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A Unified Joint Probabilistic Data Association with Multiple Models

S.J. Davey and S.B. Colegrove

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Surveillance Systems Division
Electronics and Surveillance Research Laboratory

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ABSTRACT

This paper presents the theory and examples of performance for a new algorithm that tracks targets using a Multiple Model Unified Joint Probabilistic Data Association (MM-UJPDA) filter. The models in the MM-UJPDA can be set to the ambiguity velocities encountered when initiating tracks on a sensor that has ambiguous velocities in its measurements. Alternately, the models can be set for tracking manoeuvring targets. Thus each parallel filter in the MM-UJPDAF is assigned one of a range of possible target model parameters. The term 'unified' summarizes a number of key features in the algorithm. These are: multiple non-uniform clutter regions, a model for a visible target to compute track confidence for track promotion, and measurement selection based on a fixed number of nearest measurements. The filter formulation used a new approach to create track clusters for determining the nearby tracks that share measurements. The filter's performance is demonstrated with track initiation using the multiple model and multiple target approach while for established tracking only the multiple target approach is used.

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Executive Summary

The filter described in this report extends the Multiple Model Unified Probabilistic Data Association Filter (MM-UPDAF) that is currently used for track initiation on data from the Jindalee Over-the-Horizon Radar at Alice Springs. The term "Unified" is now used to summarize the following features that are included in this tracker: an event for track initiation and termination; the selection of the nearest measurements to each tracking filter's predicted position; and, an *a priori* model for the clutter distribution which is represented by multiple clutter zones that have both uniform and nonuniform distributions.

The current tracking system uses the MM-UPDAF to resolve velocity ambiguities during track initiation and when a track is promoted to the established state the UPDAF is used. The extension to the MM-UPDAF that is reported here is part of a staged development activity to further improve performance. The MM-UPDAF and the UPDAF do not consider the possibility that measurements can be from another target. That is, they treat tracks as isolated entities. Thus in dense target conditions, instances occur where a track can transfer to another nearby target. Also a track estimate can be midway between closely spaced multimode returns with the same Doppler and range but different azimuth. At this stage, the adaption of the MM-UPDAF to handle multiple manoeuvre models has not been made. This was not done because targets which manoeuvre are more likely to do so in the proximity of other nearby targets. Therefore since the MM-UPDAF and the UPDAF do not include other targets, no attempt was made to enhance manoeuvre performance over the current capability.

The extension to the MM-UPDAF contained in this paper involves a new target model which defines, about each track, a cluster of nearby tracks. Thus each track becomes a reference track and the other tracks that have selected the same measurements are linked to form a track cluster. With the inclusion of multiple target models, excessively large track clusters are formed in dense target conditions. This problem lead to the definition of a new clustering rule to constrain the growth of a cluster about a reference track. Even so, cluster sizes increase to about 10 and the computation of the number of measurement to target assignments or events becomes large. To evaluate these events, a joint event tree which factorizes the events, gives an efficient implementation which allows real-time tracking.

The common name given for a filter of this type is the Multiple Model Joint Probabilistic Data Association Filter or MM-JPDAF[8]. While this filter has already been formulated, the approach taken here significantly differs in the ways described above. It also incorporates the advanced features in the MM-UPDAF, namely the measurement selection approach, the inclusion of track initiation and the multiple non-uniform clutter zones. So in this paper, this new multiple target filter is termed the MM-UJPDAF.

This filter was implemented to run on the Jindalee replay system and limited testing was undertaken using datasets that contained closely spaced targets. These were particularly challenging to the current tracker. The implementation only used the MM-UJPDAF for track initiation while for promoted tracks, it used the UJPDAF. Thus no attempt was made to try multiple manoeuvre models for promoted tracks. At this stage the primary emphasis has been to improve track initiation and tracking in dense target conditions. Specific examples showed how this new filter correctly tracked closely spaced targets.

Authors

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Samuel Davey completed his BE in Electrical and Electronic Engineering at the University of Adelaide in 1995. In 1998 he completed a Master's Degree in Signal and Information Processing at the University of Adelaide. He has been employed as a Professional Officer at DSTO since 1995 where he has worked in the areas of signal processing and automatic tracking. Samuel is currently conducting research towards a PhD with the Cooperative Research Centre for Sensor Signal and Information Processing at the University of Adelaide sponsored by Surveillance Systems Division. This research is focussed on extending tracking algorithms to provide improved track initiation in clutter.

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Bren Colegrove completed his PhD in Electrical Engineering at the University of Queensland in 1973. Since 1973 he has been employed as a Research Scientist by DSTO. Up to 1975 he worked on Microwave Radar clutter modelling. Since then he has worked on Over-the-Horizon Radar. This work involved the design and implementation of the Jindalee Stage B Radar. The initial emphasis of his work was radar system design based on cost minimization. Subsequent work involved signal processing, displays and automatic tracking systems. From 1979 to 1994 Dr Colegrove was the Australian National Leader of TTCP Technical Panel KTP-2 which concentrates on advancements in automatic tracking systems. Dr Colegrove's present research activities are into advance automatic tracking systems.

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A - The set of clutter region volumes $\{A_c\}_{c=1}^{C}$.

A_c - Volume of clutter region O_c.

c_i - Clutter region index for measurement y_i.

C - Number of clutter regions.

$d_{i,m,t}$ - Distance between measurement y_i and the predicted position for target model m of track t.

$\mathrm{E}\{.\}$ - Expected value or mean.

$\mathbf{F}_{m,t}(k)$ - State transition matrix for target model m of track t at time k.

$f_{m,t}(y)$ - The probability density function for measurements from a target which obeys model m of track t.

$g_c(.)$ - Probability density function for clutter measurements in the clutter region c.

$h_c^b(.)$ - Normalized histogram bin b value for clutter region c.

$\mathbf{H}_{m,t}(k)$ - Sensor measurement matrix for target model m of track t.

i - Measurement index.

I - The number of measurements preselected to update a tracking filter model. I is the same for all models and is constant.

IT - Number of unique measurements selected for all M models of T tracks.

k - Time index except when it is a subscript

$(k|j)$ - Time k given data to time j.

m - Model index.

M - The number of different target models. M is a constant.

$M_{j,t}$ - Number of models from track t that have selected measurement y_j.

n - The set of the number of measurements $\{n_c\}_{c=1}^{C}$

$n_{c,\Lambda}$ - The number of measurements within the intersection of the clutter region O_c and the region spanning all selected measurements. When the joint event list Λ is included, the number of measurements is reduced by the number of tracks in Λ.

N - The set of the number of measurements in the clutter regions $\{N_c\}_{c=1}^{C}$

N_c - Number of measurements in the clutter region O_c

O_c - Clutter region

$p_{l,m,t}$ - *A priori* transition probability for the target changing from model l to model m for track t.

$\mathbf{P}(k|j)$ - Error covariance of the state estimate $\hat{x}(k|j)$ for the reference target.

$\mathbf{P}_m(k|j)$ - Error covariance matrix given the reference target obeys model m.

Pc_c - Probability that a clutter measurement is within the intersection of the clutter region O_c and the region spanning all selected measurements.

$Pd_{m,t}$ - *A priori* probability of getting a sensor measurement or target detection given that target model m is visible for track t.

$Pm_{m,t}$ - *A priori* probability of the target being the m^{th} model for track t.

$Ps_{m,t}$ - *A priori* probability that the target measurement is within the surveillance region S and the region $SI_{m,t}$ given the target obeys the m^{th} model, is visible and detected.

Pv_t - *A priori* probability of target visibility for track t.

$\mathbf{Q}_{m,t}(k)$ - Covariance matrix for the m^{th} target model's process noise of track t.

$\mathbf{R}(k)$ - Sensor measurement noise covariance matrix.

S - The surveillance region that contains the sensor data at time k.

$SI_{m,t}$ - The multi-dimensional region contained within S and encompassing the I^{th} nearest sensor measurement to target model m of track t.

$u_{m,t}(k)$ - The m^{th} target model's process noise vector for track t.

$v(k)$ - Sensor measurement noise vector.

w_c - Filter weight used to filter the histogram bins for clutter index c.

$\mathbf{W}_m$ - Weight matrix for the m^{th} model of the reference track.

$x_{m,t}(k)$ - Target state at time k. When the subscripts m and t are present, they denote the model and track number respectively.

$\hat{x}_{i,m,t}(k|j)$ - State estimate of the target at time k given data to time j. When the subscripts i, m and t are present, they are for the i^{th} measurement being from the target satisfying model m of track t.

$y_{i,m,t}$ - The i^{th} nearest sensor measurement to model m of track t.

$\tilde{y}_{i,m,t}$ - The vector difference between the i^{th} measurement and the predicted position of the m^{th} model for track t.

$\mathrm{Y}(k)$ - The set of IM unique measurements selected by all M models.

$\mathrm{Z}(k\text{-}1)$ - Past track data.

$\beta_{i,m}$ - Probability of event $\Theta_{i,m}$ given all data.

Δ_t^{V} - *A priori* probability of the target remaining visible from k - 1 to k for track t.

$\Delta_t^{\overline{\mathrm{V}}}$ - *A priori* probability of the target becoming visible from k - 1 to k for track t.

$\Theta_{i,m}$ - The event that measurement y_i is from a target which satisfies model m of the reference track.

λ_j - Elements of the joint event index which define the track model and track source for measurement j, ie $\{m_j, t_j\}$, $j = 1, \ldots, IT$.

Λ - Joint event index list.

$\mu_{j,t}$ - Model list whose elements $\mu_{j,t}(m)$, $m = 1, M_{j,t}$ are the models from track t that selected measurement y_j.

$\Sigma_m(k|j)$ - Kalman filter error covariance matrix of the state estimate given data from reference target model m.

τ_j - Track list whose elements $\tau_j(t)$, $t = 0, \ldots T_j$ contain the tracks that selected measurement j.

Ω_m - The sub-partition of the measurement sample space containing those elements when the target satisfies model m.

1. Introduction

The fundamental problem of target tracking is measurement association. When a sensor does not provide any identification of the source of measurements, the measurement source must be inferred in order to obtain target state estimates. When the environment provides a high degree of clutter, the tracking system has to differentiate between target measurements and those produced by interference and noise. When the sensor collects measurements from several targets, the tracking system must determine which measurements were caused by each of the targets.

One solution to the problem of tracking in a cluttered environment is the Probabilistic Data Association Filter (PDAF) [8]. The PDAF assigns an association probability for each measurement selected by a track and uses these probabilities to weight these measurements for track update. The PDAF allows for the case that the target may not have formed any measurement. This is due to either a failure to detect the target signal in noise or the target not existing. The nonexistent target is modelled as an invisible target. Using the concept of target visibility [1] it is possible to formulate a PDAF which performs automatic track initiation and deletion.

The original PDAF formulation has some limitations. It assumes that all measurements are caused by either the track being updated or by clutter. Under dense target conditions, this assumption is violated and the effect of other target tracks must be considered. The Joint PDAF (JPDAF) [2] is used to account for measurements from other tracks.

One radar that forms measurements with the preceding characteristics is Over-the-Horizon Radar (OTHR). Australia's OTHR, Jindalee is a scanning sensor that provides measurements consisting of range, azimuth and Doppler. The Doppler measurement can be ambiguous due to the radar operating parameters. There are also objects that give target-like returns, such as transponders, which do not move in space but provide an observed Doppler shift. To successfully deal with these ambiguities, the current Jindalee tracker [1] uses a PDAF with target visibility, a nonuniform clutter model and multiple velocity model initiation.

In 1998 a JPDAF was implemented within the Jindalee Over the Horizon Radar (OTHR) signal processing replay frame work [4]. This filter was used only for tracks that were already established and the current system [1] was used for track initiation. This report presents an extension of this JPDAF to cope with some of the problems of track initiation noted in [1]. The JPDAF extension incorporates the multiple model initiation scheme of [1] as well as the nonuniform clutter model. The aim of this extension is to improve track initiation performance by accounting for measurements caused by nearby tracks.

This area of initiation on interfering targets occurs with closely resolved multi-path propagation for OTHR. In this case, similar propagation paths are observed as parallel tracks and may be poorly tracked by a PDAF. It is important to correctly separate these multi-path tracks because conversion from the radar measurement space into geographic co-ordinates requires a knowledge of the ionospheric path for that track.

2. Outline of Target, Sensor and Update Model

The proposed multiple-model JPDAF with target visibility (MM-UJPDAF) is intended to be an evolution of the current 1RSU multiple model PDAF with target visibility (MM-UPDAF). Therefore, the model used for the target and sensor are identical as that used in the existing MM-UPDAF tracker [1]. This model is repeated in this document for the purposes of completeness and clarity.

The model used to represent the environment, target and sensor is shown in Figure 1. Here the sensor collects returns from targets and the environment via the propagation medium. These returns are converted into sensor measurements in the receiving and measurement processes. The sensor measurements are estimates of all target like signals received by the sensor for each time step *k*. To limit the number of measurements, a threshold is used to pass likely target measurements. Here the list of measurements is assumed to be dominated by measurements from clutter and noise. The source of a measurement, ie target or clutter, is also assumed to be unknown. The measurement process also determines the extent of multiple clutter regions and updates the clutter model parameters that are used later in the track update process.

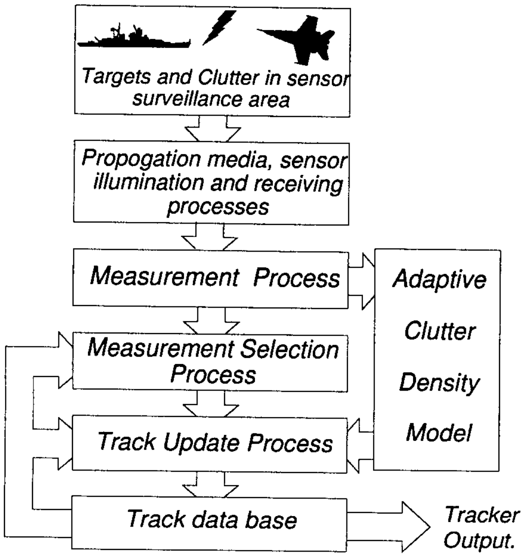


Figure 1. Sensor and environment model

The selection process in Figure 1, uses the track parameters in the track data base to select those measurements that are likely to be from the target. All targets that give measurements in the received sensor data at time *k* are assumed to be represented by tracks in the track data base. The contents of this data base are updated by the measurements and the clutter model parameters from time *k* to form new estimates that are applied to the data at the next time *k*+1.

2.1. A Priori Data

Before outlining the processing steps in Figure 1, we define the *a priori* models for the target, the clutter and track data base.

2.1.1. Target Dynamic's Model

The target is assumed to be represented by a fixed number of *M* discrete target models that describe the target dynamics. The m^{th} target model for target track *t* is given by,

$$x_{m,t}(k+1) = \mathbf{F}_{m,t}(k)x_{m,t}(k) + u_{m,t}(k), \qquad m=1,..,M \qquad (1)$$

Where $x_{m,t}(j)$ is the state of target track t with model m at time j, $\mathbf{F}_{m,t}(k)$ is the state transition matrix for the m^{th} target model of track t, and $u_{m,t}(k)$ is the m^{th} target model's process noise vector, Gaussian distributed with zero mean and covariance $\mathbf{Q}_{t,m}(k)$. The process noise accounts for target manoeuvres and model errors. Here $\mathbf{F}_{m,t}(k)$ and $\mathbf{Q}_{m,t}(k)$ are assumed to be known for each model of each track.

The target model switching is a Markov process represented by the *a priori* MxM transition probability matrix with transition probabilities $p_{l,m,l}$, where l is the model index at time k-1 and m is the model index at time k. The *a priori* model probability $Pm_{m,t}$ is therefore given by

$$Pm_{m,t} = Pm_{m,t}(k|k-1) = \sum_{l=1}^{M} p_{l,m,t}\, Pm_{l,t}(k-1|k-1) \tag{2}$$

The model transition probabilities are *a priori* and depend on the target model. In the case of track initiation on a sensor with Doppler processing, a separate target model is defined for each velocity ambiguity and there is no transition between models. However if the models represent different target manoeuvre cases, there is transition between models.

2.1.2. Clutter Model

The clutter is modelled as uniform in one or more measurement dimensions and non-uniform in the remaining dimensions. At least one dimension is assumed to have a non-uniform distribution. It is assumed that the measurements are due to more than one pdf. The surveillance region is divided in the uniform dimensions into *clutter regions* each of which has it's own non-uniform pdf in the remaining dimensions. There are C different non-uniform pdfs and hence C clutter regions. The symbol for a clutter region is O_c where $c = 1,\ldots,C$. For each measurement y_i there is an associated clutter region index c_i. The number of measurements contained in a clutter region is N_c and its area (volume) in the coordinates for the uniform distribution is given by A_c. The sets representing all clutter regions, measurements and volumes are respectively $\boldsymbol{O} = \left\{O_c\right\}_{c=1}^{C}$, $\boldsymbol{N} = \left\{N_c\right\}_{c=1}^{C}$ and $\boldsymbol{A} = \left\{A_c\right\}_{c=1}^{C}$.

The coordinates that do not have a uniform distribution, such as signal-to-noise (SNR), are represented by a continuous clutter distribution. These coordinates are identified by a separate component of the measurement vector so that $y_i = \{z_i, s_i\}$. Here z_i contains the components that have a uniform clutter distribution in the clutter region with index c_i and s_i contains the components with a nonuniform distribution.

Clutter measurements produced by the sensor are assumed to be independent of one another. The *a priori* probability distribution of a clutter measurement in the s variables given it is in a clutter region is given by the function $g_c(s)$. In the z coordinates the distribution is uniform so that the pdf of clutter measurements is given by $g_c(y) = \dfrac{g_c(s)}{A_c}$.

2.1.3. Track Data

The *a priori* track data consists of the state estimates for T targets where each is represented by M models. For each track there are M model state estimates, covariances and model probabilities. In addition the track data includes the probability of target visibility. All past data up to time k-1 are given by

$$\begin{aligned} Z(k-1) &= \left\{\left\{Z_{m,t}(k-1)\right\}_{m=1}^{M}, \ Pv_t(k-1|k-1)\right\}_{t=1}^{T} \\ &= \left\{\left\{\hat{x}_{m,t}(k-1|k-1), \mathbf{P}_{m,t}(k-1|k-1), \ Pm_{m,t}(k-1|k-1)\right\}_{m=1}^{M}, \ Pv_t(k-1|k-1)\right\}_{t=1}^{T} \end{aligned} \tag{3}$$

The target model's state estimate for time k based on past data up to time j, is denoted by $\hat{x}_{m,t}(k\,|\,j)$ and the covariance by $\mathbf{P}_{m,t}(k\,|\,j)$ where t is the track index number and m is the model index number. The collection of M estimates is called track t and the target associated with this track is designated as target t. It is assumed that all targets in the surveillance area have an associated track representation. The state estimate of target t is denoted by $\hat{x}_t(k\,|\,j)$ and the covariance of this estimate by $\mathbf{P}_t(k\,|\,j)$. These are obtained from the sum of the product of the model state and probability.

As well as the above state estimates and covariances, the track data base also has the target visibility from the previous time k-1, ie $Pv_t(k\text{-}1\,|\,k\text{-}1)$ and the model probability $Pm_{m,t}(k\text{-}1\,|\,k\text{-}1)$ for the M target models. The model probabilities are constrained by $\sum_{m=1}^{M} Pm_{m,t} = 1$ and allowance is made for the model to change with the time index k.

The model transition probability matrix gives the *a priori* probabilities for the current time k. This involves combining the states of all models that can transition to this model from time k-1. After some maths, the *a priori* state estimate and associated covariance conditioned on it being model m is obtained by the following equation.

$$\begin{aligned} \hat{x}_{m,t}(k|k-1) &= \frac{1}{Pm_{m,t}} \sum_{l=1}^{M} p_{l,m,t}\, \hat{x}_{l,m,t}(k|k-1)\, Pm_{l,t}(k-1|k-1) \\ \mathbf{P}_{m,t}(k|k-1) &= \frac{1}{Pm_{m,t}} \sum_{l=1}^{M} p_{l,m,t}\, \mathbf{P}_{l,m,t}(k|k-1)\, Pm_{l,t}(k-1|k-1) \ + \\ &\quad \frac{1}{Pm_{m,t}} \sum_{l=1}^{M} p_{l,m,t} \Big[\hat{x}_{l,m,t}(k|k-1)\hat{x}_{l,m,t}(k|k-1)^{\mathrm{T}} \ - \\ &\quad \hat{x}_{m,t}(k|k-1)\hat{x}_{m,t}(k|k-1)^{\mathrm{T}}\Big] Pm_{l,t}(k-1|k-1) \end{aligned} \tag{4}$$

Where $\hat{x}_{l,m,t}(k|k-1) = \mathbf{F}_{m,t}\, \hat{x}_{l,t}(k-1|k-1)$ and $\mathbf{P}_{l,m,t}(k|k-1) = \mathbf{F}_{m,t}\mathbf{P}_{l,t}(k-1|k-1)\mathbf{F}_{m,t}^{\mathrm{T}} + \mathbf{Q}_{m,t}$.

For the special case of track initiation where velocity ambiguities exist in the sensor measurements, a separate target model is allocated to each velocity ambiguity. During initiation the ambiguity does not change, so there is no switching between models. This then makes the probability transition matrix equal to the identity matrix and from equation (2), $Pm_{m,t}(k|k\text{-}1) = Pm_{m,t}(k\text{-}1|k\text{-}1)$. So equation (4) with an identity matrix for the probability transition matrix gives the standard prediction equations, namely.

$$\begin{aligned} \hat{x}_{m,t}(k|k-1) &= \mathbf{F}_{m,t}\,\hat{x}_{m,t}(k-1|k-1) \\ \mathbf{P}_{m,t}(k|k-1) &= \mathbf{F}_{m,t}\mathbf{P}_{m,t}(k-1|k-1)\mathbf{F}^{\mathrm{T}}_{m,t} + \mathbf{Q}_{m,t} \end{aligned} \tag{5}$$

2.2. Propagation and Receiving Process

The model for the propagation and receiving process in Figure 1 employs the notion of target visibility, ie a target is either visible or not visible (invisible) to the sensor. *Visible* targets give sensor measurements that satisfy the sensor's target and measurement models. Conversely, *Invisible* targets give no measurements that satisfy any of these models.

The factors that determine the transition between the visible and invisible states of a target are independent of any target model. Following the approach in [1], the *a priori* probability that the target is visible is given by,

$$\begin{aligned} Pv_t &= Pv_t(k-1|k-1)\Delta_t^{\mathrm{V}}(k) + \left(1 - Pv_t\ (k-1|k-1)\right)\Delta_t^{\overline{\mathrm{V}}}(k) \\ &= \left(\Delta_t^{\mathrm{V}}(k) - \Delta_t^{\overline{\mathrm{V}}}(k)\right)Pv_t(k-1|k-1) + \Delta_t^{\overline{\mathrm{V}}}(k). \end{aligned} \tag{6}$$

Where $Pv_t(.)$ is the probability of target visibility for target *t*. $\Delta_t^{\mathrm{V}}(k)$ is the *a priori* probability of target *t* remaining visible from time *k*-1 to *k*, and $\Delta_t^{\overline{\mathrm{V}}}(k)$ is the *a priori* probability of target *t* changing from being invisible at time *k*-1 to being visible at time *k*.

For notational simplicity Pv_t represents the *a priori* probability $Pv_t(k|k\text{-}1)$. When denoting the *a posteriori* probability, the time index is included, ie $Pv_t(k|k)$.

2.3. Measurement Process

The data input to the measurement process is the received and processed signal in each resolution cell. The signal is the sum of the returns from targets, clutter and noise. Clutter and noise are collectively called clutter. The assumed functions of the measurement process are:

(1) Convert the receiving system output into measurements that contain estimates of parameters such as range and bearing

(2) Partition the receiving system output into the C clutter regions in which measurements from clutter are represented by a common probability density function (pdf). Determine the clutter region parameters and update the data for the clutter region pdf models.

The details of the above steps are given in[5] [6] and [7].

The measurements output from step (1) are represented by the set $\{y_i\}_{i=1}^{N}$, where y_i is the i^{th} measurement and N is the number of measurements at time k. Here the time index is omitted for clarity.

Step (2) replaces the commonly used uniform pdf for clutter by multiple nonuniform clutter regions. It determines the clutter region shape $O = \{O_c\}_{c=1}^{C}$, and the clutter region volume $A = \{A_c\}_{c=1}^{C}$ for the dimensions where the clutter is uniformly distributed. For each measurement y_i it determines the associated clutter region index c_i. It also calculates N_c the number of measurements contained in the clutter region O_c. Using the measurements y_i, this step updates the *a priori* distributions for the clutter regions.

2.3.1. Target Probabilities

The target probabilities relate to when the target is visible and are derived from target signal fluctuations and the conditional tests in the measurement process. The measurements are random variables and are independent at each time k and the measurement process can stop the formation of a target measurement. The mechanism that causes a failure to form a target measurement is different to that which causes a target to be invisible. The action of getting a target measurement is called detection. The *a priori* probability of detection, $Pd_{m,t}$, is the probability of obtaining a sensor measurement when target t is visible and obeys the m^{th} model.

The sensor measurements from Visible targets are related to the target state by,

$$y(k) = \mathbf{H}_{m,t}(k) x_{m,t}(k) + v(k). \tag{7}$$

Where $y(k)$ is the measurement vector, $\mathbf{H}_{m,t}(k)$ is the measurement matrix for model m of track t and $v(k)$ is the measurement noise vector, Gaussian distributed with zero mean and covariance $\mathbf{R}(k)$. Here $\mathbf{H}_{m,t}(k)$ and $\mathbf{R}(k)$ are assumed known at time k. The model index for $\mathbf{H}$ is included to allow for cases such as fixed transponder signals in Doppler radars. Here the measured Doppler bears no relation to target range-rate.

The pdf for target measurements is defined given the target obeys the m^{th} model. It is expressed with respect to the predicted position of the track. The distance of a measurement from this position is normalised by the innovation covariance for the m^{th} target model $\mathbf{S}_{m,t}(k)$, where $\mathbf{S}_{m,t}(k) = \mathbf{H}_{m,t}(k) \mathbf{P}_{m,t}(k|k-1) \mathbf{H}_{m,t}^{\mathrm{T}}(k) + \mathbf{R}(k)$. The equation for calculating this distance is,

$$d_{i,m,t}^{2} = \tilde{y}_{i,m,t}^{\mathrm{T}} \, \mathbf{S}_{m,t}^{-1} \, \tilde{y}_{i,m,t} \tag{8}$$

Where $\tilde{y}_{i,m,t}$ is the vector between the track predicted position for model m, ie $\hat{x}_{t,m}(k|k-1)$, and the i^{th} measurement, ie

$$\tilde{y}_{i,m,t} = y_{i,m,t} - \mathbf{H}_{m,t}\hat{x}_{m,t}(k\,|\,k-1).$$

The pdf of target measurements is Gaussian distributed and given by,

$$f_{m,t}(y_{i,m,t}) = \frac{e^{(-d_{i,m,t}^2/2)}}{(2\pi)^{\mu/2}\sqrt{|\mathbf{S}_{m,t}|}}. \tag{9}$$

Where μ is number of degrees of freedom, ie the number of components in the measurement vector. This distribution is not truncated and therefore integrates to one over all y.

2.3.2. Clutter Probabilities

The clutter density is correlated with time, so the clutter density model in Figure 1 stores filtered histograms for each of the C clutter regions. A simple low pass filter for each clutter region c with weight $w_c(k)$ is applied to the raw histogram data from the measurements $\{y_i\}_{i=1}^{N}$. The estimated histogram for bin b is given by[6]:

$$\hat{h}_c^b(k) = \hat{h}_c^b(k-1) + w_c(k)\left[h_c^b(k) - \hat{h}_c^b(k-1)\right] \tag{10}$$

where $h_c^b(k)$ is the number of measurements at time k for bin b and clutter region c. This histogram data is used to compute a sample cumulative distribution. This distribution is then used with the Kolomogorov-Smirnov test statistic to estimate the clutter density $g_c(s)$ as described in [6]. This density is considered *a priori* because it is derived from filtered past data and because the number of measurements used for its estimate are much greater than the number selected by any track.

2.4. Measurement Selection Process

The M models for each track t select a fixed number of I measurements from the set $\{y_i\}$ based on their distance $d_{t,m,i}$ away from the predicted model position. It is assumed that the probability of any measurement outside of the I nearest being caused by the model of interest is zero. Using a fixed number of neighbours is computationally efficient and it allows the selection region to expand when the predicted position is not near to any measurements (as would be the case for a manoeuvring target).

Due to the non-linear relationship of the MM-UJPDAF computational complexity with the number of target models and tracks, it is advantageous to divide the tracks into clusters. We will define a track cluster as a collection of measurements and tracks that are assumed to form a complete set for probability calculations. Each cluster is treated individually by the MM-UJPDAF. Models of tracks within the same cluster may or may not have an impact on the probability calculations for the track model of interest, but it is assumed that all tracks outside the cluster have a negligible impact. There are many ways that these track clusters may be formed. Ultimately, the size of the clusters is governed by the real-time processing constraints.

The process of forming the track clusters is referred to as clustering. The most desirable clustering method is to group all tracks whose models select common measurements. This approach, however, leads to clusters which are too large. Pruning techniques are therefore applied to keep the cluster size manageable. Because of the pruning performed, clustering must be repeated for each track where each track is treated as a reference track of the cluster.

Clustering for this filter follows the basic principle that only tracks which are likely to be the true source of measurements selected by a model from the reference track are added to that track's cluster. All the selected measurements for each model of each track in the cluster are included in the cluster. This approach limits the cluster size to a more manageable level.

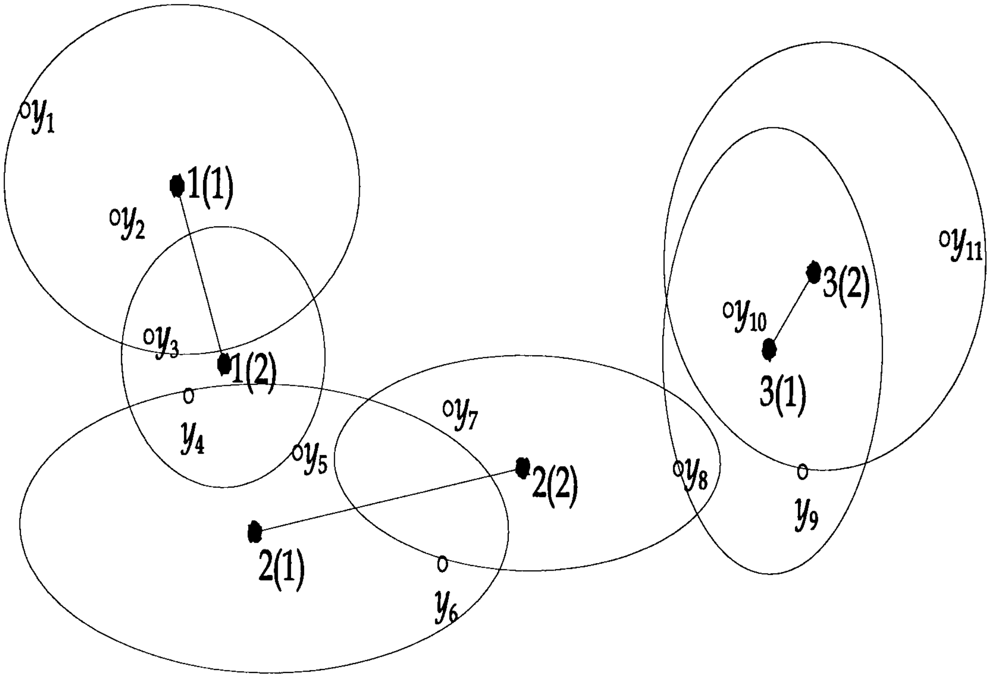


Figure 2. Example Cluster

Consider the example illustrated in Figure 2. Here, tracks are denoted by crosses, measurements with circles and an ellipse has been drawn to show which measurements are the l nearest to each track. In this example, l is three. There are two models for each track (this was chosen for simplicity of the diagram). Each of the models is labelled $t(m)$ indicating model m for track t. When we form the cluster for track 1 we include track 2 but not track 3. Clearly this is sub-optimal since track 3 will impact on the association probabilities for track 2. This approximation is, however, necessary in order to calculate the probability terms and it is assumed that the contribution of tracks like track 3 are a second order effect. The approximation is particularly necessary in high target density conditions where many tracks select the same measurements and when the JPDAF is incorporated with the multiple-model initiation filter.

The cluster size is also restricted using a predefined distance threshold. Tracks are only added to the cluster if the distance between the track and a peak selected by the reference track is less than this predefined threshold. This threshold controls the cluster size and is a design parameter. The distance between the track and a peak is the same normalised distance used for measurement association and so can be viewed as a threshold measured in standard deviations of the track pdf.

The clustering process is then summarised by three steps:

- Choose a reference track
- Find all tracks that are less than a predefined distance away from any of the measurements selected by the reference track
- Find all the measurements selected by any track in the cluster

Once the cluster has been formed, it is common [8] to form a *Validation Matrix* for the cluster. The validation matrix shows which measurements have been selected by each track. The matrix can be used to determine the joint events [8]. The approach adopted by the author does not make use of a validation matrix, but instead uses track lists. These lists enable the event probabilities to be written in a factorised form and hence be efficiently calculated. The *Validation Matrix* is not described here to avoid unnecessary complication. The track lists are described in more detail in the following section.

3. Event Space

The event space consists of all possible track and model measurement associations for a particular track cluster at time k. Define IT and T to be the number of measurements and tracks in the cluster respectively. Let y refer to a measurement in the cluster and t refer to a track. Recall that the number of models for each track is M. The reference track is track 1. Each model of each track selects I measurements as possible association candidates. These I are the closest measurements to the a priori model position and it is assumed that all other measurements have a negligible probability of being caused by this particular model.

3.1. Event Space Partitioning

Define the joint events as a partition of the event space such that within that partition every measurement in the cluster has a unique identified source. The source may be either clutter (noise) or one of the models from one of the target tracks in the cluster. No more than one measurement can have the same track source since each target produces at most one measurement. The joint events are of interest because it is possible to calculate their probability.

Let the symbol θ_Λ denote a particular joint event which defines the track and model source for each of the IT measurements. The subscript Λ is the joint event index. Of these IT measurements, n_c are contained in the clutter region c and $n_{c,\Lambda}$ is the number of measurements assigned to targets for joint event Λ in this clutter region. Λ is an IT element list whose elements $\lambda_j = (m_j, t_j)$ are indices defining the model and track that are the source of the measurement y_j under the particular joint event. Let $\lambda_j = (0\,,0)$ represent the j^{th} measurement being due to clutter and $\lambda_j = (m, t)$ represent the j^{th} measurement being due to model m of track t. Each track element t_j of Λ takes a value in the range $[0, T\,]$. The model elements, m_j each take a value in the range $[0, M]$. Since each target may cause only one measurement, no two track elements t_j of Λ can have the same non-zero value. In fact, the range of possible values for λ_j is also limited by the measurement selection process. Since each model may only consider associations with the I nearest measurements, λ_j can only take the value (m,t) if measurement y_j is one of the I nearest measurements selected by model m of track t. To quantify which models from each track select each measurement two further lists are defined.

Define T_j as the number of tracks selecting measurement y_j. To describe the possible values of λ_j, define the track list $\tau_j = \left\{\tau_j(t)\right\}_{t=0}^{T_j}$ whose elements are the tracks which have selected measurement y_j. Let the first element $\tau_j(0)$ be zero (denoting that clutter is a possible source of the measurement) and element numbers $1..T_j$ take values in $[1, T]$ corresponding to the tracks which have selected y_j.

The symbol $M_{j,t}$ defines the number of models from track t that have selected measurement y_j. $M_{j,t}$ is zero for all tracks not in the list τ_j. Define the model list $\mu_{j,t} = \left\{\mu_{j,t}(m)\right\}_{m=1}^{M_{j,t}}$ whose elements are the models from track t that have selected measurement y_j. For clutter, let $\mu_{j,0} = \{0\}$ with $M_{j,0} = 1$ for all measurements. These lists are required when we wish to enumerate the joint events.

3.2. Marginal events

Based on the solution of the MM-UPDAF filter [1], the MM-UJPDAF solution requires the probability of each of the $I + 2$ events for each model of the reference track. These events are composed of the joint events described above and are referred to here as marginal events. This name is used because these events are a union over the joint events analogous to forming the marginal pdf for one variable of a multi-variable joint pdf.

Denote the i^{th} marginal event for the m^{th} model of the reference track as $\Theta_{i,m}$. Let the pointer $\kappa(i,m)$ identify the i^{th} closest measurement to the m^{th} model of the reference track. That is, $y_{\kappa(i,m)}$ is the i^{th} closest measurement to the m^{th} model of the reference track. For notational brevity, the dependence of κ on the model m and the index i is dropped such that $\kappa \equiv \kappa(i,m)$ and $y_\kappa \equiv y_{\kappa(i,m)}$.

The I+2 marginal events for each model of a reference track are:

$\Theta_{-1,m}$	the model m is the correct model and the target is not visible
$\Theta_{0,m}$	the model m is the correct model and the target is visible but none of the I measurements selected by it are caused by the track
$\Theta_{i,m}$	the model m is the correct model and the target is visible and has caused the i^{th} closest measurement to it (that is measurement y_κ). Here i must lie in the range $[1, I\,]$

To derive an estimate of the track for a particular model m, the reference track model event Ω_m is defined. This is given by $\Omega_m = \bigcup_{i=-1}^{I} \Theta_{i,m}$.

3.3. Relationship between Joint and Marginal events

For the marginal event $\Theta_{i,m}$, model m from the reference track is the source of measurement y_κ when i lies in the range $[1, I\,]$. This event is the union over all joint events where model m from the reference track is the source of measurement y_κ. That is, all joint events where $\lambda_\kappa = (m,1)$.

The union relationship is mathematically expressed as:

$$\Theta_{i,m} = \bigcup_{\Lambda \,|\, \lambda_\kappa = (m,1)} \theta_\Lambda \tag{11}$$

where,

$$\bigcup_{\Lambda \,|\, \lambda_\kappa = (m,1)} \theta_\Lambda \equiv \Cup_1 \ldots \Cup_{\kappa-1} \Cup_{\kappa+1} \ldots \Cup_{IT} \;\theta_\Lambda \tag{12}$$

with

$$\Cup_j \equiv \bigcup_{t_j = \tilde{\tau}_j(0)}^{\tilde{\tau}_j(\tilde{T}_j)} \;\; \bigcup_{m_j = \mu_{j,t_j}(1)}^{\mu_{j,t_j}(M_{j,t_j})} \tag{13}$$

and

$$\Lambda = \begin{bmatrix} (m_1, t_1) \\ \vdots \\ (m_{\kappa-1}, t_{\kappa-1}) \\ (m, 1) \\ (m_{\kappa+1}, t_{\kappa+1}) \\ \vdots \\ (m_{IT}, t_{IT}) \end{bmatrix} \tag{14}$$

The modified track lists are defined by

$$\tilde{\tau}_j = \{0\} \cup \left\{ \tau_j \cap \overline{\{1, t_1, \ldots t_{j-1}\}} \right\} \tag{15}$$

and

$$\tilde{T}_j = card\,(\tilde{\tau}_j) - 1 \tag{16}$$

The modified track list $\tilde{\tau}_j$ is simply the track list for measurement y_j without the tracks that have already been associated with a previous measurement. Using the modified track lists ensures that no track is associated with more than one measurement for any particular joint event.

Note that the union denoted $\Cup_j$ is simply an enumeration of all the track models that measurement y_j can associate with, given the associations already made by measurements 1 . . j-1. The order of the unions cannot be interchanged because of this dependance on the associations with previous measurements. This is reflected in the recursive nature of the measurement modified track list $\tilde{\tau}_j$.

Since the joint events are mutually exclusive, it is apparent that the posterior probability of a marginal event is simply the sum of the posterior probability of the appropriate joint events.

The events corresponding to $i = 0$ and $i = -1$ must be treated slightly differently. The reason for this is that the joint events cannot differentiate between the two. The joint event index determines that the reference track has caused no measurement but it does not describe which model is correct or whether the target is visible. The union of all joint events where the reference track causes no measurement is then equivalent to the union over all models of the events $\Theta_{0,m}$ and $\Theta_{-1,m}$. The expression for this union is given below:

$$\bigcup_{m=1}^{M} \Theta_{0,m} \cup \Theta_{-1,m} = \bigcup_{\Lambda \mid \lambda_{j,t} \neq \{m,1\} \ \forall m,j} \theta_{\Lambda} \tag{17}$$

where,

$$\bigcup_{\Lambda \,|\, \lambda_{j,t} \neq \{m,1\} \; \forall m,j} \theta_{\Lambda} \equiv \Cup_{1} \ldots \Cup_{IT} \; \theta_{\Lambda} \tag{18}$$

and

$$\Lambda = \begin{bmatrix} \left(m_1, t_1\right) \\ \vdots \\ \left(m_j, t_j\right) \\ \vdots \\ \left(m_{IT}, t_{IT}\right) \end{bmatrix} \tag{19}$$

The modified indices are the same form as previously presented in equation (15) and the unions $\Cup_j$ are as presented in equation (13)

Once again, the posterior probability of the union of $\Theta_{-1,m}$ and $\Theta_{0,m}$ is calculated by summing the posterior probabilities of the relevant joint events. To calculate their individual probabilities, the conditional probabilities $Pr\{\Theta_{-1,m} | \Theta_{-1,1} \cup \Theta_{0,1} \cup \ldots \cup \Theta_{-1,M} \cup \Theta_{0,M}\}$ and $Pr\{\Theta_{0,m} | \Theta_{-1,1} \cup \Theta_{0,1} \cup \ldots \cup \Theta_{-1,M} \cup \Theta_{0,M}\}$ are required.

3.4. The Joint Event Tree

One way to better understand the operation of equations (12) and (18) is to use a graphical aid to visualise the event enumeration. It is appropriate to use a tree structure as this graphical aid. The formation of this joint event tree is illustrated by way of example.

Consider a cluster that contains three tracks and four measurements. For the moment, let each track have only one model. The measurement selections of the tracks are described by the track lists below.

$\tau_1 = \{0, 1, 2\}$ with $T_1 = 2$
$\tau_2 = \{0, 1, 2, 3\}$ with $T_2 = 3$
$\tau_3 = \{0, 2, 3\}$ with $T_3 = 2$
$\tau_4 = \{0, 1, 3,\}$ with $T_4 = 2$

In this example, $I = 3$, $IT = 4$ and $T = 3$

Let us now consider the case where track 1 causes no measurement, ie equation (17) applied to this cluster. Starting with the first list, $\tau_1 = \{0,1,2\}$, equation (15) is used to determine $\tilde{\tau}_1$ as follows.

$$\begin{aligned} \tilde{\tau}_1 &= \{0\} \cup \left\{ \{0,1,2\} \cap \overline{\{1\}} \right\} \\ &= \{0\} \cup \left\{ \{0,1,2\} \cap \{0,2,3\} \right\} \\ &= \{0,2\} \end{aligned}$$

Since the first measurement is not selected by track 3, only clutter and track 2 are considered. These values are then taken in turn to generate the modified track lists $\tilde{\tau}_2$. Using equation (15) as above, the lists and the associated value for $\tilde{\tau}_1$ are:

$$\tilde{\tau}_2 = \begin{cases} \{0,2,3\} & \text{when } t_1 = \tilde{\tau}_1(0) = 0 \\ \{0,3\} & \text{when } t_1 = \tilde{\tau}_1(1) = 2 \end{cases}$$

A simplified way to undertake these steps and determine the track lists is to use the joint event tree shown in Figure 3. Each node on this tree gives the track associated with a measurement and each layer corresponds to a particular measurement. The leaf nodes of the tree contain the source of the measurement y_{lT} and the path through the tree to reach the leaf node indicates the source for each of the other measurements. The children at each node represent the list of possible track associations, $\tilde{\tau}_j$.

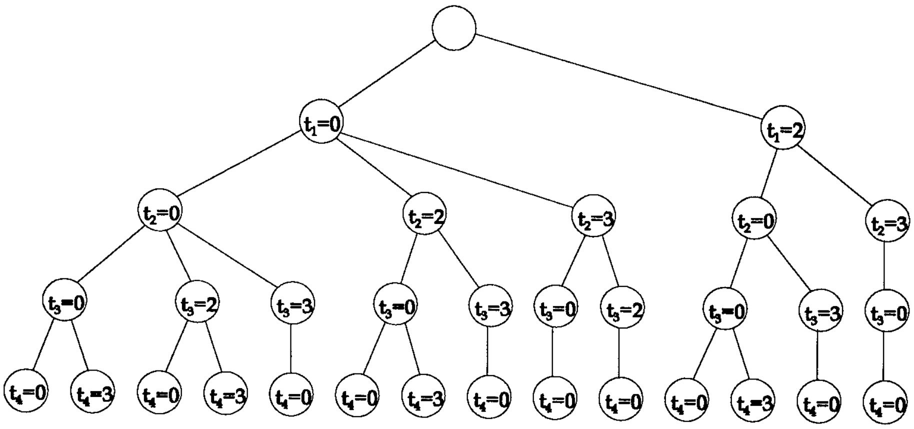


Figure 3. Example joint event tree

The example above shows that the branches available for selecting the value of t_2 depend on what value t_1 has taken. This is also highlighted by the modified track lists preceding Figure 3. From the joint event tree, the left most path gives $\Lambda = \{0,0,0,0\}$, ie all measurements are from clutter. Likewise the rightmost path gives $\Lambda = \{2,3,0,0\}$.

Adding more models to the tracks does not change the tree structure. When there is more than one model, each node on the tree represents a track choice. After the track is chosen, a model from that track is chosen using $\mu_{j,t}$ to determine which models have selected that measurement. The models add breadth to the tree, however, since each track may only have one true model, the lower branches of the tree are the same for each model

The joint event tree is a useful tool not only because it presents a formalised exhaustive enumeration scheme but also because of the factorisation it represents. Using the tree type approach to enumerating the events is much like the *divide and conquer* strategy used in such algorithms as *fast fourier transform* and *quicksort* [10]. This factorisation is important in the calculation of the marginal event probabilities as described in the following section.

4. Target State Estimate

The posterior state estimate for the JPDA filter is the same as that used for the single target multiple model PDA filter in ref with the exception of the event probability calculations. However, the state estimate is repeated here for completeness and clarity. To simplify the equations, the track index *t* is omitted.

From Section 2, the data provided by the sensor with each multiple model track consists of the selected measurements *Y*(*k*), the regions containing the *I* nearest measurement *SI*, the clutter region parameters at time *k*, ie *n*, *N*, *O* and the past track data Z(*k*-1). For notational clarity the time dependence of the sensor data is not shown. From the defined target and sensor model in Section 2 and the marginal events defined in Section 3.2, the optimal estimate of the target state $x(k)$ is,

$$\begin{aligned} \hat{x}(k|k) &= \mathrm{E}\{x(k)|\mathrm{Y},SI,n,N,O,Z\} \\ &= \sum_{m=1}^{M} \sum_{i=-1}^{I} \beta_{i,m}\, \mathrm{E}\{x_m(k)|\Theta_{i,m},\mathrm{Y},SI,n,N,O,Z\}. \end{aligned} \tag{20}$$

Where $\beta_{i,m}$ = $\Pr\{\Theta_{i,m}|\mathrm{Y},SI,n,N,O,Z\}$

$\Theta_{i,m}$ = is the marginal event as defined in Section 3.2

The evaluation of $\mathrm{E}\{x_m(k)|\Theta_{i,m},Y,SI,n,N,O,Z\}$ for $i > 0$ is the Kalman filter solution for updating the predicted state, $\hat{x}_m(k|k-1)$, of a target represented by model *m*, with measurement y_i. Thus, equation (20) may be written as the following summation.

$$\hat{x}(k|k) = \sum_{m=1}^{M} \sum_{i=-1}^{I} \beta_{i,m}\hat{x}_{i,m}(k|k) \tag{21}$$

Where $\hat{x}_{i,m}(k|k)$ = $\hat{x}_m(k|k-1) + \mathbf{W}_m\tilde{y}_{i,m}$ $i = 1,..,I$

$\hat{x}_{i,m}(k|k)$ = $\hat{x}_m(k|k-1)$ $i = -1, 0$

$\mathbf{W}_\mathrm{m}$ = $\left(\mathbf{F}_m\mathbf{P}_m(k-1|k-1)\mathbf{F}_m^\mathrm{T} + \mathbf{Q}_m\right)\mathbf{H}^\mathrm{T}\mathbf{S}_m^{-1}$

ie the Kalman gain matrix for the m^{th} model.

From equation (21), each model is similar to a PDA filter. These remain separate with the contribution of a model to the state estimate tending to zero as its probability approaches zero. This filter therefore combines the approaches taken in PDA and track split.

The selection process requires the estimate of each model's state given it is valid. This is obtained by writing equation (20) in the following form.

$$\hat{x}(k|k) = \sum_{m=1}^{M} \beta_m \hat{x}_m(k|k). \tag{22}$$

Where β_m = $\Pr\{\Omega_m|Y,SI,n,N,O,Z\} = \sum_{i=-1}^{I} \beta_{i,m}$

$\hat{x}_m(k|k)$ = $E\{x_m(k)|\Omega_m,Y,SI,n,N,O,Z\}$.

From equations (20) and (22), the estimate of each model's state is then given by,

$$\hat{x}_m(k|k) = \hat{x}_m(k|k-1) + \mathbf{W}_m\,\tilde{y}_m(k|k). \tag{23}$$

Where $\tilde{y}_m$ = $\dfrac{\sum_{i=1}^{I} \beta_{i,m}\tilde{y}_{i,m}}{\beta_m}$

Now the covariance of $\hat{x}(k|k)$ is a weighted sum of Gaussians where each is derived from the PDAF approximation for each model. The event probabilities for each model require the covariance for $\hat{x}_m(k|k)$ which is denoted by $\mathbf{P}_m(k|k)$. This is derived in Appendix A and is given by the expression.

$$\begin{aligned}\mathbf{P}_m(k|k) = {} & \frac{\beta_{-1,m}}{\beta_m}\mathbf{P}_{-1,m}(k|k-1) + \frac{\beta_{0,m}}{\beta_m}\Sigma_m(k|k-1) + \left(1 - \frac{\beta_{-1,m} + \beta_{0,m}}{\beta_m}\right)\Sigma_m(k|k) \\ & + \mathbf{W}_m\left(\sum_{i=1}^{I}\frac{\beta_{i,m}}{\beta_m}\tilde{y}_{i,m}\tilde{y}_{i,m}^{\mathrm{T}} - \tilde{y}_m\tilde{y}_m^{\mathrm{T}}\right)\mathbf{W}_m^{\mathrm{T}}\end{aligned} \tag{24}$$

Where $\Sigma_m(k|j)$ is the Kalman filter covariance for model m at time k given data to time j.

$\mathbf{P}_{-1,m}(k|j)$ is the covariance for model m when the target is invisible at time k given data to time j.

The display of track position to an operator requires the state estimate $\hat{x}(k|k)$ while multi-sensor track fusion uses the covariance $\mathbf{P}(k|k)$. This is obtained from the model covariance $\mathbf{P}_m(k|k)$ using the following expression.

$$\mathbf{P}(k|k) = \sum_{m=1}^{M} \beta_m[\mathbf{P}_m(k|k) + (\hat{x}_m - \hat{x})(\hat{x}_m - \hat{x})^{\mathrm{T}}] \tag{25}$$

The derivation of the event probabilities $\beta_{i,m}$ required for equations **(20)** and (24) is given in Appendix A. These probabilities simplify to,

$$\beta_{i,m} = \frac{b_{i,m}}{\sum\limits_{l=1}^{M} \sum\limits_{j=-1}^{I} b_{j,l}} . \tag{26}$$

Where

$$\begin{aligned} b_{i,m} = \psi_m(y_\kappa, 1) \sum_{t_1=\tilde{\tau}_1(0)}^{\tilde{\tau}_1(\bar{T}_1)} \psi(y_1, t_1) \Bigg\{ \ldots \sum_{t_{\kappa-1}=\tilde{\tau}_{\kappa-1}(0)}^{\tilde{\tau}_{\kappa-1}(\bar{T}_{\kappa-1})} \psi(y_{\kappa-1}, t_{\kappa-1}) \\ \left[\sum_{t_{\kappa+1}=\tilde{\tau}_{\kappa+1}(0)}^{\tilde{\tau}_{\kappa+1}(\bar{T}_{\kappa+1})} \psi(y_{\kappa+1}, t_{\kappa+1}) \left(\ldots \left(\sum_{t_{IT}=\tilde{\tau}_{IT}(0)}^{\tilde{\tau}_{IT}(\bar{T}_{IT})} \psi(y_{IT}, t_{IT}) \prod_{c=1}^{C} \binom{N_c - n_{c,\Lambda}}{n_c - n_{c,\Lambda}} \right) \ldots \right) \right] \Bigg\} \end{aligned}$$

$$b_{0,m} = \frac{Pm_{1,m} Pv_1 (1 - Pd_{1,m} Ps_{1,m})}{P\bar{t}_1} \sum_{t_1=\tilde{\tau}_1(0)}^{\tilde{\tau}_1(\bar{T}_1)} \psi(y_1, t_1) \left\{ \ldots \left(\sum_{t_{IT}=\tilde{\tau}_{IT}(0)}^{\tilde{\tau}_{IT}(\bar{T}_{IT})} \psi(y_{IT}, t_{IT}) \prod_{c=1}^{C} \binom{N_c - n_{c,\Lambda}}{n_c - n_{c,\Lambda}} \right) \ldots \right\}$$

$$b_{-1,m} = \frac{Pm_{1,m} (1 - Pv_1)}{P\bar{t}_1} \sum_{t_1=\tilde{\tau}_1(0)}^{\tilde{\tau}_1(\bar{T}_1)} \psi(y_1, t_1) \left\{ \ldots \left(\sum_{t_{IT}=\tilde{\tau}_{IT}(0)}^{\tilde{\tau}_{IT}(\bar{T}_{IT})} \psi(y_{IT}, t_{IT}) \prod_{c=1}^{C} \binom{N_c - n_{c,\Lambda}}{n_c - n_{c,\Lambda}} \right) \ldots \right\}$$

in the above equations,

$$P\bar{t}_i \equiv \sum_{m=1}^{M} Pm_{i,m} \left\{ 1 - Ps_i \;\; Pd_{i,m}\, Pv_i \right\}$$

$$\psi_m(y,t) \equiv \begin{cases} \dfrac{Pd_{m,t}\, Pm_{m,t}\, Pv_t\, f_{m,t}(y)}{g_{c_y}(y)\;\; I\;\; P\bar{t}_t} & m \in [1,M],\; t \in [1,T] \\ 1 & m=0,\; t=0 \end{cases}$$

$$\psi(y_i, t_j) \equiv \sum_{m=\mu_{i,j}(0)}^{\mu_{i,j}(M_{i,j})} \psi_m(y_i, t_j)$$

The b equations above are somewhat complex and are more easily understood by revisiting the joint event tree. Consider the calculation of $b_{0,m}$ for the cluster example given in section 3.4. The joint event tree for this cluster is shown in Figure 3. To calculate the event probability, we use the joint event tree and place values on each node. The root node takes the value $\frac{Pm_{m,1}\,Pv_1\,(1 - Pd_{m,1}\,Ps_{m,1})}{P\bar{t}_1}$ and the leaf nodes take the values $\psi(y_{IT}, t_{IT})\ \prod_{c=1}^{C}\binom{N_c - n_{c,\Lambda}}{n_c - n_{c,\Lambda}}$. The intermediate nodes take a value of $\psi(y_j, t_j)$. This new tree is shown in Figure 4. The event probability is then calculated by summing the leaf nodes and multiplying the sum by the value of the parent node. The parent node value is replaced by this product and the leaf nodes are discarded. This process is repeated until all the nodes of the tree have been removed and the resulting value is the un-scaled event probability $b_{0,m}$

In Figure 4 only the left branch of the tree is shown to simplify the diagram. The clutter associations have also been filled with the value $\psi(y_j, 0) = 1$. The number of clutter zones is assumed to be one.

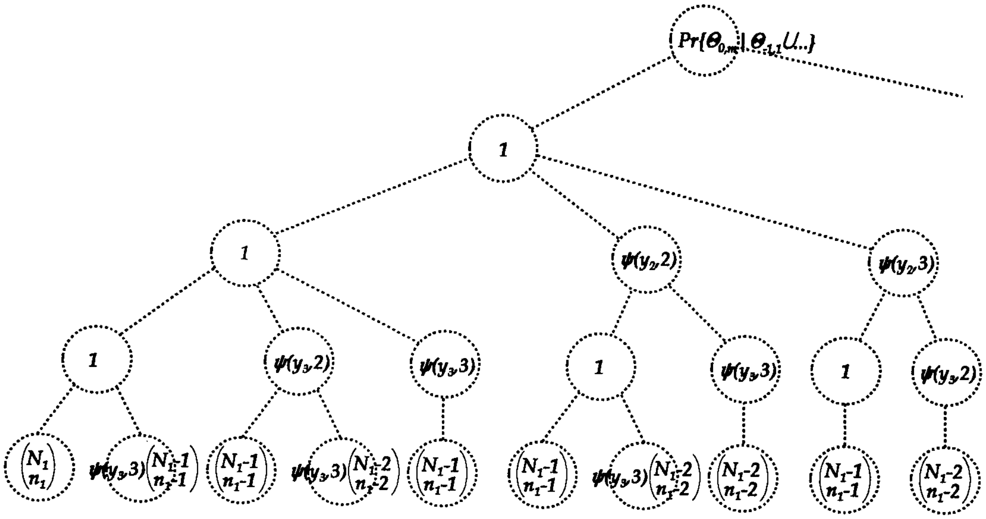


Figure 4. Joint event tree with node values

The *a priori* visibility Pv is given by equation (6). From the event probabilities $\beta_{i,m}$ the *a posteriori* target visibility is given by,

$$Pv(k|k) = 1 - \sum_{m=1}^{M} \beta_{-1,m} \tag{27}$$

This visibility is then used in equation (6) as the past data in the next time step with the *a priori* transition probabilities.

5. Filter Performance

The MM-UJPDAF filter has been implemented to operate on the Jindalee Over-the-Horizon Radar (1RSU). The implementation of this filter differs from the standard and most intuitive method. It factorises the event probability equations to give a considerable computation advantage. The advantages of the factorised approach are now considered. This is then followed by examples of performance on OTHR data.

5.1. Computation Load

The brute force method of implementing the JPDAF is to enumerate all possible joint events and calculate their event probabilities. When the event probability is calculated, the algorithm determines which marginal events this joint event contributes to and accumulates their values. As has been described in section 4 this is not the way that the JPDA algorithm for 1RSU has been implemented. Instead, by factorisation, a closed expression for each marginal probability has been found and the probability is calculated directly. This leads to a reduction in the computation requirement and the functional complexity.

There are a number of existing 'fast JPDAF' algorithms which have been designed to reduce the computational requirements of the filter. These algorithms generally take a similar form to the brute force approach but intelligently use previous event calculations to reduce the effort required to calculate the probability of similar joint events. One such algorithm is the LSA of [9]. This algorithm takes advantage of factorisation to compute the joint event probabilities. The LSA is a more efficient method than the MM-UJPDAF for calculating the marginal probabilities in the absence of multiple models. However, the complexity of the LSA rises sharply when multiple models are introduced.

The reason why the LSA is more efficient than the presented form of the MM-UJPDAF is that the MM-UJPDAF forms a separate cluster for each track and calculates the joint events for the cluster. When several tracks form the same cluster, they will share common joint events, however the MM-JPDAF does not take this into account. Remember that the cluster rules for the MM-UJPDAF limit the cluster for the case of a reference track. When these constraints are relaxed, larger cluster sizes are likely with shared common joint events. It is difficult to compare the event calculation because of the difference in clustering. However, we examine the LSA load with tracks and models.

A bound on the number of multiplication operations required for the core of the LSA method is given by [9] as:

$$N_{mult} < \sum_{L=2}^{T} C_T^L I^{L-1} (I+L-2) \tag{28}$$

assuming that each track selects I measurements.

When each track consists of M models, this expression becomes:

$$N_{mult} < \sum_{L=2}^{T} C_T^L (I \times M)^{L-1} (I \times M + L - 2) \tag{29}$$

The above bound is loose, but it is significant to notice that the computational complexity is polynomial of order $I{\times}M^{T-1}$. For a particular cluster size (fixed I and T) the number of computations required grows at a rate determined by an order T-1 polynomial in M.

The MM-UJPDAF approach requires a separate tree calculation for each model, however, each tree is independent of the number of models since the tree nodes contain values which have been summed over all models for the corresponding association. Also, models from a track are likely to share measurements. Since the root of the tree is the only point dependent on the model, tree calculations for common measurements can be shared. The result is that the MM-UJPDAF algorithm grows less than linearly with the number of models.

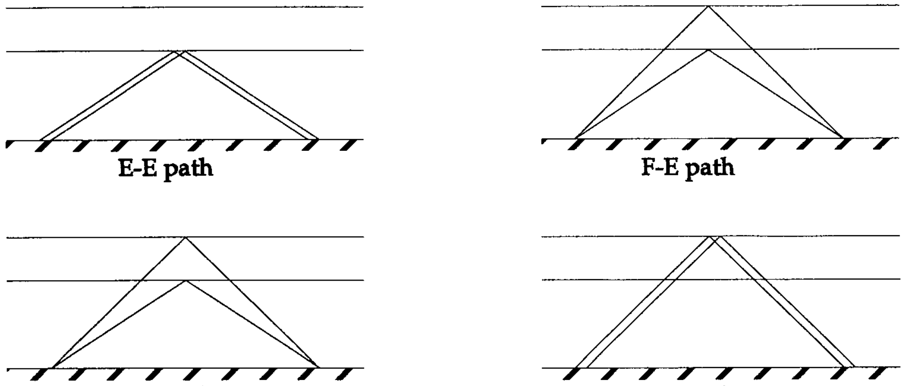
It is infeasible to compare the measured computation performance of the MM-UJPDAF with existing approaches since considerable effort would be required to implement these alternative algorithms.

5.2. Performance examples

If the number of tracks in the cluster is only one, then the marginal event probabilities simplify to the single target filter equations as presented in [1] (as one would expect). Having observed this, it is apparent that the performance of the JPDAF is the same as the single target PDAF for cases where the target tracks are well separated and the clustering process does not add neighbouring tracks into a cluster. It is useful then to only consider the performance of the filter in circumstances where neighbouring tracks do interfere with each reference track.

5.2.1. Multi-path Performance

In OTHR, the most common source for closely spaced target returns is the multi-path propagation medium. Under most conditions, the ionosphere provides at least two possible propagation paths between the radar and the target. In the simplest model, the ionosphere can be considered as multiple reflecting planes. The two main propagation layers are called the E and F layers. With these two layers present, there exist four propagation paths as shown in Figure 5.



E-E path

F-E path

E-F path

F-F path

Figure 5. Multi-path propagation in OTHR

The radar measures range using the measured time delay so that different paths appear separated in range due to the different path length. The closest paths are the E-F and F-E paths, called the *mixed mode* paths. The paths both give the same time delay (that is group or slant range) but because they are received at different elevation angles the coning effect of a linear array causes them to be measured at different azimuths. The elevation of the transmitted signal also causes slightly different radial velocity measurements. Now, since the radar does not have a measurement of tangential speed, the tracks must be initialised with fairly large azimuthal covariances. The nature of the ionosphere also tends to make the azimuthal measurement more noisy than the range and radial velocity measurements. These factors conspire to produce a tracking environment where it is common to find parallel tracks with the same range and speed but separated in azimuth. Due to various environmental factors, these propagation paths tend to fade and return with the result that a single target tracker may not adequately track these paths.

The JPDA has been demonstrated to improve tracking under this condition. An example of this improvement is shown in Figure 6. The single target PDAF performance is shown in the left plot and the multi-target JPDAF performance is shown on the right. The vertical axis is range and the horizontal axis is azimuth. Here the single target tracker fails to correctly track the two mixed mode target returns. The tracks formed by the single target filter cross over when the propagation modes fade and the tracks are lost. In contrast, the JPDAF holds the two tracks along their separate traces.

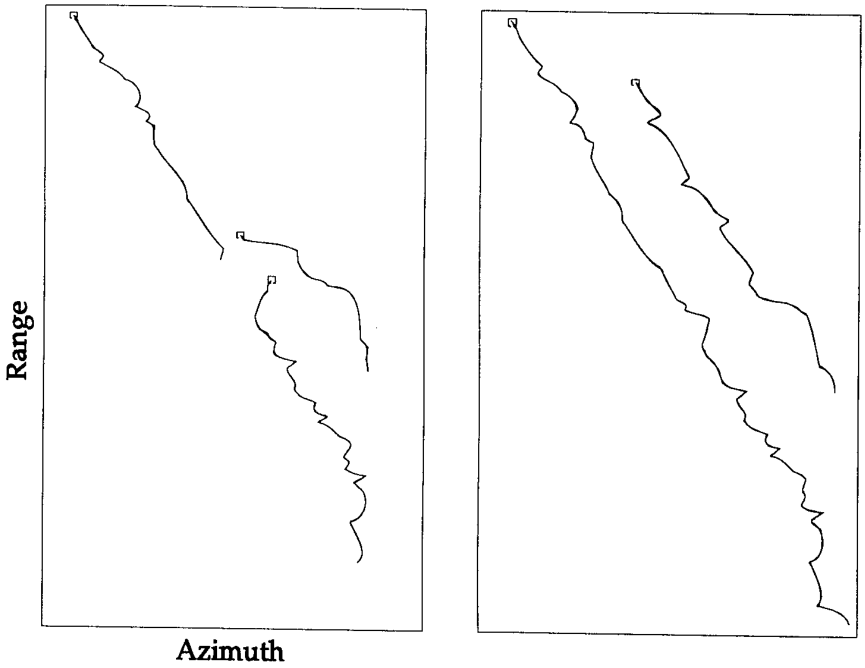


Figure 6. Multi-path tracking performance

5.2.2. True Multi-Target Performance

The filter also improves the tracking performance for more traditional multi-target tracking scenarios. Figure 7 below shows the improved performance for the case of two crossing target tracks. In this example, the two tracks have similar radial velocities, but their tangential speeds are different. When the two tracks cross, the single target filter drops one track and the other track switches to the trajectory of the dropped track. The multi-target filter correctly follows the crossing trajectories. As in Figure 6 the single target filter is shown on the left and the multi-target filter on the right. The tracks are plotted as range against azimuth.

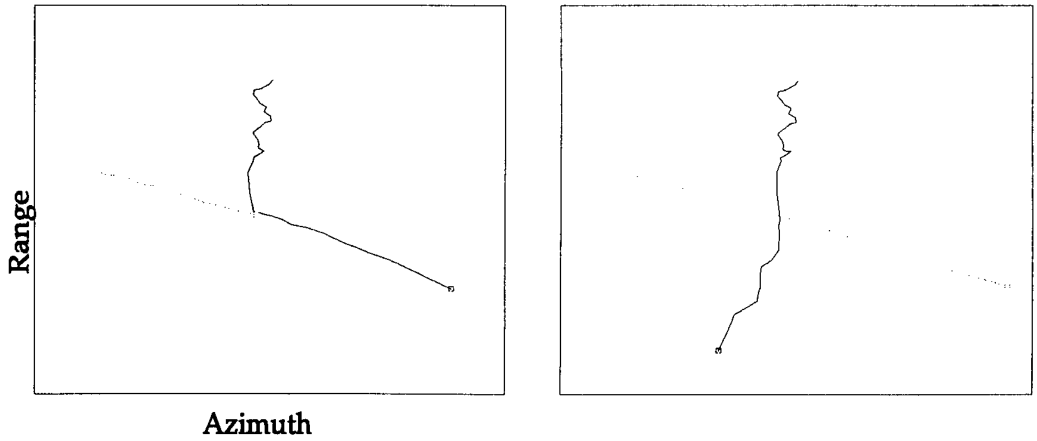


Figure 7. Crossing Target Scenario

The third example of the filter performance is shown in Figure 8. This is a much more congested target situation. In this case there are three main targets each being propagated over number of modes. These targets are moving at slightly different speeds, but they are very close geographically. In addition to these targets, there are two slower targets at close range. These two targets are actually the tracks illustrated in Figure 7. Due to the dense traffic, it is uninformative to view the tracks on a range-azimuth plot as in Figure 6 and Figure 7. Instead, the tracks are presented in Figure 8 as range against time. Each of the propagation modes can be observed at a different range and for each mode there should be three tracks - one for each of the targets. As can be seen, the JPDAF improves the performance on this data considerably and separates the individual targets well. The single horizontal trace at lower range is a synthetic calibration signal injected at the radar receiver.

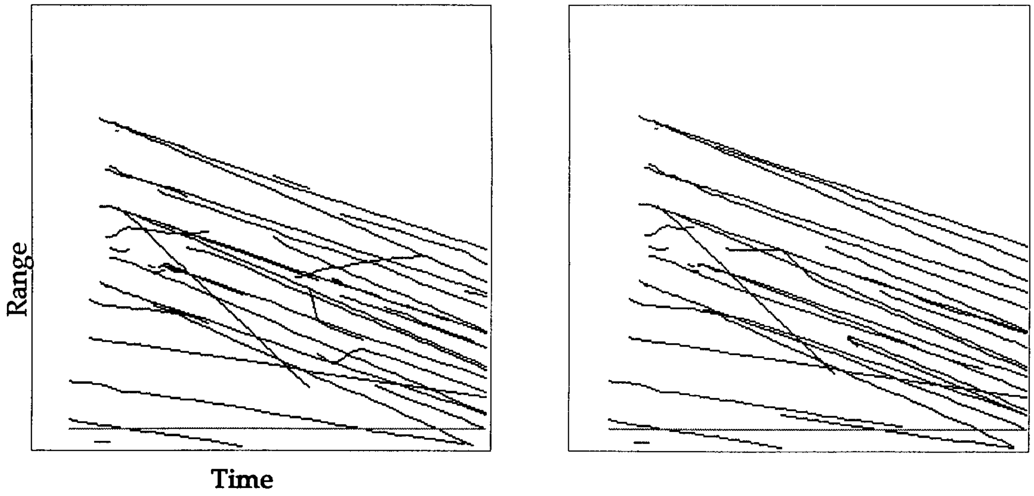


Figure 8. Single Target and Multi-Target Performance for High Density Targets

6. Conclusion

This paper has developed the theory for the multiple target extension to a multiple model filter that was originally developed for track initiation with an OTHR. Including multiple targets with multiple models greatly complicates the evaluation of the event probabilities. To simplify this step, the event space was developed with the aid of a new joint event tree. The event tree allows the efficient real-time evaluation of the event probabilities by factorizing the event space. This new approach is more efficient than current methods, particularly when dealing with multiple models. This system has demonstrated real-time operation when the track cluster size reaches 10 with over 4 models.

The multiple target extension incorporates all the previous features in the multiple model filter, ie target visibility, multiple non-uniform clutter zones and selection of measurements based on nearest neighbours. The new filter was tested on Jindalee OTHR recorded data. It was implemented for the tracking stage following track initiation. Once a track became established, a JPDA filter was used. This tracking system demonstrated enhanced performance when targets become closely spaced. With distant targets, the same performance is obtained as for the original filter without multiple targets. So this new filter extends the performance envelope of the previous filter and differences are only noted under conditions of closely spaced targets.

Future work is planned to add the multiple model filter extension to the JDPA filter for tracking manoeuvring target during established target tracking. This will be considered with other possible enhancements.

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Appendix A Derivation of Event Probabilities

To calculate the track posterior state estimate we require the posterior probabilities of the marginal events. These probabilities are calculated from the sum of joint event posterior probabilities (refer to Section 3). To derive an expression for the marginal events, we begin by seeking an expression for the posterior probability of an arbitrary joint event.

Firstly let us revisit the definitions of the cluster and the clutter. There are *IT* measurements and *T* tracks in the cluster. Each track has *M* models. Let *U* denote the whole cluster. The clutter zone associated with measurement y_j is c_j. *O* is the set of clutter zones each of which contains N_c measurements respectively. Of these, n_c are found in the track cluster and α_c of these are associated with tracks under the event θ_Λ.

The posterior probability of the a joint event can be expanded using Bayes' Rule to give:

$$\begin{aligned} P\left(\theta_\Lambda \mid U,O,Z\right) &= \frac{P\left(\theta_\Lambda,U \mid Z,O\right)}{P(U \mid Z,O)} \\ &= \frac{P\left(\theta_\Lambda,U \mid Z,O\right)}{\sum\limits_{\theta_\Lambda} P\left(\theta_\Lambda,U \mid Z,O\right)} \end{aligned} \qquad \textbf{(A1)}$$

The denominator of the above expression can be considered as a normalising factor and is the sum of the numerator over the whole event space. We need therefore only consider the numerator. Recognise that the cluster consists of the tracks, the measurements, the number of each of these and the track selection boundaries. Since the track positions and the number of tracks is fully determined by the past data these features are not probabilistic. Thus *U* is expanded into the measurements *Y*, the number of measurements *n* and the cluster boundary *S*. The numerator is a joint probability that can be expanded as a product of conditional probabilities (again using Bayes' Rule).

$$\begin{aligned} P\left(\theta_\Lambda,U \mid Z,O\right) &= P\left(\theta_\Lambda,Y,n,S \mid Z,O\right) \\ &= P\left(Y \mid n,\theta_\Lambda,S,Z,O\right)\ P\left(n \mid \theta_\Lambda,S,Z,O\right)\ P\left(\theta_\Lambda \mid S,Z,O\right)\ P(S \mid Z,O) \end{aligned} \qquad \textbf{(A2)}$$

The first term is the probability of the measurements given the number of measurements and the source of each measurement as defined by the joint event. Since the measurements are independent, this is simply the product over all measurements of the measurement density for each measurement. Thus, the first term in equation **(A2)** is given by:

$$P\left(Y \,|\, n, S, \theta_\Lambda, Z, O\right) = \prod_{\lambda_l \in \Lambda | t_l \neq 0} \tilde{f}_{\lambda_l}(y_l) \prod_{\lambda_j \in \Lambda | t_j = 0} \tilde{g}_{c_j}(y_j)$$

$$= \prod_{\lambda_l \in \Lambda | t_l \neq 0} \frac{\tilde{f}_{\lambda_l}(y_l)}{\tilde{g}_{c_l}(y_l)} \prod_{j=1}^{IT} \tilde{g}_{c_j}(y_j) \qquad \textbf{(A3)}$$

where $\tilde{g}_{c_j}(y_j)$ is the probability density function for measurement y_j being from clutter given it is within $C_{c_j} \cap S$,

ie $\dfrac{g_{c_j}(y_j)}{Pc_{c_j}}$, with $Pc_{c_j} = \int_{y \in S} g_{c_j}(y)\, dy$

$\tilde{f}_{\lambda_l}(y_l)$ is the probability density function for a measurement caused by model m_l of target t_l visible, detected and within the region containing the I nearest measurements SI,

ie $\dfrac{f_{\lambda_l}(\lambda_l)}{Ps_{\lambda_l}}$, with $Ps_{t,m} = \int_{y \in S} f_{t,m}(y)\, dy$

The second term is equation **(A2)** is the probability of observing n_c-α_c measurements from clutter zone c in the selection area given the total measurement density and the size of the selection area defined by N_c, A_c and a_c. This is given by the binomial distribution and is written:

$$P\left(n \,|\, \theta_\Lambda, S, Z, O\right) = \prod_{c=1}^{C} \binom{N_c - n_{c,\Lambda}}{n_c - n_{c,\Lambda}} Pc_c^{\,n_c - n_{c,\Lambda}} \left(1 - Pc_c\right)^{N_c - n_c} \qquad \textbf{(A4)}$$

The third term in equation **(A2)** is the prior probability of the event. This is the prior probability that each track causes the output described by the event index Λ. Since the tracks are independent, this is the product over all tracks of the prior probability for that track.

For the tracks which cause measurements under Λ we require the prior probability that $\lambda_{l,m}$ is the correct model for track $\lambda_{l,t}$ and that this track is visible, detected and has formed the i^{th} measurement inside S. This probability is given by the expression:

$$\frac{Ps_{m,t}\, Pd_{m,t}\, Pm_{m,t}\, Pv_t}{I}$$

where,

$$Ps_{m,t} \equiv P\left(S_{m,t} | D_{m,t}, M_t, V_t, Z\right) \qquad Pd_{m,t} \equiv P\left(D_{m,t} | M_t, V_t, Z\right)$$

$$Pm_{m,t} \equiv P\left(M_{m,t} | V_t, Z\right) \qquad Pv_t \equiv P\left(V_t \,| Z\right)$$

For the tracks that cause no measurement, this is the sum over all models of the probability that model m is the correct model for track t but that the track is either invisible, not detected or did not cause any of the I nearest measurements. This quantity is given by:

$$\sum_{m=1}^{M} Pm_{m,t}\left\{ (1-Ps_{m,t})\ Pd_{m,t}\ Pv_t + (1-Pd_{m,t})\ Pv_t + (1-Pv_t)\right\}$$

Expanding the bracketed expressions, this simplifies to:

$$\sum_{m=1}^{M} Pm_{m,t}\left\{ 1-Ps_{m,t}\ Pd_{m,t}\ Pv_t\right\}$$

This quantity is represented by the symbol $P\bar{t}_t$.

Thus the prior probability of the event is written as:

$$P\left(\theta_\Lambda | S,Z,O\right) = \prod_{\lambda_l \in \Lambda | t_l \neq 0} \frac{Ps_{\lambda_l}\ Pd_{\lambda_l}\ Pm_{\lambda_l}\ Pv_{t_l}}{I\ P\bar{t}_{t_l}} \prod_{j=1}^{T} P\bar{t}_j \tag{A5}$$

Substituting equations **(A3)** **(A4)** and **(A5)** into **(A2)** gives:

$$P\left(\theta_\Lambda, U | Z,O\right) = \prod_{\lambda_l \in \Lambda | t_l \neq 0} \frac{\tilde{f}_{\lambda_l}(y_l)\ Ps_{\lambda_l}\ Pd_{\lambda_l}\ Pm_{\lambda_l}\ Pv_{t_l}}{\tilde{g}_{c_l}(y_l)\ I\ P\bar{t}_{t_l}} \prod_{c=1}^{C} \binom{N_c - n_{c,\Lambda}}{n_c - n_{c,\Lambda}} Pc_c^{-n_{c,\Lambda}}$$
$$\left\{ \prod_{c=1}^{C} Pc_c^{n_c} (1-Pc_c)^{N_c - n_c} \prod_{j=1}^{IT} \tilde{g}_{c_j}(y_j) \prod_{t=1}^{T} P\bar{t}_t\ P(S|Z,O) \right\} \tag{A6}$$

The bracketed expression in **(A6)** is constant for all events and is given the symbol K. The noise density term in the initial product may be rewritten as $\frac{g_{c_l}(y_l)}{Pc_{c_l}}$. The Pc_{c_l} in the denominator of this expression then cancels with the $Pc_c^{-\alpha_c}$ term in the second product. Similarly, the Ps_{λ_l} term in the first product is cancelled by the denominator of the target density function $\tilde{f}_{\lambda_l}(y_l)$. Thus equation **(A6)** simplifies to:

$$P\left(\theta_\Lambda, U | Z,O\right) = \mathrm{K} \prod_{\lambda_l \in \Lambda | t_l \neq 0} \frac{f_{\lambda_l}(y_l)\ Pd_{\lambda_l}\ Pm_{\lambda_l}\ Pv_{t_l}}{g_{c_l}(y_l)\ I\ P\bar{t}_{t_l}} \prod_{c=1}^{C} \binom{N_c - n_{c,\Lambda}}{n_c - n_{c,\Lambda}} \tag{A7}$$

define
$$\psi_m(y_j,t) \equiv \begin{cases} \dfrac{f_{m,t}(y_j)\, Pd_{m,t}\, Pm_{m,t}\, Pv_t}{g_{c_j}(y_j)\ I\ P\bar{t}_t} & t\in[1,T],\ m\in[1,M] \\ 1 & t=0,\ m=0 \end{cases}$$

then we may write:

$$P\left(\theta_\Lambda, U \,|\, Z, O\right) = \mathrm{K} \prod_{l=1}^{IT} \psi_{m_l}(y_l, t_l) \prod_{c=1}^{C} \frac{(N_c - n_{c,\Lambda})!}{(N_c - n_c)!\ (n_c - n_{c,\Lambda})!} \tag{A8}$$

Substituting equation **(A8)** into **(A1)** then gives the posterior probability of the joint event defined by the index Λ.

We now proceed to derive an expression for the marginal event probabilities.

Recall that for $i\in[1,I]$

$$\Theta_{i,m} = \bigcup_{\Lambda\,|\,\lambda_\kappa = (1,m)} \theta_\Lambda$$

so,

$$\begin{aligned} Pr\left(\Theta_{i,m} \,|\, U,O,Z\right) &= Pr\left(\bigcup_{\Lambda\,|\,\lambda_\kappa = (1,m)} \theta_\Lambda \,\middle|\, U,O,Z \right) \\ &= \sum_{\Lambda\,|\,\lambda_\kappa = (1,m)} Pr\left(\theta_\Lambda \,|\, U,O,Z\right) \\ &\propto \sum_{\Lambda\,|\,\lambda_\kappa = (1,m)} \prod_{l=1}^{IT} \psi_{m_l}(y_l, t_l) \prod_{c=1}^{C} \frac{(N_c - n_{c,\Lambda})!}{(N_c - n_c)!\ (n_c - n_{c,\Lambda})!} \end{aligned} \tag{A9}$$

and similarly,

$$Pr\left(\bigcup_{m=1}^{M} \Theta_{0,m} \cup \Theta_{-1,m} \right) \propto \sum_{\Lambda\,|\,t_j \neq 1\ \forall j} \prod_{l=1}^{IT} \psi_{m_l}(y_l, t_l) \prod_{c=1}^{C} \frac{(N_c - n_{c,\Lambda})!}{(N_c - n_c)!\ (n_c - n_{c,\Lambda})!} \tag{A10}$$

If we define $b_{i,m}$ such that

$$Pr\left(\Theta_{i,m} \middle| U,O,Z\right) = \frac{b_{i,m}}{\sum_{m=1}^{M}\sum_{j=-1}^{I} b_{j,m}}$$

then a solution for $b_{i,m}$ for $i \in [1,I]$ as:

$$b_{i,m} = \sum_{\Lambda \mid \lambda_\kappa = (1,m)} \prod_{l=1}^{IT} \psi_{m_l}(y_l,t_l) \prod_{c=1}^{C} \frac{(N_c - n_{c,\Lambda})!}{(N_c - n_c)!\ (n_c - n_{c,\Lambda})!} \qquad \textbf{(A11)}$$

The summation above is expanded using the track lists described in section 3.1 to give:

$$b_{i,m} = \sum_{t_1}\sum_{m_1}\cdots\sum_{t_{\kappa-1}}\sum_{m_{\kappa-1}}\ \sum_{t_{\kappa+1}}\sum_{m_{\kappa+1}}\cdots\sum_{t_{IT}}\sum_{m_{IT}} \prod_{l=1}^{IT} \psi_{m_l}(y_l,t_l) \prod_{c=1}^{C} \frac{(N_c - n_{c,\Lambda})!}{(N_c - n_c)!\ (n_c - n_{c,\Lambda})!} \qquad \textbf{(A12)}$$

For compactness the summation ranges in **(A12)** have been suppressed. The summation ranges implicitly demonstrate the nested summation structure since the range for each measurement depends on the assignments of all the preceding measurements. These summation ranges are defined as follows:

$$\sum_{t_j}\sum_{m_j} \equiv \sum_{t_j=\tilde{\tau}_j(0)}^{\tilde{\tau}_j(\tilde{T}_j)} \ \sum_{m_j=\mu_{j,t_j}(1)}^{\mu_{j,t_j}(M_{j,t_j})}$$

Now, it is possible to factorise **(A12)** since the $\psi_{m_l}(y_l,t_l)$ depend only on the summations prior to l. That is, m_l and t_l are independent of all m_p t_p for $p > l$. Also, due to the marginal event of interest, m_κ and t_κ are always m and 1 respectively independent of all other m_p t_p. So we may rewrite **(A12)** as:

$$\begin{aligned} b_{i,m} = {} & \psi_m(y_\kappa,1)\sum_{t_1}\sum_{m_1}\psi_{m_1}(y_1,t_1)\ldots\sum_{t_{\kappa-1}}\sum_{m_{\kappa-1}}\psi_{m_{\kappa-1}}(y_{\kappa-1},t_{\kappa-1})\ \sum_{t_{\kappa+1}}\sum_{m_{\kappa+1}}\psi_{m_{\kappa+1}}(y_{\kappa+1},t_{\kappa+1}) \\ & \ldots\sum_{t_{IT}}\sum_{m_{IT}}\psi_{m_{IT}}(y_{IT},t_{IT}) \prod_{c=1}^{C} \frac{(N_c - n_{c,\Lambda})!}{(N_c - n_c)!\ (n_c - n_{c,\Lambda})!} \end{aligned} \qquad \textbf{(A13)}$$

Now, since m_p t_p are independent of all model associations m_l for other tracks, we may collect together the ψ_m as:

$$\psi(y_l,t_l) = \sum_{m_l=\mu_{i,t_l}(0)}^{\mu_{i,t_l}(M_{i,t_l})} \psi_{m_l}(y_l,t_l)$$

Substituting this gives the final expression for $b_{i,m}$ as:

$$b_{i,m} = \psi_m(y_\kappa,1)\sum_{t_1} \psi(y_1,t_1)\ldots\sum_{t_{\kappa-1}} \psi(y_{\kappa-1},t_{\kappa-1}) \sum_{t_{\kappa+1}} \psi(y_{\kappa+1},t_{\kappa+1}) \ldots\sum_{t_{IT}} \psi(y_{IT},t_{IT}) \prod_{c=1}^{C} \frac{(N_c-n_{c,\Lambda})!}{(N_c-n_c)!\ (n_c-n_{c,\Lambda})!} \qquad \textbf{(A14)}$$

Similarly, for the other events we obtain:

$$\begin{aligned} b_{0,m} &= Pr(\Theta_{0,m}|\Theta_{-1,1}\cup\ldots)\sum_{t_1} \psi(y_1,t_1)\ \ldots\sum_{t_{IT}} \psi(y_{IT},t_{IT})\ \prod_{c=1}^{C} \frac{(N_c-n_{c,\Lambda})!}{(N_c-n_c)!\ (n_c-n_{c,\Lambda})!} \\ &= \frac{Pm_{1,m}\,Pv_1\,(1 - Pd_{1,m}\,Ps_{1,m})}{P\bar{t}_1} \sum_{t_1} \psi(y_1,t_1)\ \ldots\sum_{t_{IT}} \psi(y_{IT},t_{IT})\ \prod_{c=1}^{C} \frac{(N_c-n_{c,\Lambda})!}{(N_c-n_c)!\ (n_c-n_{c,\Lambda})!} \end{aligned} \qquad \textbf{(A15)}$$

and

$$\begin{aligned} b_{-1,m} &= Pr(\Theta_{-1,m}|\Theta_{-1,1}\cup\ldots)\ \sum_{t_1} \psi(y_1,t_1)\ \ldots\sum_{t_{IT}} \psi(y_{IT},t_{IT})\ \prod_{c=1}^{C} \frac{(N_c-n_{c,\Lambda})!}{(N_c-n_c)!\ (n_c-n_{c,\Lambda})!} \\ &= \frac{Pm_{1,m}\,(1 - Pv_1)}{P\bar{t}_1} \sum_{t_1} \psi(y_1,t_1)\ \ldots\sum_{t_{IT}} \psi(y_{IT},t_{IT})\ \prod_{c=1}^{C} \frac{(N_c-n_{c,\Lambda})!}{(N_c-n_c)!\ (n_c-n_{c,\Lambda})!} \end{aligned} \qquad \textbf{(A16)}$$

As previously, the summation ranges have been suppressed for compactness. It is important to note that these equations are nested summations and not simply the product of sums. That is:

$$\sum_{t_1} \psi(y_1,t_1)\Bigg(\sum_{t_2} \psi(y_2,t_2)\bigg(\sum_{t_3} \psi(y_3,t_3)\bigg(\ldots\sum_{t_{IT}} \psi(y_{IT},t_{IT})\prod_{c=1}^{C} \frac{(N_c-n_{c,\Lambda})!}{(N_c-n_c)!\ (n_c-n_{c,\Lambda})!}\bigg)\ldots\bigg)\Bigg)$$

rather than

$$\Big(\sum_{t_1} \psi(y_1,t_1)\Big)\Big(\sum_{t_2} \psi(y_2,t_2)\Big)\Big(\sum_{t_3} \psi(y_3,t_3)\Big)\ldots\Big(\sum_{t_{IT}} \psi(y_{IT},t_{IT})\Big)\prod_{c=1}^{C} \frac{(N_c-n_{c,\Lambda})!}{(N_c-n_c)!\ (n_c-n_{c,\Lambda})!}$$

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19. ABSTRACT

This paper presents the theory and examples of performance for a new algorithm that tracks targets using a Multiple Model Unified Joint Probabilistic Data Association (MM-UJPDA) filter. The models in the MM-UJPDA can be set to the ambiguity velocities encountered when initiating tracks on a sensor that has ambiguous velocities in its measurements. Alternately, the models can be set for tracking manoeuvring targets. Thus each parallel filter in the MM-UJPDAF is assigned one of a range of possible target model parameters. The term 'unified' summarizes a number of key features in the algorithm. These are: multiple non-uniform clutter regions, a model for a visible target to compute track confidence for track promotion, and measurement selection based on a fixed number of nearest measurements. The filter formulation used a new approach to create track clusters for determining the nearby tracks that share measurements. The filters performance is demonstrated with track initiation using the multiple model and multiple target approach while for established tracking only the multiple target approach is used.

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