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ABSTRACT

In his book *Electronic Intelligence: the Analysis of Radar Signals*, author Richard Wiley analyses a triangulation scenario for two stationary receivers with one stationary emitter using bearings-only knowledge. In this report his work is extended to the case of more receivers using a least squares approach. We describe how this is done, plotting results for some representative scenarios as well as extending the work to the moving emitter case. The result is that accurate geolocation estimates can be produced with some very compact computer code, especially when taking advantage of modern fast techniques for numerical computations.

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EXECUTIVE SUMMARY

The Australian Defence Force has a need to pinpoint an enemy based purely on the reception of radio signals, without the need for using radar. In contrast, locating an enemy actively with radar is to be avoided, since it draws unwanted attention to the platform operating the radar. The measurement of an emitter's position using electronic support (ES) sensors is termed passive geolocation, and plays an important part both in electronic support and electronic attack.

In this report a triangulation approach is used to analyse geolocation and tracking problems using passive bearing information only. Unlike the more generally used Kalman filter, this technique needs no initial estimate of where the emitter is. Likewise, to track a moving emitter there is no need to supply initial estimates of the motion's parameters.

Triangulating the position of a stationary emitter based on the *noiseless* bearings taken by two stationary receivers is straightforward in principle. But with more than two receivers and noisy bearings, simple triangulation will not yield a unique location at all, since the lines along which the emitter is thought to lie will in general not intersect at one point. In this report we solve the matrix equation that describes the triangulation setup quite directly in the least squares sense, by making use of the pseudo-inverse of the matrix that encodes the receiver positions. The analysis is compact and easily converted to code that works well in an emitter location simulation.

We also treat the case of a moving emitter, sometimes thought to be beyond the grasp of simple triangulation. This is done by allocating the emitter with a fixed set of numbers, so that it is described by a well-defined, fixed point in a higher number of dimensions.

In practice the computations are done using Matlab and are very fast, comparable to using a Kalman filter. For the stationary emitter case, a typical result is shown in figure 7. There a ring of receivers is being built up one by one around an emitter. The graph shows that once we have placed three or four receivers (i.e. subtending an angle of around 30°–40° at the emitter), any extra do not make substantially more contribution to the accuracy of the location. Results for the case of a moving emitter can be seen in figure 9 for a constant velocity emitter and figure 10 for a constant acceleration emitter: the estimated track is quite accurate. These results show that batch techniques using matrix pseudo-inverses are alive and well in the field of geolocation.

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Don Koks completed a doctorate in mathematical physics at Adelaide University in 1996, with a dissertation describing the use of quantum statistical methods to analyse decoherence, entropy and thermal radiance in both the early universe and black hole theory. He holds a Master of Science from the University of Auckland in applied accelerator physics (proton-induced X-ray and γ-ray emission for trace element analysis), and has worked on the accelerator mass spectrometry programme at the Australian National University in Canberra. He has also worked in commercial internet development. Currently he is a Research Scientist with the Aerospace Systems group at DSTO, specialising in geolocation.

Contents

1 The Need for Geolocation and Layout of the Problem

The Australian Defence Force has a need for a passive geolocation ability, which is the pinpointing of an enemy platform based purely on the reception of signals, without the need for using radar. In contrast, using radar to locate the enemy actively is something to be avoided, since it draws unwanted attention to the platform that is operating the radar.

Geolocation plays an important part both in electronic support and electronic attack. In a support role it is required for threat warning, and can be applied in the air and on ground to pinpoint the source of a signal, the information of which can then be handed off to the appropriate attack platform. Network-centric warfare also makes heavy use of geolocation in that several widely separated receivers can use their inherently large baseline to triangulate the source of a radio emission very precisely. Ground to air communication in such warfare has the added advantage of providing a vastly improved three dimensional location of the source.

The problem of geolocating an emitter has been approached through many well-known methods, such as the Gauss-Newton routine and the Kalman filter. These are usually discussed in the context of one receiver, which must then be moving if it is to geolocate even the simplest case: a stationary emitter. In principle these methods can be used to consider the case of multiple receivers if we construct a hypothetical receiver that is coincident with each of the multiple receivers as they make their measurement. In practice though, this will lead to a highly erratic path for that one receiver, and make modelling its motion difficult.

An alternative approach is discussed for just two receivers by Richard Wiley [1], who performs a bearings-only analysis using two stationary receivers (positions assumed well known) geolocating one stationary emitter. The approach uses simple trigonometry to locate the emitter at the intersection of two lines, as in figure 1. (Note that Wiley's θ_2 is the supplementary angle to the θ_2 used in that figure and throughout this note.)

Wiley then expresses the range to this intersection point from each receiver as a function of the bearings, and from this estimates the error in that point's position as a function of the observed bearing errors. He does this by a first order approach of differentiating these ranges with respect to each bearing angle, using

$$\Delta \text{ range} \simeq \frac{\partial \text{ range}}{\partial \theta_1} \Delta\theta_1 + \frac{\partial \text{ range}}{\partial \theta_2} \Delta\theta_2 \tag{1}$$

Similar calculations are done in Wiley's companion volume [2]. There he uses the two range errors on each bearing line to define a small parallelogram shape, within which is plotted what is taken to be a 1-σ error contour. (The parallelogram is drawn by essentially scribing two short arcs from each of the two receivers with a compass, to define the small parallelogram with curved sides.)

Although this approach is reasonable for the two-receiver case, it cannot be extended as is, to higher numbers of receivers. The reason is simply that while there is just one intersection of the bearing lines for the two-receiver case, there will be multiple intersections for the case of more receivers. Even for the single emitter case that we study in this paper, error contours can no longer be constructed by drawing many parallelograms, because there is no longer any clear meaning for whatever regions those parallelograms delineate.

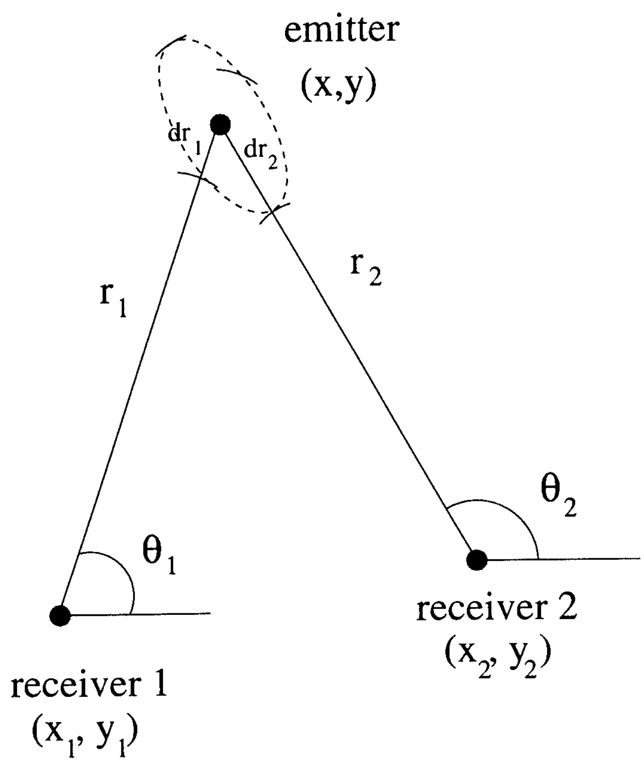

Figure 1: The case of two receivers. What is measured are the bearings, θ_1 and θ_2. For the remainder of this text, receivers are coloured blue, the true *position or track of the emitter is coloured red, while the* estimated *emitter position or track is in black.*

What we do here is show how Wiley's calculation can be done in a manner that better suggests the multiple emitter case, and then we extend it to that case.

2 Beginning with Two Receivers

The two receiver case is quite straightforward, which we will indicate here. Wiley references each of his two bearings θ_1 and θ_2 to different directions, which is not a good procedure for generalising the problem as it does not suggest how additional receivers should define *their* bearings. In contrast, all of our bearings *are* defined consistently: being measured counterclockwise from east, as shown in figure 1. Given θ_1, θ_2 and knowing the location of each receiver, we can write the trigonometric relations

$$\begin{aligned} r_1 \cos\theta_1 &= x - x_1 \quad , \quad r_2 \cos\theta_2 = x - x_2 \\ r_1 \sin\theta_1 &= y - y_1 \quad , \quad r_2 \sin\theta_2 = y - y_2 \end{aligned} \tag{2}$$

which on eliminating the unknowns x, y becomes

$$\begin{bmatrix} \cos\theta_1 & -\cos\theta_2 \\ \sin\theta_1 & -\sin\theta_2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ -y_1 + y_2 \end{bmatrix} \tag{3}$$

We can solve for r_1, r_2 (and hence x, y), then differentiate these to obtain dr_1, dr_2 as functions of $d\theta_1$, $d\theta_2$. Wiley does similarly, sets $d\theta_1$, $d\theta_2$ to be the bearing noise (e.g. one degree), and then calculates a skewed error "ellipse" by making its bounds dr_1 on either side of the emitter point in the direction seen by receiver 1, and dr_2 on either side of the emitter point in the direction seen by receiver 2.

3 Extending to Three or More Receivers

For more than two receivers the situation is not so easily interpreted. The bearing lines will in general not intersect in a single point, and the simple idea of a skewed error ellipse is no longer applicable. However, we can proceed by using a least squares approach. With n receivers, the corresponding equation to (3) becomes the overdetermined set:

$$\begin{bmatrix} \cos\theta_1 & -\cos\theta_2 & 0 & 0 & \dots & 0 \\ \sin\theta_1 & -\sin\theta_2 & 0 & 0 & \dots & 0 \\ 0 & \cos\theta_2 & -\cos\theta_3 & 0 & \dots & 0 \\ 0 & \sin\theta_2 & -\sin\theta_3 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \dots & & \cos\theta_{n-1} & -\cos\theta_n \\ 0 & 0 & \dots & & \sin\theta_{n-1} & -\sin\theta_n \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ -y_1 + y_2 \\ -x_2 + x_3 \\ -y_2 + y_3 \\ \vdots \\ -x_{n-1} + x_n \\ -y_{n-1} + y_n \end{bmatrix} \tag{4}$$

If we write this set as

$$A \begin{bmatrix} r_1 & r_2 & \dots & r_n \end{bmatrix}^t = B \tag{5}$$

where A, B are just the left hand matrix and right hand vector respectively in (4) (and the transpose has been used just to save space in this report), then the overdetermined nature translates to mean that A will not be square, and so can't be inverted. However, (4) can be inverted in the least squares sense by asking not for the $\boldsymbol{x}$ that makes $A\boldsymbol{x} = B$ (since that doesn't exist), but rather the $\boldsymbol{x}$ that minimises the distance between the two n-dimensional points $A\boldsymbol{x}$ and B.

This least squares solution to (4) is a standard result of linear algebra, and can be written in terms of the "pseudo-inverse" of A in the following way (using overbars to show we are dealing with a least squares estimate now):

$$\begin{bmatrix} \bar{r}_1 & \bar{r}_2 & \dots & \bar{r}_n \end{bmatrix}^t = (A^t A)^{-1} A^t B \tag{6}$$

where the pseudo-inverse of A is $(A^t A)^{-1} A^t$. We can then form a set of n estimates to the emitter position by locating each estimate along the sighted direction to the emitter from each receiver:

$$\begin{aligned} \text{estimate } 1 &= (x_1, y_1) + \bar{r}_1(\cos\theta_1, \sin\theta_1) \\ &\vdots \\ \text{estimate } n &= (x_n, y_n) + \bar{r}_n(\cos\theta_n, \sin\theta_n) \end{aligned} \tag{7}$$

Actually there are many different forms of (4), since we are at liberty to combine our trigonometric relations in different ways: this equates to forming different linear combinations of the rows of A and B. We have chosen a form for A that is as diagonal as possible, in the hope that any numerical routine that uses this matrix will be better optimised or produce better results for block diagonal matrices.

It might be thought that this combining of the equations that comprise (4) should not affect the least squares estimates of $r_1, \ldots, r_n$. In fact it does. The reason is that in the presence of noise, the original set strictly speaking has no solution anyway, apart from some "best" one like a least squares fit. This measure of what is best is affected by what we do with the data. What the least squares fit is doing is minimising the distance between two points in an n-dimensional space. Altering the equations is a procedure that leads ultimately to different points in this space, so that the problem has changed enough to produce a slightly different solution. For the scenarios we consider here, using a slightly different set of linear combinations of the equations comprising (4) produces results that differ by one or two percent from those of (4). The least squares procedure is certainly not guaranteed to give a single result independently of how the problem is posed.

4 Estimating the Emitter's Position Error

In general, it's too complex a problem to solve (4) via (6) analytically, and then differentiate $\bar{r}_1, \ldots, \bar{r}_n$ to find the dependence on small changes in the bearing angles as was done in the two receiver case. Here is a different, numerical approach that gives us an estimate, albeit a computationally expensive one.

With each of our n receivers, we can form a sort of 1-σ error estimate by adding or subtracting the bearing error from the measured bearing. Each change to a bearing gives a new version of (4), which then produces a new set of n estimates to the emitter position by way of (7). With n receivers, adding and subtracting the bearing error from each in all combinations will thus lead to 2^n sets of n estimates. This is really too much data to deal with and to plot. However, we can certainly plot the centroid of each set of n estimates, so that for n receivers we will produce 2^n centroid points. Each is a sort of 1-σ estimate of the emitter position. The placement and clustering of this set will give us an idea of the error estimate that we can expect n receivers to produce.

Notice that producing 2^n centroids will, and does, make for repeated patterns that at first might look overly regular. For example, consider the various stars and circles in figure 5. In the top right hand plot we have three receivers. The main calculation given by equations (6) and (7) has produced three estimates of the emitter position, and the centroid of these is the black star. Each of the 2^n green circles was found in exactly the same way as the black star, except that a different set of data was used: whereas the black star used the original set of bearings, each of the green circles represents the black star we would plot *if* we altered the bearings, either adding or subtracting the bearing error from each. Since there are three receivers, adding or subtracting from each bearing leads to 8 new sets of bearings, with their estimates from (6, 7) plotted as 8 green circles.

But a closer inspection of these shows that they are naturally divided into two sets of four bearings. After all, the top receiver can add its error to its bearing, or subtract its error from its bearing; but in both cases it will be combining this information with the same set of bearings from the other two receivers. What this means is that the four green circles produced in either case will be affected by the change in the top receiver information, but affected all in approximately the same way. The change in the top information perturbs the set of four green circles produced by the other two receivers, producing a new set of four circles that is slightly displaced from the other set of four circles.

Of course there is nothing special about the top receiver, so that it's equally valid to divide the eight green circles into sets of four in a different way (corresponding to the perturbing being done by one of the other receivers). The nett result is a good deal of regularity in the placement of the green circles that is entirely nonaccidental.

5 A Different Approach to the Least Squares Calculation

A slightly different version of (4) presents itself. Since ultimately we really want to know the emitter position (x, y), we can just as well put these two variables into the vector of unknown ranges in (4). Because this alternative approach can be easily extended to the case of a moving emitter, we will study it here.

So, rewrite (4) by including x, y as unknowns:

$$\begin{bmatrix} \cos\theta_1 & 0 & 0 & \dots & 0 & -1 & 0 \\ \sin\theta_1 & 0 & 0 & \dots & 0 & 0 & -1 \\ 0 & \cos\theta_2 & 0 & \dots & 0 & -1 & 0 \\ 0 & \sin\theta_2 & 0 & \dots & 0 & 0 & -1 \\ \vdots & \vdots & \vdots & & & & \vdots \\ 0 & 0 & 0 & \dots & \cos\theta_n & -1 & 0 \\ 0 & 0 & 0 & \dots & \sin\theta_n & 0 & -1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \\ x \\ y \end{bmatrix} = \begin{bmatrix} -x_1 \\ -y_1 \\ -x_2 \\ -y_2 \\ \vdots \\ -x_n \\ -y_n \end{bmatrix} \tag{8}$$

As with (5, 6) we can rewrite this (employing a transpose to make the notation more compact) as

$$M \begin{bmatrix} r_1 & r_2 & \dots & r_n & x & y \end{bmatrix}^t = N \tag{9}$$

so that the least squares solution is

$$\begin{bmatrix} \bar{r}_1 & \dots & \bar{y} \end{bmatrix}^t = (M^t M)^{-1} M^t N \tag{10}$$

The only two variables that really interest us are the least squares estimates of the emitter position, which we can pick out by premultiplying the entire vector of least squares estimates by a $2 \times n$ matrix composed mostly of zeroes:

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} 0 & \dots & 0 & 1 & 0 \\ 0 & \dots & 0 & 0 & 1 \end{bmatrix} (M^t M)^{-1} M^t N \tag{11}$$

It should be added that this sparse matrix is more than just a flowery way of singling $\bar{x}, \bar{y}$ out from the left hand vector of (10). Because matrix multiplication is associative, we can multiply this matrix first by $(M^tM)^{-1}$, which will make the next multiplication (by M^tN) much easier and faster. However whether or not a mathematics software package such as Matlab optimises its multiplications in this way might not be known, although we can at least put parentheses in the code which might force such a package to multiply in the required order—depending on the level of run-time optimisation that the package might use.

6 Examples: Analysing Simple Scenarios for a Stationary Emitter

In the following graphs we present examples of the geolocation done using several receivers. The graphs are grouped in sets: one set per figure. In each set we start with two receivers present, with each graph in the set being produced by adding one more receiver. Each receiver is marked by a blue cross. The actual emitter is always at the origin, marked by a red star. The centroid of the n least squares estimates to the emitter position is marked by a black star. Each of the 2^n centroids described above is marked as a small green circle.

It's also useful to establish just how well localised the emitter becomes as the number of receivers grows. What we wish to do then, is plot a measure of the emitter-finding accuracy versus the number of receivers. As figures 2–5 show, the emitter-finding accuracy is well represented by the position of the black star: i.e. the estimate that is made from the original (noisy) bearings, prior to producing the 1-σ estimates as described above; at no point have we considered it necessary to calculate the centroid of the 2^n estimates. In the last graph of each figure, we plot the distance of this estimate from the actual emitter (that is, the distance between black and red stars) as a function of the number of receivers, for the receiver configuration of that set. The graphs shown do display an increasing accuracy as the number of receivers increases, but it is also not uncommon to find a statistically allowed temporary decrease in accuracy, perhaps for one point on a graph like this. This is not to say the accuracy really decreases; rather, the noise in the bearings has just temporarily pushed the latest emitter estimate further away from the true emitter position.

Note that figures 2–8 all use the first least squares approach described, i.e. section 3 as opposed to section 5. The results of the latter section are similar enough to be excluded. The utility of section 5 is that it extends well into the approach followed in section 7.

Figure 2 shows from two to six receivers with a 1° bearing error, together with the plot showing how the error decreases with receiver number. The error falls markedly once we introduce the fifth receiver, which sits apart from the rest of the set.

Figure 3 is similar, but the receivers are now much closer to the emitter. The reduction in error for the fifth receiver is still marked.

Figure 4 is again similar but with 2° bearing error. With this extra bearing noise, there is no longer any marked reduction in error with the addition of the fifth receiver.

Figure 5 has 5° bearing error with a configuration surrounding the emitter. Since the receivers are spaced more symmetrically, it's not surprising that the 1-σ estimates are also distributed more evenly than was the case for figures 2–4. The accuracy worsens temporarily with the addition of the fourth (bottommost) receiver, but this is just a statistical fluctuation as repeated simulations show that the accuracy can also just as well worsen for any one of the other receivers.

We can also introduce many more emitters, concentrating now on how the error drops as a function of receiver number.

Figures 6–8 each show a ring of 30 receivers all with 1° bearing error, starting from the top and adding receivers one by one counterclockwise around the circle. In each plot the error bottoms out at about 6 receivers, which therefore subtend an angle of approximately 70° at the true emitter position.

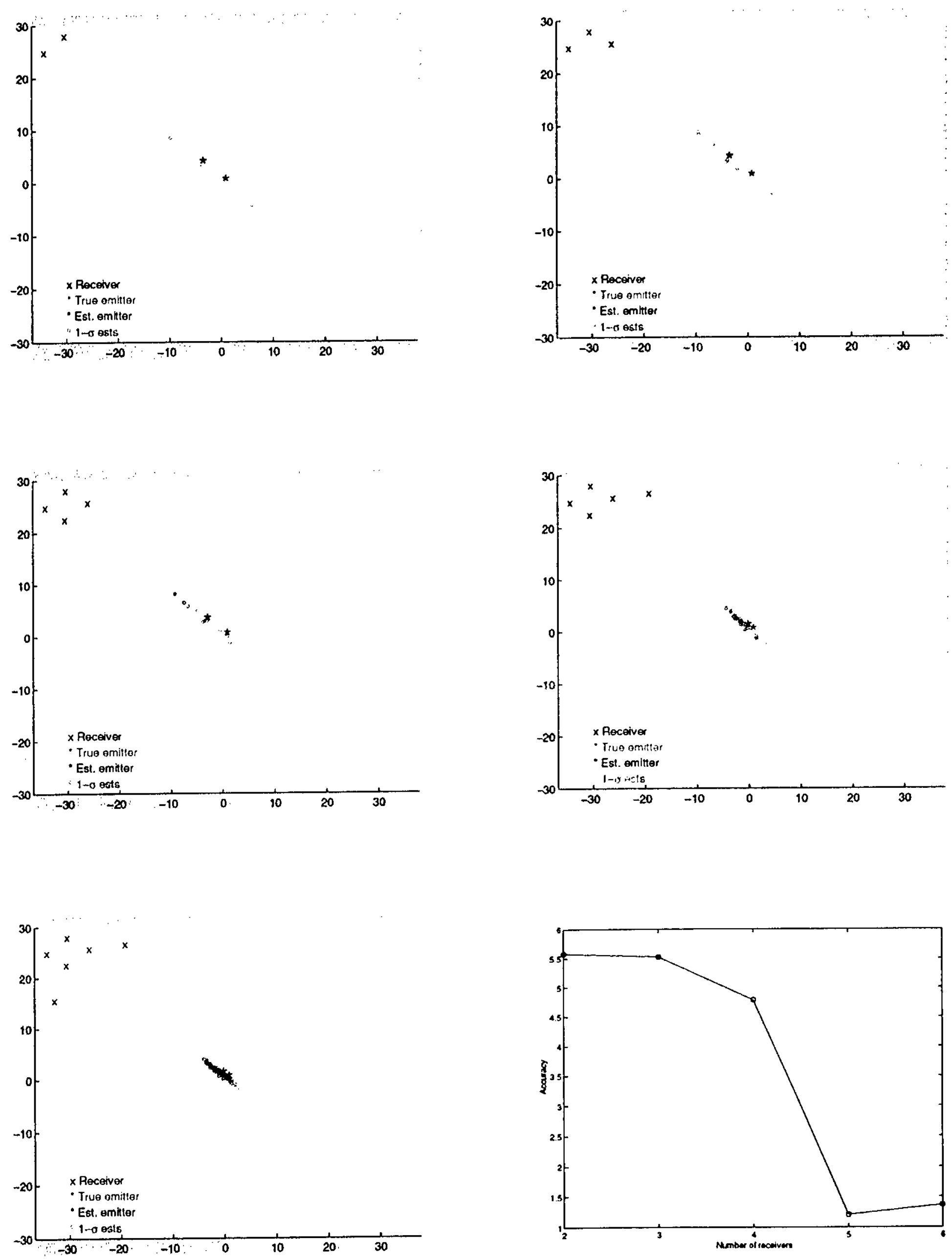

Figure 2: Building up from two to six receivers with 1° bearing error, far from emitter.

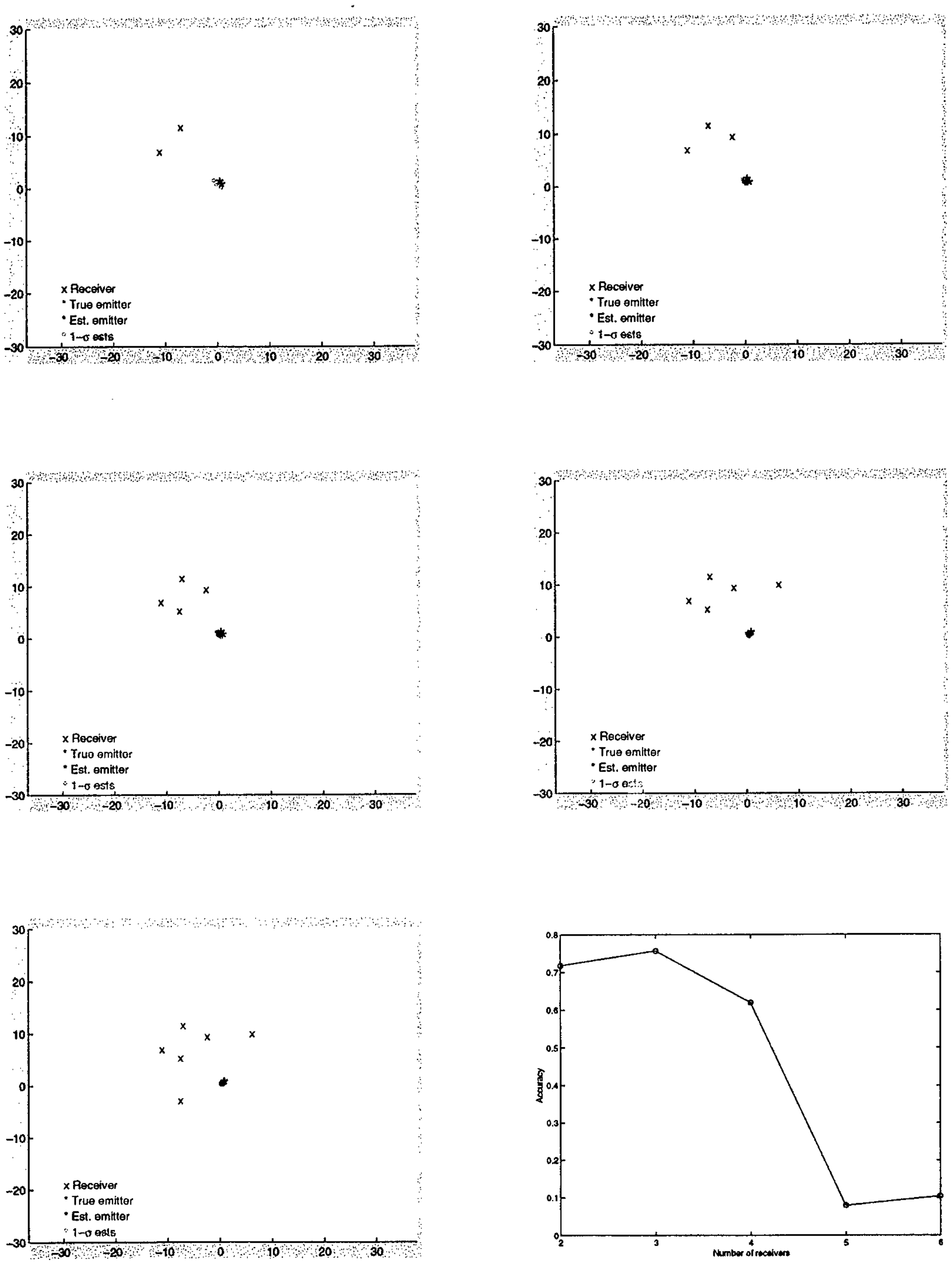

Figure 3: Building up from two to six receivers with 1° bearing error, closer to emitter.

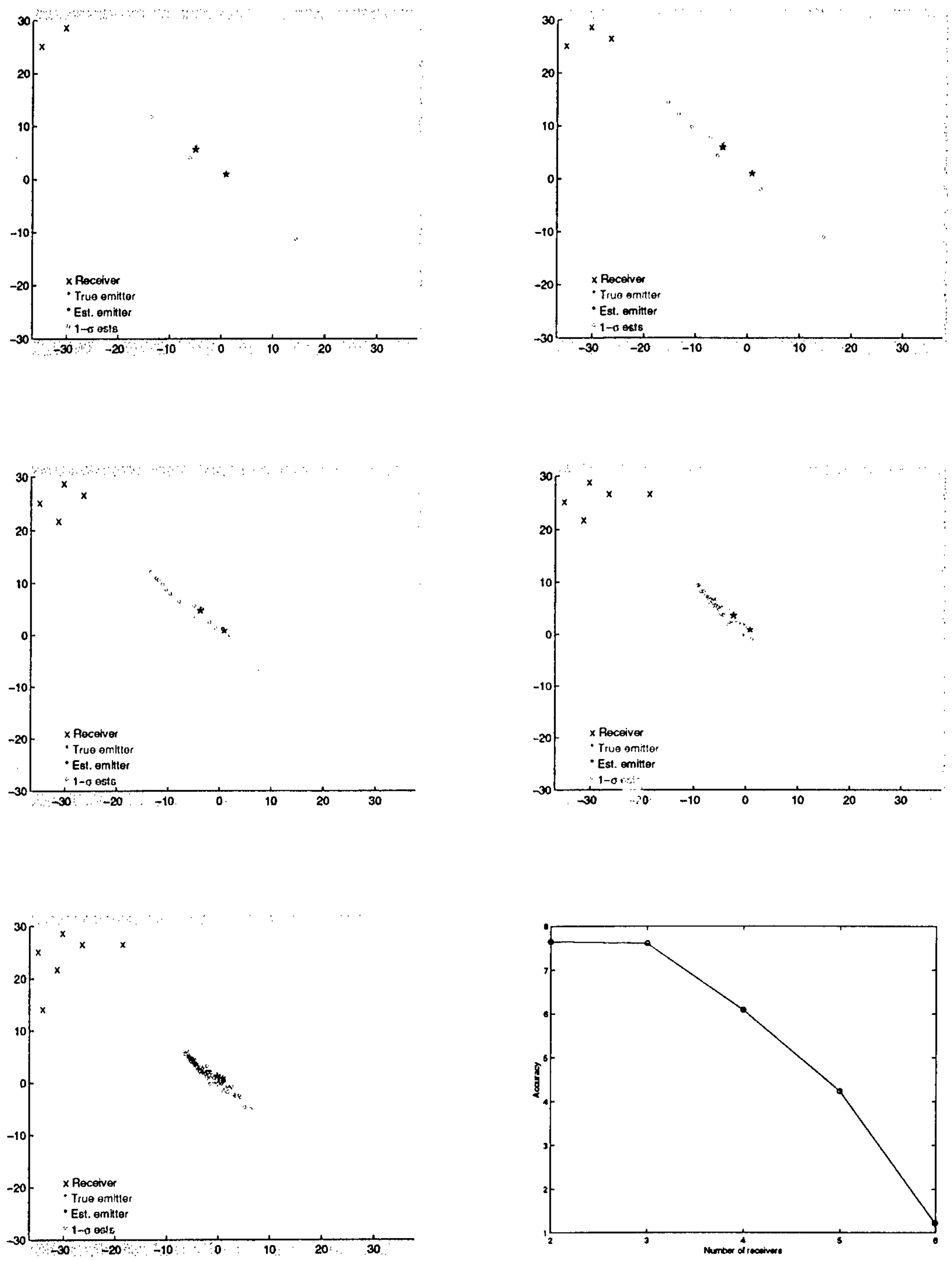

Figure 4: Building up from two to six receivers with 2° bearing error, far from emitter.

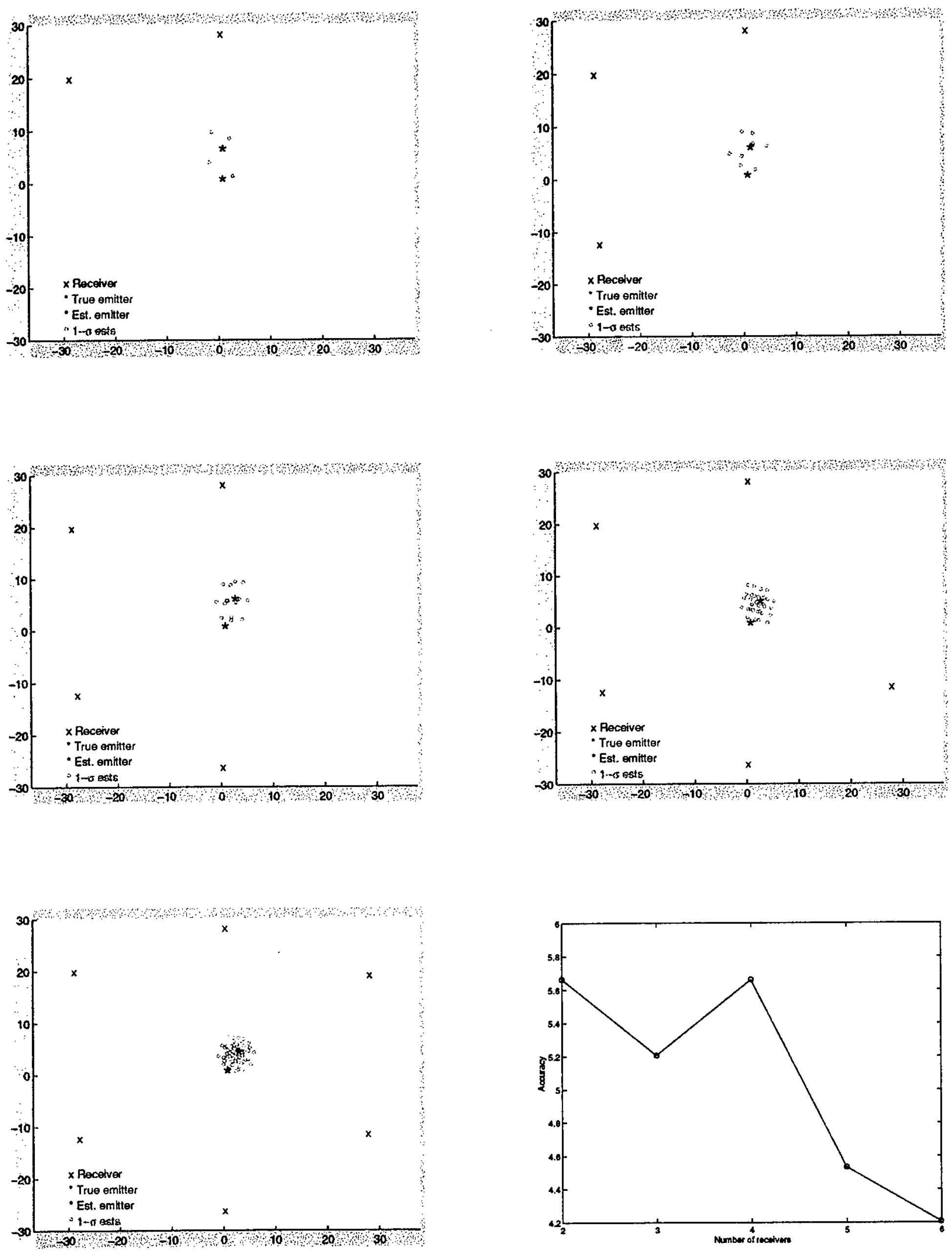

Figure 5: Building up from two to six receivers with 5° bearing error, surrounding emitter. For a discussion of the regularity in the green circles, see page 4.

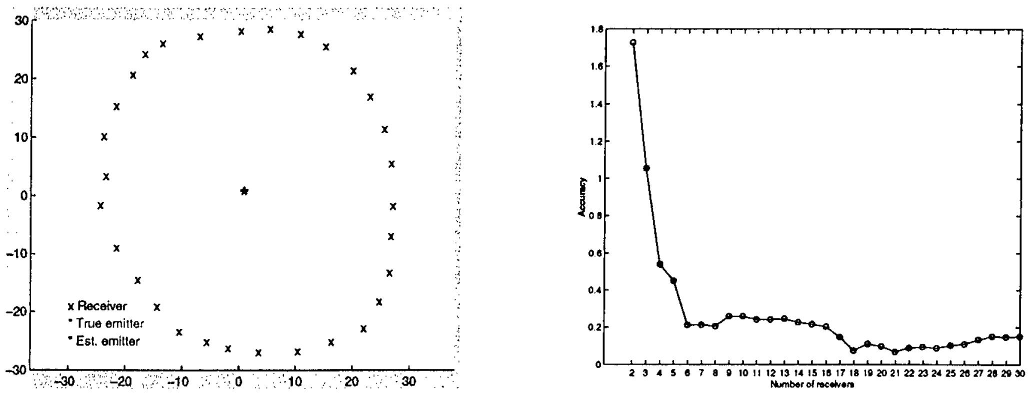

Figure 6: Thirty receivers with 1° bearing error.

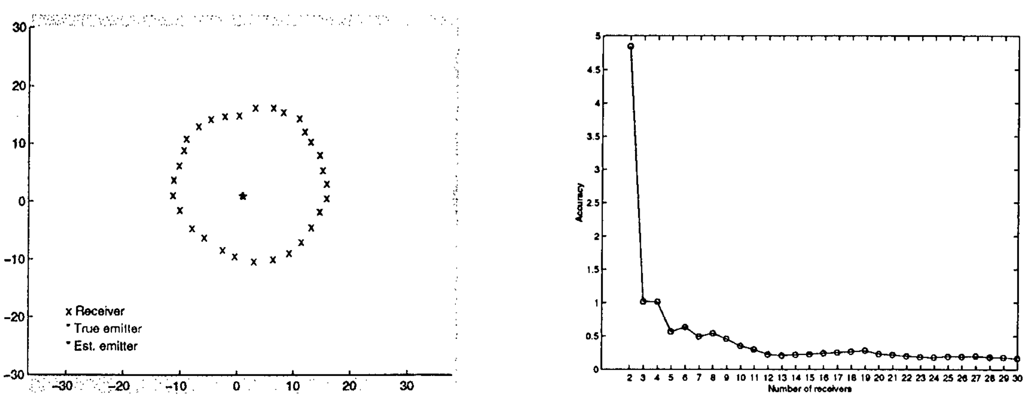

Figure 7: Thirty receivers with 1° bearing error, closer in.

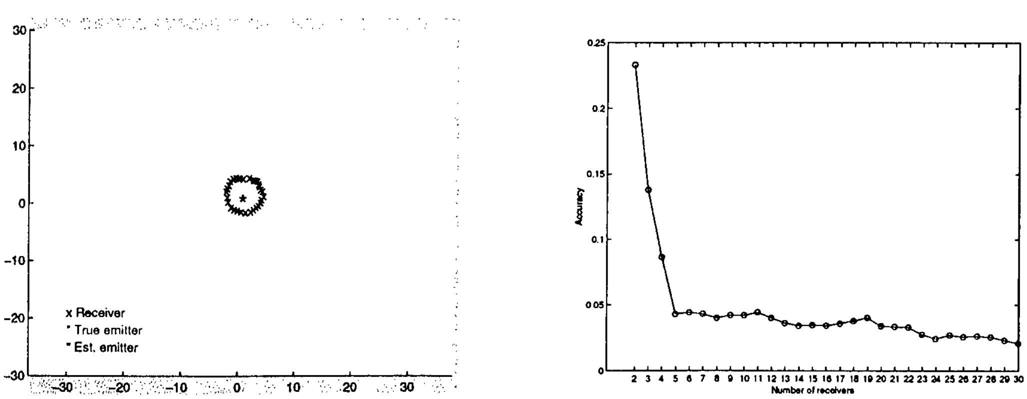

Figure 8: Thirty receivers with 1° bearing error, still closer in.

7 Extending to the Moving Emitter Case

The modified approach of section 5 can be extended easily to the cases of a constant velocity or constant acceleration emitter, or in fact all higher derivatives.

Constant velocity emitter If the emitter has initial position $s_0 \equiv (s_{0x}, s_{0y})$ and constant velocity $v \equiv (v_x, v_y)$ then its position at any time is given by the vector equation

$$s(t) = s_0 + vt \tag{12}$$

Suppose that the n receivers take snapshots of the emitter's direction at times that they record, one per receiver, so that receiver i takes its bearing measurement θ_i at time t_i. Receivers are certainly allowed to make measurements simultaneously. The analogous set of equations to (2) is then, for $i = 1$ to n:

$$\begin{aligned} r_i \cos\theta_i - s_{0x} - v_x t_i &= -x_i \\ r_i \sin\theta_i - s_{0y} - v_y t_i &= -y_i \end{aligned} \tag{13}$$

so that

$$\begin{bmatrix} \cos\theta_1 & 0 & 0 & \dots & 0 & -1 & 0 & -t_1 & 0 \\ \sin\theta_1 & 0 & 0 & \dots & 0 & 0 & -1 & 0 & -t_1 \\ 0 & \cos\theta_2 & 0 & \dots & 0 & -1 & 0 & -t_2 & 0 \\ 0 & \sin\theta_2 & 0 & \dots & 0 & 0 & -1 & 0 & -t_2 \\ \vdots & \vdots & \vdots & & & & \vdots & & \\ 0 & 0 & & \dots & \cos\theta_n & -1 & 0 & -t_n & 0 \\ 0 & 0 & & \dots & \sin\theta_n & 0 & -1 & 0 & -t_n \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \\ s_{0x} \\ s_{0y} \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -x_1 \\ -y_1 \\ -x_2 \\ -y_2 \\ \vdots \\ -x_n \\ -y_n \end{bmatrix} \tag{14}$$

Writing the left and right hand matrices in (14) as M_1 and N_1 respectively, we obtain least squares estimates to the emitter's initial conditions, from which its path can be reconstructed with ease:

$$\begin{bmatrix} \bar{s}_{0x} & \bar{s}_{0y} & \bar{v}_x & \bar{v}_y \end{bmatrix}^t = \left[\begin{pmatrix} 4 \times n \\ \text{matrix of zeroes} \end{pmatrix} \begin{pmatrix} 4 \times 4 \\ \text{identity matrix} \end{pmatrix} \right] (M_1^t M_1)^{-1} M_1^t N_1 \tag{15}$$

and the same remarks written after (11) apply to (15).

Constant acceleration emitter If the emitter now has initial position s_0, initial velocity $v_0 \equiv (v_{0x}, v_{0y})$ and constant acceleration $a \equiv (a_x, a_y)$, the vector equation for its position at time t is

$$s(t) = s_0 + v_0 t + at^2/2 \tag{16}$$

The details of the calculation are very similar to the constant velocity case and so have been omitted. Least squares estimates of the initial position, velocity and constant acceleration are

$$\begin{bmatrix} \bar{s}_{0x} & \bar{s}_{0y} & \bar{v}_{0x} & \bar{v}_{0y} & \bar{a}_x & \bar{a}_y \end{bmatrix}^t = \left[\begin{pmatrix} 6 \times n \\ \text{matrix of zeroes} \end{pmatrix} \begin{pmatrix} 6 \times 6 \\ \text{identity matrix} \end{pmatrix} \right] (M_2^t M_2)^{-1} M_2^t N_2 \tag{17}$$

where

$$M_2 = \begin{bmatrix} \cos\theta_1 & 0 & \dots & 0 & -1 & 0 & -t_1 & 0 & -t_1^2/2 & 0 \\ \sin\theta_1 & 0 & \dots & 0 & 0 & -1 & 0 & -t_1 & 0 & -t_1^2/2 \\ 0 & \cos\theta_2 & \dots & 0 & -1 & 0 & -t_2 & 0 & -t_2^2/2 & 0 \\ 0 & \sin\theta_2 & \dots & 0 & 0 & -1 & 0 & -t_2 & 0 & -t_2^2/2 \\ \vdots & \vdots & & \vdots & & & \vdots & & \vdots & \\ 0 & 0 & \dots & \cos\theta_n & -1 & 0 & -t_n & 0 & -t_n^2/2 & 0 \\ 0 & 0 & \dots & \sin\theta_n & 0 & -1 & 0 & -t_n & 0 & -t_n^2/2 \end{bmatrix} \qquad N_2 = \begin{bmatrix} -x_1 \\ -y_1 \\ -x_2 \\ -y_2 \\ \vdots \\ -x_n \\ -y_n \end{bmatrix} \tag{18}$$

8 Examples of the Moving Emitter Case

In the next few figures we show examples of how equations (15) and (17) can be used to analyse the two cases of constant velocity and constant acceleration emitters respectively.

We have set the scenarios up in the following way in a Matlab programme. For each case an "actual" emitter track is shown (always in red). Each of these has been specified by its initial conditions, chosen arbitrarily. The emitter will fly this track for ten time units (say seconds, although the simulation is not being run in real time). We specify how many receivers we want, and on entering this number, the clock starts.

Each time we place a receiver with the mouse, the time is recorded, eventually normalised so that the final receiver is placed at the ten second mark. Each receiver is then given a noisy bearing, which is where it saw the emitter at the (normalised) time of its creation. These bearings θ_i, the times t_i and the receiver positions (x_i, y_i) are then used to generate the least squares estimates to the emitter's initial conditions via (14, 15) for the constant velocity emitter and (17, 18) for the constant acceleration one.

The estimated emitter track is then drawn—but not for ten seconds. Rather, the first receiver has taken its bearing some time into the ten second emitter run, so the receivers all take their clocks to start at zero with this first bearing observation. So, if five receivers are present and take their bearing measurements at times 3, 3.1, 4, 6 and 10 of the emitter run (all normalised to ensure the last is 10), then the estimated initial conditions are only run forward in time for 7 seconds.

So we hope that the red "actual" track and the black "estimated" track will be very close at their final point, since this marks where the emitter was at the time of the last bearing measurement. But we certainly don't expect the paths to necessarily line up at their beginnings; the black path hopefully *will* lie more or less on the red path if the techniques here have been successful.

Figure 9 shows six receivers that take their 1° noisy bearings in the order shown at reasonably uniform intervals. The emitter moves with constant velocity from left to right as shown by the red line. The black estimate matches this line quite well.

Figure 10 again shows six receivers with 1° noisy bearings in the order shown. The emitter moves with constant acceleration from left to right on the red curve. Again, the black estimate matches, although not quite as well as for the constant velocity case. This is expected since higher derivatives in the emitter motion are always much more susceptible to bearing noise.

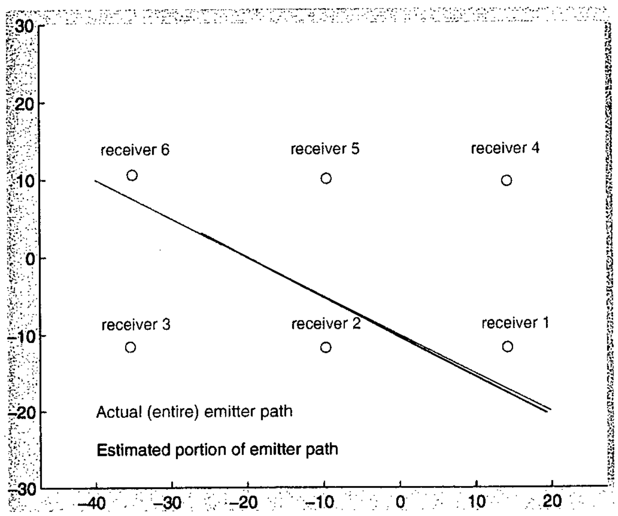

Figure 9: Six receivers with 1° bearing error; emitter moves at constant velocity from top left to bottom right. Emitters take their bearings in numbered order.

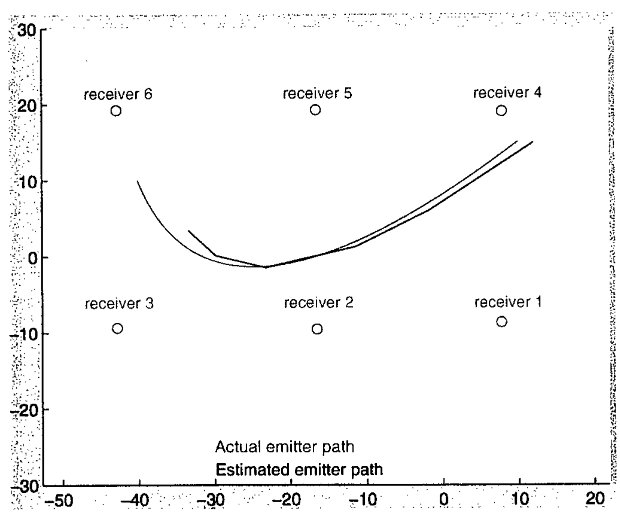

Figure 10: Six receivers with 1° bearing error; emitter moves at constant acceleration from left to right. Emitters take their bearings in numbered order.

9 Concluding Remarks

The extension of a simple trigonometrical approach by introducing the ideas of least squares allows us to analyse a static multiple receiver scenario comparatively easily. As computing speed continues to increase, numerical approaches which once would be considered too complex now are becoming more routine, and software like Matlab is easily able to manipulate large matrices such as found in (4).

Batch approaches, such as we have considered in this report, treat a lot of data all together, and were once considered to be wasteful in terms of computing resources. But it turns out that recursive techniques such as the Kalman filter actually use a similar number of computations anyway, because while they might not be concerned with large batches of data, they do require more internal computations to calculate the various factors (such as the weighting between new data and the current estimate).

Computing advances also continue to make it more practical and reasonable to quantify geolocation errors using only numerical calculations as opposed to algebraic error analyses, since such analyses are often difficult or perhaps even impossible to do analytically.

References

1. Richard G. Wiley, *Electronic Intelligence: the Analysis of Radar Signals*, 2^{nd} edition. Artech House Inc. (1993)

2. Richard G. Wiley, *Electronic Intelligence: the Interception of Radar Signals*, 1^{st} ed. Artech House Inc. (1985). Lone partial derivative symbols such as $\partial x, \partial y$ as used in the calculations of appendix B—to mean small increases in x or y—are actually quite meaningless in that context; they should more correctly be replaced by either $\Delta x, \delta x$ or even dx.

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19. ABSTRACT

In his book *Electronic Intelligence: the Analysis of Radar Signals*, author Richard Wiley analyses a triangulation scenario for two stationary receivers with one stationary emitter using bearings-only knowledge. In this report his work is extended to the case of more receivers using a least squares approach. We describe how this is done, plotting results for some representative scenarios as well as extending the work to the moving emitter case. The result is that accurate geolocation estimates can be produced with some very compact computer code, especially when taking advantage of modern fast techniques for numerical computations.

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