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14. ABSTRACT

The asymptotic behavior of solutions in solid mechanics is a broad topic of considerable mathematical and technological interest. Safe efficient operation of aircraft structures and components requires accurate assessment of the rate of diffusion of end effects, particularly for anisotropic and composite materials. This requires study of the spatial decay of solutions of elliptic partial differential equations (or systems of equations). In this research, we have investigated a sequence of boundary-value problems for second-order and fourth-order elliptic partial differential equations. Both linear and nonlinear, isotropic and anisotropic problems have been considered. The results of such investigations have widespread impact on the AFOSR mission. In particular, rigorously obtained asymptotic estimates for the rate of load diffusion in solids are immediately applicable in engineering analysis and design and have been used, for example, by the Boeing Commercial Airplane Group in application to composite structures.

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Submitted to:
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LOAD DIFFUSION IN LINEAR AND NONLINEAR SOLID MECHANICS

The asymptotic behavior of solutions in solid mechanics is a broad topic of considerable mathematical and technological interest. Safe efficient operation of aircraft structures and components requires accurate assessment of the rate of diffusion of end effects, particularly for anisotropic and composite materials. This requires study of the spatial decay of solutions of elliptic partial differential equations (or systems of equations). In this research, we have investigated a sequence of boundary-value problems for second-order and fourth-order elliptic partial differential equations. Both linear and nonlinear, isotropic and anisotropic problems have been considered. The results of such investigations have widespread impact on the AFOSR mission. In particular, rigorously obtained asymptotic estimates for the rate of load diffusion in solids are immediately applicable in engineering analysis and design and have been used, for example, by the Boeing Commercial Airplane Group in application to composite structures.

1. SUMMARY OF RESEARCH RESULTS:

This work was concerned with research on the fundamental mechanics and mathematics of load diffusion in solids and structures relevant to the US Air Force. Thin-walled structures such as aircraft fuselages, rocket casings, helicopter blades and containment vessels are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. The extent to which local stresses, such as those produced by fasteners and at joints, can penetrate girders, beams, plates and shells must be understood by the designer. Thus a distinction must be made between global structural elements (where Strength of Materials or other approximate theories may be used) and local elements which require more detailed (and more costly) analyses based on exact elasticity. The neglect of end effects is usually justified by appeals to some form of *Saint-Venant's principle* and years of experience with *homogeneous isotropic* elastic structures has served to establish this standard procedure. Saint-Venant's principle also is the fundamental basis for static mechanical tests of material properties. Thus property measurements are made in a suitable gage section where uniform stress and strain states are induced and local effects due to clamping of the specimen are neglected on invoking Saint-Venant's principle. Such traditional applications of Saint-Venant's principle require major modifications when strongly anisotropic, or inhomogeneous or composite materials are of concern. For such materials, local stress effects can persist over distances *far greater* than is typical for isotropic metals.

Explicit analytic results of the type developed in this research are crucial to the complete analysis of local or end effects in anisotropic or composite materials and structures. Previous work has shown that such end effects decay *much more slowly* than in isotropic materials. For transversely-isotropic (or specially orthotropic) materials, our earlier work has led to specific design formulas for the distance beyond which Saint-Venant edge effects can be neglected. The results, which have important implications for the experimental techniques used to measure material properties, have led to modifications of the ASTM standard test and are now quoted routinely in text- and handbooks on mechanics of composite materials. The results have been used in a technology application by the Boeing Commercial Airplane Company (See 4 B below). Our recent research deals with more complicated degrees of anisotropy, *functionally-graded materials* (FGMs) and with effects of nonlinearity. We have recently initiated a new research thrust on solution of boundary-value problems in nonlinear elasticity using constitutive models which model material behavior at large strain. These models take into account that rubber-like materials, because of limiting chain inextensibility at the molecular level, cannot sustain unlimited straining. A singularity develops in the stress response at this limiting state. *The classical models of hyperelasticity are unable to account for this large strain behavior.* One of the more tractable new models is due to A.N. Gent (1996), of the University of Akron and Goodyear Company. Recently, we have obtained explicit analytic solutions to a variety of problems for this class of materials, including an analysis of the anti-plane shear (Mode III) crack. Analytic results of the type obtained here *are crucial complements to large-scale computational analyses*. Such results are extremely valuable in building an intuitive base for developing refined material modeling strategies and assessing results from finite element analyses. We have initiated interaction with Dr. Gent at Goodyear on development of analytical methods for these new constitutive models. Potential applications of vital importance to the US Air Force are numerous, in particular to the important issue of fracture of rubber leading to tire degradation.

In [1], we have written a comprehensive invited review book Chapter on the practical implications of our results on end effects in anisotropic and composite structures. The Chapter was specifically written for practicing engineers in the aerospace and aircraft industries where such issues are of crucial importance. In particular, we have had extensive interaction with the Boeing Commercial Airplane Company on the implications of our work to the testing and design of composite sandwich panels.(See further detail in 4B below). As this report is being written, the NTSB is investigating the crash of an American Airlines Airbus A300 on November 12, 2001 and preliminary findings suggest a problem with the composite tail fin, manufactured in France. Over the last several years, the P. I. has pioneered the rigorous mathematical study of stress diffusion in composites and their practical implications and has made a specific effort to communicate the results to the Air Force and aeronautics community at large. *The P. I. is convinced that much more basic research of this type needs to be done in order to assess properly the predictions of large scale finite element codes in structural analysis of composites.*

In [2-4], the purpose is to further investigate the effects of material inhomogeneity on the decay of Saint-Venant end effects. Saint-Venant decay rates for self-equilibrated edge loads are examined in the context of anti-plane shear for linear inhomogeneous isotropic elasticity [2, 3]. The problem is governed by a second-order, linear, elliptic partial differential equation with variable coefficients. In linear elasticity, Saint-Venant's principle is used to show that self-equilibrated loads generate local stress effects that quickly decay away from the loaded end of a structure. For homogeneous isotropic linearly elastic materials this is well-documented. Self-equilibrated loads are a class of load distributions that are statically equivalent to zero, i.e., have zero resultant force and moment. When Saint-Venant's principle is valid, pointwise boundary conditions can be replaced by more tractable resultant conditions. It is shown in [2, 3] that material inhomogeneity , just as is the case with material anisotropy, significantly affects the practical application of Saint-Venant's principle to an important class of new materials, namely, *functionally graded materials* (FGMs). Such materials are now used in a variety of technological applications e.g. to provide superior oxidation and thermal shock resistances. Thermal residual stresses can be relaxed in a metal-ceramic layered material by inserting a functionally graded interface layer between the metal and the ceramic. FGMs are thus finding major application as thermal barrier coatings. It is shown in [2, 3] that the inhomogeneity can *enhance or inhibit load diffusion in such materials.* Transient heat conduction for three-dimensional FGMs is investigated in [4]. The results in [2-4] provide analytical guidelines for material tailoring and have clear AFOSR relevance e.g., to aircraft engine and wing design.

Other problems for FGMs have been investigated in [5-8]. In [5], the torsion problem for functionally graded elastic bars has been considered. It is shown that, in contrast to the classical isotropic case, the maximum shear stress can occur in the interior of the bar. Implications for material failure are immediate. Vibrations of inhomogeneous strings, rods and membranes are considered in [6]. Some closed -form exact solutions are developed in [6], which are useful as benchmark problems to assess the accuracy of numerical schemes. It is also shown that an integral-equation based technique is very accurate in obtaining lower bounds for the fundamental frequency. In [7], the pressurized hollow cylinder or disk problem is considered for a class of functionally graded materials that are inhomogeneous in the radial direction. It is shown that the stress response is radically different from the classic homogeneous isotropic case. In particular, for internally pressurized tubes, the location of the maximum hoop stress *is no longer at the inner boundary but can be in the interior or at the outer boundary !* The implications for material failure in military components, e.g. gun barrels, aircraft components, are significant. The stress response of solid rotating disks composed of radially graded materials is examined in [8]. Again, the results are radically different from those in the homogeneous isotropic case. The maximum radial and hoop stresses are no longer at the center. The results have potential application to flywheel components in the EM-gun and EM Integrated Launch Package under development at ARL, Aberdeen Proving Ground, Maryland.

As described earlier, a comprehensive invited review book chapter [1] on the research on Saint-Venant end effects in anisotropic and composite structures has been published . This chapter appears in Volume 5 of the 6 volume book series entitled " Comprehensive Composite Materials", which reviews the developments in composites technology in the century just past and sets the stage for the new millennium. This Chapter summarizes our work over several years, partially supported by AFOSR. In [14], an invited book chapter on exact solutions for boundary-value problems in compressible nonlinear elasticity has been published. The absence of the usual kinematic assumption of incompressibility (zero volume change) makes such solutions very difficult to obtain. The results are applicable to foam-rubber type materials as well as to biological materials.

The new thrust area in nonlinear elasticity concerned with hyperelastic materials modeling limiting chain extensibility at the molecular level has led to several publications [9-13] and to interaction with the Goodyear Company. The problems solved in [9, 10, 11] involve torsion , axial shear and azimuthal (or circular) shear, respectively. The stress response is shown to be radically different from that which occurs using classical hyperelastic models. The stresses induced in rotating rubber-like cylinders is considered in [12] while [13] is concerned with anti-plane shear deformations . In particular, the anti-plane shear (Mode III) crack is investigated in [13] where it is shown that , due to the limiting chain extensibility feature of the model, the stresses, though large, *are bounded at the crack tips*, in contrast to the predictions of linear elasticity and some nonlinear elasticity models. Since the standard computer codes e.g. ABACUS employ mostly classic hyperelastic models only, the current practice in stress analysis of rubber components *using large scale computation needs immediate reassessment.* The recent instances of catastrophic tire degradation e.g. Concorde crash and Firestone tire problems, point out the urgent need for such re-examination.

2. PERSONNEL SUPPORTED:

P.I.: C. O. Horgan, Wills Johnson Professor

Graduate Students (Financially supported by non-AFOSR sources):

Alice M. Chan (MS May 1999) : Ms. Chan (US citizen) is now an aerospace engineer with British Aerospace Engineering, San Diego, CA

Dom Galic (MS May 2002, PhD 2004)
Mr. Galic (US citizen) has a BS from U. C. Berkeley and is making rapid progress in research.

3. PUBLICATIONS:

(List of papers supported by AFOSR.)

1. Saint-Venant end effects for anisotropic materials (C. O. Horgan and L. A. Carlsson), *Comprehensive Composite Materials*, Vol.5, (ed. by A. Kelly and C. Zweben), Elsevier Publishers, 2000, pp. 5-21.
2. End effects in anti-plane shear for an inhomogeneous isotropic linearly elastic semi-infinite strip (A. M. Chan and C. O. Horgan), *J. of Elasticity*, 51, 1998, 227-242.

3. Saint-Venant end effects in anti-plane shear for functionally graded linearly elastic materials (C. O. Horgan and R. Quintanilla), *Mathematics and Mechanics of Solids* , 6, 2001, 115-132.

4. Spatial decay of transient end effects in functionally graded heat conducting materials (C. O. Horgan and R. Quintanilla), *Quarterly of Applied Mathematics* , 59, 2001, 529-542.

5. Torsion of functionally graded isotropic linearly elastic bars (C. O. Horgan and A. M. Chan), *J. of Elasticity*, 52, 1999, 181-199.

6. Vibrations of inhomogeneous strings, rods and membranes (C. O. Horgan and A. M. Chan), *J. of Sound and Vibration*, 225, 1999, 503-513.

7. The pressurized hollow cylinder or disk problem for functionally graded isotropic linearly elastic materials (C. O. Horgan and A. M. Chan), *J. of Elasticity* 55, 1999, 43-59.

8. The stress response of functionally graded isotropic linearly elastic rotating disks (C. O. Horgan and A. M. Chan), *J. of Elasticity* 55, 1999, 219-230.

9. Simple torsion of isotropic hyperelastic incompressible materials with limiting chain extensibility (C. O.Horgan and G. Saccomandi), *J. of Elasticity* 56, 1999, 159-170.

10. Pure axial shear of isotropic incompressible nonlinearly elastic materials with limiting chain extensibility (C. O. Horgan and G. Saccomandi), *J. of Elasticity* 57, 1999, 305-319.

11. Pure azimuthal shear of isotropic, hyperelastic incompressible materials with limiting chain extensibility (C. O. Horgan and G. Saccomandi), *International J. of Nonlinear Mechanics* 36, 2001, 465-476.

12. Large deformations of a rotating solid cylinder for non-Gaussian isotropic, incompressible hyperelastic materials (C. O. Horgan and G. Saccomandi), *J. of Applied Mechanics* 68, 2001, 115-117.

13. Anti-plane shear deformations for non-Gaussian isotropic, incompressible hyperelastic materials (C. O. Horgan and G. Saccomandi), *Proceedings of the Royal Society of London*, Series A, 457, 2001, 1999-2017.

14. Equilibrium solutions for compressible nonlinearly elastic materials (C. O. Horgan). In *Nonlinear Elasticity: Theory and Problems* (ed. by Y. B. Fu and R. W. Ogden), Cambridge University Press, 2001, pp. 135-159.

4. INTERACTIONS/TRANSITIONS:

A. Participation/Presentations at Meetings, Seminars, Etc.:

"Effects of curvilinear anisotropy on radially symmetric stresses in anisotropic linearly elastic solids" (with Sarah C. Baxter). SECTAM XIX, Nineteenth Annual Meeting of the Southeastern Conference on Theoretical and Applied Mechanics, Florida Atlantic University, May 1998.

"Saint-Venant end effects for sandwich structures". Invited lecture at the 4th International Conference on Sandwich Construction, Royal Institute of Technology, Stockholm, Sweden, June 1998.

“Structural integrity of sandwich panels: edge effects.” 15th US Army Symposium on Solid Mechanics, Myrtle Beach, S.C., April 1999.

"Anisotropy induced singularities in linear elasticity". Keynote lecture at Symposium in honor of J.L. Nowinski, University of Delaware, May 1999.

"Anisotropy induced singularities in linear elasticity". Invited lecture at Special Session on “Recent Developments in Anisotropic Elasticity”, 1999 ASME Mechanics and Materials Conference, Virginia Tech., Blacksburg, VA, June 1999.

"Anisotropy induced singularities in linear elasticity". Invited lecture at mini-symposium on " Mathematical Methods in Solid Mechanics", 4th International Congress on Industrial and Applied Mathematics, ICIAM 4, Edinburgh, Scotland, July 1999.

“ Recent mathematical developments for functionally graded linearly elastic materials”. Keynote lecture at IMSE 2000 (The Sixth International Conference on “ Integral Methods in Science and Engineering”), Banff, Alberta, Canada, June 2000.

“Extremal stresses in some boundary-value problems for functionally graded linearly elastic materials”. Invited lecture at mini-symposium at STAMM 2000, “ Symposium on Trends in

Applications of Mathematics to Mechanics", ISIMM, National University of Ireland, Galway, July 2000.

" Boundary-value problems for functionally graded linearly elastic materials". Invited lecture at pre-nominated Lecture Session on " Functionally Graded Materials", ICTAM 2000, 20th International Congress on Theoretical and Applied Mechanics (IUTAM), Chicago, August 2000.

" Boundary-value problems for functionally graded linearly elastic materials". Invited lecture at mini-symposium on " Current Developments in Elasticity", 37th Annual Meeting of the Society of Engineering Science, University of South Carolina, Columbia, SC, Oct. 2000.

"Boundary-value problems for functionally graded materials ". Presented at US Army Research Laboratory, Weapons and Materials Research Directorate, Aberdeen Proving Ground, Maryland, August 2000.

Invited colloquium lectures at the University of California, Berkeley (March 2001), the University of Ferrara, Italy (May 2001) and the University of Lecce, Italy (June 2001).

B. Consultative and Advisory Functions:

The work on end effects in anisotropic and composite structures is being used by the Boeing Commercial Airplane Group, Seattle Washington in the Boeing (NASA) Advanced Technology Composite Aircraft Structures Program (ATCAS). Main Contact : Dr. W. B. Avery. The P. I. has made several visits to Boeing to further this technology transfer. (See the attached letters from Boeing on the relevance of Horgan's work).

Extensive contact with Prof. B.A. Szabo at Washington University, St. Louis on relevance of Horgan's work to finite element studies performed by Szabo's group under AFOSR support.

Dr. Horgan has continued to interact with Dr. Nicholas Pagano, Wright Patterson AFB, concerning relevance of Horgan's new work on functionally graded materials (FGMs) to Air Force problems. One of Pagano's colleagues, Dr. V. A. Buryachenko, consulted with Horgan on August 24, 2000 regarding Horgan's invited lecture on FGMs at ICTAM 2000, The 20th International Congress on Theoretical and Applied Mechanics, Chicago, September 1, 2000.

Extensive contact with Dr. A. N. Gent of Goodyear, Akron, Ohio concerning Horgan's recent analytical work on a constitutive model proposed by Gent (1996) which accounts for limiting chain extensibility at the molecular level. Such a model captures the strain hardening observed in rubber undergoing large strain, in contrast to classical hyperelastic models. The work has potential relevance to tire degradation problems of crucial Air Force concern. (See the attached letter from Dr. Gent (June , 2000)).

Dr. Horgan visited the Army Research Laboratory, Aberdeen Proving Ground, Maryland on August 21, 2000. He presented a seminar entitled " Boundary –value problems for functionally graded materials " to personnel at the Weapons and Materials Research Directorate. Extensive discussions were carried out on the relevance of Horgan's work to design of flywheel components in the Composite Rotor for the EM-gun and the EM Integrated Launch Package under development at ARL. The potential application of functionally graded materials to the design of armature/sabot gun components were also discussed. Main contacts: Dr. Jerome Tzeng, Dr. Michael Scheidler, Dr. Timothy Wright.

5. INVENTIONS/PATENTS:
None

6. HONORS/AWARDS:

C. O. Horgan has held the title of Wills Johnson Professor of Applied Mathematics & Mechanics, University of Virginia since July 1, 1994. In May 2000, he was awarded the Civil Engineering Teaching Award for the 1999-2000 academic year.

Dr. Horgan is a Fellow of the American Society of Mechanical Engineers (ASME) and the American Academy of Mechanics. He is the author or co-author of 7 book chapters, over 125 refereed archival journal articles and numerous Conference proceedings publications on Applied Mathematics and Mechanics.

Dr. Horgan serves on the Editorial Board of the following journals: *International Journal of Nonlinear Mechanics, J. of Elasticity, SIAM Journal on Applied Mathematics*. He is a founding editor of the journal *Mathematics and Mechanics of Solids*. He served as Associate Editor for *Applied Mechanics Reviews* (1985-2000).

Dr. Horgan was an invited keynote speaker at the Sixth International Conference on "Integral Methods in Science and Engineering", IMSE2000, Banff, Alberta, Canada, June 2000.

Dr. Horgan presented an invited lecture at the Pre-nominated Lecture Session on "Functionally Graded Materials" at ICTAM 2000, The 20th International Congress on Theoretical and Applied Mechanics, Chicago, August 2000.

Prof. Horgan was a visiting Professor at the University of Ferrara and the University of Lecce, Italy during May/June 2001, engaged in collaborative research and presenting invited lectures.

The Boeing Company
P.O. Box 3707
Seattle. WA 98124-2207

December 20 1993
BY84B-JTO-M93-122

Professor Cornelius Horgan
Department of Applied Mathematics
University of Virginia
Thornton Hall
Charlottesville, VA 22903-2442

BOEING

Dear Professor Horgan,

I am writing to you in response to your letter of November 12, 1993 to Scott Finn. Thank you for sending copies of your papers on St. Venant effects. Mike Nemeth is correct in his perception that Boeing is interested in St. Venant effects in composite structures. In fact, I independently searched the literature for papers on St. Venant effects and came up with many authored by you and your colleagues. I would like to take this opportunity to give you a brief background on our project and then describe some of the technical issues that I think might interest you.

Boeing is currently funded under NASA's Advanced Composites Technology (ACT) program to develop the materials, structures, and manufacturing technology necessary to build a fuselage section for a widebody commercial transport. Boeing calls its program the Advanced Technology Composite Aircraft Structure (ATCAS) program. In this program we have chosen for study a section of a widebody aircraft (244" dia.) just aft of the wing/body intersection. The section is approximately 32 feet long. This section is chosen because its geometry presents most of the technical challenges of producing a composite fuselage structure. It contains landing gear cutouts, cargo door cutouts, window cutouts, and complex loading due to its location with respect to the wing. The program started in May of 1989 and Phase B is scheduled to conclude in 1995. During these 2 phases we have designed, built and tested several subcomponent composite structures. Most of the early work concentrated on the fuselage crown, which is a skin/stringer design with relatively thin skins. More recently, we have moved on to the keel and side structure. These structure are much more difficult to design due to the presence of a complex load state and the number of cutouts. In 1995 Phase C is scheduled to begin. During Phase C Boeing will be contracted to build a full barrel Section 46.

Although we chose skin/stringer as the structure type for the crown, we have chosen sandwich structure for the side and keel. We are using Hercules' AS4/8552 for the skin and Hexcel's HRP honeycomb core. In the past few months we have been conducting compression tests of sandwich structure with impact damage, notches, and undamaged. All of the specimens have been instrumented with strain gages and on some we have collected photoelastic data.

As you are probably aware it is common practice to use relatively short coupons for compression tests in order to prevent global buckling. Boeing's compression-after-impact (CAI) coupon is 6" x 4", which presents an aspect ratio L/W which is only 1.5. During our program we conducted a compression test of a sandwich specimen with a 1" hole. The dimensions of that coupon were 18" x 12". We did not observe the hole size strength effect we expected and came to the conclusion that the load wasn't evenly introduced into the specimen; hence, the hole never saw the expected stress concentration. In another experiment we performed compression testing of 5" x 7" and 5" x 12" open hole (1") solid laminate coupons. Photoelastic data was collected for both specimens. The results showed that better load introduction was obtained in the longer specimen. These seem to point to St. Venant effects and/or specimen fixturing conditions as the reasons for uneven load introduction. In any event, it appears that L/W ratios on the order of 1.5 are insufficient for credible test data. We have somewhat arbitrarily tried to use L/W rations of at least 2 on this program.

I would be most interested in having some informal discussions with you on these issues when you visit Seattle next year. Perhaps you can give us some insight on St. Venant effects in sandwich composite structure, and maybe you might leave with some ideas for future research. I'm looking forward to meeting you.

Sincerely,

WBAvery

William B. Avery, Ph.D
M/S 6H-PJ
206-234-0444

Boeing Commercial Airplane Group
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July 5, 1996
BYH20-BFB-L96-043

Dr. Arje Nachman
Program Director, Applied Analysis
AFOSR, 110 Duncan Ave., Suite B115
Bolling AFB
Washington, DC 20332-0001

BOEING

Dear Dr. Nachman,

I few days ago Cornelius Horgan from the University of Virginia asked me to write you and explain the relevance of his work to Boeing. I am happy to do so. This letter is the response to that request.

In 1989 Boeing started work on the Advanced Composite Technology Aircraft Structure (ATCAS) program, which is funded by NASA's Advanced Composite Technology (ACT) initiative. In this program Boeing has been developing the materials, structures, and manufacturing technology for a composite fuselage for a widebody aircraft. The goal is to design the structure such that there is significant savings in both cost and weight.

Early in the program we identified sandwich structure as having a high potential to save cost because the tooling and manufacturing processes for skin-stringer structures are expensive. Consequently we baselined sandwich construction for the fuselage keel and side panels.

When we started collecting our structural database we compression tested several solid laminate and sandwich coupons, many of them with holes. The failure loads were higher than expected, and we realized that we may have not been getting the full stress concentrations at the edges of the holes. This was confirmed through some photoelastic analyses of additional coupons. Part of the problem was test method related. But a review of some of Dr. Horgan's work helped us understand how St. Venant's effects are different for anisotropic and sandwich structures. His work became useful in guiding us in sizing test coupons based on degree of anisotropy and the particular configuration of the sandwich structure. Essentially, we found that we needed a longer specimen in order to get uniform load into the test coupon. Dr. Horgan's analyses helped us quantify that increase in length.

I appreciate Dr. Horgan's effort to keep in touch with Boeing and solicit ideas for research. I find him very "customer oriented". His work has been helpful to us. If you have any questions please feel free to contact me.

Regards,

William Avery, Ph.D.
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June 15, 2000

Professor Cornelius O. Horgan
Department of Civil Engineering, School of Engineering and Applied Science
Thornton Hall
University of Virginia
CHARLOTTESVILLE VA 22903-2442

Dear Niall:

Thank you for your letter and interesting paper. In response to your questions:

1. I assigned a value to $\mathbf{J}_m$ based on the experimental observation that a typical soft rubber can be stretched in simple extension to a maximum strain of about 700% ($\lambda_m = 8$). Allowing for some problems in reaching the theoretically possible elongation, for example, due to flaws or defects, I assumed that the ultimate maximum possible stretch ratio of the molecules was somewhat larger, about 10X ($\lambda_m = 10$). [From a molecular standpoint, assuming Gaussian molecular chains with **n** rigid, freely-jointed segments in a typical network strand, this corresponds to about $\mathbf{n} = 100$ (because $\lambda_m = \mathbf{n}^{1/2}$), a reasonable value in view of the actual length of a network strand in a typical crosslinked molecular network.] Thus putting $\lambda_m = 10$ in simple extension and taking $\mathbf{J}_m = 97.2$ was my attempt to portray a representative crosslinked rubber. These values were not taken from a best-fit to a particular set of experimental data.

2. I assume that the same value of $\mathbf{J}_m$ would describe the behavior in other states of strain. That is, I assume that $\mathbf{J}_m$ is a material parameter. Again, from a molecular standpoint, the actual value for $\mathbf{J}_m$ should be approximately proportional to n (at least, for large n) and thus $\mathbf{J}_m$ should vary inversely with the shear modulus. In other words, the behavior of a set of Gaussian chains should be describable, in principle, with only one fitting constant, the shear modulus. In Rivlin and Saunders' experiments, because the material was probably crosslinked somewhat more than I have taken as typical, with a somewhat higher modulus and a lower breaking elongation (probably about 500%), I would expect a better fit using a smaller value of $\mathbf{J}_m$, say around 50-60.

3. But you are correct – the elastic behavior at small strains, less than 100%, say, cannot be accounted for solely by terms in $\mathbf{J}_1$. Unfortunately the Mooney-Rivlin formulation with a constant ($\mathbf{C}_2$) term multiplying $\mathbf{J}_2$, although apparently an improvement, is not a good representation of the behavior under different types of strain. Thomas proposed another one-constant term to take the place of the term $\mathbf{C}_2\,\mathbf{J}_2$ in the strain energy function. He and I approximated it by $\mathbf{K}_2 \,\mathbf{Ln}\, [1 + (\mathbf{J}_2/3)]$ (A. N. Gent and A. G. Thomas, J. Polymer Sci. **28**, 625 (1958)). This term succesfully accounts for deviations from simple kinetic theory at small strains in all cases: extension, compression and shear, and is thus superior to the term $\mathbf{C}_2\,\mathbf{J}_2$. But I have not been able to devise a satisfactory physical explanation for this form of the additional term.

4. Here is a copy of a recent paper using the "Gent" strain energy function to calculate some instability problems [unstable states appear to provide a quite stringent test of the validity of a strain-energy function] and a paper on surface instabilities which you might find interesting.

Thank you again for your letter. I hope the above comments are helpful and look forward to seeing more interesting solutions from you.

Sincerely

Alan

A. N. Gent
Dr. Harold A. Morton Professor Emeritus of Polymer Physics
and Polymer Engineering