

**NAVAL POSTGRADUATE SCHOOL**  
**Monterey, California**



**THESIS**

**APPLYING TWO-SIDED MATCHING PROCESSES TO  
THE UNITED STATES NAVY  
ENLISTED ASSIGNMENT PROCESS**

by

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**APPLYING TWO-SIDED MATCHING PROCESSES TO THE  
UNITED STATES NAVY ENLISTED ASSIGNMENT PROCESS**

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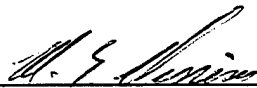


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## **ABSTRACT**

The existing Navy detailing process is characterized by bilateral negotiations between detailers and sailors. In this process, detailers match sailors with billets while attempting to satisfy, to the maximum extent possible, the needs and preferences of each group. The current process requires the effort of approximately 294 detailers and results in assignments that do not always satisfy the competing preferences of sailors and billets.

This thesis explores various two-sided matching processes as currently used in some markets, as possible alternative means of assigning sailors. The similarities and differences between existing two-sided matching processes and the Navy's assignment process are examined. Various modifications to the assignment process and the matching algorithm are proposed to enable the matching algorithm to be suitably applied to the Navy's situation.

It is found that the application of a two-sided matching process would significantly reduce the number of detailers required, while simultaneously improving the overall quality of assignments. Furthermore, to enhance the process, a means of including incentives to encourage sailors to accept difficult to fill positions is provided.

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## **I. INTRODUCTION**

### **A. BACKGROUND**

With an active-duty personnel force of 371,800 (FY 2001 DoN Budget), the US Navy has considerable manpower assets to manage. A significant part of managing the Navy's manpower assets involves assignment processes, which seek to place the right person in the right job at the right time. An assignment process that achieves this optimal mix will maximize fleet readiness and ensure sailors' careers and skills are developed and utilized to potential. An assignment process that fails to achieve the optimal mix will lead to a force that suffers decreased levels of readiness and has dissatisfied sailors. In the current economic climate dissatisfied sailors may be more inclined to show their dissatisfaction by seeking more desirable employment elsewhere.

The assignment process involves three key groups of players. These are the sailors to be assigned, the commands seeking sailors and the detailers who advise both sailors and commands. Detailers are also the primary decision-makers in the assignment process. To manage the careers and assignments of 314,450 enlisted personnel (FY 2001 DoN budget) requires considerable effort from approximately 294 detailers (Short, 2000, p38). At times, detailers are required to force sailors into assignments that the sailors have neither asked for nor desire. Such assignments can lead to considerable dissatisfaction for the sailor with resulting morale problems. Furthermore, the prospect of future unwanted assignments is not conducive to encouraging sailors to re-enlist.

### **B. MATCHING PROCESSES IN USE**

The Navy is not the only organization to face problems matching elements of its workforce with positions. Roth (1990) explores in detail the situation facing the market

for medical interns prior to the 1950's. At that time, medical students would consider the list of hospitals offering internships and based on personal preferences, would submit applications for internship to their most highly favored hospitals. Hospitals would then consider the medical students who applied and offer internship positions to those students the hospital preferred most highly. Problems arose when students were informed that they were offered an internship at a hospital, but were on a waiting list for a hospital that they ranked higher. Such a student would then be inclined to follow one of two patterns of behavior. Firstly, rather than immediately accepting the less preferred offer, the student might delay the decision as long as possible in the hope that the more preferred position would become available. Alternatively, the student might accept the less preferred offer but renege on the commitment if a better offer arose later. Both of these patterns of behavior caused problems for the many hospitals attempting to confirm internships throughout the country. As a result, it was determined that a centralized matching process should be tried.

Under the proposed central matching process for the intern market, both medical students and hospitals submit ranked preferences to a central agency that matches the students with hospitals and informs all parties of the results. This central agency became known as the National Resident Matching Program (NRMP), which has been matching the preferences of medical students with intern programs since 1952. The NRMP uses an algorithm known as the National Intern Matching Program (NIMP) to arrange the optimal matching. In the year 2000, the NRMP received the preference lists for 3,769 programs offering 22,722 positions, and sought to match these with 33,528 medical applicants. The results of this match were that 72.3% of programs were filled and 74.7% of active

applicants were matched. Inactive applicants are those who withdrew from the process or were not ranked. (National Resident Matching Program, About the NRMP)

Organizations other than NRMP are now also offering matching services. For example, San Francisco Matching Programs offers medical residency and fellowship positions (San Francisco Matching Programs), while National Matching Services Inc administers articling positions with law firms in addition to medical and dental residencies (National Matching Services).

Each of the organizations detailed above uses a process known as two-sided-matching to perform the assignment roles. It is known as two-sided-matching because there are two groups of agents, medical students and intern positions for example, within the market. It is believed that the Navy assignment situation, consisting of sailors to be matched with commands, resembles the two-sided-matching markets described above.

### **C. OBJECTIVES**

The primary objective of this thesis is to examine the issues relating to the implementation of a two-sided matching algorithm that may be used to assign sailors to commands. In addition, it explores using incentives to entice sailors to fill otherwise less desirable billets. This matching process offers potential to reduce the number of detailers, simplify the detailing process and increase the satisfaction of sailors and commands.

### **D. RESEARCH QUESTIONS**

The following research questions are addressed in this thesis:

- What two-sided-matching process most closely resembles the Navy assignment situation?

- What issues are important in the Navy assignment process with respect to applying a two-sided-matching algorithm?
- How can incentives be incorporated into the matching process?

## **E. ORGANIZATION OF THESIS**

In Chapter II, a literature review of two-sided-matching markets examines the different types of markets, algorithms used in such markets and issues that require special consideration when creating a match. The Navy assignment process is explored in Chapter III with an emphasis on issues that relate to two-sided-matching. Chapter III concludes with a discussion of factors relating to the Navy assignment situation that would preclude a direct application of two-sided matching. Chapter IV proposes solutions that will enable a two-sided matching process to be used for assigning the Navy's enlisted sailors. A discussion of implementing incentives is also provided in Chapter IV. Chapter V follows with conclusions and recommendations.

## II. TWO-SIDED-MATCHING MODELS

### A. BACKGROUND

A two-sided-matching market is represented by two distinct categories of agents, with agents from each category seeking a match with agents from the other category. As an analogy, these agents may be thought of as workers and firms, where the workers have preferences over firms where they wish to be employed, and firms have preferences over workers they wish to employ.

A two-sided-matching model is a process that seeks to match these workers and firms so that the match is acceptable to each agent and the agents are collectively satisfied with the outcome. If such a situation occurs, the match is considered stable. The issue of match stability is a key concept in two-sided matching models, and a more precise definition is presented later.

In order to determine an acceptable match, it is necessary to know the preferences of each individual agent over agents from the other side of the market. That is, each worker must indicate his or her preferences over the firms for which he / she would like to work, and similarly, each firm must indicate its preferences over workers they would like to employ. It is not necessary to provide a ranked preference list that includes every acceptable agent from the opposing side of the market. However, it will be shown later that longer preferences lists can only improve the chances of being matched.

Two-sided-matching models are described according to the market characteristics to which the model applies. The two basic matching models of interest are one-to-one and many-to-one matching models, and these are discussed in the following sections.

## **B. ONE – TO – ONE MATCHING MODELS**

### **1. Situation**

In certain markets, each individual agent is matched with at most one agent from the other group of agents. In such situations, the analogy of workers and firms does not fit well since it is assumed that firms generally employ more than a single worker. A more appropriate analogy describes these markets as marriage markets where a group of single males and females are seeking partners in marriage. The males represent agents from one side of the market, while females are the agents from the opposite side of the market. Being a one-to-one matching market, each male may only be matched with at most one female, and each female may only be matched with at most one male. Each agent, male or female, has the option of remaining single rather than being forced into marriage with a partner they do not desire.

Prior to the match, each agent, both male and female, submits a list of preferences over agents from the opposite side of the market. For each individual agent, the preference list will be a rank ordering of those agents from the other side of the market that the individual regards as being acceptable. Non-acceptable agents are those agents from the other side of the market with which an individual does not wish to be matched under any circumstances. In the marriage problem for example, a woman is unacceptable to a man if the man would prefer to remain single rather than be married to the woman. These definitions of acceptable and non-acceptable partners are important for defining stability later.

In determining the match, it is necessary to choose one set of agents to do the proposing. As indicated by Gale and Shapley (1962, p13), the solutions may be different

depending upon which group of agents does the proposing. Furthermore, the result will be optimal for the men when the men propose, and the result will be optimal for the women when the women propose. Although the women may not find the matching optimal if the men propose, the women will agree that the matching is acceptable and hence stable, and vice versa for when women propose. It should be noted that it is possible to obtain stable matches other than those obtained when either males or females propose.

Irving, et al. (1987) propose an alternative algorithm that maximizes the average satisfaction of all people involved in the process. In this situation, satisfaction is measured by the position of each person's assigned partner in his or her preference list. Furthermore, the idea of a weighted preference list is introduced with the aim being to maximize satisfaction while accounting for the weights people apply to their preferences.

It is now important to discuss the issue of stability in more detail.

## **2. Stability**

A matching is "stable if it is not blocked by any individual or any pair of agents" (Roth, 1990, p21). Since an individual has the option of remaining single, an individual may block a match if the proposed match would force the individual into an arrangement that he / she does not find acceptable. That is, if a match proposes to marry Male A to Female Y, but Male A (Female Y) would rather remain single than marry Female Y (Male A), then Male A (Female Y) individually blocks the match, and the match is regarded as unstable.

To understand a situation where a pair of agents may block a matching, consider a proposed match where Male A is paired with Female X and Male B is paired with Female Y. That is:

Match :	Male A	Male B
	Female X	Female Y

If Male A prefers Female Y to his proposed mate Female X, and similarly if Female Y prefers Male A to her proposed mate Male B, then both Male A and Female Y will block the proposed match. The match is considered to be unstable since both Male A and Female Y have an incentive to obstruct the matching process and seek a match with each other.

### **3. The Matching Procedure**

Gale and Shapley (1962, p12) provide a procedure for obtaining matches in one-to-one matching markets. The algorithm assumes that the males propose and is conducted as follows:

To start, let each boy propose to his favorite girl. Each girl who receives more than one proposal rejects all but her favorite from among those who have proposed to her. However, she does not accept him yet, but keeps him on a string to allow for the possibility that someone better may come along later.

We are now ready for the second stage. Those boys who were rejected now propose to their second choices. Each girl receiving proposals chooses her favorite from the group consisting of the new proposers and the boy on her string, if any. She rejects all the rest and again keeps the favorite in suspense.

We proceed in the same manner. Those who are rejected at the second stage propose to their next choices, and the girls again reject all but the best proposal they have had so far.

The authors of this deferred acceptance algorithm and Roth (1990, p27) prove that this procedure will ensure a stable match is obtained. This algorithm is illustrated below

in two examples. The first example demonstrates the steps involved in the deferred acceptance algorithm, while the second example shows that different results may be obtained when men or women propose.

#### **4. Example - The Matching Procedure Applied**

Consider a group of three men (A, B and C) and three women (X, Y and Z), all of whom are single. For Male A, his preferences over the females are such that his preferred partner for marriage is Female Y, however he would accept marriage with Female X even though she is not as highly favored as Female Y. Furthermore, Male A does not wish to be married to Female Z and would prefer to remain single rather than be married to Female Z. The preferences for Male A are therefore listed as follows:

$$P(\text{Male A}) = [\text{Female Y, Female X}]$$

Using the same format, the preferences for the remaining members of this market are indicated as:

$$P(\text{Male B}) = [\text{Female X, Female Y, Female Z}]$$

$$P(\text{Male C}) = [\text{Female Y, Female Z, Female X}]$$

$$P(\text{Female X}) = [\text{Male A, Male B, Male C}]$$

$$P(\text{Female Y}) = [\text{Male C, Male A, Male B}]$$

$$P(\text{Female Z}) = [\text{Male B, Male C}]$$

Using the Deferred Acceptance algorithm with the males proposing, the following steps are followed:

Round 1. The males each propose to their first preferences. Female X tentatively accepts Male B, and Female Y tentatively accepts her preferred proposer Male C.

Round 2. Having been rejected by Female Y in Round 1, Male A proposes to his second choice, which is Female X. Female X prefers Male A to her tentative assignment of Male B, and therefore rejects Male B and now tentatively accepts Male A.

Round 3. Male B now proposes to his second choice which is Female Y, however Female Y prefers her tentative assignment of Male C and rejects Male B. Male B now proposes to his third choice which is Female Z. At this stage Female Z tentatively accepts Male B.

Now that all males have made offers and are matched, the process is complete and all tentative acceptances become final. If the females were to propose in this example the matches would be the same as those obtained above. However symmetry does not always apply as seen in the following example.

### 5. Example - Different Solutions When Men or Women Propose

If the preferences for a group of men and women are:

$P(\text{Male A}) = (\text{Female X}, \text{Female Y}, \text{Female Z})$

$P(\text{Male B}) = (\text{Female Y}, \text{Female X}, \text{Female Z})$

$P(\text{Male C}) = (\text{Female X}, \text{Female Z}, \text{Female Y})$

$P(\text{Female X}) = [\text{Male B}, \text{Male A}, \text{Male C}]$

$P(\text{Female Y}) = [\text{Male C}, \text{Male A}, \text{Male B}]$

$P(\text{Female Z}) = [\text{Male A}, \text{Male B}, \text{Male C}]$

The matching obtained when the men propose is as follows:

Matching =	Male A	Male B	Male C
	Female X	Female Y	Female Z

However the matching obtained when the women propose is:

Matching =	Male A	Male B	Male C
	Female Z	Female X	Female Y

This example shows the importance of determining who will be proposing. Although the men and women agree that the two different solutions to Example 2 are acceptable and stable, the men and women differ in opinion over which solution is optimal. In situations such as this, the previously mentioned extension of the algorithm by Irving et al. can maximize the total satisfaction of all people involved in the matching process rather than favoring one group of agents at the expense of the other.

## **C. MANY – TO – ONE MATCHING**

### **1. Situation**

In the case of many-to-one matching markets, many agents from one side of the market seek to be placed with individual agents from the other side. A suitable analogy drawn in Chapter I was that of workers and firms, where a single firm may employ many workers, but each worker may work for only one firm. Just as the participants in one-to-one matching markets have the option of remaining single, individually unacceptable employment contracts will not occur in many-to-one matching markets, since workers can opt to remain unemployed and firms can opt to have a position remain vacant.

Each agent, both workers and firms, must create a list that specifies their preferences over agents from the opposite side of the market. The definition of acceptable and non-acceptable agents is the same as that given for one-to-one matching markets. That is, a worker regards a firm as unacceptable if the worker would prefer to remain unemployed rather than be employed by the firm, and the worker will not list that firm on

his or her list of preferences. Similarly, a firm that finds a worker unacceptable would rather have a position vacant than employ the unacceptable worker.

Rather than have a firm provide a ranked list of preferences over all applicants, Brissenden (1974) suggests that the firms may simplify the procedure by using a weak ordering rather than the strict ordering. In the example of university admissions, the universities may agree that high school grades will be the measure by which student applicants are ranked. However, Brissenden acknowledges that such a weak ordering destroys the notion of stable or unstable as previously defined. Although Brissenden does explain how to deal with weakly ordered preferences, the more traditional situation of strict preferences will be examined here.

In practice, many-to-one matching models are most frequently applied to the markets for medical residents, college admissions and college sororities. It should be noted that these are regarded as entry-level markets, where agents from one side of the market, such as medical students, are entering the market for the first time and do so as a collective group. Blum et al. (1997) examines the situation of senior-level positions where positions become vacant and are filled at irregular intervals. The creation of a vacancy due to a retirement or other reason can create a vacancy chain, which is a series of vacancies caused when a person moves to fill a vacant position, thereby leaving behind a new vacant position. Such a vacancy chain will continue until a position remains unfilled, or someone external to the system enters and fills a vacancy without leaving a vacancy in the system. Blum et al. also look at the destabilizing effect that new entrants or the creation of vacancies in a senior-level market will cause, since the workers and firms who were previously in a stable market, now have new options over the new set of

agents in the market. In a similar vein, Crawford (1991, p391) remarks that the introduction of a new worker can be expected to “weaken the competitive position of other workers (even those for whom he is only an imperfect substitute), and to strengthen the competitive position of firms.”

## **2. Stability Issues**

The basic stability issues that were outlined in the marriage problem remain applicable in many-to-one matching models. For the case of stability in many-to-one matching, Roth (1990, p129) defines that a matching is “stable if it is not blocked by any individual agent or any college – student pair.” The ability of individuals or pairs of agents to block a match were discussed for the marriage problem so will not be elaborated upon here. However a new concept, that of group stability is now introduced.

Group stability refers to the ability of a coalition of students or universities to block a matching. Roth (1990, p130) defines that a matching “ $\mu$  is blocked by some coalition A of colleges and students if, by matching among themselves, the students and colleges in A could all get an assignment preferable to  $\mu$ .” Provided any individual, any pair of agents or any coalition does not block a matching, then the matching is regarded as stable.

Experience shows that achieving stability is important to ensuring the success of the matching process. Roth (1991) investigated seven centralized matching systems for physicians within regional markets in the United Kingdom. He found that two of the systems produced stable matches and continued to be used, while the other five had the potential to produce unstable matches. Of the five matching systems that can result in unstable matches, three were abandoned while the other two systems continued to

operate. The reason for the failure of the potentially unstable procedures was the increasing number of physicians and hospitals choosing to negotiate matches outside of the central matching system. This led Board (1994, p562) to suggest that “whether an unstable matching procedure succeeds or fails appears to be linked to the number of participants likely to be placed in unstable pairs.” Furthermore, in order for instability to be a problem, agents on each side of the market need to have knowledge of the preferences of agents from the opposing side of the market. That is, a physician and hospital can only be a blocking pair and cause instability if each prefers the other and is aware that the other prefers them when compared to the assigned match. If information regarding preferences is difficult to obtain, then although the matching may be theoretically unstable, the resultant match may be satisfactory in reality. It is likely that the success or failure of an unstable system will be related to both the number of people placed in unstable pairs, and the degree of knowledge of opposing agents’ preferences.

### **3. The Matching Procedure**

Prior to commencing the matching procedure, it is necessary to edit the lists submitted by the agents. Describing the matching procedure as a market consisting of medical students to be matched with hospitals, Roth describes the editing procedure as follows. The lists are:

edited by removing from each hospital program’s rank-order list any student who has marked that program as unacceptable, and by removing from each student’s list any hospital which has indicated he is unacceptable...The edited lists are thus rank orderings of acceptable alternatives. (Roth, 1990, p135)

In describing the process relating to college admissions, Gale and Shapley provide a description of the matching process leading to the applicant-optimal assignment:

First, all students apply to the college of their first choice. A college with a quota of  $q$  then places on its waiting list the  $q$  applicants who rank highest, or all applicants if there are fewer than  $q$ , and rejects the rest. Rejected applicants then apply to their second choice and again each college selects the top  $q$  from among the new applicants and those on its waiting list, puts these on its new waiting list, and rejects the rest. The procedure terminates when every applicant is either on a waiting list or has been rejected by every college to which he is willing and permitted to apply. At this point each college admits everyone on its waiting list and the stable assignment has been achieved. (Gale and Shapley, 1962, pp. 13-14)

With respect to the algorithm described above, Gale and Shapley show that “Every applicant is at least as well off under the assignment given by the deferred acceptance procedure as he would be under any other stable assignment.” (Gale and Shapley, 1962, p14). Therefore this procedure may be described as the applicant-optimal algorithm. Just as it is possible to have both a male and female-optimal algorithm for one-to-one matching, so too is it possible in many-to-one matching to have both applicant and firm-optimal assignments. However, unlike in the marriage model where the procedure stays the same but the agents who propose changes, in the many-to-one model a different procedure is required to obtain the firm-optimal assignments. Roth (1990) describes the procedure for obtaining the firm-optimal match, known as the NIMP (National Intern Matching Program) algorithm, which after editing as described previously, proceeds as follows:

These lists are entered into what may be thought of as a list-processing algorithm consisting of a matching phase and a tentative-assignment-and-update phase. The first step of the matching phase (the 1:1 step) checks to see if there are any students and hospital programs which are top-ranked in one another's ranking. (If a hospital  $H_i$  has a quota of  $q_i$  then the  $q_i$  highest students in its ranking are top-ranked). If no such matches are found, the matching phase proceeds to the 2:1 step, at which the second ranked hospital program on each student's ranking is compared with the top-ranked students on that hospital's ranking. At any step when no matches are found, the algorithm proceeds to the next step, so the generic  $k:1$  step of the matching phase seeks to find student-hospital pairs such that the

student is top-ranked on the hospital's ranking and the hospital is  $k$ th ranked by the student. At any step where such matches are found, the algorithm proceeds to the tentative-assignment-and-update phase.

When the algorithm enters the tentative-assignment-and-update phase from the  $k:1$  step of the matching phase, the  $k:1$  matches are tentatively made; i.e., each student who is a top ranked choice of his  $k$ th hospital is tentatively assigned to that hospital. The rankings of the students and hospitals are then updated in the following way. Any hospital which a student  $s_j$  ranks lower than his tentative assignment is deleted from his ranking (so the updated ranking of a student  $s_j$  tentatively assigned to his  $k$ th choice now lists only his first  $k$  choices) and student  $s_j$  is deleted from the ranking of any hospital which was deleted from  $s_j$ 's ranking (so the updated rankings of each hospital now include only those applicants who haven't yet been tentatively assigned to a hospital they prefer). When the rankings have been updated in this way, the algorithm returns to the start of the matching phase, which examines the updated rankings for new matches. Any new tentative matches found in the matching phase replace prior tentative matches involving the same student. The algorithm terminates when no new tentative matches are found, at which point tentative matches become final. (Roth, 1990, p135)

An example demonstrating the firm-optimal procedure is now described.

#### 4. Example – The Firm-Optimal Matching Algorithm Applied

Let there be three firms X, Y and Z who have vacancies and preferences over workers A, B, C and D as follows:

	<u>Firm X</u>	<u>Firm Y</u>	<u>Firm Z</u>
Vacancies	2	1	1
Workers	C	B	B
	B	C	C
	A	D	A
	D	A	D

Then let the preferences of the workers over the firms be as follows:

	<u>Worker A</u>	<u>Worker B</u>	<u>Worker C</u>	<u>Worker D</u>
Firms	Y	X	Y	Y
	X	Y	X	Z
		Z		X

An initial editing of the lists is required because both Workers A and C regard Firm Z as unacceptable. Therefore, Workers A and C need to be removed from Firm Z's preference list. This yields the firms' revised preference lists, with each firms' top preferences, i.e. those corresponding to its quota limit, shown in bold:

	<u>Firm X</u>	<u>Firm Y</u>	<u>Firm Z</u>
Workers	<b>C</b>	<b>B</b>	<b>B</b>
	<b>B</b>	C	D
	A	D	
	D	A	

The firm-optimal matching procedure proceeds as follows:

Round 1. Each worker proposes to the first firm listed on their respective lists, with the firms only accepting the workers proposal if the worker is amongst the firm's top preferences. This results in a single tentative match between Firm X and Worker B. Now the algorithm enters the tentative-assignment-and-update phase, where Worker B deletes his / her lower preferences Y and Z, and Firms Y and Z delete Worker B from their preferences. This provides the following new preference lists:

	<u>Firm X</u>	<u>Firm Y</u>	<u>Firm Z</u>
Workers	<b>C</b>	<b>C</b>	<b>D</b>
	<b>B</b>	D	
	A	A	
	D		

and,

	<u>Worker A</u>	<u>Worker B</u>	<u>Worker C</u>	<u>Worker D</u>
Firms	Y	X	Y	Y
	X		X	Z
				X

With {Firm X and Worker B} tentatively assigned.

Round 2. Returning again to the start of the matching phase, each worker proposes to the first firm listed on their respective lists. This time Firm Y and Worker C are added to the list of tentative matches. After updating the lists now look as follows:

	<u>Firm X</u>	<u>Firm Y</u>	<u>Firm Z</u>
Workers	<b>B</b>	<b>C</b>	<b>D</b>
	A	D	
	D	A	

and,

	<u>Worker A</u>	<u>Worker B</u>	<u>Worker C</u>	<u>Worker D</u>
Firms	Y	X	Y	Y
	X			Z
				X

With [Firm X and Worker B] and [Firm Y and Worker C] tentatively assigned.

Round 3. Once more, each worker proposes to the first firm on their respective lists. However no new matches are created in this manner, and the process now proceeds to look at the second preferences for the workers who are as yet unmatched. Doing so leads to the following final assignments:

<u>Firm X</u>	<u>Firm Y</u>	<u>Firm Z</u>
Worker B	Worker C	Worker D
Worker A		

## 5. Incentive and Behavior Issues

A great deal of consideration has been given to the incentives that agents on each side of the market have to falsify or lie about their true set of preferences. Dubbins and Freedman (1981) show that a student cannot do better by falsifying their true preferences when using the Gale – Shapley algorithm that produces the applicant-optimal assignment. They show furthermore that if several students collude in the same matching process, with each using a false rank ordering, they cannot all get better universities relative to

each student's true rank ordering. It should be stressed that these results are only true for the case when the Gale-Shapley algorithm is used to produce the applicant-optimal match. However, using the NIMP algorithm that produces the college or hospital-optimal match, "in general there will be some students who can get into a preferred hospital by suitably falsifying their preferences." (Gale and Sotomayor, 1985, p262). However, manipulating the system by falsifying preferences requires some knowledge of the other agents' preferences within the system.

## **6. Optimality and Stability**

Roth (1990) makes an important distinction between the stability and optimality of the matches that are obtained by using the hospital or student-optimal matching procedure. The student-optimal procedure produces a stable matching that is weakly Pareto optimal and for which every student likes the match as well as any other stable matching. However, for the hospital-optimal procedure, while every hospital likes the match as well as any other stable match, the hospital-optimal matching need not be even weakly Pareto optimal. That is, when examining the match obtained using the hospital-optimal matching procedure, it may be possible to obtain other matches that make some hospitals better off without making any hospitals worse off. However these alternative matches are not stable. To highlight the difference, the example used by Roth consists of three hospitals  $H_1$ ,  $H_2$  and  $H_3$ , and four students  $s_1$ ,  $s_2$ ,  $s_3$  and  $s_4$ . Hospital  $H_1$  has two positions available while the other two hospitals have one each. The preferences are shown as follows:

$$P(s_1) = H_3, H_1, H_2 \quad P(H_1) = s_1, s_2, s_3, s_4$$

$$P(s_2) = H_2, H_1, H_3 \quad P(H_2) = s_1, s_2, s_3, s_4$$

$$P(s_3) = H_1, H_3, H_2 \quad P(H_3) = s_3, s_1, s_2, s_4$$

$$P(s_4) = H_1, H_2, H_3$$

For the preferences shown above only one stable matching exists, and this can be arrived at using either the student or hospital-optimal procedure. The matching is as follows:

$$\begin{array}{l} \text{Stable Match:} \\ \begin{array}{ccc} H_1 & H_2 & H_3 \\ s_3 \ s_4 & s_2 & s_1 \end{array} \end{array}$$

For this stable matching, it is not possible to make any student better off without making some other student worse off, so the solution is Pareto optimal from the students' perspective (in fact, each student receives his or her first preference). However, each hospital agrees that the following match is preferred to the above listed stable match:

$$\begin{array}{l} \text{Alternative Match:} \\ \begin{array}{ccc} H_1 & H_2 & H_3 \\ s_2 \ s_4 & s_1 & s_3 \end{array} \end{array}$$

Therefore the stable match given first is not Pareto optimal from the hospitals' perspective. However, this alternative match is not stable due to the existence of blocking pairs  $\{s_3, H_1\}$  and  $\{s_1, H_1\}$ . So although hospitals may collectively agree that a better matching exists than the matching obtained by the hospital-optimal procedure, the best stable matching for the hospitals is the one obtained by the hospital-optimal procedure.

## 7. The Impact of Married Couples on the NIMP Procedure

By the mid-1970's, the number of married couples participating in the NIMP matching procedure began increasing, posing a new problem for the administrators of the match. The initial solution to accommodate married couples in the matching procedure

required one member of the couple to be designated as the "leading member," but with each member continuing to submit a preference list. The leading member of the couple was then matched as part of the normal procedure, with the preference list for the partner then edited to remove geographically distant positions. Then the partner was matched, if possible, to a position in the same vicinity as the leading member. However, this procedure can be unstable because it does not take into account the couple's preferences over pairs of positions. Roth shows that for the "hospital-intern problem with couples, the set of stable matchings may be empty." (Roth, 1990, p141). Due to instability problems, the participation rate of married couples in the NIMP procedure is much lower than for single medical school graduates. At this stage, the issue of processing married couples appears unresolved; Roth completes the discussion by stating that "for large markets containing married couples the problem of even determining if a stable matching exists may be computationally infeasible" (Roth, 1990, p143).

### **8. The Impact of the Length of Preference Lists**

Since 1996, the National Resident Matching Program has been collecting data to determine if a relationship exists between the length of an agent's preference list and the likelihood of being matched. The data shows that matched applicants and filled programs consistently have longer preference lists than unmatched applicants and unfilled programs. Averaging the data from 1996 to 2000, the average length of the rank order list for filled programs was 52.91 as compared to an average length for unfilled programs of 21.40. Over the same period, the average length of the rank order list for matched applicants was 7.07 as compared to an average length for unmatched applicants of 5.07 (National Resident Matching Program, Guide to the Main Match). These findings are

logical since a longer preference list does not reduce an agent's chance of being matched with a higher preference, but rather the extended preference list provides further matching options for the agent, in the event that the higher options are infeasible.

#### **D. SUMMARY OF FINDINGS**

Two-sided-matching procedures exist to obtain stable matches between two groups of agents within a market. Depending on the approach taken, these procedures generally result in a stable match, which one group of agents regards as the best possible outcome while the other group regards it as the worst acceptable outcome. Regardless of the approach, however, participants in a two-sided matching process can only enhance their opportunity of being matched by increasing the length of their preference list. Irving et al. (1987) proposed the use of weighted preference lists in a method that maximizes the average satisfaction of all agents involved.

Although matching procedures that lead to stable matches are likely to have the greatest opportunity for success, Roth (1991) showed that there are situations in which markets using unstable matches continue to operate. In the case of senior level markets, Blum et al. (1997) show that it may be difficult to avoid instability due to the creation of vacancy chains.

Dubbins and Freedman (1981), and Gale and Sotomayor (1985) show that in certain situations, agents in a market may obtain better partners if they lie about their true preferences. However, to do so requires certain knowledge about the behavior of other agents in the market. Without such information, agents may be expected to act truthfully.

### **III. THE NAVY'S ASSIGNMENT PROCESS**

#### **A. AN OVERVIEW OF MANPOWER AND PERSONNEL PROCESSES**

##### **1. Manpower, Personnel and Training Processes**

This thesis develops a new process for assigning sailors that will reduce the number of detailers, simplify the detailing process and increase the satisfaction of sailors and commands. In order to propose a new process, it is essential to have an adequate understanding of the current processes.

The Manpower, Personnel and Training Process involves four sub-processes, as shown in Figure 3.1. The first of these sub-processes is known as the Manpower Requirements process, and it is here that the basic force size and shape is determined based upon the national military strategy, which considers threats to the nation and the desired military capabilities. The second sub-process is Manpower Programming, and it determines end strength and budget figures based on the manpower requirements. These first two sub-processes consider "spaces" within the force, as compared to the next two sub-processes that work with real people or "faces" to meet the requirements of the first two sub-processes. The third sub-process is Personnel Planning, and involves recruiting, training and strength planning based on the personnel who comprise the Navy. The final sub-process is Personnel Distribution, and it is this sub-process upon which this thesis focuses. The Personnel Distribution sub-process will now be discussed in more detail.

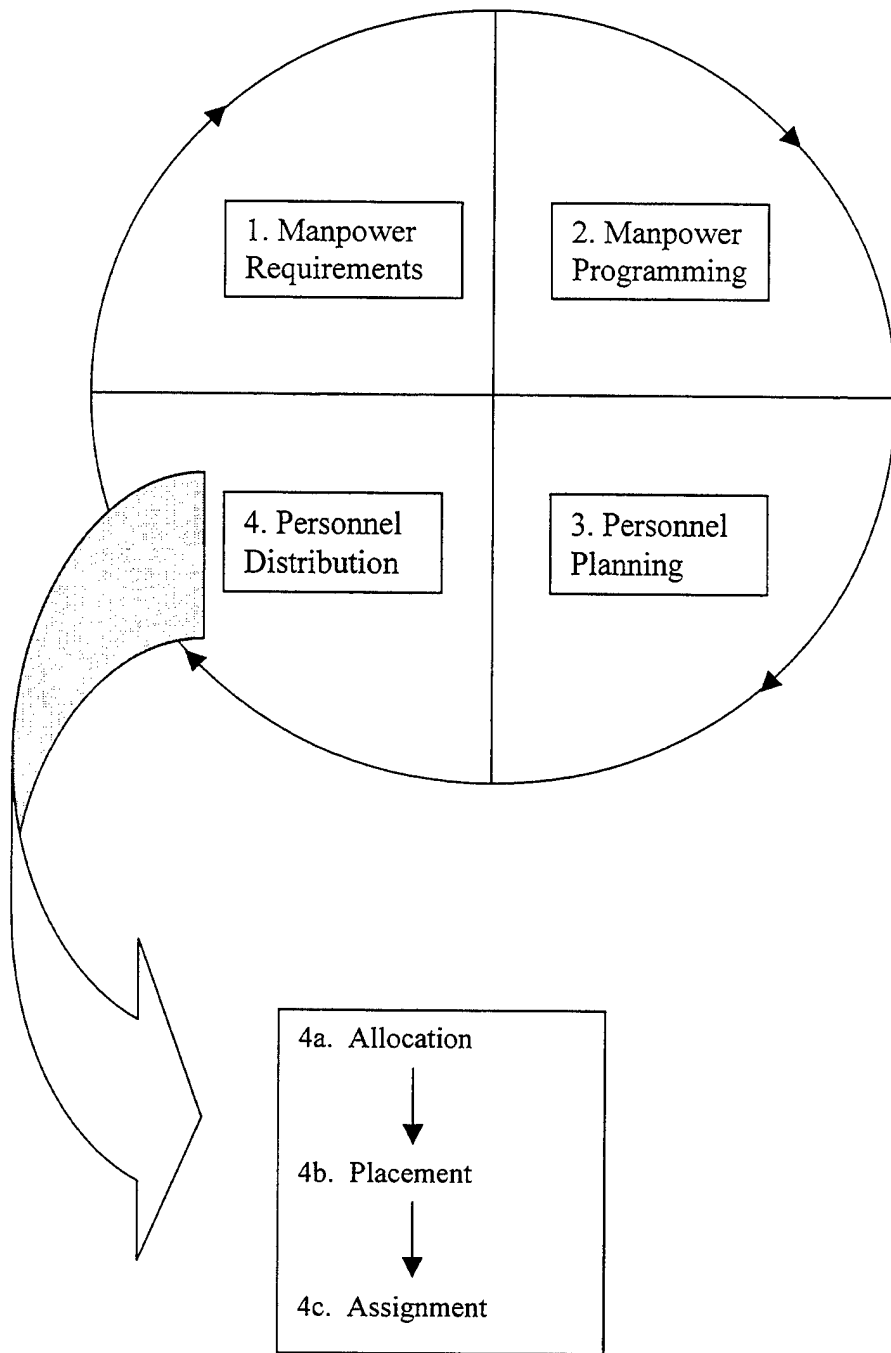


Figure 3.1. The Manpower, Personnel and Training Processes

## **2. The Personnel Distribution Sub-Process**

The Personnel Distribution sub-process is itself made up of three processes that are shown in the lower portion of Figure 3.1. However, before examining these processes it is first necessary to learn some of the unique terminology applied to the Navy's manpower processes.

When assigned to a billet within the Navy, a sailor is given a Projected Rotation Date (PRD) which indicates the window of time within which the sailor's next assignment is expected to occur. A move is usually restricted to the period starting three months prior to the PRD and extending to four months after the PRD.

Each sailor is identified by the Navy's information systems nine months prior to his or her PRD. Once identified, the sailors are divided into one of two mutually exclusive classes: Distributable Inventory or Non-Distributable Inventory. Unless an identified sailor is expected to leave the Navy, be in training or not be available for assignment due to other reasons (e.g. medical restrictions), the identified sailor will become part of distributable inventory. The distributable inventory is therefore all sailors approaching PRD and capable of filling the Navy's requirements in the coming nine-month period.

The first of the three Personnel Distribution processes is Allocation and involves apportioning distributable inventory to the four Manning Control Authorities (MCAs): Pacific Fleet, Atlantic Fleet, Reserve Forces and Bureau of Personnel. The allocations are made to the MCAs in order to ensure that there is a prioritized balance of distributable sailors in the fleet.

Once the distributable inventory has been divided between the MCAs, the Placement process equitably spreads the projected strength across each of four activities; sea, submarine, air and shore. Although the placement process is conducted centrally and remains external to the MCAs, it is in the placement process that the needs of each command are addressed.

Finally, once the distributable inventory has been divided between the MCAs (allocation) and the activities within each MCA (placement), the Assignment process allocates specific sailors to existing or projected vacancies. The assignment process is the first time where sailors' preferences are considered in the overall process.

## **B. THE ASSIGNMENT PROCESS**

### **1. The Procedure**

The entire personnel distribution process has been explored in detail by Short (2000). In exploring the assignment process, this section will not go into the level of detail as examined by Short, but rather focus on the issues most relevant to applying a two-sided matching process to the assignment process.

In order to post a sailor to a billet, detailers must have a means for identifying the projected future vacancies. The vacancies are determined through requisitions, with a requisition being created when the projected personnel at a command falls below the requirements. The requisitions are generated with the aid of various information systems that consider PRD and other factors that would remove a sailor from his or her job.

The assignment process occurs on a fortnightly cycle. During this cycle, upcoming vacancies are advertised online through the Job Advertising Selection System (JASS). After reviewing JASS, sailors, or career counselors as the sailors'

representatives, submit preferences, or applications, to detailers during the fortnightly cycle. Once the cutoff time for applications has been reached, JASS is temporarily closed to external viewing by sailors, and detailers commence the assignment process.

The benefit of extending the detailing process to a fortnightly cycle is highlighted by Short (2000, p29) who states, "No longer is the detailing process a first-come, first-serve assignment process. Detailing involves batch processing, thereby leveling the playing field for all sailors." Once assignments are made, JASS is updated with the new requisitions, and sailors without a future assignment can again view JASS to submit their preferences for the next fortnightly cycle.

Information provided from the Enlisted Placement Management Center (EPMAC) provides two reasons for the above-described cycle occurring on a fortnightly basis rather than a monthly cycle that received some consideration. Firstly, sailors indicate that they desire a more frequent opportunity to look at the requisitions. Secondly, the more frequent process allows changes (e.g. advancements and unplanned losses) in the distribution system to be measured more frequently.<sup>1</sup>

A brief analysis of the process involved in assigning sailors has been explored. The next section details the policies and considerations that detailers must take into account when assigning sailors.

## **2. Assignment Considerations**

The foremost considerations when assigning a person to a vacancy are their rate and rating, for it is these that determine in the first instance whether a person is eligible for a position. Enlisted personnel are defined by both a rate, which refers to a person's

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<sup>1</sup> Lee O'Quinn, EP46 EPMAC, email dated Jan 11, 2001.

pay grade, and rating which refers to an occupation specialty. For example, a person may be an E-6 Petty Officer First Class (the rate) Boatswain's Mate (the rating).

In the Navy assignment process, the use of eligibility criteria (rate and rating) for position vacancies is a significant difference from the situation facing the markets described in Chapter II. In the case of both the college admissions and the medical intern markets, the individual agents are from a group with a high degree of homogeneity with respect to skills and enter the market at entry-level positions. In the Navy assignment situation however, the sailors and the associated vacancies are not all at the rate of Seaman for sailors entering the market together for the first time. A detailer cannot assign a sailor at the rate of E-2 to fill the vacancy of an E-7, and it would be far from desirable for an E-7 to fill the position of an E-2. Therefore, rate is a significant consideration when assigning personnel. Besides a sailor's rate, the rating is generally the other crucial criterion that determines eligibility for a vacancy. For example, an Aviation Electronics Mate (rate) cannot do the work of a Fire Control Technician (rate) and vice versa.

In some instances, where the number of vacancies for sailors of a particular rate and rating exceed the number of sailors matching those criteria, there may be leeway for varying the position requirements. For example, a ship requiring an E-6 Electronics Warfare Technician may accept an E-5 or E-7 of the same rating if there is a shortage of sailors at the rank of E-6 in that particular rating. Furthermore, an E-7 of that rating may be preferable to an E-5, since the E-7 would possess all the skills required for the E-6 position, whereas the E-5 would not. Although it is not desirable to vary the position requirements as described, in some situations it may be necessary.

The simplest solution in applying a two-sided matching model to the Navy assignment process may be to consider the assignment process as many matching markets, with each market narrowly defined by rate and rating. For example, one of the markets could be defined for E-7 Gunner's Mates. To define markets in this manner would require many markets to operate concurrently, however this is not the primary concern with such a simplified procedure. The concern relates to the overlap between markets, because as described previously, there may be some leeway for some vacancies to accept a variety of rates or ratings. Therefore, the Navy situation may be considered to be many markets with some possible overlap between the various markets.

While rate and rating are often the foremost criteria for matching sailors with vacancies, detailers must consider a variety of other factors. Factors that make a sailor eligible or otherwise for a position include:

- Gender. There are some positions for which females are ineligible to apply to. Examples of such positions are any billets aboard submarines or in SEAL units.
- Projected Rotation Date (PRD). A sailor may only be moved to a new position once they are within the time window three months prior to four months after PRD.
- Sea / Shore Rotation. Sailors must achieve a balance between time spent at sea and time spent ashore.

Further to the eligibility policies described above, there are a number of other policies and factors that provide guidance to detailers when assigning sailors. Some of these include:

- Requisition Priority. Some billets, due to their critical nature, are assigned a higher priority for filling than are others. In order of precedence, the CNO may nominate billets or a command as having Priority 1 or Priority 2, while the MCAs may designate a Priority 3 to a position or command. Positions with a higher priority must be filled, potentially at the expense of positions with lower or no priority attached.
- JASS Preference. Sailors have the ability to indicate their preferences and apply online for positions through the use of JASS.
- Married Couples. Where possible, the Navy is required to co-locate married service members.
- PCS Cost. The Navy does not have an unlimited budget to fund the costs of moving sailors from one location to another. As a result, it is necessary for detailers to consider the cost of relocating a sailor. Although the cost of PCS moves are always a consideration, if funding for PCS moves becomes particularly limited, detailers may need to re-consider assignments so as to minimize moving costs.
- Distributable Inventory Balance. As described previously, the personnel distribution sub-process is required to ensure a balanced distribution of distributable inventory between each MCA. The assignment process must work within the confines of ensuring a balanced distribution of sailors.

The above points are not an exhaustive list of policies pertaining to the assignment of sailors. Short (2000, pp49-53) lists the various policy documents and references that guide detailers in assigning sailors.

It is clear that there are a significant number of policies that impact on matching sailors with commands. Existing two-sided matching markets do not face the same type of policy restrictions as described in relation to the Navy assignment process. The only restriction detected in relation to the intern-matching program is the need to co-locate married couples, while not even this restriction exists in the college admissions program. Given Roth's (1990, p143) previously quoted statement that "for large markets containing married couples the problem of even determining if a stable matching exists may be computationally infeasible", the computational complications created by introducing so many more rules can only be speculated.

### **3. Preferences**

When deciding between various billet vacancies, a sailor is required to mentally evaluate a variety of aspects relating to each position. Sailors must consider such factors as the location, duty type (sea or shore) and implications alternative positions have for promotion. Each sailor will attach different levels of importance to these various aspects. For example, a sailor with a spouse employed in the local area may put a high value on remaining in the same location, even if it means spending more time at sea and away from home. On the other hand, a single sailor who seeks to make the Navy a long-term career would be more likely to accept a position in an undesirable location if it enhances the prospects for promotion.

When sailors have given due consideration to factors such as those given above, they can specify their preferences by contacting their detailer or submitting through JASS.

Just as a sailor needs to consider what he or she regards as the more important aspects of a billet (e.g. location, duty type, promotion potential), so too must the Navy consider what it regards as important when assigning sailors to the various billets. For example, once basic eligibility requirements have been met, what aspect is next most important with respect to determining which sailor goes where. Is minimization of PCS cost the most important factor, or is sailor career development more important? Alternatively, is the need to have the most highly qualified and competent sailor on board a ship about to deploy to the Persian Gulf the overriding consideration? For example, consider two sailors, A and B, who are under consideration for a vacancy in Billet X. If by filling Billet X, Sailor B would receive greater benefit in terms of career development than Sailor A, then how much higher a price in PCS dollars would the Navy be willing to pay for that career development. Prior to determining this, a more fundamental question needs to be asked – “how do we quantify these measures?” That is, how is a value placed on aspects such as career development or the enhanced capability of a ship that results from having better performing sailors onboard?

Detailers are regularly required to make subjective assessments regarding the value of factors such as career development when assigning sailors. Furthermore, there is the perception that such subjective decision making may be influenced by personal biases. For example, a ship’s commanding officer may contact a detailer to request a particular sailor. To remove the potential for personal bias and make the process more accountable, it is necessary to develop metrics to quantitatively express what each command regards as important. The development of such metrics into a utility function would assist in creating preference lists for each billet.

In Table 3.1 the utilities of having any one of three sailors assigned to a notional billet are calculated. The utility is based on just three aspects in this example, although other aspects could be included. Each aspect is awarded a level of importance that relates to how important the given aspect is in relation to the billet under consideration. Each sailor is then rated on a scale of 1 to 10 according to how they measure up to the following criteria:

1. PCS cost. The PCS cost to move a sailor from their current location to the notional billet is calculated. A value of 1 indicates a high cost to move (perhaps from East coast USA to Japan) while a value of 10 indicates a low cost to move (remain in same locality).
2. Skills and Training Level. Each billet has a list of skills or a training level that a sailor should have accomplished in order to be able to fulfill the billet's requirements. In some cases it may be difficult to find sailors that have completed all the necessary training and possess all the required skills. A value of 1 would indicate that the sailor possesses virtually none of the billet's stated skill, while a value of 10 would indicate the sailor possesses all of the stated skills.
3. Measure of past performance. Based on previous performance history, sailors could be rated from a value of 1 (substandard performance) to 10 (outstanding performance). In some billets it will be more important than in others to have high performing sailors. For example, it is more important to have higher performing sailors on a ship that deploys to the Persian Gulf than in a clerk's position on shore.

Each sailor's score for each attribute is multiplied by an importance factor. These importance factors indicate the relative importance that the particular command attaches to the various criteria.

Evaluating utility in a manner that considers factors such as those given above will make it possible to determine whether, for a given billet, it is better to accept a sailor who has not yet completed all the required training, but performs to a high standard as measured by fitness reports.

The utility table may look as follows:

<b>Metric</b>	<b>Sailor A</b>	<b>Sailor B</b>	<b>Sailor C</b>	<b>Importance</b>
<b>PCS Cost</b>	3	7	10	Low
<b>Skills and Training Level</b>	8	6	8	Critical
<b>Past Performance</b>	5	6	3	High
<b>Score</b>	12.6	12.2	12.4	
<b>Preference List Ranking</b>	1	3	2	

Importance Weights: Low = 0.2 Medium = 0.5 High = 0.8 Critical = 1.0

Table 3.1 – Utility Table

The utility table in Figure 3.1 shows that Sailor A is the first preference for the notional billet, with Sailors C and B being the second and third preferences respectively. So although the cost to move to the new billet would be significantly higher for Sailor A than for Sailor B, and although Sailor A has a slightly lower performance history than Sailor B, the high level of importance attached to the skills and training in this billet leads Sailor A to be the first preference.

If the assignment process is to be automated using computer technology, utility functions are essential for determining preference lists. In the example given above, a scalar system is used that rates sailors along a scale of 1 to 10 according to various factors. Alternatively, equations of various functional forms may be used to calculate utility.

Once a process for calculating utility is determined, this could also become the measure by which the Navy can evaluate its quality of fit. That is, what average level of utility has the Navy been able to obtain using its existing assignment process, and what level of utility is possible if the assignment process is accomplished using two-sided matching technology. The impact of changing the process would also need to be assessed with regard to sailors' utilities.

## **C. THE ASSIGNMENT PROCESS AS A TWO-SIDED MATCHING PROCESS**

### **1. A Matching Model for the Assignment Process**

The second chapter of this thesis explores one-to-one and many-to-one matching models, each of which resembles some characteristics of the Navy's assignment process. The difference in framing the process centers on whether every Navy billet is considered an individual agent with its own set of preferences over all sailors (a one-to-one process), or whether there are groups of billets that seek identically qualified sailors and therefore have the same preferences over those groups of sailors (a many-to-one process). In each case, it is the sailors who are considered the "one," since each sailor is individually free to make his or her own choice with respect to the jobs they prefer. Since the sailor agents will be treated identically regardless of which model is used, discussion for now will

center on the differences between treating every billet as an individual agent or whether some billets may be grouped.

## **2. The Assignment Process as a One-to-One Market**

As discussed above, if the assignment process is framed as a one-to-one or marriage model, then every billet is considered an individual agent and must produce a preference list over sailors. Thus, each billet must consider each of the sailors who form part of the distributable inventory, firstly to determine their eligibility for the billet, and secondly to determine how those eligible sailors are ranked for the preference list. While computers would automate much of this process, in instances where commands require many sailors of the same qualification, this process would involve considerable repetition and therefore needless processing. The situation may be more difficult and confusing for sailors who desire a particular billet at a command that has multiple billets of the type that is sought. In this case, the sailor would need to ensure that each of the identical billets is included on his or her preference list, and determine if it makes a difference in which order the identical billets are placed. While the order in which preferences for identical billets are listed does not make a difference, to sailors who may be somewhat skeptical of the entire process, this situation may make them hesitant to embrace the process.

It is shown by Roth (1990, pp. 131 – 134) that it is possible to consider a college admissions problem as a related marriage market. Translating the terminology of Roth's college example into the Navy situation under consideration, for each ship  $S$  with a quota of  $q_s$  identical billets, it is possible to break each  $S$  into  $q_s$  pieces of itself, each with a quota of one. In the new market so formed, each sailor and each billet is an individual

agent. Each of the  $q_s$  positions for each ship are denoted  $s(1), s(2) \dots s(q_s)$ , and each of these positions has preferences over sailors that are identical to those of  $S$ . For a sailor considering one of the  $q_s$  positions on ship  $S$ , there is indifference between each of the positions  $s(1), s(2) \dots s(q_s)$  on the ship. To keep the process simplified, strict preferences will be expressed, and it is therefore possible for each sailor to replace his or her preference  $S$  by the string of preferences  $s(1), s(2) \dots s(q_s)$  in that order. When the process is structured in such a manner, Roth (1990) shows that the matching obtained by the one-to-one process will correspond to the matching obtained by the many-to-one process.

### **3. The Assignment Process as a Many-to-One Market**

Based on the above comparison between one-to-one and many-to-one matching models, it may be questioned whether there is any difference between the two and why one would be chosen one over the other. Although there are a number of similarities between the two models, they are in fact not equivalent as detailed by Roth (1985, pp277 - 288), and due to differences in strategic behavior, it is incorrect to simplify a many-to-one market as a one-to-one market. Although the details of these differences are beyond the scope of this thesis, the differences relate to the fact that ships or commands with quotas greater than one will have preferences over the groups of sailors assigned rather than simply preferences over the individual sailors. Furthermore, there is a difference in the incentive behavior of agents to state their true preferences.

Due to the strategic issues presented above, the correct representation of the Navy's assignment process therefore appears to be in the form of a many-to-one matching process. However, the many eligibility and policy requirements discussed

earlier in this chapter necessitate significant modifications to the many-to-one model if it is to be used for the assignment process. These will be discussed in the next section of this chapter.

It was detailed in Chapter II that there are two approaches to the many-to-one matching process, described as the applicant-optimal and the firm-optimal procedures. Relating to the Navy's situation, the applicant-optimal procedure would provide each sailor with the best possible billet that he or she can have from all possible stable matches. However this procedure also assigns the worst acceptable sailor to each billet. Alternatively, the firm-optimal procedure would provide each billet with the best possible sailor from all possible stable matches. This procedure correspondingly assigns sailors to their worst acceptable billet. Consideration must be given as to whether it is preferable to implement a system that would favor sailors or commands. This question is left open, and attention will now be given examining the shortcomings that may occur if the many-to-one model is applied without modification to the Navy's assignment process.

#### **D. SHORTCOMINGS OF THE MANY-TO-ONE MODEL**

If the assignment process is modeled as a many-to-one process, it is important to realize that some modifications are required to account for the eligibility and policy requirements, and that some practices associated with the assignment process may invalidate the stability issues. These issues are detailed in the remaining sections of this chapter. Possible resolutions to these issues will be discussed in Chapter IV .

##### **1. Vacancy Chains and Inherent Instability**

Unless radical changes are implemented to the way in which sailors are assigned to billets, there will be instability in the system. To understand this, consider an initial

group of sailors and vacant billets that are matched using a two-sided matching process. It will be assumed that the sailors and billets are matched in a stable manner and will remain matched for three years until their specified PRD. Based on current timelines, new requisitions are created and JASS preferences submitted in the ensuing two-week period after the initial match, with the matching process occurring at the end of the two-week period.

The emergence of new billet vacancies and new sailors entering the assignment consideration period (the time window starting nine months prior to PRD) will create instability for the sailors and billets most recently matched. For example, if Sailor A was matched to Billet X in the first round and Sailor B to Billet Y in the second round, instability will exist if Sailor A prefers Billet Y to X, and Billet Y prefers Sailor A to B. However the matches are fixed until the specified PRD, therefore the billets and sailors do not have a means for avoiding this type of instability. So although each round of matches may be stable when considered individually, all enlisted personnel and billets cannot be in a stable match using the current procedures that involve frequent matches and lengthy assignment contracts. The situation resembles the vacancy chains in senior level markets described in Chapter II, except that sailors not approaching PRD cannot be considered for reassignment. This situation is not considered in existing two-sided matching markets such as the college-admissions or medical-intern markets, since they are restricted to entry level markets.

## **2. Frequency of Assignments**

As previously indicated, the assignment process is repeated approximately every two weeks, a significant difference from the annual process that generally occurs in

existing two-sided matching markets. At this stage it is uncertain what impact may result if two-sided matching technology is employed for the Navy and the frequency of assignments remains fortnightly.

Currently sailors have the opportunity to contact their detailers nine months prior to their PRD, with the assignment window starting three months prior and ending four months after the PRD. This gives the sailor multiple windows of opportunity for assignments. That is, approximately every two weeks from a time nine months prior to the PRD a sailor has the opportunity to be matched with a billet that is currently or projected to be vacant.

### **3. Requirement to Match all Sailors and Fill Priority Billets**

In the year 2000, the National Resident Matching Program (NRMP) attempted to match 33,528 medical students with 3,769 programs offering 22,722 positions. Of the medical students, 25,056 remained active indicating that they did not withdraw and were not unranked by hospitals. On completion of the match, 72.3% of the programs were filled and 74.7% of active applicants were matched to a position.

For the Navy assignment situation, to have approximately 75% of applicants matched in a stable manner by an automated process would be an impressive result. Such a process would remove a considerable workload from detailers. However, to have approximately 25% of sailors without a match would be an unacceptable situation. In the case of the NRMP, unmatched applicants apply personally to various programs to obtain a match. A similar result is unacceptable for the Navy. Sailors cannot be expected to spend their time appealing to various commands in search of employment. Therefore, if a

two-sided process is applied to the Navy's assignment process, it is necessary to implement a mechanism that will ensure that 100% of sailors are matched.

While some of the Navy's billets will be vacant at various times, there are certain critical billets that cannot go unfilled. Such billets are indicated by a variety of priority levels that indicate the necessity of filling the billets. If a two-sided matching process is implemented, it will be necessary to include a mechanism that allows higher priority to be attached to certain billets, thus ensuring that on completion of the match high priority billets are not left vacant.

#### **4. Impact of Married Couples**

Similar to the situation in the NRMP, the Navy has the complication of needing to send married couples to the same location.

#### **5. Inability to Force Matches**

One of the key assumptions of the theory of two-sided matching is that agents cannot be forced into a match that they do not find acceptable. That is, there are some instances where an agent would prefer to remain unmatched rather than be forced into a match with an undesirable partner. To ignore this vital element of two-sided matching theory would lead to unstable matches and the demise of the process.

One of the problems facing the Navy is the need to sometimes send sailors to billets that they do not desire. If two-sided matching is applied to the assignment process, unless the issue of stability is abandoned, it is not possible to send sailors to locations that they do not indicate on their preference lists. However, as indicated above, to force undesirable matches and abandon stability issues diminishes the power of the matching process. Therefore, it is necessary to find a process that encourages sailors to include

positions that are generally regarded as less desirable on their preference lists. Even in the absence of a two-sided matching process, it would be beneficial to find a means of encouraging sailors to accept positions that are generally regarded as undesirable. For sailors who occupy undesirable billets, this would enhance their overall level of satisfaction with the Navy.

It should be noted that encouraging sailors to accept undesirable billets should not be the first step in improving satisfaction with such billets. The primary objective should be to determine the factors that make certain billets undesirable and address the fundamental issues. This may include considering such factors as assignment location, work conditions or hours of work. If such issues cannot be resolved or it is impractical to do so, then, as described above, other means should be considered to make these billets more desirable. This may involve the use of 'incentive credits' or payments to sailors to encourage them to accept such billets. These issues will be discussed in Chapter IV.

## **IV. SUGGESTED SOLUTIONS**

The last section of Chapter III discusses various issues that complicate the process of applying two-sided matching to the Navy's assignment process. This chapter proposes a variety of mechanisms that may be used to resolve the various issues raised in Chapter III. Primarily, these mechanisms relate to changes in the assignment process and the application of the algorithm, rather than changes to matching algorithm itself

### **A. RESOLVING THE INHERENT INSTABILITY**

Two elements combine to create the 'inherent instability' described in Chapter III. Firstly, assignments occur frequently with each new round presenting new options to already-matched sailors and commands, with the new options serving to undermine any stability that may have existed beforehand. Secondly, once assigned to a billet, it is only under exceptional circumstances that a sailor may leave the billet prior to the projected rotation date (PRD). Even though an already assigned sailor may prefer an alternate vacant billet, and the vacant billet in return prefers this sailor to its proposed match, unless the sailor is due for re-assignment, the preferred match cannot take place and both the sailor and billet will be dissatisfied. It is therefore the frequency of assignments and the inability to leave a billet prior to PRD that erode the possibility for obtaining Navy-wide system stability. The question then becomes, what can be done about these two issues to enhance stability?

#### **1. Changing the Frequency of Assignments**

An over-simplified solution would be to assign and re-assign all personnel at the same time, say every two to three years, however such a system is clearly infeasible. The frequency with which sailors enter and leave the service, become promoted or change

specialty, the variations in assignment duration and the changing demands of the service are some of the reasons why this is not an option for consideration. However, an assignment process that occurs less frequently than the current fortnightly system may offer some advantages. If the assignment process were to occur monthly or bimonthly, the first noticeable difference would be an increase in the number of requisitions and sailors to be matched each time. Under the current system this may be a disadvantage because an increase in the number of requisitions and sailors seeking matches imposes a greater workload on detailers. Furthermore, due to the variety of needs and desires of both sailors and commands, there is a limit to how many sailors and requisitions a detailer may consider at any one time. However, if the process is automated using two-sided matching technology, a decreased frequency of assignments would be advantageous since it offers a greater range of options to sailors and billets alike.

The effects of changing the frequency of assignments will require more detailed analysis and experimental study. However, an assignment process that occurs either monthly or bimonthly should be considered. By doing so, a sailor (billet) would not be restricted to considering only the requisitions (sailors) that have become available in a two-week period, but would be able to choose from all requisitions (sailors) in a one or two-month period. This would also remove the possibility of a sailor receiving an assignment, only to find a preferable billet available two weeks later.

## **2. Variable Assignment Duration**

The second issue relates to a sailor being compelled to remain at an assigned billet despite a preferable billet becoming available. While strong arguments, such as the need for team cohesiveness or deployment preparation, may argue for lengthy fixed duration

assignments, in some situations the corrosive effect of a dissatisfied sailor at a command may outweigh the potential benefit gained. What then would be the situation if the Navy disposed of the notion of PRDs and allowed its sailors to move at any time after assignment? Would there be chaos with sailors moving at every opportunity, or would there be some self-imposed factors that regulate the extent to which sailors would move?

The following points describe a variety of options aimed at providing greater flexibility in assignment duration. After these have been outlined, a discussion of the possible costs and benefits of each option is presented. It is assumed that the proposal from the previous section, that of bimonthly assignments, has been adopted. The options are as follows:

1. Sailors are free to be re-assigned at any time the assignment board meets.
2. Sailors are required to serve a minimum of six months at an assigned location before being eligible for reassignment.
3. Sailors are required to serve a minimum of six months before being eligible for reassignment, and may not serve longer than four years at an assigned location.

The options listed above are not a complete or exhaustive list. These options have been chosen because they demonstrate three basic strategies in freeing up the labor market for sailors. The first strategy creates almost total freedom in the assignment process, with sailors available to move at any time they desire. The second and third strategies provide greater freedom than currently exists, but in the case of option two puts constraints on minimum service time at a billet, and in the case of option three an additional constraint of maximum service time at a billet is added.

*a. Option One – Freedom to Move at Any Time*

The first option will provide sailors and commands with the greatest flexibility and therefore is most likely to bring stability to the assignment process. It leaves the potential however for some sailors to frequently change billets; in some cases, the less desirable billets might also have a high turnover of sailors. Factors that would deter sailors from moving frequently are the costs, both implicit and explicit, that sailors face when moving. That is, sailors moving to a new location, must, where applicable, face the costs of leaving friends behind, moving children to new schools, losing income until a spouse is able to find new employment and bearing the general stress associated with moving to a new location. While not all of these are financial costs, they are factors that would be considered by sailors when considering whether to move to a new location. It is acknowledged that there would be minimal implicit or explicit costs involved with changing assignment in the same locality. There are other factors, however, that would tend to decrease the frequency of changing assignments, even for sailors remaining in the same location.

In order for a sailor to be able to move, it is necessary for the sailor to find a vacant billet that he or she prefers and for that billet to prefer that sailor to any other sailor applying for the billet. Only if this situation occurs may a sailor move. Furthermore, if such a situation arises, it indicates that both the sailor and the sailor's new billet are more satisfied.

If sailors are permitted to move frequently, it may be necessary to give consideration to the quality of match in the billet that the sailor vacated. That is, if the sailor moves to a new billet and a high quality of fit is achieved at the new billet, what consideration should be given to the quality of fit at the billet the sailor departed? If the

billet that the sailor departed is not able to find a suitable quality sailor and the job suffers, would it be preferable for the sailor to remain in the original billet. At the extreme case, the recently departed billet may not be matched with any sailor and could remain vacant for an unspecified time. Therefore, when the Navy considers whether to allow a sailor to move, it may be necessary to consider quality of fit at both the old and new locations. Such a procedure would likely require multiple iterations of the matching process since the Navy would not be able to determine the new quality of fit until a new sailor is tentatively assigned to the billet.

*b. Option Two – Freedom to Move After Minimum Service Period*

The second option differs from the first only by requiring sailors to serve a minimum time period in an assigned billet; in this case a six-month minimum period of service. After remaining in a billet for six months, a sailor becomes eligible for consideration at any upcoming assignment board. Once assigned to a new billet, the sailor must again wait six-months before applying for any other billet. This proposed option takes away the possibility of some billets having very high turnover and the possible deleterious effect that this may have on task accomplishment for those billets.

The minimum period to be served prior to eligibility for re-assignment could be varied, however a longer duration increases the potential dissatisfaction that sailors may face. On the other hand, a shorter duration leads to the possibility of higher turnover in some billets. For these reasons, a six-month duration is considered a reasonable starting point for consideration. However, experimentation may be required to determine the impact that other time periods would have.

*c. Option Three – Freedom within Minimum and Maximum Service Periods*

The third option incorporates both a minimum and maximum time for which a sailor may serve in a billet. The addition of a maximum time limit ensures that sailors cannot remain in the most desirable billets indefinitely, and thereby ensures that other sailors have the opportunity to be assigned to the desirable billets. Different billets may have different upper time limits. For example, for billet types that are relatively rare and highly sought after, it may be desirable to place an upper time limit of two or three years. For other more common billets, a four-year upper time limit may be more appropriate.

For a sailor who serves the maximum time allowed in a particular billet, there would be nothing to preclude the sailor from attempting to re-match with the same billet when the maximum time is complete. It may be that the sailor wishes to remain in the billet, and the billet prefers that sailor to any other sailors applying to the billet. In such a situation the sailor and billet could be re-matched as it is in the best interests of both to do so. To prevent such a re-match would not only disadvantage both the sailor and the billet, but would unnecessarily introduce instability into the system.

*d. Impact on PCS Costs*

Due to the explicit and implicit costs of moving, a sailor should perceive an alternate billet as having a significantly higher personal value to warrant the cost of moving. However if a sailor is sufficiently dissatisfied with his or her current assignment, then this proposal may offer a means to obtain a more desirable assignment, a factor that would improve the sailors satisfaction while potentially decreasing the likelihood of the Navy losing a sailor.

If the potential for more frequent moves would increase the annual cost of PCS moves, funding for PCS moves may be restricted to only those sailors who have served a minimum time, say two years, in a billet. This would not impact on the notion of stability because the cost of a PCS move would then be included in sailors' consideration of preferences. This is in much the same manner in which medical students fund their own travel to hospitals at which they gain residency. Medical students therefore consider the costs of this travel when determining their preferences.

*e. Benefits to the Navy of a Flexible Labor Market*

The notion of variable assignment duration as discussed above would provide considerable career freedom to sailors and is referred to here as a flexible labor market within the Navy. Such a flexible labor market would provide sailors with more freedom to manage their careers than currently exists, and it would in some ways make the Navy's labor market more closely aligned with civilian labor markets. The benefit of using two-sided matching technology in conjunction with variable assignment durations is that the additional freedom to sailors is not associated with a loss to the Navy. That is, because two-sided matching uses preferences of agents on each side of the market, the process can give more consideration to the desires of sailors than currently exists, while continuing to acknowledge the needs of the Navy. It is believed that the freedom and control sailors would receive over their careers from a flexible labor market would be viewed positively and would increase sailors' satisfaction with the Navy. Such an increase in satisfaction may have positive effects on re-enlistment rates.

In addition to increasing sailors' satisfaction, there are potentially many other benefits that may be obtained from creating a flexible labor market within the Navy. These benefits arise primarily from the feedback information that could be

obtained from monitoring the market if such a system is implemented. The Navy would have the ability to monitor the frequency with which sailors move on from each command or billet type. High turnover in certain billet types would indicate that attention needs to be paid to the type of work or conditions associated with that billet type. If it is not possible to change the work or conditions, alternative means may be necessary to encourage sailors to remain in the high turnover positions. An incentive system to accomplish this will be discussed later in this chapter.

In some situations, monitoring turnover rates may indicate changes in turnover associated with the arrival and departure of particular Commanding Officers (COs). If the system indicates that turnover increases at every command to which a CO is posted, it could indicate that the CO is not suitable for command positions and may be better suited to serve the Navy in other ways. Thus, this system could assist selection for promotion and higher command positions.

Monitoring the frequency with which sailors move between billets may also indicate which duty types are better suited to certain sailors. Furthermore, a billet that prefers its sailors to remain for a longer period may rate lower on its preferences sailors that have a history of moving frequently, and increase the priority on job specific experience.

*f. Possible Complications*

Implementing a flexible labor market would obviously require extensive testing. One potential concern is that in narrowly defined categories for which there are relatively few billets, sailors may not be able to move when desired. This may occur if a sailor wishing to move into a new billet is unable to find a vacant billet or a billet that another sailor is departing. If there are no vacant billets then the sailor must wait until

another sailor vacates a billet of the type that is sought. However, it may be that the situation would not be significantly different from that which currently exists with the use of a fixed PRD.

Discussion here has focussed on one-sided as opposed to two-sided flexibility. The one-sided flexibility refers to the exclusive ability of sailors to determine if and when they wish to seek a new assignment. Two-sided flexibility would indicate that commands have similar flexibility in choosing if and when to seek new sailors. The implications of implementing two-sided flexibility into the process would require further consideration.

## **B. THE NEED TO MATCH ALL SAILORS**

As indicated in Chapter III, existing matching markets often match high percentages of applicants. However, not all applicants are matched. If a two-sided matching process is implemented, it is essential that a means be determined to ensure all sailors are assigned to a billet with adequate notice prior to their move.

In the year 2000 NRMP, 33,528 medical students applied for 22,722 positions. With so many applicants in excess of positions available it is not surprising that there were a number of medical students left unmatched on completion of the process. However, if the number of medical students were halved so that there were considerably fewer medical students than positions and all other factors remained unchanged, would all students receive a match? The answer is likely to be no and is related to the length of preference lists submitted by both students and positions. However, if the preference lists were sufficiently long, possibly to the extent of including all agents from the other side of the market, then it would be possible for all students to be matched. It should be noted

that to include all agents from the opposing side of the market indicates that there are no agents from the opposing side of the market that are regarded as unacceptable.

It therefore appears at first glance that all sailors could be matched provided the number of billets is in excess of sailors and that preference lists are sufficiently long. However this does not take into account some important factors. First, billets' preference lists cannot include all sailors because some sailors are ineligible for certain billets. For example, a billet with the requirement for an E-5 Aircraft Mechanic cannot list an E-3 General Duty sailor as a preference, no matter how long the list of preferences. In a similar vein, an E-3 General Duty sailor cannot create an extensive preference list that includes positions such as an E-5 Aircraft Mechanic. Furthermore, even in cases where a sailor may be eligible for certain positions, there may be compelling reasons that the sailor cannot be assigned to certain billets. For example, a sailor may be listed under the Exceptional Family Member (EFM) program; the sailor's family circumstances may limit the locations to which the sailor may be assigned. For such reasons, it is not possible for sailors or commands to simply extend their preference lists to include all agents from the opposing side of the market.

#### **1. Matching Sailors with Special Needs**

There are two approaches that may resolve the situation of sailors with special needs such as the EFM program. These are as follows:

- The first approach would be to manually pre-arrange matches of EFM program sailors prior to the main match. In this way the EFM program sailors are guaranteed of being placed with a billet that they require. Once EFM program sailors are placed at the necessary billets the remainder of

sailors would be matched using a two-sided matching process. The disadvantage of this method is that it potentially increases the instability and is a subjective process, something that this entire review of the matching process seeks to avoid.

- The second approach is to re-structure the preference lists of commands to include a factor that accounts for special considerations such as EFM programs. That is, any sailor with special needs will be rated more highly than sailors without special needs. Therefore if the billet rates the sailor with special needs highest, and in return the sailor rates the billet highest then the chances of matching the sailor with a billet that he or she needs is high. Although it is likely that special needs sailors will be matched with a billet that is suitable to them, there is no guarantee since it depends on the number of vacancies appearing in billets able to accommodate special needs sailors. Therefore, in certain situations it may be necessary for detailers to resort to manual intervention.

## **2. Extending Preference Lists to Maximize the Number of Sailors Matched**

Even in the absence of EFM program sailors, it is difficult to ensure that the remainder of sailors are matched after completing the process without forcing sailors or commands to extend their preference lists to include 'unacceptable' agents. At this stage, the only possible solution is to maximize the length of preference lists. This will not guarantee that 100 percent of sailors are matched, but will ensure the highest possible rate of matching.

It should be noted that the proportion of sailors matched will be relative to the balance between the number of billets of various types and the inventory of sailors who have the required rate and rating to fill those billets. If there are insufficient sailors to fill the number of billets of a particular type, matching all sailors is not likely to be a concern. For example, if there are 500 E-6 Electronics Warfare Technician billets, yet only 400 sailors of that category, matching all the sailors is not likely to be a problem. However, if the number of sailors in a particular category exceeds the number of billets that they may fill, there is clearly a problem that cannot be resolved by two-sided matching.

From the commands' perspective, billets must extend their preference lists to include all sailors who are eligible for the billets, and then allow some latitude in certain eligibility criteria at lower preference levels. That is, determine the set of sailors who are strictly eligible for the billet and then rank them accordingly. Once this is done, allow variation in certain eligibility criteria and then rank the next set of sailors. For example, a billet that requires an E-6 Electronics Warfare Technician would select initially only E-6 Electronics Warfare Technicians and rank them on the preference list according to specified criteria such as past performance and training level. At the end of the preference list so determined, additional sets of sailors would be added with each set internally ranked according to the specified criteria (performance etc.). For example the second set of sailors may be E-7 Electronics Warfare Technicians, followed by a set of E-5 Electronics Warfare Technicians. However, the preference list for E-6 Electronics Warfare Technician would not be extended to include sailors unable to complete the work required in the billet.

Sailors would be required to extend their preference lists to include all billets for which they are qualified, as well as positions that they either over-qualify for (e.g. an E-5 over-qualifies for an E-4 billet), or positions for which they almost qualify (e.g. an E-6 may almost qualify for an E-7 billet). To ease the process of sailors producing such extensive preference lists, it would be possible to individually specify certain billets, then allow a computer process to rank other billets according to criteria that the sailor specifies. For example, a sailor may list six billets that her or she particularly desires. After those billets, the sailor may indicate that the next most important factor to consider is location followed by duty type. That is, the preferences may be ordered as shore duty at San Diego, sea duty at San Diego, shore duty at Hawaii, sea duty at Hawaii and so on. Based on the sailor's selection of factors such as location and duty type, a computer process could then automatically rank and add to the preference list all billets for which the sailor qualifies. The sailor may then make final adjustments as desired.

It is emphasized that extending preference lists as detailed above will not guarantee that all sailors are matched. It will, however, ensure that the maximum possible number of sailors are matched. Without destroying the notion of stability, a central concept in two-sided matching, it is not possible to alter the matching process to ensure all agents are matched. Experimentation will be required to determine the percentage of sailors that can be matched by extending preference lists as described above. Provided that there is a balance between the inventory of sailors and the billets in each category defined by rate and rating, there should not be any significant numbers of sailors remaining unmatched.

### **3. Multiple Opportunities for Matching**

The final factor that will assist to ensure that all sailors receive an assignment is that there are multiple windows of opportunity for sailors to be matched. After a sailor enters the matching process that commences nine months prior to re-assignment, there are multiple requisition cycles during which the sailor may be matched.

#### **C. ENSURING PRIORITY BILLETS ARE FILLED**

As explained in Chapter III, the Navy assigns priority levels to critical billets to ensure they are filled in preference to less critical billets. It is important to ensure that the two-sided matching process accounts for preferences and fills critical billets.

If priority billets are left vacant after completing the two-sided matching process, this may be due to two reasons. The first, and hopefully less likely reason, is that none of the sailors who are listed on the priority billets' preference lists have nominated the priority billets. Unless all sailors universally regard the priority billet as undesirable, this is not likely to occur. The second reason is that non-priority billets have 'poached' sailors from the priority billets. In the following example, it will be seen that although each sailor lists the priority billet amongst his or her preferences, the priority billet remains vacant after completing on the match.

##### **1. Example - Priority Billet Vacancy**

In this example, Commands X and Z are non-priority commands and have two vacancies each. Command Y has one vacancy and is nominated as a priority billet. Preferences of each of the commands and four sailors (A, B, C and D) are as shown:

Commands	<u>X(2)</u>	<u>Y(1)</u>	<u>Z(2)</u>	
	C	B	A	
	B	C	D	
	A	D	B	
	D	A	C	
Sailors	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>
	Z	Z	Z	X
	Y	X	X	Z
	X	Y	Y	Y

Thus, there are four sailors seeking assignment, and three commands offering a total of five vacancies. To obtain the command optimal solution, the NIMP algorithm detailed in Chapter II is used. At the 1:1 step, Command Z is matched with Sailor A, with Sailor A's second and third preferences deleted in the tentative-assignment-and-update phase. In the next round, Command Z remains matched with Sailor A and no further 1:1 matches are made. The process then proceeds to the 2:1 phase and all sailors are matched. The resultant match provides Command Z with Sailors A and D, and Command X with Sailors B and C. This leaves the priority billet at Command Y vacant. It is now necessary to seek a means of manipulating the matching process so that the billet at Command Y is not vacant after completing the matching process.

## 2. Option One – Match Priority Billets First

An option towards ensuring that priority billets are filled is to divide the billets into two groups; those that are designated as high priority and those that are not. The priority billets will then be matched against the entire set of sailors, and any sailors remaining unmatched after completing this first matching round will enter the second round to be matched with the non-priority billets.

There are two ways to conduct the match with the priority billets. The first is as described above where all priority billets are matched against the entire group of sailors.

Alternatively, the priority billets may be broken down into groups of Priority 1, 2 and 3 billets, with each priority level being matched in order.

The first process is preferred since there is a larger group of billets against which the sailors may be matched, thus leading to a better quality match. However, there is a potential for lower priority billets to be filled and higher priority billets left vacant. For this reason, the second proposal is suggested whereby Priority 1 billets would be matched first, then the unmatched sailors would be matched with Priority 2 billets. The process would be repeated for Priority 3 billets and finally non-priority billets, each time using the sailors who remain unmatched from the previous round.

It is not necessary to immediately commence with the second process and break the priority billets down into subsets. A match may be conducted using the first process, and then if the higher priority billets remain vacant, the second process may be used. If the second process fails to fill the high priority billets, then a close examination of sailors' preferences may be required since such a situation would indicate that few sailors include the high priority billets on their preference lists.

### **3. Option Two – Manipulate the Match to Fill Priority Billets**

This option does not divide billets into priority and non-priority billets, but rather conducts a match using all sailors and billets available at the time. After completing this match, sailors are adjusted from non-priority billets to priority billets. The process will be demonstrated using the earlier example that indicates a priority billet left vacant.

The preference lists as used previously are as follows:

Commands	<u>X(2)</u>	<u>Y(1)</u>	<u>Z(2)</u>	
	C	B	A	
	B	C	D	
	A	D	B	
	D	A	C	
Sailors	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>
	Z	Z	Z	X
	Y	X	X	Z
	X	Y	Y	Y

The final match obtained using the command optimal, NIMP algorithm is:

Match:	Command X	Command Y	Command Z
	Sailors B, C		Sailors A, D

In this example, Command Y with one vacancy is the priority billet.

It is necessary to look at the sailors' lower preferences to determine who should be moved to the priority billet. It is seen that each of Sailors A, B, C and D rank Billet Y one preference lower than their current assignment. Since a straight listing of billets does not indicate the degree to which the sailors prefer their current assignment to Billet Y, the only possible interpretation without additional information is that the decrease in utility (satisfaction) is the same for each sailor (since each sailor ranks Billet Y one rank lower than their current match). Therefore, in this case, Sailor B would be chosen to move to Billet Y since Billet Y prefers Sailor B out of the four potential sailors.

#### 4. Option Three – Multiple Opportunities to Match

The final approach to ensuring that priority billets are filled is the simplest method, and involves waiting, possibly over multiple requisitions cycles, for the priority billets to be filled. If the matching process remains fortnightly, or even changes to a monthly cycle, provided that a vacancy is identified well in advance, there will be multiple windows of opportunity to match the priority billets. That is, if the priority

billets are not filled in the first month that they are part of the matching process, they should be at some time over the next few matches. If the billet remains unmatched after several windows of opportunity, then one of the previously described processes may be implemented. It may also be necessary to consider whether sailors are all ranking the priority billet at the lowest levels on their preference lists, in which case it may be necessary to offer an incentive to entice sailors to seek the billet.

#### **D. PROVIDING INCENTIVES TO SAILORS FOR LESS**

##### **DESIRABLE BILLETS**

As described previously, each billet has certain attributes that sailors consider when making their preferences. These attributes may be the billet's location, duty type, typical work hours or the potential for promotion or career enhancement. Depending on the sailor who is considering the attributes and the billet to which they relate, these attributes may be regarded as either providing satisfaction or dissatisfaction. What one sailor regards as providing satisfaction, for example a billet located in Monterey, California, another may regard as yielding dissatisfaction. If a particular sailor finds that the combined effect of the dissatisfying attributes is greater than the combined effect of satisfying attributes for a given billet, then that sailor will regard the billet as undesirable. Assuming that all sailors consider a billet to be undesirable, the degree to which the billet is regarded as undesirable will vary for every sailor. As a result, the level of incentive required to encourage sailors to accept an undesirable billet will vary across sailors.

From the above consideration, it is clear that some sailors will require smaller incentives to apply for certain undesirable billets than will other sailors. The offer of an incentive to accept a billet will encourage those sailors with the least level of

dissatisfaction to apply for the billet. As higher levels of incentives are offered, more sailors will apply for the billet.

The objective of an incentive is not for every sailor to apply for the undesirable billets, but for sufficient sailors to apply so that the matching process may continue. If only a single sailor includes the undesirable billet on his or her preference list, there is no guarantee that the sailor will be matched with the billet, unless the sailor and billet list each other as their first preferences. However, greater numbers of sailors applying to an undesirable billet will not only increase the chances of the billet being filled, but will also increase the likelihood of a higher quality fit for the billet.

In determining the appropriate level of incentive, it is necessary to balance the marginal cost of increasing the incentive against the marginal benefit obtained by increasing the incentive. Since offering an incentive is a cost to the Navy, an increase in the incentive offered is regarded as the marginal cost, or the additional cost. Assume the Navy offers a sufficient level of incentive to encourage one sailor to apply for an undesirable billet, and that this sailor then becomes matched with the billet. If the Navy increases the incentive then more sailors will apply to the billet, thus increasing the quantity of sailors that the billet may consider, and potentially increasing the quality of match for the billet. Thus the marginal benefit to the Navy from an increase in the incentive level is the additional quality of the sailor who is matched to the billet, and is related to the number of sailors encouraged to apply to a billet. The Navy must seek to increase the number of qualified sailors applying for a billet by increasing the incentives offered, until the marginal cost of increasing the incentive equals the marginal benefit obtained. While the variety of sailors' qualities will make it impossible to determine

exactly how many sailors should be encouraged to apply for a billet, simulation and experimentation may provide an indicative number.

There are two aspects of the incentive scheme to be considered when determining how to encourage sailors to apply for less desirable billets. These aspects are the types of incentives and the mean by which the incentives are applied.

### **1. Types of Incentives**

There are a wide variety of incentives that may be offered to sailors. Some of the possibilities are:

- Monthly cash payments,
- Additional annual leave,
- Promotion points that increase the opportunity for early promotion, or
- Assignment points that allow a sailor to obtain a better future assignment if currently accepting an undesirable assignment.

When assessing the various alternatives, it is necessary to consider the reinforcement value of the incentive. That is, does the incentive frequently remind the sailor of the benefit he or she receives in return for the hardships? Failure to provide an ongoing reminder of the benefit may cause the sailor to become dissatisfied during the assignment. A monthly cash bonus works well from this perspective, because the sailor sees that he or she is being rewarded for working at the particular billet on a monthly basis. On the other hand, promotion points or receiving additional consideration for a future assignment do not work well from this perspective; the reward is only received at a future time and even then may not be guaranteed.

Another aspect from which the alternatives need to be considered is the ability to vary the incentive according to the nature of the billet to which it is applied. For example, a billet that is generally regarded as only marginally undesirable needs less of an incentive than a billet that is strongly disliked by all sailors. From this perspective, cash payments work well because it is easy to vary levels of bonuses. A billet that is only marginally disliked may only offer a \$50 per month bonus; other billets more strongly disliked may offer \$800 per month, with various levels in between. Offering annual leave credits allows some limited flexibility in the level of incentives offered to various billets, while promotion or assignment considerations provide the least flexibility.

When compared to the other incentive types briefly discussed here, cash incentives are the easiest to tailor to the situation and also provide the greatest positive reinforcement. Therefore, cash incentives will be used as the basis for the remainder of the discussion regarding implementing incentives.

## **2. Applying Cash Incentives**

There are a variety of ways in which to implement cash based incentives. At the simplest level it is possible to categorize all naval activities into a limited number of classes, such as sea, shore CONUS or shore OCONUS. Based on which of these classes is most difficult to fill, incentives could be applied to anyone who applies to the relevant activity class. Such an application however is very broad-based and assumes that every billet within a class offers the same level of satisfaction or dissatisfaction. For example, if shore OCONUS billets are generally regarded as being difficult to fill, then all shore OCONUS billets would be offered the same level of incentive even though some billets may in fact be regarded as desirable. For the desirable billets within the shore OCONUS

class, it would not be necessary to offer any incentive; to do so using a broad-based application would be inefficient and wasteful. A more appropriate alternative is to target the incentives to particular billets.

Targeting incentives to individual billets allows each and every billet in the Navy to have its own incentive level. It is hoped that the majority of billets would not require any incentive, while some would require modest amounts and others more substantial incentives, as the situation dictates.

Having decided that targeted incentives are more appropriate, it is necessary to determine the level of incentive to be set for each billet. From an economic perspective, it is preferable to minimize the constraints on the system and allow market forces to determine the levels of incentives. However, while strong arguments exist for such a free market system, consideration must be given to the culture and work situation of the organization to which the incentives are being applied. As a result, for the reasons outlined below, there are strong arguments for restricting the flexibility of the system:

- Any system to be implemented must be easily understood by the sailors who use it. If incentive levels frequently vary by considerable amounts, the sailors must consider the impact of stating their preference now as opposed to some time in the future. For example, if a billet becomes available and it appears that it is unlikely to be filled in the first requisition cycles in which it is offered, a sailor must consider if it is better to wait until later requisition cycles and hope that the offered incentive increases as the Navy becomes more desperate to fill the billet.

- If incentives are varied frequently, it may be necessary for sailors to update their preferences on a daily basis. This would in turn require sailors to have more frequent access to the latest information, and would increasingly distract them from their work.
- If the amount of incentive offered for a particular type of billet is allowed to vary considerably over time, there is the likelihood that two sailors doing identical work within the same command would be paid different incentive levels because they arrived at different times. From a horizontal equity perspective, this is not a desirable outcome.

For the reasons described above, it is preferable that incentives do not vary by significant amounts over time.

### **3. Setting Incentive Levels**

It is envisioned that a staff of manpower planners would determine the necessary incentive levels based on a variety of factors. This approach would allow the Navy's manpower planners to consider the degree of difficulty in filling billets, and apply incentives, of varying amounts, to those billets. For example, a billet that is moderately difficult to fill could offer a cash incentive of \$100 per month, while billets that are consistently difficult to fill could offer a cash incentive of \$600 per month. Therefore, within guidelines, setting incentives for the various billets would be at the discretion of Navy planners. These planners could work within a constrained budget and seek to fill the maximum number of billets, on a prioritized basis, for the budget available.

Possibly the greatest difficulty would be initially implementing the process, when it is necessary to determine the initial incentives for the various difficult-to-fill billets. If

the initial levels are set incorrectly and require significant changes, then the equity concerns raised previously will be a problem. To initially determine appropriate incentive levels, sailors may be surveyed to ask what monthly payment they would require to place a particular billet as first preference. That is, a sailor considers his or her most favored billet that does not offer any cash incentive and determines the level of satisfaction that would be obtained by going to that billet. The sailor then considers various undesirable billets, and determines, for each in turn, the cash payment necessary to generate a level of satisfaction equal to his or her most preferred billet. That is, what cash payment is necessary to make the undesirable billet the first ranked preference?

Based on survey results, it is not necessary for the incentive to be set so high that the majority of sailors apply. Rather, the incentive should be set at a level where, on average, a reasonable number of sailors will apply for the billet as first preference, with other sailors listing the billet as a lower preference. For example, assume 20 sailors indicate that they would list a billet as first preference if the incentive was \$300 per month, and the remainder of sailors, who are suitably qualified for the billet, indicate various bonuses above \$300. If the incentive is set at \$300 per month, 20 sailors could be expected to list the billet as their first preference. Another sailor, who indicated that a payment of \$400 per month would be required to induce him or her to list the billet as first preference, may be willing to list the billet as a fifth preference, for example.

Once survey results are obtained, the planning staff would determine a reasonable first estimate of the incentives to set for each billet. These estimates, however, should undergo further testing and verification to confirm how a sample group of sailors would rank the various billets when the estimated incentives are applied. It is necessary to return

to the sailors to verify the estimates. To illustrate, the sailors were initially asked what payment would be required to encourage them to list a billet as a first preference, when compared to their most preferred non-incentive-paying billet. Suppose a sailor indicated payments of \$300 for Billet A and \$400 for Billet B, but the planning staff decided to provide incentives of \$400 and \$450 to billets A and B, respectively. The sailor is no longer willing to list Billet B as first preference since Billets A and B provide economic rent of \$100 and \$50 per month respectively, and assuming the sailor is rational, he or she will choose the billet offering the higher economic rent. Therefore, because a sailor may obtain varying levels of economic rent from the various billets offering incentives, the number of sailors that place a billet as first preference may differ from what is expected from the initial survey results. For such reasons, considerable testing and verification of the incentive levels should be conducted prior to implementation.

Once a best estimate of the necessary incentives are determined, the incentives may be applied to billets. Once applied there are a variety of options as to how incentives may vary over time. One option would be to conduct an annual review of incentives and increase or decrease the incentives based on the degree of difficulty of filling the billets. While such a process ensures a relatively constant incentive level and addresses the behavioral and equity issues discussed previously, it does not allow variation in situations where a priority billet remains vacant over a number of requisition cycles. An alternative approach may be to set annual limits for the various billets, but allow small increases to the incentive if the billet is a high priority and remains unfilled after say three requisition cycles. In such situations, raising the incentive by small amounts may be sufficient to obtain more volunteers and therefore enable the two-sided-matching process to remain

voluntary. Furthermore, provided the increments are minimal and do not occur frequently, the previously described disadvantages of a free market system may not be encountered.

## **V. CONCLUSIONS AND RECOMMENDATIONS**

### **A. CONCLUSIONS**

#### **1. The Situation**

This thesis provides an indication of the complex task facing the Navy's detailers. Approximately 294 detailers are responsible for managing the detailing process of a force that comprises 314,450 active duty sailors. In addition to sailors' preferences over each command's attributes, and the commands' preferences over each sailor's attributes, the detailers must consider eligibility criteria as well as numerous policies pertaining to the assignment of sailors. This creates an incredibly complex task whereby detailers must tradeoff the various preferences. Furthermore, because no standard procedure has been identified to assign sailors to commands in an optimizing manner, the quality of the assignments is largely dependent on the ability of detailers to mentally tradeoff sailors' and commands' preferences.

#### **2. Two-Sided Matching Theory**

In response to the concerns raised above regarding the complexity of the assignment process, this thesis examines two-sided matching processes as an alternative approach for assigning sailors to commands. These processes systematically match agents from two distinct groups based on the preferences of each. Two-sided matching processes are employed in a variety of situations. In each situation, participation in the process is voluntary. It may therefore be assumed that all agents who participate in the process find the match results acceptable, for otherwise the process would not continue operation due to the voluntary nature. This relates to the concept of stability, a key concept in two-sided matching procedures. The two-sided matching procedures examined

in this thesis guarantee stable matches. That is, when provided with the results from the matching procedure, stability ensures that there will be no participants able to find mutually preferable matches.

This thesis examines one-to-one and many-to-one matching procedures and explores the issues pertaining to the application of these procedures to the Navy's assignment process. Although the two procedures are quite similar in many aspects, it is determined that many-to-one matching procedures are likely to be more appropriate to the Navy's situation due to strategic differences between the two processes.

The potentially different results obtainable from selecting either the command or sailor optimal procedure were discussed. No recommendations are made as to whether the command or sailor optimal procedure should be used, as the issues relating to this are beyond the scope of this thesis. However, reference to a process that seeks to maximize the average satisfaction of both sailors and commands is provided.

### **3. Application of Two-Sided Matching**

Sailors are currently able to indicate their desires for various billets over the Internet using the Job Advertising Selection System (JASS). If a two-sided matching process is implemented as a Web-based means for assigning sailors, sailors would be able to indicate their preferences in a similar manner to the current JASS system. However, no process currently exists for commands to indicate their preferences over sailors. This thesis provides a potential system whereby commands could specify various sailor attributes, and then indicate the relative importance of each attribute. Each sailor would be scored against the variety of attributes, with the final preference ranking being derived from the score each sailor receives for each attribute.

#### **4. The Requirement to Modify the Two-Sided Matching Process**

If a two-sided matching procedure is applied to the Navy's assignment process, there are a number of differences between the Navy's situation and other markets where two-sided matching is employed. Due to these differences, the two-sided matching procedures can not be directly applied to the Navy's assignment process. Therefore, changes to the matching procedure may be required for the following reasons:

1. The fixed assignment period will create instability for sailors and billets. This occurs because sailors (billets) who prefer billets (sailors) that are available and seeking a match, are unable to seek the preferred billet (sailor) because of the fixed duration of their existing assignment.
2. The impact of repeated matching cycles on the incentive behavior is unknown. Currently, the Navy's assignment process is repeated every two weeks, with various billets and sailors remaining in the process for subsequent assignment periods. The impacts of applying a two-sided matching process every two weeks without clearing the market is unknown. The existence of multiple periods of opportunity may result in undesirable gaming behavior from sailors.
3. Two-sided matching procedures are not able to ensure that all agents from one side of the market receive matches. Therefore, it is not possible to ensure that all sailors receive matches.
4. Two-sided matching procedures do not have integral mechanisms to ensure priority billets are filled.

5. In many situations, the Navy's current assignment process is not voluntary. That is, although sailors may indicate their preferences for various billets, sailors are often required to accept undesirable billets. Since one of the central concepts of two-sided matching is the voluntary aspect of the procedure, if the Navy continues to enforce undesirable matches, the stability concept will be invalidated and the process weakened.

### **5. Modifications to the Two-Sided Matching Process**

Based on the reasons detailed above, this thesis considers implementing the assignment process and ways in which it would be necessary or desirable to modify the assignment process. Some of the suggested modifications to the process include:

1. Changing the frequency with which assignments occur. This would potentially increase the quality of assignments made due to the greater pool of sailors and billets being considered at each assignment round.
2. Allowing sailors flexibility in the time at which they move between billets. This will increase the level of control sailors have over their careers as well as decreasing the instability that would exist if fixed duration assignments continue.
3. Ensuring priority billets are filled. This thesis explored various approaches to filling priority billets, including using a conventional matching process but initially including only priority billets, a conventional matching process that includes all billets but with the results manipulated after

completing the matching process, or relying on the multiple windows of opportunity to obtain a match.

4. Providing incentives to encourage sailors to apply for the less desirable billets. This would ensure the two-sided matching process remains voluntary as well as increasing the satisfaction of sailors. A discussion of the rationale for applying incentives, the desired qualities of the incentives and means of implementing the incentives are included.

## **6. Benefits of Implementing Two-Sided Matching**

There are potentially many benefits to be obtained from implementing a two-sided matching process to assign sailors within the Navy. First, the system would considerably reduce the number of detailers required to assign sailors, which would allow considerable cost savings. Second, the algorithm is able to more systematically and accurately tradeoff the needs and preferences of commands and sailors. This will enable better quality matches for sailors and commands when considered as a whole. Third, the two-sided matching process is not subject to any personal bias since the matches are based entirely on the preferences of sailors and commands, not on the contacts that a person has. Finally, integrating an incentive system into the process will provide the Navy a means of obtaining volunteers for difficult-to-fill positions, while increasing the satisfaction of the sailors filling those billets.

## **B. RECOMMENDATIONS**

This thesis provides a two-sided matching process as a potential alternative process for assigning the Navy's active duty sailors. Implementing such a process requires various modifications, primarily in applying the process. Rigorous testing of the

proposed process and its implementation is required to confirm the optimal manner in which the process should be implemented as well as to prove the benefits to the Navy's decision-makers.

It is recommended that experimentation and simulation be undertaken to verify the following:

1. The potential improvement in match quality that may be obtained from utilizing a two-sided matching process. Other thesis work being undertaken concurrently with this thesis seeks to explore the improvements in match quality by using experimental subjects in a simplified scenario. More realistic research in this area is warranted. This could conceivably be conducted by obtaining the results of assignments by enlisted detailers and comparing this to the results obtainable if the two-sided matching process were used. This will require collecting detailed information on the attributes that sailors and commands value, and the relative importance each attaches to these attributes. This is required so that a utility function may be specified to create preference lists for the algorithm as well as for evaluating the quality of the results.
2. The variation in match quality obtained by varying the frequency with which assignments are made.
3. The impact of discontinuing current fixed term assignment duration and allowing sailors the flexibility to negotiate new assignments at more frequent intervals. This thesis proposed three alternative approaches to this

process and each of these should be considered in both experimental and simulation environments.

4. The impact of multiple assignment rounds. Simulations could determine whether multiple assignment rounds ensure all sailors receive matches and whether priority billets are filled as required. Experimentation would be required to determine if multiple assignment rounds would introduce any undesirable gaming behavior from sailors.
5. The optimal method to ensure priority billets are matched. This thesis discussed three methods and each of these requires further exploration to determine the optimal method.
6. An analysis of various methods by which commands may specify their preferences over sailors.
7. The optimal method for introducing incentives into the process. This would cover the following aspects; the ability of incentives to entice sailors to accept otherwise undesirable billets, the levels of incentives sailors would require to modify their preferences and the preferred means of applying incentives. Much of this work would require surveys of sailors to determine the levels of incentives required for various billets.
8. The preferred manner for assigning married couples. As an initial approach, simulation should be undertaken to determine the results obtainable in the Navy assignment context if the NIMP solution to this problem, as discussed in Chapter II, is implemented.

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