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13. ABSTRACT (Maximum 200 Words) The objectives of research sponsored by the AFOSR grant F49620-98-1-0146 were 1) To develop new control theory and dynamical systems tools for the problems of active control of fluid flow instabilities and control of mixing and 2) To apply the developed theoretical tools to problems of active control of instabilities of compression systems of jet engines and control of mixing in combustion chambers of jet engines. To achieve these objectives, the following research topics was done: a new model for axial compression systems of jet engines, that included the effects of air injection actuators was developed. Control design for the resulting evolution equation was pursued. Effect of finite-dimensional control was taken into account by designing a no-control-spillover finite dimensional control law. Theory of mixing was advanced to include topics in three-dimensional shear flow mixing and mixing in aperiodic flows. Control of mixing in vortex element systems was pursued. Ergodic theory concepts proved useful in that study and lead to investigation of a novel model-validation methodology for nonlinear systems with noise.					
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FINAL TECHNICAL REPORT:  
**Dynamics and Control of Instabilities  
and Mixing in Complex Fluid Flows;  
Applications to Jet Engines**  
AFOSR grant F49620-98-C-0006.

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January 1, 1998 to January 1, 2001.

## 1 Objectives

The objective of our research is to

- Develop new control theory and dynamical systems tools for the problems of active control of fluid flow instabilities and control of mixing.
- To apply the developed theoretical tools to problems of active control of instabilities of compression systems of jet engines and control of mixing in combustion chambers of jet engines.

## 2 Development of Effort

### 2.1 1998-1999

This project is partly a continuation of the project "Dynamics and Control of Infinite-Dimensional Models of Jet Engine Compression Systems", AFOSR grant F49620-97-1-0293 that lasted from May 1, 1997 to December 31, 1998. We have thus continued the work on modeling and control of axial compression systems and pursued new research on the control of mixing. At the PI's department, 3 faculty members (including the PI), 5 graduate students and a postdoctoral fellow were involved in some aspect of the research. There was

an active exchange of information and collaboration with the members of the AFOSR-PRET team on Robust Nonlinear Control of Aeroengines and jet engine specialists from UTRC (most intensely with Andrzej Banaszuk). The PI presented talks at UTRC both on the advances in stall modelling and control and the control of mixing. A joint project on nonlinear identification with UTRC was started. Another transition to industry of the part of the project on the control of mixing was made through the PI's involvement in combustion and mixing research at the Propulsion Research Institute in Carpinteria (subsidiary of Turbodyne Inc.).

## 2.2 1999-2000

We have continued the work on modeling and control of axial compression systems and the control of mixing. At the PI's group, 1 graduate student was working on stall instability problem supported by the AFOSR grant, but the exchange of information and ideas with the other members of the group (5 students) was intense. The PI pursued a collaboration with George Haller and Andrew Poje of the Division of Applied Mathematics at Brown University on mixing in aperiodic flows. There was an active exchange of information and collaboration with jet engine specialists from UTRC (most intensely with Andrzej Banaszuk and Bernd Noack). The PI presented a number of talks at UTRC both on the advances in stall modelling and control and the control of mixing. A joint project on nonlinear identification with UTRC was completed. A new joint project on control of mixing was started.

In the indicated period the following was achieved:

- The PI has developed a new theory of statistical properties of control systems using the ergodic-theoretical formalism of skew products. This formalism has applications both in model validation (which was pursued jointly with Andrzej Banaszuk of UTRC in an AFOSR-sponsored project) and control of mixing (which is being pursued jointly with Andrzej Banaszuk and Bernd Noack of UTRC in another AFOSR-sponsored project).
- Greg Hagen has advanced the study of nonlinear controllers for the one-dimensional stall instability models begun in 1998-1999.
- With Greg Hagen we started analyzing properties of the two-dimensional stall instability model that the PI has developed. It was found that part-span stall can be analyzed within the context of this model. This is the first time that a model has produced part-span stall.

## 2.3 2000-2001

We have continued the work on modeling and control of axial compression systems and the control of mixing. At the PI's group, 1 graduate student (Greg Hagen) was working on the stall instability problem supported by the AFOSR grant. The PI pursued a collaboration with J. Ottino's group at Northwestern

University on theoretical, computational and experimental aspects of mixing in 3D. There was an active exchange of information and collaboration with UTRC. The joint project (with Andrzej Banaszuk and Bernd Noack) on control of mixing in a *fluid flow induced by a vortex in a corner* was completed.

The new developments in this period were: 1) Framework for *control of flows induced by vortex elements* [12], 2) *Continuation of study of nonlinear model identification of complex systems using ergodic theory concepts* [10], 3) *Analysis and control of stall instabilities* and discussion of control spillover in reaction-diffusion equations [4] and 4) *Analysis and control of mixing* [3, 12].

Overall in this project we developed a set of models of interest, initiated the study of nonlinear model validation based on ergodic theory and provided an analysis and control study for them. These developments, described in more detail below, in turn defined new problems that we will study in the future. The focus of the future work will be analysis, model validation and identification and control: we have defined types of models that we want to study as 1) *Vortex element models* and 2) *A set of nonlocal evolution equations governing stall dynamics*.

### 3 Accomplishments

The research achievements of the project have been:

- The new model of axial compressor dynamics was derived [8] based on the first principles. This model is simple and suitable for application of a variety of analysis and control techniques. The stall cell size was predicted analytically from the infinite-dimensional evolution equation. The prediction fits the experimental data very well. We have derived a comprehensive, large-scale theory of axial compression system dynamics. We have pursued the dimensional analysis of the problem to be able to do order of magnitude estimates for the dynamics. We have then found that the large-scale dynamics is governed by equations that reduce to the Moore-Greitzer equations in the limit when:
  - The average speed of the fluid in rotor-stator rows is assumed to be one half of the rotor speed.
  - The fluid is incompressible.
  - The force on blades has the Moore-Greitzer characteristic shape.

This last issue is the most important within our findings: the concept of the compressor characteristic is replaced by the average force on the blade, which makes it a measurable quantity. The differences with the Moore-Greitzer model occur because the structure of the flow in the stator and rotor rows is treated more precisely. Thus, we find that the speed of the stall cell does not have to be strictly one-half of the rotor speed, but depends on the shape of rotor and stator blades. Axial extent of the compressor was also taken into account in the newly developed models.

### 3.1 The ergodic-theoretical analysis of control systems

Motivated by the problems arising in control of flows in jet engines, the PI has made contributions in ergodic theory of control systems. For complex flow problems such as stall instability and control of mixing the issue of validation of reduced order models arises. In the context of control of mixing the study of ergodic properties of a sequence of transformations is needed. We have pursued the related applied developments in collaboration with Andrzej Banaszuk (UTRC; model validation and control of mixing, AFOSR supported), Bernd Noack (UTRC; control of mixing) and George Haller and Andrew Poje (Brown University; mixing in aperiodic flows).

### 3.2 Distributed parameter control of axial compression systems using air injection

A new model for axial compression systems with air injection actuators was developed and we designed locally optimal and globally stabilizing controllers. The system is of the form

$$\frac{\partial y}{\partial t} = a^2 \frac{\partial^2 y}{\partial \theta^2} + f(y + M) - \overline{f(y + M)} + B_1 y_1 + B_2 u \quad (1)$$

$$\frac{\partial y_1}{\partial t} = u \quad (2)$$

In the extension of this work, we used the sector condition of the compressor characteristic function to derive sufficient conditions to guarantee asymptotic stability of this nonlinear evolution equation system. From these sufficient conditions we derived two different control laws. The nonlinear control law reshapes the nonlinear compressor characteristic function to satisfy a sector condition that is sufficient for asymptotic stability. The linear control law was designed to stabilize the system with the original compressor characteristic function and its given sector properties. We have also numerically analyzed the properties of these controllers using numerical methods that we have developed.

To alleviate some of the problems created by the coupling of modes through the nonlinearity of the system, in [4] we avoid directly decomposing the PDE into Fourier modes, and instead consider a control Lyapunov function (CLF) for the original infinite-dimensional system. By extending the sector property [6] of the system nonlinearity to the infinite dimensional case [5] we obtained an upper bound on the CLF derivative in terms of the Fourier coefficients of the state. This allowed us to consider the high modes as stable despite the inherent coupling, which, in turn, provides for the direct construction of a finite-dimensional controller of the low modes of the system. The CLF derivative shows explicitly how the high modal content of the controller can destabilize the entire system. Upper bounds of the high modes of the CLF derivative were obtained to quantify the effects of control spillover. An LQR control design is used to make the CLF derivative negative with these upper bounds. Furthermore we stated

relations between the system and LQR parameters that are sufficient for global stability and robustness with respect to control spillover.

### 3.3 Investigation of the two-dimensional axial compression system model

Previous models of axial compression systems were not able to produce part-span solutions because they involved only one independent spatial variable. Our model derived by the PI in [8] is fully three-dimensional. The evolution equation for the axial velocity decouples and is function of radius and angle only. We performed numerical simulations of the two-dimensional model assuming linear change in the compressor characteristic in the radial direction. By changing hub to tip ratio we were able to obtain part-span stall with particular initial conditions.

### 3.4 Investigation of mixing in aperiodic fluid flows

The relationship between statistical and geometric properties of particle motion in aperiodic, two-dimensional flows was examined. Finite-time invariant manifolds associated with transient hyperbolic trajectories were shown to divide the flow into distinct regions with similar statistical behavior. In particular, numerical simulations indicate that there exists a close correlation between such geometric structures and *patchiness* [7] plots which describe the distribution of Lagrangian average velocity over initial conditions. We found that Eulerian velocity correlation time scales are significantly longer than their Lagrangian counterparts indicating the existence of well defined Lagrangian structures. Identification of such structures shows a similar, close relationship between the invariant manifold geometry and patchiness calculations at intermediate time scales where anomalous dispersion rates were found.

### 3.5 Controlling vortex motion and chaotic advection

In the study [12] we pursued the question of *optimizing and controlling* transport and mixing in two-dimensional flows. Akin to first chaotic advection studies [13] we chose a simple vortex model - a single vortex in a corner subject to a potential field. In the absence of time-dependent forcing (introduced by modulating the potential field) the vortex is either at a stable equilibrium position or is moving on a periodic trajectory in the plane.

The first question that we asked is if whether the vortex motion is controllable. The answer to it was shown to be positive using the transformation to *flat coordinates* [11, 2]. Having this result, we proceeded to search over all the trajectories of the vortex in a bounded domain of the plane and determine the optimal one with respect to a suitable measure. In this work we chose this measure to be the flux through the separatrix, thus linking control theory and chaotic advection theory. Once the optimal trajectory was found, we stabilized it using a feedback law. The search for the optimal trajectory - which was

restricted to lie within a circle to preserve stability of the recirculation zone - was pursued using the numerical simplex method. Various characterizations of mixing utilizing dynamical systems concepts were pursued.

### 3.6 Mixing in three-dimensional maps and flows

An important class of three-dimensional volume-preserving maps and flows arises as a perturbation from integrable action-action-angle maps and flows. For example, three-dimensional, time dependent perturbations of vortex rings with no swirl lead to such Poincaré maps. We studied properties of this class of maps and flows. While action-angle-angle volume-preserving maps admit an analogue of the KAM theorem, general results on non-existence of two-dimensional invariant manifolds of action-action-angle maps are proven in [9]. Non-existence of such two-dimensional invariant manifolds means possibility of global transport and a mechanism for such transport - the local mechanism of resonance induced dispersion [1] - was studied perturbatively. Resonance induced dispersion was shown to arise from the existence of periodic orbits of saddle-focus type that survive perturbation at places where two-dimensional invariant manifolds break down. This mechanism can lead to complete mixing even if the size of the perturbation to an integrable map is very small. The mechanism was experimentally confirmed in [3].

### 3.7 Nonlinear model validation

In both of the problems described above the issue of comparison of models with experimental data arises. Both the stall evolution equations and the vortex models are nonlinear complex systems. The dynamics of such systems is best described in terms of statistics. In [10] a method of model validation based on ergodic theory treatment of statistical properties of deterministic systems was proposed.

While in numerical experiments and analytical work the full state of a system is an observable, in experiments this is typically not the case. Usually the value of one observable - a function on the phase space - is measured. This observation lead to the development of Takens embedding theorem [14]. In [10] we developed a similar approach using time averages of functions. In particular, we *constructively* proved, using time-averages of certain functions on the phase-space, that ergodic partitions and invariant measures of systems can be compared using a single observable. We developed pseudometrics on spaces of dynamical systems using this result.

In some contexts though, comparing invariant measures is not enough. Consider, for example two systems that have a (geometrically) identical globally attracting limit cycle, but on the limit cycle of the first system the dynamics is given by  $\dot{\theta} = \omega_1$  and on the limit cycle of the second system the dynamics is given by  $\dot{\theta} = \omega_2$ , where  $\omega_1 \neq \omega_2$ . While these two systems have identical invariant measures supported on the same geometrical object, their *asymptotic speed* is different. We proposed a formalism based on harmonic analysis that extends

the concept of comparing the invariant measures. We showed that information beyond that obtained using time averages can be acquired by taking harmonic averages if the system has a factor that is a rotation on the circle. Using these methods, reconstruction of an invariant measure from a single variable leads to distinction between a nonlinear, limit-cycling system and linear, lightly damped one.

## 4 Personnel supported:

**Faculty:** Igor Mezić,

Department of Mechanical and Environmental Engineering,  
University of California, Santa Barbara.

**Postdoctoral fellows:** Dr. Kaixia Zheng,

Department of Mechanical and Environmental Engineering,  
University of California, Santa Barbara.

**Graduate students:** Gregory Hagen.

Department of Mechanical and Environmental Engineering,  
University of California, Santa Barbara.

## 5 Interactions/transitions:

### 5.1 Academic interactions

Mixing in aperiodic flows was investigated with George Haller and Andrew Poje from Brown University. There was an interaction with the AFOSR-PRET group on Nonlinear Robust Control of Aeroengines. A number of graduate students from PI's group and other research groups in the department participated in the various aspects of the research. The PI gave a number of invited lectures on the topics of research described here, for example at UCLA, University of Illinois, Urbana-Champaign, Harvard, Courant Institute, Princeton, etc. The PI and graduate student, Greg Hagen, attended the 1999 SIAM Conference on Dynamical Systems and Applications, where Hagen gave a talk on pattern formation in axial compressors. He also gave a talk at the 1999 ACC in San Diego on control of flow in axial compression systems using air injection. The PI gave a talk on Control of Mixing at the Workshop on Flow Control in San Diego.

Mixing in 3D flows was investigated with J. Ottino from Northwestern University. The PI gave a number of invited lectures on the topics of research described here, for example at Caltech, Institute for Fluid Mechanics at *ETH Zürich*, *MIT Gas Turbine Lab*, etc. The PI and graduate student, Greg Hagen, attended the 1999 APS DFD meeting in New Orleans, where Hagen gave a talk on control of stall in axial compressors and the PI gave a talk on 3D mixing. Hagen also gave a talk at the 2000 ACC in Chicago on control spillover. The PI gave a talk on Nonlinear Model Validation at the Workshop on Lagrangian Methods in Fluid Mechanics in San Diego. The PI was a member of a subpanel

of the Future Directions in Control and Dynamical Systems panel meeting in Washington DC.

## 5.2 Industrial interactions

There was a strong connection with UTRC's Control and Dynamics group on mixing, jet engine instabilities and model validation.

## 5.3 Transitions

### 1. *Performer:* I. Mezić

*Customer:* United Technologies Research Center, Hartford, Connecticut.

*Contact:* Dr. Andrzej Banaszuk, (860) 610-7381.

*Result:* A presentation and discussion with the UTRC research team on the control of mixing.

*Application:* The UTRC research team was acquainted with chaotic advection techniques that could be useful for combustion in combustion chambers of jet engines.

### 2. *Performer:* I. Mezić

*Customer:* United Technologies Research Center, Hartford, Connecticut.

*Contact:* Dr. Andrzej Banaszuk, (860) 610-7381.

*Result:* Joint work with Andrzej Banaszuk and Bernd Noack on control of mixing for combustion.

### 3. *Performer:* I. Mezić

*Customer:* United Technologies Research Center, Hartford, Connecticut.

*Contact:* Dr. Andrzej Banaszuk, (860) 610-7381.

*Result:* Joint work with Andrzej Banaszuk on model validation.

### 4. *Performer:* I. Mezić

*Customer:* United Technologies Research Center, Hartford, Connecticut.

*Contact:* Dr. Satish Narayanan, (860) 610-7381.

*Result:* Discussion of methods of analysis and experimentation for jet noise reduction

### 5. *Performer:* I. Mezić

*Customer:* United Technologies Research Center, Hartford, Connecticut.

*Contact:* Dr. Andrzej Banaszuk, (860) 610-7381.

*Result:* Joint work with Andrzej Banaszuk and Bernd Noack on control of mixing for combustion.

## 6 Honors/Awards

The PI has been awarded the Sloan Research Fellowship in Mathematics, the NSF CAREER Award in 1999 and the Axelby Outstanding Paper Award by the IEEE Society in 2000 for the paper on control of mixing with D'Alessandro and Dahleh sponsored by this grant.

## 7 Publications

### 7.1 Journal articles and conference proceedings published and accepted for publication

1. G. Haller and Igor Mezić, "Reduction of three-dimensional, volume-preserving flows with symmetry," *Nonlinearity*, 11, 319-339 (1998)
2. Igor Mezić and Stephen Wiggins, "A Method for Visualization of Invariant Sets of Dynamical Systems Based on the Ergodic Partition", *Chaos*, 9,213-218, (1999).
3. A. Banaszuk and H. A. Hauksson and Igor Mezić "A Backstepping Controller for a Nonlinear Partial Differential Equation Model of Compression System Instabilities". *SIAM Journal of Control and Optimization*, 37, No. 5. pp. 1503-1537 (1999).
4. Igor Mezić, Stephen Wiggins and David Betz, "Residence-time distributions for chaotic flows in pipes", *Chaos*, 9,173-182, (1999).
5. Andrew Poje, George Haller and I. Mezić, "The Geometry and Statistics of Mixing in Aperiodic Flows", *Physics of Fluids*, 11, 2963-2968 (1999).
6. D. D'Alessandro, M. Dahleh, and I. Mezić, "Control of mixing in fluid flow: A maximum entropy approach". *IEEE Transactions on Automatic Control*. 44: (10), 1852-1863 (1999).
7. J. Fountain, D. Khakhar, I. Mezić and J. Ottino, "Chaotic Mixing in a Bounded 3D Flow". *J. Fluid. Mech.*, 417, 265-301 (2000)
8. G. Hagen and I. Mezić, "Control spillover in dissipative evolution equations". *Proceedings of the American Control Conference* (2000).
9. S. Narayanan and I. Mezić "Overview of Some Theoretical & Experimental Results on Modeling and Control of Shear Flows," *Proceedings of Conference on Decision and Control*, Sydney (2000).
10. Bernd R. Noack, I. Mezić, and Andrzej Banaszuk, "Controlling vortex motion and chaotic advection," *Proceedings of Conference on Decision and Control*, Sydney (2000).

11. I. Mezić and Andrezej Banaszuk, "Comparison of Systems with Complex Behavior," Proceedings of Conference on Decision and Control, Sydney (2000).
12. I. Mezić, "Dynamics and transport in a class of volume-preserving maps and flows". *Physica D* (2001).
13. I. Mezić, "Chaotic advection in bounded Navier-Stokes flows". *Journal of Fluid Mechanics* 431, 347-370 (2001).
14. D. D'Alessandro, I. Mezić and M. Dahleh, "Statistical Properties of Controlled Fluid Flows with Applications to Control of Mixing". To appear in *Systems and Control Letters* (2001).
15. I. Mezić, "Controllability of group translations". Proceedings of the American Control Conference (2001).

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