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Modeling the Operation of a Platoon of Amphibious Vehicles for Support of Operational Test and Evaluation (OT&E)

by

Donald P. Gaver
Patricia A. Jacobs
Jesse A. Kemp

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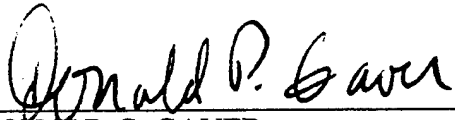
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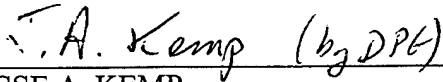
This report was prepared by:



DONALD G. GAVER
Distinguished Professor of
Operations Research

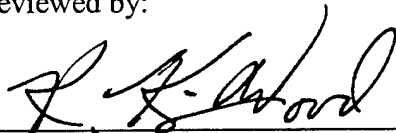


PATRICIA A. JACOBS
Professor of Operations Research



JESSE A. KEMP
Captain, U.S. Marine Corps

Reviewed by:



R. KEVIN WOOD
Associate Chairman for Research
Department of Operations Research

Released by:



JAMES N. EAGLE
Chairman
Department of Operations Research



DAVID W. NETZER
Associate Provost and Dean of Research

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Modeling the Operation of a Platoon of Amphibious Vehicles for Support of Operational Test and Evaluation (OT&E)

Donald P. Gaver
Distinguished Professor of
Operations Research
Operations Research Department
Naval Postgraduate School
Monterey, California
dgaver@nps.navy.mil

Patricia A. Jacobs
Professor of Operations Research
Operations Research Department
Naval Postgraduate School
Monterey, California
pajacobs@nps.navy.mil

Jesse A. Kemp
Captain, U. S. Marine Corps
Operational Logistics Curriculum
Naval Postgraduate School
Monterey, California
jakemp@nps.navy.mil

Abstract

This report documents an analytical model to assist in the planning of the Operational Test and Evaluation (OT&E) of the Marine Corps' prospective Advanced Amphibious Assault Vehicle (AAAV). The model's emphasis is on suitability issues such as Operational Availability in an on-land (after ocean transit) mission region. The model predicts strong sensitivity to the *form* of an assumed distribution of times to vehicle breakdown, and hence recommends that appropriate test data be obtained to reveal that form (the *mean* alone is inadequate). Removal of design faults likely to cause early failures is encouraged.

The AAAV design is for a relatively lightweight but technologically advanced (mobile and lethal) amphibious vehicle that operates in platoons. If individual vehicles break down they must often be *transported* for repair, for example, to their point of origin, a ship. One option is to allow another platoon member to tow; another is to assign an auxiliary vehicle to transport. Such duties may seriously diminish the platoon productive mission availability (the towing/transport agents may themselves fail). OT&E should be designed to fully test for the *system-wide effect* of force (platoon, and beyond) sustainability requirements.

Executive Summary

This report documents dynamical and probabilistic models to assist in planning the Operational Testing of the Marine Corps' Advanced Amphibious Assault Vehicle (AAAV). The model is programmed in rapidly executable software available from the authors; it runs on a personal computer (PC to readers of this report). The software allows an analyst to quantitatively study the sensitivity of the operational availability of one (or more) platoons, each of 12 vehicles. Among sensitive features or properties are:

- The statistical nature (inferred distribution, *not* just the "mean") of the times from a repair completion to subsequent failure. For sensitivity to distribution *form* see Sections 4 and 6.
- The times to repair failed vehicles (including the often significant time and resources required to transport those vehicles to the repair facilities).
- The numbers, types, and concepts of employment (COE) of the necessary support vehicles (e.g., landing craft and helicopters).

The motivation is this: modern military equipments tend to be designed to be lighter, more transportable, and quickly mobile, but also more combat effective than their predecessors. An example is the Marine Corps' AAAV, which is intended to replace the present Amphibious Assault Vehicle (AAV). Although it is smaller than the AAV, the new AAAV is faster and more technologically advanced. It possesses more sophisticated sensors and more lethal weaponry and is expected to be more combat effective than the present AAV.

Such technological improvements are, however, likely to be characterized, at least initially, by increased fragility, and certainly less self-contained on-board sustainment (repair) capability. For the AAAV, the advanced technology that results in greater speed and lethality comes at the expense of external support requirements in case of onboard equipment failures (e.g., of sensors and communications gear), and particularly in case of mechanical breakdowns. In general, Operational Testing of new, sophisticated, potentially highly capable and effective, *but* potentially failure-prone assets should be done in the context of their entire essential support system. Such tests, if done entirely in the field, are costly and time-consuming at best. Consequently, a trustworthy, model-based preview of the entire system in operational action should be of great suggestive value. Focused, smaller-scale tests along with appropriate data acquisition and analysis, can be used iteratively to parameterize overall models; such Model-Test-Model philosophy and practice promise to enlighten decision-makers' judgments concerning the new system's likely operational contribution. This report describes such an approach, and quantitative results from them.

1. Problem Setting

The purpose of this paper is to formulate a model for the operational analysis of a group of mobile amphibious vehicles that co-operate on-land in a remote location, having been launched from the sea (littoral/ocean). The specific application is to preview and extend or enhance Operational Test and Evaluation of the Marine Corps' Advanced Amphibious Assault Vehicle (AAAV), but the model capability and usefulness extends beyond that objective. In particular, the model can be used to analyze logistical and maintenance support required to carry out an operation in a timely manner. The model is implemented in software, obtainable from the authors, which is executable on a personal computer. The software is used to illuminate logistical and maintenance issues.

Suppose a platoon of n (e.g., $n = 12$) nominally identical *amphibious vehicles* (AV here an abbreviation for the AAAV, but also for other vehicle types), begins at time $t = 0$, to traverse an ocean or littoral environment towards a *beach/shoreline* (B) a distance d (miles) away from a *mother ship* (MS) source. Operating at high speed, the vehicles are designed to travel at speed V_T (in miles per hour, which may vary, depending on sea state, etc.). The *ideal* is that they reach their first destination, B, without interruption; this ideal is unrealistic, so there is subsequent discussion of failures and recovery during transit and their impact on arrival time at the B. At this point, they prepare to become land vehicles by activating a tread system and effectively becoming the rough, functional equivalent of Army Armored Personnel Carriers (APCs). Their individual payloads are platoons of Marines. The AVs are of smaller capacity than APCs, and are less well armored, but carry more sophisticated payloads (sensors, communications, and weapons). They carry out operational assignments on land, and eventually return to a destination and begin again. See Buckles (1999) for further discussion and references.

1.1 Reliability Considerations

Realistically, the AV is made up of failure-prone subsystems: propulsion, drive train, navigation, communications, weapons, etc.; and each subsystem may well contain one or more *design faults* that cause failure. Failures of any of these subsystems in transit degrades the mission capability of the AV, and hence of the AV platoon.

The impact of some failure modes differs from others: certain failures of the engine or drive train, or other major propulsion system elements, incapacitate a platform, rendering it immobile or “quiescent” (“dead in the water”), and require the platform’s removal to a maintenance facility. The Marine passengers are also transferred off during such a phase. Other failures, for instance of a sensor or part of the navigation system, may allow the platform to keep up with the remainder of the platoon, but in less than fully capable condition. A quiescent period may end after a delay when an auxiliary vehicle delivers spare parts. The present sophisticated design of an AV platform appears to make a great many different failures potentially possible, or even likely, *and* to require off-platform assistance for rectifying such failures. *Thus, the failure-recovery subsystem cannot be ignored, and should itself be tested as part of the overall system.* Runs of the proposed models will help to indicate the effects of unreliability upon overall end-to-end operational availability. Operational data acquisition during Operational Testing (OT) can thus be suggested.

1.2 Purpose of the Paper

The major objective of this paper is to reveal the sensitivities of overall AV system operational effectiveness to failure events and alternative concepts of a recovery-repair operation. It will be seen that the basic system of platforms, a platoon, is one of potentially mutually *dependent* subsystems, the platforms themselves *plus* auxiliary support. It can also be seen that if the basic system is realistically expanded to include support agents such as landing craft and *helicopters*, that themselves have competing operational functions, then the major surface and airborne subsystems can exhibit strong,

interrelated dynamics caused by reliability-maintainability system properties and requirements. Thus, careful acquisition and analysis of OT data is essential to show that there is an acceptable overall system performance when such an interdependent system is finally delivered to the ultimate users.

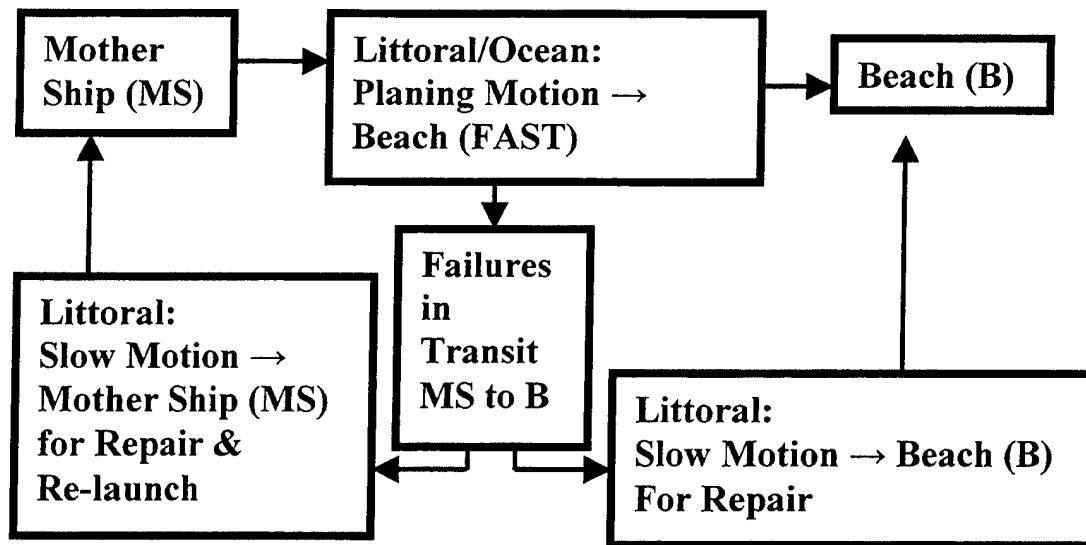
Since it is unlikely that an entire platoon of AVs and its support system will be operationally tested, it is important that evaluation be supported by comprehensive and adequate system-levels models, and that these models be parameterized in a manner consistent with field experience.

2. Model Outline

Preliminary simplified models are presented, in which numbers of individual platforms in various conditions, so-called *states*, are represented as functions of each other, and of elapsed time measured from platoon launch from the MS. The basic structure of the initial model (D-1) is chosen to illustrate a plausible, dynamic version that can be interpreted as a rough "expected value" (or deterministic fluid) model. A "better" model would be stochastic, i.e., crafted to explicitly represent variability in, say, the times between platform failures and times to repair failed platforms. A simplified, analytically tractable stochastic model (S-1) is provided here. A related, more detailed model, to be investigated by computer simulation (Monte Carlo), will be the subject of a Modeling and Simulation (M&S) OR thesis by Capt Jesse A. Kemp, USMC.

Next follows a flow chart for the transitions of platforms/vehicles among their various states, as recognized in the quite simplified models (D-1) and (S-1). *Note:* In (D-1) we ignore the effects of explicit distance or range from the MS. Spatial considerations will be brought in subsequently, and are explicitly present in the Kemp thesis.

2.1 Flow Chart



2.2 State Variables

$A(t)$: Number of active, mission-capable vehicles in the ocean-littoral transitioning at high surface (planing) speed across water at time t .

$L(t)$: Number of *unavailable* (i.e., in transport service) auxiliary vehicles, e.g., landing craft of type LCU(R) (here abbreviated LC), in the Littoral ocean at time t . *Note*: Available LCs number $L - L(t)$ for the present, L being the total number available. There are *other types* of LC—the LCAs—that are *not* capable of towing broken-down AVs. They are not considered at this point.

$Q_T(t)$: Number of *quiescent* AVs (e.g., immobile or “dead in the water”), and those not fully mission-capable at time t . These are destined to be transported, or otherwise proceed at low speed, to either the MS or the B for repair.

$R_A(t)$: Number of AVs under transport by a companion AV towards the MS (“in reverse”) at t , (note that $R_A(t)$ counts the number of vehicles in *pairs*

(the tow-er “alive,” the tow-ee “dead”). Also note that this number *can always be zero if, by edict, AVs do not tow.*

$R_L(t)$: Number of AVs under transport by an auxiliary agent (e.g., landing craft, such as an LCU(R)) towards the MS.

$F_A(t)$: Number of AVs under transport (here tow) by another AV towards the B (“forward”) at t (again, this counts individuals in pairs). Note that this number can be zero. AVs *may* not tow, or may do so only if LCs (LCUs or LCU(R)s) are all busy transporting other failed AVs, or are engaging in other activities.

$F_L(t)$: Number of AVs under transport by an auxiliary agent (e.g., landing craft) towards the B.

$M_S(t)$: Number of AVs undergoing or awaiting (queuing for) maintenance on the MS at t .

$M_B(t)$: Number of AVs undergoing or awaiting maintenance on the B at t .

$B(t)$: Number of operational AV platforms accumulated on the B at t . Some have arrived without failure (perhaps after being towed to the MS, repaired and relaunched); some have experienced maintenance on the B after arriving there.

2.3 Parameters

Here are parameters minimally describing the dynamic evolution of the system of platforms, as it transitions (planes) from the MS to the B.

λ : Failure rate per platform while running at fast speed; $1/\lambda$ is the “mean time to failure” (MTTF), measured from any time point at which a platform is up and running. (Units are $1/\text{time} = \text{e.g., } 1/\text{hours}$). *Note:* The value of λ may depend upon speed, sea state, condition of the littoral ocean environment, and other explanatory variables; this can be written as

$\lambda(\bullet)$. Further, a model that permits *infant mortality* (early post-launch failure) can easily be adapted from the present setup. Exercise of the resulting model will show the possible considerable sensitivity of outcomes to a natural departure from the usual constant-failure-rate (exponential) assumption.

p_i : Probability of “infant failure,” meaning that a platform fails after launch, to attain planing speed. It, therefore, immediately joins a queue of AVs awaiting maintenance at the MS.

v_{PA} : Rate at which quiescent/failed AVs are picked up (placed under transport, i.e., towed) by fellow AVs (this can be omitted/made zero, thus eliminating the AV-tow option by setting $v_{PA} = 0$). (Units are 1/time = e.g., 1/hours).

v_{PL} : Rate at which quiescent/failed vehicles are picked up (placed under transport, i.e., towed) by auxiliary vehicles (e.g., LCs, such as the LCU(R)).

$v_P = v_{PA} + v_{PL}$: Total rate of tow-er pickup by tow-ee.

v_T : Rate at which individual platforms reach the B; $1/v_T$ is the mean time to transition to the B while in planing motion (given that no failure occurs). (Units are 1/time = e.g., 1/hours). v_T can be set equal to V_T/d to calibrate the present model to one with deterministic motion, distance to the beach d , speed velocity V_T , and no failures (see Section 4 for related discussion).

$\mu_S(\bullet)$: Rate at which the MS maintenance facilities can return a failed platform to fully active capability.

Note 1: For the present, MS maintenance is treated as a saturable, single-server repair system.

Note 2: When there is infant mortality, i.e., $p_i > 0$, and a platform fails essentially just after being launched, then the *effective repair rate*, $\mu_s(\bullet)$, equals an actual repair rate multiplied by the probability that no infant failure occurs:

$$\mu_s(\bullet) = (1 - p_i)\mu_s^\#(\bullet);$$

in other words, the effective mean repair time (effective repair time random variable is denoted by R_{eff}) with infant failures present, with probability p_i , is

$$E[R_{eff}] = E[R] / (1 - p_i).$$

This is because an average of $1/(1 - p_i)$ repair times must be performed before the infant-failure mode is avoided (the present model does not allow for that mode's permanent removal; see Gaver *et al.* (2000) for plausible reliability growth alternatives).

Further Note: The “•” notation refers to the influence of other variables, such as feedback, to enhance service if a big backlog develops. The default in all cases is a constant parameter value.

- $\mu_B(\bullet)$: Maintenance rate at the B.
- $p_{AR}(\bullet)$: Probability that a failed vehicle is towed to the MS (reverse) by a fellow AV (two AVs in complex, one “alive” and one “dead”).
- $p_{AB}(\bullet)$: Probability that a failed vehicle is towed to the B by a fellow AV.
Note: $p_{AR}(\bullet) + p_{AB}(\bullet) = 1$.
- $p_{LR}(\bullet)$: Probability that a failed vehicle is transported to the MS (reverse) by an auxiliary vehicle (e.g., LC).
- $p_{LB}(\bullet)$: Probability that a failed vehicle is transported to the B by an auxiliary vehicle (e.g., LC). *Note:* $p_{LR}(\bullet) + p_{LB}(\bullet) = 1$.

- v_{RA} : Rate at which a towed vehicle complex (two AVs, one “alive” and one “dead”) reaches the MS; $1/v_{RA}$ is the mean time for the towed complex to reach the MS.
- v_{RL} : Same, when transport is by auxiliary vehicles (e.g., LC).
- v_{BA} : Rate at which a towed vehicle complex reaches the B; $1/v_{BA}$ is the mean time for the towed complex to reach the B.
- v_{BL} : Same, when transport is by auxiliary vehicles (e.g., landing craft).
- L : Total number of landing craft used for AV recovery (LCU(R)s, not to be confused with LCACs) available in the ocean littoral (assume fixed for present).

Simplification 1. *Pyramiding/cascading failure possibilities are not modeled here.* If a failed and quiescent vehicle must be transported or towed *without failure*, it is presumed transported or towed to the MS or to the B, depending upon which destination is perceived as the best from the viewpoint of total vehicle soonest availability at the B waypoint. The problem is potentially made more complex by the possibility that previously failed towed vehicle complexes (e.g., two AVs, or an AV and LC, may be ahead of a new breakdown, and hence virtually queued up ahead of that breakdown in time). Consequently, a breakdown that occurs near the MS might still be advantageously towed *to the B* if there are other (higher priority) transported or towed complexes between the current breakdown and the MS.

Simplification 2. It is assumed here that *a transported or towed complex does not break down between its origin point and destination* (MS or B). This is optimistic, and calls for future amendment, although the model then becomes more complex and difficult to manage. Failures that are not total breakdowns can be handled in terms of the current model.

Simplification 3. For the purposes of this discussion it will simply be assumed that if a breakdown/failure occurs it is towed to the MS with probability $p_{\cdot R}$, and to the B with probability $p_{\cdot B}$, where “ \cdot ” refers to the tow-er type $p_{AR(\cdot)}$, ..., $p_{LB(\cdot)}$ (see 2.3). A convenient, but very preliminary assumption is that these probabilities are 1/2. However, it is clear that more realistic probabilities should depend minimally on the *time* that has elapsed since platoon launch from the MS; the probabilities can also be made to depend upon repair backlogs and delays, possibly due to past availability at the two possible destinations. In general, a more refined description of the system state should lead to a better decision, but at a cost in model complexity.

Clearly the decision as to where to tow the failed AV should ultimately depend upon how soon it can be in service near (e.g., inland of) the B.

Simplification 4. The mean time to be towed to either the MS or the B *should* depend systematically and predictably upon the actual breakdown *location*, although in practice there may be great variability. As a surrogate, the mean time could depend on elapsed time, t ; presumably as elapsed time t increases, the distance of the platoon from B decreases (*but not* if early breakdowns persistently occur near the MS, since the repair facility there may become overloaded). Our present assumption of a *constant mean* time is thus crude, since it merely represents an order-of-magnitude delay. The assumption can be relaxed by segmenting the Littoral region into several range bands from the MS to the B (and beyond), and counting platforms in various states in each. This refinement proliferates the state equations, and is left for a second round of modeling. It will subsequently be shown that the simple Markovian type of fluid model presented here, is in quite good agreement with a more limited, but stochastic, model.

Many of the simplifications mentioned have been avoided in a Monte Carlo computer simulation model implemented by Capt Jesse. A. Kemp (USMC) as part of his Masters thesis in Operations Research.

3. Deterministic Analytical (Mathematical) Model (D-1)

Here is a set of dynamic stock-and-flow differential equations that describe the time-dependent numbers of AV platforms in the various states. The present model "stops at the beach," i.e., at the B. The model is continued/amended in Section 4, to include the on-land mission segment.

$$\begin{aligned}
 \underbrace{\frac{dA(t)}{dt}}_{\text{Rate of "increase" of number of active AVs in the Littoral}} &= - \underbrace{\lambda A(t)}_{\text{Rate of failure of active AVs}} - \underbrace{v_{PA} Q_T(t) A(t)}_{\text{Rate of tow initiation for quiescent (broken-down) AVs}} - \underbrace{v_T A(t)}_{\text{Rate of "free" active AVs reaching B}} \\
 &+ \underbrace{\mu_S(\bullet) \frac{M_S(t)}{1 + M_S(t)}}_{\text{Rate at which AVs under maintenance at MS become active in the Littoral}} + \underbrace{v_{RA} R_A(t)}_{\text{Rate at which AVs under tow by fellow AVs reach the MS (reverse path); tow-er released immediately}}
 \end{aligned} \tag{3.1}$$

$$\begin{aligned}
 \underbrace{\frac{dQ_T(t)}{dt}}_{\text{Rate of "increase" of number of quiescent AVs awaiting tow}} &= \underbrace{\lambda A(t)}_{\text{Rate of towable-failure events}} - \underbrace{v_{PA} Q_T(t) A(t)}_{\text{Rate of tow initiation by active AVs}} - \underbrace{v_{PL} Q_T(t) [L - L(t)]^+}_{\text{Rate of removal initiation by available auxiliary vehicles (e.g., LCs)}}
 \end{aligned} \tag{3.2}$$

Note: $L(t) = R_L(t) + F_L(t)$. Also, $0 \leq L(t) \leq L$,

$$\text{so } [L - L(t)]^+ = \max[L - L(t), 0]$$

Note: Suitable initial conditions are

$$A(0) = (1 - p_i)n$$

$$M_s(0) = p_i n$$

where n is the number of AVs launched from the MS

$$\begin{aligned}
 \underbrace{\frac{dR_A(t)}{dt}}_{\text{Rate of "increase" of number of AVs (tow-er and tow-ee) being towed ("reverse") to the MS}} &= \underbrace{2p_{AR}(\bullet) v_{PA} Q_T(t) A(t)}_{\text{Rate of initiation of tow (complex of two vehicles) towards the MS}} - \underbrace{v_{RA} R_A(t)}_{\text{Rate of dropoff of AVs for maintenance at the MS}} - \underbrace{v_{RA} R_A(t)}_{\text{Rate of tow-er return to active status, and to hover rate towards the B}}
 \end{aligned} \tag{3.3}$$

Note: When an active vehicle picks up and tows a failed, quiescent vehicle, this means that two vehicles (AVs) form a complex and proceed at rate v_{PA} . Hence, the "2."

$$\underbrace{\frac{dR_L(t)}{dt}}_{\text{Rate of "increase" of the number of AVs being transported to the MS by an auxiliary vehicle}} = \underbrace{p_{LR}(\bullet)v_{PL}Q_T(t)[L-L(t)]^+}_{\text{Rate of pickup of failed AVs by available landing craft: headed for the MS}} - \underbrace{v_{RL}R_L(t)}_{\text{Rate of delivery of landing craft towed AVs to the MS}} \quad (3.4)$$

$$\underbrace{\frac{dF_A(t)}{dt}}_{\text{Rate of "increase" of number of AVs in tow ("forward") to the B}} = \underbrace{2p_{AB}(\bullet)v_{PA}Q_T(t)A(t)}_{\text{Rate of initiation of tow (complex of 2 vehicles) towards the B}} - \underbrace{v_{BA}F_A(t)}_{\text{Rate of dropoff for maintenance at the B}} - \underbrace{v_{BA}F_A(t)}_{\text{Rate of tow-er return to active status}} \quad (3.5)$$

Note: In this case, both tow-er and tow-ee remain at the B, but the tow-ee may enter repair, encountering delay. $F_A(t)$ counts both tow-ers and tow-ees.

$$\underbrace{\frac{dF_L(t)}{dt}}_{\text{Rate of "increase" of number of AVs in tow to B by LC}} = \underbrace{p_{LB}(\bullet)v_{PL}Q_T(t)[L-L(t)]^+}_{\text{Rate of initiation of transport by LC}} - \underbrace{v_{BL}F_L(t)}_{\text{Rate of dropoff of LC tows at B}} \quad (3.6)$$

$$\underbrace{\frac{dM_S(t)}{dt}}_{\text{Rate of "increase" of number of AVs in MS maintenance}} = \underbrace{v_{RA}R_A(t)}_{\text{Rate of dropoff for maintenance at the MS from AV tow}} + \underbrace{v_{RL}R_L(t)}_{\text{Rate of dropoff at MS from LC transport}} - \underbrace{\frac{\mu_S(\bullet)M_S(t)}{1+M_S(t)}}_{\text{Rate of maintenance completion at the MS}} \quad (3.7)$$

$$\underbrace{\frac{dM_B(t)}{dt}}_{\text{Rate of "increase" of number of AVs in B maintenance}} = \underbrace{v_{BA}F_A(t)}_{\text{Rate of AV dropoff for maintenance at the B by AV tow-ers}} + \underbrace{v_{BL}F_L(t)}_{\text{Rate of AV dropoff for maintenance at the B by LC tow-ers}} - \underbrace{\frac{\mu_B(\bullet)M_B(t)}{1+M_B(t)}}_{\text{Rate of maintenance completion at the B}} \quad (3.8)$$

$$\underbrace{\frac{dL(t)}{dt}}_{\text{Rate of increase of LCs unavailable for failed AV transport (in use)}} = - \underbrace{v_{RL}R_L(t)}_{\text{Rate of dropoff of AVs by LCs at the MS}} - \underbrace{v_{BL}F_L(t)}_{\text{Rate of dropoff of AVs by LCs at the B}} + \underbrace{v_{PL}Q_T(t)[L-L(t)]^+}_{\text{Rate of pickup of quiescent/failed AVs by available LCs}} \quad (3.9)$$

$$\underbrace{\frac{dB(t)}{dt}}_{\substack{\text{Rate of "increase" \\ of active, available \\ AVs at the B}}} = \underbrace{\nu_T A(t)}_{\substack{\text{Rate of "free" \\ active AVs \\ reaching the \\ B}} + \underbrace{\nu_{BA} F_A(t)}_{\substack{\text{Rate of active \\ tow-er AVs \\ reaching the \\ B}} + \underbrace{\frac{\mu_B(\bullet) M_B(t)}{1 + M_B(t)}}_{\substack{\text{Rate of maintenance \\ completion of the B}} \quad (3.10)$$

4. The On-Land Mission. Helicopters (For Transport and AV Rescue-Recovery). Model D-2.

An important aspect of Marine-Navy cooperation to dominate and secure a land region "from the sea" is (in present concept) to use AVs and Helicopters (Hs for short), or some other mobile platform, cooperatively. A primary function of an H force is to transport personnel and supplies to inland points. An important secondary role is to support AVs, when the latter fail, while in the on-land mission phase.

We now aim to more explicitly include in the model the contribution of Hs to AV dynamics, and ultimately to AV availability on the B. Conversely, we also study the impact of AV requirements for support on H availability. There will clearly be a tradeoff that can be dependent upon conflict conditions ashore. The present model does not reflect the time and event-dependent character of such needs, but only general trends that result from a consistent control policy.

4.1 Augmentation of the Basic Model to Represent AV Operational Region (On-Land) Availability

The focus here is on AVs that may fail when on land, and can be repaired on-site. This accounts, at least crudely, for the use of Hs to furnish needed parts to a stranded AV.

Now consider the availability of AVs that have eventually and successfully reached the B, and undergone successful repair, if needed. The state variable $B(t)$ of (3.10) counts that number at time t , *provided* none are yet dispatched to carry actual in-land operational missions. But, such are the ultimate objectives of the AVs, so we now add features to the model to represent this phase of the AV operational cycle.

4.2 Additional State Variables

These augment those of Section 2.2 to represent on-land conditions (the subscript O denotes this on-land operational phase throughout):

$A_O(t)$: Number of active, mission-capable vehicles (AVs) on land, transitioning among waypoints at time t . These are assumed to be failure-prone with rate $\lambda_o(\bullet)$.

$Q_O(t)$: Number of failed, quiescent AVs (e.g., in place on land) and not fully mission capable at t . These require recovery assistance; we assume it is furnished on site by helicopters or other support vehicles. Other vehicles (e.g., for transport) can also be accommodated in the model, but are not here.

Let there be h (e.g., five or 10) helicopters (Hs) or other mobile support vehicles exclusively dedicated to assist failed on-land AVs.

Let

$H_o(t)$: Number of Hs *outgoing* to assist failed AVs at t .

$H_i(t)$: Number of Hs *incoming* from having assisted an on-the-spot repairable AV, at t .

Note: It is assumed here that H transit times, both outbound and incoming, are much greater than the time on site, which is taken to be negligible.

$h-H_o(t)-H_i(t)$: Number of Hs available to be sent on AV recovery/assistance missions, at t .

4.3 Parameters

Let

- $\lambda_o(\bullet)$: The failure rate of AVs on land. As before, the notation “ \bullet ” stands for the influence of explanatory variables, such as terrain state and speed of advance. Here the default is a constant.
- $\nu_{H_o}(\bullet)$: Rate at which outbound Hs encounter failed/quiescent AVs on land.
- $\nu_{H_i}(\bullet)$: Rate at which inbound Hs return to H pool, and become available for reassignment to AV recovery.
- $\nu_L(\bullet)$: Rate at which active AVs leave mission region (return to the B or reach a destination); $1/\nu_L$ is the mean mission duration.

Note: The h Hs are considered to be exclusively assigned to provide on-land, on-site recovery assistance to failed AVs. They are themselves assumed immune to crashes and failures (an unrealistically optimistic assumption in hostile territory, but one that can be relaxed). They are also assumed to make only one trip—and that successful—per failed AV.

Let

$m(B(t'), 0 \leq t' \leq t)$: (Denotes) the rate at which available AVs on the B enter on-land mission status at time t .

Here are state equations to describe the on-land mission availability of the AVs.

$$\begin{aligned}
 \underbrace{\frac{dA_o(t)}{dt}} &= \underbrace{m(B(t'), 0 \leq t' \leq t)}_{\text{Rate of mission force creation from those on B}} - \underbrace{\lambda_o(\bullet)A_o(t)}_{\text{Rate of failure of active on-land AVs}} - \underbrace{\nu_L A_o(t)}_{\text{Rate of depletion of AVs in mission region}} \\
 &+ \underbrace{\nu_{H_o}(\bullet)Q_o(t)H_o(t)}_{\text{Rate of H-assisted recovery of failed-quiescent AVs}}
 \end{aligned}
 \tag{4.1}$$

$$\underbrace{\frac{dQ_o(t)}{dt}} = - \underbrace{\nu_{H_o}(\bullet)Q_o(t)H_o(t)}_{\text{Rate of H-assisted recovery of failed-quiescent AVs}} + \underbrace{\lambda_o(\bullet)A_o(t)}_{\text{Rate of failure of active AVs}} \quad (4.2)$$

$$\underbrace{\frac{dH_o(t)}{dt}} = - \underbrace{\nu_{H_o}(\bullet)Q_o(t)H_o(t)}_{\substack{\text{Rate of H-assisted} \\ \text{recovery of failed AVs} \\ \text{(rate of outgoing H} \\ \text{release)}}} + \underbrace{\lambda_o(\bullet)A_o(t)I[h - H_o(t) - H_i(t)]}_{\text{Rate of launch of Hs in response to failures}} \quad (4.3)$$

$$\text{Note: } I[h - H_o(t) - H_i(t)] = \begin{cases} 1 & \text{if } h - H_o(t) - H_i(t) \geq 1; \\ 0 & \text{if } h - H_o(t) - H_i(t) < 1 \end{cases} \quad (4.3a)$$

That is, an H recovery mission begins only if at least one or more Hs are available.

$$\underbrace{\frac{dH_i(t)}{dt}} = \underbrace{\nu_{H_o}(\bullet)Q_o(t)H_o(t)}_{\substack{\text{Rate of H-assisted} \\ \text{recovery of failed AVs} \\ \text{(rate of outgoing H} \\ \text{release)}}} - \underbrace{\nu_{H_i}(\bullet)H_i(t)}_{\text{Rate of return of Hs to base}} \quad (4.4)$$

4.4 AV Beach Accumulation and Land Mission

Although the platoon of n (e.g., $n=12$) AVs departs from the MS essentially as a concentrated group, its members accumulate gradually at the B. In the fluid approximation model used here, the accumulated number on the B *approaches*, but never actually reaches, n ; this is because a continuous “fluid” approximation is being used. We adopt the convention that “all” AVs are effectively at the B when $B(t)$ reaches nf_m , where $0 < f_m \leq 1$, but f_m is taken to be large, e.g., 0.95 or 0.99 (choice of the analyst). At this point, the on-land platoon-strength mission is launched.

Thus, (a) calculate the launch time, t_m :

$$B(t_m) = nf_m;$$

then (b) begin integrating (4.1) at $t = t_m$ and invoke the initial condition $A_o(t_m) = nf_m$, so

Initial Conditions (I)

<u>$0 \leq t \leq t_m$</u>	<u>$t = t_m$</u>	<u>$t > t_m$</u>
$A_o(t) = 0$	$A_o(t) = n f_m$	Solve differential equations
$Q_o(t) = 0$	$Q_o(t) = 0$	
$H_o(t) = 0$	$H_o(t) = 0$	
$H_i(t) = 0$	$H_i(t) = 0$	

4.5 Upper Bound on Number Available During On-Land Mission Segment

In order to obtain a deterministic upper bound on the on-land availability of the platoon, pick $f_m = 1 - 1/2n$, which stops $B(t)$ accumulation when $B(t_m) = n - 1/2$, an approximation to the mean first-passage time to n . Thus, (a) launch a mission at t_m , where

$$B(t_m) = n t_m = n - 1/2,$$

and use the initial condition $A_o(t_m) = n$. Then, (b) solve (4.1) – (4.4). These define Initial Conditions (II).

The two simple initial conditions specified in Subsections 4.4 and 4.5, allow the general term in $(B(t'), 0 \leq t' \leq t)$ in (4.1) to be omitted; integration starts at time t_m with a given level of platoon strength. *Note:* The above specifies a particular and plausible initial condition for the on-hand mission segment of the AV platoon. Other modes of dispatch of the AVs into the on-land environment can be represented by straightforward changes.

4.6 Numerical Illustration of the Sensitivity of Operational Availability to the Form/Shape of the Time-to-Failure Distribution

In this subsection we compare results for an exponential time to failure to those of a mixed failure time distribution, in which there is a probability p_i that the failure time is 0 and a probability $(1-p_i)$ that the failure time is exponential. The exponential and the mixed exponential have the same means, but their *shapes* are different. The deterministic model of Sections 3 and 4 is used. There are 12 AVs in a platoon. Model parameters common to all the cases are as follows:

AVs traveling to the B:

Distance from the MS to the B = 40 nautical miles (nm)

All failed AVs in water are towed back to the MS

AV velocity in water = 25nm/hr

Towing velocity in water = 5nm/hr

If an AV does not fail, the time to reach the B is 1.6 hours

Arrival rate at the B, $v_T = 25/40 = 0.625$

Towing rate back to the MS if failed, $v_{PA} = 5/20 = 0.25$

Stop at the B until 10 of the AVs are available at the B; then the available AVs start the land portion of mission

AVs on Land:

Distance from the B to last destination = 100nm

AV velocity on land = 25nm/hr

Travel time on land without failure = 4 hours

There are five waypoints; an AV pauses 10 minutes at each waypoint

Total time spent at waypoints = 50 min.

AVs are subject to failure at the waypoints

Total time an AV transits on land if no failure = 4.83 hours

Rate at which an AV arrives at land destination, $v_L = 1/4.83 = 0.21$

Helicopter:

Velocity = 50nm/hour

Mean time to fly to failed AV = $(40+(100/2))/50 = 1.8$ hours

There is a two-hour administrative logistic down time (ALDT) prior to each H mission to repair a failed AV on land

Rate of arrival of an H from the MS to a failed AV, $\nu_{H_i} = 1/(2+1.8) = 0.26$

Rate of return of an H to the MS, $\nu_{H_o} = 1/1.8 = 0.56$

There are two Hs

Results for the following AV time to failure models are compared; all models for time to failure for an AV have a mean of 72 hours. Two of the failure-time models have infant failure modes as described in Section 3.

Exponential model:

Mean time to failure=72 hours

Mean repair time on the MS=three hours

Failure rate, $\lambda=1/72=0.014$

Rate of repair completion on the MS= $1/3=0.33$

Initial condition: all 12 AVs are available at time 0

Mixed model 1:

With $p_i = 0.25$ the failure time is 0, and with probability $(1-p_i) = 0.75$ the time to failure is exponential with mean 96 hours; the mean time to failure is 72 hours

Failure rate, $\lambda=1/96 = 0.010$

Rate of repair completion on MS (includes repair extensions due to additional failures at time 0) = $0.75/3 = 0.25$

Initial condition: nine AVs are available at time 0, and three are waiting for or are being repaired.

Mixed model 2:

With $p_i = 1/12$ the failure time is 0, and with probability $(1-p_i) = 11/12$ the time to failure is exponential with mean 78.5; the mean time to failure is 72 hours.

Rate of repair completion on MS (includes repair extensions due to additional failures at time 0) = $(11/12)/3 = 0.31$

Initial condition: 11 AVs are available at time 0, and one is being repaired.

The following figures display the accumulated mean number of AVs that arrive at the B; the mean number of AVs in repair at the MS; and the mean number of AVs in transit on land to the final land destination.

Note: The form of the failure distribution greatly influences the time at which the platoon reaches the final land destination. The assumption of a "pure exponential" distribution tends to be quite optimistic if an "infant failure" type of alternative holds.

Conclusion: OT data acquisition and analysis should allow a propensity for early failures, as compared to the exponential, to reveal itself. Automatic assumption of a simple exponential may be dangerously optimistic. See Section 6 for a similar message.

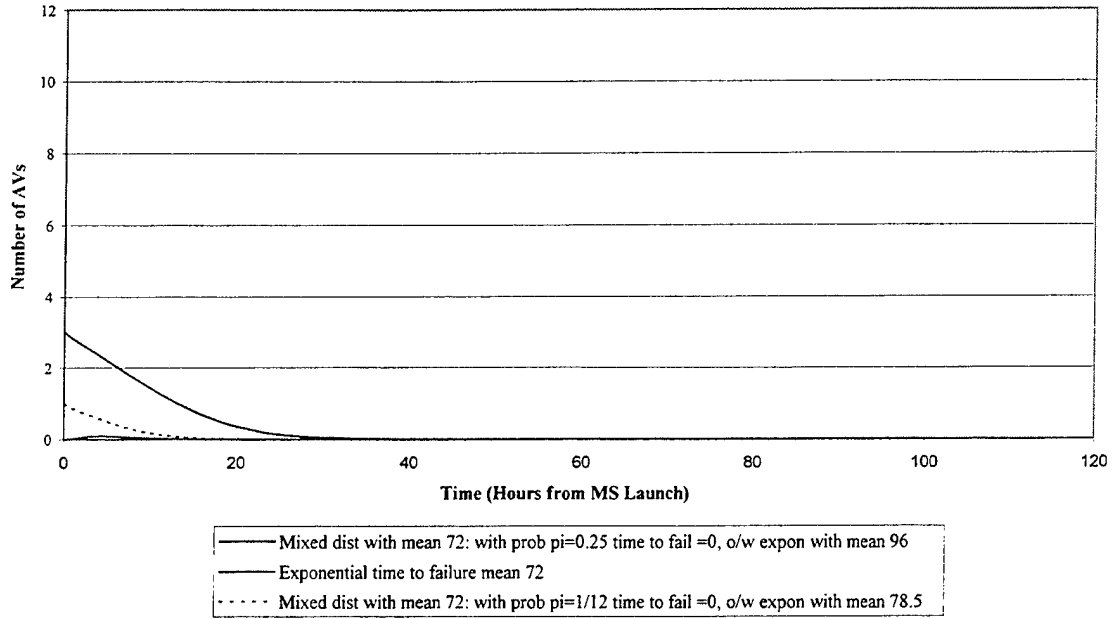


Figure 1.
Number of AVs in Repair at the Mother Ship (MS).

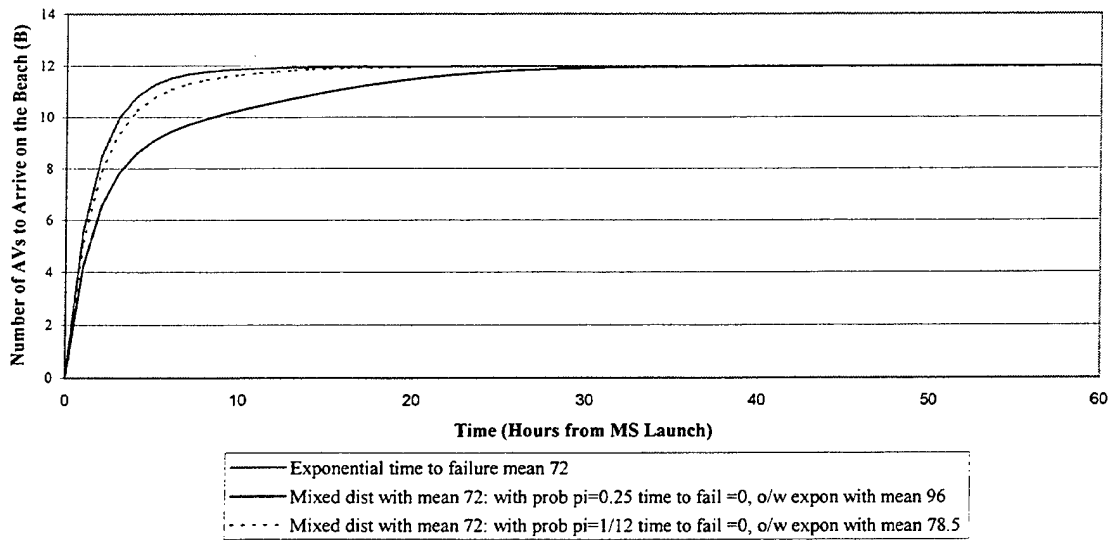


Figure 2.
Cumulative Number of AVs that Arrive on the Beach (B).

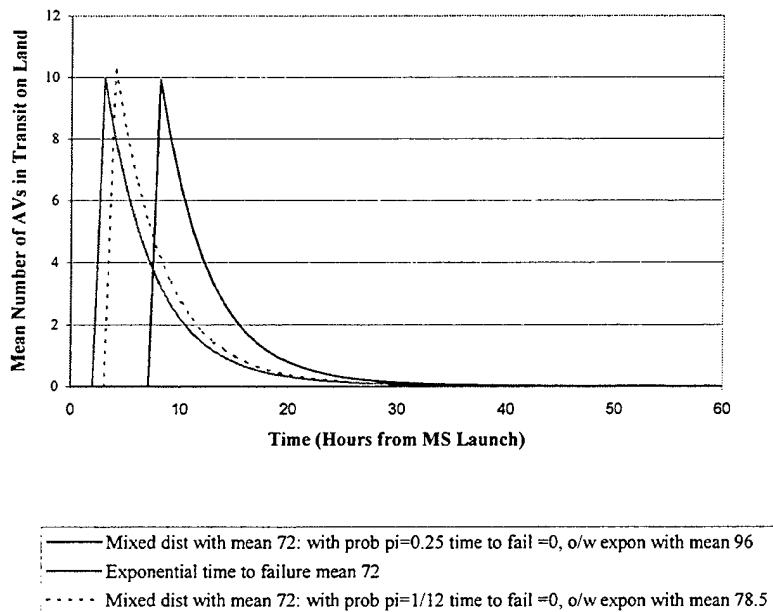


Figure 3.
The Mean Number of AVs in Transit on Land.

5. A Stochastic Model for Platform Transit Time

It is easily possible to extend the previous analytical model to a tractable *stochastic* version that allows for *general* (not just Markov/exponential constant failure rate) random times to failure. Restriction to the situation in which individual platforms operate independently is immediately tractable, and quite informative: it shows quantitatively that the *ultimate transit time to B*, T_d may be quite sensitive to the form of the *time-to-failure distribution*. Facile, knee-jerk-conventional assumption of “the exponential” to describe time to breakdown, T_d , can considerably understate the (expected) time for a platform to ultimately reach the B from the MS.

Consequently, it is important to tabulate, study, and incorporate actual, observed times to platform failure, particularly those that require unanticipated early return to the MS, and subsequent repair and re-launch. An important objective of testing is to identify and remove the causes of such so-called “infant failures.”

It can be seen intuitively that a particular *mean* time to failure can be consistent with (a) a great preponderance of short time values, balanced by some (enough—even one!)

very long observations (sometimes called the infant mortality situation) and (b) an apparently exponential-like scatter, or (c) numerous alternatives. Clearly option (a), with its many repairs and re-starts, can radically extend the time to waypoint B by inducing many failures that must await assistance or return to the MS, experience maintenance and re-launch. An important exception occurs if, after each such “infant” short-time failure, a serious attempt is made to *remove a design fault that causes the failure, and that attempt is successful* (and does not somehow otherwise disrupt the design). This highly reasonable concept of design evolution and testing (“test-analyze-fix-test”) can, it is hoped, result in non-homogeneous time-to-failure data that exhibits a general increase in operating times to failure (so-called “reliability growth”). Certainly the summary of such data by a simple mean, and the invocation of a constant failure rate $\hat{\lambda} = 1/\text{mean}$, are overly simplistic and quite possibly misleading. (*Note:* This indicates that our first model (D-1) of Section 3 must be viewed with caution; however, it can be readily altered to reflect infant mortality, or relatively many premature-compared-to-exponential failures).

As was the case in Model (D-1), some failures are *serious*, and require slow transit, e.g., back to the MS, or forward to the B, for repair. This can be accomplished by towing by a fellow non-disabled AV, or transported by LCU(R), as represented in Model (D-1) in Section 3. In the present “independent” model, the failed AV is assumed to proceed under its own or totally independent power, or with LCU(R) transport assistance, so the AV platoon is not depleted for towing. This represents the situation in which there are always ample LCU(R)s or other vehicles for transport of failed AVs. Its failure-induced slow-return velocity is $V_R \ll V_T$. *Minor* (not serious) failures can, in principle, occur while a seriously failed AV is being returned to the MS, and thus adding the maintenance burden, and hence delay at the MS. An interesting Concept of Employment (COE) policy question is whether to (a) service on-site by LCU(R), minor on-site failures soon after occurrence, without transporting the AV to the MS or the B, or (b) to wait until the AV reaches the MS or the B, and service any *serious* failure and the accumulated minor

casualties at that time. Of course, the same issue arises if serious failures are towed to the B for repair: should auxiliary, e.g., LCU(R), support be used to service minor failures as they occur (or *en route*), or should the service of accumulated minor plus serious failures be delayed until the B is reached? This latter COE reduces the burden on auxiliary vehicles, but might jeopardize partially disabled AV safety, and also potentially submits the auxiliary vehicles to failure. The control policy should depend, in part, upon current need for auxiliary vehicles for other missions, *and* on the maintenance backlog, hence delay at the MS or the B. Logistics (spare parts and diagnostics equipment) requirements must be considered as well, and may be addressed in terms of the current models and natural variations on them. Many such are possible and remain to be analyzed.

5.1 Stochastic Model (S-1)

Here is a *simple* stochastic model for recovery of a remotely failed AV during ocean transit. A single generic platform/vehicle travels at constant velocity V_T (miles/hour) from the MS to the B, a distance d . This distance can be covered in precise time $t = d/V_T$, unless there are interrupts caused by system failures, which we must account for. *Note:* Here we assume that actual time of transit when/if no failures occur is a *constant*, whereas in the previous model that time is *mean* (of an exponential random variable).

Notation

T_d : Random time for a representative platform to reach the B from the MS for the first time, having started at launch (time = 0), and possibly experienced various numbers of failures and repairs *en route*. Discovery of the probabilistic properties of this random variable is the objective of the model. Model runs can then guide analyses of alternative COEs.

T_b : The random time, measured from a launch from the MS until breakdown/failure, while attempting to transit to the B; $P\{T_b > t\} = 1 - F_{T_b}(t = d/V_T)$, where $F_{T_b}(\bullet)$ is the distribution of the

(arbitrary) time to breakdown (under stable design conditions), more generally $F_{T_b}(\bullet; \theta)$, where introducing a generic parameter θ can represent any quantified set of conditioning or explanatory variables, or it can represent a random environmental effect as well. A differently distributed T_b will prevail over land. The present model pertains to the ocean-crossing segment of the transit only.

R_O : Random time to rectify a failure that is susceptible to on-the-spot repair, given spare parts and diagnostic capability. This incorporates the time for, say, LC transit to the failed AV, and even possibly necessary revisits.

R_M : Random time to repair a serious failure that cannot be rectified on-site, i.e., must be removed to MS (here assumed to occur under own propulsion), but at (slow) velocity $V_R \ll V_T$. As will be discussed later, R_M might include repair or minor failures that accumulated during the reverse motion.

(Note: We do not model here the option of a slow progress to the B for repair).

$R_m(T(0)) \equiv R_m(N(T(0)))$: Random time to repair all minor failures ($N(T(0))$ in number) accumulated during generic random transit time $T(0)$. This can represent (a) failures of sensors (FLIR) or communications equipment until breakdown ($T(0)=T_b$), or (b) (any such “minor”) failures accumulated during major-failure-free transit from the MS to the B, in time $t=d/V_T$.

5.2 The Basic Stochastic Model for Time from the MS to the B

A platform/vehicle plans to travel at velocity, V_T from the MS to the B, a distance d . If the vehicle breaks down before covering d , it proceeds at velocity ($V_R \ll V_T$) back to the MS, where it is repaired and re-launched.

Hence

$$T_d = \frac{d}{V_T} + R_m(t) \text{ with probability } 1 - F_{T_b}(t) \equiv \bar{F}_{T_b}(t),$$

where it is assumed that $R_m(t)$ minor failure repairs are

accumulated while in transit without serious failure. These (5.1)

are repaired at the B.

$$= \left(1 + \frac{V_T}{V_R}\right)s + R_M + R_m \left(\left(1 + \frac{V_T}{V_R}\right)s \right) + T_d^\# \text{ with probability } F_{T_b}(ds), 0 \leq s < t$$

where we assume that minor failures occur Poisson-wise, until either trip completion or a serious breakdown occurs, inducing a re-start. *Note:* It has been tacitly assumed here that minor failure repairs at the MS take the same amount of time as at the B. This certainly need not be true: one can specify $R_{m, MS}(t)$ and $R_{m, B}(s)$ as being quite different random processes.

In (5.1), $T_d^\#$ refers to an independent random replica of T_d . If failure-inducing fault removals occur, we obtain a system of such equations; this important feature is omitted for the present.

Also in (5.1), the accumulated minor failure repair times can be modeled as a random sum of independent random variables, where the random number of summands depends upon the time of exposure until either the MS-B transit completes, or a breakdown and reverse transit occurs, starting at T_b . For time of exposure $T(0)=x$, and $\{N(y), 0 \leq y < x\}$ a Poisson process with rate $\eta_m(\bullet)$, and S_j a random minor failure service duration,

$$R_m(x) = \sum_{j=0}^{N(x)} S_j. \quad (5.2)$$

Comments: The above model formulation is illustrative and can readily be changed, although not without a price in additional model complexity. A simpler version would ignore all failures except the major ones, but require auxiliary transport. Such a version

already shows the amplifying impact of in-transit failures on overall measures of transit time, such as its mean, $E[T_d]$. This effect becomes more pronounced if it becomes necessary to repair accumulated minor failures. It has also—and importantly—been assumed that such repairs can be accomplished *flawlessly* during the visit to the MS to repair any major failure, but clearly this may not occur. Also, in this model no attempt is made to explicitly represent delays in queue for repair at the MS. These delays can introduce a substantial increase in the time, T_d , and hence an increase in the time until a platoon of n (here treated as independent and identical units) assembles at the B. The reader is reminded that the stochastic behavior of the individual vehicles (AVs) in a platoon is modeled as independent, which is the consequence of having plentiful support by LCU(R)s and by maintenance facilities at the MS and the B. Hence, results from the present model are almost certainly optimistic. They can, however, provide useful checks on more detailed simulation models.

5.3 Expectation/Mean of T_d

A general formula for the expectation/mean of the time for an AV to complete the transit, T_d , is obtained by conditional expectation from (5.1); first condition on T_b :

$$E[T_d|T_b] = \left(t + E[R_m(t)] \right), \text{ if } T_b \geq t$$

$$= \left\{ \begin{array}{l} T_b \left(1 + \frac{V_T}{V_R} \right) + E[R_M] \\ + E \left[R_m \left(T_b \left(1 + \frac{V_T}{V_R} \right) \right) | T_b \right] + E[T_d] \end{array} \right\}, \text{ if } T_b < t \quad (5.3)$$

Consequently, removing the condition on T_b ,

$$E[T_d] = \frac{C_1}{C_2}$$

where

$$C_1 = [t + E[R_m(t)]] \left[1 - F_{T_b}(t) \right] + E[R_M] F_{T_b}(t) \quad (5.4)$$

$$+ \int_0^t \left[s \left(1 + \frac{V_T}{V_R} \right) + E \left[R_m \left(s \left(1 + \frac{V_T}{V_R} \right) \right) \right] \right] dF_{T_b}(s)$$

$$C_2 = 1 - F_{T_b}(t)$$

5.5 Special (Tractable) Case: Poisson Minor Failures

For illustration, let minor failures occur according to a stationary Poisson process, with rate λ_m , with repair durations $\{S_j, j = 1, 2, \dots, N(T(0))\}$ independently and identically distributed, and with mean $E[S]$.

The result is

$$\begin{aligned} E[T_d] &= \frac{1}{1 - F_{T_b}(t)} \left[(t + \lambda_m t E[S]) (1 - F_{T_b}(t)) \right] \\ &+ \frac{1}{1 - F_{T_b}(t)} E[T_b; t] \left(1 + \frac{V_T}{V_R} \right) \{1 + \lambda_m E[S]\} \\ &+ \frac{1}{1 - F_{T_b}(t)} E[R_M] F_{T_b}(t) \end{aligned} \quad (5.5)$$

Note that the expectations with respect to T_b are over the set $0 \leq T_b \leq t$, e.g.,

$$E[T_b; t] = \int_0^t x dF_{T_b}(x); \quad (5.6)$$

this applies to the later expectations, $E[g(T_b);t]$, as well, with $g(\bullet)$ being an arbitrary function (of T_b).

5.6 Exponentially Distributed T_b

If T_b is distributed exponentially with rate parameter λ (a very special case) then

$$E[T_b; t] = \int_0^t x e^{-\lambda x} \lambda dx = \frac{1}{\lambda} [1 - (1 + \lambda t) e^{-\lambda t}]$$

and (5.7)

$$1 - F_{T_b}(t) = e^{-\lambda t}.$$

For very small λ this gives

$$E[T_d] = t [1 + \lambda_m E[S] + \lambda E[R_M]] + \frac{1}{2} \lambda t^2 \left[\left(1 + \frac{V_T}{V_R} \right) \{1 + \lambda_m E[S]\} \right]. \quad (5.8)$$

AVs in transit from the MS to the B may, in principle, break down and require transport back to the MS for repairs several times before successful transit is accomplished. The effect on eventual transit time T_d , of such return-to-MS transports and re-repairs can become quite large; the time to (slowly) return for repairs adds to the total time, and the repairs congest the repair facility, adding delay. The present infant-failure model does not permit an early (infant) failure to get far from the MS, but it does compound the repair burden and increases turnaround delay, *and* therefore extends the transit time to the B.

6. Comparison of Deterministic and Stochastic Models

In this section we compare results from the stochastic model and a simplified deterministic model. The deterministic model considered in this section is:

$$\begin{aligned}
 \underbrace{\frac{dA(t)}{dt}}_{\text{Rate of "increase" of number of active AVs in the Littoral}} &= - \underbrace{\lambda A(t)}_{\text{Rate of failure of active AVs}} - \underbrace{\nu_T A(t)}_{\text{Rate of "free" active AVs reaching B}} \\
 &+ \underbrace{\mu_S(\bullet) \frac{M_S(t)}{1 + M_S(t)}}_{\text{Rate at which AVs under maintenance at MS become active in the Littoral}}
 \end{aligned} \tag{6.1}$$

$$\begin{aligned}
 \underbrace{\frac{dQ_T(t)}{dt}}_{\text{Rate of "increase" of number of quiescent AVs awaiting tow}} &= \underbrace{\lambda A(t)}_{\text{Rate of towable-failure events}} - \underbrace{\nu_{PL} Q_T(t)}_{\text{Rate of removal initiation by available auxiliary vehicles (e.g., LCs)}}
 \end{aligned} \tag{6.2}$$

$$\begin{aligned}
 \underbrace{\frac{dR_L(t)}{dt}}_{\text{Rate of "increase" of the number of AVs being transported to the MS by an auxiliary vehicle}} &= \underbrace{\nu_{PL} Q_T(t)}_{\text{Rate of pickup of failed AVs by available landing craft; headed for the MS}} - \underbrace{\nu_{RL} R_L(t)}_{\text{Rate of delivery of landing craft towed AVs to the MS}}
 \end{aligned} \tag{6.3}$$

$$\begin{aligned}
 \underbrace{\frac{dM_S(t)}{dt}}_{\text{Rate of "increase" of number of AVs in MS maintenance}} &= \underbrace{\nu_{RL} R_L(t)}_{\text{Rate of dropoff at MS from LC transport}} - \underbrace{\frac{\mu_S(\bullet) M_S(t)}{1 + M_S(t)}}_{\text{Rate of maintenance completion at the MS}}
 \end{aligned} \tag{6.4}$$

$$\begin{aligned}
 \underbrace{\frac{dB(t)}{dt}}_{\text{Rate of "increase" of active, available AVs at the B}} &= \underbrace{\nu_T A(t)}_{\text{Rate of "free" active AVs reaching the B}}
 \end{aligned} \tag{6.5}$$

In this model, all disabled AVs are towed back to the MS. There is always an LC available for towing.

The corresponding stochastic model for one AV is that of (5.1), with the minor failure rates $\lambda_m = 0$. The variance of the time for a single AV to arrive at the B for the stochastic model of this section can be computed using

$$T_d^2 = \begin{cases} (d/V_T)^2 & \text{if } T_b > d/V_T \\ \left(\left[1 + \frac{V_T}{V_R} \right] T_b + R_M + T_d^\# \right)^2 & \text{if } T_b < d/V_T \end{cases}$$

$$E[T_d^2] = \frac{A}{\bar{F}_{T_b}(d/V_T)}$$

where

(6.6)

$$A = \left[\frac{d}{V_T} \right]^2 \bar{F}_{T_b} \left(\frac{d}{V_T} \right) + \left[1 + \frac{V_T}{V_R} \right]^2 \int_0^{d/V_T} x^2 F_{T_b}(dx) + 2[E[R_M] + E[T_d]] \left[1 + \frac{V_T}{V_R} \right] \int_0^{d/V_T} x F_{T_b}(dx) + (E[R_M^2] + 2E[R_M]E[T_d]) F_{T_b} \left(\frac{d}{V_T} \right)$$

and $\bar{F}_{T_b}(t) = 1 - F_{T_b}(t)$

We approximate the distribution of the AV arrival time at the B for an exponential time to failure as follows: if no failure occurs, then the time for an AV to reach the B is d/V_T hours; the probability of at least one serious failure in transit is $p_f = 1 - \exp\left\{-\lambda \frac{d}{V_T}\right\}$.

Then we approximate the distribution of the time for an AV to reach the B as

$$X = \frac{d}{V_T} + Y \text{ where} \tag{6.7}$$

$$Y = \begin{cases} Z & \text{with prob } p_f \\ 0 & \text{with prob } 1 - p_f \end{cases}$$

and Z has an exponential distribution with mean

$$E[Z] = \left(E[T_d] - \frac{d}{V_T} \right) / p_f. \tag{6.8}$$

Note that

$$E[X] = E[T_d].$$

The variance for the approximation (6.7) is

$$Var[X] = Var[Y] = Var[Z]p_f + E[Z]^2 p_f(1 - p_f). \tag{6.9}$$

The stochastic model corresponding to the deterministic model for multiple vehicles is that each of the vehicles travels independently to the B; all the AVs start at time zero.

Thus the number of vehicles that have arrived at the B by time s has a binomial distribution, with number of trials being the number of AVs, and the probability of success being $P\{X \leq s\}$.

6.1 Numerical Example

Below are displayed results for a numerical example. The parameter values are:

Deterministic Model:

Rate of transit completion to the beach for a working AV is $\nu_T = 20/25$ per hour

Rate of commencing towing for failed AV is $\nu_{PL} = 2$ per hour

Rate of tow completion to mother ship (MS) is $\nu_{RL} = 1/5$ per hour

Rate of repair for failed vehicle at MS is $\mu_S = 0.5$ per hour

Rate of failure for a working AV is $\lambda = 1/3$ per hour

Stochastic Model:

Distance to B, d is 25nm

Planing velocity, V_T , for a working AV is 20nm per hour

Towing velocity, V_R , to the MS is 5nm per hour

Mean repair time at the MS is two hours

Variance of a repair time at the MS is one hour²

Mean time to serious failure is three hours

We assume first that the time to a serious failure has an exponential distribution. If *no serious failure occurs*, then the time for an AV to reach the B is 1.25 hours. In the stochastic model, allowing for failures as above, the mean time for an AV to reach the beach, T_d , is 3.79 hours, and the standard deviation of the time is 4.59 hours. The probability of at least one serious failure in transit is $p_f = 1 - \exp\left\{-\frac{1}{3}1.25\right\} = 0.34$.

We will approximate the distribution of the time for an AV to reach the B as

$$\begin{aligned} \mathbf{X} &= 1.25 + \mathbf{Y} \text{ where} \\ \mathbf{Y} &= \begin{cases} \mathbf{Z} & \text{with prob } p_f \\ 0 & \text{with prob } 1 - p_f \end{cases} \end{aligned} \quad (6.10)$$

and \mathbf{Z} has an exponential distribution with mean

$$E[\mathbf{Z}] = (E[T_d] - 1.25) / p_f. \quad (6.11)$$

The mean of the approximate time is 3.79 hours, the same as that of T_d . However, its standard deviation is 5.60 hours, which is 22% greater than that of T_d .

Figure 4 below displays the mean and quantiles for the binomial stochastic model and the mean from the deterministic model.

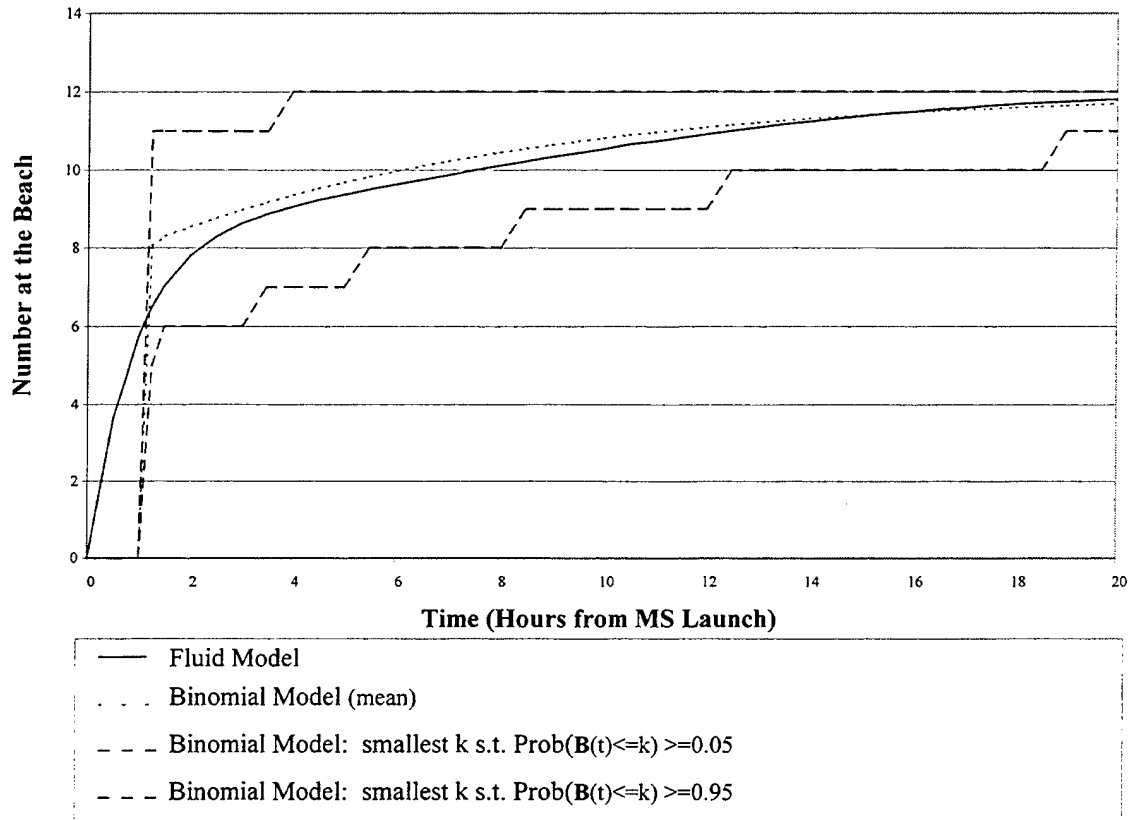


Figure 4.
Number of AVs at the Beach Time from Launch.

Comments: It can be seen from Figure 4 that the deterministic and stochastic model means agree well when the time exceeds one hour. The stochastic model permits probability limits to be created; these suggest considerable variability in the number of platoon elements (AVs) that have arrived at the B by any fixed time t . For instance, at $t = 6$ hours, the probability that eight or more out of 12 have arrived is $\approx 95\%$, while the mean number is ≈ 10 . It is likely that such variability is actually too small.

In the results displayed below we explore the effect of different distributions of the time to failure on the time for an AV to arrive at the B. We use the stochastic model of this section. We compare the mean and standard deviation of the arrival time at the B, T_d , for times to serious failure having a mixed exponential distribution: for $x > 0$,

$$F_{T_b}(x) = p[1 - e^{-\lambda_1 x}] + (1 - p)[1 - e^{-\lambda_2 x}]. \quad (6.12)$$

The following tables display the mean and standard deviations for the AV arrival times at the B. *Note:* Once again, the form of the distribution of the time to serious failure can have a dramatic effect on the mean and standard deviation of the time to arrive at the B. The form of that distribution—specifically its tendency to exhibit small values—will also influence logistics requirements, and their spatial location. Comparison of the mean and standard deviation of the operationally important time to arrive at the B, suggests that the eventual arrival time distribution, T_d , will be approximately exponential in the case of frequent, serious failures of the AV.

Distribution of Time to Failure	Velocity When not Failed (nm/Hour)	Time to Arrive at Destination if No Failure (Hours)	Mean Time to Arrive at Destination (Hours)	Standard Deviation of Time to Arrive at Destination (Hours)
Exponential Mean 3	5	5.00	29.36	27.69
	10	2.50	9.31	9.45
	15	1.67	5.40	6.01
	20	1.25	3.79	4.59
	25	1.00	2.91	3.80
	30	0.83	2.36	3.30
	35	0.71	1.99	2.94
	40	0.63	1.72	2.68
Mixed Exponential Mean 3 $p=0.5; \lambda_1=1, \lambda_2=1/5$	5	5.00	25.85	23.81
	10	2.50	11.38	11.40
	15	1.67	7.26	7.91
	20	1.25	5.29	6.21
	25	1.00	4.15	5.20
	30	0.83	3.41	4.53
	35	0.71	2.89	4.05
	40	0.63	2.50	3.68

Table 1.

Moments for Time of Individual AV to Arrive at Destination, Beach (B), which is 25nm away. The velocity of return to Mother Ship (MS) if failed is five nm per hour; the mean repair time is two hours and the variance of the repair time is one hour.

Distribution of Time to Failure	Velocity When not Failed (nm/Hour)	Time to Arrive at Destination if No Failure (Hours)	Mean Time to Arrive at Destination (Hours)	Standard Deviation of Time to Arrive at Destination (Hours)
Exponential Mean 6	5	5.00	13.21	11.45
	10	2.50	5.34	5.15
	15	1.67	3.33	3.58
	20	1.25	2.41	2.86
	25	1.00	1.89	2.43
	30	0.83	1.56	2.14
	35	0.71	1.32	1.94
	40	0.63	1.15	1.78
Mixed exponential Mean 6 $p=0.5; \lambda_1=1, \lambda_2=1/11$	5	5.00	14.86	12.57
	10	2.50	8.32	8.17
	15	1.67	5.81	6.33
	20	1.25	4.45	5.24
	25	1.00	3.56	4.51
	30	0.83	2.97	4.00
	35	0.71	2.54	3.63
	40	0.63	2.22	3.33

Table 2.

Moments for Time of Individual AV to Arrive at Destination, Beach (B), which is 25nm away. The velocity of return to Mother Ship (MS) if failed is five nm per hour; the mean repair time is two hours and the variance of the repair time is one hour.

Distribution of Time to Failure	Velocity When not Failed (nm/Hour)	Time to Arrive at Destination if No Failure (Hours)	Mean Time to Arrive at Destination (Hours)	Standard Deviation of Time to Arrive at Destination (Hours)
Exponential Mean 9	5	5.00	9.86	7.89
	10	2.50	4.29	3.87
	15	1.67	2.73	2.77
	20	1.25	2.00	2.24
	25	1.00	1.58	1.92
	30	0.83	1.31	1.70
	35	0.71	1.11	1.54
	40	0.63	0.97	1.42
Mixed exponential Mean 9 $p=0.5; \lambda_1=1, \lambda_2=1/17$	5	5.00	12.48	10.04
	10	2.50	7.52	7.30
	15	1.67	5.41	5.88
	20	1.25	4.18	4.95
	25	1.00	3.39	4.32
	30	0.83	2.84	3.85
	35	0.71	2.44	3.50
	40	0.63	2.14	3.22

Table 3.

Moments for Time of Individual AV to Arrive at Destination, Beach (B), which is 25nm away. The velocity of return to Mother Ship (MS) if failed is five nm per hour; the mean repair time is two hours and the variance of the repair time is one hour.

Distribution of Time to Failure	Velocity When not Failed (nm/Hour)	Time to Arrive at Destination if No Failure (Hours)	Mean Time to Arrive at Destination (Hours)	Standard Deviation of Time to Arrive at Destination (Hours)
Exponential Mean 18	5	5.00	7.17	4.73
	10	2.50	3.34	2.52
	15	1.67	2.18	1.85
	20	1.25	1.62	1.51
	25	1.00	1.28	1.31
	30	0.83	1.07	1.17
	35	0.71	0.91	1.07
	40	0.63	0.79	0.98
Mixed exponential Mean 18 $p=0.5; \lambda_1=1/30, \lambda_2=1/6$	5	5.00	8.61	6.43
	10	2.50	3.97	3.44
	15	1.67	2.57	2.52
	20	1.25	1.90	2.06
	25	1.00	1.51	1.78
	30	0.83	1.25	1.58
	35	0.71	1.06	1.44
	40	0.63	0.93	1.33

Table 4.

Moments for Time of Individual AV to Arrive at Destination, Beach (B), which is 25nm away. The velocity of return to Mother Ship (MS) if failed is five nautical miles (nm) per hour; the mean repair time is two hours and the variance of the repair time is one hour.

References and Bibliography

1. Buckles, Brian K., "Case Study of the United States Marine Corps Advanced Amphibious Assault Vehicle (AAAV) Program Test and Evaluation Strategy," Naval Postgraduate School Thesis for the MS degree in Management (1999).
2. Gaver, D.P., "Point Process Problems in Reliability," *Stochastic Point Processes: Statistical Analysis and Applications*, ed. by Peter A.W. Lewis, 774-800, John Wiley & Sons, Inc., New York, NY, 1972.
3. Gaver, D.P., Glazebrook, K.D., Jacobs, P.A., Seglie, E.A., "Probability Models for Sequential-Stage System Reliability Growth via Failure Mode Removal," Proceedings of MMR' 2000: *Second International Conference on Mathematical Methods in Reliability*, University Victor Segalen, Bordeaux, France, July 4-7, 2000.
4. Gaver, D.P., Jacobs, P.A., Seglie, E.A., "Stochastic Models for Promoting and Testing System Reliability Evolution," ESREL 2001: European Safety and Reliability International Conference, Torino, Italy, September 16-20, 2001.
5. Kemp, Jesse A., "Modeling and Simulation in Support of Operational Test and Evaluation for the Advanced Amphibious Assault Vehicle (AAAV)," The Naval Postgraduate School Thesis for the MS degree in Operations Research (2001).

Reference Web sites concerning the AAAV:

1. <http://www.gdps.com/programs/aaav.html> (provides physical system specifications)
2. <http://www.atc.army.mil/globe/content98/Aug98/articles/page5.htm>

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