NPS-OR-01-011

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Stochastic Models for Promoting and Testing System Reliability Evolution

by

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September 2001

Prepared for: Naval Postgraduate School
Monterey, California 93943

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This report was prepared for and funded by the Director, Operational Test and Evaluation (DOT&E), The Pentagon (Room 3E318), Washington, DC 20301-1700. Research also supported by the Institute of Joint Warfare Analysis (IJWA) and the Modeling, Virtual Environments and Simulation (The MOVES) Institute at the Naval Postgraduate School.

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REPORT DOCUMENTATION PAGE

Form approved

OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	September 2001	Technical Report

4. TITLE AND SUBTITLE
Stochastic Models for Promoting and Testing System Reliability Evolution

5. FUNDING
MIPR NO. DVAM10001

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943

8. PERFORMING ORGANIZATION REPORT NUMBER
NPS-OR-01-011

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of the Director, Operational Test and Evaluation (DOT&E)
The Pentagon (Room 3E318)
Washington, DC 20301-1700

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words.)

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Our model allows quick numerical assessment of TFT operating characteristics, given defining parameter values. It thus provides a planning tool for test designers.

14. SUBJECT TERMS
Operational test and evaluation; reliability growth; software testing; system with substages

15. NUMBER OF PAGES
15

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

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Many systems and systems-of-systems function in sequential-stage fashion, and are constantly *on* when operative, but are failure-susceptible. Communication systems, power generation and transmission, and vehicular transportation systems tend to fall into this category. We propose a reliability growth model for such systems that is based on design defect removal under a Test-Fix-Test (TFT) protocol: a system is assembled and put under test, for example for a fixed mission time, or multiple thereof. If the system fails during the test time its failure source in some stage is diagnosed, the stage is re-designed, and the new prototype system reassembled (system design is "fixed") and the system is re-tested. The test (TFT) process is repeated until a pre-determined test period elapses with no failures. This is analogous to the run-test criteria analyzed for one-shot devices [1]. In this model we also allow for occasional defective re-design: response to a test failure can actually (and realistically) *increase* the number the number of failure-generating design defects.

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1. Introduction and Model Formulation

Mathematical models are formulated for the *reliability evolution* (desirably growth [2], [3], [4], [5], but also occasional realistic decay) of a continuously operating ("always on") system that is tested, fixed (partially re-designed) if it fails, re-tested, etc, until a specified stopping condition is achieved. The stopping rule utilized here is analogous to a run test [1], [6]; here the entire system must survive without any failure for a time τ in order to pass the test, have its design frozen, and be eligible for operational testing and eventual usage in the field.

Two test measures of effectiveness (MOEs) are analytically evaluated:

(a) the probability that the system survives in the field, i.e., after the end-to-end testing period of specified duration τ is survived without failure, and the design is frozen; and

(b) the expected duration of such a test.

It is also possible to analytically evaluate other such measures by our backward equation technique: the variance of test duration, the probability distribution of remaining design defects or faults, and so forth. All of these measures are evaluated in terms of basic parameters, such as the initial number of design-fault-susceptible modules per stage (d_i for stage i) the maximum number per stage (m_i), the rate of design fault activation, hence failure per design fault module (λ_i), the number of sequential stages (S), the duration of the fault-free test interval that must be survived in order to pass the test (τ) (specified in advance by the planner/analyst), the probability of effective re-design/fault removal (ρ_i), and the probability of ineffective re-design/fault addition (α_i). In the present model study the analyst must furnish values for these basic "what if" parameters, and the model then evaluates the MOEs

(a) and (b). The model is also extended to account for test-to-test environmental variability both random and systematic.

Our model can also form the basis for statistical inference concerning stage-wise fault population parameters. Given observations on failures at various stages, a likelihood function can be written down and analyzed, possibly making use of Bayesian methodology. This will be a topic for future research.

2. Generic Situation: Staged Systems in Continuous Time Under Test-Fix-Test

Consider a system, $\mathcal{S}$, that is made up of S stages, $\mathcal{S}_1$, $\mathcal{S}_2$, ..., $\mathcal{S}_i$, ..., $\mathcal{S}_S$, the i^{th}, (i=1, ... S), of which has a maximum number of modules, m_i, all of which must operate for the i^{th} stage $\mathcal{S}_i$ to be operative. However, stage i initially has d_i, $(1 \le d_i \le m_i)$, design defects, i.e., improperly designed failure prone modules. These are presumed to activate independently and randomly as exposure (test, or field operation) time elapses. Initially we presume the time to (activation/failure) of each design defect in stage i to be exponentially distributed, with rate λ_i. The $m_i - d_i$ modules without defects at $\mathcal{S}_i$ are assumed (for now) not to be failure-susceptible.

It is here assumed that if the system $\mathcal{S}$ is put *on test* at $t=0$ it operates successfully *until the first design-defective module in any stage activates/fails*; when that module fails, $\mathcal{S}$ fails (no redundancy). *Occurrence* of such activation is an opportunity for re-design (permanent or temporary repair) of the failed module. If this step is (*i*) *positively effective* the module is no longer activation/failure-prone, i.e., d_i is decreased by one; if this step is (*ii*) *negatively effective*, the re-design is not only ineffective, it adds a defective module, so the net number of defects is increased by

one; otherwise the re-design is (*iii*) *ineffective*, meaning that there is no change in the number of defective modules. Note that the above defect-removal/re-design option is only available at the testing (developmental, early operational) stage.

2.1 Test Protocol

We analyze properties of a no-mission time failure test protocol: specify a test time, τ, and test the system for that time. Each such test event is called a subtest. If a failure occurs during that subtest, perform re-design and test again, continuing until the system survives for time τ without failure. At this moment the test is complete and the design is frozen. This is clearly analogous for the *run of r* criteria analyzed [1].

There are two simple versions of this protocol.

(A) The subtests all last for the basic test time τ, even if a failure occurs during a subtest and the subtest has failed at that point. For the present we consider just one failure to be possible during a subtest. Generalizations will be furnished later.

(B) The subtests each last until the time to first failure or time τ, whichever occurs first.

This requires that the system be constantly monitored in real time to discover failure occurrence; if this is feasible it is undoubtedly more time efficient. But operational circumstances may compel the use of (A). It is the version we analyze first.

3. Test Protocol Modeling and Measures of Test Effeciveness Under Protocol (A)

The system model and test protocol yield expressions that allow numerical evaluation of measures of system test-fix, *etc*, effectiveness.

3.1 Probability of Fielded (Design-Frozen) Success

Let $p_\tau(d_1,...,d_i,...,d_S) =$ Probability that the tested and accepted (design-frozen) system survives without failure for time τ_F.

Then by probability arguments that proceed from the first subtest (backward equation approach) we obtain

$$p_\tau(d_1,...,d_i,...,d_S) = \underbrace{\left(e^{-\sum_{i=1}^{S}\lambda_i d_i \tau}\right)}_{\substack{\text{probability no failures, so no re-designs}}}\underbrace{\left(e^{-\sum_{i=1}^{S}\lambda_{iF} d_i \tau_F}\right)}_{\text{probability no field failures}} +$$

$$+\left(1-e^{-\sum_{i=1}^{S}\lambda_i d_i \tau}\right)\sum_{i=1}^{S}\frac{\lambda_i d_i}{\sum_{k=1}^{S}\lambda_k d_k}\Bigg[\underbrace{\rho_i(d_i)p_\tau(d_1,...,d_i-1,...,d_S)}_{\text{defect in module i removed}} +$$

$$+\underbrace{\alpha_i(d_i)p_\tau(d_1,...,d_i+1,...,d_S)}_{\text{new defect introduced by "redesign"}} + \tag{3.1}$$

$$+\underbrace{\left(1-\rho_i(d_i)-\alpha_i(d_i)\right)p_\tau(d_1,...,d_i,...,d_s)\Big]}_{\text{no change in number of defects}}$$

where λ_{iF} is the failure rate in the field of a remaining design defect in stage i.

The conditional probability of defect removal (ρ_i) and addition (α_i) are assumed to be

$$\begin{aligned}\rho_i(d_i) &= \rho_i \quad for\ 1 \le d_i \le m_i \\ &= 0 \quad otherwise\end{aligned} \tag{3.2,a}$$

$$\begin{aligned} \alpha_i(d_i) &= \alpha_i \quad for\ 1 \le d_i \le m_i - 1 \\ &= 0 \quad otherwise \end{aligned} \tag{3.2,b}$$

In the present model $d_i \le m_i$ for all stages, where m_i is the specified maximum number of defects in stage i. The above expression may be recursively solved, starting with $p_\tau(0, 0, \ldots 0) = 1$; (3.2,b) prevents the number of defects from exceeding m_i in stage i.

3.2 Expected Test Duration, Protocol (A) (Each Subtest Requires Time τ)

Let $w_\tau(d_1, d_2, \ldots, d_i, \ldots, d_S) =$ Expected/mean time to complete a test that terminates with system first failure-free survival of time τ.

Then again by arguing from the first subtest

$$\begin{aligned} & w_\tau(d_1, d_2, \ldots, d_i, \ldots, d_S) = \tau + \\ & + \left(1 - e^{-\sum_{i=1}^{S} \lambda_i d_i \tau}\right) \sum_{i=1}^{S} \frac{\lambda_i d_i}{\sum_{k=1}^{S} \lambda_k d_k} \left[\rho_i(d_i) w_\tau(d_1, \ldots, d_i - 1, \ldots, d_S) + \right. \\ & + \alpha_i(d_i) w_\tau(d_1, \ldots, d_i + 1, \ldots d_s) + \\ & \left. + (1 - \rho_i(d_i) - \alpha_i(d_i)) w_\tau(d_1, \ldots, d_i, \ldots, d_S)\right] \end{aligned} \tag{3.3}$$

Here the initial/boundary condition is $w_\tau(0, \ldots, 0, \ldots 0) = \tau$.

3.3 Generalization for Between-Test Variability

It is possible to explicitly account for an additional likely source of variability: subtest environmental variation, represented by a sequence of positive independent identically distributed random variables, $\{\theta_t; t = 1, 2, \ldots\}$ where t denotes the subtest number. Illustrate by generalizing (4.1). Conditional on the values $\theta_1, \theta_2, \ldots$, and deconditioning subtest by subtest,

$$p_\tau(d_1,d_2,\ldots,d_S)=E\left[p_\tau(d_1,\ldots,d_S,\theta_1,\ldots,\theta_t,\ldots)\right]=$$

$$=E\left[e^{-\left(\sum_{i=1}^{S}\lambda_i d_i\tau\right)\theta}\right]E\left[e^{-\left(\sum_{i=t}^{S}\lambda_{iF}d_i\tau_F\right)\theta_F}\right]$$

$$\left(1-E\left[e^{-\left(\sum_{i=1}^{S}\lambda_i d_i\tau\right)\theta}\right]\right)\sum_{i=1}^{S}\frac{\lambda_i d_i}{\sum_{k=1}^{S}\lambda_k d_k}\left[\rho_i(d_i)p_\tau(d_1,\ldots,d_i-1,\ldots,d_S)+\right. \quad (4.4)$$

$$+\alpha_i(d_i)p_\tau(d_1,\ldots,d_i+1,\ldots,d_S)+$$

$$\left.+(1-\rho_i(d_i)-\alpha_i(d_i))p_\tau(d_1,\ldots,d_i,\ldots,d_S)\right]$$

4. Test Protocol Modeling Under Protocol (B) (Each Subtest Requires the Time to Failure or τ, Whichever Occurs First)

In this protocol it is possible to correctly detect a failure in Stage i when it occurs, without waiting until the end of the test.

4.1 Probability of Fielded (Design Frozen) Success

If $p_\tau(d_1,d_2\ldots,d_i,\ldots d_S)$ is defined as in Section 4.1, then the backward equation for this function is the same as in (4.1). Furthermore, the expression (4.4) that incorporates independent between-test variability holds for this situation also.

4.2 Expected Test Duration, Protocol (B)

Define $w_\tau(d_1,d_2\ldots,d_i,\ldots d_S)$ to be the mean time to test termination (after the system survives time τ). Then in this situation the backward equation becomes

$$w_\tau(d_1,d_2\ldots,d_i,\ldots d_S)=\tau e^{-\sum_{i=1}^{S}\lambda_i d_i\tau}+$$

$$+\sum_{i=1}^{S}\int_0^\tau e^{-\sum_{i=1}^{S}\lambda_i d_i x}\lambda_i d_i dx\left[x+\rho_i(d_i)w_\tau(d_1,\ldots,d_i-1,\ldots,d_S)+\right.$$

$$+\alpha_i(d_i)w_\tau(d_1,\ldots,d_i+1,\ldots,d_S)+$$

$$\left.+(1-\rho_i(d_i)-\alpha_i(d_i))w_\tau(d_1,\ldots,d_i,\ldots,d_S)\right].$$

This simplifies to

$$w_\tau(d_1,d_2\ldots,d_i,\ldots d_S)=\frac{1}{\sum_{i=1}^{S}\lambda_i d_i}\left[1-e^{-\sum_{i=1}^{S}\lambda_i d_i\tau}\right]+$$

$$+\left[1-e^{-\sum_{i=1}^{S}\lambda_i d_i\tau}\right]\sum_{i=1}^{S}\frac{\lambda_i d_i}{\sum_{k=1}^{S}\lambda_k d_k}\left[\rho_i(d_i)w_\tau(d_1,\ldots,d_i-1,\ldots,d_S)+\right. \qquad (5.1)$$

$$+\alpha_i(d_i)w_\tau(d_1,\ldots,d_i+1,\ldots,d_S)+$$

$$\left.+\left(1-\rho_i(d_i)-\alpha_i(d_i)\right)w_\tau(d_1,\ldots,d_i,\ldots,d_S)\right].$$

To generalize (5.1) to account for between-test variability it is only necessary to replace the first term $\frac{1}{\sum_{i=1}^{S}\lambda_i d_i}\left[1-e^{-\sum_{i=1}^{S}\lambda_i d_i\tau}\right]$, by $E\left[\frac{1-e^{-\left(\sum_{i=1}^{S}\lambda_i d_i\tau\right)\theta}}{\theta\sum_{i=1}^{S}\lambda_i d_i}\right]$ and, in the second term, $e^{-\sum_{i=1}^{S}\lambda_i d_i\tau}$ by $E\left[\left(e^{-\left(\sum_{i=1}^{S}\lambda_i d_i\tau\right)\theta}\right)\right]$, where the expectation is on θ.

Any distribution having positive support and with an explicit Laplace-Stieltjes transform provides tractable closed form models for Protocol (A). To obtain a closed-form expression for Protocol (B) it must be possible to *integrate* the Laplace transform of θ from zero to a finite limit $\sum_{i=1}^{S}\lambda_i d_i\tau$.

5. Illustrative Numerical Example

The backward equations may be solved iteratively to provide numerical insights into system performance under the TFT testing protocol. Here is a brief, isolated, but suggestive example.

5.1 A Test-Stage Situation

The parameters used are the following:

$\lambda_1 = 0.01, \lambda_2 = 0.05$	*(Test defect activation rate (hours)$^{-1}$)*
$\rho_1 = \rho_2 = 0.75$	*(Defect rectification/correction probability)*
$\alpha_1 = 0.20, \alpha_2 = 0.10$	*(Defect mis-identification/addition of one defect probability)*
$m_1 = m_2 = 4$	*(Maximum number of defects in each stage)*
$\tau_F = 100$	*(The field mission time (hours))*
$\lambda_{1F} = 0.05, \lambda_{2F} = 0.05$	*(Field defect activation rate (hours)$^{-1}$)*

(A) Examine the effect of the basic sub-test time, τ, on the probability of surviving a field operation without failure. The numbers in the small table below indicate the surprisingly systematic effect of test duration on probability of successful field operation.

Table 1: Probability of Surviving τ_F (Field Operation)

Initial Defects (d_i)		Test Time (τ)			
d_1	d_2	50	100	200	300
0	1	0.89 (119) [70]	0.99 (252) [127]	1.00 (508) [229]	1.00 (761) [329]
2	2	0.25 (276) [106]	0.52 (651) [204]	0.83 (1462) [375]	0.94 (2264) [510]
2	4	0.29 (426) [120]	0.55 (945) [215]	0.84 (2035) [380]	0.94 (3119) [513]
4	4	0.16 (519) [139]	0.45 (1201) [261]	0.81 (2625) [453]	0.93 (4021) [593]

( )=Expected Test Time, Protocol (A)
[ ]=Expected Test Time, Protocol (B)

However, the required number of tests tends to increase substantially particularly under Protocol (A). If the test can be stopped as soon as a failure occurs, considerable time can be saved. The moral is that only by considerable testing and fixing (in an

error-prone "fix" environment) can we eventually hope to have a highly reliable (small, two-stage) system.

Software that can be activated to exercise programs to evaluate various situations (and parameter variations) appears at http://www.nps.navy.mil/opnsrsch/testeval/.

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