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**ANALYTICAL AND EXPERIMENTAL STUDIES INTO  
STRUCTURAL HEALTH MONITORING**

AFOSR GRANT NUMBER F49620-97-1-0129

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18 December 2000

## **ANALYTICAL AND EXPERIMENTAL STUDIES INTO STRUCTURAL HEALTH MONITORING**

### **Abstract**

The research objectives of this project were to develop a "toolbox" of methods for monitoring the "health" of representative structural systems encountered in aerospace and aircraft structures subjected to arbitrary dynamic environments. The research was focused on developing methods suitable for use with structural response measurements from flexible structural components and assemblages that may incorporate elements undergoing significant nonlinear (i.e., not-necessarily-linear) deformations. Furthermore, the research developed innovative approaches for coping with the wide spectrum of situations arising in the aircraft and aerospace field. It did this by formulating a general approach for using structural vibration measurements to analyze and quantify the extent and location of relatively small changes in the structural parameters, which may be precursors of serious damage to the structure being monitored.

### **Accomplishments**

The research goals of this project were accomplished by performing analytical and experimental studies in the following areas:

#### ***On-Line Identification of Hysteretic Systems***

Problems involving the identification of structural systems exhibiting inelastic restoring forces with hereditary characteristics are widely encountered in the aerospace and applied mechanics field. Representative examples involve joint-dominated deployable structures or flexible civil structures undergoing strong earthquake ground motion. Due to the hysteretic nature of the restoring force in such situations, the nonlinear force cannot be expressed in the form of an algebraic function involving the instantaneous values of the state variables of the system. Consequently, much effort has been devoted by numerous investigators to develop models of hysteretic restoring forces and techniques to identify such systems.

This part of the research was concerned with the development of an efficient method for the on-line identification of hysteretic restoring forces. This is crucial for the practical implementation of the control and monitoring of nonlinear systems. In the present approach, an analytical and experimental study was made of adaptive techniques for the on-line identification of hysteretic systems.

Making use of the form of a general purpose hysteretic model (Wen, 1989), and assuming that the disturbance and the inertia properties of the system are known or easily estimated, the time history of the underlying hysteretic forces is obtained. Subsequently, the hysteretic restoring force in the nonlinear system under discussion is then parameterized linearly with respect to the coefficients appearing in the nonlinear differential equation which governs the evolution of the hereditary forces in the model. Using discrete time measurements, the system estimator is then expressed in a form suitable for evaluation by efficient techniques found in the adaptive estimation and adaptive control literature. The adaptive law used guarantees that all the signals will remain bounded and that the estimation error will asymptotically approach zero if the mathematical model is a good representation of the unknown physical system. The problem formulation was generalized so as to handle more complicated situations: (1) cases where the system mass as well as the restoring forces need to be identified, and (2) cases of multidegree-of-freedom systems with certain topologies.

Further details concerning the results of this research are available in the following publications:

1. Housner, G.W., Bergman, L.A., Caughey, T.K., Chassiakos, A.G., Claus, R.O., Masri, S.F., Skelton, R.E., Soong, T.T., Spencer, B.F., and Yao, J.T.P., (1997), "Structural Control: Past, Present and Future," *ASCE Journal of Engineering Mechanics*, (Special Issue), Vol 123, No 9, Sept 1997, pp 897-971. (Awarded the 1999 ASCE Norman Prize "Best Paper Award")
2. Smyth, A W; Masri, S F; Chassiakos, A G; and Caughey, T K, (1998), "On-Line Parametric Identification of MDOF Nonlinear Hysteretic Systems," *ASCE Journal of Engineering Mechanics*, vol 125, no 2, pp 133-142.
3. Smyth, A.W., Masri, S. F., Chassiakos, A.G., Caughey, T.K., (1999), "An Efficient Parametric Identification Method for Detection of Changes in Nonlinear Systems," *Proceedings of the First Romanian American Workshop in Structural Engineering: Information Processing and Damage Detection*, 25-30 June 1999, Iasi, Romania.

### ***Damage Detection on the Basis of Influence Coefficients***

This method uses a time-domain identification procedure to detect structural changes on the basis of noise-polluted measurements. The method of approach requires the use of excitation and acceleration response records to develop an equivalent multi-degree-of-freedom (MDOF) mathematical model, whose order is compatible with the number of sensors used. Application of the identification procedure under discussion yields the optimum value of the elements of equivalent linear system matrices (influence coefficients). By performing the identification task before and after potential structural changes (damage) in the physical system have occurred, quantifiable changes in the identified mathematical model may be detected by analyzing the probability density functions of the identified system matrices.

This approach exploits the physics of the class of problems usually encountered in the structural dynamics field by embedding some information about the physical model structure (the form of the equations of motion) into the identification procedure, thus endeavoring to improve the sensitivity of the system identification results to small changes in the physical system parameters.

The usefulness of the identification procedure under discussion for damage detection was demonstrated by means of tests on a complex mechanical system exhibiting significant nonlinear characteristics. This system was used to conduct experiments to generate data sets that are subsequently analyzed to determine the mean, variance, and probability density functions corresponding to each element of the identified system matrices. To gain some insight into the reliability of the proposed detection scheme, physically different versions of the test article were investigated, in which the location as well as the magnitude of the "damage" was varied.

The method provided clear indication of structural alteration through changes in the identified parameters. Two dimensionless measures (involving dimensionless change in each system element as well as the variance of each identified element) prove to be particularly useful in detecting dominant changes. Because the method under discussion is a parametric identification scheme based on a linearized model, its ability to capture the overall dynamic response of nonlinear systems is of course limited. Further parametric or nonparametric nonlinear identification techniques can be employed to obtain a complete linear/nonlinear dynamic model of the system dynamics, by either treating the residual error as the nonlinear response component or by treating the entire dynamic response generally.

On the basis of this exploratory study, it appears that determining the probability density functions of the identified system matrices may furnish useful indices, that can be conveniently extracted during an experimental test, to quantify changes in the characteristics of physical systems without the need for virtually any information about the topology of the system or the nature of the underlying physical phenomena that are being observed.

Further details concerning the results of this research are available in the following publications:

1. Smyth, A.W., Masri, S.F., Abdel-Ghaffar, A.M. and Nigbor, R.N., (2000), "Development of a Nonlinear Multi-Input/Multi-output Model for the Vincent Thomas Bridge Under Earthquake Excitations," Paper # 2211. Proceedings of the 12th World Conference on Earthquake Engineering (12WCEE), Auckland, New Zealand, 31 January - 4 February 2000.
2. Smyth, A.W. and Masri, S.F., (2000), "Reduced-Order Nonlinear Modeling of the Vincent Thomas Bridge Based on Measurements of its Dynamic Response to Earthquakes," Proc

of ASCE 14th Engineering Mechanics Conference (EM2000), 21-24 May 2000, Austin, TX, p 172.

3. Smyth, A. W., and Masri, S.F., (2000), "An Approach for the Development of Reduced-Order Nonlinear Multi-Input/Multi-Output Models for the Control of Distributed Systems Under Nonuniform Base Excitation," Second European Conference on Structural Control, Paris, July 2000.
4. Smyth, A. W., Masri, S.F., Caughey, T.K., and Hunter, N.F., (2000), "Surveillance of Intrinsic Mechanical Systems on the Basis of Vibration Signature Analysis," ASME Journal of Applied Mechanics, vol 67, No 3, (September), pp 540-551.

### ***Damage Detection Using Neural Networks***

While there are many approaches that have been investigated or are still being developed for signature-based NDE of structures, the class of health monitoring approaches that do not require detailed knowledge of the vulnerable parts of the structure, or of the failure modes of the structure, have a significant advantage in that they have the potential to cope with unforeseen failure patterns. Furthermore, health monitoring techniques that rely on nonparametric system identification approaches, in which a priori information about the nature of the model is not needed, have a significant advantage when dealing with real-world situations where the selection of a suitable class of parametric models to be used for identification purposes is quite often a daunting task. Among the structure-unknown identification approaches that have been receiving growing attention recently are neural networks. A study by Masri et al (1993) has demonstrated that neural networks are a powerful tool for the identification of systems typically encountered in the structural dynamics field.

In conventional identification approaches employed in the applied mechanics community, modal information or information about the model of the structure is needed to accomplish the identification and subsequent "damage" detection. Assumptions regarding the linearity or nonlinearity of the underlying physical process (structural behavior) will have drastic effects on the model selection and the accompanying identification scheme.

On the other hand, not only do neural networks not require information concerning the phenomenological nature of the system being investigated, but they also have fault tolerance, which makes them a robust means for representing model-unknown systems encountered in the real world. Neural networks do not require any prior knowledge of the system to be identified. Whether the physical structure being investigated is linear or not, will not require any changes in a neural network with proper architecture; i.e., neural networks can treat both linear and nonlinear systems with the same formulation. This property makes the scheme particularly attractive in the aerospace engineering field where the nonlinear nature of structures has often to be considered.

One of the accomplishments of this project involves the development of a nonparametric structural damage detection methodology, which is based on nonlinear system identification approaches for the health monitoring of structure-unknown systems. In its general form, the method requires no information about the topology or the nature of the physical system being monitored. The approach relies on the use of vibration measurements from a "healthy" system to train a neural network for identification purposes. Subsequently, the trained network is fed comparable vibration measurements from the same structure under different episodes of response in order to monitor the health of the structure and thereby provide a relatively sensitive indicator of changes (damage) in the underlying structure. For systems with certain topologies, the method can also furnish information about the region within which structural changes have occurred.

The approach was applied to an intricate mechanical system which incorporates significant nonlinear behavior typically encountered in the applied mechanics field. The system was tested in its "virgin" state as well as in "damaged" states corresponding to different degrees of parameter changes. It was shown that the proposed method is a robust procedure and a practical tool for the detection and overall quantification of changes in nonlinear structures whose constitutive properties and topologies are not known. It was also shown that the non-uniqueness of "optimal" neural network models of the dynamics of nonlinear systems proves to be a major impediment in attributing physical modification with a change in neural network parameter.

Further details concerning the results of this research are available in the following publications:

1. Nakamura, M., Masri, S.F., Chassiakos, A.G., and Caughey, T.K., (1998), "Nonparametric Damage Detection in a Multistory Building Through the Use of Neural Networks," Proc of the *Second World Conference on Structural Control*, Kyoto, Japan, 28 June - 1 July 1998.
2. Kosmatopoulos, E.B.; Smyth, A.W.; Masri, S.F.; and Chassiakos, A.G., (2000), "Adaptive Neural Identification of Nonlinear Structural Systems," Proc of ASCE 14th Engineering Mechanics Conference (EM2000), 21-24 May 2000, Austin, TX, p 142.
3. Masri, S.F., Smyth, A.W., Chassiakos, A.G., Caughey, T.K., and Hunter, N.F., (2000), "Application of Neural Networks for Detection of Changes in Nonlinear Systems," ASCE Journal of Engineering Mechanics, July, 2000
4. Kosmatopoulos, E.B.; Smyth, A.W.; Masri, S.F.; and Chassiakos, A.G., (2000), "Robust Neural Estimation of Internal Forces in Nonlinear Structures Under Random Excitation." Presented at the 20th International Congress on Theoretical and Applied Mechanics (ICTAM'2000), Chicago, IL, 27 August - 2 September 2000, ICTAM Ref No 2012.

### ***Training Neural Networks by Adaptive Random Search Techniques***

A relatively simple stochastic optimization procedure based on the Adaptive Random Search (ARS) algorithm was developed to train artificial neural networks of the type encountered in applied mechanics applications. The essential features of the algorithm that influence its search efficiency were evaluated and a procedure was outlined for replacing the back-propagation training approach by the new method. This was done to train networks involving high-dimensional parameter vectors. The method was successfully used in conjunction with a multilayer network involving a parameter vector of very high dimension.

It was shown that the ARS approach shifts the training effort from the user to the computer by exchanging additional computer search effort for easier training tasks on the part of the user. Extensive simulation studies were conducted to provide statistically significant results related to the characteristics of the stochastic training approach. Guidelines were formulated for applying the method to generic neural network training episodes.

Further details concerning the results of this research are available in the following publications:

1. Masri, S F; Smyth, A W; Chassiakos, A G; Nakamura, M; and Caughey, T K, (1998), "Training Neural Networks by Adaptive Random Search Techniques," *ASCE Journal of Engineering Mechanics*, vol 125, no 2, pp 123-132.
2. Bedrossian, H. and Masri, S.F., (2000), "Monte Carlo Simulation Studies of a Class of Stochastic Optimization Techniques," *Proceedings of the International Conference on Monte Carlo Simulation MCS\_ 2000, Monte Carlo/Monaco, 18 - 21 June 2000*

### ***Probabilistic Representation and Transmission of Nonstationary Random Loads***

Since the quantification and transmission of uncertainties in the dynamic environment of structural systems is crucial to the development of a robust structural health monitoring methodology based on system identification approaches, it is essential to develop data-based approaches for the probabilistic representation and transmission of nonstationary processes in multi-degree-of-freedom systems. A relatively simple and straight-forward procedure was developed for representing nonstationary random process data in a compact probabilistic format which can be used as excitation input in MDOF analytical random vibration studies.

The method involves two main stages of compaction. The first stage is based on the spectral decomposition of the covariance matrix by the orthogonal Karhunen-Loeve expansion. The dominant eigenvectors are subsequently least-squares fitted with orthogonal polynomials to yield an analytical approximation. This

compact analytical representation of the random process is then used to derive an exact closed-form solution for the nonstationary response of general linear multi-degree-of-freedom dynamic systems.

The approach was illustrated by the use of an ensemble of free-field acceleration records from the 1994 Northridge earthquake to analytically determine the covariance kernels of the response of a two-degree-of-freedom system resembling a commonly encountered problem in the structural control field. Spectral plots of the extreme values of the rms response of representative MDOF systems under the action of the subject earthquake were also presented. It was shown that the proposed random data-processing method is not only a useful data-archiving and random process (such as earthquakes or turbulence) feature-extraction tool, but also provides a probabilistic measure of the average statistical characteristics of the process corresponding to a spatially distributed region. Further details concerning the results of this research are available in the following publications:

1. Masri, S.F., Smyth, A.W., and Traina, M.I. (1998), "Probabilistic Representation and Transmission of Nonstationary Processes in Multi-Degree-of-Freedom Systems," ASME Journal of Applied Mechanics, vol 65, June, pp 398-409.
2. Smyth, A.W. and Masri, S.F., (2000), "Representation and Transmission of Stochastic Loads for Nonlinear Systems Response and Control," Presented at the 20th International Congress on Theoretical and Applied Mechanics (ICTAM'2000), Chicago, IL, 27 August - 2 September 2000), ICTAM 2000 Abstract 2296.

#### **Acknowledgment/Disclaimer**

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#### **Publications:**

The following papers, which are supported in part by this research effort, have been published:

1. Housner, G.W., Bergman, L.A., Caughey, T.K., Chassiakos, A.G., Claus, R.O., Masri, S.F., Skelton, R.E., Soong, T.T., Spencer, B.F., and Yao, J.T.P., (1997), "Structural Control: Past, Present and Future," ASCE Journal of Engineering Mechanics, (Special Issue), Vol 123, No 9, Sept 1997, pp 897-971. (Awarded the 1999 ASCE Norman Prize "Best Paper Award")
2. Masri, S.F., Smyth, A.W., and Traina, M.I. (1998), "Probabilistic Representation and Transmission of Nonstationary Processes in Multi-Degree-of-Freedom Systems," ASME Journal of Applied Mechanics, vol 65, June, pp 398-409.

3. Nakamura, M., Masri, S.F., Chassiakos, A.G., and Caughey, T.K., (1998), "Nonparametric Damage Detection in a Multistory Building Through the Use of Neural Networks," Proc of the *Second World Conference on Structural Control*, Kyoto, Japan, 28 June - 1 July 1998.
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6. Smyth, A.W., Masri, S. F., Chassiakos, A.G., Caughey, T.K., (1999), "An Efficient Parametric Identification Method for Detection of Changes in Nonlinear Systems, " Proceedings of the First Romanian American Workshop in Structural Engineering: Information Processing and Damage Detection, 25-30 June 1999, Iasi, Romania.
7. Smyth, A.W., Masri, S.F., Abdel-Ghaffar, A.M. and Nigbor, R.N., (2000), "Development of a Nonlinear Multi-Input/Multi-output Model for the Vincent Thomas Bridge Under Earthquake Excitations," Paper # 2211, Proceedings of the 12th World Conference on Earthquake Engineering (12WCEE), Auckland, New Zealand, 31 January - 4 February 2000.
8. Kosmatopoulos, E.B.; Smyth, A.W.; Masri, S.F.; and Chassiakos, A.G., (2000), "Adaptive Neural Identification of Nonlinear Structural Systems," Proc of ASCE 14th Engineering Mechanics Conference (EM2000), 21-24 May 2000, Austin, TX, p 142.
9. Smyth, A.W. and Masri, S.F., (2000), "Reduced-Order Nonlinear Modeling of the Vincent Thomas Bridge Based on Measurements of its Dynamic Response to Earthquakes," Proc of ASCE 14th Engineering Mechanics Conference (EM2000), 21-24 May 2000, Austin, TX, p 172.
10. Bedrossian, H. and Masri, S.F., (2000), "Monte Carlo Simulation Studies of a Class of Stochastic Optimization Techniques," Proceedings of the International Conference on Monte Carlo Simulation MCS\_ 2000, Monte Carlo/Monaco, 18 - 21 June 2000
11. Smyth, A. W., and Masri, S.F., (2000), "An Approach for the Development of Reduced-Order Nonlinear Multi-Input/Multi-Output Models for the Control of Distributed Systems Under Nonuniform Base Excitation," Second European Conference on Structural Control, Paris, July 2000.
12. Masri, S.F., Smyth, A.W., Chassiakos, A.G., Caughey, T.K., and Hunter, N.F., (2000), "Application of Neural Networks for Detection of Changes in Nonlinear Systems," *ASCE Journal of Engineering Mechanics*, July, 2000
13. Kosmatopoulos, E.B.; Smyth, A.W.; Masri, S.F.; and Chassiakos, A.G., (2000), "Robust Neural Estimation of Internal Forces in Nonlinear Structures Under Random Excitation." Presented at the 20th International Congress on Theoretical and Applied Mechanics (ICTAM'2000), Chicago, IL, 27 August - 2 September 2000, ICTAM Ref No 2012.
14. Smyth, A.W. and Masri, S.F., (2000), "Representation and Transmission of Stochastic Loads for Nonlinear Systems Response and Control," Presented at the 20th International Congress on Theoretical and Applied Mechanics (ICTAM'2000), Chicago, IL, 27 August - 2 September 2000), ICTAM 2000 Abstract 2296.

15. Smyth, A. W., Masri, S.F., Caughey, T.K., and Hunter, N.F., (2000), "Surveillance of Mechanical Systems on the Basis of Vibration Signature Analysis," *ASME Journal of Applied Mechanics*, vol 67, No 3, (September), pp 540-551.

**Interactions/Transitions:**

Results of the research are directly applicable to modeling and health monitoring of civil infrastructure systems as well as general nonlinear systems encountered in the aerospace field.

Data sets obtained from tests conducted on an aerospace system in collaboration with Dr. Norman Hunter of the Analysis and Testing Group WX-11 at the Los Alamos National Laboratory, were used to calibrate the efficiency of the analytical results of this project. Two technical papers have been prepared on the basis of the analysis results, and have been published.

**New Discoveries, Inventions, or Patent Disclosures:** None

**Honors/Awards:** None

**References**

1. Masri, S.F., Chassiakos, A.G. and Caughey, T.K., (1993), "Identification of Nonlinear Dynamic Systems Using Neural Networks," *Journal of Applied Mechanics*, Trans. ASME, Vol 60, March, pp 123-133.
2. Wen, Y.K., (1989), "Methods of Random Vibration for Inelastic Structures," *Appl Mech Rev* vol 43 No (2), pp.39-52.