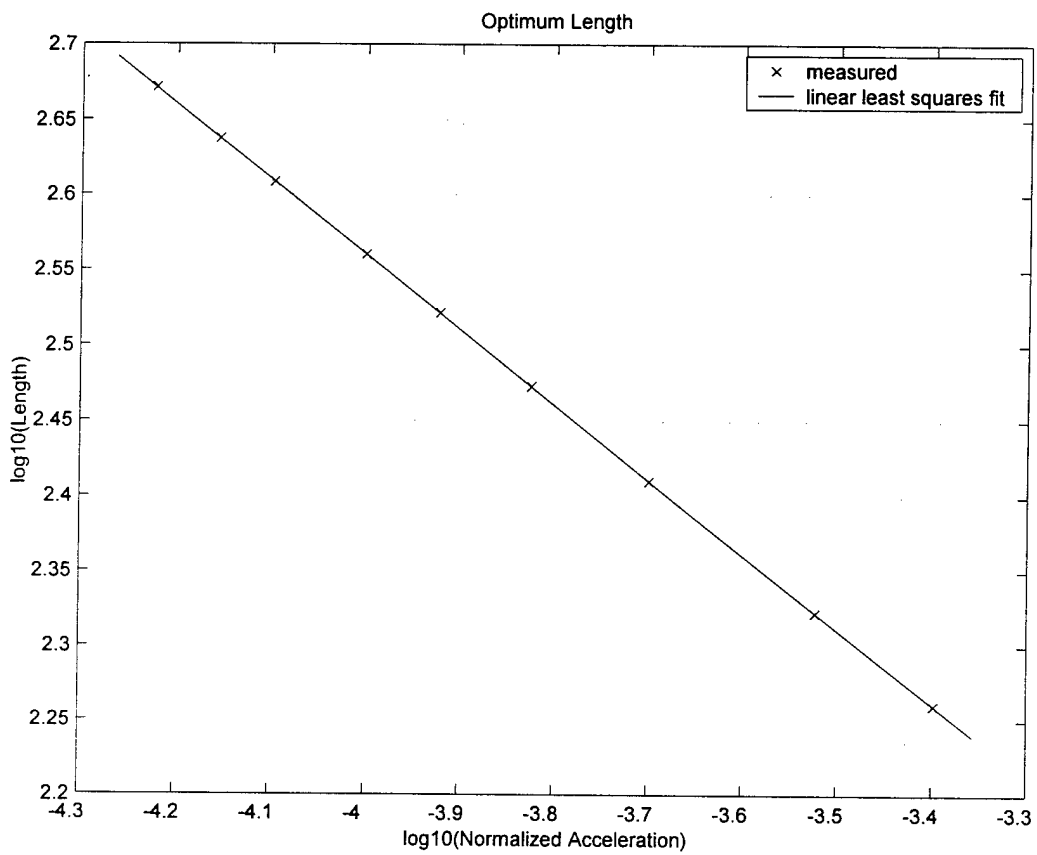


Figure 8: The dependence of the  $\log_{10}(\text{optimum length})$  on the  $\log_{10}(\text{acceleration})$ .

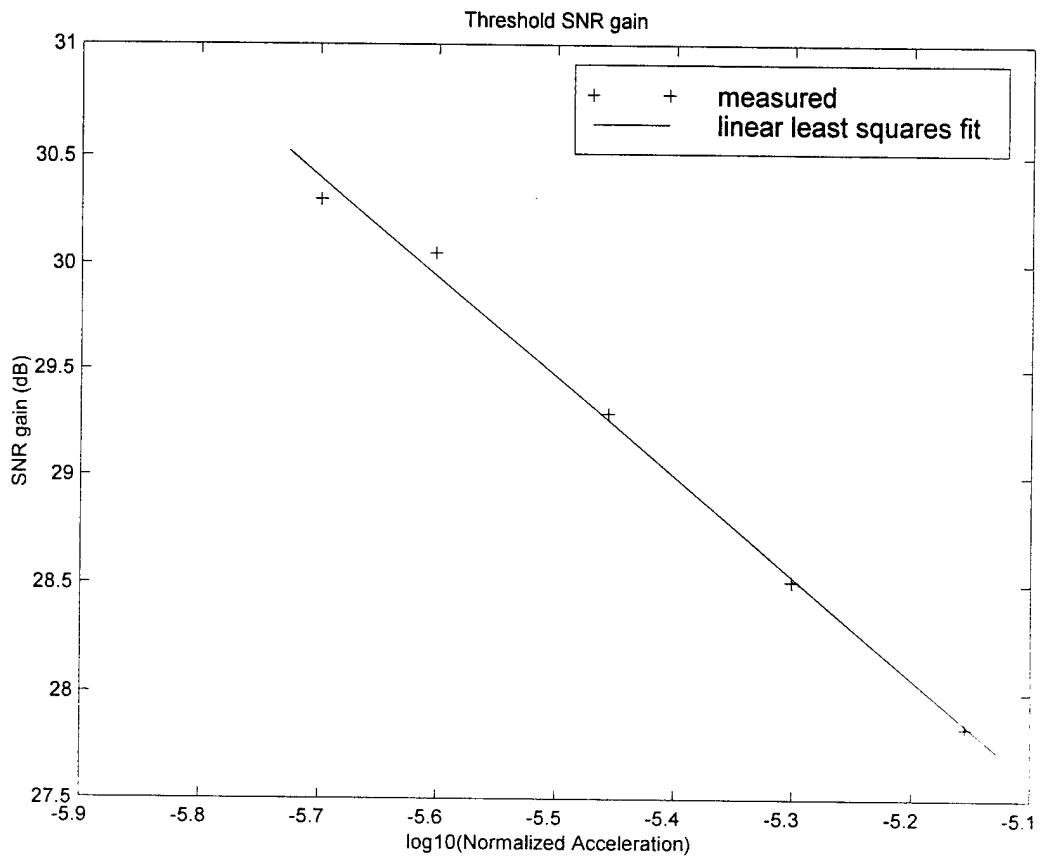


Figure 9: The dependence of the  $\log_{10}(\text{threshold SNR})$  on the  $\log_{10}(\text{acceleration})$ .

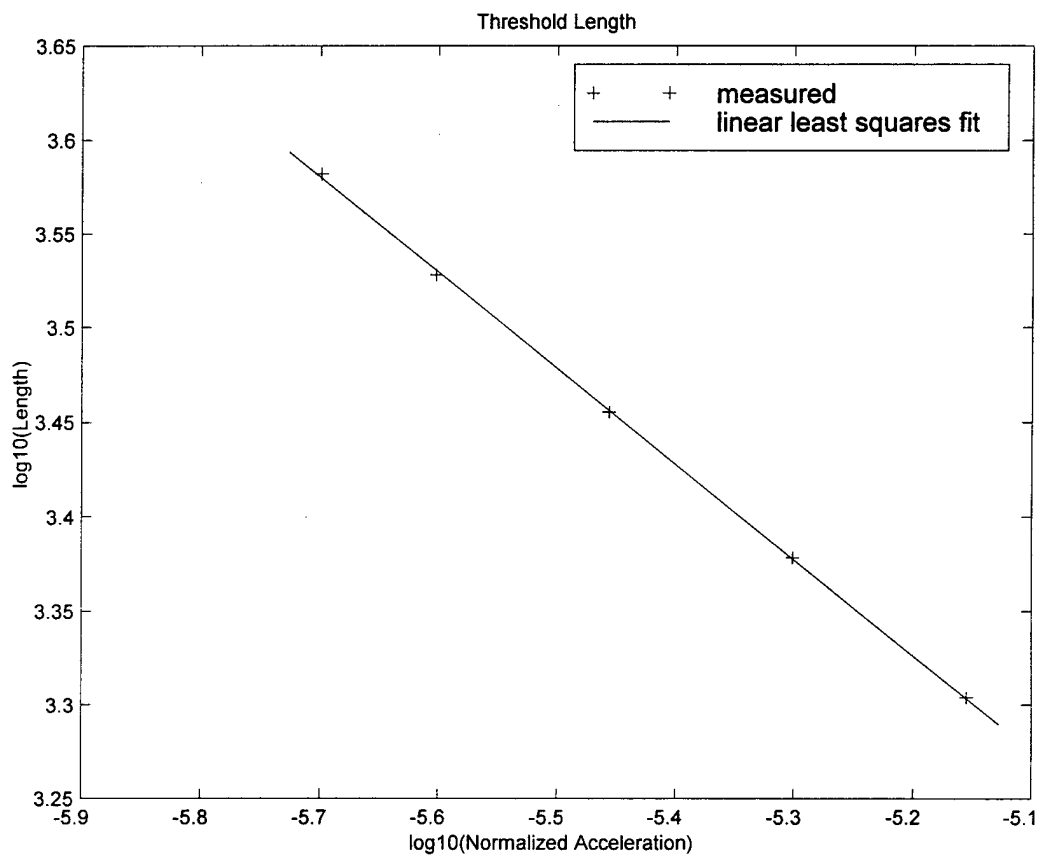


Figure 10: The dependence of the  $\log_{10}(\text{threshold length})$  on the  $\log_{10}(\text{acceleration})$ .

## 7 Application to GLRT

In the case of an accelerating target, the GLRT method seeks to maximize  $\Lambda(c_1, c_2) = \frac{1}{N} \left| \sum_{n=0}^{N-1} r(n) e^{-j(c_1 n + \frac{1}{2} c_2 n^2)} \right|^2$  with respect to  $c_1$  and  $c_2$  [Whalen, 1971; Kay, 1993]. A crude way of doing this is to evaluate  $\Lambda(c_1, c_2)$  on a rectangular grid in the  $c_1$ - $c_2$  plane and choose the maximum. The results of the previous section imply that the grid spacing in the  $c_2$  dimension cannot be less than a critical value if the method is to work for all values of  $b_2$ .

To see this, first note that, for a fixed  $c_2$ , maximizing  $\Lambda(c_1, c_2)$  with respect to  $c_1$  is equivalent to using the Fourier method to detect a target with initial velocity  $b_1$  and acceleration  $(b_2 - c_2)$ . Suppose the grid spacing in the  $c_2$  dimension is  $2\Delta$ . Then for every  $b_2$  there is a  $c_2$  grid value such that  $|b_2 - c_2| \leq \Delta$ . For this fixed  $c_2$  value, the noise-free version of  $\Lambda(c_1, c_2)$  will have a unique maximum with respect to  $c_1$  provided the threshold length  $N_t$  corresponding to acceleration  $|b_2 - c_2|$  is greater than  $N$ . To ensure uniqueness of the maximum for every  $b_2$ , we require  $\Delta \leq \Delta_{\text{crit}}$ , where  $\Delta_{\text{crit}}$  is the acceleration for which the given  $N$  is the threshold length.

The  $\text{SNR}_{\text{gain}}$  is at most  $N$ , and depends on  $|b_2 - c_2|$ . We can use the graphs to choose  $\Delta$  so that the loss of  $\text{SNR}_{\text{gain}}$  is less than a given value.

## 8 Conclusions

The Fourier transform method, or Doppler processing method, has been generally used in HF radar to detect targets that are moving with constant radial acceleration. Examples of accelerating targets are manoeuvring aircrafts and missiles. In this report we show that there are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing.

We considered using the Fourier method for detecting constantly accelerating targets and estimating their median velocities, and empirically showed that there are decreasing functions  $N_{opt}(b_2)$  and  $N_t(b_2)$  of the acceleration  $b_2$ , termed the optimum length and the threshold length, respectively, with the following properties:

1. in the region  $N < N_t$  the Fourier power spectrum of the noise-free signal has a unique maximum at the median velocity, and outside of which it has at least two maxima; thus  $N_t$  implicitly defines the maximum allowable acceleration for a given  $N$ .
2. in the region  $N < N_t$  the  $\text{SNR}_{\text{gain}}$  varies as a concave function of  $N$  achieving a maximum at  $N = N_{opt}$ ; moreover  $N_{opt} < N_t$ .

Formulae were given for  $N_{opt}$  and corresponding SNR in terms of the acceleration. Similarly, Formulae were given for  $N_t$  and corresponding SNR in terms of the acceleration. These formulae might lead to the design of detectors that are of manageable complexity yet have near optimum performance.

As an application, we considered a GLRT based detector that applies a search over a grid. We showed that the grid spacing must be less than a critical value. We also showed how to choose the grid spacing so as to achieve a given  $\text{SNR}_{\text{gain}}$ .

## Acknowledgements

The work was carried out as part of a collaborative research contract from Defence Research Establishment Ottawa to Raytheon Canada (DREO Ref. W7714-8-0183/001/SV, RSCL Ref. DND339).

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. ORIGINATOR (the name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in section 8.) Defence Research Establishment Ottawa Department of National Defence Ottawa, Ontario, Canada K1A 0Z4		2. SECURITY CLASSIFICATION (overall security classification of the document, including special warning terms if applicable)  UNCLASSIFIED	
3. TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C or U) in parentheses after the title.)  Limitations and strengths of the Fourier transform method to detect accelerating targets (U)			
4. AUTHORS (Last name, first name, middle initial)  Thayaparan, Thayananthan and Yasotharan, Ambighairajah			
5. DATE OF PUBLICATION (month and year of publication of document)  November 2000	6a. NO. OF PAGES (total containing information. Include Annexes, Appendices, etc.)  24	6b. NO. OF REFS (total cited in document)  09	
7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)  DREO TECHNICAL MEMORANDUM			
8. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include the address.) Defence Research Establishment Ottawa Department of National Defence Ottawa, Ontario, Canada K1A 0Z4			
9a. PROJECT OR GRANT NO. (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant)  05AB11		9b. CONTRACT NO. (if appropriate, the applicable number under which the document was written)	
10a. ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document.)  DREO TECHNICAL MEMORANDUM 2000-078		10b. OTHER DOCUMENT NOS. (Any other numbers which may be assigned this document either by the originator or by the sponsor)	
11. DOCUMENT AVAILABILITY (any limitations on further dissemination of the document, other than those imposed by security classification)  <input checked="" type="checkbox"/> Unlimited distribution <input type="checkbox"/> Distribution limited to defence departments and defence contractors; further distribution only as approved <input type="checkbox"/> Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved <input type="checkbox"/> Distribution limited to government departments and agencies; further distribution only as approved <input type="checkbox"/> Distribution limited to defence departments; further distribution only as approved <input type="checkbox"/> Other (please specify):			
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(U) There are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing. In using a Pulse Doppler Radar to detect a non-accelerating target in additive white Gaussian noise and to estimate its radial velocity, the Fourier method provides an output signal-to-noise ratio (SNR) that increases linearly with the number of pulses. When the target is accelerating, the Fourier method may still be used to detect the target and estimate its median velocity, provided the acceleration is small enough in the sense described in the paper. For a given acceleration, when the number of pulses is increased, the output SNR of the Fourier method varies as a concave function, increasing to a maximum and then decreasing, before the method fails catastrophically. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance. Empirical formulae for the dependence of the optimum SNR and the optimum number of pulses on the acceleration are given. The results are shown to be relevant to the design of Generalized Likelihood Ratio Test (GLRT) based detectors that apply a search over a grid.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

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Fourier Transform  
Doppler Processing  
Accelerating Targets  
Doppler Radar  
Doppler Smearing  
Generalized Likelihood Ratio Test  
Generalized Velocity  
Generalized Acceleration