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THESIS

**PERFORMANCE ANALYSIS OF PILOT-AIDED
FORWARD CDMA CELLULAR CHANNEL**

by

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September 2001

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CELLULAR CHANNEL**

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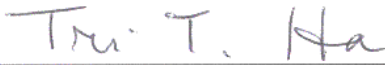
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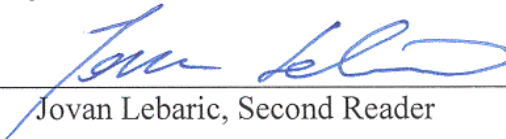
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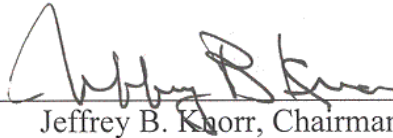
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ABSTRACT

In this thesis we analyze the performance of the forward channel of a DS-CDMA cellular system operating in a Rayleigh-fading, Lognormal-shadowing environment. We develop an upper bound on the probability of bit error, including all the participating interference. In addition, various techniques such as sectoring and forward error correction in the terms of convolutional encoding are applied to optimize the performance. We further improve the performance by applying a narrow bandpass filter in the pilot tone branch of the demodulator. We then adjust the bandwidth of the filter in the means of the interference power passing through and observe the effects on the probability of bit error of the system. Moreover, pilot tone power control is added to enhance the demodulation. Finally, in this thesis a simple single cell system functioning as a port-to -port network communication between very small numbers of users is analyzed.

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EXECUTIVE SUMMARY

An increasing demand for high data rate applications and greater mobility has led to the development of a third generation of service (3G). The existing second-generation system was originally designed for wireless voice communications and thus could not afford applications such as wireless full internet access or high quality image and video transmission. The third generation mobile cellular system employs Code Division Multiple Access (CDMA) that can increase the capacity many times over the present systems. Wideband CDMA systems are expected to offer high data rate services, up to an outstanding 2 Mbps, which currently cannot be provided by existing cellular systems. However, unlike Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA), that are bandwidth limited, Wideband CDMA (W-CDMA) systems are interference limited. The primary interference sources are intracell and intercell interference, however additive white Gaussian noise (AWGN) is also considered. In order to maintain an acceptable quality of service and enhance performance, some forms of interference reduction are utilized in W-CDMA systems. Thus, the performance of such cellular systems taking into account all the interference parameters had to be explored.

Accordingly, we set up a forward channel for a DS-SS-CDMA cellular system. We built an information signal and we propagated it through the medium channel applying all the appropriate losses, effects and interferences. We use the extended Hata model to predict the large-scale path loss and we further incorporate lognormal shadowing to express the power fluctuations between users at same distance from the base station. Moreover, we use Rayleigh fading to express small-scale propagation effects, caused by multipath and Doppler shift of the signal. We set the receiving mobile user at the edge of the center cell, assuming the worst-case scenario. Finally, we form the total received signal by the examined user including the intracell and intercell interference as well as the Additive White Gaussian Noise (AWGN).

A significant factor in determining the quality of service is the Signal to Noise plus Interference Ratio (SNIR). Thus, we demodulate the received signal and we develop the SNIR. We develop an upper bound on the probability of bit error for the forward channel and therefore we explore the performance of an unfiltered system that takes into account all the received interference. We then optimize the performance using various techniques.

Accordingly, we incorporate Forward Error Correction (FEC) and we develop an upper bound on the bit error probability for the coded system. We simulate the probability of bit error using Monte Carlo simulation method and we compare the performance results with previous work done. The large amount of interference imported from the pilot recovery branch, is responsible for the quite poor performance that is achieved. Thus, in order to limit the power of the interference terms down, a narrowband filter is applied at the pilot tone recovery branch. Further reduction of the intercell interference is acquired by antenna sectoring and thus we achieve an acceptable performance for the coded DS CDMA system. However, whenever we increase the amount of interference passing through the filter, apply heavy shadowing conditions or augment the number of users per cell, the performance of the system diminishes, below the standards. Therefore, we induce power allocation at the form of power control of the pilot tone channel. We derive a relation between the power allocated to the pilot tone and the other channels, and we develop the probability of bit error for the power-controlled system.

Finally, we explore the performance of a simple single cell system operating in a Rayleigh fading, Lognormal shadowing environment. In particular we examine the functionality of this system as a port to-to-port communication between two to five users.

I. INTRODUCTION

A. BACKGROUND

Nowdays, the increasing demand for high data rate applications and greater mobility has led to the development of a third generation of service (3G). The existing second-generation system was originally designed for wireless voice communications and thus could not afford application as wireless full Internet access and high quality image and video transmission. The third generation mobile cellular system employs code division multiple access (CDMA) that can increase the capacity many times over the present systems. Wideband CDMA systems are expected to offer high data rate services, up to 2 Mbps, which currently cannot be provided by existing cellular systems. However speed connection may drop to 144 Kbps for faster moving users, which is much faster than a wired Internet modem connection (56 Kbps). The third generation system is already been used in Japan and is going to be implemented in Europe in 2002. However for the United States is not expected to be on line before 2004.

B. OBJECTIVE

Unlike Frequency division multiple access (FDMA) and Time division multiple access (TDMA) that are bandwidth limited, wideband CDMA systems are interference limited. The primary interference sources for the forward channel of such systems are intracell and intercell interference, as well as the additive white Gaussian noise (AWGN). Therefore, the objective of this thesis is to define a comprehensive Signal to Noise plus Interference Ratio (SNIR), a significant factor of the quality of service experienced by the user. Using that, we aim to develop an upper bound on the probability of bit error for the forward channel of a CDMA cellular system operating in a slow flat Rayleigh fading environment affected by Lognormal shadowing. In order to maintain an acceptable quality of service and capacity, we intend to utilize some form of interference reduction. Specifically, we can get advantage of the flexibility of Wideband CDMA and incorporate novel features that can optimize the system performance and limit the effects of the interference. Such features are convolution coding, sectoring, pilot tone filtering, and pilot tone power allocation. Finally, we object to analyze the performance of a single cell

system operating as a port-to port network communication between small numbers of users.

C. RELATED WORK

There are a lot of related researches on the DS-CDMA channel. However most of the work done was focused on the reverse channel, which is generally much different than the forward. A very comprehensive analysis of a DS-CDMA forward channel has been done in [1]. While in this investigation both Lognormal shadowing and Rayleigh fading effects using convolutional encoding are considered, the interference from the pilot recovery channel is being ideally filtered out and is not taken into account.

Furthermore, [1] optimizes power using fast power control instead of pilot tone power control. There are several other relative publications that investigate the DS-CDMA performance. However in these researches either Nakagami or Ricean fading is considered as in [2] and [3] respectively, or FEC in the form of Golay codes is applied, as in [4]. Moreover, the single cell performance analysis has not yet been analytically investigated, so the related work is quite limited.

Summarizing, we can conclude that previous analysis of the forward DS-CDMA cellular system didn't consider the effect of the interference from the pilot recovery channel. Therefore a comprehensive work that would include and extend previous research needs to be accomplished.

D. THESIS OUTLINE

In Chapter II we set up a forward channel for the DS-CDMA cellular system. We also build an information signal, which we propagate through the medium channel applying all the appropriate losses effects and interferences such as path loss, shadowing, fading or noise. Finally we form the total received signal by the examined user.

In Chapter III we set the mobile user in a position in the center cell of the seven-cell cluster assuming the worst-case scenario. We demodulate the received by the user signal and we develop the Signal to Noise plus Interference Ratio (SNIR), taking into account all the interfering terms. We then incorporate convolutional encoding and find an upper bound on the bit error probability for the coded system. We simulate the probability of bit error using Monte Carlo simulation method and we compare the

performance results with previous work done. Next we apply filtering at the pilot tone recovery branch and we revise the already developed probability of error by limiting the interference terms' power. Finally we further reduce interference by implying sectoring to the antennas and examine the resultant performance for various channel conditions.

In Chapter IV, we further optimize the performance of the system by introducing power control to the pilot tone channel. We derive a relation between the power allocated to the pilot channel and the other users and we compare the resultant probability of bit error with previous work done.

In Chapter V, we present a simple case of a single cell environment, where a port-to-port communication between two or three users is required. Therefore, we adopt the already developed probability of bit error for the seven-cell cluster, revising it to a much simpler form where intercell interference is eliminated. We simulate the probability of bit error and we compare the results for a small number of users and different shadowing conditions. We optimize the receiver adding a narrowband filter at the pilot tone acquisition branch channel and we examine the performance for a larger number of users.

Finally in Chapter VI we summarize our conclusions and provide areas of further research.

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II. FORWARD CHANNEL MODEL

In this chapter we are going to examine analytically the forward channel, which is the traffic channel that carries the signal from the base station to the mobile user. This channel, as discussed earlier, is very important, much more than the reverse channel, due to the increased need for downloading very large amounts of data at high-speed rates.

Accordingly we'll first set up a forward DS-CDMA channel, and then an information signal that we will propagate through the medium channel, applying all the appropriate losses, effects and interferences, such as path loss, shadowing, fading or noise. [1].

A. BUILDING THE DS-CDMA FORWARD CHANNEL

A typical seven-cell cluster is shown in Figure 2.1. The user we are going to examine is user #1 of the center cell. The layout of the base stations and cells is assumed to be as shown. We know that in practice, the cells are circular overlapping each other. However for practical reasons, we will use the hexagonal cells cluster in our model, which is commonly used in theory.

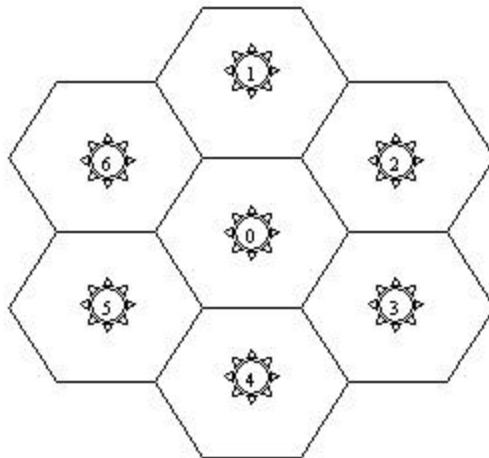


Figure 2.1. Typical Seven-Cell Cluster.

Building-up the forward signal, we will try to comply with the notation set in [1], so that a comparison of our results and formulas with previous work can be done.

We represent the information signal for the mobile user k as $b_k(t) \in \{\pm 1\}$, with bit duration T . Each bit is spread by a factor of N , using orthogonal Walsh functions W_N , $N \in (0, 1 \dots 127)$, resulting to chip duration of $T_c = T/N$. We should note that the spreading of each binary sequence is not the same, but varies according to the Walsh function W_N that is used. Furthermore, the orthogonality of the Walsh functions assures that intracell interference is eliminated.

In order to ensure equal spreading for all the information signals we will use PN sequences, apart from the Walsh spreading. We call $c(t)$ the PN sequence for the center cell and $c_i(t)$, $i=1, 2 \dots 6$, for the other cells respectively. All the PN sequences have the same length $N=128$, acquiring equal spreading of the information bits and minimizing intercell interference as well.

Finally, after spreading, the information signal is BPSK modulated and finally is ready for transmission.

Summarizing all that, the transmitted signal for the k -th user can be described as [Th]:

$$s_k(t) = \sqrt{2P_{t,k}} b_k(t) w_k(t) c(t) \cos(2\pi f_c t), \quad (2.1)$$

where

k = mobile user or channel k in the center cell,

$P_{t,k}$ = the average transmitted power in the k -th channel ,

$b_k(t)$ = the information signal for the k -th user channel in the center cell,

$w_k(t)$ = Walsh function for the k -th user channel in the center cell,

$c(t)$ = PN spreading signal for the center cell, and

f_c = the carrier frequency of the signal,

The sum of all the signals transmitted by the base station of the center cell to all the users in the cell is:

$$s_0(t) = \sum_{k=0}^{K-1} s_k(t) = \sum_{k=0}^{K-1} \sqrt{2P_{t,k}} b_k(t) w_k(t) c(t) \cos(2\pi f_c t), \quad (2.2)$$

where K is the number of the active channels in the center cell.

Next we will describe the effects and phenomena that take part in the propagation of the signal.

B. PROPAGATION IN THE MOBILE RADIO CHANNEL

The transmitted signal suffers different type of losses and effects during its propagation from the base station to the mobile user. These are the path loss due to the distance between the base and the user, the lognormal shadowing effect due to the different levels of clutter on the propagation path, and the small scale fading due to multipath.

1. Large Scale Path Loss

The power of a signal propagating at a large distance d decreases logarithmically with distance, using a path loss exponent n related to the characteristics of the environment. In general, the average path loss can be expressed after [5] as:

$$\bar{L}_n(d) = \bar{L}(d_0) + 10n \log\left(\frac{d}{d_0}\right) \text{ (in dB)}, \quad (2.3)$$

where $\bar{L}(d_0)$ is the average path loss at the reference distance d_0 calculated using the Friis free space equation.

For cellular communications, the extended Hata model is commonly employed to predict the median path loss L_H in dB as follows:

$$L_H = 46.3 + 33.9 \log \frac{f_c}{\text{MHz}} - 13.82 \log \frac{h_{base}}{m} - a(h_{mobile}) \\ + (44.9 - 6.55 \log \frac{h_{base}}{m}) \log \frac{d}{\text{km}} + C_\gamma, \quad (2.4)$$

where

$$a(h_{mobile}) = (1.1 \log \frac{f_c}{\text{MHz}} - 0.7) \frac{h_{mobile}}{m} - (1.56 \log \frac{f_c}{\text{MHz}} - 0.8) \text{ (in dB)},$$

and

$$C_M = \begin{cases} 0 \text{ dB, for medium sized city and suburban areas} \\ 3 \text{ dB, for metropolitan centers.} \end{cases}$$

The extended Hata Loss model is restricted to the following range of parameters [Rap]:

$$f_c = 1500 \text{ MHz to } 2000 \text{ MHz,}$$

$$h_{base}=30 \text{ m to } 200 \text{ m,}$$

$$h_{mobile}=1 \text{ m to } 20 \text{ m,}$$

$$d=1 \text{ Km to } 20 \text{ Km.}$$

Accordingly in our model we are going to use parameters that lie in between these restrictions, and mostly near the worst-case limits, such as:

$$f_c=2000 \text{ MHz,}$$

$$h_{base}=30 \text{ m ,}$$

$$h_{mobile}=1 \text{ m ,} \tag{2.5}$$

$$d=1 \text{ Km.}$$

$$C_M=3 \text{ dB, for a metropolitan center.}$$

2. Log-Normal Shadowing

The formula we used in (2.4) for the path loss, does not consider the fact that the surrounding environmental clutter may vary between two locations with the same distance. This phenomenon is known as shadowing.

Eventually the path loss $L_X(d)$ at a particular location is random and is distributed lognormally [5]. So we have:

$$L_X(d) = L(d)X, \tag{2.6}$$

where X is a lognormal random variable $X \sim \mathcal{LN}(\mu, \sigma)$, with mean $\mu = \mu_{dB} = 0$ and variance σ_{dB} , with $\sigma = \ln 10 / 10$, as defined in [1].

Accordingly, when the extended Hata model is employed, we can add the lognormal shadowing in (2.4) and the median path loss can be calculated as

$$L_X(d) = L_H(d)X \quad (2.7)$$

We further assume that the base station transmits a limited amount of total power P to all the channels. If we assume that all channels will be transmitted with a base line signal power P_t then we can relate the signal power $P_{t,k}$ in each channel k to P_t using the power factor f_k as follows:

$$P_{t,k} = f_k P_t \quad (2.8)$$

where, the power factor will be $f_k=1$ for all channels in a uniform power allocation. However, in the pilot control case that we examine in Chapter III, we'll need to increase the pilot tone power factor f_0 in order to enhance synchronization between the base station and the mobile user.

If we apply the Hata-lognormal losses of the channel and simplify the antenna gains and the system losses to one, the received power P_k from the k^{th} channel can be defined as:

$$P_k = \frac{P_{t,k}}{L_H(d)X} = \frac{f_k P_t}{L_H(d)X} \quad (2.9)$$

where

- f_k = the power factor used to adjust the power in the k^{th} channel,
- P_t = the baseline signal power,
- L_H = the median path loss using the Hata model ,
- X = the Lognormal random variable $(0, \mathbf{I} \mathbf{s}_{dB})$.

As shown in [1], P_k is a lognormal random variable, with $P_k \sim \Lambda(\mathbf{m}_{P_k}, \mathbf{I} \mathbf{s}_{dB})$, where $\mathbf{m}_{P_k} = \ln(f_k P_t / L_H)$.

3. Small Scale Fading due to Multipath

Small scale fading is the amplitude fluctuations of the signal caused by interference between two or more copies of the transmitted signal, arriving at the mobile user at slightly different times after bouncing off various obstacles and get time delayed or Doppler shifted.

Our model as we've already mentioned deals with high data rates. So the channel impulse response changes at slower rates than the transmitted signal. In this case as seen in [5], the channel may be assumed to be static over one or several reciprocal bandwidth intervals. Therefore as proved in [1], the signal undergoes slow fading.

On the other hand, the mobile radio channel has a constant gain and a linear phase response over the bandwidth of the transmitted signal. So as defined in [5], the received signal undergoes flat fading. The most common amplitude distribution for a flat fading channel is the Rayleigh distribution. Respectively we will assume as in [1], that the amplitudes are distributed as a Rayleigh random variable R .

Summarizing, we are going to use the Rayleigh slow flat fading channel model to represent the small scale fading due to multipath.

C. BUILDING THE RECEIVED SIGNAL IN THE RAYLEIGH-LOGNORMAL CHANNEL

In this section we are going to combine the phenomena analyzed separately in the previous part and form a slow-flat-Rayleigh fading channel, with lognormal shadowing, and path loss defined by the Hata model, setting up the signal received by the mobile user.

1. The Forward Signal $s_0(t)$

As already discussed, the transmitted signal $\xi(t)$ is affected by small scale fading modeled by the Rayleigh random variable R , and large-scale path loss with shadowing modeled by the lognormal random variable X and the median path loss L_H given by the Hata model.

Moreover, we have to introduce to the transmitted signal a phase discrepancy \mathbf{q}_d , and a time delay \mathbf{t}_d . All these are applied to the transmitted signal $s_0(t)$ of (2.2) and we obtain the forward signal as follows:

$$\begin{aligned}
 s_0(t) &= \sum_{k=0}^{K-1} R \sqrt{\frac{2P_{t,k}}{L_H X}} b_k(t - \mathbf{t}_d) w_k(t - \mathbf{t}_d) \cos(2\mathbf{p} f_c(t - \mathbf{t}_d) + \mathbf{q}_d) \\
 &= \sum_{k=0}^{K-1} R \sqrt{2P_k} b_k(t - \mathbf{t}_d) w_k(t - \mathbf{t}_d) \cos(2\mathbf{p} f_c(t - \mathbf{t}_d) + \mathbf{q}_d) \quad (2.10)
 \end{aligned}$$

We can assume that $\mathbf{t}_d = \mathbf{q}_d = 0$, since these delays are relative amongst the base stations, so the forward signal can be modified as

$$s_0(t) = \sum_{k=0}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) \cos(2\mathbf{p} f_c t) \quad (2.11)$$

2. The Co-Channel Interference $z(t)$

The signals from the adjacent base stations dedicated to the users in the other six cells of the cluster, are also received by the mobile user #1 of the center cell. The sum of these signals forms the co-channel interference and can be expressed as:

$$\mathbf{z}(t) = \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \quad (2.12)$$

where

i = the adjacent cells $i=1,2\dots 6$,

ij = mobile user or channel j in adjacent cell i ,

K_i = the number of active channels in adjacent cell i ,

R_i = Rayleigh fading random variable for signals from adjacent cell i ,

P_{ij} = Lognormal Random Variable representing the average power received from the j -th channel in adjacent cell i as defined in (2.9),

$b_{ij}(t)$ = the information signal for the j -th user channel in adjacent cell i ,

$w_{ij}(t)$ = Walsh function for the j -th user channel in adjacent cell i ,

$c_i(t)$ = PN spreading signal for the adjacent cell i ,

f_c = the carrier frequency of the signal,

\mathbf{t}_i = the time delay from adjacent cell i , relative to the time delay from the center cell base station,

\mathbf{j}_i = the phase delay from adjacent cell i , relative to the phase delay from the center cell base station .

3. The Received Signal $r(t)$

The received signal $r(t)$ is comprised of all the above mentioned signals, plus the Additive White Gaussian Noise (AWGN) $n(t) \sim N(0, N_0/2)$.

Consequently,

$$r(t) = s_0(t) + \mathbf{z}(t) + n(t) = \sum_{k=0}^{K-1} R_k \sqrt{2P_k} b_k(t) w_k(t) \cos(2\mathbf{p} f_c t) + \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \quad (2.13)$$

D. SUMMARY

In this chapter we built a DS-SSMA forward Channel model. We set up the transmitted signals and then we propagate them in the mobile radio channel. We described the phenomena taking place during the propagation, and we modeled the channel based on its distribution as a Rayleigh-Lognormal channel.

Finally we formed the total signal received by the examined mobile user of the center cell.

In the next session we are going to make an as realistic as possible performance analysis of the received signal, finding the SNIR and the BER, and then we are going to compare the results for various parameters with previous work done.

III. DS-CDMA PERFORMANCE ANALYSIS

In Chapter II we presented analytically all the parameters participating in our scenario. We built a channel model, and we introduced all the appropriate signals that constitute the received signal. In this chapter we are going to use this signal to analyze the performance of the receiver, adjusting appropriately various parameters, in order to achieve the best performance.

Starting up the analysis we have to set the mobile user at a place in the center cell. We will assume that the user is at any of the corners of the cell, which is the worst case, since its distance from the base station is maximum. A graphic representation of this scenario is shown at Figure 3.1. We call the distance of the user from the base station d , and its distance from the adjacent cells' base stations D_i . Distance D_i has been geometrically in [1] as:

$$D_i = \begin{cases} d, & i = 4,5 \\ 2d, & i = 3,6 \\ \sqrt{7}d & i = 1,2 \end{cases}$$

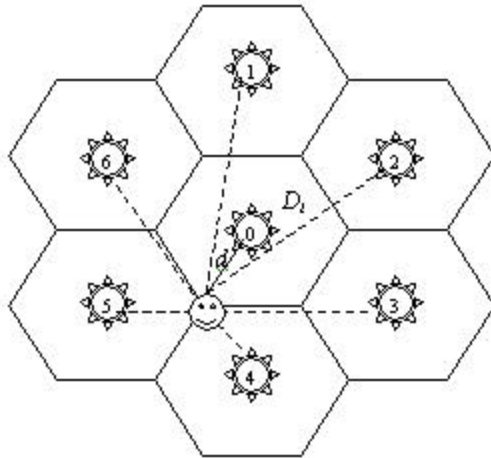


Figure 3.1. Distance of Mobile User from Base Stations.

We are going to apply these distances at the Hata model of (2.4), in order to calculate the median path losses of the transmitted signals.

A typical block diagram of the receiver of the mobile user is shown in Figure 3.2. The received signal $r(t)$ splits at the receiver into two branches. The upper branch is the information branch where the data for the user are despread. The lower branch is the pilot tone recovery branch, where a pilot signal is acquired in order to achieve the demodulation of the received information signal. Finally the demodulated signal is integrated over the bit period and forms the decision statistic Y .

Next we will develop the demodulated signal $y_2(t)$, the decision statistic Y , and eventually the SNIR in order to find the probability of bit error of the system.

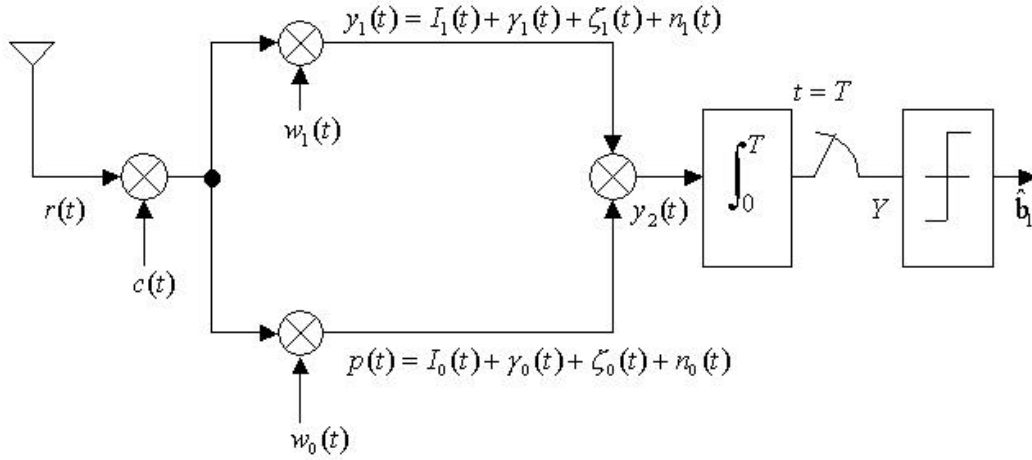


Figure 3.2. Block Diagram of the Mobile Receiver.

A. THE DEMODULATED SIGNAL $y_2(t)$

The signal received from mobile user has been defined in (2.13) as:

$$r(t) = s_0(t) + \mathbf{z}(t) + n(t)$$

$$= \underbrace{\sum_{k=0}^{K-1} R\sqrt{2P_k} b_k(t) w_k(t) c(t) \cos(2\mathbf{p} f_c t)}_{s_0(t)} +$$

$$\begin{aligned}
& + \underbrace{\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) \cos(2\mathbf{p} f_c t + \mathbf{j}_i)}_{\mathbf{z}(t)} \\
& + n(t).
\end{aligned} \tag{3.1}$$

The dispread modulated signal $y_1(t)$ at the upper branch in Figure 3.2, can be expressed as:

$$\begin{aligned}
y_1(t) &= r(t) c(t) w_1(t) = (s_0(t) + \mathbf{z}(t) + n(t)) c(t) w_1(t) \\
&= \underbrace{s_0(t) c(t) w_1(t)}_{I_1(t) + \mathbf{g}_1(t)} + \underbrace{\mathbf{z}(t) c(t) w_1(t)}_{\mathbf{z}_1(t)} + \underbrace{n(t) c(t) w_1(t)}_{h_1(t)}.
\end{aligned} \tag{3.2}$$

Next we are going to analyze these terms contained in $y_1(t)$.

The first sum $I_1(t) + \mathbf{g}_1(t)$ simplified is equal to:

$$\begin{aligned}
I_1(t) + \mathbf{g}_1(t) &= s_0(t) c(t) w_1(t) = R \sqrt{2P_1} b_1(t) w_1(t) c(t) \cos(2\mathbf{p} f_c t) w_1(t) c(t) \\
&+ \left(\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) c(t) \cos(2\mathbf{p} f_c t) \right) c(t) w_1(t)
\end{aligned} \tag{3.3}$$

The desired information signal is in $I_1(t)$:

$$\begin{aligned}
I_1(t) &= R \sqrt{2P_1} b_1(t) w_1(t) c(t) \cos(2\mathbf{p} f_c t) w_1(t) c(t) \\
&= R \sqrt{2P_1} b_1(t) \cos(2\mathbf{p} f_c t),
\end{aligned} \tag{3.4}$$

while the intracell interference in the information channel is contained in $\mathbf{g}_1(t)$:

$$\begin{aligned}
\mathbf{g}_1(t) &= \left(\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) c(t) \cos(2\mathbf{p} f_c t) \right) c(t) w_1(t) \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) w_1(t) \cos(2\mathbf{p} f_c t)
\end{aligned} \tag{3.5}$$

The term $\mathbf{z}_1(t)$ contains the intercell interference and is:

$$\begin{aligned}
\mathbf{z}_1(t) &= \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_1) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) \cos(2\mathbf{p} f_c t + \mathbf{j}_1) \right) c(t) w(t) \\
&= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_1) w_{ij}(t+\mathbf{t}_i) w_i(t) c_i(t+\mathbf{t}_1) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_1). \quad (3.6)
\end{aligned}$$

Finally, the term $\mathbf{h}_1(t)$ contains the thermal noise in the channel:

$$\mathbf{h}_1(t) = n(t) c(t) w(t). \quad (3.7)$$

Summarizing, we saw that the upper branch contains the despread modulated information signal $I_1(t)$, intracell interference $\mathbf{g}_1(t)$, intercell interference $\mathbf{z}_1(t)$ and noise $\mathbf{h}_1(t)$. So summing all that we have:

$$y_1(t) = I_1(t) + \mathbf{g}_1(t) + \mathbf{z}_1(t) + \mathbf{h}_1(t) \quad (3.8)$$

The lower branch in Figure 3.2 is the pilot recovery branch. The pilot signal $p(t)$ in this branch can be expressed as:

$$p(t) = r(t) c(t) w(t) \quad (3.9)$$

The Walsh sequence $w_0(t)$ is equal to 1 for all t , so (3.9) can be written as:

$$\begin{aligned}
p(t) &= r(t) c(t) = (s_o(t) + \mathbf{z}(t) + n(t)) c(t) \\
&= \underbrace{s_o(t) c(t)}_{I_0(t) + \mathbf{g}_0(t)} + \underbrace{\mathbf{z}(t) c(t)}_{\mathbf{z}_0(t)} + \underbrace{n(t) c(t)}_{\mathbf{h}_0(t)} = I_0(t) + \mathbf{g}_0(t) + \mathbf{z}_0(t) + \mathbf{h}_0(t) \quad (3.10)
\end{aligned}$$

Next we are going to analyze these terms contained in $p(t)$. The sum $I_0(t) + \mathbf{g}_0(t)$ simplified is equal to:

$$\begin{aligned}
I_0(t) + \mathbf{g}_0(t) &= \left(\sum_{k=0}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) c(t) \cos(2\mathbf{p} f_c t) \right) c(t) \\
&= \underbrace{\left(R \sqrt{2P_0} b_0(t) c(t) \cos(2\mathbf{p} f_c t) \right) c(t)}_{I_0(t)} + \underbrace{\left(\sum_{k=1}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) c(t) \cos(2\mathbf{p} f_c t) \right) c(t)}_{\mathbf{g}_0(t)}, \quad (3.11)
\end{aligned}$$

where we expanded the sum for $k = 0$, and $k \neq 0$.

The desired pilot tone is contained in $I_0(t)$:

$$\begin{aligned} I_0(t) &= \left(R\sqrt{2P_0}b_0(t)c(t)\cos(2\mathbf{p}f_c t) \right) c(t) \\ &= R\sqrt{2P_0}\cos(2\mathbf{p}f_c t), \end{aligned} \quad (3.12)$$

since $c^2(t) = 1$ and $b_0(t) = 1$.

The intracell interference in the pilot channel is contained in $\mathbf{g}_0(t)$:

$$\begin{aligned} \mathbf{g}_0(t) &= \left(\sum_{k=1}^{K-1} R\sqrt{2P_k}b_k(t)w_k(t)c(t)\cos(2\mathbf{p}f_c t) \right) c(t) \\ &= \sum_{k=1}^{K-1} R\sqrt{2P_k}b_k(t)w_k(t)\cos(2\mathbf{p}f_c t) \end{aligned} \quad (3.13)$$

The term $\mathbf{z}_0(t)$ contains the intercell interference in the pilot channel and is:

$$\begin{aligned} \mathbf{z}_0(t) &= \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i\sqrt{2P_{ij}}b_{ij}(t+\mathbf{t}_i)w_{ij}(t+\mathbf{t}_i)c_i(t+\mathbf{t}_i)\cos(2\mathbf{p}f_c t+\mathbf{j}_i) \right) c(t) \\ &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i\sqrt{2P_{ij}}b_{ij}(t+\mathbf{t}_i)w_{ij}(t+\mathbf{t}_i)c_i(t+\mathbf{t}_i)\cos(2\mathbf{p}f_c t+\mathbf{j}_i) \end{aligned} \quad (3.14)$$

Finally, the term $\mathbf{h}_0(t)$ contains the thermal noise in the pilot channel:

$$\mathbf{h}_0(t) = n(t)c(t) \quad (3.15)$$

Summarizing, we saw that the lower branch contains the pilot recovery signal $I_0(t)$, intracell interference $\mathbf{g}_0(t)$, intercell interference $\mathbf{z}_0(t)$ and noise $\mathbf{h}_0(t)$. So summing all these up we form the pilot signal $p(t)$:

$$p(t) = I_0(t) + \mathbf{g}_0(t) + \mathbf{z}_0(t) + \mathbf{h}_0(t), \quad (3.16)$$

where the terms are defined in (3.11) to (3.15).

Applying the pilot signal $p(t)$ to the information signal $y_1(t)$ we obtain the demodulation of the received signal. The product of them yields $y_2(t)$ as follows:

$$\begin{aligned}
y_2(t) &= y_1(t)p(t) = (I_1 + \mathbf{g}_1 + \mathbf{z}_1 + \mathbf{h}_1)(I_0 + \mathbf{g}_0 + \mathbf{z}_0 + \mathbf{h}_0) \\
&= I_1 I_0 + I_1 \mathbf{g}_0 + I_1 \mathbf{z}_0 + I_1 \mathbf{h}_0 + \mathbf{g}_1 I_0 + \mathbf{g}_1 \mathbf{g}_0 + \mathbf{g}_1 \mathbf{z}_0 + \mathbf{g}_1 \mathbf{h}_0 + \\
&\quad \mathbf{z}_1 I_0 + \mathbf{z}_1 \mathbf{g}_0 + \mathbf{z}_1 \mathbf{z}_0 + \mathbf{z}_1 \mathbf{h}_0 + \mathbf{h}_1 I_0 + \mathbf{h}_1 \mathbf{g}_0 + \mathbf{h}_1 \mathbf{z}_0 + \mathbf{h}_1 \mathbf{h}_0
\end{aligned} \tag{3.17}$$

Looking at the signal $y_2(t)$ we see that it consists of sixteen terms. Analyzing these terms individually we see that the desired information bit $b_1(t)$ is contained in the term $I_1 I_0$ defined by:

$$\begin{aligned}
I_1 I_0 &= \left(R\sqrt{2P_1} b_1(t) \cos(2\mathbf{p} f_c t) \right) \left(R\sqrt{2P_0} \cos(2\mathbf{p} f_c t) \right) \\
&= 2R^2 \sqrt{P_0 P_1} b_1(t) \cos^2(2\mathbf{p} f_c t) \\
&= R^2 \sqrt{P_0 P_1} b_1(t) (1 + \cos(4\mathbf{p} f_c t)).
\end{aligned} \tag{3.18}$$

Intracell interference is contained in the $I_1 \mathbf{g}_0$ term defined as:

$$\begin{aligned}
I_1 \mathbf{g}_0 &= \left(R\sqrt{2P_1} b_1(t) \cos(2\mathbf{p} f_c t) \right) \left(\sum_{k=1}^{K-1} R\sqrt{2P_k} b_k(t) w_k(t) \cos(2\mathbf{p} f_c t) \right) \\
&= \sum_{k=1}^{K-1} 2R^2 \sqrt{P_1 P_k} b_1(t) b_k(t) w_k(t) \cos^2(2\mathbf{p} f_c t) \\
&= \sum_{k=1}^{K-1} R^2 \sqrt{P_1 P_k} b_1(t) b_k(t) w_k(t) (1 + \cos(4\mathbf{p} f_c t))
\end{aligned} \tag{3.19}$$

Intercell interference is contained in the $I_1 \mathbf{z}_0$ term defined as:

$$\begin{aligned}
I_1 \mathbf{z}_0 &= \left(R\sqrt{2P_1} b_1(t) \cos(2\mathbf{p} f_c t) \right) \times \\
&\quad \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \\
&= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R R_i \sqrt{P_1 P_{ij}} b_1(t) b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) c(t) \frac{1}{2} (\cos(4\mathbf{p} f_c t + \mathbf{j}_i) + \cos(\mathbf{j}_i)) \\
&= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R R_i \sqrt{P_1 P_{ij}} b_1(t) b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) c(t) (\cos(4\mathbf{p} f_c t + \mathbf{j}_i) + \cos(\mathbf{j}_i))
\end{aligned} \tag{3.20}$$

Noise is also contained in the $I_1 \mathbf{h}_0$ term:

$$\begin{aligned}
I_1 \mathbf{h}_0 &= \left(R\sqrt{2P_1} b_1(t) \cos(2\mathbf{p} f_c t) \right) (n(t)c(t)) \\
&= R\sqrt{2P_1} b_1(t) n(t)c(t) \cos(2\mathbf{p} f_c t)
\end{aligned} \tag{3.21}$$

Intracell interference is contained in the $\mathbf{g}_1 I_0$ term and is defined as:

$$\begin{aligned}
\mathbf{g}_1 I_0 &= \left(\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R\sqrt{2P_k} b_k(t) w_k(t) w_1(t) \cos(2\mathbf{p} f_c t) \right) \left(R\sqrt{2P_0} \cos(2\mathbf{p} f_c t) \right) \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R^2 2\sqrt{P_0 P_k} b_k(t) w_k(t) w_1(t) \cos^2(2\mathbf{p} f_c t) \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R^2 \sqrt{P_0 P_k} b_k(t) w_k(t) w_1(t) (1 + \cos(4\mathbf{p} f_c t))
\end{aligned} \tag{3.22}$$

Intracell interference is also contained in the $\mathbf{g}_1 \mathbf{g}_0$ term:

$$\begin{aligned}
\mathbf{g}_1 \mathbf{g}_0 &= \left(\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R\sqrt{2P_k} b_k(t) w_k(t) w_1(t) \cos(2\mathbf{p} f_c t) \right) \left(\sum_{k=1}^{K-1} R\sqrt{2P_k} b_k(t) w_k(t) \cos(2\mathbf{p} f_c t) \right) \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq 1}}^{K-1} R^2 2\sqrt{P_k P_q} b_k(t) b_q(t) w_k(t) w_1(t) w_q(t) \cos^2(2\mathbf{p} f_c t) \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq 1}}^{K-1} R^2 \sqrt{P_k P_q} b_k(t) b_q(t) w_k(t) w_1(t) w_q(t) (1 + \cos(4\mathbf{p} f_c t)) \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq 1}}^{K-1} R^2 \sqrt{P_k P_q} b_k(t) b_q(t) w_{k \oplus 1}(t) w_q(t) (1 + \cos(4\mathbf{p} f_c t)) \\
&= \left(\underbrace{\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R^2 \sqrt{P_k P_{k \oplus 1}} b_k(t) b_{k \oplus 1}(t)}_{q=k \oplus 1} + \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq k \oplus 1}}^{K-1} R^2 \sqrt{P_k P_q} b_k(t) b_q(t) w_{k \oplus 1}(t) w_q(t) \right) (1 + \cos(4\mathbf{p} f_c t)) \\
&= \left(\underbrace{R^2 \sqrt{P_0 P_{0 \oplus 1}} b_0(t) b_{0 \oplus 1}(t)}_{k=0} + \sum_{k=2}^{K-1} R^2 \sqrt{P_k P_{k \oplus 1}} b_k(t) b_{k \oplus 1}(t) + \right.
\end{aligned}$$

$$\begin{aligned}
& + \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq k \oplus 1}}^{K-1} R^2 \sqrt{P_k P_q} b_k(t) b_q(t) w_{k \oplus 1}(t) w_q(t) \left(1 + \cos(4\mathbf{p} f_c t) \right) \\
& = \left(R^2 \sqrt{P_0 P_1} b_1(t) + \sum_{k=2}^{K-1} R^2 \sqrt{P_k P_{k \oplus 1}} b_k(t) b_{k \oplus 1}(t) + \right. \\
& \quad \left. \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq k \oplus 1}}^{K-1} R^2 \sqrt{P_k P_q} b_k(t) b_q(t) w_{k \oplus 1}(t) w_q(t) \right) \left(1 + \cos(4\mathbf{p} f_c t) \right), \tag{3.23}
\end{aligned}$$

where $w_{k \oplus 1}(t)$ is a Walsh function, defined in [1] as the product of $w_k(t)$ and $w_1(t)$.

A product of intracell and intercell interference is also contained in $\mathbf{g}_i \mathbf{z}_0$ term:

$$\begin{aligned}
\mathbf{g}_i \mathbf{z}_0 & = \left(\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) w_1(t) \cos(2\mathbf{p} f_c t) \right) \times \\
& \quad \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) \alpha(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \\
& = \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R R_i \sqrt{P_k P_{ij}} b_{ij}(t + \mathbf{t}_i) b_k(t) w_{ij}(t + \mathbf{t}_i) w_k(t) w_1(t) c_i(t + \mathbf{t}_i) \alpha(t) \times \\
& \quad (\cos(4\mathbf{p} f_c t + \mathbf{j}_i) + \cos \mathbf{j}_i) \tag{3.24}
\end{aligned}$$

Intracell interference we also have at $\mathbf{g}_i \mathbf{h}_0$ term:

$$\begin{aligned}
\mathbf{g}_i \mathbf{h}_0 & = \left(\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) w_1(t) \cos(2\mathbf{p} f_c t) \right) n(t) c(t) \\
& = \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) w_1(t) n(t) c(t) \cos(2\mathbf{p} f_c t) \tag{3.25}
\end{aligned}$$

Intercell interference is also contained in the $\mathbf{z}_1 I_0$ term and is defined as:

$$\begin{aligned}
\mathbf{z}_1 I_0 & = \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) w_1(t) c_1(t + \mathbf{t}_i) \alpha(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \left(R \sqrt{2P_0} \cos(2\mathbf{p} f_c t) \right) \\
& = \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R R_i 2 \sqrt{P_0 P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) w_1(t) c_1(t + \mathbf{t}_i) \alpha(t) \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i)
\end{aligned}$$

$$= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} RR_i \sqrt{P_0 P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) w_1(t) c_1(t + \mathbf{t}_i) c(t) [\cos(4\mathbf{p} f_c t + \mathbf{j}_i) + \cos(\mathbf{j}_i)] \quad (3.26)$$

A product of intercell and intracell interference is present in $\mathbf{z}_i \mathbf{g}_0$ term, defined as:

$$\begin{aligned} \mathbf{z}_i \mathbf{g}_0 &= \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) w_1(t) c_1(t + \mathbf{t}_i) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \times \\ &\quad \left(\sum_{k=1}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) \cos(2\mathbf{p} f_c t) \right) \\ &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} RR_i 2\sqrt{P_{ij} P_k} b_{ij}(t) b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) w_k(t) w_1(t) c_1(t + \mathbf{t}_i) c(t) \times \\ &\quad \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \\ &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} RR_i \sqrt{P_{ij} P_k} b_{ij}(t) b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) w_{k\oplus 1}(t) c_1(t + \mathbf{t}_i) c(t) (\cos(4\mathbf{p} f_c t + \mathbf{j}_i) + \cos(\mathbf{j}_i)) \quad (3.27) \end{aligned}$$

A product of intracell and intracell from the pilot recovery branch, interference is contained in the $\mathbf{z}_1 \mathbf{z}_0$ term:

$$\begin{aligned} \mathbf{z}_1 \mathbf{z}_0 &= \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) w_1(t) c_1(t + \mathbf{t}_i) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \times \\ &\quad \left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \\ &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} R_i R_p 2\sqrt{P_{ij} P_{pq}} b_{ij}(t + \mathbf{t}_i) b_{pq}(t + \mathbf{t}_p) w_{ij}(t + \mathbf{t}_i) w_{pq}(t + \mathbf{t}_p) \times \\ &\quad c_i(t + \mathbf{t}_i) c_p(t + \mathbf{t}_p) c^2(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \cos(2\mathbf{p} f_c t + \mathbf{j}_p) \\ &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} R_i R_p 2\sqrt{P_{ij} P_{pq}} b_{ij}(t + \mathbf{t}_i) b_{pq}(t + \mathbf{t}_p) w_{ij}(t + \mathbf{t}_i) w_{pq}(t + \mathbf{t}_p) \times \\ &\quad c_i(t + \mathbf{t}_i) c_p(t + \mathbf{t}_p) \frac{1}{2} [\cos(4\mathbf{p} f_c t + \mathbf{j}_i + \mathbf{j}_p) + \cos(\mathbf{j}_i - \mathbf{j}_p)] \\ &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} R_i R_p \sqrt{P_{ij} P_{pq}} b_{ij}(t + \mathbf{t}_i) b_{pq}(t + \mathbf{t}_p) w_{ij}(t + \mathbf{t}_i) w_{pq}(t + \mathbf{t}_p) \times \\ &\quad c_i(t + \mathbf{t}_i) c_p(t + \mathbf{t}_p) [\cos(4\mathbf{p} f_c t + \mathbf{j}_i + \mathbf{j}_p) + \cos(\mathbf{j}_i - \mathbf{j}_p)] \quad (3.28) \end{aligned}$$

A product of intercell interference and noise can also be found in $\mathbf{z}_1 \mathbf{h}_0$ term:

$$\begin{aligned}
\mathbf{z}_1 \mathbf{h}_0 &= (n(t)c(t)) \times \\
&\left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_1(t) c_i(t+\mathbf{t}_i) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \\
&= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_1(t) c_i(t+\mathbf{t}_i) n(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i)
\end{aligned} \tag{3.29}$$

Noise in the demodulated signal is also expressed by the $\mathbf{h}_1 I_0$ term as:

$$\begin{aligned}
\mathbf{h}_1 I_0 &= (n(t) c(t) w_1(t)) \left(R \sqrt{2P_0} \cos(2\mathbf{p} f_c t) \right) \\
&= R \sqrt{2P_0} n(t) c(t) w_1(t) \cos(2\mathbf{p} f_c t)
\end{aligned} \tag{3.30}$$

The next term contains noise and intracell interference:

$$\begin{aligned}
\mathbf{h}_1 \mathbf{g}_0 &= (n(t) c(t) w_1(t)) \left(\sum_{k=1}^{K-1} R \sqrt{2P_k} b_k(t) w_k(t) \cos(2\mathbf{p} f_c t) \right) \\
&= \sum_{k=1}^{K-1} R \sqrt{2P_k} b_k(t) n(t) w_k(t) w_1(t) c(t) \cos(2\mathbf{p} f_c t) \\
&= \sum_{k=1}^{K-1} R \sqrt{2P_k} b_k(t) n(t) w_{k \oplus 1}(t) c(t) \cos(2\mathbf{p} f_c t),
\end{aligned} \tag{3.31}$$

where $w_{k \oplus 1}(t) = w_k(t) w_1(t)$ is a Walsh function defined in [1].

Noise and intercell interference are also contained in $\mathbf{h}_1 \mathbf{z}_0$ term:

$$\begin{aligned}
\mathbf{h}_1 \mathbf{z}_0 &= (n(t) c(t) w_1(t)) \times \\
&\left(\sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_1(t+\mathbf{t}_i) c(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) \right) \\
&= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) n(t) w_{ij}(t+\mathbf{t}_i) w_1(t) c_i(t+\mathbf{t}_i) \cos(2\mathbf{p} f_c t + \mathbf{j}_i)
\end{aligned} \tag{3.32}$$

Finally, noise in the demodulated signal can be also found in the $\mathbf{h}_1 \mathbf{h}_0$ term:

$$\begin{aligned}
\mathbf{h}_1 \mathbf{h}_0 &= (n(t) c(t) w_1(t)) (n(t) c(t)) \\
&= n^2(t) c^2(t) w_1(t) \\
&= n^2(t) w_1(t),
\end{aligned} \tag{3.33}$$

since $c^2(t) = 1$.

Accordingly, the demodulated signal $y_2(t)$ from (3.17) is modified using (3.18) through (3.33), and then is sent into the integrator in order to determine the decision statistic Y , which we'll perform in the next Section.

B. THE DECISION STATISTIC Y

In this section we will develop the decision statistic Y , which would help us find the signal to noise plus interference ratio and eventually the performance of the channel.

In order to calculate Y we will integrate the demodulated signal $y_2(t)$ consisted of the 16 terms we calculated in the previous section at time $t=T$, as shown in Figure 3.2. Moreover we will condition our decision statistic Y , on the Rayleigh fading random variable $R=r$ and on the received power $P_k=p_k$ which represents the lognormal random variable. This results in

$$\begin{aligned}
Y|_{r,p_k} &= \int_0^T y_2(t) dt \Big|_{r,p_k} = \\
&= \int_0^T \left(I_1 I_0 + I_1 \mathbf{g}_0 + I_1 \mathbf{z}_0 + I_1 \mathbf{h}_0 + \mathbf{g}_1 I_0 + \mathbf{g}_1 \mathbf{g}_0 + \mathbf{g}_1 \mathbf{z}_0 + \mathbf{g}_1 \mathbf{h}_0 + \right. \\
&\quad \left. + \mathbf{z}_1 I_0 + \mathbf{z}_1 \mathbf{g}_0 + \mathbf{z}_1 \mathbf{z}_0 + \mathbf{z}_1 \mathbf{h}_0 + \mathbf{h}_1 I_0 + \mathbf{h}_1 \mathbf{g}_0 + \mathbf{h}_1 \mathbf{z}_0 + \mathbf{h}_1 \mathbf{h}_0 \right) dt \Big|_{r,p_k} = \\
&= \underbrace{\int_0^T I_1 I_0 dt}_{Y_1} \Big|_{r,p_k} + \underbrace{\int_0^T I_1 \mathbf{g}_0 dt}_{\mathbf{g}_{11}} \Big|_{r,p_k} + \underbrace{\int_0^T I_1 \mathbf{z}_0 dt}_{\mathbf{z}_{11}} \Big|_{r,p_k} + \underbrace{\int_0^T I_1 \mathbf{h}_0 dt}_{\mathbf{h}_{11}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{g}_1 I_0 dt}_{\mathbf{g}_{12}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{g}_1 \mathbf{g}_0 dt}_{\mathbf{g}_{13}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{g}_1 \mathbf{z}_0 dt}_{\mathbf{z}_{12}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{g}_1 \mathbf{h}_0 dt}_{\mathbf{h}_{12}} \Big|_{r,p_k} + \\
&\quad + \underbrace{\int_0^T \mathbf{z}_1 I_0 dt}_{\mathbf{z}_{13}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{z}_1 \mathbf{g}_0 dt}_{\mathbf{z}_{14}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{z}_1 \mathbf{z}_0 dt}_{\mathbf{z}_{15}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{z}_1 \mathbf{h}_0 dt}_{\mathbf{h}_{13}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{h}_1 I_0 dt}_{\mathbf{h}_{14}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{h}_1 \mathbf{g}_0 dt}_{\mathbf{h}_{15}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{h}_1 \mathbf{z}_0 dt}_{\mathbf{h}_{16}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{h}_1 \mathbf{h}_0 dt}_{\mathbf{h}_{17}} \Big|_{r,p_k} \\
&= Y_1 + \mathbf{g}_{11} + \mathbf{z}_{11} + \mathbf{h}_{11} + \mathbf{g}_{12} + \mathbf{g}_{13} + \mathbf{z}_{12} + \mathbf{h}_{12} + \mathbf{z}_{13} + \mathbf{z}_{14} + \mathbf{z}_{15} + \mathbf{h}_{13} + \mathbf{h}_{14} + \mathbf{h}_{15} + \mathbf{h}_{16} + \mathbf{h}_{17} \\
&= Y_1 + \sum_{i=11}^{15} \mathbf{z}_i + \sum_{i=11}^{13} \mathbf{g}_i + \sum_{i=11}^{17} \mathbf{h}_i . \tag{3.34}
\end{aligned}$$

In the next pages we will develop each component of the decision statistic Y separately:

1. $Y_1 = \int_0^T I_1 I_0 dt \Big|_{r,p_k} = \int_0^T r^2 \sqrt{p_0 p_1} b(t) (1 + \cos(4\mathbf{p} f_c t)) dt$

$$\begin{aligned}
&= r^2 \sqrt{p_0 p_1} b_1 \int_0^T (1 + \cos(4\mathbf{p} f_c t)) dt \\
&= r^2 \sqrt{p_0 p_1} T b_1,
\end{aligned} \tag{3.35}$$

where $b_k \in \{\pm 1\}$ corresponds with the time function $b_k(t)$, which is constant over the period $(0, T)$. Also we assume that the carrier frequency f_c is an integer multiple of the bit rate of the system, which means that $f_c = k/T$, so we have

$$\int_0^T \cos(4\mathbf{p} f_c t) dt = 0.$$

$$\begin{aligned}
2. \quad \mathbf{g}_{11} &= \int_0^T I_1 \mathbf{g}_0 dt \Big|_{r, p_k} = \int_0^T \sum_{k=1}^{K-1} r^2 \sqrt{p_1 p_k} b_1(t) b_k(t) w_k(t) (1 + \cos(4\mathbf{p} f_c t)) dt \\
&= \sum_{k=1}^{K-1} r^2 \sqrt{p_1 p_k} b_1 b_k \int_0^T w_k(t) dt + \sum_{k=1}^{K-1} r^2 \sqrt{p_1 p_k} b_1 b_k \int_0^T w_k(t) \cos(4\mathbf{p} f_c t) dt \\
&= 0,
\end{aligned} \tag{3.36}$$

where the first integral is zero since a Walsh function integrated over the bit period is always equal to zero and the second integral is also zero since $f_c = k/T$.

$$\begin{aligned}
3. \quad \mathbf{z}_{11} &= \int_0^T I_1 \mathbf{z}_0 dt \Big|_{r, p_k} \\
&= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} r R_i \sqrt{p_1 P_{ij}} b_1(t) b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) c(t) (\cos(4\mathbf{p} f_c t + \mathbf{j}_i) + \cos(\mathbf{j}_i)) dt \\
&= r \sqrt{p_1} \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{P_{ij}} b_1(t) b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) c(t) \cos(\mathbf{j}_i) dt
\end{aligned} \tag{3.37}$$

$$\begin{aligned}
4. \quad \mathbf{h}_{11} &= \int_0^T I_1 \mathbf{h}_0 dt \Big|_{r, p_k} = \int_0^T r \sqrt{2p_1} b_1(t) n(t) c(t) \cos(2\mathbf{p} f_c t) dt \\
&= r \sqrt{2p_1} \int_0^T b_1(t) n(t) c(t) \cos(2\mathbf{p} f_c t) dt
\end{aligned} \tag{3.38}$$

$$5. \quad \mathbf{g}_{12} = \int_0^T \mathbf{g}_1 I_0 dt \Big|_{r, p_k} = \int_0^T \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 \sqrt{p_0 p_k} b_k(t) w_k(t) w_1(t) (1 + \cos(4\mathbf{p} f_c t)) dt$$

$$\begin{aligned}
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 \sqrt{p_0 p_k} b_k \int_0^T w_k(t) w_1(t) (1 + \cos(4\mathbf{p} f_c t)) dt \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 \sqrt{p_0 p_k} b_k \left(\int_0^T w_k(t) w_1(t) dt + \int_0^T (1 + \cos(4\mathbf{p} f_c t)) dt \right) \\
&= 0,
\end{aligned} \tag{3.39}$$

where the integrals are zero due to the orthogonality of the Walsh functions and the since we assumed $f_c = k/T$.

$$\begin{aligned}
6. \quad \mathbf{g}_{13} &= \int_0^T \mathbf{g}_1 \mathbf{g}_0 dt \Big|_{r, p_k} = \int_0^T \left\{ r^2 \sqrt{p_0 p_0} b_1(t) + \sum_{k=2}^{K-1} r^2 \sqrt{p_k p_{k \oplus 1}} b_k(t) b_{k \oplus 1}(t) + \right. \\
&\quad \left. + \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq k \oplus 1}}^{K-1} r^2 \sqrt{p_k p_q} b_k(t) b_q(t) w_{k \oplus 1}(t) w_q(t) \right\} (1 + \cos(4\mathbf{p} f_c t)) dt \\
&= \int_0^T r^2 \sqrt{p_0 p_0} b_1(t) (1 + \cos(4\mathbf{p} f_c t)) dt + \\
&\quad + \int_0^T \sum_{k=2}^{K-1} r^2 \sqrt{p_k p_{k \oplus 1}} b_k(t) b_{k \oplus 1}(t) (1 + \cos(4\mathbf{p} f_c t)) dt + \\
&\quad + \int_0^T \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq k \oplus 1}}^{K-1} r^2 \sqrt{p_k p_q} b_k(t) b_q(t) w_{k \oplus 1}(t) w_q(t) (1 + \cos(4\mathbf{p} f_c t)) dt \\
&= r^2 \sqrt{p_0 p_1} \int_0^T b_1(t) dt + r^2 \int_0^T \sum_{k=2}^{K-1} \sqrt{p_k p_{k \oplus 1}} b_k(t) b_{k \oplus 1}(t) dt + \\
&\quad + r^2 \int_0^T \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{q=1 \\ q \neq k \oplus 1}}^{K-1} \sqrt{p_k p_q} b_k(t) b_q(t) w_{k \oplus 1}(t) w_q(t) dt \\
&= \underbrace{r^2 \sqrt{p_0 p_1} T b_1}_{Y_1} + r^2 T \underbrace{\left(\sum_{k=2}^{K-1} \sqrt{p_k p_{k \oplus 1}} b_k b_{k \oplus 1} \right)}_{\mathbf{g}_{14}} + 0 \\
&= Y_1 + \mathbf{g}_{14},
\end{aligned} \tag{3.40}$$

where we assumed again that $f_c = k/T$. As we see we gain another Y_1 term, which doubles the power of the desired signal, however we also get an intracell interference term in the form of \mathbf{g}_{14} .

$$7. \quad \mathbf{z}_{12} = \int_0^T \mathbf{g}_1 \mathbf{z}_0 dt \Big|_{r, p_k} = \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r R_i \sqrt{p_k P_{ij}} b_{ij}(t + \mathbf{t}_i) b_k(t) w_{ij}(t + \mathbf{t}_i) w_{k \oplus 1}(t) \times$$

$$\begin{aligned}
& c_i(t+\mathbf{t}_i)c(t)(\cos(4\mathbf{p}f_c t+\mathbf{j}_i)+\cos\mathbf{j}_i)dt = \\
& = r\int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=0}^{K-1} R_i \sqrt{p_k P_{ij}} b_{ij}(t+\mathbf{t}_i) b_k(t) w_{ij}(t+\mathbf{t}_i) w_{k\oplus 1}(t) c_i(t+\mathbf{t}_i) \alpha(t) \cos\mathbf{j}_i dt \quad (3.41)
\end{aligned}$$

$$\begin{aligned}
8. \quad \mathbf{z}_{13} &= \int_0^T \mathbf{z}_1 I_0 dt \Big|_{r,p_k} \\
&= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} r R_i \sqrt{p_o P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_1(t) c_i(t+\mathbf{t}_i) c(t) (\cos(4\mathbf{p}f_c t+\mathbf{j}_i)+\cos\mathbf{j}_i) dt \\
&= r\sqrt{p_o} \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_1(t) c_i(t+\mathbf{t}_i) c(t) \cos\mathbf{j}_i dt. \quad (3.42)
\end{aligned}$$

$$\begin{aligned}
9. \quad \mathbf{h}_{12} &= \int_0^T \mathbf{g}_1 \mathbf{h}_0 dt \Big|_{r,p_k} = \int_0^T \sum_{k=0}^{K-1} r \sqrt{2p_k} b_k(t) w_k(t) w_{1\oplus 1}(t) c(t) \cos(2\mathbf{p}f_c t) dt \\
&= r \int_0^T \sum_{k=0}^{K-1} \sqrt{2p_k} b_k(t) w_k(t) w_{1\oplus 1}(t) c(t) \cos(2\mathbf{p}f_c t) dt \quad (3.43)
\end{aligned}$$

$$\begin{aligned}
10. \quad \mathbf{z}_{14} &= \int_0^T \mathbf{z}_1 \mathbf{g}_0 dt = \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} r R_i \sqrt{P_{ij} p_k} b_k(t) b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_{k\oplus 1}(t) \times \\
& c_i(t+\mathbf{t}_i) c(t) (\cos(4\mathbf{p}f_c t+\mathbf{j}_i)+\cos\mathbf{j}_i) dt \\
&= r \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} R_i \sqrt{p_k P_{ij}} b_k(t) b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_{k\oplus 1}(t) c_i(t+\mathbf{t}_i) \alpha(t) \cos\mathbf{j}_i dt \quad (3.44)
\end{aligned}$$

$$\begin{aligned}
11. \quad \mathbf{z}_{15} &= \int_0^T \mathbf{z}_1 \mathbf{z}_0 dt \Big|_{r,p_k} \\
&= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{Kp-1} R_i R_p \sqrt{P_{ij} P_{pq}} b_{ij}(t+\mathbf{t}_i) b_{pq}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_{pq}(t+\mathbf{t}_p) \times \\
& c_i(t+\mathbf{t}_i) c_p(t+\mathbf{t}_p) [\cos(4\mathbf{p}f_c t+\mathbf{j}_i+\mathbf{j}_p)+\cos(\mathbf{j}_i-\mathbf{j}_p)] \\
&= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{Kp-1} R_i R_p \sqrt{P_{ij} P_{pq}} b_{ij}(t+\mathbf{t}_i) b_{pq}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_{pq}(t+\mathbf{t}_p) \times \\
& c_i(t+\mathbf{t}_i) c_p(t+\mathbf{t}_p) \cos(\mathbf{j}_i-\mathbf{j}_p) dt. \quad (3.45)
\end{aligned}$$

$$\begin{aligned}
12. \quad \mathbf{h}_{13} &= \int_0^T \mathbf{z}_1 \mathbf{h}_0 dt \Big|_{r,p_k} \\
&= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_1(t) c_i(t+\mathbf{t}_i) \alpha(t) \cos(2\mathbf{p}f_c t+\mathbf{j}_i) dt \quad (3.46)
\end{aligned}$$

$$13. \quad \mathbf{h}_{14} = \int_0^T \mathbf{h}_1 I_0 dt \Big|_{r,p_k} = \int_0^T r \sqrt{2p_0} n(t) \alpha(t) w_1(t) \cos(2\mathbf{p}f_c t) dt$$

$$= r\sqrt{2p_0} \int_0^T n(t) c(t) w_1(t) \cos(2\mathbf{p} f_c t) dt \quad (3.47)$$

$$\begin{aligned} 14. \quad \mathbf{h}_{15} &= \int_0^T \mathbf{h}_1 \mathbf{g}_0 dt \Big|_{r, p_k} = r \int_0^T \sum_{k=1}^{K-1} \sqrt{2p_k} b_k(t) n(t) w_k(t) w_1(t) c(t) \cos(2\mathbf{p} f_c t) dt \\ &= r \int_0^T \sum_{k=1}^{K-1} \sqrt{2p_k} b_k(t) n(t) w_{k \oplus 1}(t) c(t) \cos(2\mathbf{p} f_c t) dt \end{aligned} \quad (3.48)$$

$$\begin{aligned} 15. \quad \mathbf{h}_{16} &= \int_0^T \mathbf{h}_1 \mathbf{z}_0 dt \Big|_{r, p_k} \\ &= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) n(t) w_{ij}(t + \mathbf{t}_i) w_1(t) c_i(t + \mathbf{t}_i) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) dt . \end{aligned} \quad (3.49)$$

$$16. \quad \mathbf{h}_{17} = \int_0^T \mathbf{h}_1 \mathbf{h}_0 dt \Big|_{r, p_k} = \int_0^T n^2(t) w_1(t) dt . \quad (3.50)$$

Summarizing all of the above, we see that the demodulated information bit is expressed by two Y_1 terms, acquired by the integration of $I_1 I_0$ and $?_1 ?_0$ terms respectively. Intracell interference $?$ is expressed by \mathbf{g}_{14} term only, derived from the integration of $?_1 ?_0$ term, since \mathbf{g}_{11} and \mathbf{g}_{12} are zero, while intercell interference can be expressed by \mathbf{z} , where

$$\mathbf{z} = \sum_{i=11}^{15} \mathbf{z}_i . \quad (3.51)$$

Finally, the additive noise contribution to the decision statistic can be combined in \mathbf{h} term, where

$$\mathbf{h} = \sum_{i=11}^{17} \mathbf{h}_i . \quad (3.52)$$

We can also combine the noise, intracell and intercell interference in our decision statistic into a single term \mathbf{x} . Summing all these up, our conditioned decision statistic Y from (3.34) becomes:

$$\begin{aligned} Y \Big|_{r, p_k} &= Y_1 + Y_1 + \underbrace{\mathbf{g} + \mathbf{z} + \mathbf{h}}_{\mathbf{x}} \\ &= 2Y_1 + \mathbf{x} , \end{aligned} \quad (3.53)$$

where $\mathbf{x} = \mathbf{g} + \mathbf{z} + \mathbf{h}$.

As defined, $Y|_{r,p_k}$ is not very practical for the developing of the performance analysis of our system. In order to simplify the analysis we can use a technique called the Gaussian approximation. Accordingly, we assume that all the terms of the above equations are independent and we are going to model $Y|_{r,p_k}$ as Gaussian random variable y . with mean

$$E\{y\} = \bar{Y} = 2Y_1 = 2r^2 \sqrt{p_0 p_1} T b_1, \quad (3.54)$$

where Y_1 has been defined in (3.35), and variance the sum of the interfering terms variances defined by

$$\text{Var}\{y\} = \text{Var}\{\mathbf{g}\} + \text{Var}\{\mathbf{z}\} + \text{Var}\{\mathbf{h}\} = \text{Var}\{\mathbf{x}\} = \mathbf{s}_x^2 \quad (3.55)$$

Summarizing, we modeled our decision statistic Y as a Gaussian random variable $y \sim N(\bar{Y}, \mathbf{s}_x)$. In the next section we are going to develop the SNIR and probability error of our system.

C. SIGNAL TO NOISE PLUS INTERFERENCE RATIO

In this section we will develop a conditional SNIR for our DS-CDMA forward signal in the Rayleigh-lognormal fading channel. We will not remove the conditioning on the random variables $R=r$ and $P_k=p_k$, until we develop the probability of error. The SNIR after [6] is defined as the ratio of the average power of the message signal to the average power of the noise, both measured at the receiver output. Therefore, the SNIR can be defined as

$$\text{SNIR}|_{r,p_k} = \frac{\bar{Y}^2}{\mathbf{s}_x^2}, \quad (3.56)$$

where \bar{Y} is defined by (3.54), and \mathbf{s}_x^2 is determined in (3.55) and is going to be thoroughly defined in the next pages.

The total intercell interference I is defined in (3.51) as the sum of all the intercell interfering terms I_k . Since the co-channel interference contributions I_k are modeled as zero

mean random variables, we define the total co-channel interference variance as the sum of the variances of the contributing terms:

$$\text{Var}\{\mathbf{z}\} = \sum_{i=1}^{15} \text{Var}\{\mathbf{z}_i\} \quad (3.57)$$

We define the variances of the interfering terms \mathbf{z}_i as follows:

$$\text{Var}\{\mathbf{z}_{11}\} = \mathbf{s}_{\mathbf{z}_{11}}^2 = \frac{1}{3N} r^2 p_1 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}], \quad (3.58)$$

$$\text{Var}\{\mathbf{z}_{12}\} = \mathbf{s}_{\mathbf{z}_{12}}^2 = \frac{1}{3N} r^2 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=1 \\ k \neq i}}^{K-1} p_k E[R_i^2] E[P_{ij}], \quad (3.59)$$

$$\text{Var}\{\mathbf{z}_{13}\} = \mathbf{s}_{\mathbf{z}_{13}}^2 = \frac{1}{3N} r^2 p_0 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}] \quad (3.60)$$

$$\text{Var}\{\mathbf{z}_{14}\} = \mathbf{s}_{\mathbf{z}_{14}}^2 = \frac{1}{3N} r^2 p_1 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} p_k E[R_i^2] E[P_{ij}], \quad (3.61)$$

$$\begin{aligned} \text{Var}\{\mathbf{z}_{15}\} = \mathbf{s}_{\mathbf{z}_{15}}^2 = & \frac{1}{N} T^2 \left(\frac{1}{3} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} E[R_i^4] E[P_{ij}] E[P_{iq}] + \right. \\ & \left. + \frac{1}{4} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=0 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} E[R_i^2] E[R_p^2] E[P_{ij}] E[P_{pq}] \right), \end{aligned} \quad (3.62)$$

where the complete derivation of these terms can be found in Appendix III-A.1.

The intracell interference contribution \mathbf{g} is represented with \mathbf{g}_{14} term. We define its variance by

$$\text{Var}\{\mathbf{g}\} = \mathbf{s}_{\mathbf{g}}^2 = \mathbf{s}_{\mathbf{g}_{14}}^2 = r^4 T^2 \sum_{k=2}^{K-1} p_k p_{k \oplus 1}, \quad (3.63)$$

where the complete derivation of this term can be found in Appendix III-A.2.

Similarly, the total additive noise \mathbf{h} is defined in (3.52) as the sum of all the noise terms \mathbf{h}_i . Since the noise contributions \mathbf{h}_i are modeled as zero mean random variables, we define the total additive noise variance as the sum of the variances of the contributing terms:

$$\text{Var}\{\mathbf{h}\} = \sum_{i=1}^{17} \text{Var}\{\mathbf{h}_i\} \quad (3.64)$$

We define the variances of the contributing terms \mathbf{h}_i as follows:

$$\text{Var}\{\mathbf{h}_{11}\} = \mathbf{s}^2_{h_{11}} = \frac{1}{2} r^2 p_1 N_0 T, \quad (3.65)$$

$$\text{Var}\{\mathbf{h}_{12}\} = \mathbf{s}^2_{h_{12}} = \frac{1}{2} r^2 N_0 T \sum_{\substack{k=1 \\ k \neq 1}}^{K-1} p_k, \quad (3.66)$$

$$\text{Var}\{\mathbf{h}_{13}\} = \mathbf{s}^2_{h_{13}} = \frac{1}{2} N_0 T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}], \quad (3.67)$$

$$\text{Var}\{\mathbf{h}_{14}\} = \mathbf{s}^2_{h_{14}} = \frac{1}{2} r^2 p_0 N_0 T, \quad (3.68)$$

$$\text{Var}\{\mathbf{h}_{15}\} = \mathbf{s}^2_{h_{15}} = \frac{1}{2} r^2 N_0 T \sum_{k=1}^{K-1} p_k, \quad (3.69)$$

$$\text{Var}\{\mathbf{h}_{16}\} = \mathbf{s}^2_{h_{16}} = \frac{1}{2} N_0 T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}], \quad (3.70)$$

$$\text{Var}\{\mathbf{h}_{17}\} = \mathbf{s}^2_{h_{17}} = \frac{3N_0^2}{4}, \quad (3.71)$$

where the complete derivation of these terms can be found in Appendix III-A.3.

Using (3.57) through (3.71) we can update the variance of the decision statistic Y defined by (3.55) as follows:

$$\begin{aligned} \mathbf{s}_x^2 &= \mathbf{s}_z^2 + \mathbf{s}_g^2 + \mathbf{s}_h^2 \\ &= \sum_{i=11}^{15} \mathbf{s}^2_{z_i} + \mathbf{s}^2_{g_{14}} + \sum_{i=11}^{17} \mathbf{s}^2_{h_i} \\ &= \underbrace{\left\{ \frac{1}{3N} r^2 p_1 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}] \right\}}_{z_{11}} + \\ &+ \underbrace{\left\{ \frac{1}{3N} r^2 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} p_k E[R_i^2] E[P_{ij}] \right\}}_{z_{12}} \end{aligned}$$

$$\begin{aligned}
& + \underbrace{\frac{1}{3N} r^2 p_0 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}]}_{z_{13}} + \\
& + \underbrace{\frac{1}{3N} r^2 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} p_k E[R_i^2] E[P_{ij}]}_{z_{14}} + \tag{3.72} \\
& \left. \underbrace{\frac{1}{N} T^2 \left(\frac{1}{3} \sum_{i=0}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} E[R_i^4] E[P_{ij}] E[P_{iq}] + \frac{1}{4} \sum_{i=0}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=0 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} E[R_i^2] E[R_p^2] E[P_{ij}] E[P_{pq}] \right)}_{z_{15}} \right\} + \\
& \quad + \underbrace{r^4 T^2 \sum_{k=2}^{K-1} p_k p_{k \oplus 1}}_{g_{14}} + \\
& \quad + \left\{ \underbrace{\frac{1}{2} r^2 p_1 N_0 T}_{h_{11}} + \underbrace{\frac{1}{2} r^2 N_0 T \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} p_k}_{h_{12}} + \right. \\
& \quad + \underbrace{\frac{1}{2} N_0 T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}]}_{h_{13}} \\
& \quad + \underbrace{\frac{1}{2} r^2 p_0 N_0 T}_{h_{14}} \\
& \quad + \underbrace{\frac{1}{2} r^2 N_0 T \sum_{k=1}^{K-1} p_k}_{h_{15}} \\
& \quad \left. + \underbrace{\frac{1}{2} N_0 T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}]}_{h_{16}} + \underbrace{\frac{3N_0^2}{4}}_{h_{17}} \right\}.
\end{aligned}$$

Accordingly we modify the conditional SNIR from (3.56) as

$$SNIR|_{r,p_k} = \frac{(\bar{Y})^2}{\mathbf{s}_y^2} = \frac{(2Y_1)^2}{\mathbf{s}_z^2 + \mathbf{s}_g^2 + \mathbf{s}_h^2}$$

$$\begin{aligned}
&= \frac{(2r^2 \sqrt{p_1 p_0} T)^2}{\mathbf{s}_{z_{11}}^2 + \mathbf{s}_{z_{12}}^2 + \mathbf{s}_{z_{13}}^2 + \mathbf{s}_{z_{14}}^2 + \mathbf{s}_{z_{15}}^2 + \mathbf{s}_{g_{11}}^2 + \mathbf{s}_{h_{11}}^2 + \mathbf{s}_{h_{12}}^2 + \mathbf{s}_{h_{13}}^2 + \mathbf{s}_{h_{14}}^2 + \mathbf{s}_{h_{15}}^2 + \mathbf{s}_{h_{16}}^2 + \mathbf{s}_{h_{17}}^2} \\
&= \frac{4r^4 p_1 p_0 T^2}{\mathbf{s}_x^2}, \tag{3.73}
\end{aligned}$$

where \mathbf{s}_x^2 is defined in (3.72).

Summarizing we developed the SNIR for the uncoded DS-CDMA signal operating in a Rayleigh-Lognormal channel. A comparison can be made with the SNIR obtained in [1], where the pilot channel interference was assumed to be zero and thus only $\mathbf{s}_{z_{13}}^2$ and $\mathbf{s}_{h_{14}}^2$ variances were considered. As we see in (3.73), another Y_1 term was gained, however had to take into account considered all the interfering terms since any filtering had not been applied. The performance of the system under normal operating conditions even with the ideal filtering was proved in [1] to be quite poor ($P_e \cong 1/2$), without using any coding.

Accordingly, in the next section, in order to improve the performance of the system we will we will add forward error correction (FEC), keeping up with the analysis done in [1].

D. FORWARD ERROR CORRECTION

As seen in Section C, in order to have a meaningful analysis, Forward Error Correction (FEC) was required. For comparison reasons and compatibility we will add the same FEC with [1] to the system. Therefore an (n,k) encoder is applied, producing n coded bits for every k information bits, which gives us a coded rate $R_{cc}=k/n$, and a reduced bit duration $T_{cc}=T(k/n)$, in order to preserve the bit rate of the system.

On the other hand a decoder is applied at the output of the demodulator of the receiver in order to extract the information signal.

For simplicity purposes we will assume that the information bit transmitted is $b_1(t)=1$, for all the values of t , which is the all zero sequence. Accordingly, we will name the coded bits examined by the decoder y_{jm} , where j is the branch in the trellis of the decoder and $m=1,2,\dots,n$ is the position of the coded bit with in the j -th branch.

To decode the information we will use a way similar to that of [7], using the Viterbi Algorithm with soft decision decoding.

Accordingly, our demodulator output from (3.53) will change to:

$$y_{jm} \Big|_{r_{jm} \cdot p_{k_{jm}}} = \bar{Y}^c_{jm} + \mathbf{x}^c_{jm}, \quad (3.74)$$

where

$$\bar{Y}^c_{jm} = 2r_{jm}^2 \sqrt{p_{0,jm} p_{1,jm}} T_{cc},$$

and

$$\begin{aligned} \mathbf{x}^c_{jm} &= \mathbf{z}^c_{jm} + \mathbf{g}^c_{jm} + \mathbf{h}^c_{jm} \\ &= \sum_{i=11}^{15} \mathbf{z}^c_{i,jm} + \mathbf{g}^c_{14,jm} + \sum_{i=11}^{17} \mathbf{h}^c_{i,jm} \end{aligned}$$

The decoder output y_{jm} , conditioned on $R=r$ and $P_k=p_k$, can be modeled as a Gaussian random variable, exactly as $Y|_{r,p_k}$ in the uncoded system. Similarly, the mean value of y_{jm} adapted from the uncoded case, can be defined as

$$E\{y_{jm}\} = \bar{Y}^c_{jm} = 2r_{jm}^2 \sqrt{p_{0,jm} p_{1,jm}} T_{cc}, \quad (3.75)$$

while its variance can be defined as

$$\begin{aligned} \text{Var}\{y_{jm}\} &= \text{Var}\{\mathbf{x}^c_{jm}\} \\ &= \text{Var}\{\mathbf{g}^c_{jm}\} + \text{Var}\{\mathbf{z}^c_{jm}\} + \text{Var}\{\mathbf{h}^c_{jm}\} \\ &= \sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}^2_{z_{i,jm}} + \mathbf{s}^2_{g_{14,jm}} + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}^2_{h_{i,jm}} \end{aligned}$$

Using the same procedure as in [7], [1], the Viterbi algorithm branch metrics in each path i for branch j are:

$$\mathbf{m}^{(i)}_j = \sum_{m=1}^n y_{jm} (1 - 2c^{(i)}_{jm}), \quad (3.76)$$

where $c_{jm}^{(i)} \in \{0,1\}$ is the logical transformation of the analog information bit $b_{jm}^{(i)} \in \{\pm 1\}$, and $b_{jm}^{(i)} = 1 - 2c_{jm}^{(i)}$.

We sum the metrics over all the branches B and form the path metrics:

$$\begin{aligned} CM^{(i)} &= \sum_{j=1}^B \mathbf{m}^{(i)}_j \\ &= \sum_{j=1}^B \sum_{m=1}^n y_{jm} (1 - 2c_{jm}^{(i)}) \end{aligned} \quad (3.77)$$

If we set $i=0$ the correct path, then since it is $c_{jm}^{(0)} = 0$ for all jm , (3.77) becomes

$$CM^{(0)} = \sum_{j=1}^B \sum_{m=1}^n y_{jm} \quad (3.78)$$

On the other hand for any other competing path $i=1$, $c_{jm}^{(1)} = 1$ for a number of coded bits. At this case (3.77) can be described as

$$CM^{(1)} = \sum_{j=1}^B \sum_{m=1}^n y_{jm} (1 - 2c_{jm}^{(1)}) \quad (3.79)$$

We will denote as d bits in this competing path, the number of the bits that $c_{jm}^{(1)} = 1$. Accordingly, in the next section we will find the probability of error for any path through trellis, which is a distance d from the correct path.

E. PROBABILITY OF BIT ERROR

In order to find the probability of bit error, we will use the procedure described in [7], finding primarily the first event error probability. This is defined as the probability that another path that merges with the all zero path at node B has a metric that exceeds the metric of that all zero path for the first time. If we suppose that the incorrect path that merges with the all zero path is for example $i=1$, and differs from the all zero path in d -bits, then there are d 1's in the path $i=1$ and the rest are 0's. Then, after [1], [7] the probability of error in the pairwise comparison of the metrics $CM^{(0)}$ and $CM^{(1)}$ is

$$P_2(d) \Big|_{r_{jm}, P_{k,jm}} = \Pr\{CM^{(1)} \geq CM^{(0)}\}$$

$$\begin{aligned}
&= \Pr\{\text{CM}^{(1)} - \text{CM}^{(0)} \geq 0\} \\
&= \Pr\left\{-2 \sum_{j=1}^B \sum_{m=1}^n y_{jm} c_{jm}^{(1)} \geq 0\right\} \\
&= \Pr\left\{\sum_{j=1}^B \sum_{m=1}^n y_{jm} c_{jm}^{(1)} \leq 0\right\}. \tag{3.80}
\end{aligned}$$

If we set a new index l that runs over the set of d bits in which the two paths differ, we have $y'_l = y_{jm}$, for $c_{jm}^{(1)} = 1$. Accordingly the first event error probability can be modified as

$$\begin{aligned}
P_2(d) \Big|_{r_{jm}, p_{k,jm}} &= \Pr\left\{\sum_{l=1}^d y'_l \leq 0\right\} \\
&= \Pr\{y_l \leq 0\}. \tag{3.81}
\end{aligned}$$

where the random variable y_l is the sum of the independent Gaussian random variables y'_l . Thereafter y_l is also a Gaussian random variable. Its first moment is defined as

$$\begin{aligned}
E\{y_l\} &= \sum_{l=1}^d E\{y'_l\} \\
&= \sum_{l=1}^d 2r_l^2 \sqrt{p_{0,l} p_{1,l} T_{cc}} \tag{3.82}
\end{aligned}$$

As we defined in (2.9), the received power P_k from the k^{th} channel can be written as

$$P_k = \frac{P_{t,k}}{L_H(d)X} \tag{3.83}$$

Adjusting this equation for the coded case, we can modify the fixed terms $p_{k,l}$ as follows:

$$p_{k,l} = \frac{f_k P_t}{L_H(d)x_l}, \tag{3.84}$$

where $X_l=x_l$ is our lognormal random variable, $X_l \sim \Lambda(0, \mathbf{I} \mathbf{s}_{dB})$. Moreover it has been shown in [1] that the transformation of $\tilde{X}_l = 1/X_l$ results in another lognormal random variable $\tilde{X}_l \sim \Lambda(0, \mathbf{I} \mathbf{s}_{dB})$, with an estimate of

$$E\{\tilde{X}_l\} = E\{1/X_l\} = E\{X_l\} = \exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right), \quad (3.85)$$

where $\mathbf{m}_x = \mathbf{I} \mathbf{m}_{dB} = 0$.

Thus, we can modify the estimate of y_l from (3.82) as follows:

$$\begin{aligned} E\{y_l\} &= \bar{y}_l = \sum_{l=1}^d 2r_l^2 \sqrt{\left(\frac{f_0 P_t}{L_H(d)x_l}\right) \left(\frac{f_1 P_t}{L_H(d)x_l}\right)} T_{cc} \\ &= \frac{2\sqrt{f_0 f_1 P_t T_{cc}}}{L_H(d)} \sum_{l=1}^d r_l^2 \tilde{x}_l \end{aligned} \quad (3.86)$$

Accordingly the second moment of y_l is defined as

$$\begin{aligned} \text{Var}\{y_l\} &= \sum_{l=1}^d \text{Var}\{y_l'\} = \sum_{l=1}^d \text{Var}\{\mathbf{x}_l^c\} = \\ &= \sum_{l=1}^d \left(\text{Var}\{\mathbf{z}_l^c\} + \text{Var}\{\mathbf{g}_l^c\} + \text{Var}\{\mathbf{h}_l^c\} \right) \\ &= \sum_{l=1}^d \left\{ \sum_{i=1}^{15} \mathbf{s}_{z_{i,l}^c}^2 + \mathbf{s}_{g_{i,l}^c}^2 + \sum_{i=1}^{17} \mathbf{s}_{h_{i,l}^c}^2 \right\} \\ &= \sum_{l=1}^d \left\{ \left[\frac{1}{3N} r_l^2 p_{1,l} T_{cc}^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} E\{P_{ij}\} + \right. \right. \\ &\quad \left. \left. + \frac{1}{3N} r_l^2 T_{cc}^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} p_{k,l} E\{R_i^2\} E\{P_{ij}\} \right] \right. \\ &\quad \left. + \frac{1}{3N} r_l^2 p_{0,l} T_{cc}^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} E\{P_{ij}\} \right. \\ &\quad \left. + \frac{1}{3N} r_l^2 T_{cc}^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} p_{k,l} E\{R_i^2\} E\{P_{ij}\} \right\} \end{aligned} \quad (3.87)$$

$$\begin{aligned}
& + \frac{1}{N} T_{cc}^2 \left(\frac{1}{3} \sum_{i=0}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} E\{R_i^4\} E\{P_{ij}\} E\{P_{iq}\} + \frac{1}{4} \sum_{i=0}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=0 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} E\{R_i^2\} E\{R_p^2\} E\{P_{ij}\} E\{P_{pq}\} \right) + \\
& + r_i^4 T_{cc}^2 \sum_{k=2}^{K-1} p_{k,l} p_{k \oplus 1, l} \\
& + \left\{ \frac{1}{2} r_i^2 p_{1,l} N_0 T_{cc} + \frac{1}{2} r_i^2 N_0 T_{cc} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} p_{k,l} + \right. \\
& + \frac{1}{2} N_0 T_{cc} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} E\{P_{ij}\} \\
& + \frac{1}{2} r_i^2 p_{0,l} N_0 T_{cc} \\
& + \frac{1}{2} r_i^2 N_0 T_{cc} \sum_{k=1}^{K-1} p_{k,l} \\
& \left. + \frac{1}{2} N_0 T_{cc} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} E\{P_{ij}\} + \frac{3N_0^2}{4} \right\}.
\end{aligned}$$

Eventually, we can now use the moments of y_l that we found in (3.86) and (3.87) to find the first event error probability from (3.81), as follows,

$$\begin{aligned}
P_2(d) \Big|_{r_i, p_{k,l}} &= \Pr\{y_l \leq 0\} \\
&= Q\left(\frac{\sqrt{\bar{y}_l^2}}{\sqrt{\mathbf{s}_{y_l}^2}}\right) \\
&= Q\left(\frac{\sqrt{\frac{4f_0 f_1 P_t^2 T_{cc}^2 \left(\sum_{l=1}^d r_l^2 \tilde{x}_l\right)^2}{L_H^2(d)}}}{\sqrt{\sum_{l=1}^d \left(\sum_{i=1}^{15} \mathbf{s}_{z_{i,l}^c}^2 + \mathbf{s}_{g_{i4,l}^c}^2 + \sum_{i=11}^{17} \mathbf{s}_{h_{i,l}^c}^2\right)}}}\right) \\
&= Q\left(\frac{\sqrt{4 \left(\sum_{l=1}^d r_l^2 \tilde{x}_l\right)^2}}{\sqrt{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}_{z_{i,l}^c}^2 + \mathbf{s}_{g_{i1,l}^c}^2 + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}_{h_{i,l}^c}^2\right)}}}\right)
\end{aligned}$$

$$\begin{aligned}
&= \mathbb{Q} \left(\sqrt{\frac{4 \left(\sum_{l=1}^d r_l^2 \tilde{x}_l \right)^2}{\underbrace{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\mathbf{s}^2_{z_{i3,l}} + \mathbf{s}^2_{h_{i4,l}} \right)}_{a_1} + \underbrace{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}^2_{z_{i,l}} + \mathbf{s}^2_{g_{i,l}} + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}^2_{h_{i,l}} \right)}_{a_2}}} \right)} \\
&= \mathbb{Q} \left(\sqrt{\frac{4 \left(\sum_{l=1}^d r_l^2 \tilde{x}_l \right)^2}{a_1 + a_2}} \right) \\
&= \mathbb{Q} \left(\sqrt{\frac{4 z_d^2}{a_1 + a_2}} \right) = P_2(d) \Big|_{z_d}, \tag{3.88}
\end{aligned}$$

where we practically grouped the variances into two terms a_1 and a_2 , depending on if the contain pilot tone or not, as follows:

$$a_1 = \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\mathbf{s}^2_{z_{i3,l}} + \mathbf{s}^2_{h_{i4,l}} \right), \tag{3.89}$$

and

$$a_2 = \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}^2_{z_{i,l}} + \mathbf{s}^2_{g_{i,l}} + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}^2_{h_{i,l}} \right) \tag{3.90}$$

We also introduced in (3.88) a new random variable z_d , which is the sum of d multiplicative chi-square (with 2 degrees of freedom)-lognormal random variables given by

$$z_d = \sum_{l=1}^d r_l^2 \tilde{x}_l \tag{3.91}$$

and a second random variable w_d , which is the sum of d squared multiplicative chi-square2-lognormal random variables, defined as

$$w_d = \sum_{l=1}^d (r_l^2 \tilde{x}_l)^2 = \sum_{l=1}^d r_l^4 \tilde{x}_l^2 \tag{3.92}$$

In order to develop the first event error probability in a more practical form, we will need to expand the a_1 and a_2 terms. We will use the estimate of the lognormal random variable X_i , and \tilde{X}_i defined in (3.85), and we will also normalize the expected value of the Rayleigh fading parameters to 1, such that $E\{R_i^2\} = 1$.

We will also introduce a new variable E_c , which represents a baseline received coded bit energy without the effects of fading or shadowing, such that

$$\begin{aligned} E_c &= \left(\frac{k}{n}\right) E_b \\ &= \left(\frac{k}{n}\right) \frac{f_1 P T}{L_H(d)} = \frac{f_1 P T_{cc}}{L_H(d)}, \end{aligned} \quad (3.93)$$

where E_b is the uncoded bit energy.

Consequently, the first event error probability conditioned on r_l , x_l and consequently on z_d , w_d can be modified from (3.88) as follows:

$$P_2(d)|_{z_d, w_d} = Q\left(\sqrt{\frac{4z_d^2}{a_1 + a_2}}\right) \quad (3.94)$$

where

$$a_1 = z_d \left(\frac{\exp\left(\frac{I^2 s_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \right), \quad (3.95)$$

and

$$\begin{aligned}
a_2 = z_d & \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0}\right) \frac{L_H(d)}{L_H(D_i)} + \frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1}\right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0}\right) + \right. \\
& + \frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1}\right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{k=0}^{K-1} \left(\frac{f_k}{f_0}\right) + \frac{1}{2} \left(\frac{E_c}{N_0}\right)^{-1} \left(\frac{f_1}{f_0}\right) + \\
& \left. + \frac{1}{2} \left(\frac{E_c}{N_0}\right)^{-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0}\right) + \frac{1}{2} \left(\frac{E_c}{N_0}\right)^{-1} \sum_{k=1}^{K-1} \left(\frac{f_k}{f_0}\right) \right) + \\
& + w_d \sum_{k=2}^{K-1} \left(\frac{f_k}{f_0}\right) \left(\frac{f_{k \oplus 1}}{f_1}\right) + \\
& + d \left\{ \frac{\left[\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \right]^2}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right)^2 + \right. \\
& + \frac{\left[\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \right]^2}{4N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=1 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right) \left(\frac{L_H(d)}{L_H(D_p)}\right) + \\
& \left. + \exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \left(\frac{E_c}{N_0}\right)^{-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0}\right) \frac{L_H(d)}{L_H(D_i)} + \frac{3}{4} \left(\frac{f_1}{f_0}\right) \left(\frac{E_c}{N_0}\right)^{-2} \right\}. \tag{3.96}
\end{aligned}$$

The details of the conversion of a_1 and a_2 into the above form can be found in Appendix III-B.

For simplicity we can set $a = \frac{4z_d^2}{a_1 + a_2}$. Therefore (3.94) can now be expressed as

$$P_2(d)|_a = Q(\sqrt{a}) \tag{3.97}$$

We can now remove the conditioning in (3.97) by integrating across the pdf $p_a(a)$, as follows:

$$\begin{aligned} P_2(d) &= \int_{-\infty}^{\infty} P_2(d)|_a p_a(a) da \\ &= \int_{-\infty}^{\infty} Q(\sqrt{a}) p_a(a) da \end{aligned} \quad (3.98)$$

Summing the point estimates of $P_2(d)$ over all the possible distances d between code words, we can calculate an upper bound of the bit error probability P_e as follows [7]:

$$P_e \leq \frac{1}{k} \sum_{d=d_{free}}^{\infty} \mathbf{b}_d P_2(d), \quad (3.99)$$

where β_d is the total number of information bit errors, assuming that the correct word is the all-zero code word, and k denotes the number of information bits per level.

As we see in (3.99) for any particular convolution encoder we require a series of $P_2(d)$ for $d=d_{free}, d_{free}+1, d_{free}+2, \dots$ in order to calculate the upper bound on the probability of bit error P_e . For practical reasons we will use the first five terms only, so we will have that $d=d_{free}, d_{free}+1 \dots d_{free}+4$. In our analysis we will also consider convolutional encoder with a code rate $R_{cc}=1/2$ and constraint length $\nu=8$. Therefore we assume $d_{free}=10$, which is a typical value for such encoder. Consequently we can calculate the values of β_d for this particular convolutional code, and find $\beta_{10}=2, \beta_{11}=22, \beta_{12}=60, \beta_{13}=148, \beta_{14}=340$. Eventually we incorporate the simulated results of $P_2(d)$ for $d=10$ through 14 into (3.99), and calculate the bounded bit error probability P_e .

Therefore we developed a tight upper bound on the probability of error P_e for the coded cellular system in the Rayleigh-lognormal channel. In the next section we will use all these to explore the performance analysis of the DS-CDMA cellular system.

F. BIT-ERROR ANALYSIS OF DS-CDMA WITH FEC

In Section E we developed the probability of bit error of the DS-CDMA channel with FEC operating in a Rayleigh fading and lognormal shadowing environment. In this section we are going to analyze its performance over various interference weights.

In order to evaluate the integral in (3.98) we will use the Monte Carlo simulation method. Thus, we generate d independent samples from the chi-square2 lognormal distribution. If we sum them, we form one realization for the z_d , defined in (3.91). On the other hand, if we first square each one of the d samples and then sum them we form one realization of w_d , defined in (3.92). Consequently we replace them in (3.98) and we get one realization r_1 for $P_2(d)$. We repeat this process 10,000 times and form our point estimate \bar{r} for $P_2(d)$ as follows:

$$\bar{r} = \frac{1}{10^4} \sum_{i=1}^{10^4} r_i. \quad (3.100)$$

We then introduce the simulated first event error probability to (3.99) and get the tight upper bound of the probability of bit error. Accordingly, we simulate our model for the case of 2 and 3 users per cell and for $s_{dB}=2$, and 3 dB. Figure 3.3 depicts the resulted probability of bit error, versus the average received SNR per bit given by

$$\begin{aligned} \bar{g}_b &= E \left\{ \frac{R^2 f_1 P T}{N_0 L_H(d) X} \right\} \\ &= E \{ R^2 \} E \left\{ \frac{1}{X} \right\} \frac{f_1 P T}{N_0 L_H(d)} = E \{ X \} \frac{E_b}{N_0}, \end{aligned} \quad (3.101)$$

where we normalized $E \{ R^2 \} = 1$.

As shown in Figure 3.3, the probability of bit error using FEC is quite poor, bellow the minimum standards ($P_e \approx 10^{-3} \sim 10^{-4}$), even for a very small number of users or a light-shadowing environment. The cause of the poor performance can be focused on the great amount of interference at the pilot recovery tone, which deteriorates the demodulation of the signal.

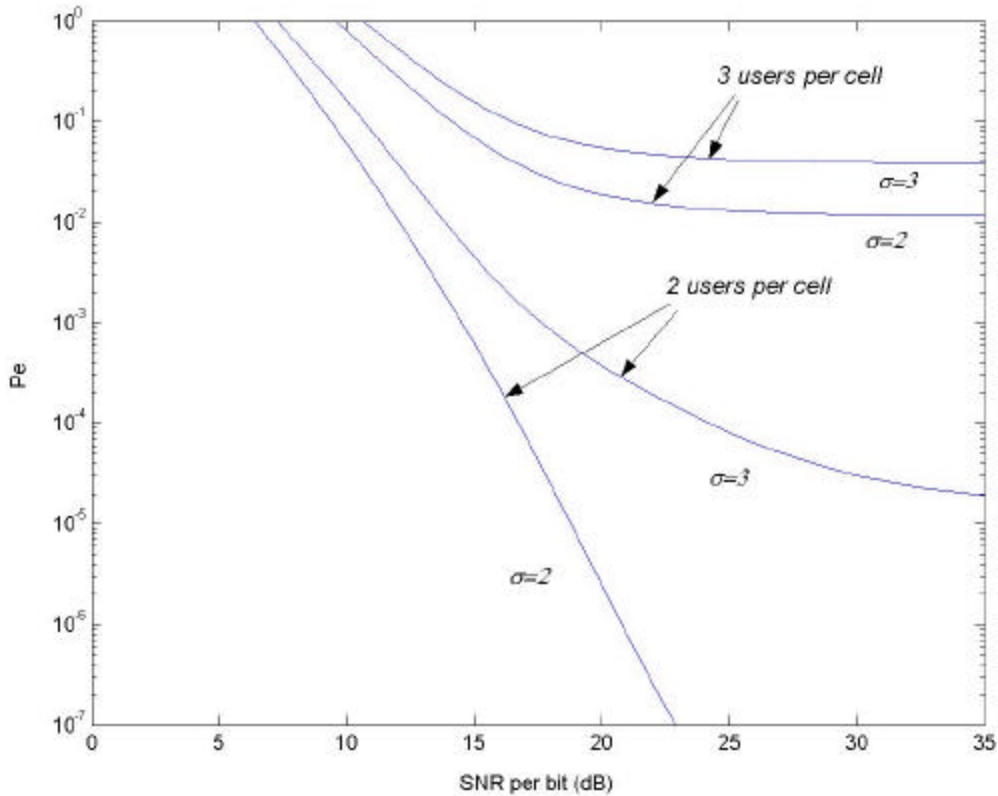


Figure 3.3. Probability of Bit Error for DS-CDMA in Various Channel Conditions with 2 and 3 Users per Cell, using a Rate $\frac{1}{2}$ Convolutional Encoder with $\nu=8$.

G. APPLYING FILTERING AT THE PILOT TONE ACQUISITION BRANCH

In Section F we analyzed the performance of the forward channel in a DS-CDMA cellular system. As we saw in the simulated results the performance of the system although we used FEC turned out to be quite poor. ($P_e \ll 10^{-3}$).

The solution to the poor performance of the system can be focused on the elimination of the interfering terms. As we saw in Section A the interfering terms $\mathbf{g}_0(t)$, $\mathbf{z}_0(t)$, $\mathbf{h}_0(t)$ in the pilot signal $p(t)$ are spread spectrum signals, compared to the narrowband component $I_0(t)$. However, as it was proved their interference at the signal performance is still very large. Therefore, in order to eliminate this we apply a narrow bandpass filter at the pilot recovery branch centered at the carrier frequency f_c , as seen in Figure 3.4.

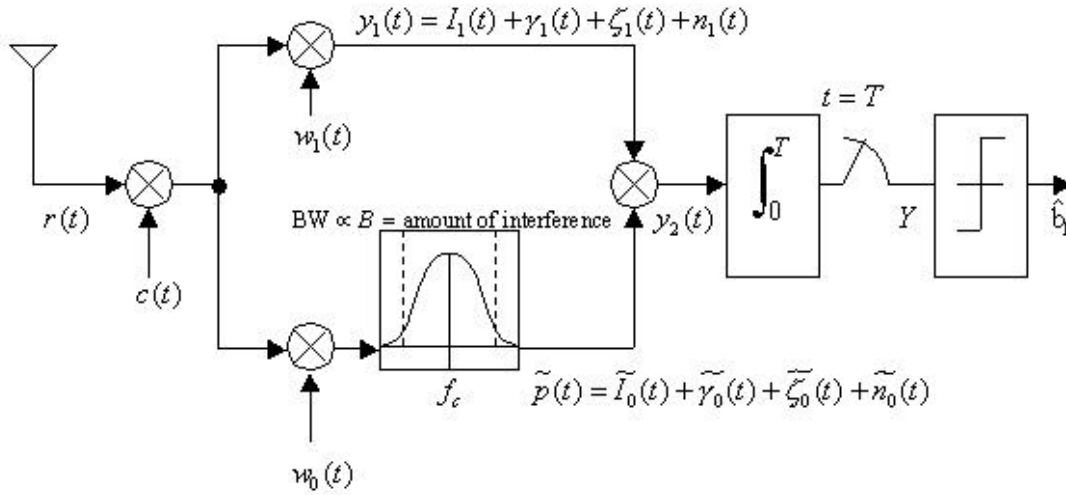


Figure 3.4. Block Diagram of the Mobile User Receiver using a Narrow Bandpass Filter at the Pilot Acquisition Branch.

Accordingly, the performance of the DS-CDMA channel would depend on the characteristics of the filter, such as type or bandwidth. For practical reasons we are not going to specify a particular type of filter. Instead we are going to let the reader decide what are the characteristics of the filter he wants for optimum performance. What we are going to specify is a practical variable \mathbf{B} , which corresponds to the power of the interference passing through the filter, and is directly proportional to the bandwidth and the type of the filter. This variable takes values from 0 to 1, corresponding to all the possible states between two cases. The first case where $\mathbf{B}=\mathbf{0}$, represents the ideal filtering case where **0% of the interference** passes through the filter, while the desired pilot tone signal $I_0(t)$ remains unchanged, and has been thoroughly analyzed in [1]. The second case where $\mathbf{B}=\mathbf{1}$, corresponds to the no-filtering case, where **100% of the interference** passes through the filter, and has already been examined in Sections C to F.

Consequently, we are going to adopt for our analysis the already developed in Section E moments of the signal y_l and eliminate the power of all the interfering terms by a value of B . The only terms that will pass unchanged through the filter are the pilot tone terms. Therefore the terms $\mathbf{z}_1 I_0$, $\mathbf{h}_1 I_0$ and consequently their integrated products \mathbf{z}_{13} , \mathbf{h}_{14} , are the only terms that will not be attenuated by the filter.

Another difference in the analysis can be spotted in the integrated intracell interference product $\mathbf{g}_1\mathbf{g}_0$, described in (3.40) as follows:

$$\begin{aligned} \mathbf{g}_{11} &= \int_0^T \mathbf{g}_1\mathbf{g}_0 dt \Big|_{r,p_k} = \underbrace{r^2 \sqrt{p_0 p_1} T \mathbf{b}_1}_{Y_1} + r^2 T \underbrace{\left(\sum_{k=2}^{K-1} \sqrt{p_k p_{k \oplus 1}} \mathbf{b}_k \mathbf{b}_{k \oplus 1} \right)}_{\mathbf{g}_{14}} \\ &= Y_1 + \mathbf{g}_{14} \end{aligned}$$

The product of the intracell interfering terms resulted to another Y_1 term, which doubled the power of the desired signal. However, using the narrowband filter on the data recovery channel, we reduce the effect of all the non-pilot terms coming from the pilot recovery channel, including the loss of the additional Y_1 term. Consequently this term will no longer aid the demodulation, but on the contrary it will act as interference. Accordingly, in order to get an as more as possible realistic analysis, we will subtract it from the information signal terms and apply it to the filtered interfering terms.

Thus, the estimate of y_l from (3.86) will have the Y_1 term from the product of the pilot terms $I_1 I_0$ only, and will be modified as follows:

$$\begin{aligned} E\{y_l\} &= \bar{y}_l = \sum_{l=1}^d r_l^2 \sqrt{\left(\frac{f_0 P_t}{L_H(d) x_l} \right) \left(\frac{f_1 P_t}{L_H(d) x_l} \right)} T_{cc} \\ &= \frac{\sqrt{f_0 f_1} P_t T_{cc}}{L_H(d)} \sum_{l=1}^d r_l^2 \tilde{x}_l \end{aligned} \tag{3.102}$$

Accordingly the second moment of y_l will be defined as

$$= r_l^4 \tilde{x}_l^2 \frac{f_0 f_1 P_t^2}{L_H^2(d)} T_{cc}^2. \tag{3.104}$$

Eventually, we can now use the moments of y_l that we found in (3.104) to modify the first event error probability from (3.88), as follows,

$$P_2(d) \Big|_{r,p_k,l} = \Pr\{y_l \leq 0\}$$

$$\begin{aligned}
&= Q \left(\frac{\bar{y}_l}{\mathbf{s}_{y_l}} \right) = Q \left(\sqrt{\frac{\bar{y}_l^2}{\mathbf{s}_{y_l}^2}} \right) \\
&= Q \left(\sqrt{\frac{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)} \left(\sum_{l=1}^d r_l^2 \tilde{x}_l \right)^2}{\sum_{l=1}^d \left\{ \mathbf{s}_{z_{13,l}}^2 + \mathbf{s}_{h_{4,l}}^2 + B \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}_{z_{i,l}}^2 + \mathbf{s}_{g_{4,l}}^2 + \mathbf{s}_{g_{15,l}}^2 + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}_{h_{i,l}}^2 \right) \right\}}} \right) \\
&= Q \left(\sqrt{\frac{\left(\sum_{l=1}^d r_l^2 \tilde{x}_l \right)^2}{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left\{ \mathbf{s}_{z_{13,l}}^2 + \mathbf{s}_{h_{4,l}}^2 + B \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}_{z_{i,l}}^2 + \mathbf{s}_{g_{4,l}}^2 + \mathbf{s}_{g_{15,l}}^2 + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}_{h_{i,l}}^2 \right) \right\}}} \right) \\
&= Q \left(\sqrt{\frac{\left(\sum_{l=1}^d r_l^2 \tilde{x}_l \right)^2}{\underbrace{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d (\mathbf{s}_{z_{13,l}}^2 + \mathbf{s}_{h_{4,l}}^2)}_{a_1} + B \times \left[\underbrace{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}_{z_{i,l}}^2 + \mathbf{s}_{g_{4,l}}^2 + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}_{h_{i,l}}^2 \right)}_{a_2} \right] + \underbrace{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}_{g_{15,l}}^2}_{a_3}}} \right) \\
&= Q \left(\sqrt{\frac{\left(\sum_{l=1}^d r_l^2 \tilde{x}_l \right)^2}{a_1 + B(a_2 + a_3)}} \right) \\
&= Q \left(\sqrt{\frac{z_d^2}{a_1 + B(a_2 + a_3)}} \right) = P_2(d) \Big|_{z_d}, \tag{3.105}
\end{aligned}$$

where

$$a_1 = \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d (\mathbf{s}^2_{z_{i3l}} + \mathbf{s}^2_{h_{i4l}}) \quad (3.106)$$

$$= z_d \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \right)$$

$$a_2 = \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}^2_{z_{i1l}} + \mathbf{s}^2_{g_{i1l}} + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}^2_{h_{i1l}} \right)$$

$$= z_d \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)} + \frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0} \right) + \right.$$

$$+ \frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{k=0}^{K-1} \left(\frac{f_k}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \left(\frac{f_1}{f_0} \right) +$$

$$\left. + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{k=1}^{K-1} \left(\frac{f_k}{f_0} \right) \right)$$

$$+ w_d \sum_{k=2}^{K-1} \left(\frac{f_k}{f_0} \right) \left(\frac{f_{k \oplus 1}}{f_1} \right) +$$

$$+ d \left\{ \frac{\left[\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \right]^2}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} \left(\frac{f_{ij}}{f_0} \right) \left(\frac{f_{iq}}{f_1} \right) \left(\frac{L_H(d)}{L_H(D_i)} \right)^2 + \right.$$

$$\left. + \frac{\left[\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \right]^2}{4N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=1 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} \left(\frac{f_{ij}}{f_0} \right) \left(\frac{f_{iq}}{f_1} \right) \left(\frac{L_H(d)}{L_H(D_i)} \right) \left(\frac{L_H(d)}{L_H(D_p)} \right) + \right.$$

$$+ \exp \left\{ \left(\frac{I^2 s_{dB}^2}{2} \right) \left(\frac{E_c}{N_0} \right)^{-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)} + \frac{3}{4} \left(\frac{f_1}{f_0} \right) \left(\frac{E_c}{N_0} \right)^{-2} \right\},$$

and

$$\begin{aligned} a_3 &= \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}^2_{g_{i5,l}} = \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d r_l^4 \tilde{x}_l^2 \frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)} \\ &= \sum_{l=1}^d r_l^4 \tilde{x}_l^2 = w_d. \end{aligned} \quad (3.108)$$

The details of the conversion of a_1 and a_2 into the above forms can be found in Appendix III-B.

The unconditioned first event error probability has been defined in (3.98). We simulate the first event probability of error the same way we did in Sections E and F, using Monte Carlo simulation method. Eventually we introduce our results to (3.99) and get an upper bound in the probability of bit error.

Before we proceed to the analysis of the BER of the filtered case we will test our simulation model. Thus, we allow only the desired pilot tone to pass through the narrow bandpass filter, while all the interfering terms are being eliminated (0% Interference- $B=0$). For comparison reasons, we simulated our model for the case of 20 users per cell and for $s_{dB}=2$ to 9. The resulted probability of error is depicted in Figure 3.5, where we observe that our simulation products track identically the results of [1], verifying both our calculations and our simulation model.

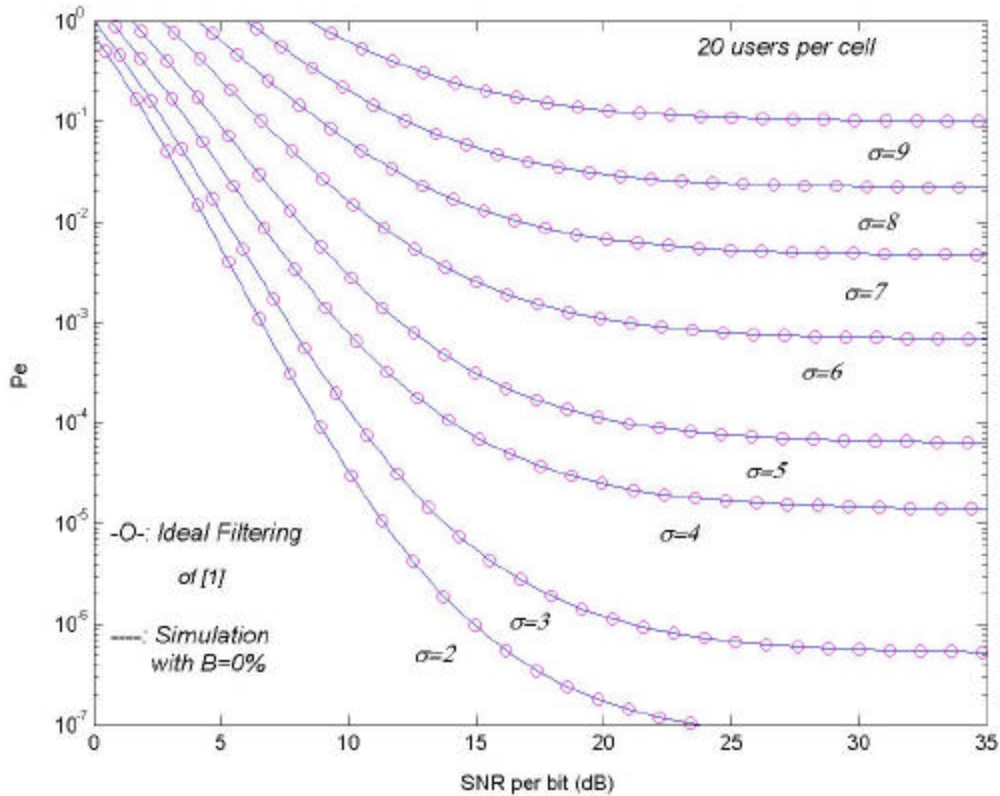


Figure 3.5. Comparison of Probability of Bit Error for DS-CDMA in Various Channel Conditions, using a Rate $\frac{1}{2}$ Convolutional Encoder with $v=8$.

In order to further improve the performance, we can limit the amount of interference by sectoring the cells into 3 or 6 sectors of 120° or 60° sectors respectively. This simply reduces the number of sectored users in the cell i to K_i/S , where S is the number of sectors. As we see in Figure 3.6, the performance of our DS-CDMA cellular system is greatly improved with sectoring. Accordingly we will use 60° sectoring to optimize the performance.

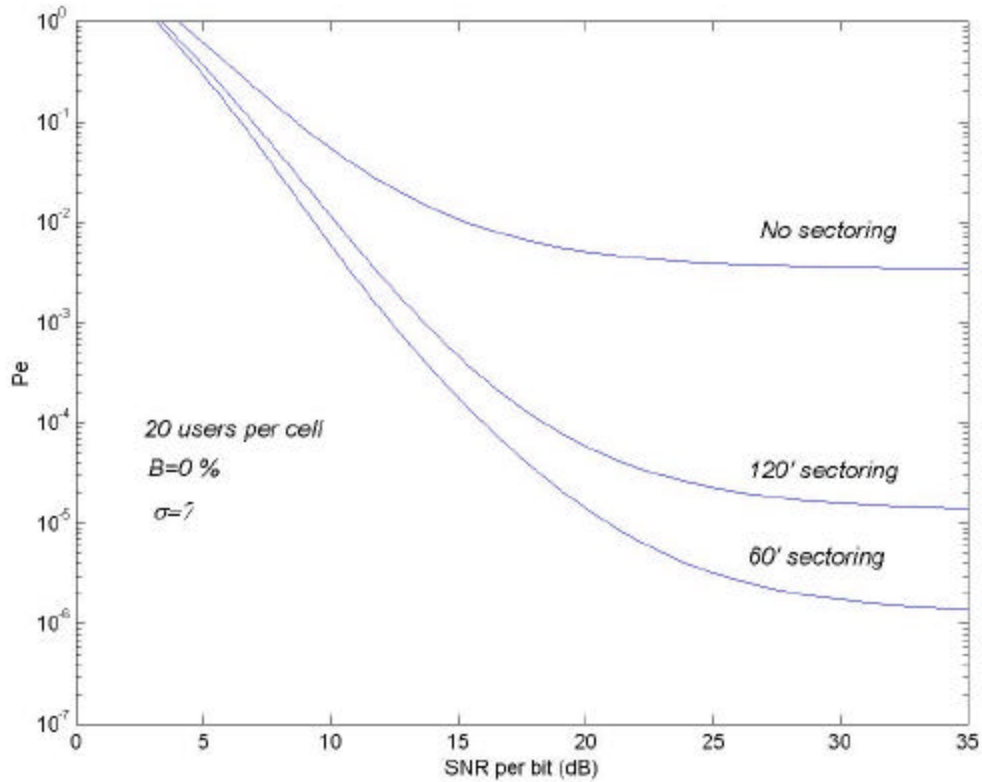


Figure 3.6. Comparison of Bit Error for DS-CDMA using Sectoring with 20 Users per Cell.

Eventually, since we tested our simulation model, we can vary the interference at the pilot recovery channel and check the effects on the bit error probability. We performed the simulation of the bit error probability for 20 users per cell in an environment with lognormal $s_{db} = 5$ dB. The resulted probability of bit error is depicted in Figure 3.7. As we see, when we increase the amount of interference that passes through the pilot filter, the probability of error drifts away from that of the ideal filtering case, while the performance of the system is reduced according to the amount of interference that gets through, or generalizing to the bandwidth of the filter.

Similar graphical results for various channel conditions are also provided in Appendix III-C.

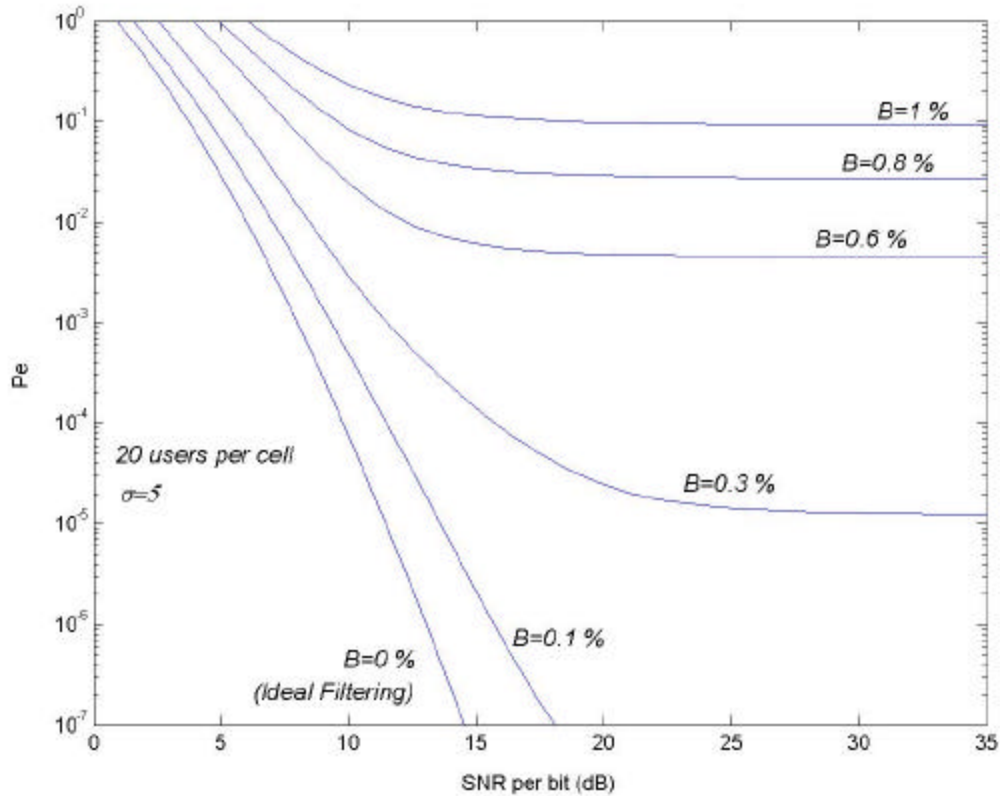


Figure 3.7. Probability of Bit Error for coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 5$) with 20 Users per Cell, using 60° Sectoring.

Summarizing, we can observe in the simulated results that an increase either in the number of users per cell and the amount of the lognormal shadowing or in the pilot channel interference (bandwidth of filter), deteriorates furthermore the performance of the system. Thus, in the next section we will try to further improve the performance by adding power control at the pilot tone channel.

APPENDIX III-A. DEVELOPING THE VARIANCES OF THE INTERFERENCE TERMS

1. Variance of Intercell Interference

$$\begin{aligned}
 \text{a. } \mathbf{z}_{11} &= r\sqrt{p_1} \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{P_{ij}} b_{ij}(t) b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) d_1(t) \cos(\mathbf{j}_i) dt \\
 &= r\sqrt{p_1} \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) d_1(t) \cos \mathbf{j}_i dt
 \end{aligned} \tag{3.109}$$

where $d_1(t) = b_1(t)c(t)$ is a spread spectrum PN sequence.

$$\text{Let } I_{ij} = \int_0^T R_i \sqrt{\frac{P_{ij}}{2}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) d_1(t) \cos \mathbf{j}_i dt, \tag{3.110}$$

be the contribution to the interference terms from the individual channel j , in the adjacent cell i .

Accordingly, (3.109) can be modified using (3.110) as follows:

$$\mathbf{z}_{11} = r\sqrt{2p_1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} I_{ij}$$

We can simplify I_{ij} as follows:

$$I_{ij} = R_i \sqrt{\frac{P_{ij}}{2}} \cos \mathbf{j}_i \int_0^T a_{ij}(t+\mathbf{t}_i) d_1(t) dt, \tag{3.111}$$

where $a_{ij}(t) = b_{ij}(t)w_{ij}(t)c_i(t)$ is also a PN sequence.

The variance of \mathbf{z}_{11} can be now defined as

$$\mathbf{s}_{\mathbf{z}_{11}}^2 = 2r^2 p_1 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \mathbf{s}_{I_{ij}}^2, \tag{3.112}$$

assuming that the contributions of the I_{ij} terms are independent each other.

Now, we'll find the moments of I_{ij} :

$$I_{ij} = R_i \sqrt{\frac{P_{ij}}{2}} \cos \mathbf{j}_i \int_0^T a_{ij}(t+\mathbf{t}_i) d_1(t) dt$$

Let

$$x = \int_0^T a_{ij}(t+\mathbf{t}_i) d_1(t) dt$$

Then,

$$\begin{aligned} E[x] &= E \left[\int_0^T a_{ij}(t + \mathbf{t}_i) d_1(t) dt \right] \\ &= \int_0^T E \left[a_{ij}(t + \mathbf{t}_i) d_1(t) \right] dt = 0 \end{aligned}$$

Accordingly,

$$\begin{aligned} E\{I_{ij}\} &= E \left\{ R_i \sqrt{\frac{P_{ij}}{2}} (\cos \mathbf{j}_i) x \right\} \\ &= E \left\{ R_i \sqrt{\frac{P_{ij}}{2}} (\cos \mathbf{j}_i) \right\} E\{x\} = 0, \end{aligned}$$

since $E\{x\} = 0$.

The variance of I_{ij} can be calculated as follows:

$$\begin{aligned} \mathbf{s}_{I_{ij}}^2 &= E\{I_{ij}^2\} = \frac{E[R_i^2] E[P_{ij}]}{2} E[\cos^2 \mathbf{j}_i] \mathbf{s}_x^2 \\ &= \frac{E[R_i^2] E[P_{ij}] \mathbf{s}_x^2}{4}, \end{aligned} \tag{3.113}$$

where we determined $E[\cos^2 \mathbf{j}_i] = 1/2$, assuming that \mathbf{j}_i is uniformly distributed between $(0, 2\pi)$.

Therefore we will find the variance of x , as follows:

$$\begin{aligned} \mathbf{s}_x^2 &= E[x^2] = E \left[\int_0^T \int_0^T a_{ij}(t + \mathbf{t}_i) d_1(t) a_{ij}(\mathbf{l} + \mathbf{t}_i) d_1(\mathbf{l}) dt d\mathbf{l} \right] \\ &= \int_0^T \int_0^T E \left[a_{ij}(t + \mathbf{t}_i) a_{ij}(\mathbf{l} + \mathbf{t}_i) \right] E[d_1(t) d_1(\mathbf{l})] dt d\mathbf{l} \end{aligned}$$

We observe that $d_1(t)$ and $a_{ij}(t)$ which we defined in (3.109) and (3.111) are PN signals independent from each other, with the same chip period T_c , and with autocorrelation

$$\mathbf{b}(\mathbf{t}) = \begin{cases} 1 - \frac{|\mathbf{t}|N}{T} & , \text{ for } |\mathbf{t}| \leq T/N \\ 0 & , \text{ elsewhere} \end{cases} \tag{3.114}$$

Accordingly, we can write the variance of x as

$$\mathbf{s}_x^2 = \int_0^T \int_0^T \mathbf{b}^2(t-I) dt dI$$

We change the variables as follows:

$$u = t - I, \text{ and } v = t + I,$$

and we solve for t, I :

$$u + v = 2t \Rightarrow t = 1/2(u + v)$$

$$u - v = -2I \Rightarrow I = 1/2(v - u)$$

We calculate the Jacobian determinant of the transformation as follows:

$$J_{tI} = \det \begin{pmatrix} \frac{\partial t}{\partial u} & \frac{\partial I}{\partial u} \\ \frac{\partial t}{\partial v} & \frac{\partial I}{\partial v} \end{pmatrix} = \det \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{2}$$

The new limits of integration are determined to be

$$-T < u < T \text{ and } |u| < v < 2T - |u|,$$

as shown in Figure 3.8.

Applying the changes of variables we find that

$$\begin{aligned} \mathbf{s}_x^2 &= \int_{-T}^T \int_{|u|}^{2T-|u|} \mathbf{b}^2(u) J_{tI} dv du \\ &= \int_{-T}^T \int_{|u|}^{2T-|u|} \mathbf{b}^2(u) \frac{1}{2} dv du. \end{aligned}$$

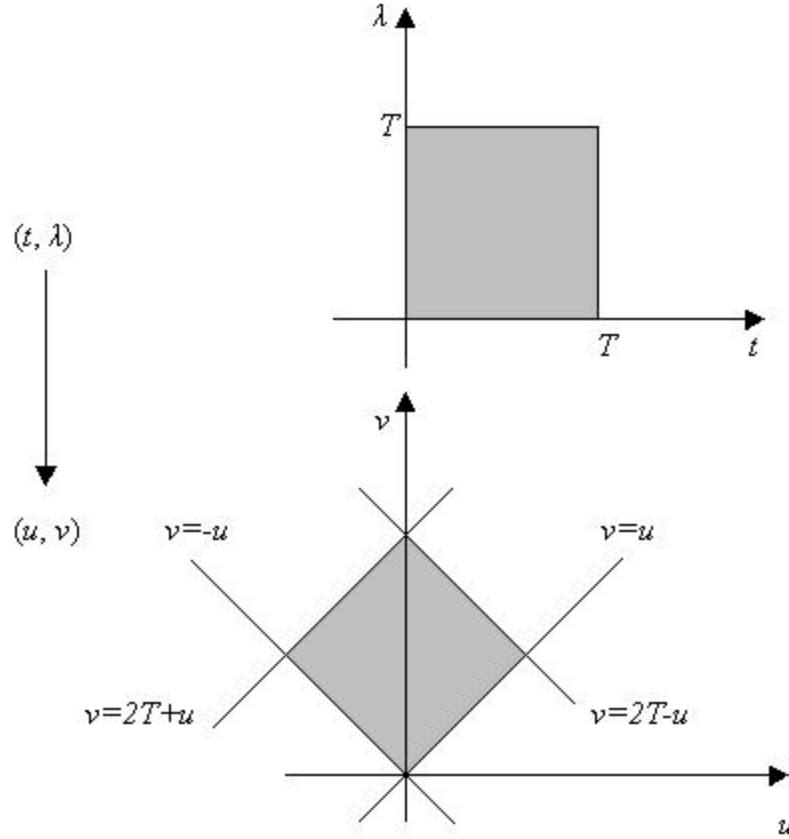


Figure 3.8. Transformation of the Limits of Integration $(t, \lambda) \rightarrow (u, v)$.

We assume that the region of integration is symmetric about the v axis, as shown in Figure 3.8, so

$$\begin{aligned}
 \mathbf{s}_x^2 &= 2 \int_0^T \int_u^{2T-u} \mathbf{b}^2(u) \frac{1}{2} dv du \\
 &= \int_0^T \mathbf{b}^2(u) (2T - 2u) du \\
 &= 2 \int_0^T \mathbf{b}^2(u) (T - u) du
 \end{aligned} \tag{3.115}$$

Accordingly, applying the autocorrelation function $\mathbf{b}(u)$ which we defined (3.114), we can modify (3.115) as follows:

$$\mathbf{s}_x^2 = 2 \int_0^{T/N} \left(1 - \frac{Nu}{T} \right)^2 (T - u) du$$

$$\begin{aligned}
&= 2 \int_0^{T/N} \left(1 - \frac{2Nu}{T} + \frac{N^2 u^2}{T^2} \right) (T-u) du \\
&= 2 \int_0^{T/N} \left(T - 2Nu + \frac{N^2 u^2}{T} - u + \frac{2Nu^2}{T} - \frac{N^2 u^3}{T^2} \right) du \\
&= 2 \left[Tu - \frac{2Nu^2}{2} + \frac{N^2 u^3}{3T} - \frac{u^2}{2} + \frac{2Nu^3}{3T} - \frac{N^2 u^4}{4T^2} \right]_0^{T/N} \\
&= \frac{2T^2}{N} - \frac{2T^2}{N} + \frac{2T^2}{3N} - \frac{T^2}{N^2} + \frac{4T^2}{3N^2} - \frac{T^2}{2N^2} \\
&= \frac{2T^2}{3N} - \frac{3T^2}{2N^2} + \frac{4T^2}{3N^2} \\
&= \frac{2T^2}{3N} - \frac{T^2}{6N^2} \approx \frac{2T^2}{3N} \quad , \tag{3.116}
\end{aligned}$$

since $N = 128 \gg 1$.

Eventually we can use (3.116) to modify (3.113) as follows:

$$\begin{aligned}
\mathbf{s}_{I_{ij}}^2 &= \frac{E[R_i^2] E[P_{ij}] \mathbf{s}_x^2}{4} \\
&= \frac{E[R_i^2] E[P_{ij}]}{4} \times \frac{2T^2}{3N} \\
&= \frac{E[R_i^2] E[P_{ij}] T^2}{6N} \tag{3.117}
\end{aligned}$$

The variance of I_{ij} given in (3.117) will be used, as we'll see in the calculation of variance of other interference terms too.

We can now use (3.117) to derive the variance of $?_{11}$ from (3.112), as follows:

$$\begin{aligned}
\mathbf{s}_{z_{11}}^2 &= 2r^2 p_1 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \mathbf{s}_{I_{ij}}^2 \\
&= 2r^2 p_1 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \frac{T^2 E[R_i^2] E[P_{ij}]}{6N}
\end{aligned}$$

$$= \frac{1}{3N} r^2 p_1 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}] \quad (3.118)$$

$$\begin{aligned} \text{b. } \mathbf{z}_{12} &= r \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K_i-1} R_i \sqrt{p_k P_{ij}} b_{ij}(t+\mathbf{t}_i) b_k(t) w_{ij}(t+\mathbf{t}_i) w_{k\oplus 1}(t) c_i(t+\mathbf{t}_i) c(t) \cos \mathbf{j}_i dt \\ &= r \int_0^T \sum_{\substack{k=0 \\ k \neq 1}}^{K_i-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{p_k P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) e_{k\oplus 1}(t) \cos \mathbf{j}_i dt, \end{aligned}$$

where $w_{k\oplus 1}(t) = w_k(t) w_1(t) w_0(t)$ is another Walsh sequence, and $e_{k\oplus 1}(t) = w_{k\oplus 1}(t) c(t)$ is a PN signal.

We consider again the contribution of each interference term individually, so we let

$$I_{ij} = \int_0^T R_i \sqrt{\frac{P_{ij}}{2}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) e_{k\oplus 1}(t) \cos \mathbf{j}_i dt$$

Accordingly \mathbf{z}_{12} can be written as

$$\mathbf{z}_{12} = r \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K_i-1} \sqrt{2p_k} I_{ijk}$$

The variance of I_{ijk} has been calculated in (3.117) and is

$$E[I_{ijk}^2] = \frac{T^2 E[R_i^2] E[P_{ij}]}{6N} \quad (3.119)$$

The variance of \mathbf{z}_{12} can be written as

$$\mathbf{s}_{\mathbf{z}_{12}}^2 = r^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} 2p_k \mathbf{s}_{I_{ij}}^2 \quad (3.120)$$

Applying (3.119) to (3.120) we can find the variance of \mathbf{z}_{12} as follows:

$$\mathbf{s}_{\mathbf{z}_{12}}^2 = r^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K_i-1} 2p_k \frac{T^2 E[R_i^2] E[P_{ij}]}{6N}$$

$$= \frac{1}{3N} r^2 T^2 \sum_{i=1}^6 \sum_{\substack{j=0 \\ k \neq 1}}^{K_i-1} \sum_{k=0}^{K-1} p_k E[R_i^2] E[P_{ij}] \quad (3.121)$$

$$\begin{aligned} \text{c. } \mathbf{z}_{13} &= r \sqrt{p_0} \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) w_1(t) c_i(t+\mathbf{t}_i) c(t) \cos(\mathbf{j}_i) dt \\ &= r \sqrt{p_0} \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) e_1(t) \cos \mathbf{j}_i dt, \end{aligned}$$

where $e_1(t) = w_1(t) c(t)$ is another PN signal.

We consider again the contribution of each interference term individually, so we let

$$I_{ij} = \int_0^T R_i \sqrt{\frac{P_{ij}}{2}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) e_1(t) \cos \mathbf{j}_i dt$$

So \mathbf{z}_{13} can be modified as

$$\mathbf{z}_{13} = r \sqrt{2p_0} \sum_{i=1}^6 \sum_{\substack{j=0 \\ k \neq 1}}^{K_i-1} \sum_{k=0}^{K-1} I_{ijk}$$

The variance of \mathbf{z}_{13} is now given by

$$\mathbf{s}_{\mathbf{z}_{13}}^2 = 2r^2 p_0 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \mathbf{s}_{I_{ij}}^2,$$

assuming that the I_{ij} terms are independent one with each other.

Using the variance of I_{ij} that has been calculated in (3.117), we can derive $\mathbf{s}_{\mathbf{z}_{13}}^2$ as follows:

$$\begin{aligned} \mathbf{s}_{\mathbf{z}_{13}}^2 &= 2r^2 p_0 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \frac{E[R_i^2] E[P_{ij}] T^2}{6N} \\ &= \frac{1}{3N} r^2 T^2 p_0 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}] \quad (3.122) \end{aligned}$$

$$\begin{aligned} \text{d. } \mathbf{z}_{14} &= r \int_0^T \sum_{i=1}^6 \sum_{\substack{j=0 \\ k \neq 1}}^{K_i-1} \sum_{k=0}^{K-1} R_i \sqrt{p_k P_{ij}} b_{ij}(t+\mathbf{t}_i) b_k(t) w_{ij}(t+\mathbf{t}_i) w_{k \oplus 1}(t) * c_i(t+\mathbf{t}_i) c(t) \cos \mathbf{j}_i dt \\ &= r \int_0^T \sum_{k=1}^{K-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{p_k P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) e_{k \oplus 1}(t) \cos \mathbf{j}_i dt, \end{aligned}$$

where $w_{k\oplus 1}(t) = w_k(t) w_1(t) w_0(t)$, is another Walsh sequence, and $e_{k\oplus 1}(t) = w_{k\oplus 1}(t)c(t)$ a PN signal. We also set

$$I_{ij} = \int_0^T R_i \sqrt{\frac{P_{ij}}{2}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) e_{k\oplus 1}(t) \cos(\mathbf{j}_i) dt$$

So \mathbf{z}_{14} can be written as

$$\mathbf{z}_{14} = r \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sqrt{2p_k} I_{ijk}$$

The variance of I_{ijk} has been calculated in (3.117) and is

$$E[I_{ijk}^2] = \frac{T^2 E[R_i^2] E[P_{ij}]}{6N}$$

Accordingly we derive the variance of \mathbf{z}_{14} as follows:

$$\begin{aligned} \mathbf{s}_{\mathbf{z}_{14}}^2 &= r^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} 2p_k \frac{T^2 E[R_i^2] E[P_{ij}]}{6N} \\ &= \frac{1}{3N} r^2 T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{k=1}^{K-1} p_k E[R_i^2] E[P_{ij}] \end{aligned} \quad (3.123)$$

$$\begin{aligned} \text{e. } \mathbf{z}_{15} &= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} R_i R_p \sqrt{P_{ij} P_{pq}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) \times \\ & b_{pq}(t + \mathbf{t}_p) w_{pq}(t + \mathbf{t}_p) c_p(t + \mathbf{t}_p) \cos(\mathbf{j}_i - \mathbf{j}_p) \\ &= \mathbf{z}_{150} + \mathbf{z}_{151}, \end{aligned} \quad (3.124)$$

where we considered two cases, \mathbf{z}_{150} for $p = i$, and \mathbf{z}_{151} for $p \neq i$.

$$\begin{aligned} \mathbf{z}_{150} &= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} R_i^2 \sqrt{P_{ij} P_{iq}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) b_{iq}(t + \mathbf{t}_i) w_{iq}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) w_1(t) \cos(\mathbf{j}_i - \mathbf{j}_i) \\ &= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} R_i^2 \sqrt{P_{ij} P_{iq}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) b_{iq}(t + \mathbf{t}_i) w_{iq}(t + \mathbf{t}_i) w_1(t) dt, \end{aligned}$$

since $c_i^2(t + \mathbf{t}_i) = 1$.

We let again

$$I_{ijq} = \int_0^T R_i^2 \sqrt{P_{ij} P_{iq}} \underbrace{b_{ij}(t + \mathbf{t}_i) b_{iq}(t + \mathbf{t}_i)}_{b_k} \underbrace{w_{ij}(t + \mathbf{t}_i) w_{iq}(t + \mathbf{t}_i)}_{a_k} w_1(t) \cos(0) dt$$

$$= \int_0^T R_i^2 \sqrt{P_{ij} P_{iq}} b_k a_k w_i(t) \cos(\mathbf{f}_i) dt,$$

where $b_k = b_{ij}(t + \mathbf{t}_i) b_{iq}(t + \mathbf{t}_i)$, $a_k = w_{ij}(t + \mathbf{t}_i) w_{iq}(t + \mathbf{t}_i)$ and $f_i = 0$.

We consider I_{ijq} as the standard form used in (3.117) with the following transformation:

$$R_i \sqrt{\frac{P_k}{2}} \rightarrow R_i^2 \sqrt{P_{ij} P_{iq}}$$

Accordingly the variance of I_{ijq} can be modified as follows:

$$E[I_{jkq}^2] = \frac{T^2 2E[R_i^4] E[P_{ij} P_{iq}]}{6N}$$

where we did the following transformation

$$R_i^2 \frac{P_k}{2} \rightarrow R_i^4 P_{ij} P_{iq}$$

So we can express \mathbf{z}_{150} in terms of I_{ijq} as follows:

$$\mathbf{z}_{150} = \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} I_{ijq},$$

while the variance of \mathbf{z}_{150} is

$$\begin{aligned} \mathbf{s}_{\mathbf{z}_{150}}^2 &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} \mathbf{s}_{I_{ijq}}^2 \\ &= \frac{1}{3N} T^2 \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} E[R_i^4] E[P_{ij}] E[P_{iq}] \end{aligned}$$

$$\mathbf{z}_{151} = \int_0^T w_1(t) \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=1 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} R_i R_p \sqrt{P_{ij} P_{iq}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) b_{pq}(t + \mathbf{t}_p) w_{pq}(t + \mathbf{t}_p) c_p(t + \mathbf{t}_p) \cos(\mathbf{j}_i - \mathbf{j}_p)$$

For simplicity we set $R_i R_p = R_p$, $w_i w_p = w_{ip}$, $\cos \mathbf{j}_{ip} = \cos(\mathbf{j}_i - \mathbf{j}_p)$

Also we set $e_{ij}(t) = b_{ij}(t) w_{ij}(t) c_i(t)$, and $e_{pq}(t) = b_{pq}(t) w_{pq}(t) c_p(t)$, which are PN sequences.

Then we modify \mathbf{z}_{151} as follows:

$$\mathbf{z}_{151} = \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=1 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} R_{ip} \sqrt{P_{ij} P_{iq}} \mathbf{j}_{ip} \int_0^T w_1(t) e_{ij}(t+\mathbf{t}_i) e_{pj}(t+\mathbf{t}_p) dt$$

We set again

$$I_{ijpq} = R_{ip} \sqrt{P_{ij} P_{iq}} \cos \mathbf{j}_{ip} \int_0^T w_1(t) e_{ij}(t+\mathbf{t}_i) e_{pq}(t+\mathbf{t}_i) dt$$

Accordingly \mathbf{z}_{151} can be expressed in terms of I_{ijpq} as

$$\mathbf{z}_{151} = \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} I_{ijpq}$$

and since the contributing terms I_{ijpq} are independent we can write that

$$\begin{aligned} \mathbf{s}_{z_{151}}^2 &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} \mathbf{s}_{I_{ijpq}}^2 \\ &= E \left[I_{ijpq}^2 \right] \\ &= E \left[R_i^2 R_p^2 P_{ij} P_{iq} \cos^2 \mathbf{j}_{ip} \int_0^T \int_0^T w_1(t) w_1(\mathbf{I}) e_{ij}(t+\mathbf{t}_i) e_{ij}(\mathbf{I}+\mathbf{t}_i) e_{pq}(t+\mathbf{t}_p) e_{pq}(\mathbf{I}+\mathbf{t}_p) dt d\mathbf{I} \right] \\ &= \underbrace{E[R_i^2] E[R_p^2] E[P_{ij}] E[P_{pq}]}_z \frac{1}{2} \int_0^T \int_0^T E[w_1(t) w_1(\mathbf{I})] E[e_{ij}(t+\mathbf{t}_i) e_{ij}(\mathbf{I}+\mathbf{t}_i)] E[e_{pq}(t+\mathbf{t}_p) e_{pq}(\mathbf{I}+\mathbf{t}_p)] dt d\mathbf{I} \\ &= z \int_0^T \int_0^T E[w_1(t) w_1(\mathbf{I})] \mathbf{b}^2(t-\mathbf{I}) dt d\mathbf{I}, \end{aligned}$$

$$\text{where for simplicity we set } z = \frac{1}{2} E[R_i^2] E[R_p^2] E[P_{ij}] E[P_{pq}]. \quad (3.125)$$

We know that the autocorrelation function $\mathbf{a}_i(t-\mathbf{I}) = E[w_i(t) w_i(\mathbf{I})]$ of any particular Walsh function is dependent on which user channel we are considering.

Accordingly in order to make our model non-channel specific we will use the average of all the autocorrelation functions, which can be calculated as [1]:

$$\frac{1}{N} \sum_{i=0}^{N-1} \mathbf{a}_i(u) = \begin{cases} 1 - \frac{|u|N}{T}, & \text{for } |u| \leq \frac{T}{N} \\ 0, & \text{elsewhere} \end{cases} \quad (3.126)$$

As we see the average $\mathbf{a}_i(u)$ is the same with $\mathbf{b}(t)$ defined in (3.114).

Accordingly we modify (3.125) as follows:

$$\mathbf{s}_{I_{ipq}}^2 = z \int_0^T \int_0^T \mathbf{b}^3(t-l) dt dl$$

We change the variables as follows:

$$\begin{aligned} u &= t - l \\ v &= t + l \end{aligned}$$

So $\mathbf{s}_{I_{ipq}}^2$ can be written as

$$\begin{aligned} \mathbf{s}_{I_{ipq}}^2 &= z \int_0^T \int_0^{2T-u} \mathbf{b}^3(u) dv du \\ &= z \int_0^T \mathbf{b}^3(u) (2T - 2u) du \\ &= 2z \int_0^T \mathbf{b}^3(u) (T - u) du \\ &= 2z \int_0^{T/N} \left(1 - \frac{Nu}{T}\right)^3 (T - u) du \\ &= 2z \int_0^{T/N} \left(1 - \frac{Nu}{T}\right) \left(1 - \frac{2Nu}{T} + \frac{N^2u^2}{T^2}\right) (T - u) du \\ &= 2z \int_0^{T/N} \left(1 - \frac{2Nu}{T} + \frac{N^2u^2}{T^2} - \frac{Nu}{T} + \frac{2N^2u^2}{T^2} - \frac{N^3u^3}{T^3}\right) (T - u) du \\ &= 2z \int_0^{T/N} \left(1 - \frac{3Nu}{T} + \frac{3N^2u^2}{T^2} - \frac{N^3u^3}{T^3}\right) (T - u) du \\ &= 2z \int_0^{T/N} \left(T - 3Nu + \frac{3N^2u^2}{T} - \frac{N^3u^3}{T^2} - u + \frac{3Nu^2}{T} - \frac{3N^2u^3}{T^2} + \frac{N^3u^4}{T^3}\right) du \\ &= 2z \int_0^{T/N} \left[\frac{N^3u^4}{T^3} - u^3 \left(\frac{N^3 + 3N^2}{T^2}\right) + u^2 + \left(\frac{3N^2 + 3N}{T}\right) - u(3N + 1) + T\right] du \\ &= 2z \left[\frac{N^3u^5}{5T^3} - \frac{(N^3 + 3N^2)}{4T^2} u^4 + \left(\frac{3N^2 + 3N}{3T}\right) u^3 - \frac{(3N + 1)}{2} u^2 - Tu \right]_0^{T/N} \end{aligned}$$

$$\mathbf{g}_{14} = r^2 T \left(\sum_{k=2}^{K-1} \sqrt{p_k p_{k\oplus 1}} b_k b_{k\oplus 1} \right)$$

We'll find the moments of \mathbf{g}_{14} :

$$\begin{aligned} E[\mathbf{g}_{14}] &= E \left\{ r^2 T \left(\sum_{k=2}^{K-1} \sqrt{p_k p_{k\oplus 1}} b_k b_{k\oplus 1} \right) \right\} \\ &= r^2 T \left(\sum_{k=2}^{K-1} E \left\{ \sqrt{p_k p_{k\oplus 1}} \right\} E \{ b_k \} E \{ b_{k\oplus 1} \} \right) = 0 \end{aligned}$$

$$\mathbf{s}_{g_{14}}^2 = E[\mathbf{g}_{14}^2] = E \left[r^4 T^2 \sum_{j=2}^{K-1} \sum_{k=2}^{K-1} \sqrt{p_j} b_j \sqrt{p_{j\oplus 1}} b_{j\oplus 1} \sqrt{p_k} b_k \sqrt{p_{k\oplus 1}} b_{k\oplus 1} \right]$$

We split the sums into two cases, $j = k$, and $j \neq k$:

$$\begin{aligned} \mathbf{s}_{g_{14}}^2 &= r^4 T^2 \left(\sum_{k=2}^{K-1} p_k E[b_k^2] p_{k\oplus 1} E[b_{k\oplus 1}^2] + \right. \\ &\quad \left. + \sum_{j=2}^{K-1} \sum_{\substack{k=2 \\ k \neq j}}^{K-1} \sqrt{p_j} E[b_j] \sqrt{p_k} E[b_k] \sqrt{p_{j\oplus 1}} E[b_{j\oplus 1}] \sqrt{p_{k\oplus 1}} E[b_{k\oplus 1}] \right) \\ &= r^4 T^2 \left(\sum_{k=2}^{K-2} p_k p_{k\oplus 1} + 0 \right) \\ &= r^4 T^2 \sum_{k=2}^{K-2} p_k p_{k\oplus 1} \end{aligned} \tag{3.128}$$

3. Variance of Noise Interference

$$\text{a. } \mathbf{h}_{11} = r \int_0^T \sqrt{2p_1} b_1(t) \chi(t) n(t) \cos 2\mathbf{p} f_c t dt$$

We'll find the moments of \mathbf{h}_{11}

$$\begin{aligned} E[\mathbf{h}_{11}] &= E \left[r \int_0^T \sqrt{2p_1} b_1(t) \chi(t) n(t) \cos 2\mathbf{p} f_c t dt \right] \\ &= r \sqrt{2p_1} \int_0^T b_1(t) \chi(t) E[n(t)] \cos 2\mathbf{p} f_c t dt = 0 \end{aligned}$$

$$\mathbf{s}_{h_{11}}^2 = E[\mathbf{h}_{11}^2]$$

$$\begin{aligned} &= r^2 2 p_1 E \left[\int_0^T \int_0^T b_1(t) b_1(\mathbf{l}) c_1(t) c_1(\mathbf{l}) n(t) n(\mathbf{l}) \cos 2\mathbf{p} f_c t \cos 2\mathbf{p} f_c \mathbf{l} dt d\mathbf{l} \right] \\ &= r^2 2 p_1 \int_0^T \int_0^T b_1(t) b_1(\mathbf{l}) c_1(t) c_1(\mathbf{l}) E[n(t) n(\mathbf{l})] \cos 2\mathbf{p} f_c t \cos 2\mathbf{p} f_c \mathbf{l} dt d\mathbf{l} \end{aligned} \tag{3.129}$$

We know that the autocorrelation function of noise is given by

$$E[n(t)n(\mathbf{l})] = \frac{N_0}{2} \mathbf{d}(t-\mathbf{l}) = \begin{cases} \frac{N_0}{2}, & \text{for } t = \mathbf{l} \\ 0, & \text{elsewhere} \end{cases}$$

Accordingly (3.129) can be modified as follows:

$$\begin{aligned} \mathbf{s}_{h_{11}}^2 &= r^2 2p_1 \int_0^T \int_0^T \frac{N_0}{2} \mathbf{d}(t-\mathbf{l}) b_1(t) b_1(\mathbf{l}) c_1(t) c_1(\mathbf{l}) \cos(2\mathbf{p} f_c t) \cos 2\mathbf{p} f_c \mathbf{l} dt d\mathbf{l} \\ &= r^2 2p_1 \int_0^T \frac{N_0}{2} b_1^2(t) c_1^2(t) \cos(2\mathbf{p} f_c t) dt \\ &= r^2 2p_1 \frac{N_0}{2} \int_0^T b_1^2(t) c_1^2(t) \cos^2(2\mathbf{p} f_c t) dt \\ &= r^2 2p_1 \frac{N_0}{2} \int_0^T b_1^2(t) c_1^2(t) \frac{1}{2} [1 + \cos 4\mathbf{p} f_c t] dt \\ &= r^2 2p_1 \frac{N_0}{4} \int_0^T (1 + \cos 4\mathbf{p} f_c t) dt \\ &= r^2 2p_1 \frac{N_0}{4} \left[t + \frac{\sin 4\mathbf{p} f_c t}{4\mathbf{p} f_c} \right]_0^T \\ &= r^2 2p_1 \frac{N_0}{4} \left[T + \frac{\sin 4\mathbf{p} f_c T}{4\mathbf{p} f_c} \right] \\ &= r^2 2p_1 \frac{N_0}{4} T \\ &= \frac{1}{2} r^2 p_1 N_0 T, \end{aligned} \tag{3.130}$$

where we assumed that $f_c = \frac{k}{T}$.

$$\begin{aligned} \text{b. } \mathbf{h}_{12} &= r \int_0^T \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sqrt{2p_k} b_k(t) w_k(t) w_{1(t)} \vartheta(t) n(t) \cos(2\mathbf{p} f_c t) dt \\ &= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r \sqrt{2p_k} \int_0^T b_k(t) w_k(t) w_{1(t)} \vartheta(t) n(t) \cos(2\mathbf{p} f_c t) dt \end{aligned}$$

We'll find the moments of \mathbf{h}_{12} :

$$E[\mathbf{h}_{12}] = E \left\{ \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r \sqrt{2p_k} \int_0^T b_k(t) w_k(t) w_{1(t)} \vartheta(t) n(t) \cos(2\mathbf{p} f_c t) dt \right\}$$

$$= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r \sqrt{2p_k} \int_0^T b_k(t) w_k(t) w_1(t) \varphi(t) E[n(t)] \cos(2\mathbf{p} f_c t) dt = 0,$$

since $E[n(t)] = 0$.

$$\mathbf{s}_{h_{12}}^2 = E[\mathbf{h}_{12}^2]$$

$$= E \left[\sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{l=0 \\ l \neq 1}}^{K-1} r^2 2p_k \int_0^T \int_0^T b_k(t) b_l(\mathbf{l}) w_k(t) w_l(\mathbf{l}) w_1(t) w_1(\mathbf{l}) n(t) n(\mathbf{l}) \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c \mathbf{l}) dt d\mathbf{l} \right]$$

$$= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{l=0 \\ l \neq 1}}^{K-1} r^2 2p_k \int_0^T \int_0^T b_k(t) b_l(\mathbf{l}) w_k(t) w_l(\mathbf{l}) w_1(t) w_1(\mathbf{l}) c(t) c(\mathbf{l}) E[n(t) n(\mathbf{l})] \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c \mathbf{l}) dt d\mathbf{l}$$

$$= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{l=0 \\ l \neq 1}}^{K-1} r^2 2p_k \int_0^T \int_0^T b_k(t) b_l(\mathbf{l}) w_k(t) w_l(\mathbf{l}) w_1(t) w_1(\mathbf{l}) c(t) c(\mathbf{l}) \frac{N_0}{2} \delta(t - \mathbf{l}) \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c \mathbf{l}) dt d\mathbf{l}$$

$$= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \sum_{\substack{l=0 \\ l \neq 1}}^{K-1} r^2 2p_k \frac{N_0}{2} \int_0^T b_k(t) b_l(t) w_k(t) w_l(t) w_1^2(\mathbf{l}) \delta(t) \cos^2(2\mathbf{p} f_c t) dt. \quad (3.131)$$

The Walsh functions $w_k(t)$ and $w_l(t)$ are orthogonal with each other, so

$$\int_0^T w_k(t) w_l(t) dt = \begin{cases} 0, & \text{for } k \neq l \\ 1, & \text{for } k = l \end{cases}$$

Accordingly we can split the integral into two cases, for $k = l$, and $k \neq l$, and rewrite (3.131) as follows:

$$\mathbf{s}_{h_{12}}^2 = \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 \frac{N_0}{2} 2p_k \int_0^T b_k^2(t) w_k^2(t) \cos^2(2\mathbf{p} f_c t) dt +$$

$$+ \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 \frac{N_0}{2} 2p_k \int_0^T b_k(t) w_k(t) b_l(t) w_l(t) \cos^2(2\mathbf{p} f_c t) dt.$$

$$= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 2p_k \frac{N_0}{2} \int_0^T \frac{1}{2} [1 + \cos 4\mathbf{p} f_c t] dt + 0$$

$$\begin{aligned}
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 2 p_k \frac{N_0}{4} \left[T + \frac{\sin 4 \mathbf{p} f_c t}{4 \mathbf{p} f_c} \right] \\
&= \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} r^2 2 p_k \frac{N_0}{4} T \\
&= \frac{1}{2} r^2 N_0 T \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} p_k, \tag{3.132}
\end{aligned}$$

where we assumed that $f_c = \frac{k}{T}$.

$$\begin{aligned}
c. \quad \mathbf{h}_{13} &= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) w_1(t) n(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) dt \\
&= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \underbrace{\int_0^T R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) w_1(t) n(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) dt}_{N_{ij}}
\end{aligned}$$

Let N_{ij} be the noise contribution to the interference terms from the individual channel j , in the adjacent cell I , where

$$N_{ij} = \int_0^T R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) w_1(t) n(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) dt$$

$$R_i \sqrt{\frac{P_k}{2}} \rightarrow R_i \sqrt{2P_{ij}} \Rightarrow R_i^2 \frac{P_k}{2} \rightarrow 2R_i^2 P_{ij}$$

Then \mathbf{h}_{13} can be expressed in terms of N_{ij} as follows:

$$\mathbf{h}_{13} = \sum_{i=1}^6 \sum_{j=0}^{K_i-1} N_{ij}$$

Accordingly, we'll find the moments of N_{ij} .

$$\begin{aligned}
E[N_{ij}] &= E \left[\int_0^T R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) w_1(t) n(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) dt \right] \\
&= \int_0^T R_i \sqrt{2P_{ij}} b_{ij}(t+\mathbf{t}_i) w_{ij}(t+\mathbf{t}_i) c_i(t+\mathbf{t}_i) w_1(t) E[n(t)] \cos(2\mathbf{p} f_c t + \mathbf{j}_i) dt \\
&= 0,
\end{aligned}$$

since $E[n(t)] = 0$.

$$\begin{aligned}
\mathbf{s}_{N_{ij}}^2 &= E[N_{ij}^2] = E\left\{R_i^2 2P_{ij} \int_0^T \int_0^T b_{ij}(t+\mathbf{t}_i)b_{ij}(\mathbf{l}+\mathbf{t}_i)w_{ij}(t+\mathbf{t}_i)w_{ij}(\mathbf{l}+\mathbf{t}_i) \times \right. \\
&\times c_i(t+\mathbf{t}_i)c_i(\mathbf{l}+\mathbf{t}_i)w_1(t)w_1(\mathbf{l})n(t)n(\mathbf{l})\cos(2\mathbf{p}f_c t+\mathbf{j}_i)\cos(2\mathbf{p}f_c \mathbf{l}+\mathbf{j}_i)dtd\mathbf{l}\left.\right\} \\
&= R_i^2 2P_{ij} \int_0^T \int_0^T b_{ij}(t+\mathbf{t}_i)b_{ij}(\mathbf{l}+\mathbf{t}_i)w_{ij}(t+\mathbf{t}_i)w_{ij}(\mathbf{l}+\mathbf{t}_i) \times \\
&\times c_i(t+\mathbf{t}_i)c_i(\mathbf{l}+\mathbf{t}_i)w_1(t)w_1(\mathbf{l})E[n(t)n(\mathbf{l})]\cos(2\mathbf{p}f_c t+\mathbf{j}_i)\cos(2\mathbf{p}f_c \mathbf{l}+\mathbf{j}_i)dtd\mathbf{l} \\
&= R_i^2 2P_{ij} \int_0^T \int_0^T b_{ij}(t+\mathbf{t}_i)b_{ij}(\mathbf{l}+\mathbf{t}_i)w_{ij}(t+\mathbf{t}_i)w_{ij}(\mathbf{l}+\mathbf{t}_i) \times \\
&\times c_i(t+\mathbf{t}_i)c_i(\mathbf{l}+\mathbf{t}_i)w_1(t)w_1(\mathbf{l})\frac{N_0}{2}\mathbf{d}(t-\mathbf{l})\cos(2\mathbf{p}f_c t+\mathbf{j}_i)\cos(2\mathbf{p}f_c \mathbf{l}+\mathbf{j}_i)dtd\mathbf{l} \\
&= 2E[R_i^2]E[P_{ij}]\frac{N_0}{2}\int_0^T b_{ij}^2(t+\mathbf{t}_i)w_{ij}^2(t+\mathbf{t}_i)c_i^2(t+\mathbf{t}_i)w_1^2(t)\cos^2(2\mathbf{p}f_c t+\mathbf{j}_i)dt \\
&= E[R_i^2]E[P_{ij}]N_0\int_0^T \frac{1}{2}(1+\cos(4\mathbf{p}f_c t+2\mathbf{j}_i))dt \\
&= E[R_i^2]E[P_{ij}]\frac{N_0}{2}\int_0^T 1+\cos(4\mathbf{p}f_c t+2\mathbf{j}_i)dt \\
&= E[R_i^2]E[P_{ij}]\frac{N_0}{2}\left[t+\frac{\sin(4\mathbf{p}f_c t+2\mathbf{j}_i)}{4\mathbf{p}f_c}\right]_0^T \\
&= E[R_i^2]E[P_{ij}]\frac{N_0}{2}\left[T+\frac{\sin(4\mathbf{p}f_c T+2\mathbf{j}_i)}{4\mathbf{p}f_c}-\frac{\sin(2\mathbf{j}_i)}{4\mathbf{p}f_c}\right]. \tag{3.133}
\end{aligned}$$

We assume again that $f_c = \frac{k}{T}$, so we can simplify the sinusoid terms in (3.133) as follows:

$$\sin\left(4\mathbf{p}\frac{k}{T}T+2\mathbf{j}_i\right)=\sin 2\mathbf{j}_i.$$

Therefore (3.133) can be written as

$$\begin{aligned}
\mathbf{s}_{N_{ij}}^2 &= E[R_i^2]E[P_{ij}]\frac{N_0}{2}\left[T+\frac{\sin(2\mathbf{j}_i)}{4\mathbf{p}f_c}-\frac{\sin(2\mathbf{j}_i)}{4\mathbf{p}f_c}\right] \\
&= E[R_i^2]E[P_{ij}]\frac{N_0}{2}T
\end{aligned}$$

Eventually, we can find the variance of \mathbf{h}_{13} , assuming that N_{ij} are independent as follows:

$$\begin{aligned}
\mathbf{s}_{\mathbf{h}_{13}}^2 &= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \mathbf{s}_{N_{ij}}^2 \\
&= \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}] \frac{N_0}{2} T \\
&= \frac{1}{2} N_0 T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}]
\end{aligned} \tag{3.134}$$

d. $\mathbf{h}_{14} = r\sqrt{2p_0} \int_0^T n(t) \alpha(t) w_1(t) \cos 2\mathbf{p} f_c t$

$$\begin{aligned}
\mathbf{s}_{\mathbf{h}_{14}}^2 &= E[\mathbf{h}_{14}^2] = \\
&= E \left[r^2 2p_0 \int_0^T \int_0^T n(t) n(\mathbf{l}) c(t) c(\mathbf{l}) w_1(t) w_1(\mathbf{l}) \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c \mathbf{l}) dt d\mathbf{l} \right] \\
&= r^2 2p_0 \int_0^T \int_0^T E[n(t) n(\mathbf{l})] c(t) c(\mathbf{l}) w_1(t) w_1(\mathbf{l}) \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c \mathbf{l}) dt d\mathbf{l} \\
&= r^2 2p_0 \int_0^T \int_0^T \frac{N_0}{2} \mathbf{d}(t-\mathbf{l}) c(t) c(\mathbf{l}) w_1(t) w_1(\mathbf{l}) \cos(2\mathbf{p} f_c t) \cos(2\mathbf{p} f_c \mathbf{l}) dt d\mathbf{l} \\
&= r^2 2p_0 \frac{N_0}{2} \int_0^T c^2(t) w_1^2(t) \cos^2 2\mathbf{p} f_c t dt \\
&= r^2 2p_0 \frac{N_0}{2} \int_0^T \frac{1}{2} [1 + \cos(4\mathbf{p} f_c t)] \\
&= r^2 2p_0 \frac{N_0}{4} \left[T + \frac{\sin(4\mathbf{p} f_c T)}{4\mathbf{p} f_c} \right] \\
&= r^2 2p_0 \frac{N_0 T}{4} \\
&= \frac{1}{2} r^2 p_0 N_0 T,
\end{aligned} \tag{3.135}$$

where we assumed that $f_c = \frac{k}{T}$.

e. $\mathbf{h}_{15} = r \int_0^T \sum_{k=1}^{K-1} \sqrt{2p_k} b_k(t) w_{k\oplus 1}(t) c(t) n(t) \cos(2\mathbf{p} f_c t) dt$

$$\begin{aligned}
&= \sum_{k=1}^{K-1} r \int_0^T \sqrt{2p_k} b_k(t) w_k(t) w_1(t) w_0(t) n(t) \cos(2\mathbf{p} f_c t) dt \\
&= \sum_{k=1}^{K-1} r \int_0^T \sqrt{2p_k} b_k(t) w_k(t) w_1(t) n(t) \cos(2\mathbf{p} f_c t) dt, \\
&\text{since } w_0(t) = 1.
\end{aligned}$$

We observe that \mathbf{h}_{15} is the same as \mathbf{h}_{12} , apart from the limits of the sum, which are not significant at the calculation of the variance. Thus working the same way as \mathbf{h}_{12} we obtain

$$\begin{aligned}
\mathbf{s}_{n_{15}}^2 &= \frac{1}{2} r^2 N_0 T \sum_{k=1}^{K-1} p_k \\
\text{f. } \mathbf{h}_{16} &= \int_0^T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} R_i \sqrt{2P_{ij}} b_{ij}(t + \mathbf{t}_i) w_{ij}(t + \mathbf{t}_i) c_i(t + \mathbf{t}_i) w_1(t) n(t) \cos(2\mathbf{p} f_c t + \mathbf{j}_i) dt
\end{aligned}$$

We observe that \mathbf{h}_{16} is equal to \mathbf{h}_{13} . Therefore from (3.134) we have that

$$\mathbf{s}_{h_{16}}^2 = \mathbf{s}_{h_{13}}^2 = \frac{1}{2} N_0 T \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E[R_i^2] E[P_{ij}] \quad (3.136)$$

$$\text{g. } \mathbf{h}_{17} = \int_0^T \mathbf{h}^2(t) w_1(t) dt$$

We'll find the moments of \mathbf{h}_{17}

$$\begin{aligned}
E[\mathbf{h}_{17}] &= E[\mathbf{h}^2(t)] \int_0^T w_1(t) dt \\
&= \frac{N_0}{2} \int_0^T w_1(t) dt = 0,
\end{aligned}$$

since we know that for any Walsh sequence $\int_0^T w_k(t) dt = 0$

$$E[\mathbf{h}_{17}^2] = E\left[\int_0^T \int_0^T n^2(t) n^2(\mathbf{l}) w_1(t) w_1(\mathbf{l}) dt d\mathbf{l} \right]$$

We set $y(t) = n^2(t)$ and we apply the square law:

$$\begin{aligned}
E[\mathbf{h}_{17}^2] &= \int_0^T \int_0^T E[y(t)y(\mathbf{I})] w_1(t)w_1(\mathbf{I}) dt d\mathbf{I} \\
&= \int_0^T \int_0^T (R_h^2(0) + 2R_h^2(t-\mathbf{I})) w_1(t)w_1(\mathbf{I}) dt d\mathbf{I} \\
&= \underbrace{\int_0^T \int_0^T R_h^2(0) w_1(t)w_1(\mathbf{I}) dt d\mathbf{I}}_{\mathbf{s}_{h_{71}}^2} + \underbrace{2 \int_0^T \int_0^T R_h^2(t-\mathbf{I}) w_1(t)w_1(\mathbf{I}) dt d\mathbf{I}}_{\mathbf{s}_{h_{72}}^2} \\
&= \mathbf{s}_{h_{71}}^2 + \mathbf{s}_{h_{72}}^2, \tag{3.137}
\end{aligned}$$

where $R_h(t-\mathbf{I})$ is the autocorrelation function of $\mathbf{h}(t)$, such that

$$R_h(t-\mathbf{I}) = E[\mathbf{h}(t)\mathbf{h}(\mathbf{I})] = \frac{N_0}{2}\mathbf{d}(t-\mathbf{I}).$$

Accordingly we can find $\mathbf{s}_{h_{71}}^2$ as follows:

$$\begin{aligned}
\mathbf{s}_{h_{71}}^2 &= \int_0^T \int_0^T R_h^2(0) w_1(t)w_1(\mathbf{I}) dt d\mathbf{I} \\
&= \int_0^T \int_0^T \left[\frac{N_0}{2}\mathbf{d}(0) \right]^2 w_1(t)w_1(\mathbf{I}) dt d\mathbf{I} \\
&= \frac{N_0^2}{4} \int_0^T \int_0^T (w_1(t)\mathbf{d}(0))(w_1(\mathbf{I})\mathbf{d}(0)) dt d\mathbf{I} \\
&= \frac{N_0^2}{4} \int_0^T \int_0^T w_1(0)\mathbf{d}(t)w_1(0)\mathbf{d}(\mathbf{I}) dt d\mathbf{I} \\
&= \frac{N_0^2 w_i^2(0)}{4} \int_0^T \int_0^T \mathbf{d}(t)(\mathbf{I}) dt d\mathbf{I} \\
&= \frac{N_0^2}{4} \tag{3.138}
\end{aligned}$$

$$\begin{aligned}
\mathbf{s}_{h_{72}}^2 &= 2 \int_0^T \int_0^T R_h^2(t-\mathbf{I}) w_1(t)w_1(\mathbf{I}) dt d\mathbf{I} \\
&= 2 \int_0^T \int_0^T \left(\frac{N_0}{2}\mathbf{d}(t-\mathbf{I}) \right)^2 w_1(t)w_1(\mathbf{I}) dt d\mathbf{I}
\end{aligned}$$

$$= \frac{N_0^2}{2} \int_0^T \int_0^T \mathbf{d}(t-\mathbf{l}) \mathbf{d}(t-\mathbf{l}) w_1(t) w_1(\mathbf{l}) dt d\mathbf{l} \quad (3.139)$$

We know that $x(t)\mathbf{d}(t) = x(0)\mathbf{d}(t)$. (3.140)

Let $x(t) = \mathbf{d}(t)$, then (3.140) can be written as

$$\mathbf{d}(t)\mathbf{d}(t) = \mathbf{d}(0)\mathbf{d}(t)$$

Accordingly we form the terms of (3.139) into groups, and we obtain

$$\begin{aligned} \mathbf{s}_{h_{72}}^2 &= \frac{N_0^2}{2} \int_0^T \int_0^T \mathbf{d}(0) w_1(t) \mathbf{d}(t-\mathbf{l}) w_1(\mathbf{l}) d\mathbf{l} dt \\ &= \frac{N_0^2}{2} \int_0^T \mathbf{d}(0) w_1(t) dt \int_0^T \mathbf{d}(t-\mathbf{l}) w_1(\mathbf{l}) d\mathbf{l} \\ &= \frac{N_0^2}{2} \int_0^T \mathbf{d}(0) w_1^2(t) dt \\ &= \frac{N_0^2}{2} w_1^2(0) \int_0^T \mathbf{d}(t) dt \\ &= \frac{N_0^2}{2}, \end{aligned} \quad (3.141)$$

since $w_1^2(t) = 1$, and $\int_0^T \mathbf{d}(t) dt = 1$.

Finally we can find the variance of \mathbf{h}_{17} adding (3.138) and (3.141), and obtain

$$\begin{aligned} \mathbf{s}_{h_{17}}^2 &= \mathbf{s}_{h_{71}}^2 + \mathbf{s}_{h_{72}}^2 \\ &= \frac{N_0^2}{4} + \frac{N_0^2}{2} = \frac{3N_0^2}{4} \\ &= \frac{3N_0^2}{4} \end{aligned} \quad (3.142)$$

APPENDIX III-B. DEVELOPING THE a_1 AND a_2 TERMS IN SNIR

In order to develop the terms a_1 and a_2 , we are going to introduce a new random variable $Z_d = z_d$, which is the sum of d multiplicative chi-square (with 2 degrees of freedom)-lognormal random variables given by

$$z_d = \sum_{l=1}^d r_l^2 \tilde{x}_l,$$

and another random variable $W_d = w_d$, which is the sum of d squared multiplicative chi-square (with 2 degrees of freedom)-lognormal random variables, given by

$$w_d = \sum_{l=1}^d r_l^4 \tilde{x}_l^2 = \sum_{l=1}^d (r_l^2 \tilde{x}_l)^2$$

We are also going to use the identity for the lognormal random variable X_i from (3.85):

$$E\{\tilde{X}_i\} = E\{1/X_i\} = E\{X_i\} = \exp\left(\frac{I^2 \mathbf{s}_{dB}^2}{2}\right),$$

and we will normalize the Rayleigh random variables R_i , such that

$$E\{R_i^4\} = E\{R_i^2\} = E\{R_i\} = 1$$

We finally introduce a new random variable $E_c = f_1 P_t T_{cc} / L_H(d)$, which represents a baseline received coded bit energy without the effects of fading or shadowing, or $E_c = (k/n)E_b$, where E_b is the uncoded bit energy.

The term a_1 defined in (3.89) can be written as

$$\begin{aligned} a_1 &= \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d (\mathbf{s}^2_{z_{i3l}} + \mathbf{s}^2_{h_{i4l}}) \\ &= \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}^2_{z_{i3l}} + \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}^2_{h_{i4l}} \end{aligned} \quad (3.143)$$

Expanding each term of a_1 separately we get:

$$\begin{aligned}
1. \quad & \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}^2_{z_{i3,l}} = \frac{\sum_{l=1}^d \mathbf{s}^2_{z_{i3,l}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
& = \frac{\sum_{l=1}^d r_l^2 \tilde{x}_l f_0 P_t T_{cc}^2}{L_H(d)} \left[\frac{1}{3N} \sum_{i=0}^6 \sum_{j=1}^{K_i-1} E\{R_i^2\} \frac{f_{ij} P_t}{L_H(D_i)} E\left\{\frac{1}{x_i}\right\} \right] \\
& = \frac{\sum_{l=1}^d r_l^2 \tilde{x}_l f_0 P_t T_{cc}^2}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
& = z_d \left[\frac{1}{3N} \sum_{i=0}^6 \sum_{j=1}^{K_i-1} E\{R_i^2\} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} E\{x_i\} \right] \\
& = z_d \left[\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} \right] \tag{3.144}
\end{aligned}$$

$$\begin{aligned}
2. \quad & \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}^2_{h_{i4,l}} = \frac{\sum_{l=1}^d \mathbf{s}^2_{h_{i4,l}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
& = \frac{\sum_{l=1}^d r_l^2 \tilde{x}_l f_0 P_t T_{cc}^2}{L_H(d)} \\
& = \frac{L_H(d)}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \times \frac{1}{2} \left(\frac{E_c}{N_0} \right) \\
& = z_d \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \tag{3.145}
\end{aligned}$$

So after (3.144) and (3.145) the term a_1 from (3.143) can be expressed as:

$$a_1 = \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\mathbf{s}^2_{z_{i3,l}} + \mathbf{s}^2_{h_{i4,l}} \right)$$

$$= z_d \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \right) \quad (3.146)$$

The term a_2 defined in (3.90) can be written as

$$\begin{aligned} a_2 &= \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}^2_{z_{i,l}} + \mathbf{s}^2_{g_{i,l}} + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}^2_{h_{i,l}} \right) \\ &= \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \left(\mathbf{s}^2_{z_{11,l}} + \mathbf{s}^2_{z_{12,l}} + \mathbf{s}^2_{z_{14,l}} + \mathbf{s}^2_{z_{15,l}} + \mathbf{s}^2_{g_{11,l}} + \mathbf{s}^2_{h_{11,l}} + \right. \\ &\quad \left. + \mathbf{s}^2_{h_{12,l}} + \mathbf{s}^2_{h_{13,l}} + \mathbf{s}^2_{h_{15,l}} + \mathbf{s}^2_{h_{16,l}} + \mathbf{s}^2_{h_{17,l}} \right). \end{aligned} \quad (3.147)$$

Expanding each term of a_2 separately we get:

$$\begin{aligned} 1. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}^2_{z_{11}} &= \frac{\sum_{l=1}^d \mathbf{s}^2_{z_{11}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\ &= \frac{\sum_{l=1}^d \frac{r_l^2 \tilde{x}_l f_0 P T_{cc}^2}{L_H(d)} \left[\frac{1}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} E\{P_{ij}\} \right]}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\ &= z_d \left(\frac{1}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)} E\{x_i\} \right) \\ &= z_d \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)} \right) \end{aligned} \quad (3.148)$$

$$2. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \mathbf{s}^2_{z_{12}} = \frac{\sum_{l=1}^d \mathbf{s}^2_{z_{12}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}}$$

$$\begin{aligned}
&= \frac{\sum_{\ell=1}^d r_\ell^2 T_{cc}^2}{f_0 f_1 P_t^2 T_{cc}^2} \left(\frac{1}{3N} \sum_{i=0}^6 \sum_{j=1}^{K_i-1} E\{R_i^2\} \frac{f_{ij} P_t}{L_H(D_i)} E\{x_i\} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \frac{f_k P_t}{L_H(d) x_\ell} \right) = \\
&= \sum_{\ell=1}^d r_\ell^2 \tilde{x}_\ell \left(\frac{1}{3N} \sum_{i=0}^6 \sum_{j=1}^{K_i-1} E\{R_i^2\} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} E\{x_i\} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0} \right) \right) \\
&= z_d \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0} \right) \right) \tag{3.149}
\end{aligned}$$

$$\begin{aligned}
3. \frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)} \sum_{\ell=1}^d \mathbf{s}^2_{z_{14}} &= \frac{\sum_{\ell=1}^d \mathbf{s}^2_{z_{14}}}{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2}} \\
&= \frac{\sum_{\ell=1}^d r_\ell^2 T_{cc}^2}{f_0 f_1 P_t^2 T_{cc}^2} \left(\frac{1}{3N} \sum_{i=0}^6 \sum_{j=1}^{K_i-1} E\{R_i^2\} \frac{f_{ij}}{L_H(D_i)} P_t E\{x_i\} \sum_{k=0}^{K-1} \frac{f_k P_t}{L_H(d) x_\ell} \right) = \\
&= \sum_{\ell=1}^d r_\ell^2 \tilde{x}_\ell \left(\frac{1}{3N} \sum_{i=0}^6 \sum_{j=1}^{K_i-1} E\{R_i^2\} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} E\{x_i\} \sum_{k=0}^{K-1} \left(\frac{f_k}{f_0} \right) \right) \\
&= z_d \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1} \right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{k=0}^{K-1} \left(\frac{f_k}{f_0} \right) \right) \tag{3.150}
\end{aligned}$$

$$\begin{aligned}
4. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{z_{15}} &= \frac{\sum_{\ell=1}^d \mathbf{s}^2_{z_{15}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
&= \frac{\sum_{\ell=1}^d T_{cc}^2}{f_0 f_1 P_t^2 T_{cc}^2} \left(\frac{1}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} E\{R_i^4\} \frac{f_{ij} P_t}{L_H(D_i)} E\left\{\frac{1}{x_i}\right\} \frac{f_{ij} P_t}{L_H(D_i)} E\left\{\frac{1}{x_i}\right\} \right) +
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=1 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} E\{R_i^2\} E\{R_p^2\} \frac{f_{ij} P_t}{L_H(D_i)} E\left\{\frac{1}{x_i}\right\} \frac{f_{ij} P_t}{L_H(D_p)} E\left\{\frac{1}{x_i}\right\} = \\
& = d \frac{1}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} E\{R_i^4\} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right)^2 (E\{x_i\})^2 + \\
& + d \frac{1}{4N} \sum_{i=1}^6 \sum_{\substack{j=0 \\ p \neq i}}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} E\{R_i^2\} E\{R_p^2\} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right) \left(\frac{L_H(d)}{L_H(D_p)}\right) E\{x_i\} E\{x_p\} \\
& = d \times \frac{\left[\exp\left(\frac{I^2 \mathbf{s}_{dB}^2}{2}\right)\right]^2}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right)^2 + \\
& + d \times \frac{\left[\exp\left(\frac{I^2 \mathbf{s}_{dB}^2}{2}\right)\right]^2}{4N} \sum_{i=1}^6 \sum_{\substack{j=0 \\ p \neq i}}^{K_i-1} \sum_{p=1}^6 \sum_{q=0}^{K_p-1} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right) \left(\frac{L_H(d)}{L_H(D_p)}\right) \quad (3.151)
\end{aligned}$$

$$\begin{aligned}
5. \quad & \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{h_{11}} = \frac{\sum_{\ell=1}^d \mathbf{s}^2_{h_{11}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
& = \frac{\sum_{\ell=1}^d \frac{1}{2} r_i^2 x_i \frac{f_1 P_t T_{cc}}{L_H(d)}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} N_0 = \frac{1}{2} z_d \frac{L_H(d) N_0}{f_0 P_t T_{cc}} = \\
& = \frac{1}{2} z_d \frac{L_H(d) N_0}{f_1 P_t T_{cc}} \left(\frac{f_1}{f_0}\right) = \frac{1}{2} z_d \left(\frac{E_c}{N_0}\right)^{-1} \left(\frac{f_1}{f_0}\right) \quad (3.152)
\end{aligned}$$

$$\begin{aligned}
6. \quad & \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{h_{12}} = \frac{\sum_{\ell=1}^d \mathbf{s}^2_{h_{12}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
& = \frac{\sum_{\ell=1}^d \frac{1}{2} r_i^2 \tilde{x}_i \frac{P_t T_{cc} N_0}{L_H(d)} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} f_k}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} = \frac{1}{2} z_d \frac{P_t T_{cc} N_0}{L_H(d)} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} f_k
\end{aligned}$$

$$= \frac{1}{2} z_d \left(\frac{E_c}{N_0} \right)^{-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \begin{pmatrix} f_k \\ f_0 \end{pmatrix} \quad (3.153)$$

$$\begin{aligned} 7. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{h3} &= \frac{\sum_{\ell=1}^d \mathbf{s}^2_{h3}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\ &= \frac{\sum_{\ell=1}^d \frac{1}{2} N_0 T_{cc} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} \frac{f_{ij} P_t}{L_H(D_i)} E \left\{ \frac{1}{x_i} \right\}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} = \\ &= d \frac{1}{2} \left(\frac{E_{cc}}{N_0} \right)^{-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)} E\{x_i\} \\ &= d \frac{\exp\left(\frac{I^2 \mathbf{s}^2_{dB}}{2}\right)}{2} \left(\frac{E_{cc}}{N_0} \right)^{-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)} \end{aligned} \quad (3.154)$$

$$\begin{aligned} 8. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{h5} &= \frac{\sum_{\ell=1}^d \mathbf{s}^2_{h5}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\ &= \frac{\sum_{\ell=1}^d \frac{1}{2} r_i^2 N_0 T_{cc} \sum_{k=1}^{K-1} \frac{f_k P_t}{L_H(d)} x_l}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} = \\ &= \frac{1}{2} z_d \left(\frac{E_c}{N_0} \right)^{-1} \sum_{k=1}^{K-1} \begin{pmatrix} f_k \\ f_0 \end{pmatrix} \end{aligned} \quad (3.155)$$

$$\begin{aligned} 9. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{h6} &= \frac{\sum_{\ell=1}^d \mathbf{s}^2_{h6}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\ &= \frac{\sum_{\ell=1}^d \frac{1}{2} N_0 T_{cc} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} \frac{f_{ij} P_t}{L_H(D_i)} E \left\{ \frac{1}{x_i} \right\}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \end{aligned}$$

$$\begin{aligned}
&= d \frac{1}{2} \left(\frac{E_{cc}}{N_0} \right)^{-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)} E \left\{ \frac{1}{x_i} \right\} \\
&= d \frac{\exp\left(\frac{I^2 \mathbf{s}_{dB}^2}{2}\right)}{2} \left(\frac{E_{cc}}{N_0} \right)^{-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} E\{R_i^2\} \left(\frac{f_{ij}}{f_0} \right) \frac{L_H(d)}{L_H(D_i)}
\end{aligned} \tag{3.156}$$

$$\begin{aligned}
10. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{h_{17}} &= \frac{\sum_{\ell=1}^d \mathbf{s}^2_{h_{17}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
&= \frac{\sum_{\ell=1}^d \frac{3N_0^2}{4}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} = d \frac{3}{4} \frac{N_0^2}{f_1^2 P_t^2 T_{cc}^2} \left(\frac{f_1}{f_0} \right) \\
&= d \frac{3}{4} \left(\frac{f_1}{f_0} \right) \left(\frac{E_c}{N_0} \right)^{-2}
\end{aligned} \tag{3.157}$$

$$\begin{aligned}
11. \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{\ell=1}^d \mathbf{s}^2_{g_{14}} &= \frac{\sum_{\ell=1}^d \mathbf{s}^2_{g_{14}}}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \\
&= \frac{\sum_{\ell=1}^d r_l^4 T_{cc}^2}{\frac{f_0 f_1 P_t^2 T_{cc}^2}{L_H^2(d)}} \sum_{k=2}^{K-1} p_k p_{k\oplus 1} = \frac{\sum_{\ell=1}^d r_l^4 \tilde{x}_l^2 \frac{P_t^2}{L_H^2(d)} \sum_{k=2}^{K-1} f_k f_{k\oplus 1}}{\frac{f_0 f_1 P_t^2}{L_H^2(d)}} \\
&= w_d \sum_{k=2}^{K-1} \left(\frac{f_k}{f_0} \right) \left(\frac{f_{k\oplus 1}}{f_1} \right).
\end{aligned} \tag{3.158}$$

So after (3.148) through (3.158), the term a_2 from (3.147) can be expressed as:

$$a_2 = \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\sum_{\substack{i=11 \\ i \neq 13}}^{15} \mathbf{s}^2_{z_{i,l}^e} + \mathbf{s}^2_{g_{i,l}^e} + \sum_{\substack{i=11 \\ i \neq 14}}^{17} \mathbf{s}^2_{h_{i,l}} \right)$$

$$\begin{aligned}
a_2 = & z_d \left(\frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0}\right) \frac{L_H(d)}{L_H(D_i)} + \frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1}\right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0}\right) + \right. \\
& + \frac{\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right)}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_1}\right) \frac{L_H(d)}{L_H(D_i)} \times \sum_{k=0}^{K-1} \left(\frac{f_k}{f_0}\right) + \frac{1}{2} \left(\frac{E_c}{N_0}\right)^{-1} \left(\frac{f_1}{f_0}\right) + \\
& \left. + \frac{1}{2} \left(\frac{E_c}{N_0}\right)^{-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0}\right) + \frac{1}{2} \left(\frac{E_c}{N_0}\right)^{-1} \sum_{k=1}^{K-1} \left(\frac{f_k}{f_0}\right) \right) + \\
& + w_d \sum_{k=2}^{K-1} \left(\frac{f_k}{f_0}\right) \left(\frac{f_{k \oplus 1}}{f_1}\right) + \\
& + d \left\{ \frac{\left[\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \right]^2}{3N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{q=0}^{K_i-1} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right)^2 + \right. \\
& + \frac{\left[\exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \right]^2}{4N} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \sum_{\substack{p=1 \\ p \neq i}}^6 \sum_{q=0}^{K_p-1} \left(\frac{f_{ij}}{f_0}\right) \left(\frac{f_{iq}}{f_1}\right) \left(\frac{L_H(d)}{L_H(D_i)}\right) \left(\frac{L_H(d)}{L_H(D_p)}\right) + \\
& \left. + \exp\left(\frac{\mathbf{I}^2 \mathbf{s}_{dB}^2}{2}\right) \left(\frac{E_c}{N_0}\right)^{-1} \sum_{i=1}^6 \sum_{j=0}^{K_i-1} \left(\frac{f_{ij}}{f_0}\right) \frac{L_H(d)}{L_H(D_i)} + \frac{3}{4} \left(\frac{f_1}{f_0}\right) \left(\frac{E_c}{N_0}\right)^{-2} \right\} \tag{3.159}
\end{aligned}$$

APPENDIX III-C COMPARISON OF PROBABILITY OF BIT ERROR FOR THE RAYLEIGH-LOGNORMAL CHANNEL USING 60° ANTENNA SECTORING, FEC AND PILOT TONE FILTERING

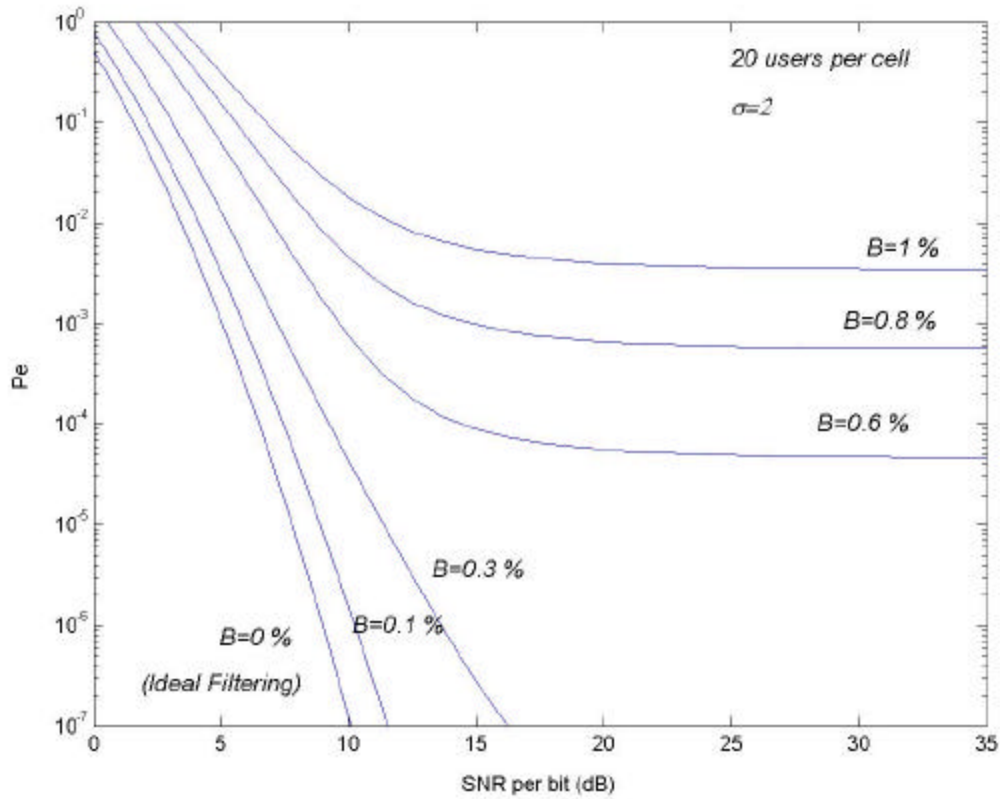


Figure 3.9. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 2$) with 20 Users per Cell, using 60° Sectoring.

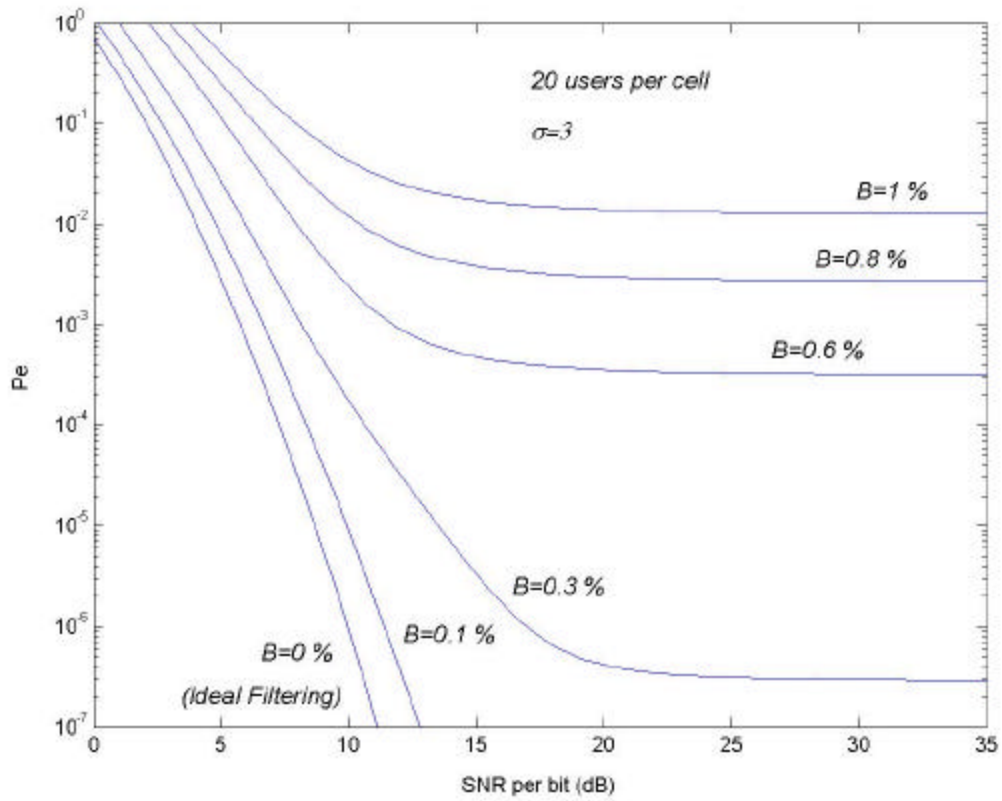


Figure 3.10. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 3$) with 20 Users per Cell, using 60° Sectoring.

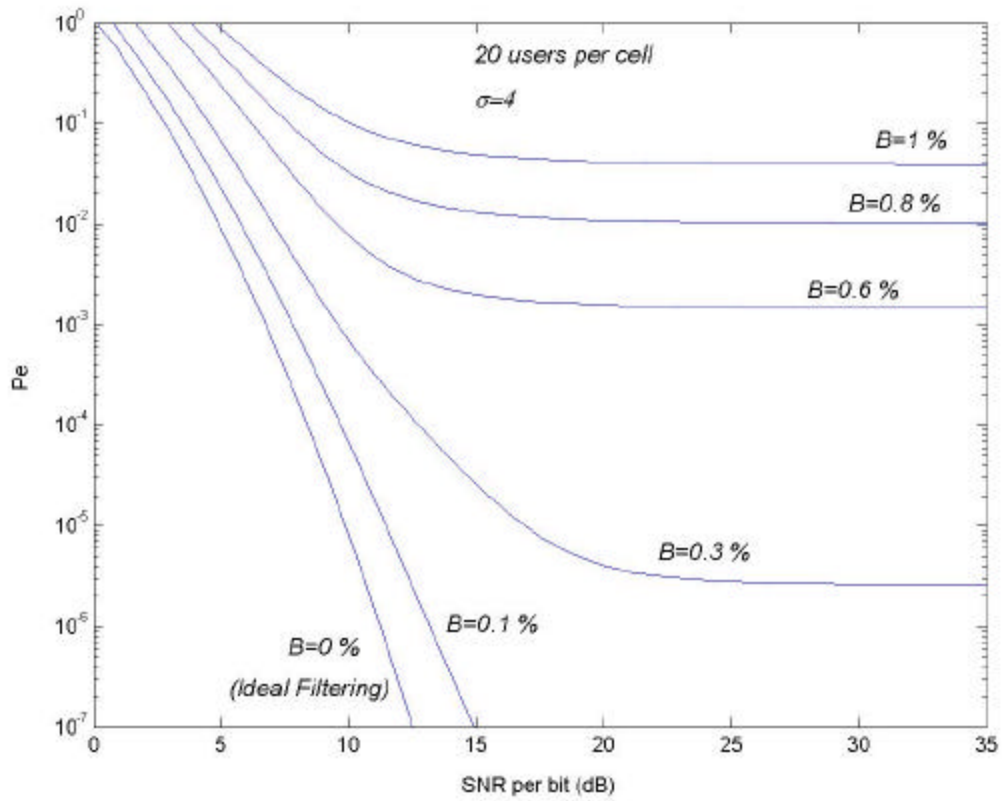


Figure 3.11. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 4$) with 20 Users per Cell, using 60° Sectoring.

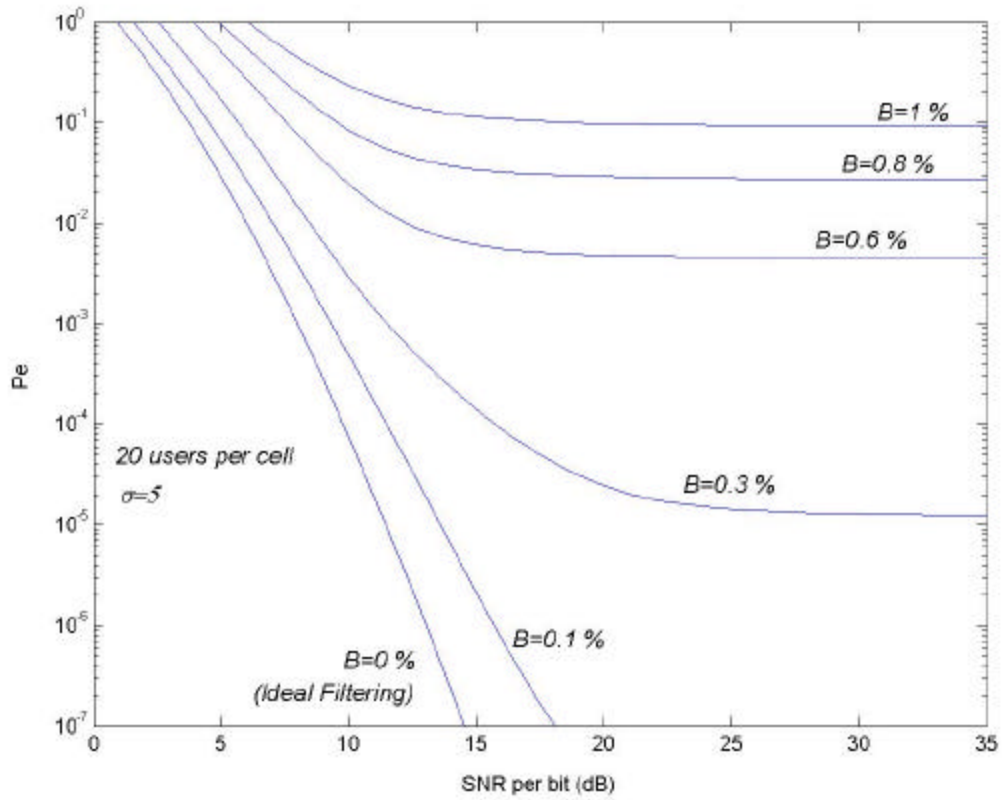


Figure 3.12. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 5$) with 20 Users per Cell, using 60° Sectoring.

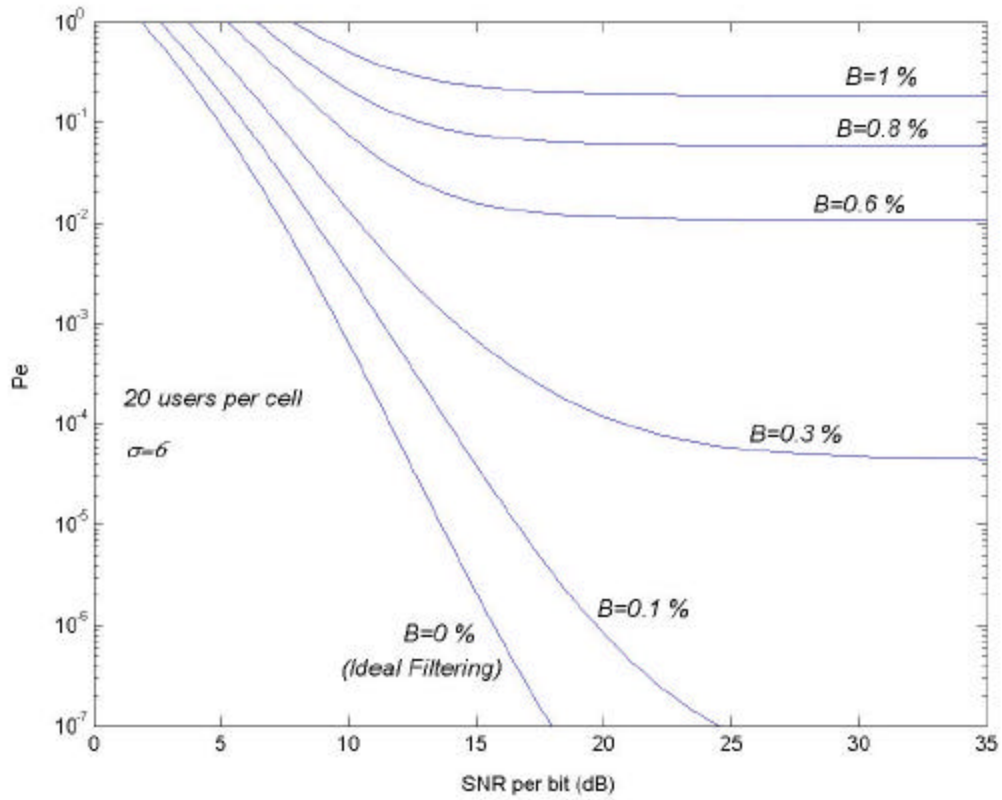


Figure 3.13. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 6$) with 20 Users per Cell, using 60° Sectoring.

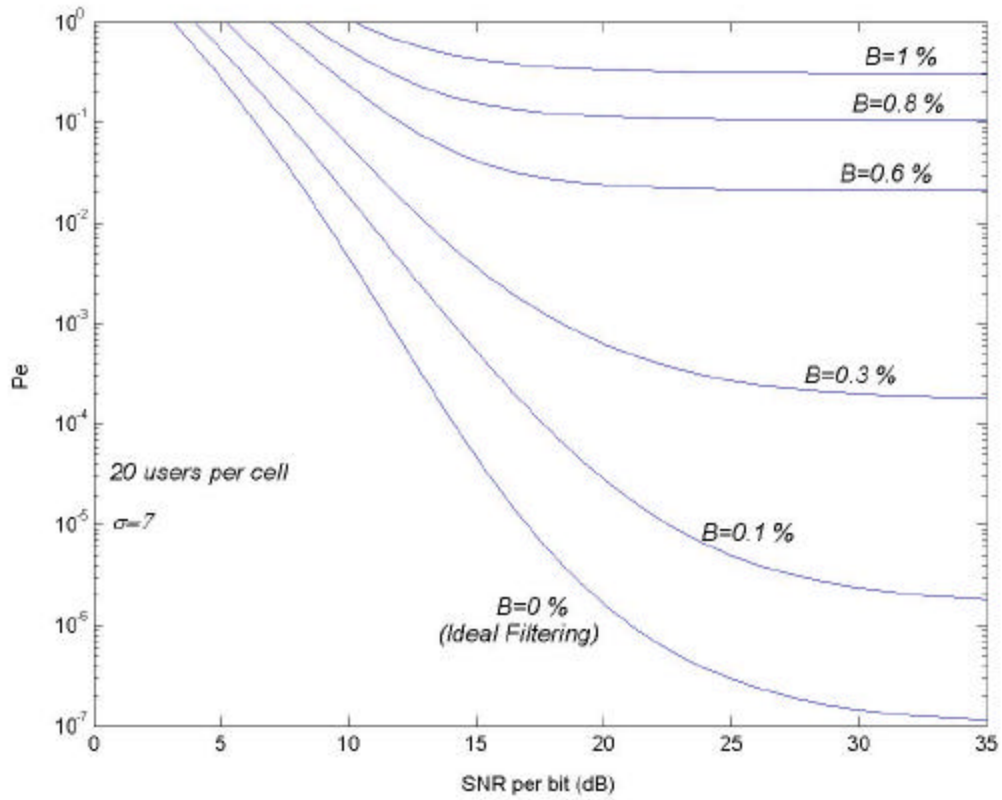


Figure 3.14. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 7$) with 20 Users per Cell, using 60° Sectoring.

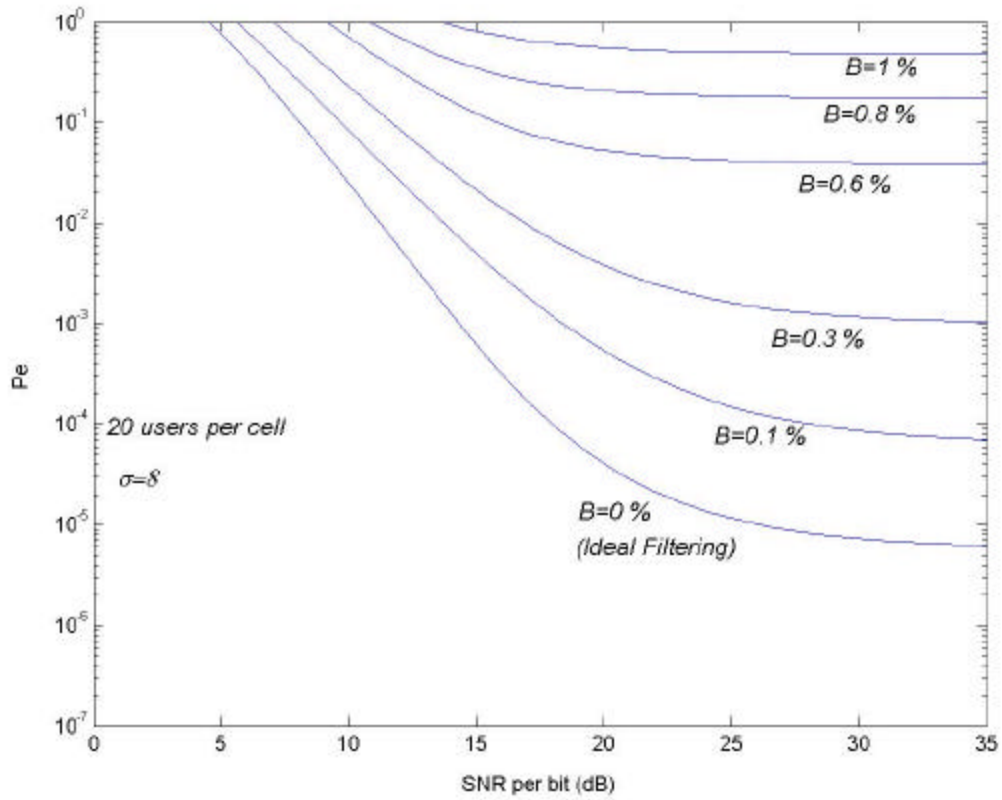


Figure 3.15. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 8$) with 20 Users per Cell, using 60° Sectoring.

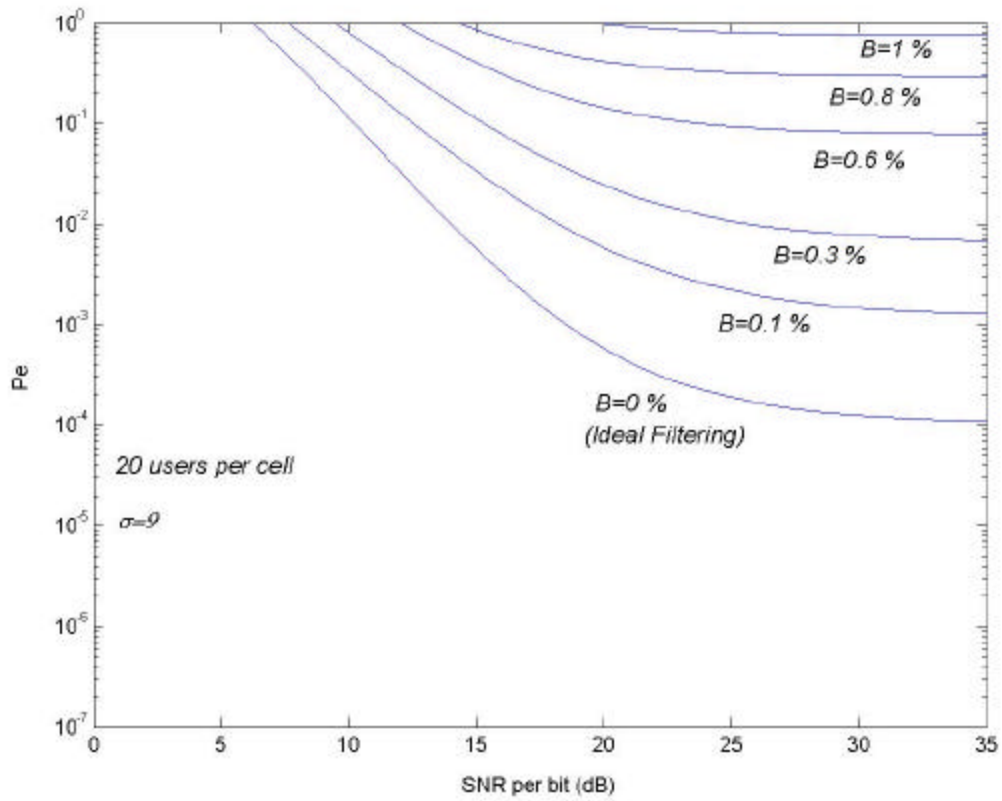


Figure 3.16. Probability of Bit Error for Coded DS-CDMA with Rayleigh Fading and Lognormal Shadowing ($s_{dB} = 9$) with 20 Users per Cell, using 60° Sectoring.

IV. APPLYING POWER CONTROL AT THE PILOT TONE SIGNAL

In Chapter III we analyzed the performance of the forward channel in a DS-CDMA cellular system implying a narrow bandpass filter at the pilot recovery branch. We developed an upper bound of the probability of bit error in Rayleigh fading lognormal shadowing environment with forward error correction. As we saw in the simulated results, when we increased either the number of users in the cell, or the amount of interference that passes through the filter, the performance of the system turned out to be quite poor.

In this chapter we will try to optimize the performance by adjusting the pilot tone power in the center cell. Increasing the power in the pilot channel will enhance synchronization between the base station and the mobile user and therefore help the demodulation of the information signal.

As seen in (2.8) the signal power $P_{t,k}$ in each channel k is defined as:

$$P_{t,k} = f_k P_t, \quad (4.1)$$

where

$$\begin{aligned} f_k &= \text{the power factor used to adjust} \\ &\quad \text{the power in the } k^{\text{th}} \text{ channel,} \\ P_t &= \text{the baseline signal power} \end{aligned}$$

If we assume that the base station transmits a limited amount of total power P_T to all the channels, then we can write

$$\begin{aligned} P_T &= \sum_{k=0}^{K_i-1} f_k P_t \\ &= (f_0 + \sum_{k=1}^{K_i-1} f_k) P_t \end{aligned} \quad (4.2)$$

In our analysis we assume that the power transmitted by the base station is equal for all the channels, except the pilot tone channel, which means that $f_k=1$ for all $k \neq 0$. Moreover there is a constant C_p , such that $P_T = C_p P_t$. Thus we can modify (4.2) as follows:

$$C_p P_t = (f_0 + (K_i - 1)f_k) P_t$$

or

$$C_p = f_0 + (K_i - 1)f_k \quad (4.3)$$

At the equal power case that we examined in Chapter III we assumed that the signal powers were equal for all the channels, which using (4.1) implies that $f_0=f_k=1$. Applying that to (4.3) we can find the constant C_p as follows:

$$C_p = 1 + K_i - 1 = K_i \quad (4.4)$$

As we see the constant C_p equals to the number of users or channels in the cell. Applying (4.4) to (4.3), we get a general form for the power factor f_k that adjusts the power at the other channels, given by

$$f_k = \frac{K_i - f_0}{K_i - 1} \quad (4.5)$$

The amount of power allocated to the pilot channel can be derived from (4.1) and is

$$P_{t,0} = f_0 P_t = f_0 \frac{P}{C_p} = \left(\frac{f_0}{K_i} \right) P_T \quad (4.6)$$

We also see in (4.6) that the ratio of the allocated power at the pilot tone $P_{t,0}$ to the total power P_T is equal to

$$R_{f_0} = \frac{P_{t,0}}{P_T} = \frac{f_0}{K_i}. \quad (4.7)$$

In our analysis we assumed that the transmitted power is limited. Therefore, when the pilot tone power is increased, the power distributed to the other channels is going to be reduced by

$$\begin{aligned}
\Delta P_{t,k} &= P_{t,k} - P'_{t,k} \\
&= f_k P_t - f'_k P_t = (f_k - f'_k) P_t \\
&= \left(1 - \frac{K_i - f_0}{K_i - 1} \right) P_t = \left(\frac{f_0 - 1}{K_i - 1} \right) P_t \\
&= \left(\frac{R_{f_0} K_i - 1}{K_i - 1} \right) P_t. \tag{4.8}
\end{aligned}$$

Normalizing the reduction to the baseline signal power we have:

$$\frac{\Delta P_{t,k}}{P_t} = \frac{R_{f_0} K_i - 1}{K_i - 1}. \tag{4.9}$$

As we'll see later in our analysis, in order to improve performance sometimes it is required to allocate up to 30 % of the total power at the pilot tone depending on the channel conditions, which means that $R_{f_0} = 0.3$. Accordingly the reduction in the power allocated to the other channels given by (4.9) is going to be minor, assuming a large number of users in the cell. Thus the credit for any improvement in the performance analysis would exclusively belong to the pilot tone power allocation.

For a certain percentage (ratio) of power allocated at the pilot channel we can calculate f_0 from (4.7) and then introduce the result to (4.5) and find f_1 . We apply these values to first event bit error probability calculated in (3.105) and simulate the integral of (3.98) exactly as we did in Chapter III. Accordingly, we find the tight upper bound on the bit error probability defined in (3.99).

Figure 4.1 compares the probability of bit error for a cellular system using pilot power control with the equal power case analyzed in Section III. In an average case of 10% of pilot channel interference, and 20 users per cell in a shadowing environment with $\mathbf{s}_{dB} = 7$ dB, we observe that the advantage in the performance using power control is

considerable. However, there is a cut-off power, where the power control no longer provides an improvement in the performance. As we see the cut-off occurs when we allocate to the pilot tone around 40 % of the total power, depending on the channel conditions.

Appendix IV provides graphical results similar to Figure 4.1 for various channel conditions verifying the statement that power control in the pilot channel dramatically improves the performance.

Figure 4.2 depicts the performance of the DS-CDMA system summarizing all the three cases we've already analyzed. As we can see in figure, originally the performance is quite poor, even if we take into account a small amount of interference at the pilot channel. However when we add power control to the pilot channel the probability of bit error we achieve is quite satisfactory for an average SNR of 15 dB ($P_e \approx 10^{-4}$).

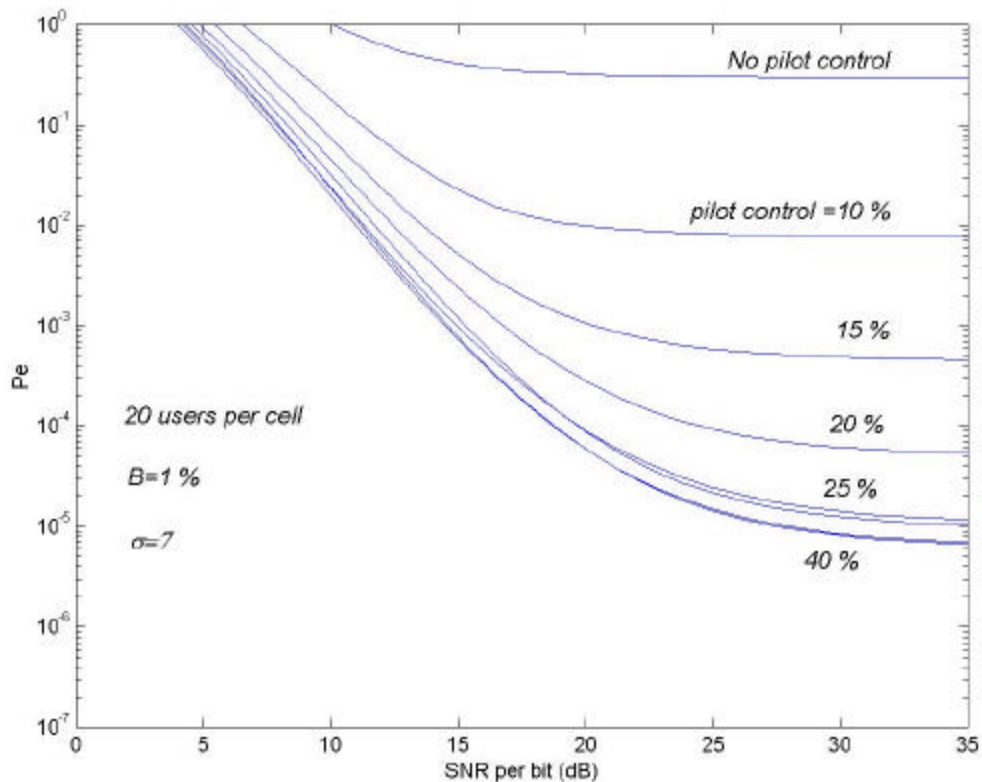


Figure 4.1. Comparison of Probability of Bit Error for DS-CDMA with Rayleigh-Lognormal ($\sigma_{dB} = 7$) Channel and FEC ($R_{cc}=1/2$ and $\gamma=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

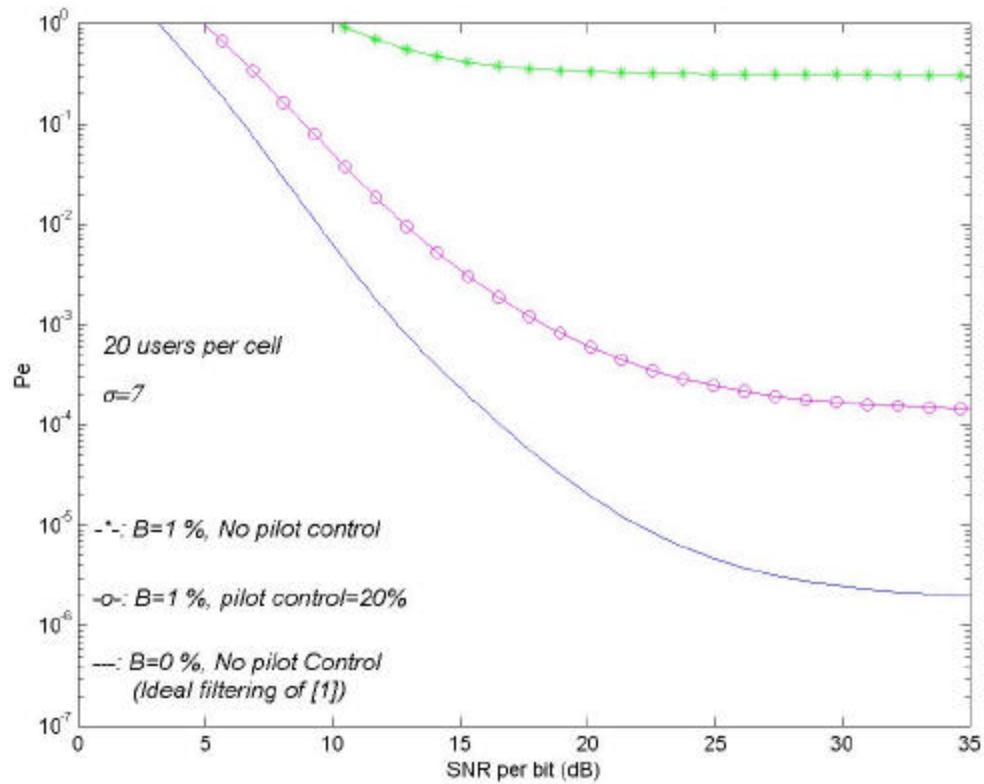


Figure 4.2. Comparison of Probability of Bit Error for DS-CDMA with Rayleigh-Lognormal ($s_{dB} = 7$) Channel and FEC ($R_{cc}=1/2$ and $\gamma=8$), 20 Users/Cell and 60° Sectoring.

Accordingly, we have shown that by carefully adding power control to the pilot channel, we can greatly improve the performance of our DS-CDMA cellular system operating in a Rayleigh-Lognormal channel.

APPENDIX IV. COMPARISON OF PROBABILITY OF BIT ERROR FOR RAYLEIGH-LOGNORMAL CHANNEL USING 60° SECTORING, FEC AND PILOT TONE POWER CONTROL

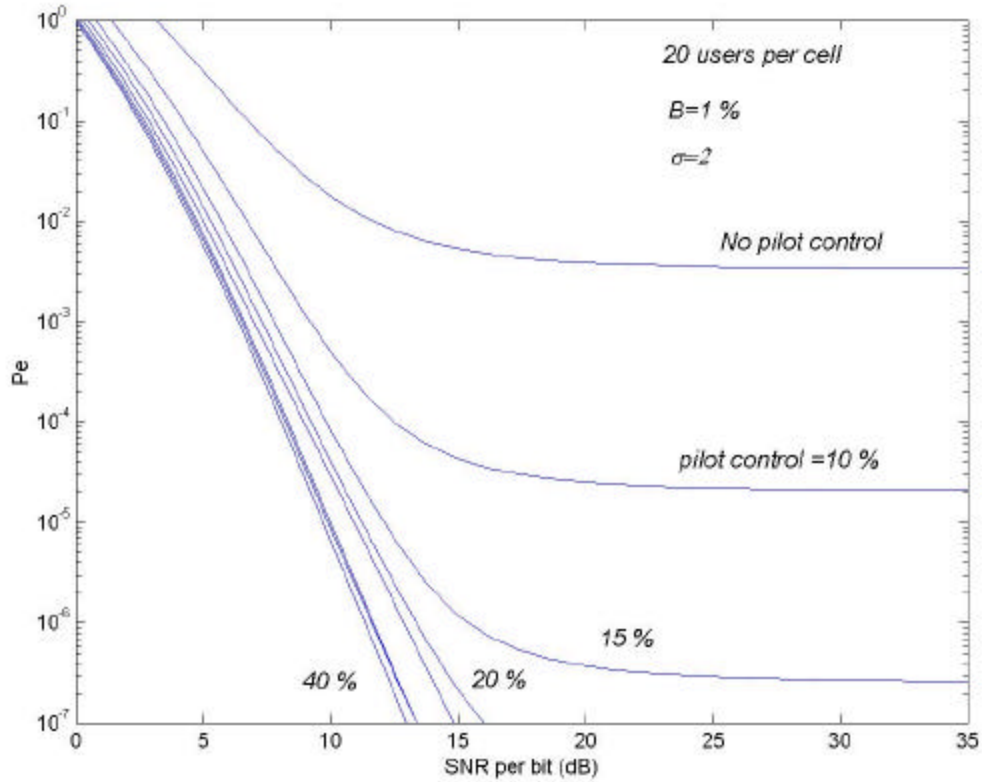


Figure 4.3. Comparison of Probability of Bit Error for DS-SS-SSM with Rayleigh-Lognormal ($s_{dB} = 2$) Channel and FEC ($R_{cc}=1/2$ and $\gamma=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

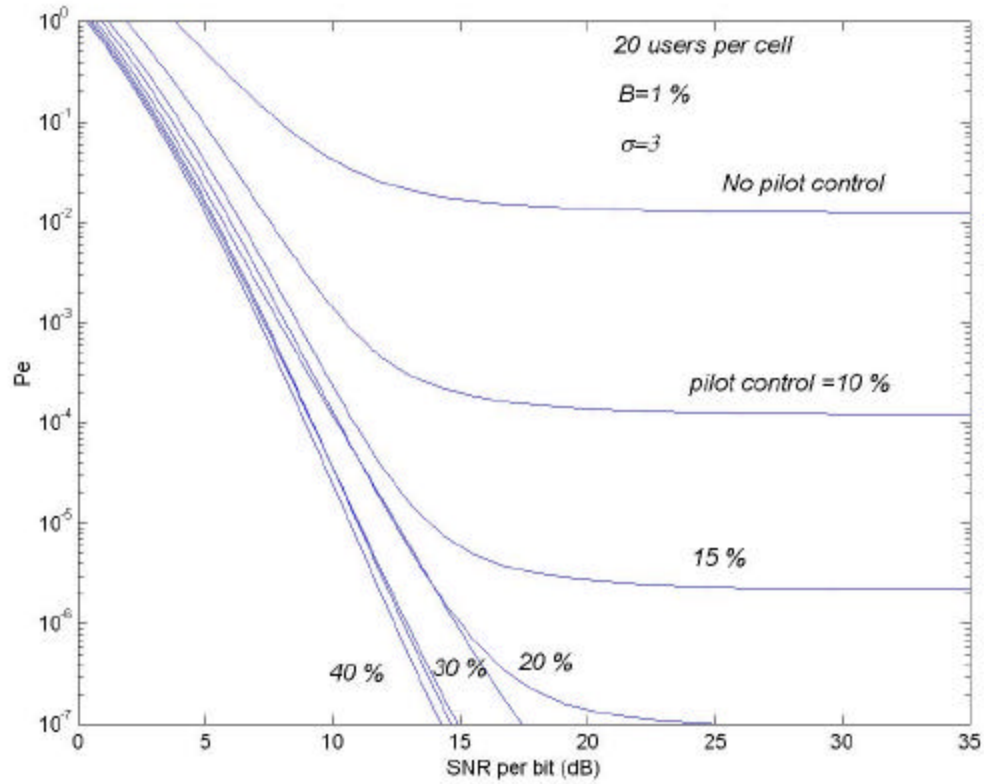


Figure 4.4. Comparison of Probability of Bit Error for DS-SS-SSM with Rayleigh-Lognormal ($\sigma_{dB} = 3$) Channel and FEC ($R_{cc}=1/2$ and $K=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

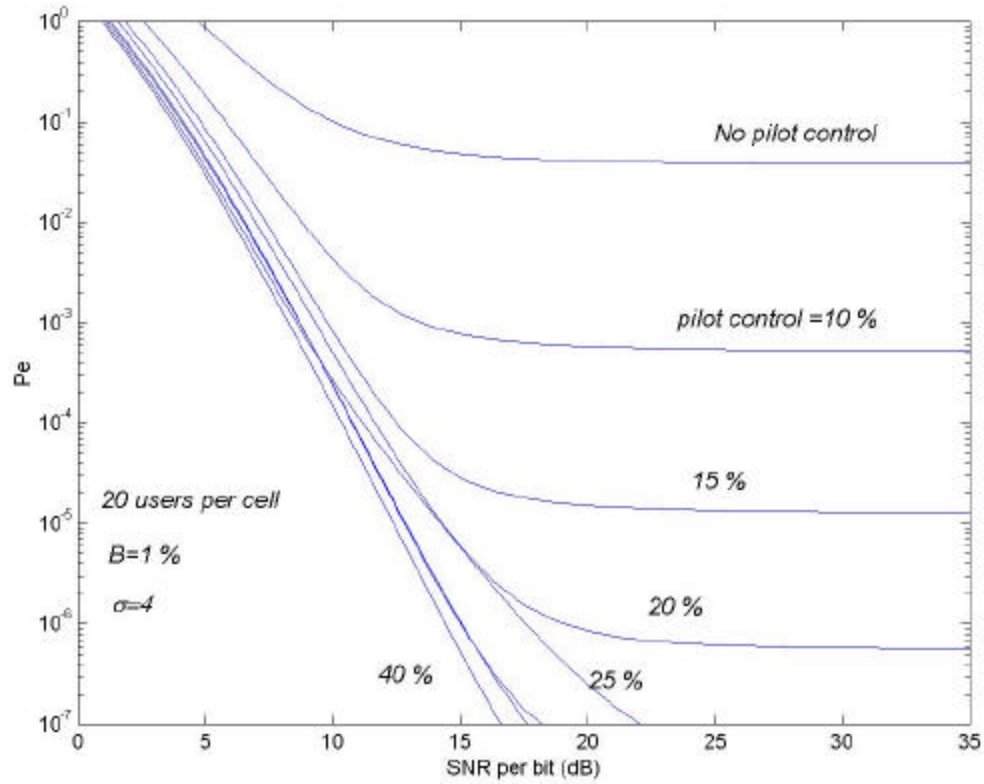


Figure 4.5. Comparison of Probability of Bit Error for DS-SS-SSMA with Rayleigh-Lognormal ($\sigma_{dB} = 4$) Channel and FEC ($R_{cc}=1/2$ and $\gamma=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

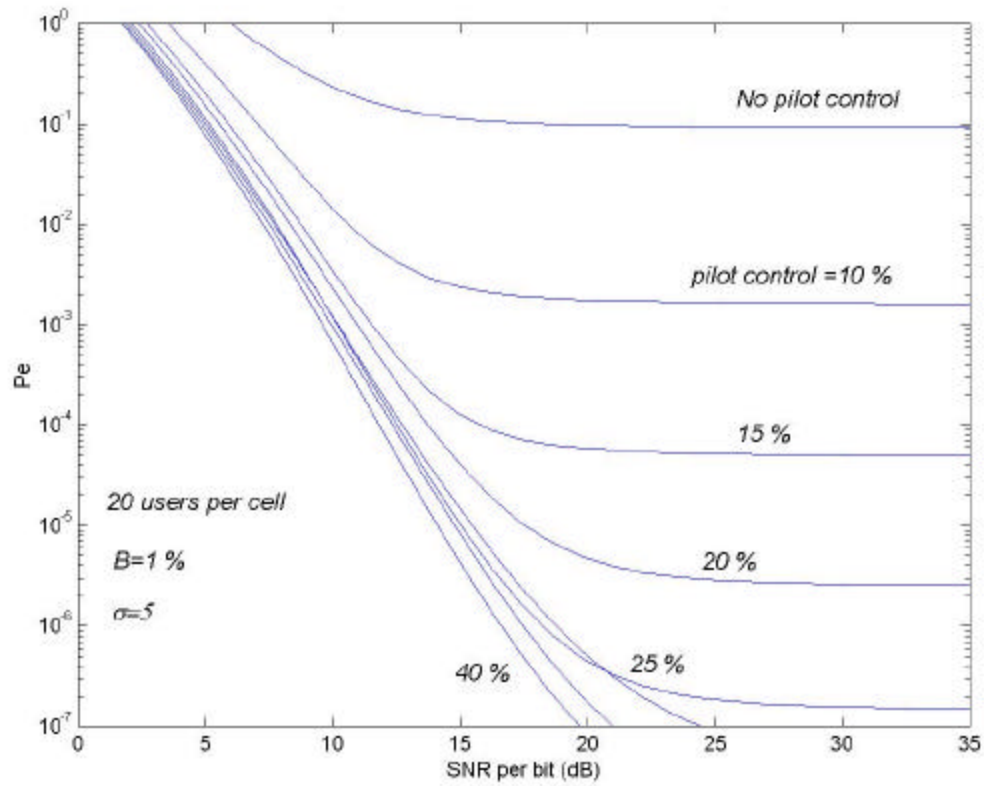


Figure 4.6. Comparison of Probability of Bit Error for DS_CDMA with Rayleigh-Lognormal ($s_{dB} = 5$) Channel and FEC ($R_{cc}=1/2$ and $\gamma=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

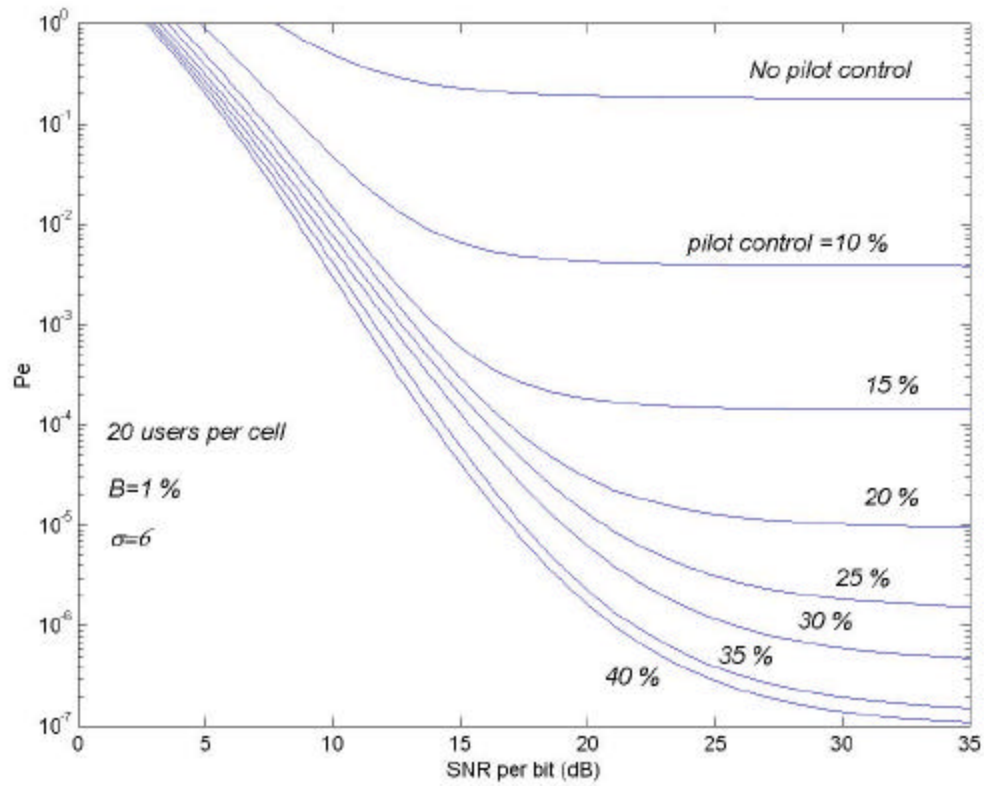


Figure 4.7. Comparison of Probability of Bit Error for DS-SS-SSM with Rayleigh-Lognormal ($s_{dB} = 6$) Channel and FEC ($R_{cc}=1/2$ and $\gamma=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

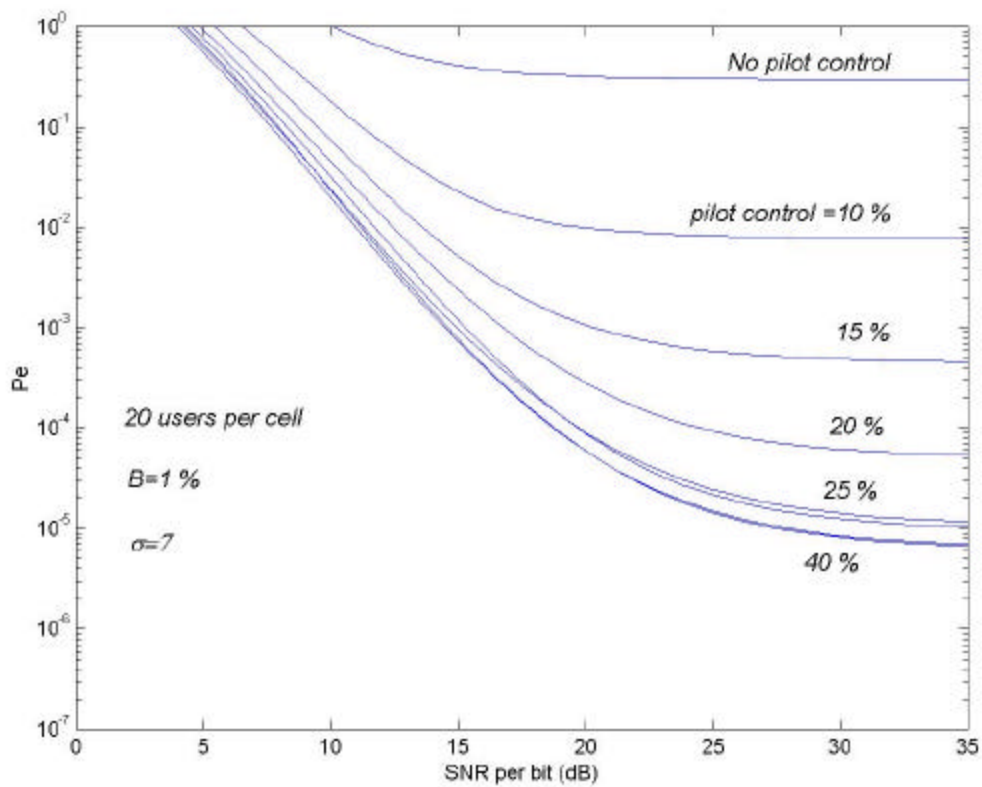


Figure 4.8. Comparison of Probability of Bit Error for DS-SS-SSMA with Rayleigh-Lognormal ($\sigma_{dB} = 7$) Channel and FEC ($R_{cc}=1/2$ and $K=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

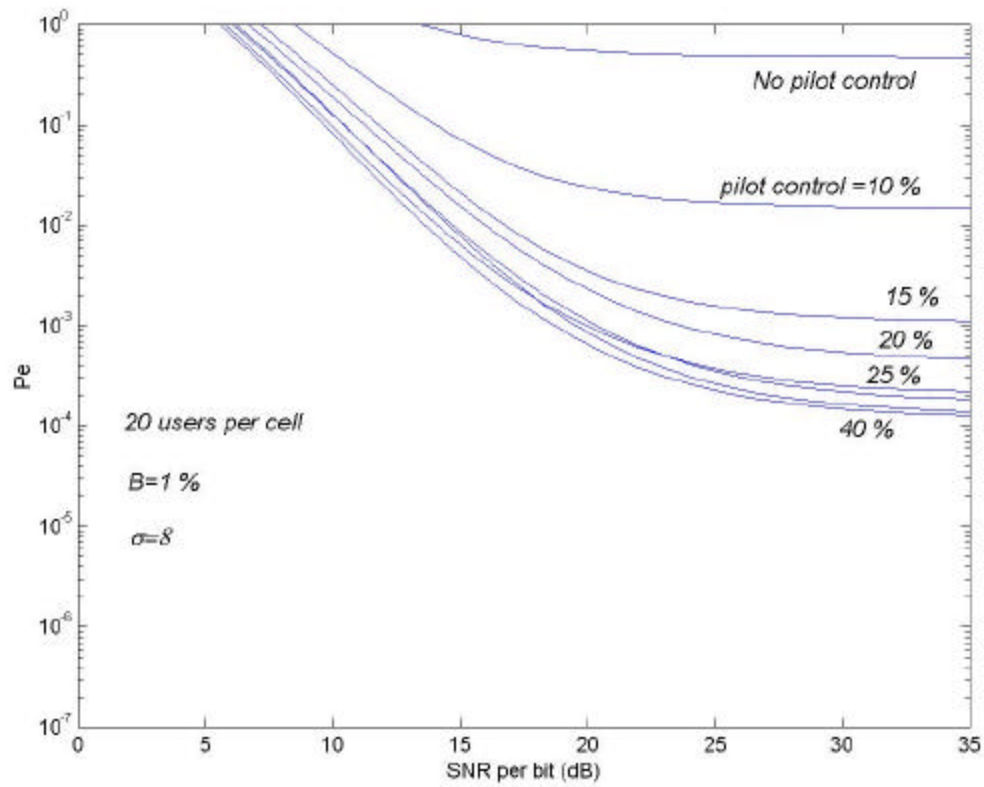


Figure 4.9. Comparison of Probability of Bit Error for DS-SS-SSMA with Rayleigh-Lognormal ($S_{dB} = 8$) Channel and FEC ($R_{cc}=1/2$ and $\sigma=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

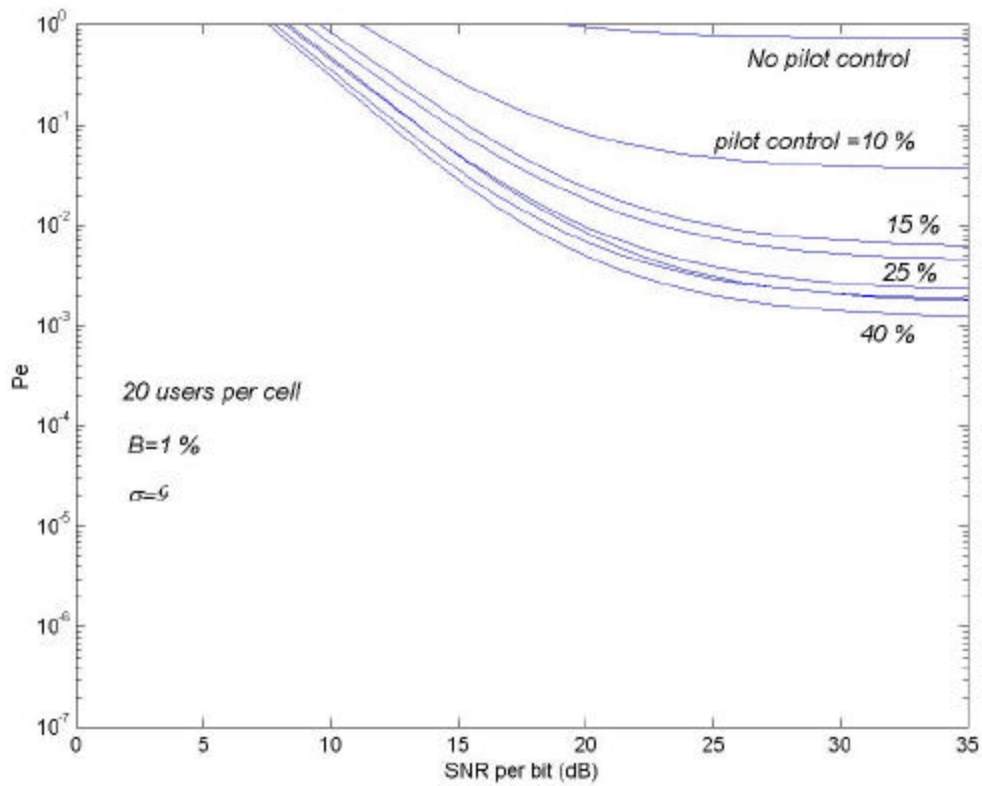


Figure 4.10. Comparison of Probability of Bit Error for DS-SS-SSM with Rayleigh-Lognormal ($S_{dB} = 9$) Channel and FEC ($R_{cc}=1/2$ and $\alpha=8$), assuming 1% Pilot Channel Interference, 20 Users/Cell and using 60° Sectoring.

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V. SINGLE CELL MODEL PERFORMANCE ANALYSIS

In Chapter III we analyzed the performance of the forward channel in a DS-CDMA cellular system for a large number of users, using a hexagonal seven-cell cluster model. However there are cases such as in academic, industrial or military environments where port-to-port communication between very small numbers of users is required. In this case, since the use of a complex seven-cell cluster is not necessary we reduce the number of cells to one only, establishing a type of an Intracell Network, while the advantages of DS-CDMA such as the high-speed connection, are preserved. In this section we are going to analyze the performance of the single-cell environment, adapting the analysis we've already done for the seven-cell cluster to the single cell case.

A. PROBABILITY OF BIT ERROR FOR SINGLE-CELL DS-CDMA

In a single-cell DS-CDMA system, as its name denotes, there is only one cell used, thus there is no co-channel interference from adjacent cells. Accordingly, the received signal $r(t)$ is going to contain the traffic intended for the mobile user, the interfering signals for the other users and the AWGN only. Accordingly, we can modify (2.13) as follows:

$$\begin{aligned} r(t) &= s_0(t) + n(t) \\ &= \sum_{k=0}^{K-1} R\sqrt{2P_k} b_k(t) w_k(t) c(t) \cos(2\pi f_c t) + n(t). \end{aligned} \quad (5.1)$$

The despread signal $y_1(t)$ at the information signal branch can be expressed as:

$$\begin{aligned} y_1(t) &= r(t) c(t) w_1(t) \\ &= \underbrace{s_0(t) c(t) w_1(t)}_{I_1(t) + g_1(t)} + \underbrace{n(t) c(t) w_1(t)}_{h_1(t)} \\ &= I_1(t) + g_1(t) + h_1(t), \end{aligned} \quad (5.2)$$

where I_1 contains the information signal, g_1 the intracell interference and h_1 the AWGN, following the procedure we analytically described in Chapter A of Section III.

The despread signal $p(t)$ in the pilot recovery branch can be expressed by

$$\begin{aligned}
p(t) &= r(t) c(t) w_0(t) \\
&= \underbrace{s_0(t)c(t)w_0(t)}_{I_0(t)+\mathbf{g}_0(t)} + \underbrace{n(t)c(t)w_0(t)}_{\mathbf{h}_0(t)} \\
&= I_0(t) + \mathbf{g}_0(t) + \mathbf{h}_0(t)
\end{aligned} \tag{5.3}$$

In the single cell case when small number of users is present, the amount of total interference is not expected to be so large compared with the seven-cell case. Therefore, we will first try to analyze the performance without using the narrow bandpass filter in the pilot recovery branch.

Accordingly, the demodulated signal $y_2(t)$ is going to be expressed by

$$\begin{aligned}
y_2(t) &= y_1(t) p(t) \\
&= I_1 I_0 + I_1 \mathbf{g}_0 + I_1 \mathbf{h}_0 + \mathbf{g}_1 I_0 + \mathbf{g}_1 \mathbf{g}_0 + \mathbf{g}_1 \mathbf{h}_0 + \mathbf{h}_1 I_0 + \mathbf{h}_1 \mathbf{g}_0 + \mathbf{h}_1 \mathbf{h}_0
\end{aligned} \tag{5.4}$$

As we observe, $y_2(t)$ is the same with the seven-cell case defined in (3.17), if the co-channel interference products are eliminated.

Similarly, the decision statistic Y can be defined as follows:

$$\begin{aligned}
Y|_{r,p_k} &= \int_0^T y_2(t) dt \Big|_{r,p_k} = \\
&= \underbrace{\int_0^T I_1 I_0}_{Y_1} \Big|_{r,p_k} + \underbrace{\int_0^T I_1 \mathbf{g}_0}_{\mathbf{g}_{11}} \Big|_{r,p_k} + \underbrace{\int_0^T I_1 \mathbf{h}_0}_{\mathbf{h}_{11}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{g}_1 I_0}_{\mathbf{g}_{12}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{g}_1 \mathbf{g}_0}_{\mathbf{g}_{13}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{g}_1 \mathbf{h}_0}_{\mathbf{h}_{12}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{h}_1 I_0}_{\mathbf{h}_{14}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{h}_1 \mathbf{g}_0}_{\mathbf{h}_{15}} \Big|_{r,p_k} + \underbrace{\int_0^T \mathbf{h}_1 \mathbf{h}_0}_{\mathbf{h}_{17}} \Big|_{r,p_k}
\end{aligned}$$

where the integrals comprising Y have been thoroughly analyzed in (3.35) through (3.50).

Moreover, we apply the same Forward Error Correction ($R_{cc}=1/2, v=8$) in order to improve the performance. Consequently the first event error probability from (3.94) can be adjusted as follows:

$$P_2(d) \Big|_{z_d, w_d} = Q \left(\sqrt{\frac{4z_d^2}{\frac{L_H^2(d)}{f_0 f_1 P_r^2 T_{cc}^2} \sum_{l=1}^d (\mathbf{s}^2_{h_{4,l}^c} + \mathbf{s}^2_{g_{14,l}} + \mathbf{s}^2_{h_{11,l}^c} + \mathbf{s}^2_{h_{12,l}^c} + \mathbf{s}^2_{h_{15,l}^c} + \mathbf{s}^2_{h_{17,l}^c})}} \right) \tag{5.5}$$

where the variances of the intercell interfering terms have been ignored, while the terms appearing in the denominator have been thoroughly calculated in Appendix III-A, B. Accordingly the first event error probability can be expressed by

$$P_2(d)|_{z_d, w_d} = Q \left(\sqrt{\frac{4z_d^2}{z_d \left\{ \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \left(\frac{f_1}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{k=1}^{K-1} \left(\frac{f_k}{f_0} \right) \right\} + w_d \sum_{k=2}^{K-1} \left(\frac{f_k}{f_0} \right) \left(\frac{f_{k \oplus 1}}{f_1} \right) + d \frac{3}{4} \left(\frac{f_1}{f_0} \right) \left(\frac{E_c}{N_0} \right)^{-2}} \right) \quad (5.6)$$

For simplicity we can set $a = \frac{4z_d^2}{a_1 + a_2}$. Therefore (5.6) can now be expressed as

$$P_2(d)|_a = Q(\sqrt{a}) \quad (5.7)$$

We can now remove the conditioning in (3.97) by integrating across the pdf $p_a(a)$ as follows:

$$\begin{aligned} P_2(d) &= \int_{-\infty}^{\infty} P_2(d)|_a p_a(a) da \\ &= \int_{-\infty}^{\infty} Q(\sqrt{a}) p_a(a) da \end{aligned} \quad (5.8)$$

Accordingly, we will simulate the first event error probability exactly as we did in Chapter III, using 10,000 Monte Carlo simulation trials, and consequently we'll find the upper power on the bit error probability P_e as follows:

$$P_e \leq \frac{1}{k} \sum_{d=d_{free}}^{d_{free}+4} \mathbf{b}_d P_2(d) \quad (5.9)$$

We should also note that in the simple single cell case there is no need to implement sectoring to the antennas, since we don't have any intercell interference.

In Figure 5.1 the probability of bit error versus the average SNR defined in (8.1), is represented for a Rayleigh-Lognormal environment with $s_{dB} = 3$. As we observe, the performance of the single cell system proved to be quite satisfactory for 2 users in the cell. However when we increase the number of users to 3, the performance deteriorates dramatically due to the introduced intracell interference.

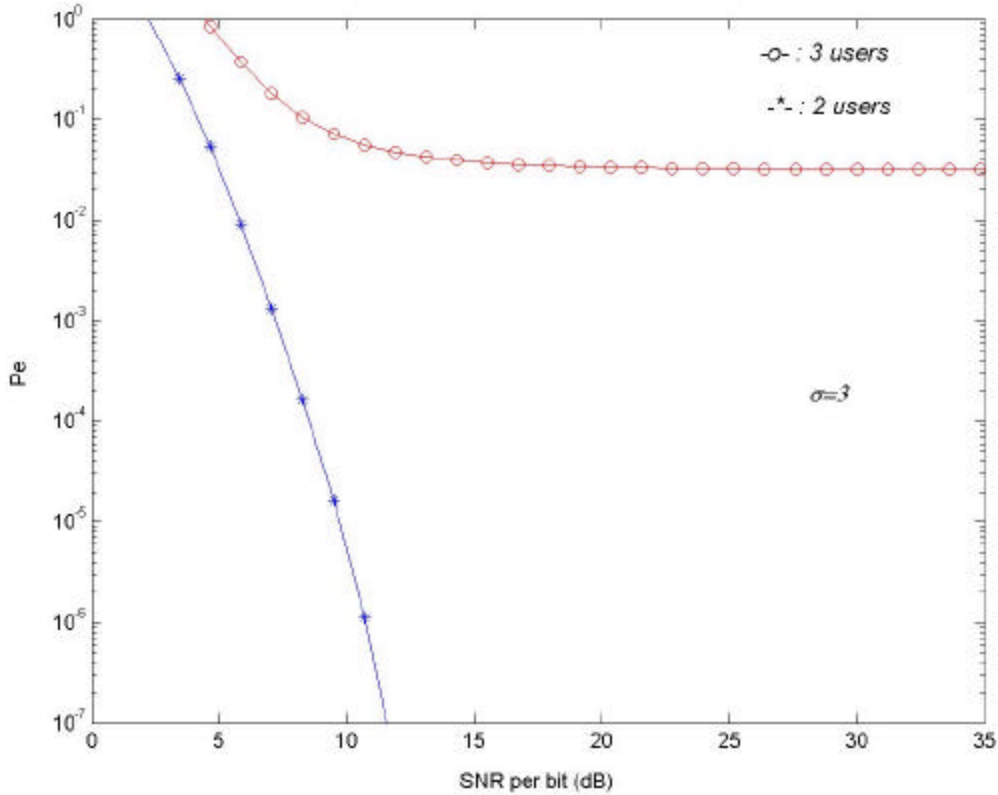


Figure 5.1. Probability of Bit Error for Single Cell DS-CDMA in a Rayleigh-Lognormal ($s_{dB} = 3$) Channel using FEC ($R_{cc}=1/2$ and $v=8$).

Figure 5.2 depicts performance results for various lognormal shadowing conditions with two users in the cell. As we see the probability of bit error is quite satisfactory for two users in the cell in almost all the channel conditions.

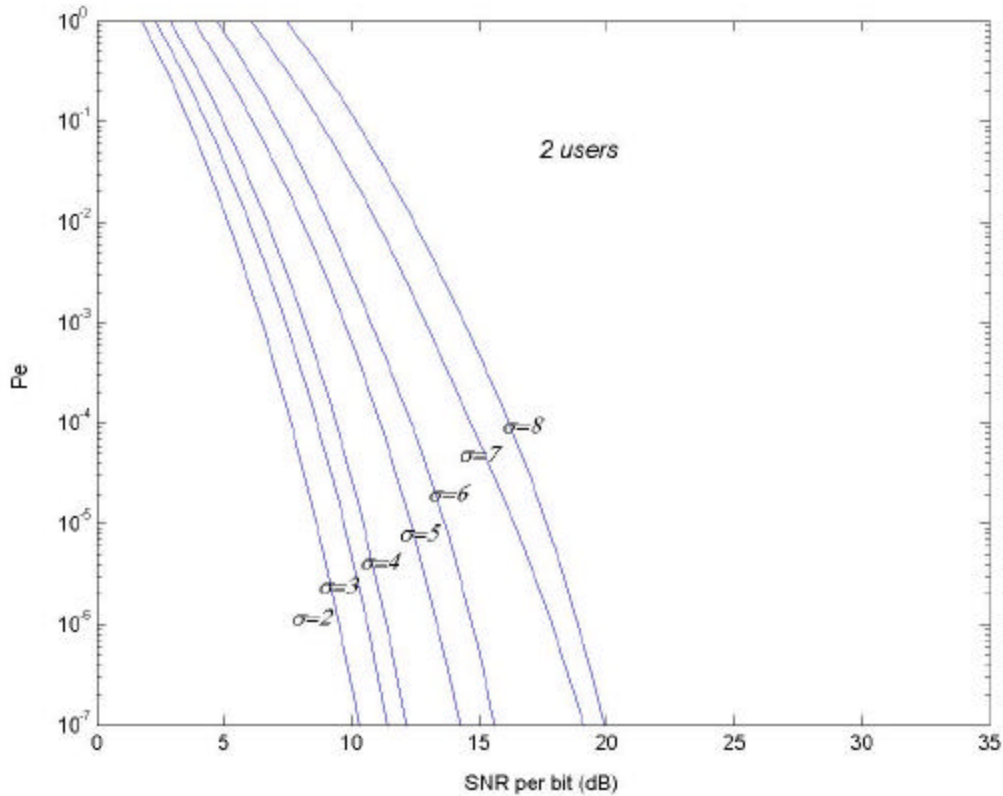


Figure 5.2. Probability of Bit Error for a Single-Cell DS-CDMA in a Rayleigh Fading Lognormal Shadowing Channel with 2 Users in the Cell, using FEC ($R_{cc}=1/2$ and $\nu=8$).

Accordingly we showed that communication between two users in a single cell DS-CDMA Rayleigh fading and Lognormal shadowing channel can be quite effective. However the performance of the system turned out to be quite poor when the number of the users in the cell was further increased. Consequently, in the next chapter we will try to increase the capacity of the single cell system cutting down the interference from the other users with a narrowband filter.

B. APPLYING FILTERING AT THE PILOT TONE ACQUISITION BRANCH

As we saw in Section B the performance of the single cell system turned out to be quite poor even for three users per cell. Therefore, in this chapter we will add a narrowband filter at the pilot tone acquisition branch to limit down the interference from the other users and the noise.

We will follow the analysis we did in Section G of Chapter III using the variable B to define the amount of interference passing through the filter, which is directly proportional to the bandwidth and the type of the filter. Consequently, we can adopt the first event error probability from (3.105), and ignore the contribution of the intercell interfering terms as follows:

$$\begin{aligned}
P_2(d) \Big|_{z_d, w_d} &= \\
&= Q \left(\sqrt{\frac{z_d^2}{\frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \mathbf{s}^2_{h_{i4,j}} + B \times \left\{ \frac{L_H^2(d)}{f_0 f_1 P_t^2 T_{cc}^2} \sum_{l=1}^d \left(\mathbf{s}^2_{g_{i3,l}^c} + \mathbf{s}^2_{g_{i5,l}^c} + \mathbf{s}^2_{h_{i1,l}^c} + \mathbf{s}^2_{h_{i2,l}^c} + \mathbf{s}^2_{h_{i5,l}^c} + \mathbf{s}^2_{h_{i7,l}^c} \right) \right\}}} \right) \\
&= Q \left(\sqrt{\frac{z_d^2}{z_d \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} + B \times \left\{ z_d \left[\frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \left(\frac{f_1}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{k=1}^{K-1} \left(\frac{f_k}{f_0} \right) \right] + \right.} \right) \\
&\quad \left. \left[+ w_d \sum_{k=2}^{K-1} \left(\frac{f_k}{f_0} \right) \left(\frac{f_{k \oplus 1}}{f_1} \right) + d \frac{3}{4} \left(\frac{f_1}{f_0} \right) \left(\frac{E_c}{N_0} \right)^{-2} + w_d \right]} \right) \tag{5.10}
\end{aligned}$$

For simplicity we can set

$$a = \frac{z_d^2}{\left(z_d \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} + B \times \left\{ z_d \left(\frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \left(\frac{f_1}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{\substack{k=0 \\ k \neq 1}}^{K-1} \left(\frac{f_k}{f_0} \right) + \frac{1}{2} \left(\frac{E_c}{N_0} \right)^{-1} \sum_{k=1}^{K-1} \left(\frac{f_k}{f_0} \right) \right) + \right.} \\ \left. + w_d \sum_{k=2}^{K-1} \left(\frac{f_k}{f_0} \right) \left(\frac{f_{k \oplus 1}}{f_1} \right) + d \frac{3}{4} \left(\frac{f_1}{f_0} \right) \left(\frac{E_c}{N_0} \right)^{-2} + w_d \right\} \right) \quad (5.11)$$

Therefore (5.10) can now be expressed as

$$P_2(d)|_a = Q(\sqrt{a}) \quad (5.12)$$

We can now remove the conditioning in (5.12) by integrating across the pdf $p_a(a)$ as follows:

$$\begin{aligned} P_2(d) &= \int_{-\infty}^{\infty} P_2(d)|_a p_a(a) da \\ &= \int_{-\infty}^{\infty} Q(\sqrt{a}) p_a(a) da \end{aligned} \quad (5.13)$$

Accordingly, we'll simulate the first event error probability exactly as we did in Chapter III, using 10,000 Monte Carlo simulation trials, and consequently we'll find the upper power on the bit error probability P_e as follows:

$$P_e \leq \frac{1}{k} \sum_{d=d_{free}}^{d_{free}+4} \mathbf{b}_d P_2(d)$$

The performance results of the system using the filter turned out to be quite good. As depicted in Figure 5.3, the probability of bit error decreases dramatically as the interference diminishes, allowing the system to operate effectively at worse channel conditions and greater capacities.

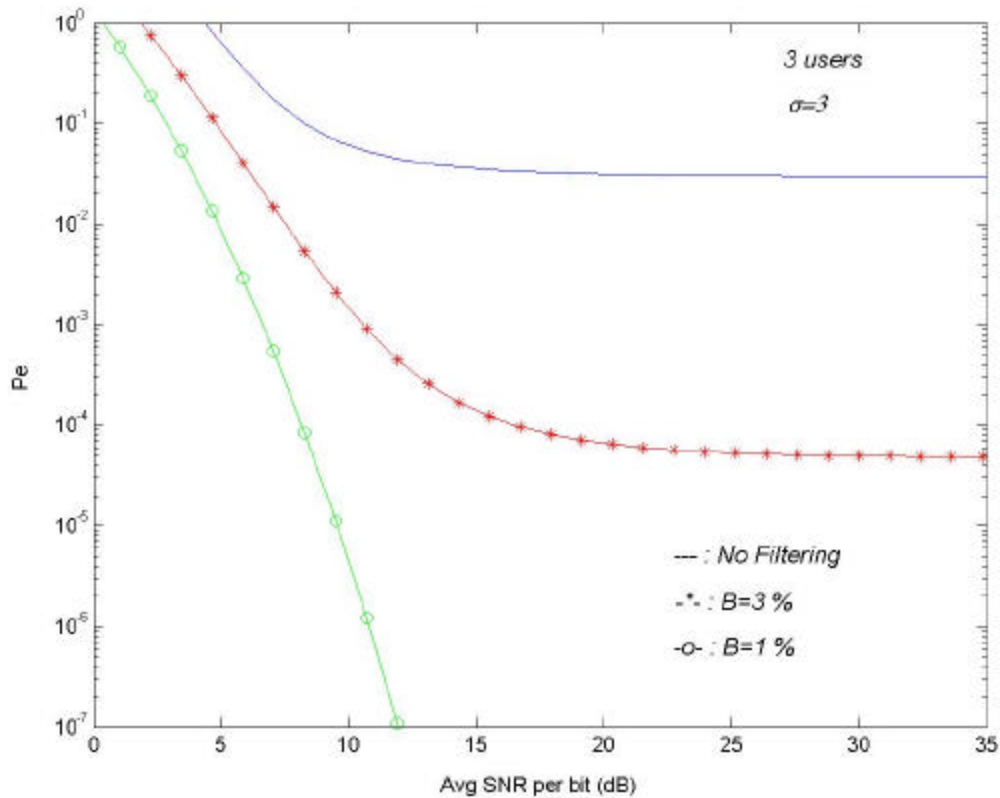


Figure 5.3. Comparison of Probability of Bit Error for a Single-Cell DS-CDMA in a Rayleigh Fading Lognormal Shadowing Channel with 3 Users in the Cell, using FEC ($R_{cc}=1/2$ and $\nu=8$).

Therefore, we use a narrower filter and check the performance of the system in all the channel conditions for a larger number of users. As we see in Figure 5.4, the system can now operate with an acceptable performance in an environment with 5 users per cell limiting the amount of interfering power at 1%.

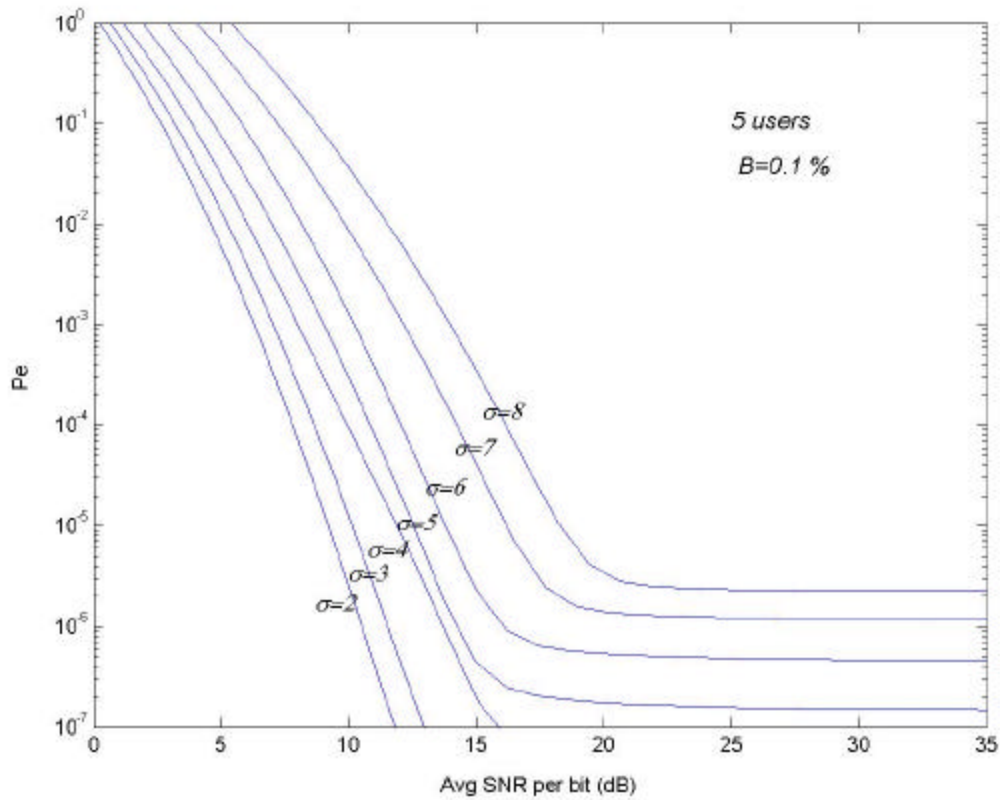


Figure 5.4. Probability of Bit Error for a Single-Cell DS-CDMA in a Rayleigh Fading Lognormal Shadowing Channel with 5 Users in the Cell, using FEC ($R_{cc}=1/2$ and $\nu=8$) and Pilot Tone Filtering.

Accordingly, we've shown that the communication between small numbers of users in a single cell environment is effective, and that the use of narrowband filtering at the pilot tone can increase the performance and the capacity of the system.

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VI. CONCLUSIONS AND FUTURE WORK

In this thesis we analyzed the performance of the forward channel of a DS CDMA cellular system operating in a Rayleigh-fading, Lognormal-shadowing environment. We optimized the performance using various techniques, such as pilot tone filtering, sectoring, convolutional encoding and pilot channel power control. Finally, we presented a simple case of DS-CDMA system operating in only one cell in the form of port-to-port communication between small numbers of users.

A. CONCLUSIONS

In Chapter II we set up a forward channel for the DS-CDMA cellular system. We also built an information signal and we propagated it through the medium channel, applying all the appropriate losses, effects and interferences. We used the extended Hata model to predict the large-scale path loss and we further incorporated lognormal shadowing. Moreover, we used Rayleigh fading to include small-scale propagation effects. Finally we formed the total received signal by the examined user, including the intracell and intercell interference, as well as the Additive White Gaussian Noise (AWGN).

In Chapter III we set the mobile user in a position in the center cell of the seven-cell cluster assuming the worst-case scenario. We demodulated the received signal and we developed a Signal to Noise plus Interference Ratio (SNIR), taking into account all the interfering terms. We then incorporated Forward Error Correction (FEC) and developed a tight upper bound on the bit error probability for the coded system. We simulated the probability of bit error using Monte Carlo simulation method and we compared the performance results with previous work done. The resulted performance was found quite poor, even for a small number of users due to the large amount of interference imported from the pilot recovery branch. Therefore, we applied a narrowband filter in order to limit down the power of the interference terms and we revised the already developed probability of error. Finally we further reduced the intercell interference by adding antenna sectoring. The performance we achieved was quite acceptable. However, whenever we increased the amount of interference passing through

the filter, (in other words, the bandwidth of the filter), applied “heavy” shadowing conditions, or augmented the number of users per cell, the performance of the system diminished, much below the acceptable standards.

In Chapter IV we further optimized the performance of the system by introducing power control to the pilot tone channel. We derived a relation between the power allocated to the pilot channel and the other channels and we simulated again the probability of bit error. The performance of the system was greatly improved using pilot tone power control. However in heavy conditions or when we use increased the bandwidth of the filter, 20 or even 30 % of the total power needed to be allocated at the pilot channel for optimum results. Finally a comparison the resulted probability of bit error with previous and related work done is done.

In Chapter V, we presented a simple case of a single cell environment, where a port-to-port communication between two or three users is required. We adopted the already developed probability of bit error for the seven-cell cluster, revising it to a much simpler form where intercell interference is eliminated. Moreover, the use of antenna sectoring or narrowband pilot tone filtering was not required. We developed the probability of bit error and we simulated it using Monte Carlo simulation method. The performance of the system turned out to be acceptable for two users in the cell and “light” shadowing conditions. Further improvement in the performance was achieved by using a bandpass filter at the pilot tone branch. The capacity of the cell increased to five users for an average 1% of interference passing through the filter.

B. FUTURE WORK

The analysis we followed could be easily adapted for a lot of research in the DS-CDMA cellular systems. For example, performance analysis in a Nakagami or a Ricean instead of a Rayleigh fading channel could be done, using the same procedure and the same probability of error that we derived.

Moreover, in Chapter V we implemented pilot tone power control to enhance the performance. Fast power control could be applied instead, and a comparison of the performance could be made.

Furthermore, in our analysis we placed the receiving mobile user at the edge of the hexagonal cell examining the worst-case scenario. A probability of error based on different user distribution could also be derived.

Finally, a not so practical performance analysis choosing a particular type of filter at the pilot tone branch could be done in Chapter IV, where the resulted probability of error would be directly proportional to the bandwidth of the filter.

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