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14. ABSTRACT A workshop was held on the topic of quantum transport in semiconductor devices. This workshop brought together 17 lecturers and 35 other attendees for this purpose.					
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Advanced Research Workshop on Quantum Transport in Semiconductors

Hotel Villa del Mare, Maratea, Italy 17-22 June 2001

Session 1

Moderator: John Barker, Glasgow University, "Status of the physics and modeling of ultrasmall devices"

- 1A Greg Timp, University of Illinois, "Making small MOSFETs: ballistic and quantum effects"
- 1B Max Fischetti, IBM Research, "Modeling small MOSFETs-the role of quantum and many-body effects"
- 1C Steve Goodnick, Arizona State University, "Full Band Structure Calculations for Transport in Wide-Band-Gap Semiconductors"

Session 2

Moderator: Max Fischetti, IBM Research, "Semi-classical modeling of small semiconductor devices"

- 2A Asen Asenov, Glasgow University, "Discrete impurities and quantum potentials in MOSFET modeling"
- 2B Richard Akis, Arizona State University, "Effective potentials for quantum effects in MOSFETs"

Session 3

Moderator: Antti-Pekka Jauho, Danish University of Technology, "Introduction to Quantum Transport"

- 3A David Ferry, Arizona State University, "Wave function approaches for self-consistent computations of transport in quantum dots and arrays"
- 3B John Barker, Glasgow University, "Trajectories in quantum mechanics"
- 3C Michael Bonitz, Rostock University, "Non-equilibrium Green's functions: transient phenomena and the role of the initial state for devices"

Session 4

Moderator: Chihiro Hamaguchi, Osaka University, "The Metal-Insulator Transition"

- 4A Günther Bauer, University of Linz, "The metal-insulator transition in $d = 2$ "
- 4B Jonathan Bird, Arizona State University, "The metal-insulator transition in open quantum dots and arrays"

Session 5

Moderator: Michael Bonitz, Rostock University, "Spins and few particle problems in Semiconductors"

- 5A David Awschalom, UC Santa Barbara, "Spin coherence and optical measurements"
- 5B Daniel Loss, University of Basel, "Quantum computing in semiconductor systems"
- 5C Sankar Das Sarma, University of Maryland, "Few electron (and few impurity) systems-small system effects"

Session 6

Moderator: Gerhard Klimeck, Jet Propulsion Laboratory, "Applications of quantum transport in devices"

- 6A Carlo Jacoboni, Modena University, "The Wigner function and quantum transport"
- 6B Harold Grubin, SRA, Inc., "Modeling resonant tunneling diodes with Wigner functions and density matrices"
- 6C Dejan Jovanovic, Motorola, "Non-equilibrium Green's functions for MOSFET modeling"

Session 7

Moderator: Karl Hess, University of Illinois, "Applications of quantum transport in optics"

- 7A Tilmann Kuhn, University of Münster, "Quantum kinetics and the femtosecond time scale in optical excitation of semiconductors"
- 7B Rolf Haug, University of Hannover, "Single-electron charging effects in quantum dot arrays"

Session 8

Moderator: Anant Anantram, NASA Ames, "Novel New Concepts"

- 8A Karl Hess, University of Illinois, "EPR Experiments: the Assumptions, and the Failure of Bell's Theorem"
- 8B Antti-Pekka Jauho, Danish University of Technology, "Scanning probe measurements of biomolecules on silicon surfaces"
- 8C Mark Lundstrom, Purdue, "The Landauer approach in device modeling"
- 8D Wolfgang Windl, Motorola, "Diffusion and clustering of impurities-a problem that cannot be ignored"

Poster Presentations

Poster Group 1

- 1-1 "Analytical theory for the low frequency transport and noise in Q1D conductors," G. Gomilla and L. Reggiani, Universidad Polytechnica Catalunya.
- 1-2 "Monte Carlo simulations of Wigner function tunneling: Role of the effective potential," L. Shifren and D. K. Ferry, Arizona State University.
- 1-3 "3D device Monte Carlo modeling with discrete impurities," S. Barraud, S. Galdin, P. Dollfus, Institut d'Electronique Fondamentale.
- 1-4 "3D modeling of imperfect interfaces and edges in decanano MOSFETs," S. Kaya, Ohio

University, and A. R. Brown, S. Roy, and A. Asenov, Glasgow University.

- 1-5 "Comparison of first-order quantum correction schemes in 3D drift-diffusion simulations in sub-0.1 micron MOSFETs," J. R. Watling, A. R. Brown, A. Asenov, and D. K. Ferry, Glasgow University.
- 1-6 "Quantum potential in the presence of heterointerfaces," J. R. Watling, R. C. W. Wilkins, and J. R. Barker, Glasgow University.
- 1-7 "Performance of scaled double delta doped PHEMTs," K. Kalna, J. R. Watling, A. Brown, and A. Asenov, Glasgow University.
- 1-8 "Electron transport and infrared photoconductivity in quantum dot structures," V. Ryzhii and V. Mitin, University of Aizu.
- 1-9 "Photoluminescence from hot electrons in low dimensional systems," H. Momose, Y. Inui, M. Itoh, and C. Hamaguchi, Osaka University.
- 1-10 "Nonlinear transport through an array of quantum dots," G. Kiesslich, A. Wacker, and E. Schöll, Technische Universität Berlin.

Poster Group 2

- 2-1 "Quantum algorithms in the frame of a coupled-quantum-wires physical system," S. Reggiani, A. Bertoni, R. Brunetti, and M. Rudan, University of Bologna.
- 2-2 "Quantum waves in the coupled-wire Qubit," J. Harris, R. Akis, and D. K. Ferry, Arizona State University.
- 2-3 "Quantum mechanical study of nanoscale MOSFET," A. Svizhenko, M. P. Anantram, and T. R. Govindam, NASA Ames Research Laboratory.
- 2-4 "Effects of lead population on magnetic flux controlled dissipative electron transport through coupled quantum dots," N. Horing, Stevens Institute of Technology.
- 2-5 "Transport in quantum dot arrays," N. Mori, Y. Takamura, T. Ishida, and C. Hamaguchi, Osaka University.
- 2-6 "Non-equilibrium transport in nanostructures," J. Fransson, University of Uppsala.
- 2-7 "Tunneling spectroscopy of exchange-correlation interaction of electrons in Schottky barrier in quantizing magnetic field," A. Shulman, Institute of Radioengineering and Electronics.
- 2-8 "Stochastic simulation of the Barker-Ferry equation," M. Nedjalkov, Technical University of Vienna.
- 2-9 "Simulation of entanglement in semiconductor quantum wires," A. Bertoni, R. Ionicioiu, P. Zanardi, F. Rossi, and C. Jacoboni, University of Modena.
- 2-10 "Mesoscopic fluctuations of Coulomb drag between quasi-ballistic 1D wires," N. A. Mortensen, K. Flensburg, and A.-P. Jauho, Danish Technical University.

Poster Group 3

- 3-1 "Spin dynamics in III-V quantum wells using Monte Carlo simulation," A. Bournel and P. Hesto, Université Paris Sud.
- 3-2 "Electron-spin-phonon coupling and relaxation dynamics in a double quantum dot," V. Puller, L. Mourokh, and N. Horing, Stevens Institute of Technology.
- 3-3 "Quantum computing with spin qubits in semiconductor structures," V. Privman, Clarkson University.
- 3-4 "High-field transport through semiconductor heterostructures," M. Morifuji, Osaka University.
- 3-5 "Effect of electronic disorder on the phonon-drag," V. Mitin, Wayne State University.
- 3-6 "Wigner function dynamics in presence of an infinite potential barrier," P. Bordone and C. Jacoboni, Modena University.
- 3-7 "Quantum transport in 2D MOSFETs," A. P. Anantram, A. Svizhenko, and T. R. Govindan,

NASA Ames Research Center.

- 3-8 "Density-matrix modeling of terahertz photon-assisted tunneling in resonant tunneling diodes," M. Asada, Tokyo Institute of Technology.
- 3-9 "Hyperfine interaction between nuclear and electronic spins," I. D. Vagner, Holon Academic Institute of Technology.
- 3-10 "Metal-insulator transition in 2D few-electron systems," A. Filinov, M. Bonitz, and Yu. Lozovik, University of Rostock.
- 3-11 "Quantum dot modeling using NEMO-3D," G. Klimeck, F. Oyafuso, R. C. Bowen, T. B. Boykin, T. A. Cwik, E. Huang, and E. Vinyard, Jet Propulsion Laboratory.

Advanced Research Workshop
on

Quantum Transport in Semiconductors

Introduction and Overview

John Barker
University of Glasgow

Hotel Villa del Mare, Merate, Italy
17-22 June, 2001

Advanced Research Workshop
on

Quantum Transport in Semiconductors

Organisation

David Kane
John Ballie

Charles Foxton

Hal Grubb, Mark Lee, David Rees, David Smith

Advanced Research Workshop
on

Quantum Transport in Semiconductors

Our long-suffering sponsors!

Office of Naval Research
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Origins

1 Day Workshop on Quantum Transport Theory
(Glasgow, May 2000)

Present state of play in the field

Encourage new workers

Map out new opportunities

Flag new directions

Format

Gordon Conference Style

- Introductory/tutorial material
- Advanced Topics
- Pre-publication ethics

Sessions

- Moderator: overview and introduction (5-10 minutes)
- Speakers: 45 minutes (questions welcomed in talks)
- Questions & Discussion: 10-15 minutes per lecture

Time Table

- Breakfast 8 00
- Morning session: 9 00-12 30 (coffee 11 10)
- Lunch 13 00
- Ad hoc sessions
- Poster session: 16 00-17 00
- Evening session: 1700-1930
- Dinner: 20 00

Wednesday

- 14 00
- Excursion: Monastery visit + shopping in Maratea
- Please sign up: Rosella Brunetti
- Dinner: 20 00

Session 1: Monday, June 18

Ultra-small devices

Status of Physics and Modelling of ultra-small devices

- Moderator: John Barker, University of Glasgow
- Greg Timp, University of Illinois
- Max Fischetti, IBM T J Watson Research Lab
- Steve Goodnick, Arizona State University

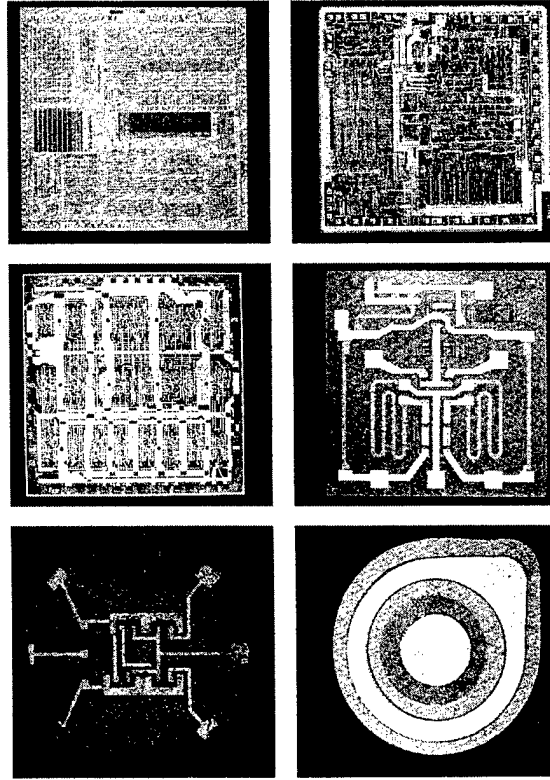


Fig. 1.1.1. Evolution of device structure from simple gates to complex multi-layered architectures.

Moores Law
history
since
1970

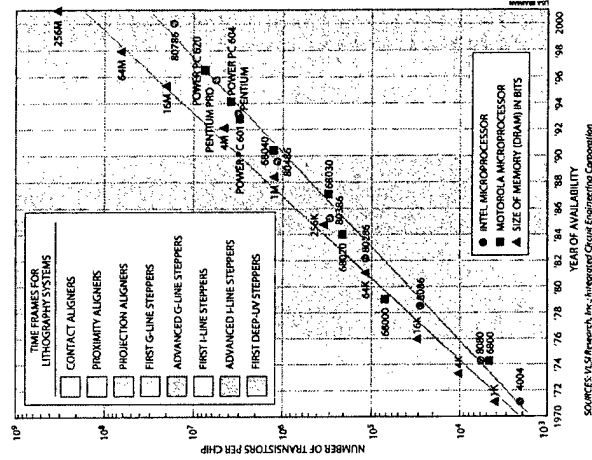


Fig. 1.1.2. Moore's Law: The number of transistors on a chip doubles every 1.5 years.



Status of the physics and modelling of ultra-small Semiconductor devices

John. R. Barker
Nanoelectronics Research Centre
Department of Electronics and Electrical Engineering
University of Glasgow



1. The future of space exploration is in the hands of the people who are working on the ground.



Advanced Research Workshop
on
Quantum Transport in Semiconductors

The dinosaur model: silicon will eventually become extinct



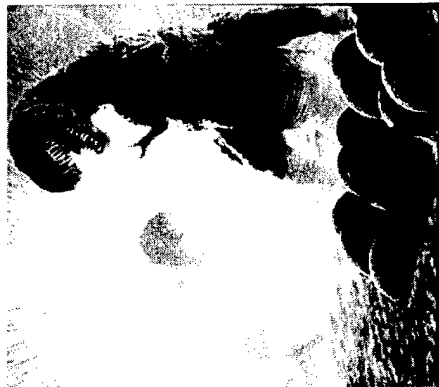
.... due to impending catastrophe

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After the catastrophe, a long hiatus will occur and silicon's role will be replaced by new species : quantum, molecular...

QuickTime™ and a
Photo-JPEG decompressor
are needed to see this picture.

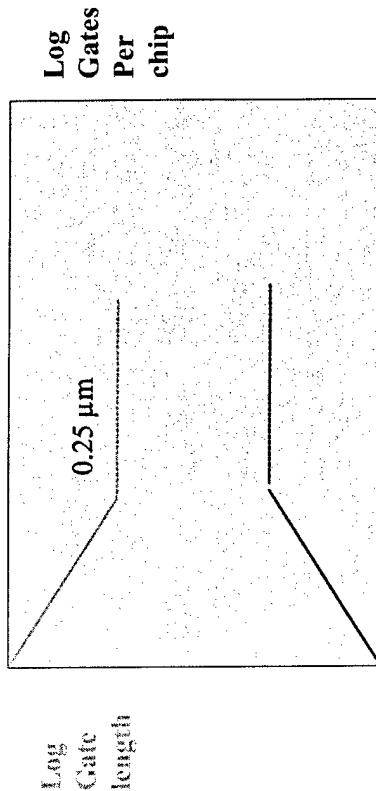
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Projected Breakdown of Moore's Law

1977



Time
(challenged by JRB/DKF
In 1979-25 nm)

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The guiding philosophy behind semiconductor quantum transport theory strategy has often been :

- Moore's empirical Law will eventually breakdown at small enough physical scales.
- Quantum effects will likely become dominant by then.
- Prepare for new devices based on novel quantum concepts:
 - discrete energy levels
 - wave interference devices
 - tunnel devices
 - single electronics
 - quantum computing...

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What has actually happened

Year	1999	2001	2004	2008	2011	2014
MPU Gate Length (nm)	140	100	70			
Oxide thickness (nm)	19-25	15-19	12-15			
Drain extensions (nm)	42-70	30-50				

- Solution exists
- Solution Being Pursued
- No Known Solutions

Scaling of MOSFETs to decanano dimensions

(International Roadmap for Semiconductors - 1999 Edition)

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The silicon MOSFET scene

Conventional device modelling:
Empirical
Drift Diffusion
Hydrodynamic
Monte Carlo
Self-consistent via Poisson equation

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- The first moves:
- Ballistic transport
- Serious non-equilibrium effects
- Schrödinger-Poisson
- Quantum levels in channel
- Density gradient or Quantum potential
- Corrections
- Tunnelling & Tunneling
- Keep charge away from interfaces

And very recently, the road map has speeded up by 5 years!!

Robert Chau (Intel) 2001

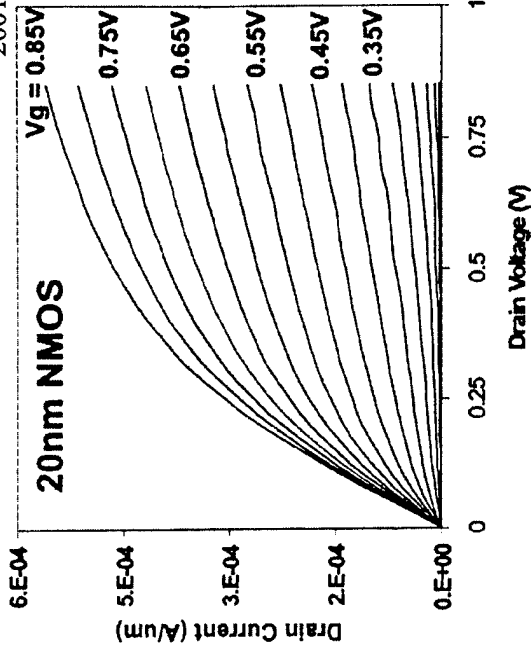
“High performance 30 nm gate length CMOS transistors operating at $V_{cc} = 0.85$ V demonstrated for 65 nm logic technology node, which will be ready for production in 2005.”

Part of Intel Road Map:

Year	2001	2003	2005	2007	2009
Node	150	90	65	45	30
Gate Length	70	50	30	20	15

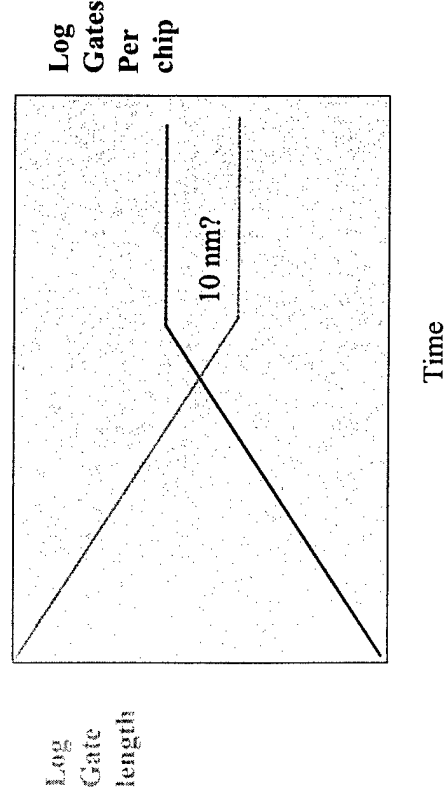
In production

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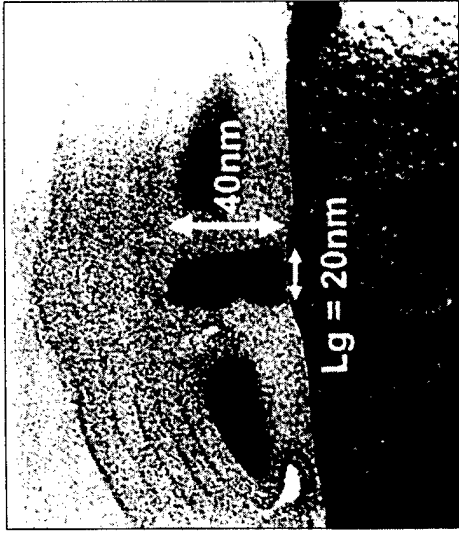


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Where is the limit and what QTT is needed if any?



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TEM Cross section of a 20 nm gate length NMOS transistor

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Single prototype MOSFETS devices have also been demonstrated with healthy I-V characteristics with gate lengths in 7-14 nm scales.

- What quantum/many-body effects occur in these devices and how do we model them?
- What are the implications of the dinosaur refusing to die for quantum devices?

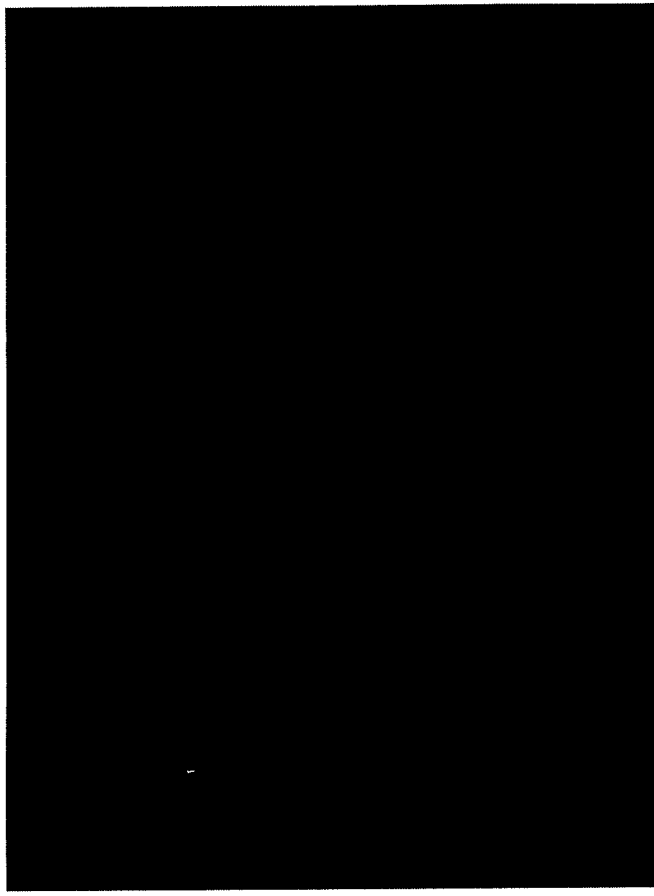
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Certainly, below 50 nm we expect strong **atomistic effects**.

The predicted high carrier densities also suggest strong **many-body effects**: Coulombic, correlations/fluctuations.

But at what level do we need to move to a true atomistic model, and where does the need for computational **chemistry methods** come in?

These questions are addressed in today's sessions and the role of the fundamental theoretical frameworks that underpin true QTT and fully quantum devices are discussed later.



The Nano-transistor

J. Bude, F. Baumann[‡], K. Evans-Lutterodt, A. Ghetti, S. Goodnick[†], J. Graul[‡],
M. Green, S. Hillenius, Y. Kim, J. Lyding⁺, W. Mansfield, D. Muller[‡], T. Sorsch,

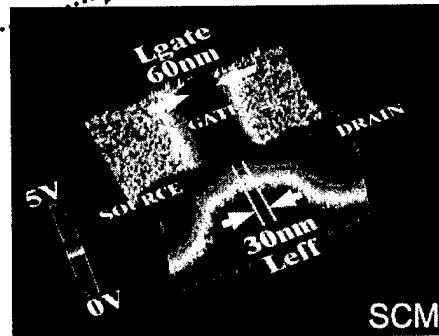
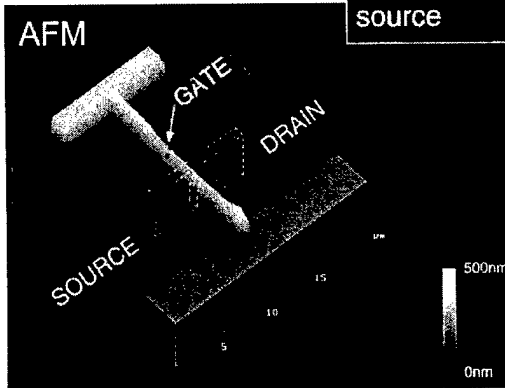
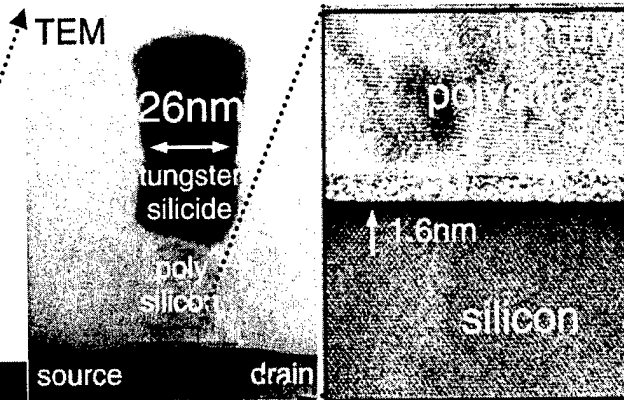
K. Timp⁺, R. Timp⁺, J. Yu⁺.

*Agere, Arizona State University[†], Bell Laboratories, Lucent Technologies[‡],
University of Illinois⁺*

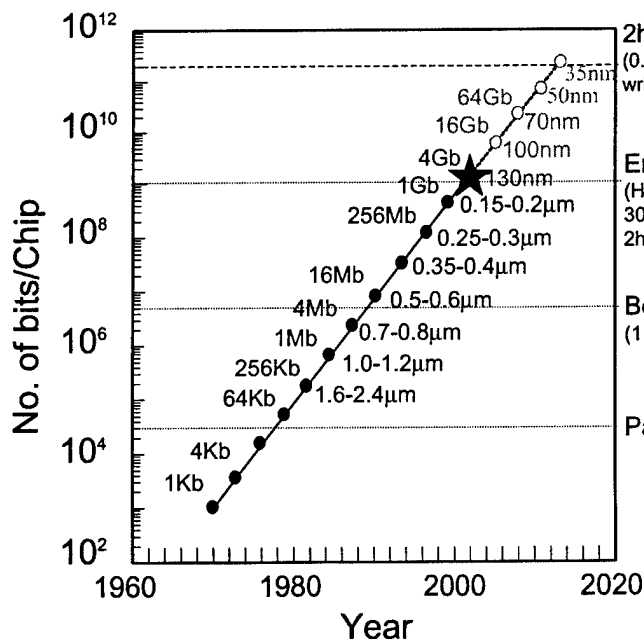
G. Timp
University of Illinois

Nano-transistor

$$\begin{aligned} L_{eff} &< 25\text{nm} \\ t_{ox} &= 1.3-1.6\text{nm} \\ r_f &< 20-30\text{nm} \end{aligned}$$



The Secret Behind Moore's Law



2hrs. HDTV
(0.01% of all
written knowledge)

Encyclopedia
(Human DNA,
30sec. HDTV,
2hrs. Audio CD)

Book
(1 min. Audio CD)

Page

Chip-to-chip
connection:
~1¢

On-chip
connection:
~40μ¢

2000 ITRS National Roadmap Projections

30nm research is targeted at a ~15 year horizon

Technology Generations

YEAR OF FIRST PRODUCT SHIPMENT	2001	2004	2008	2014
TECHNOLOGY GENERATIONS				
DENSE LINES (DRAM Half-Pitch) (nm)	130	90	60	30
Logic (High-Volume: MPU)				
Logic Transistors/chip (including SRAM)	67.3M	135M	539M	4.3B
Chip Frequency (GHz)				
On-chip local clock, high performance	2.1G	3.5G	7.1G	14.9G
TECHNOLOGY REQUIREMENTS				
Min. Logic Vdd(V) (desktop)	1.5	1.2	0.9	0.6
Nominal Ion (n/pMOS)@25C(mA/ μ m)	0.75/0.35		0.75/0.35	0.75/0.35
Nominal Ioff @25C (nA/ μ m)	5		80	160
tox equivalent (nm)	1.5-1.9		0.8-1.2	0.5-0.6
junction depth xj (nm)			16-26	8-13

Solutions exist

Solutions
being pursued

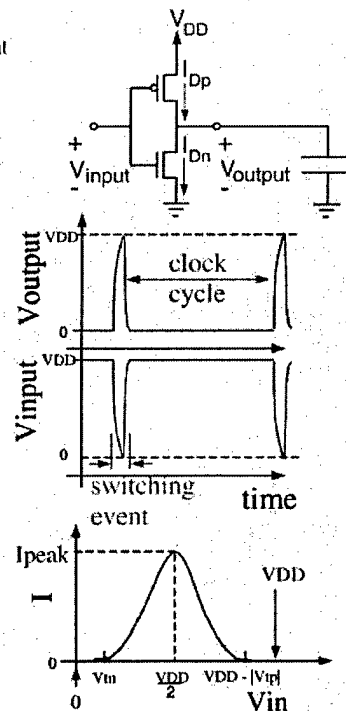
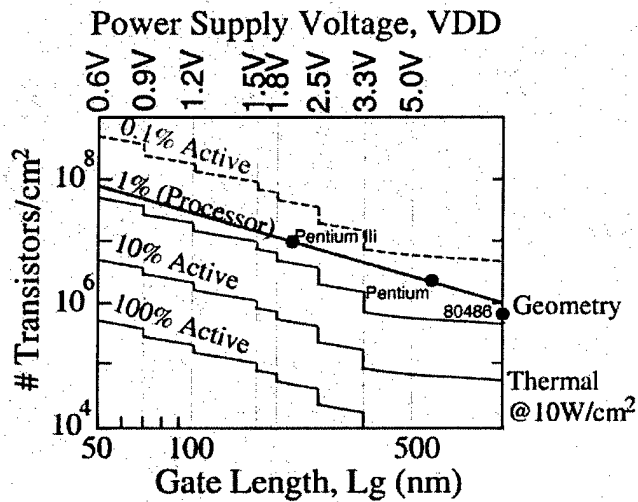
No known
solutions

*Drive current independent of technology

One of the Secrets Behind the Success of CMOS

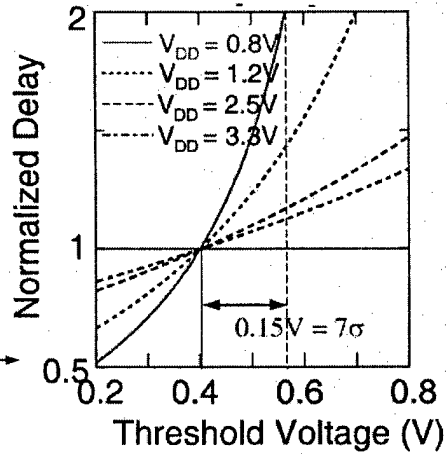
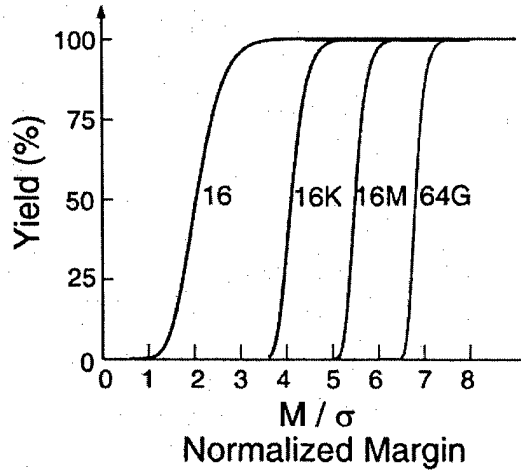
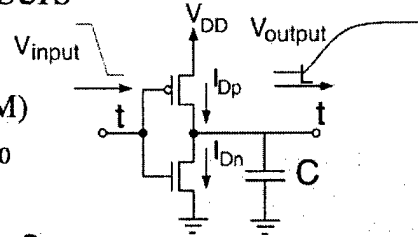
Active Power Dissipation: $f_c C V_{DD}^2 \sim \eta V_{DD} I_{Dsat}$

Stand-by Power Dissipation: $V_{DD} I_{leakage}$

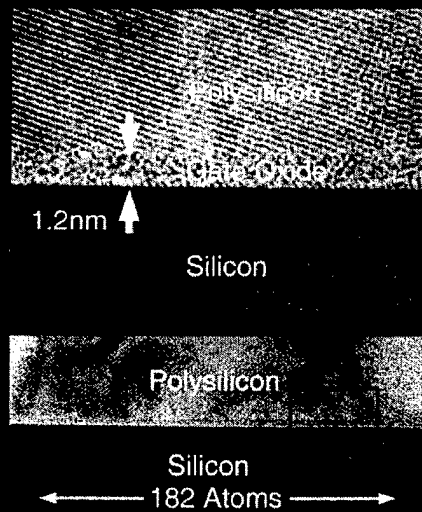
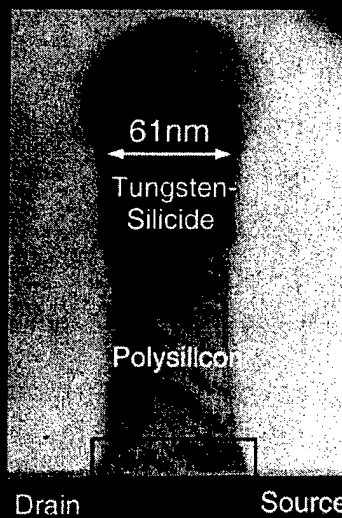


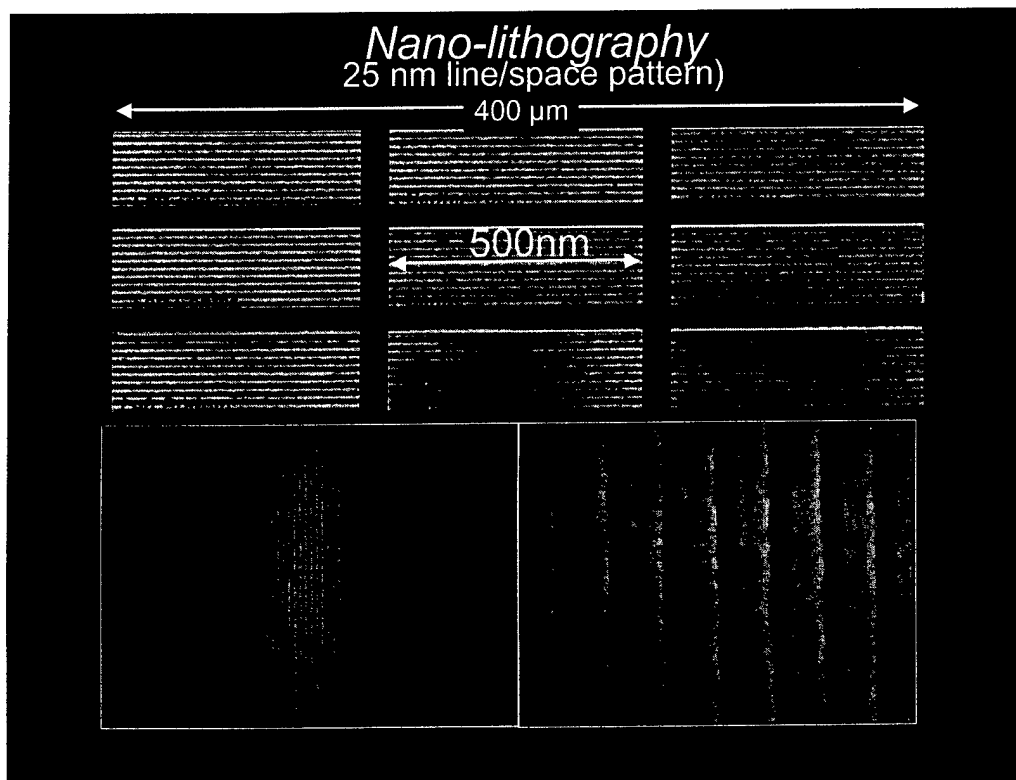
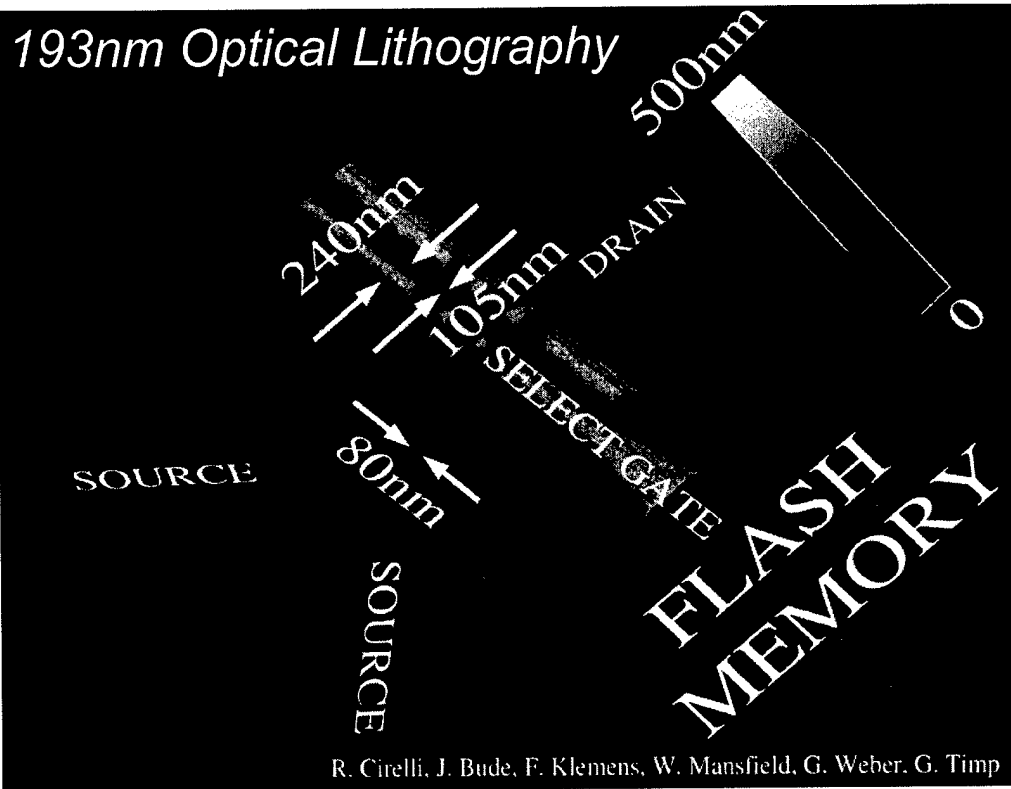
The Tyranny of Large Numbers

circuit margin M : $I_{D0}(1-M) < I_D < I_{D0}(1+M)$
 process control: standard deviation $= \sigma I_{D0}$

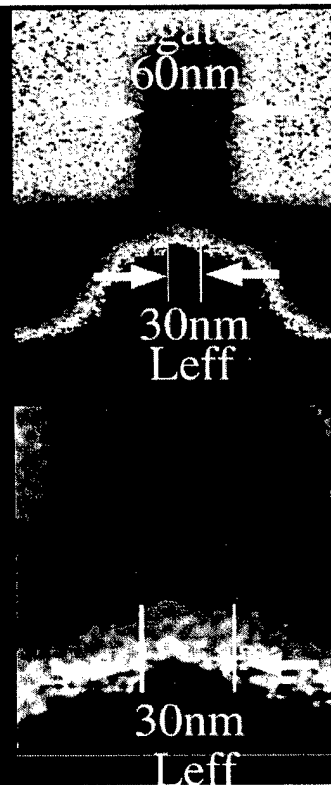
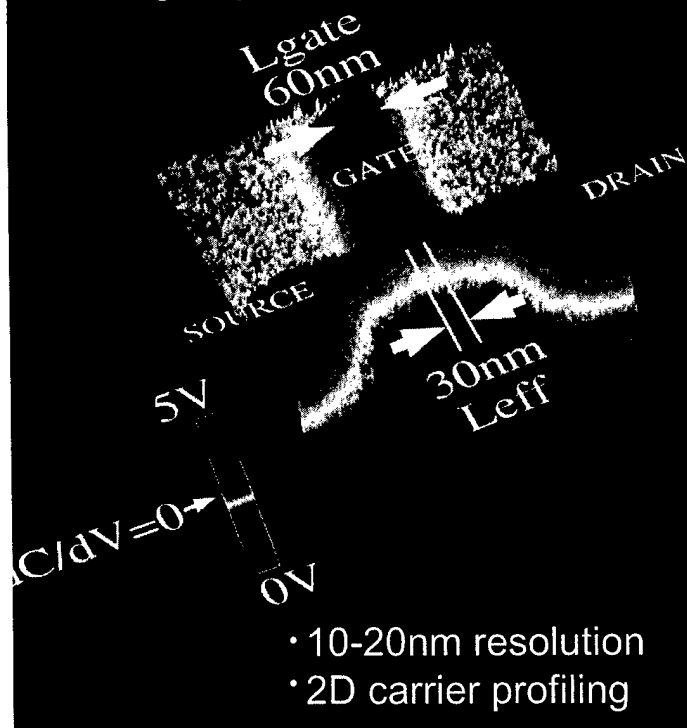


Nano-Transistor

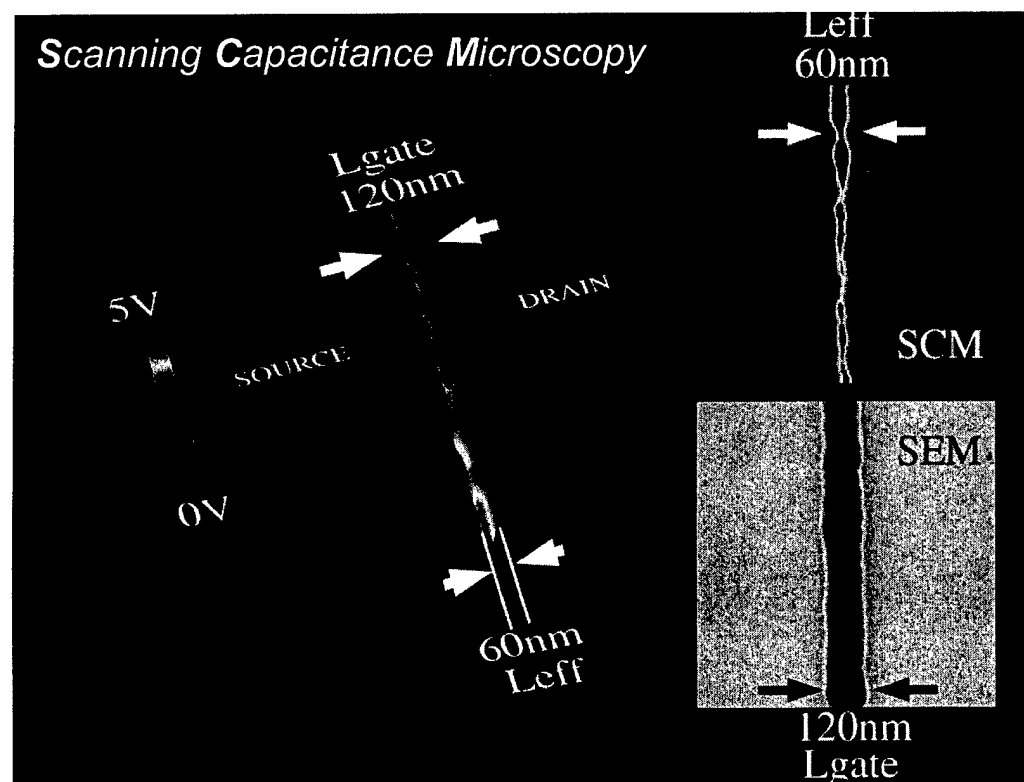




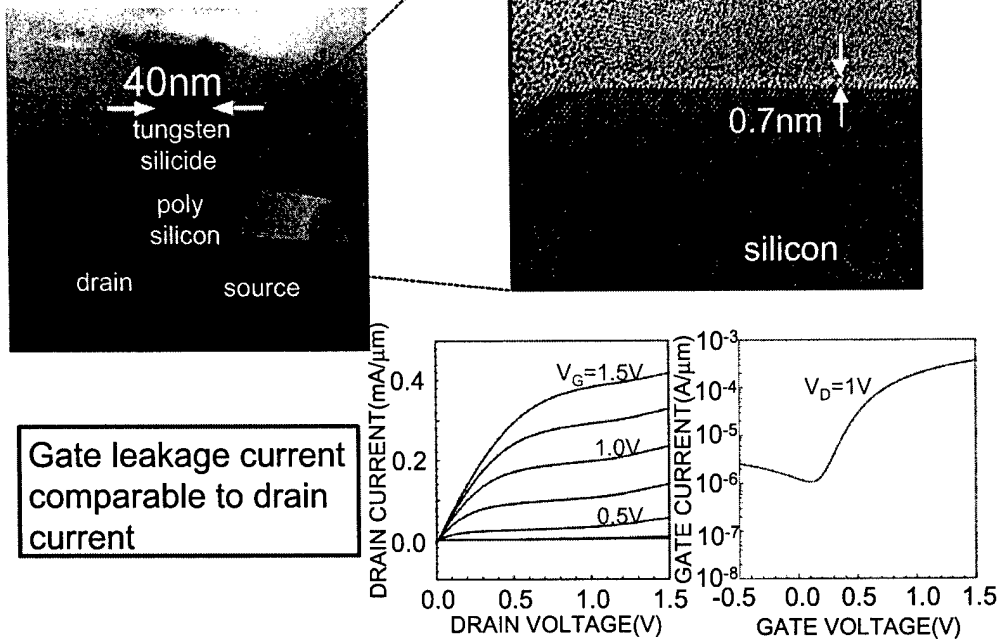
Scanning Capacitance Microscopy



Scanning Capacitance Microscopy

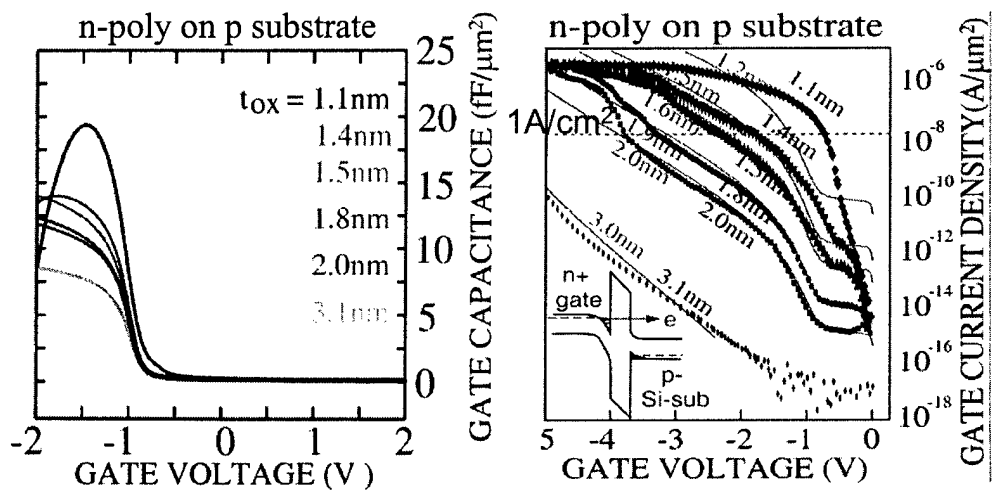


The Nanotransistor: Hyper-thin Oxide

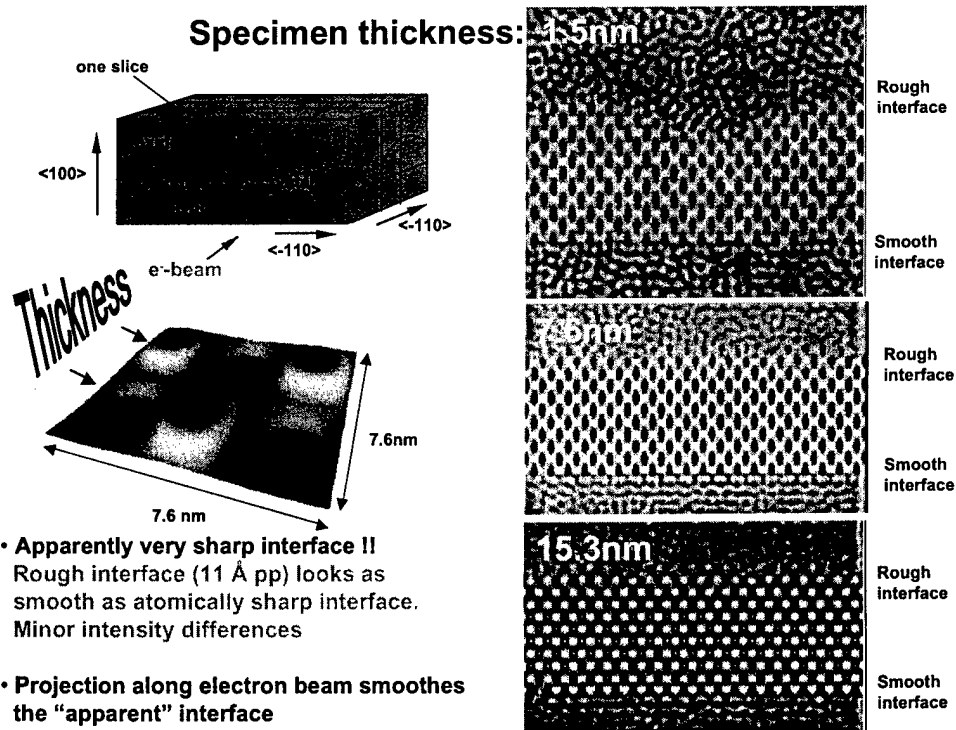
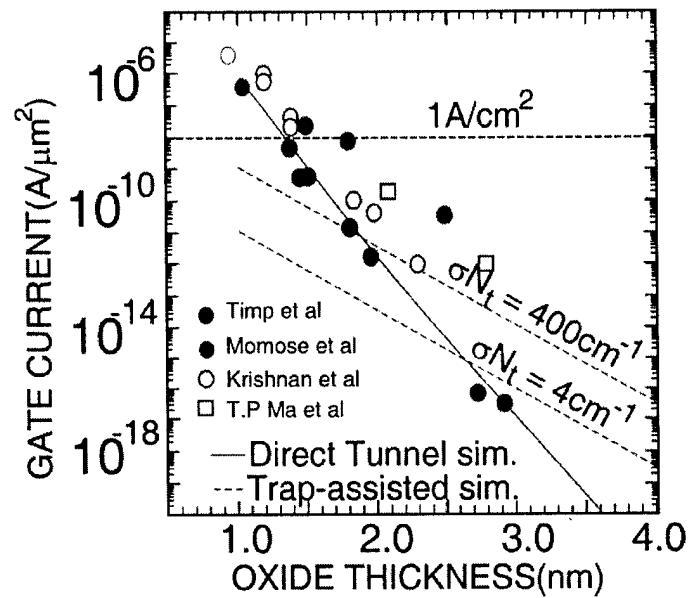


Capacitance Improvement with decreasing t_{ox}

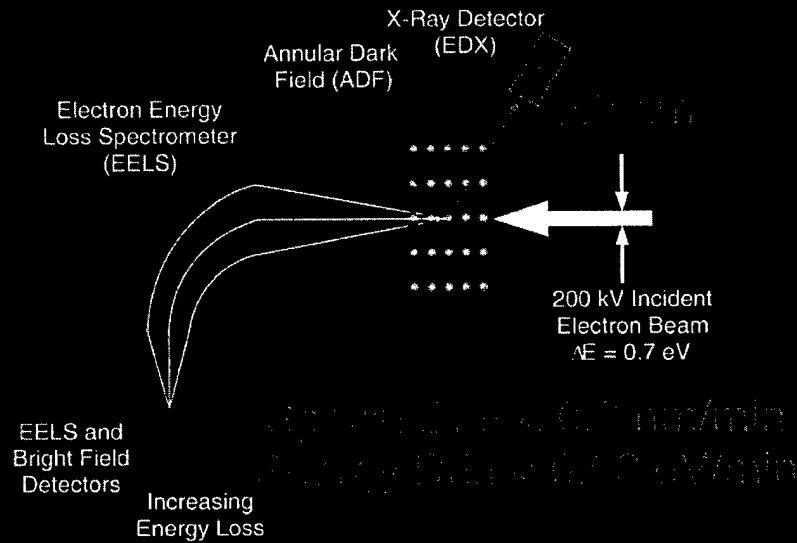
$$I_D = WC_G(V_G - V_T)v_o$$



- exponential increase in leakage with increasing V_G
- exponential increase in leakage with t_{ox}

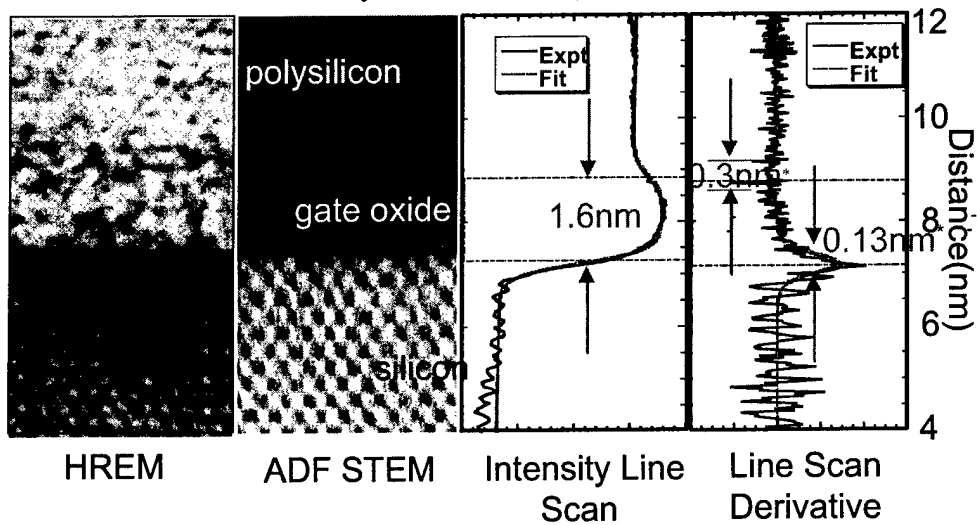


Scanning Transmission Electron Microscopy/ Electron Energy Loss Spectroscopy

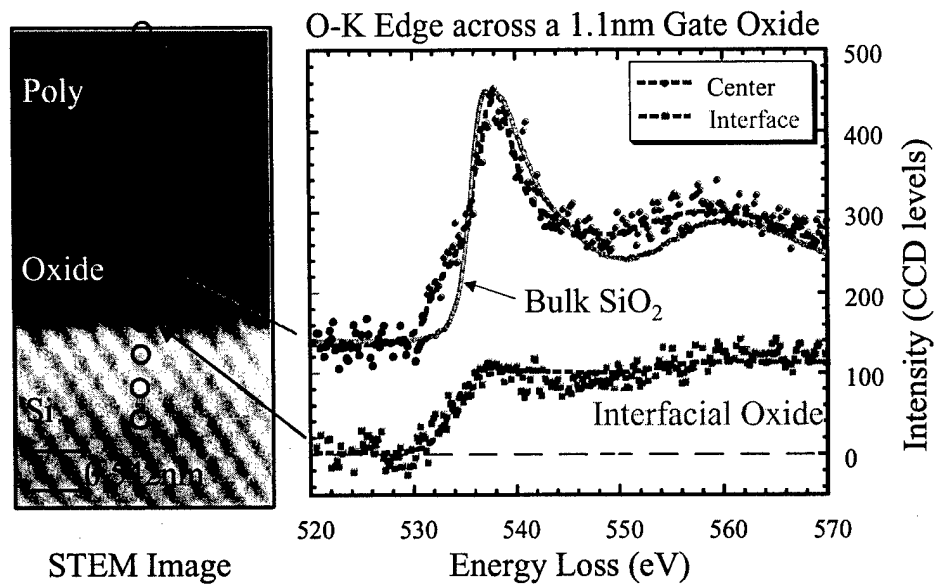


What does $t_{ox} = 1\text{nm}$ look like?

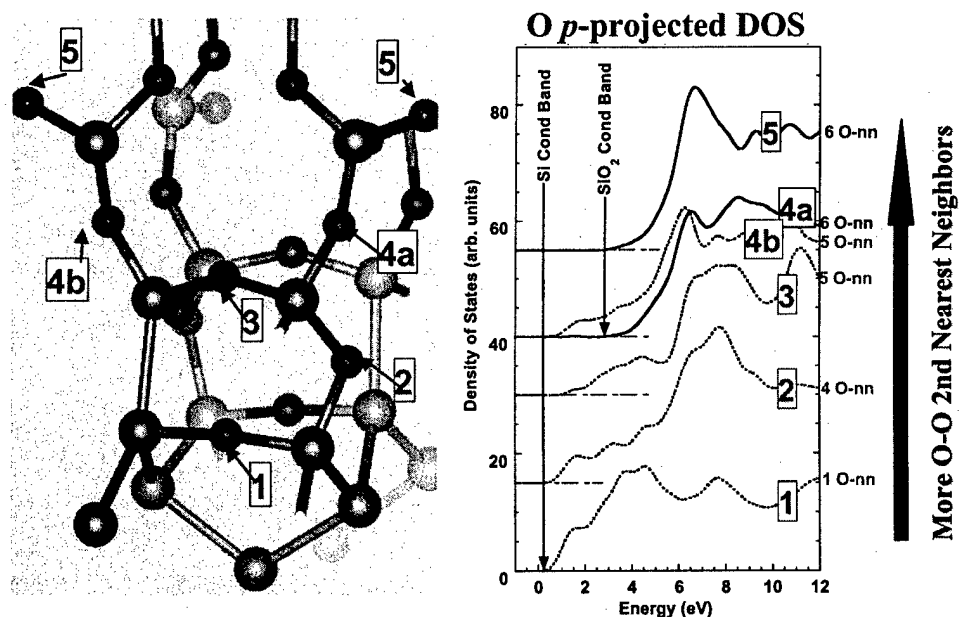
Nominally 1.3nm thick gate oxide



Oxygen Bonding from STEM/EELS (Chemistry on an Atomic Scale)



A Model Si/SiO₂ Interface



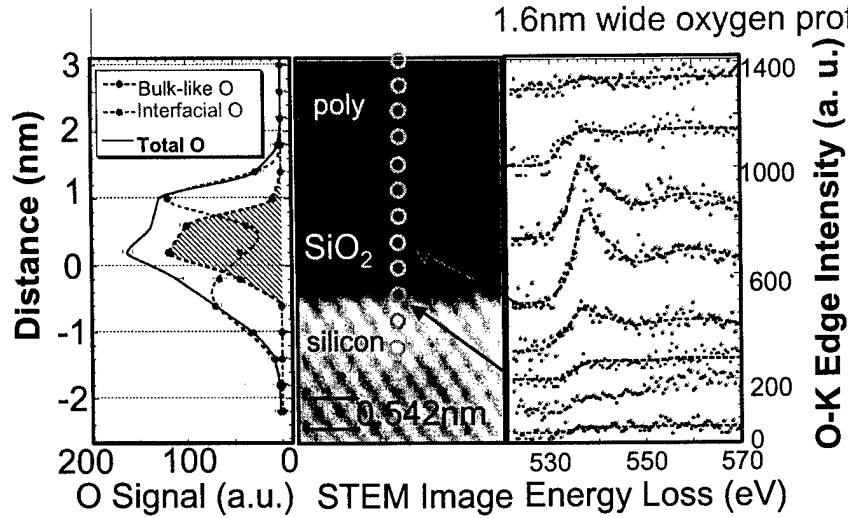
(Model-3 of A. Pasquarello, M. S. Hybertson, R. Car, PRB)

When is $t_{\text{OX}} = 1\text{nm}$?

STEM/EELS

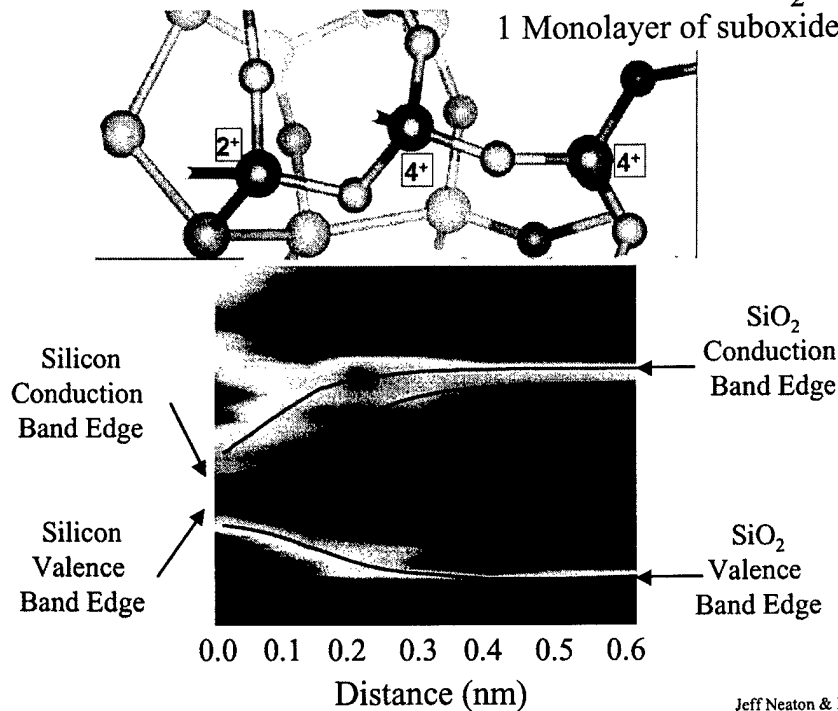
Nominally 1.1nm thick gate oxide: 0.8-1.0nm SiO_2

1.6nm wide oxygen profile

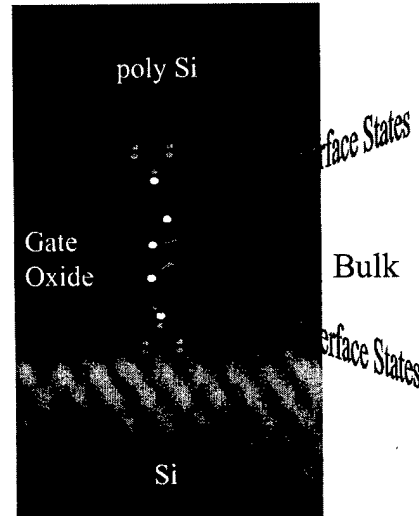
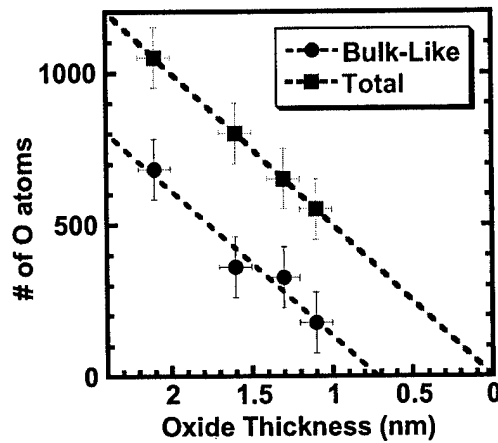


Evolution of the O-DOS from Si to SiO_2

1 Monolayer of suboxide

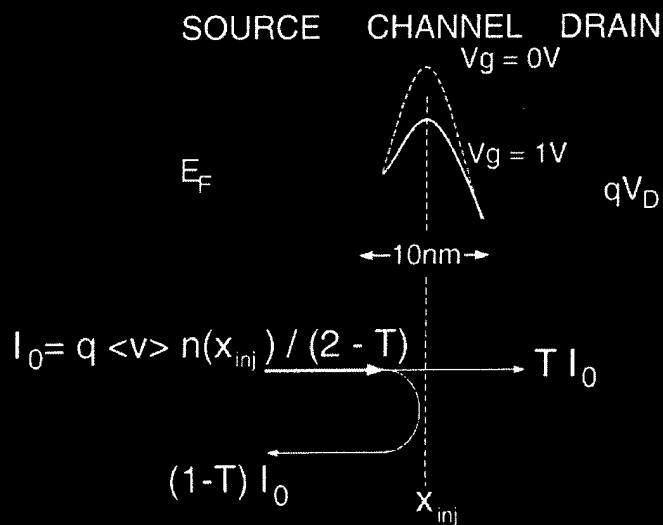


The End of the Roadmap for SiO₂



There will be no more bulk-like bonding
when the Oxide is less than 0.7 nm

The MOSFET from Landauer's Perspective

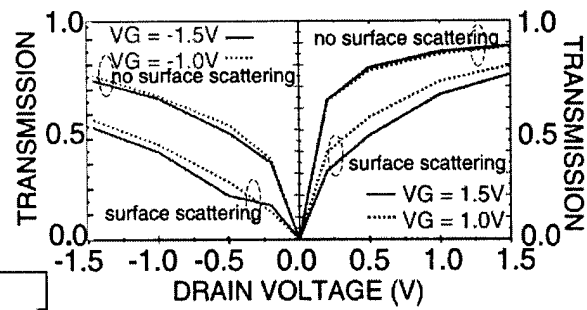
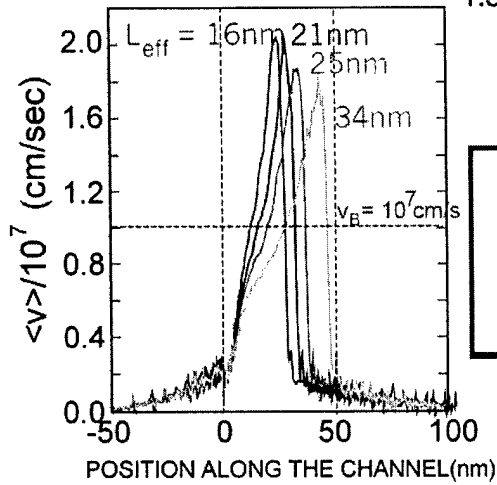


$$I_D / W = T I_0 = q \langle v \rangle n(x_{inj}) T / (2 - T)$$

Lundstrom, S. Datta, F. Assad,
Frank, Laux and Fischetti

The Ballistic Nanotransistor

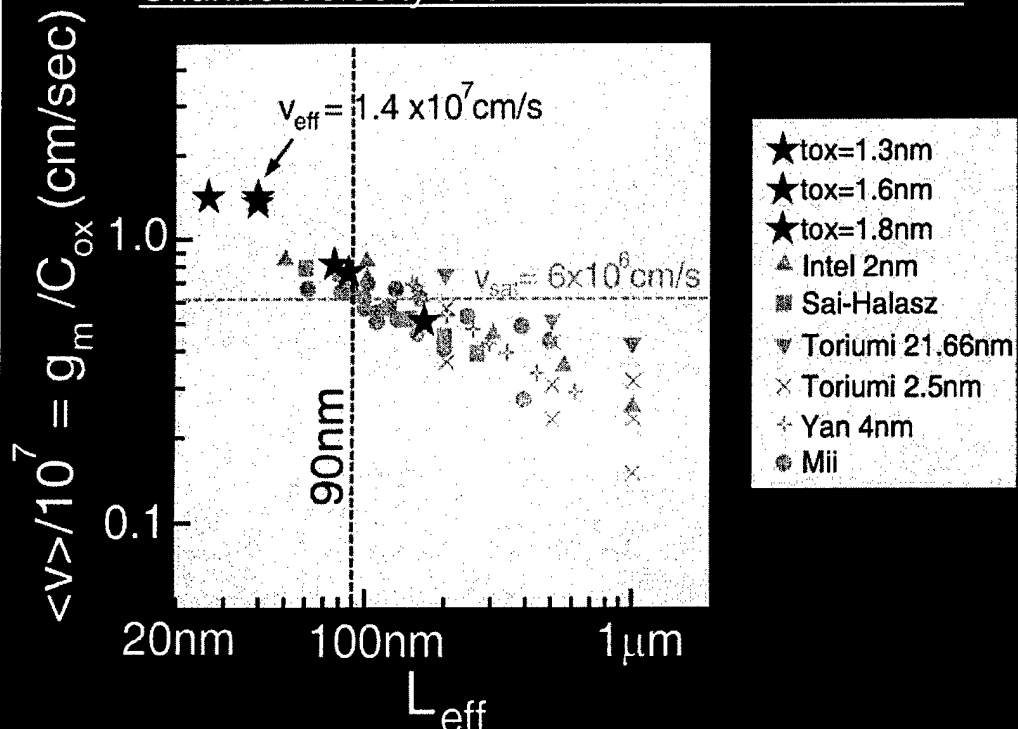
($V_g = V_{ds} = 1V$)



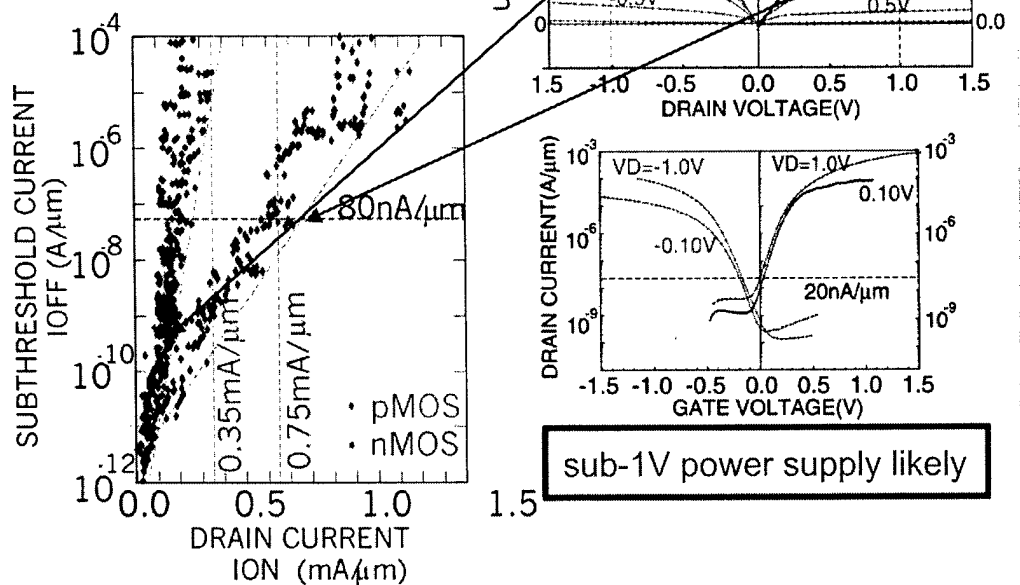
$\langle v \rangle$ exceeds bulk saturation

the velocity (averaged over L_{eff}) increases as L_{eff} decreases.

Channel velocity exceeds bulk saturation value

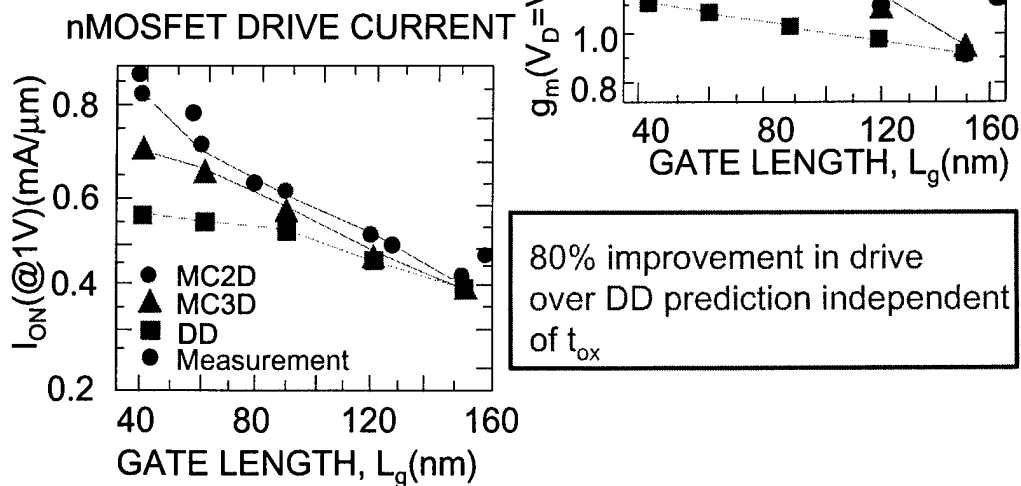


Nanotransistor DC Performance



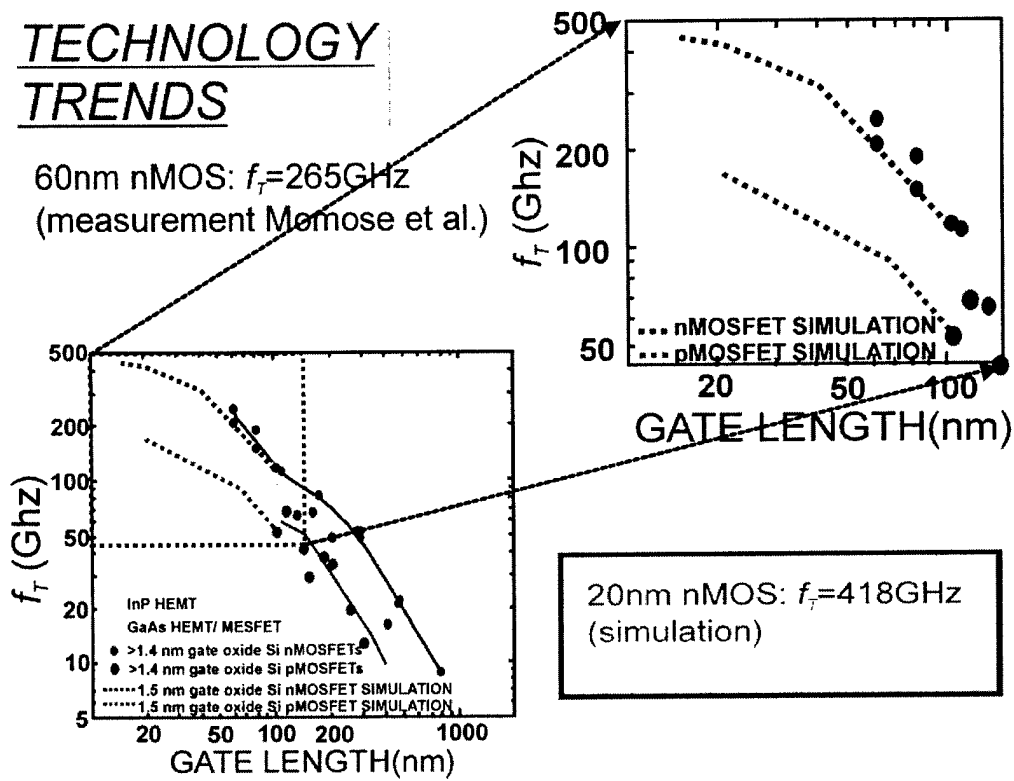
Simulation versus Measurement

$V_{DD} = 1V @ V_T = 0.4V$;
 $t_{ox} = 1.5nm$ (ellips.)



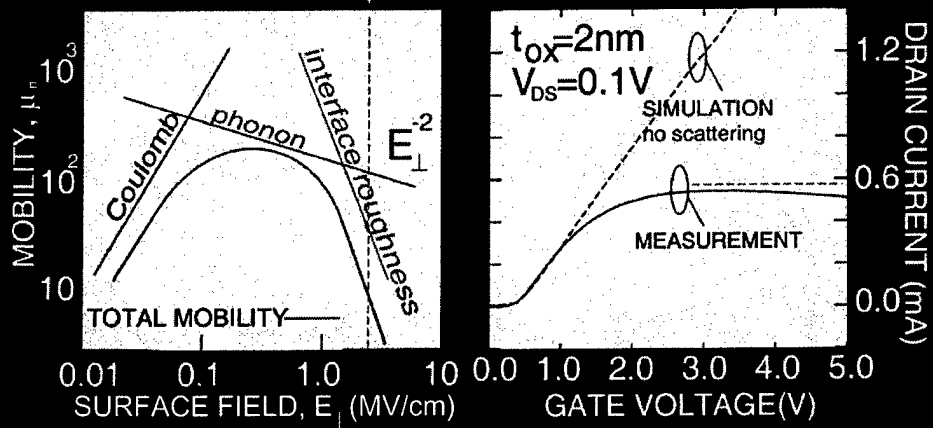
TECHNOLOGY TRENDS

60nm nMOS: $f_T=265\text{GHz}$
(measurement Mamose et al.)



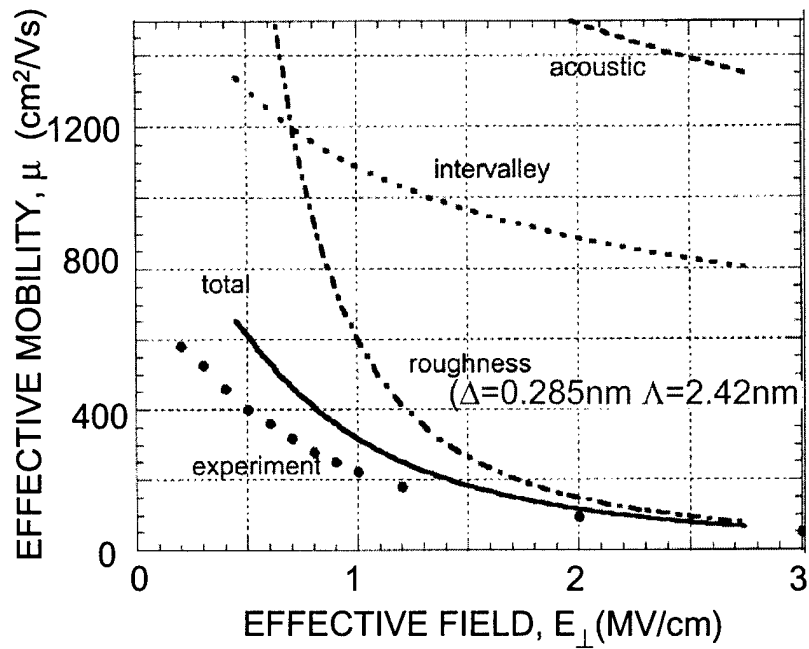
Roughness Scattering and Mobility

50nm



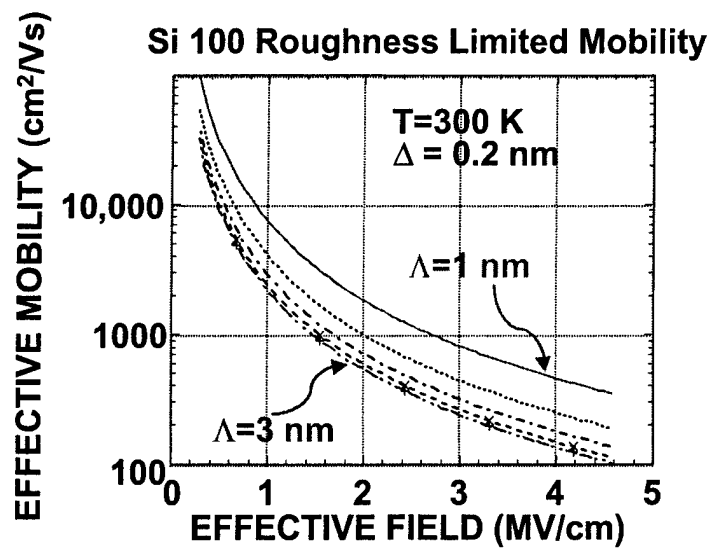
• ostensibly roughness scattering predominates for $E_{\perp} > 0.5\text{MV/cm}$

MOBILITY RESOLVED

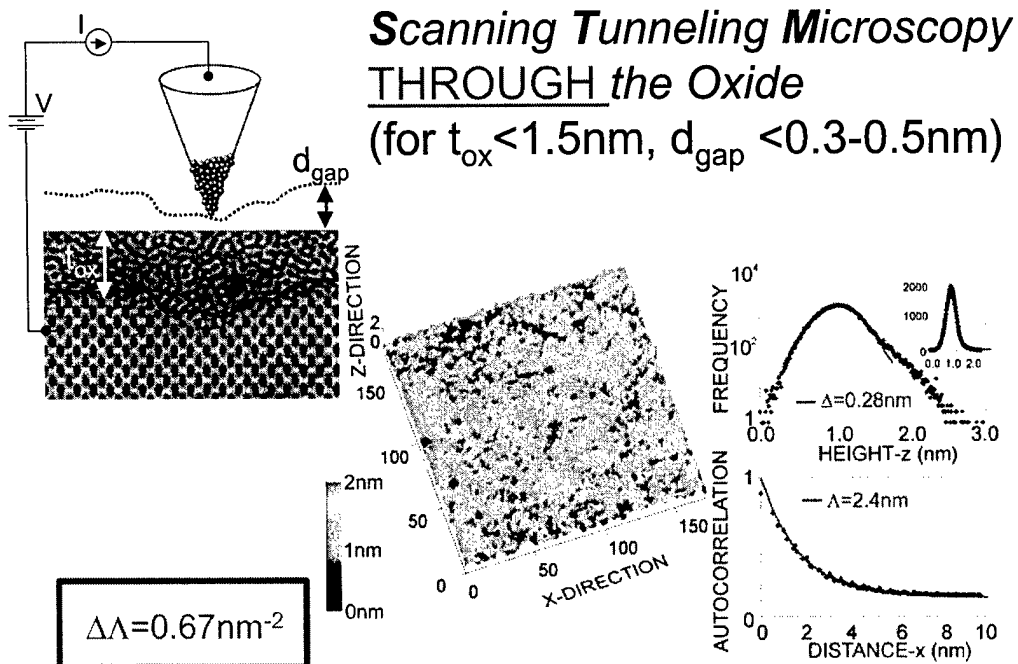


S. Goodnick

Roughness limited mobility decreases as inverse square of surface field, and as $(\Delta\Lambda)^2$:

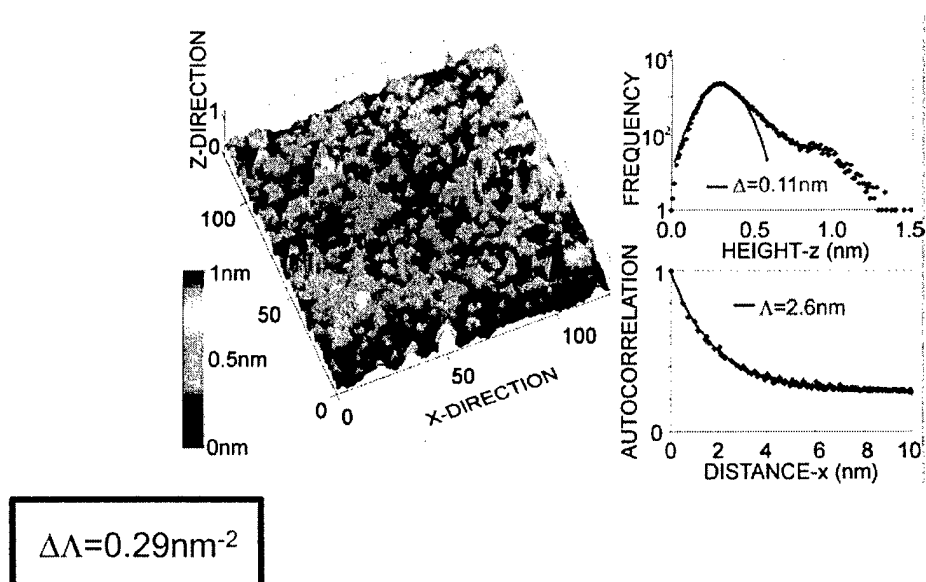


S. Goodnick

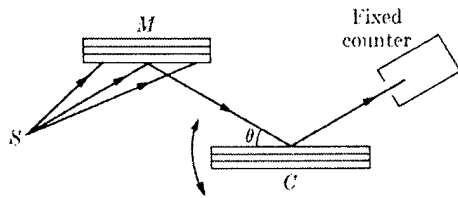


J. Yu and J. Lyding

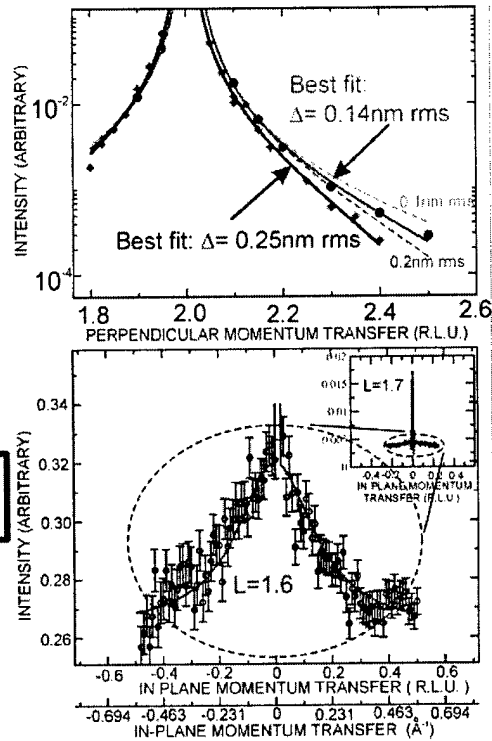
Scanning Tunneling Microscopy
THROUGH the Oxide



X-ray DIFFRACTION



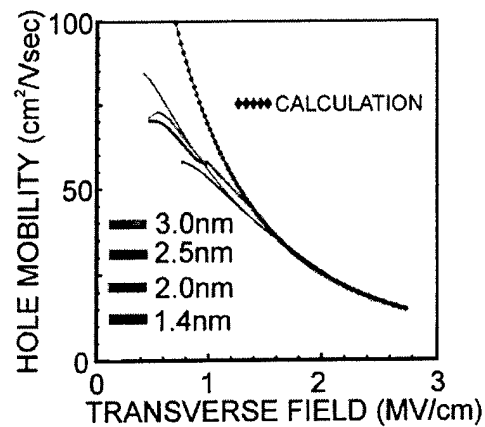
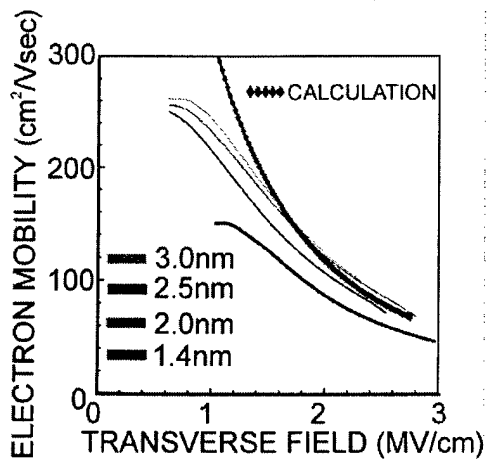
$$\Delta\Lambda = 0.67 \text{ nm}^{-2} \Rightarrow 0.35 \text{ nm}^{-2}$$



K. Evans-Lutterodt

MOBILITY MEASUREMENTS VS. CALCULATION

$\Delta = 0.28 \text{ nm}$, $\Lambda = 2.4 \text{ nm}$



Completely constrained mobility Calculation on "rough" MOSFETS correlates well with measurements.

The Highlights

- *Homeric challenges face bulk silicon technology, but the same is true of all the pretenders to the throne.*
- *Some of these challenges stem from “atomic” sensitivity (e.g. the gate oxide)*
- *As process control decreases; circuit margins must increase at an even faster rate.*

Long-range Coulomb interactions, coupled plasmon/insulator-TO-phonon modes, and electron mobility in Si inversion layers

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IBM Research Division
T. J. Watson Research Center
Yorktown Heights, NY 10598

in collaboration with Deborah Neumayer and Eduard Cartier

June 2001

Outline

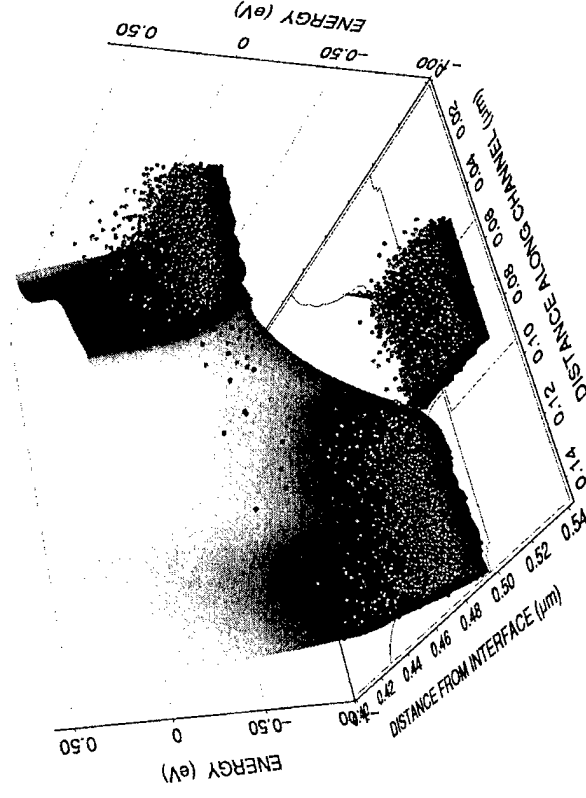
- Long-range Coulomb interactions (a short review):
 - g_m and v_{eff} degradation in short-channel/thin-oxide MOSFETs (semiclassical)
 - mobility degradation in thin-oxide MOSFETs (quantum)
- High- κ insulators:
 - Simple picture
 - Dispersion of coupled insulator-TO-phonons/plasmon modes
 - Material parameters
 - Scattering strength
 - Electron mobility in inversion layers
 - Effect of SiO₂ interfacial layers

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Long-range Coulomb interactions in small MOSFETs

- Source, drain, and gate regions are high-density electron gases
- S/D separation (i.e., channel length) is shrinking below the Debye length of the channel
- Gate needs to be 1 nm (or less!) away from the channel
- Collective 'fluctuations' in S/D perturb electrons in the channel (electron/bulk-plasmon interactions)
- Collective fluctuations in gate (interface plasmons) cause Coulomb drag

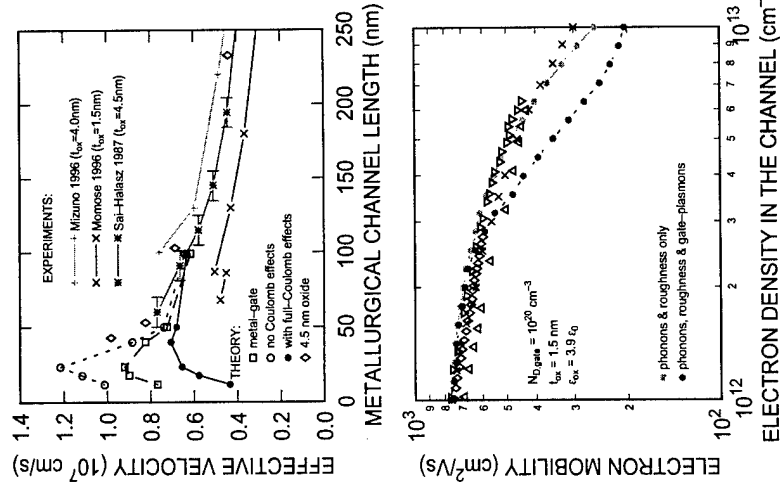


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Coulomb interactions and device speed

- S/D interactions thermalize carriers, build high-energy tails, increase momentum-loss *indirectly*
- Gate-induced Coulomb-drag subtracts momentum *directly*
- Lower transconductance, lower mobility



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High- κ insulators

- VLSI scaling demands insulator thicknesses t approaching 1 nm for (near) future devices. But it's hard to go below 1.5 nm:
 - Gate tunneling draws too much stand-by power
 - Reliability (wear-out, breakdown) is a debated issue, but 1 nm seems to be 'too much' even to the optimist
- Lot of activity on insulators with high dielectric constant ($\epsilon_{ox}^0 = \kappa \epsilon_0$), so that the gate capacitance

$$C_g = \frac{\epsilon_{ox}^0}{t} = \frac{\epsilon_{SiO_2}^0}{t_{eq}}$$

corresponds to the required small t_{eq} .

- Materials considered:
 - Metal-oxides (HfO_2 , ZrO_2 , Al_2O_3 ...), with $\kappa \approx 10$ -30
 - Silicates ($ZrSiO_4$, $HfSiO_4$), with $\kappa \approx 15$ -25
 - Perovskites, rare-earth oxides, with $\kappa \approx 100$
 - Nitrides (AlN , Si_3N_4), with $\kappa \approx 10$
 - Ferroelectrics ($\kappa \sim 10^3$)... but that's another story...

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High- κ insulators: A simple picture

- $\epsilon_{ox}^0 = \epsilon_{ion}^0 + \epsilon_{el}^0$
- $\epsilon_{el}^0 \propto E_{gap}^{-1}$... so, not much room left for insulators (must have large E_{gap} !)
- Large ϵ_{ion}^0 due to polarizable bonds (typically, metal-oxygen), 'soft' bonds associated with 'soft' TO-phonons
- ϵ_{ion}^0 not too large (ions move slowly)
- So, for large- κ materials, ϵ_{ox}^0 much larger than ϵ_{ox}^∞
- Insulator-TO-modes at interface (surface-optical modes, SO) scatter electrons with strength $\propto$:

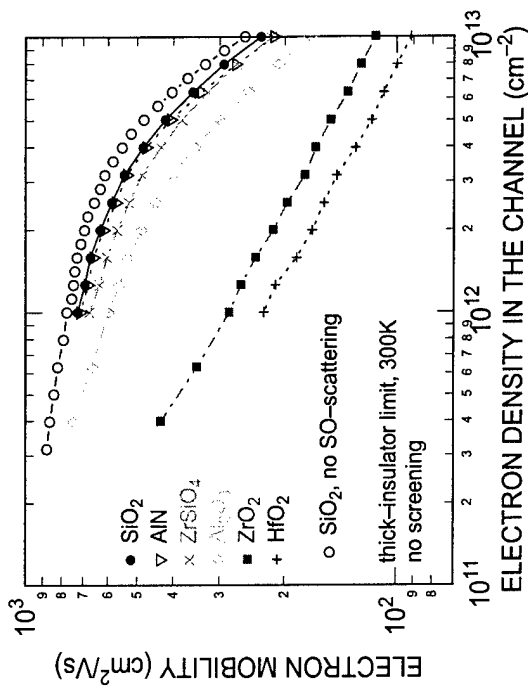
$$\hbar\omega_{SO} \left[\frac{1}{\epsilon_{Si}^\infty + \epsilon_{ox}^\infty} - \frac{1}{\epsilon_{Si}^\infty + \epsilon_{ox}^0} \right]$$

(Fröhlich + images, Wang-Mahan, 72)

- Small effect ($\sim 5\%$) in SiO_2 :
 - ϵ_{ox}^0 (≈ 3.9) not much larger than ϵ_{ox}^∞ (≈ 2 to 2.5) because Si-O bond is 'hard'
 - Small Fröhlich-like coupling constant
 - Hard bonds imply 'stiff' TO-modes, $\hbar\omega_{TO} \gg k_B T$: too energetic to be emitted by thermal electrons, too energetic to be thermally excited (no absorption).
- But large effects in high- κ materials:
 - Large ϵ_{ox}^0 due to polarization of 'soft' bonds
 - Large Fröhlich-like coupling constant
 - Soft bonds imply 'soft' TO-modes, $\hbar\omega_{TO} \sim k_B T$: energy low enough to allow emission by thermal electrons, low enough to be thermally excited (absorption possible).

High- κ insulators: A simple picture

- Use Wang-Mahan (Hess-Vogl, 79; Moore-Ferry, 80) matrix element
- Single insulator/Si interface (infinitely-thick insulator)
- No electronic screening
- Add Si-phonons and (shamelessly) empirical surface-roughness (Matthiessen's rule to fit experiments for SiO_2 at $n_s = 10^{13} \text{ cm}^{-2}$)



Interface modes: A better model for thin-insulator MOSFETs

- Consider a depleted-Si/insulator/inverted-Si MOS system
- Look for longitudinal eigenmodes (TM-modes or p -waves) of Maxwell's equations: $\epsilon_{tot}(\mathbf{Q}, \omega) = 0$.
These give $\mathbf{E} \neq 0$ even when $\mathbf{D} = 0$, while transverse modes ($\epsilon_{tot}(\mathbf{Q}, \omega) = \infty$) give $\mathbf{E} = 0$ for any $\mathbf{D}$, so no scattering field.
- Use non-retarded limit (OK for wavelength $\sim \lambda_F$), so unknown is electrostatic potential satisfying Laplace equation.
- Chose model dielectric functions (long-wavelength-limit, two insulator TO modes).
- Impose continuity of $\mathbf{E}_{\parallel}$ and $\mathbf{D}_{\perp}$ at two interfaces
→ homogeneous linear problem of 4 equations in 4 unknowns
→ 3 unknowns expressed in terms of the remaining one (to be fixed).
- Secular equation (i.e. vanishing of determinant) gives dispersion of coupled modes (4+2 of them).
- Determine phonon- and plasmon-content of each mode.
- Normalize potential (i.e. determine last unknown) somehow... (semiclassically). This defines the scattering potential.
- Account for Landau-damping (lost in taking a real ϵ in the long-wavelength limit).
- Determine the subband structure (triangular-well).
- Compute the mobility (Kubo-Greenwood).

Interface modes

- Consider MOS system:
 - (poly-)Si gate, $\epsilon_g(\mathbf{q}, \omega)$, for $z \leq 0$
 - insulator, $\epsilon_{ox}(\omega)$, for $0 < z \leq t$
 - Si 2DEG, $\epsilon_s(\mathbf{Q}, \omega)$, for $z > t$
- Write electrostatic potential (non-retarded limit):

$$\phi(\mathbf{R}, z, t) = \sum_{\mathbf{Q}} \phi_{\mathbf{Q}, \omega}(z) e^{i\mathbf{Q} \cdot \mathbf{R}} e^{i\omega t}$$

with

$$\phi_{\mathbf{Q}, \omega}(z) = \begin{cases} a_{\mathbf{Q}, \omega} e^{Qz} & (z < 0) \\ b_{\mathbf{Q}, \omega} e^{-Qz} + c_{\mathbf{Q}, \omega} e^{Qz} & (0 \leq z < t_{ox}) \\ d_{\mathbf{Q}, \omega} e^{-Qz} & (z \geq t_{ox}) \end{cases}$$

- Solve Laplace equation:

$$\frac{d^2 \phi_{\mathbf{Q}, \omega}(z)}{dz^2} - Q^2 \phi_{\mathbf{Q}, \omega}(z) = 0$$

with boundary-conditions (BCs):

$$\begin{cases} E_{\parallel, \omega}(\mathbf{R}, z = 0^-, t) = E_{\parallel, \omega}(\mathbf{R}, z = 0^+, t) \\ E_{\parallel, \omega}(\mathbf{R}, z = t_{ox}^-, t) = E_{\parallel, \omega}(\mathbf{R}, z = t_{ox}^+, t) \\ D_{z, \omega}(\mathbf{R}, z = 0^-, t) = D_{z, \omega}(\mathbf{R}, z = 0^+, t) \\ D_{z, \omega}(\mathbf{R}, z = t_{ox}^-, t) = D_{z, \omega}(\mathbf{R}, z = t_{ox}^+, t) \end{cases}$$

- BCs become:

$$\begin{cases} a_{\mathbf{Q}, \omega} \\ b_{\mathbf{Q}, \omega} e^{-Qt} + c_{\mathbf{Q}, \omega} e^{Qt} \\ \epsilon_g(\mathbf{Q}, \omega) a_{\mathbf{Q}, \omega} \\ \epsilon_{ox}(\omega) [b_{\mathbf{Q}, \omega} e^{-Qt} - c_{\mathbf{Q}, \omega} e^{Qt}] \end{cases} \begin{aligned} &= b_{\mathbf{Q}, \omega} + c_{\mathbf{Q}, \omega} \\ &= d_{\mathbf{Q}, \omega} e^{-Qt} \\ &= \epsilon_{ox}(\omega) (c_{\mathbf{Q}, \omega} - b_{\mathbf{Q}, \omega}) \\ &= \epsilon_s^{(2D)}(\mathbf{Q}, \omega) d_{\mathbf{Q}, \omega} e^{-Qt}, \end{aligned}$$

- Eigenmodes $\omega(\mathbf{Q})$ given by secular equation:

$$\begin{aligned} &e^{Qt} [\bar{\epsilon}_g(\mathbf{Q}, \omega) + \epsilon_{ox}(\omega)] [\bar{\epsilon}_s^{(2D)}(\mathbf{Q}, \omega) + \epsilon_{ox}(\omega)] \\ &- e^{-Qt} [\bar{\epsilon}_g(\mathbf{Q}, \omega) - \epsilon_{ox}(\omega)] [\bar{\epsilon}_s^{(2D)}(\mathbf{Q}, \omega) - \epsilon_{ox}(\omega)] = 0 \end{aligned}$$

"Effective" dielectric functions

- Gate:
 - RPA:

$$\bar{\epsilon}_g(Q, \omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} d\left(\frac{q_z}{Q}\right) \frac{\epsilon_g(Q, q_z; \omega)}{1 + (q_z/Q)^2} \approx \epsilon_g(q = \sqrt{2}Q, \omega)$$

- Long-wavelength limit:

$$\epsilon_g(\omega) = \epsilon_{Si}^{\infty} \left(1 - \frac{\omega_{p,g}^2}{\omega^2} \right)$$

Getting $\omega_{p,g}$ is not trivial: Electron density in the depleted gate is at least z -dependent. Use either surface-density or Q -dependent average.

- Substrate:
 - RPA:

$$\bar{\epsilon}_s^{(2D)}(Q, \omega) = \epsilon_{Si}^{\infty} \left\{ 1 + e^{2Qt} \sum_{\mu\mu'} \frac{\beta_{\mu\mu'}(Q, \omega)}{2Q} \Phi_{Q,\omega;\mu\mu'} \Phi_{Q,\mu\mu'}^{(0)} \right\}$$

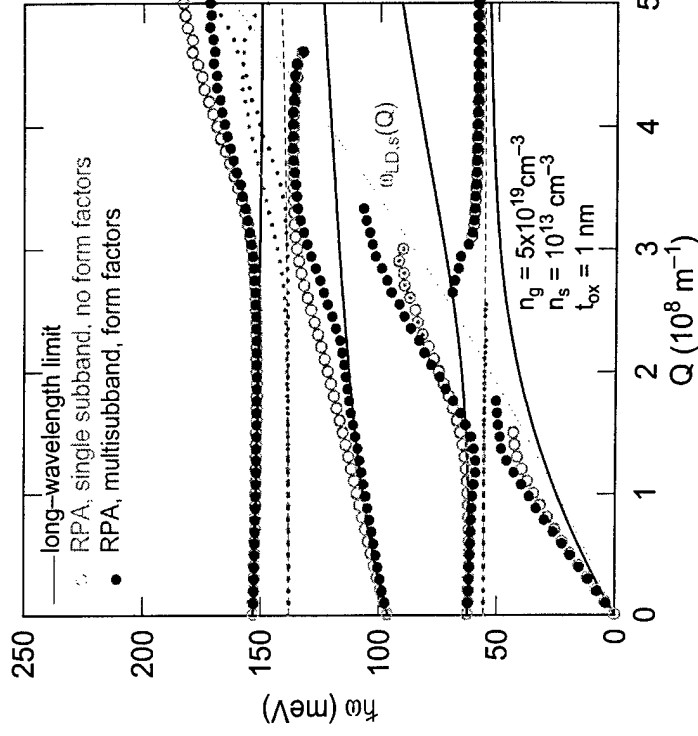
- Long-wavelength limit:

$$\epsilon_s^{(2D)}(Q, \omega) = \epsilon_{Si}^{\infty} \left[1 - \frac{\omega_{p,s}(Q)^2}{\omega^2} \right]$$

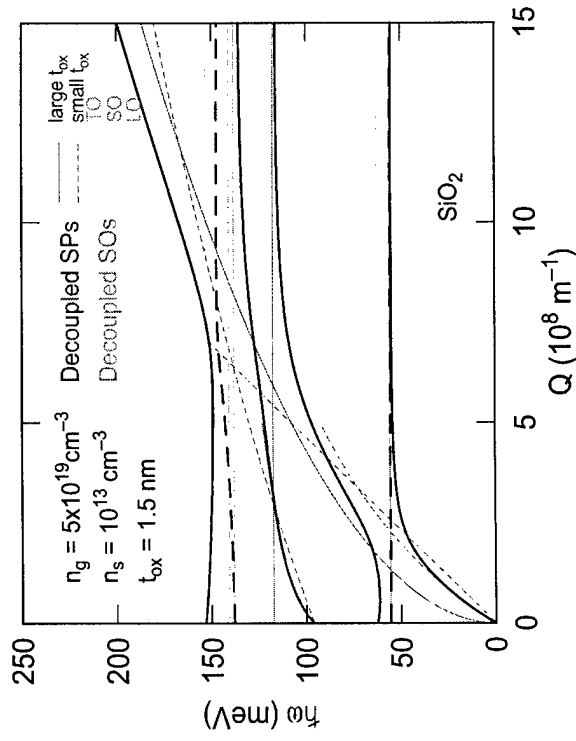
- Insulator (taking only 2 TO modes):

$$\epsilon_{ox}(\omega) = \epsilon_{ox}^{\infty} + (\epsilon_{ox}^i - \epsilon_{ox}^{\infty}) \frac{\omega_{TO2}^2}{\omega_{TO2}^2 - \omega^2} + (\epsilon_{ox}^0 - \epsilon_{ox}^i) \frac{\omega_{TO1}^2}{\omega_{TO1}^2 - \omega^2}$$

RPA vs long-wavelength-limit for Si/SiO₂/Si



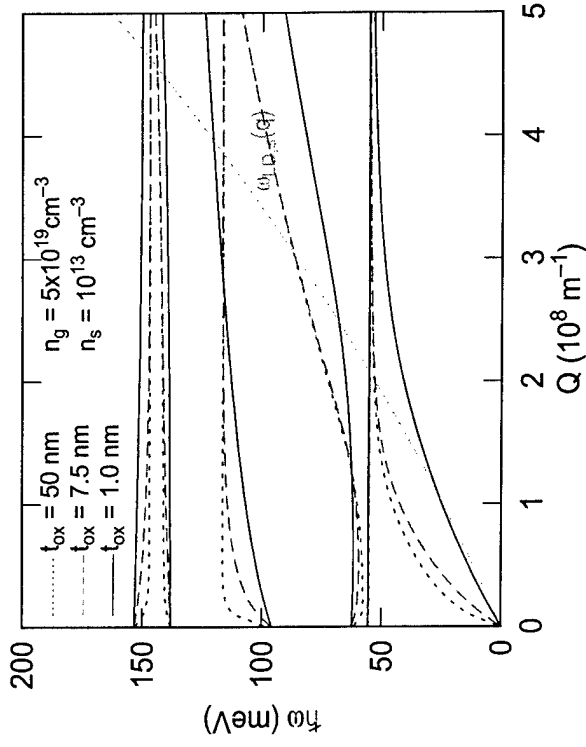
**Coupled modes for Si/SiO₂/Si system:
Effect of coupling**



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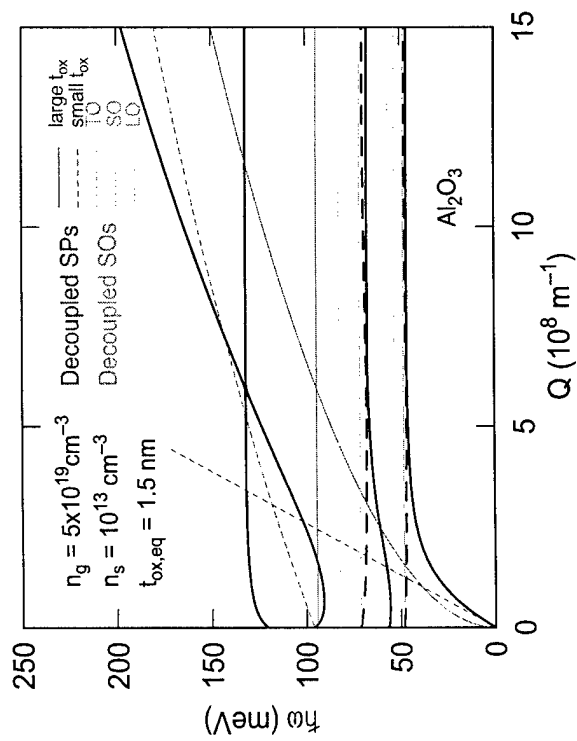
**Coupled modes for Si/SiO₂/Si system:
Thickness dependence**



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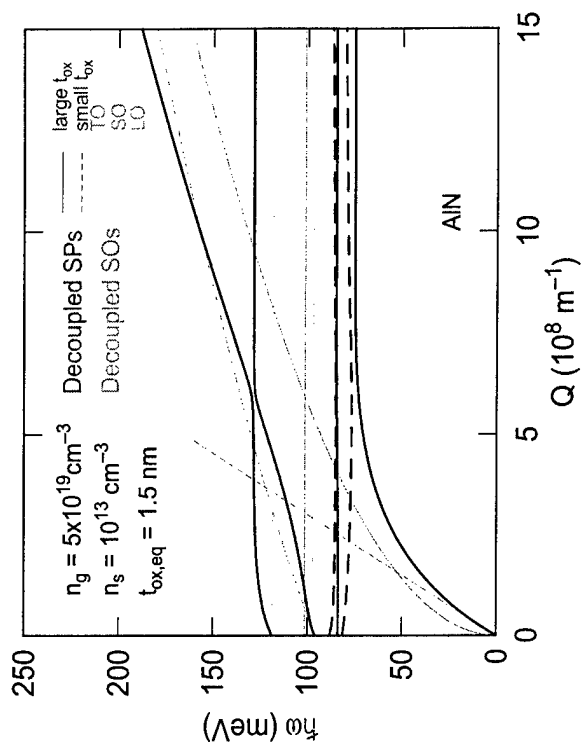
Coupled modes for Si/Al₂O₃/Si system



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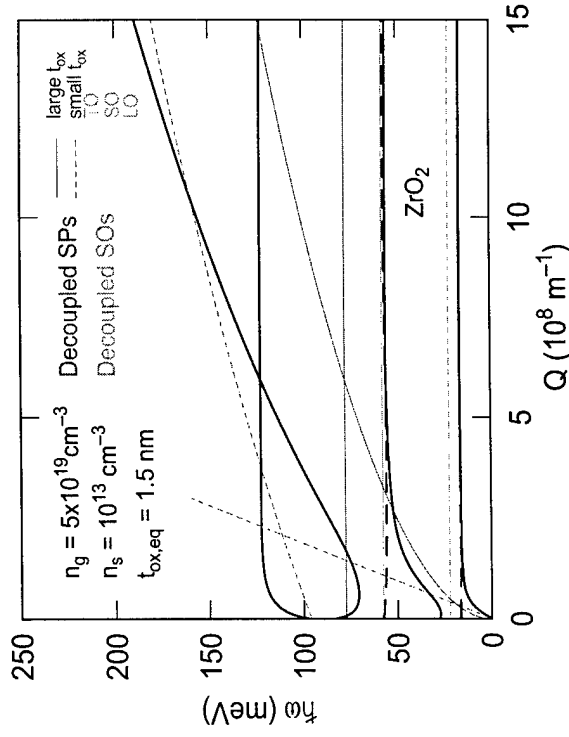
Coupled LO-plasmons modes for Si/AIN/Si system



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Coupled LO-plasmons modes for Si/ZrO₂/Si system



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TO- and plasmon content Landau damping

- Define 3 dispersions $\omega_Q^{(-\alpha,j)}$ ($j=1,3$) by ignoring mode α ($\alpha = \text{TO1, TO2, gate-plasma, substrate-2D-plasma}$)

- Mode- α content of mode $\omega_Q^{(i)}$ (for 4 modes):

$$\Pi^{(\alpha)}(\omega_Q^{(i)}) \approx$$

$$\left| \frac{(\omega_Q^{(i)})^2 - \omega_Q^{(-\alpha,1)}{}^2}{(\omega_Q^{(i)})^2 - \omega_Q^{(j)}{}^2} \frac{(\omega_Q^{(i)})^2 - \omega_Q^{(-\alpha,2)}{}^2}{(\omega_Q^{(i)} - \omega_Q^{(k)})^2} \frac{(\omega_Q^{(i)})^2 - \omega_Q^{(-\alpha,3)}{}^2}{(\omega_Q^{(i)} - \omega_Q^{(l)})^2} \right|$$

with i, j, k, l cyclical.

- Content normalized so that:

$$\sum_{i=1}^4 \Pi^{(\alpha)}(\omega_Q^{(i)}) = 1$$

$$\Pi^{(G)}(\omega_Q^{(i)}) + \Pi^{(S)}(\omega_Q^{(i)}) + \Pi^{(TO1)}(\omega_Q^{(i)}) + \Pi^{(TO2)}(\omega_Q^{(i)}) = 1$$

- Landau damping (poor man's way): Ignore response of gate-plasma when gate-plasmon-like mode is in Landau-damped region; same for substrate plasma

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Scattering strength

- Field $\phi_{Q,\omega_Q}^{(i)}(z)$ defined up to a multiplicative constant $a_{Q,\omega_Q}^{(i)}$:

$$b_{Q,\omega_Q}^{(i)} = \frac{\epsilon_{ox}(\omega_Q^{(i)}) - \epsilon_g(\omega_Q^{(i)})}{2\epsilon_{ox}(\omega_Q^{(i)})} a_{Q,\omega_Q}^{(i)}$$

$$c_{Q,\omega_Q}^{(i)} = \frac{\epsilon_{ox}(\omega_Q^{(i)}) + \epsilon_g(\omega_Q^{(i)})}{2\epsilon_{ox}(\omega_Q^{(i)})} a_{Q,\omega_Q}^{(i)}$$

$$d_{Q,\omega_Q}^{(i)} = \frac{\epsilon_{ox}(\omega_Q^{(i)}) - \epsilon_g(\omega_Q^{(i)})}{\epsilon_{ox}(\omega_Q^{(i)}) + \epsilon_s(Q, \omega_Q^{(i)})} a_{Q,\omega_Q}^{(i)}$$

- Fix $a_{Q,\omega_Q}^{(i)}$ via second-quantization or semiclassically.
- Semiclassically (not-so-trivial extension of Kittel 63):
 - Re-write potential as:

$$\phi_Q^{(i)}(\mathbf{R}, z, t) = \phi_{Q,\omega_Q}^{(i)}(z) \cos(\mathbf{Q} \cdot \mathbf{R} - \omega_Q^{(i)} t)$$

- Write interface charge-density as:

$$\rho_Q^{(i)}(\mathbf{R}, z, t) = \{ \delta(z) [\epsilon_{gate}(\omega_Q^{(i)}) a_{Q,\omega_Q}^{(i)} + \epsilon_{insulator}(\omega_Q^{(i)}) (b_{Q,\omega_Q}^{(i)} - c_{Q,\omega_Q}^{(i)})] + \delta(z-t) [\epsilon_{insulator}(\omega_Q^{(i)}) (c_{Q,\omega_Q}^{(i)} e^{Qt} - b_{Q,\omega_Q}^{(i)} e^{-Qt}) + \epsilon_{substrate}(Q; \omega_Q^{(i)}) d_{Q,\omega_Q}^{(i)} e^{-Qt}] \} Q \cos(\mathbf{Q} \cdot \mathbf{R} - \omega_Q^{(i)} t).$$
- $\epsilon_{gate}(\omega)$, $\epsilon_{insulator}(\omega)$, and $\epsilon_{substrate}(Q, \omega)$ to be determined: whatever response is accounted for by these functions, is not accounted for by the potential

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- Get total energy (potential+kinetic, including self-energy):
 $\langle W_Q^{(i)} \rangle = 2 \langle U_Q^{(i)} \rangle$

$$= \frac{2}{\Omega} \left\langle \int_{\Omega} d\mathbf{R} \int_{-\infty}^{\infty} dz \phi_Q^{(i)}(\mathbf{R}, z, t) \rho_Q^{(i)}(\mathbf{R}, z, t) \right\rangle$$

$$= Q \epsilon_{TOT}(Q, \omega_Q^{(i)}) \left[\frac{\epsilon_{ox}(\omega_Q^{(i)}) - \epsilon_g(\omega_Q^{(i)})}{\epsilon_{ox}(\omega_Q^{(i)}) + \epsilon_s(Q, \omega_Q^{(i)})} \right]^2 a_{Q,\omega_Q}^{(i)^2} e^{-2Qt}$$

with

$$\epsilon_{TOT}(Q, \omega) = \epsilon_{gate}(\omega) \left[\frac{\epsilon_{ox}(\omega) + \epsilon_s(Q, \omega)}{\epsilon_{ox}(\omega) - \epsilon_g(\omega)} \right]^2 e^{2Qt}$$

$$+ \epsilon_{insulator}(\omega) \left\{ \left[\frac{\epsilon_{ox}(\omega) + \epsilon_s(Q, \omega)}{2\epsilon_{ox}(\omega)} \right]^2 (e^{2Qt} - 1) + \left[\frac{\epsilon_{ox}(\omega) - \epsilon_s(Q, \omega)}{2\epsilon_{ox}(\omega)} \right]^2 (1 - e^{-2Qt}) \right\} + \epsilon_{substrate}(Q, \omega)$$

- Finally:

$$\phi_{Q,\omega_Q}^{(i)} = \left[\frac{\hbar \omega_Q^{(i)}}{2 Q \epsilon_{TOT}(Q, \omega_Q^{(i)})} \right]^{1/2} e^{-Q(z-t)}$$

- Setting:
 - * $\epsilon_{gate}(\omega) = \epsilon_g(\omega)$
 - * $\epsilon_{insulator}(\omega) = \epsilon_{ox}(\omega)$
 - * $\epsilon_{substrate}(Q, \omega) = \epsilon_s^{(2D)}(Q, \omega)$
 the secular equation is equivalent to $\epsilon_{TOT}(Q, \omega) = 0$

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• Scattering field due to plasmon- and phonon-components:

– Plasmon: Set

- * $\epsilon_{substrate}(Q, \omega) = \epsilon_{Si}$, $\epsilon_{gate}(\omega) = \epsilon_{Si}$
(full plasma response lumped into the field)
- * $\epsilon_{insulator}(\omega) = \epsilon_{ox}^0$
(TO-response excluded from the field and fully lumped into the dielectric function) in $\epsilon_{TOT}(Q, \omega)$
- * Call it $\epsilon_{TOT}^{(PL)}(Q, \omega)$

Then, field due to gate-plasmons:

$$\phi_{Q, \omega_Q}^{(i, g, PL)}(z) = \left[\frac{\hbar \omega_Q^{(i)}}{2 Q \epsilon_{TOT}^{(PL)}(Q, \omega_Q^{(i)})} \Pi^{(G)}(\omega_Q^{(i)}) \right]^{1/2} e^{-Q(z-t)}$$

– TO1-mode:

- * Let substrate and gate plasmas respond:
 $\epsilon_{substrate}(Q, \omega) = \epsilon_s(Q, \omega)$, $\epsilon_{gate}(\omega) = \epsilon_g(\omega)$ (so excluding plasmas from the potential)
- * Define $\epsilon_{TOT}^{(TO1)}(Q, \omega_Q^{(i)})$ by setting

$$\epsilon_{insulator}(\omega) = \epsilon_{ox} \frac{\omega_{LO2}^2 - \omega^2}{\omega_{TO2}^2 - \omega^2}$$

(phonon 2 responds at the frequency ω , while phonon 1 does not respond).

- * Define $\epsilon_{TOT, low}^{(TO1)}(Q, \omega_Q^{(i)})$ by setting

$$\epsilon_{insulator}(\omega) = \epsilon_{ox} \frac{\omega_{LO2}^2 - \omega^2}{\omega_{TO2}^2 - \omega^2} \left(\frac{\omega_{LO1}}{\omega_{TO1}} \right)^2$$

(phonon 2 responds at the frequency ω , while phonon 1 responds fully).

Then, amplitude of (scattering) field due only to TO1 is:

$$\phi_{Q, \omega_Q}^{(i, PH1)}(z) = e^{-Q(z-t)} \times \left\{ \frac{\hbar \omega_Q^{(i)}}{2Q} \left[\frac{1}{\epsilon_{TOT, high}^{(TO1)}(Q, \omega_Q^{(i)})} - \frac{1}{\epsilon_{TOT, low}^{(TO1)}(Q, \omega_Q^{(i)})} \right] \Phi^{(TO1)}(\omega_Q^{(i)}) \right\}^{1/2}$$

- Note: Modes 5 and 6 are SO-modes at the far (gate-insulator) interface. At small K_F , large n_g , they are screened by gate-electrons, at large K_F their effect on the mobility is depressed by a factor $\exp(-2K_F t) \dots$ so, ignore them.

- Interesting limits:

- TO-mode (no plasma) in bulk: Scattering field $\propto$

$$\left[\frac{\hbar\omega_{LO}}{2q^2} \left(\frac{1}{\epsilon^\infty} - \frac{1}{\epsilon^0} \right) \right]^{1/2}$$

(usual Fröhlich coupling)

- Coupled TO-plasma modes in bulk:

$$\epsilon_{TOT}(Q, \omega) = \epsilon^\infty [(\omega^2 - \omega_{LO}^2)/(\omega^2 - \omega_{TO}^2) - (\omega_P/\omega)^2]$$

with dispersion:

$$\omega_\pm^2 = \frac{1}{2} \{ \omega_{LO}^2 + \omega_P^2 \pm [(\omega_{LO}^2 + \omega_P^2)^2 - 4\omega_{TO}^2 \omega_P^2]^{1/2} \}$$

and scattering field:

$$\left\{ \frac{\hbar\omega_\pm}{2q^2} \left[\frac{1}{\epsilon^\infty - \epsilon^\infty \omega_P^2/\omega_\pm^2} - \frac{1}{\epsilon^0 - \epsilon^\infty \omega_P^2/\omega_\pm^2} \right] \left| \frac{\omega_\pm^2 - \omega_P^2}{\omega_\pm^2 - \omega_-^2} \right| \right\}^{1/2}$$

(cf. Varga 65, Kim-Das-Senturia 78, Ridley 88, Sanborn 95)

- Single TO-mode at Si-insulator interface:

$$\epsilon_{TOT} = \epsilon_{ox}(\omega) + \epsilon_{Si}^\infty$$

with dispersion

$$\omega_{SO} = \omega_{TO} \left[\frac{\epsilon_{ox}^0 + \epsilon_{Si}^\infty}{\epsilon_{ox}^\infty + \epsilon_{Si}^\infty} \right]^{1/2}$$

and scattering strength $\propto$

$$\phi_{Q, \omega_{SO}}^{(PH)}(z) = \left\{ \frac{\hbar\omega_{SO}}{2Q} \left[\frac{1}{\epsilon_{Si}^\infty + \epsilon_{ox}^\infty} - \frac{1}{\epsilon_{Si}^\infty + \epsilon_{ox}^0} \right] \right\}^{1/2} e^{-Q(z-t)}$$

(cf Wang-Mahan 72) which is also the unscreened $Qt \rightarrow \infty$ -limit of the general formula

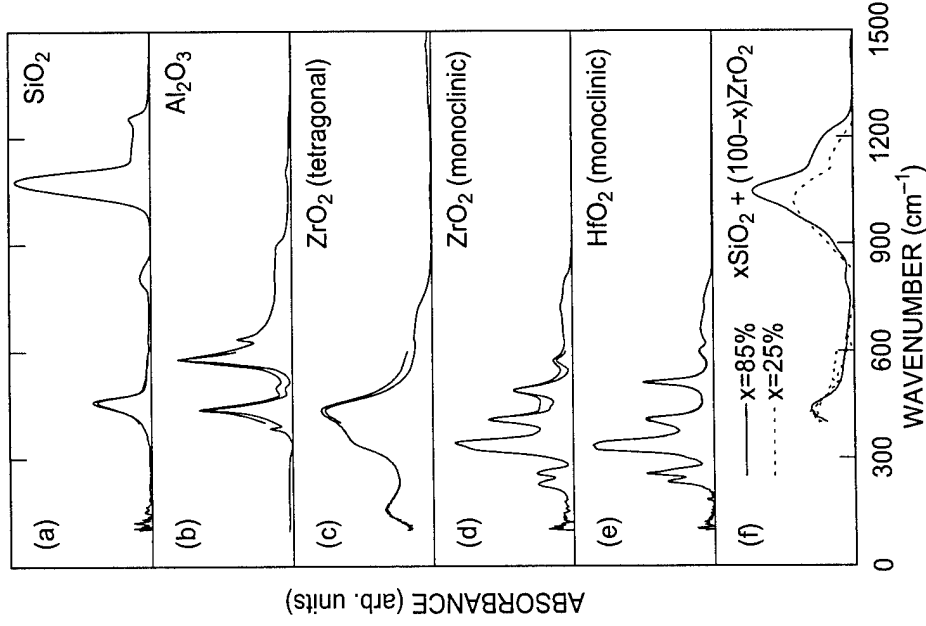
Insulator parameters

- Select/compromise:
 - from literature (theory, experiments: FTIR, Raman, tunneling spectroscopy)
 - From 'in-house' FTIR
- Complications:
 - several modes: lump 2 stronger groups/bands into 2 discrete TO modes
 - several forms (cubic, hexagonal, monoclinic, amorphous): find closest to 'in-house' FTIR-measured films
 - process dependence (e.g., Al-ions in non-stoichiometric Al_2O_3): assume 'ideal' films
 - interfacial layers (SiO_2 for oxides and silicates, Si_3N_4 for nitrides and oxynitrides): estimate effect

Material Quantity (units)	SiO_2	Al_2O_3	AlN	ZrO_2	HfO_2	ZrSiO_4
ϵ_{ox}^0 (ϵ_0)	3.90	12.53	9.14	24.0	22.00	11.75
ϵ_{ox}^t (ϵ_0)	3.05	7.27	7.35	7.75	6.58	9.73
ϵ_{ox}^∞ (ϵ_0)	2.50	3.20	4.80	4.00	5.03	4.20
ω_{TO1} (meV)	55.60	48.18	81.40	16.67	12.40	38.62
ω_{TO2} (meV)	138.10	71.41	88.55	57.70	48.35	116.00
α_1	0.0248	0.0788	0.0248	0.2504	0.3102	0.0322
α_2	0.0113	0.0814	0.0423	0.0779	0.0362	0.2942

$$\alpha_i = \frac{e^2}{4\pi\hbar} \left(\frac{m_t}{2\hbar\omega_{SOi}} \right)^{1/2} \left(\frac{1}{\epsilon_{Si}^\infty + \epsilon_{ox}^\infty} - \frac{1}{\epsilon_{Si}^\infty + \epsilon_{ox}^0} \right)$$

FTIR spectra (D. Neumayer)



Mobility

- Scattering rate:

$$\frac{1}{\tau_{\mu\nu}(\mathbf{K})} = \frac{2\pi}{\hbar} \sum_{\mathbf{Q}} |V_{\mu\nu}(\mathbf{Q})|^2 \delta[E_{\mu}(\mathbf{K}) - E_{\nu}(\mathbf{K} + \mathbf{Q}) \pm \Delta E(\mathbf{Q})]$$

- Momentum relaxation rate (along x-axis):

$$V_{\mu\nu}(\mathbf{Q}) = \int_0^{\infty} dz \zeta_{\mu}^*(z) \phi_{\mathbf{Q}}(z) \zeta_{\nu}(z)$$

$$\frac{1}{\tau_{\mu\nu}^{(p,x)}(\mathbf{K})} \approx \frac{2\pi}{\hbar} \sum_{\mathbf{Q}} |V_{\mu\nu}(\mathbf{Q})|^2 \left(\frac{Q_x}{K} \right)$$

- Mobility along x-axis (assuming spherical dispersion):

$$\times \delta[E_{\mu}(\mathbf{K}) - E_{\nu}(\mathbf{K} + \mathbf{Q}) \pm \Delta E(\mathbf{Q})]$$

$$\mu_{x,x} = \sum_{\nu} \mu_{\nu} \frac{n_{\nu}}{n_s} =$$

$$\sum_{\nu} \frac{en_{\nu}}{m_{x,\nu} k_B T n_s} \int_{E_{\nu}}^{\infty} dE (E - E_{\nu}) \rho_{\nu}(E) \tau_{\nu}^{(p,x)}(E) f_{\nu}(E) [1 - f_{\nu}(E)]$$

- Screening (dynamic):

$$V_{\mu\nu}^{(s)}(\mathbf{Q}, \omega) = V_{\mu\nu}(\mathbf{Q}) - \sum_{\lambda\lambda'} \frac{\beta_{\lambda}(\mathbf{Q}, \omega)}{Q} g_{\mu\nu;\lambda\lambda'}(\mathbf{Q}) V_{\lambda\lambda'}^{(s)}(\mathbf{Q}, \omega)$$

$$g_{\mu\nu;\lambda\lambda'}(\mathbf{Q}) = \int_0^{\infty} dz \int_0^{\infty} dz' \zeta_{\mu}^*(z) \zeta_{\nu}^*(z') G_{\mathbf{Q}}(z, z') \zeta_{\lambda}(z) \zeta_{\lambda'}(z')$$

where $G_{\mathbf{Q}}(z, z')$ is the Poisson Green's function.

In matrix form:

$$V^{(s)}(\mathbf{Q}, \omega) = [1 - \Pi(\mathbf{Q}, \omega)]^{-1} V(\mathbf{Q})$$

Scattering processes

- Bulk Si phonons:

$$|V_{\mu\nu}(\mathbf{Q})|^2 \propto \frac{\Delta_Q^2}{\hbar\omega_Q} F_{\mu\nu}(Q)$$

- Example: for acoustic, intravalley, one-subband, variational wavefunction

$$\frac{1}{\tau(p)} \approx \frac{3m_z b \Delta_{ac}^2}{64\hbar^3 \rho c_s^2} \rightarrow \mu_{ph} \propto b^{-1} \propto n_s^{-1/3}$$

- In general, more complicated:

1. Deformation potential is anisotropic:

$$\Delta_{LA} \rightarrow \Xi_d + \Xi_u \cos^2 \eta_Q$$

$$\Delta_{TA} \rightarrow \Xi_u \cos \eta_Q \sin \eta_Q$$

2. Subband dispersion is nonparabolic
3. Subband dispersion is non-spherical
4. Intervalley scattering (projected onto 2D BZ)
5. Are bulk Si phonons OK near the interface?
6. What about screening?

- Coulomb scattering (with dopants, oxide charges, etc):

$$V_{\mu\nu}(\mathbf{Q}) \propto \frac{e^2 N_C^{1/2}}{Q^2} \tilde{G}_{\mu\nu;\nu\nu}(Q)$$

(must be screened!) and

$$\mu_C \propto \frac{n_s^{4/3}}{N_C}$$

- Surface roughness:

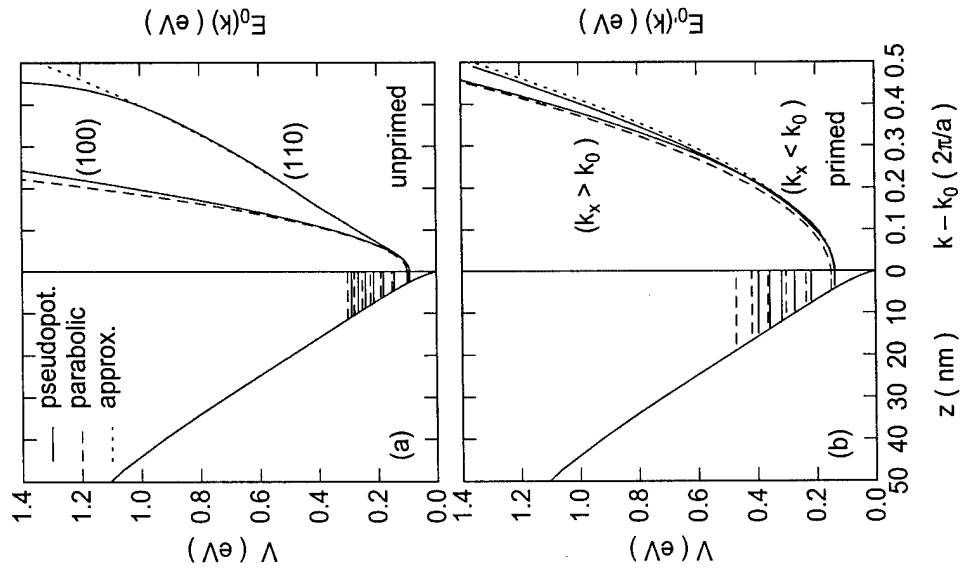
Ando's model assumes 1st-order perturbation given by steps at interface of rms height Δ and correlation-distance Λ .

$$V_{\mu\nu}(\mathbf{Q}) \propto \Delta \Lambda F_s \left\{ \frac{e^{-Q^2 \Lambda^2/4}}{(1 + Q^2 \Lambda^2/2)^{-1}} \right.$$

(must be screened) and

$$\mu_{SR} \propto \Delta^2 \Lambda^2 n_s^{-2}$$

Subbands



Triangular-well approximation

- Energy levels:

$$E_\mu = \left(\frac{\hbar^2}{2m_z} \right)^{1/3} \left[\frac{3\pi F_s}{2} \left(\mu + \frac{3}{4} \right) \right]^{2/3}$$

- Wavefunctions:

$$\zeta_\mu(z) = N A_0 \left[\left(\frac{2m_z e F_s}{\hbar^2} \right)^{1/3} \left(z - \frac{E_\mu}{e F_s} \right) \right]$$

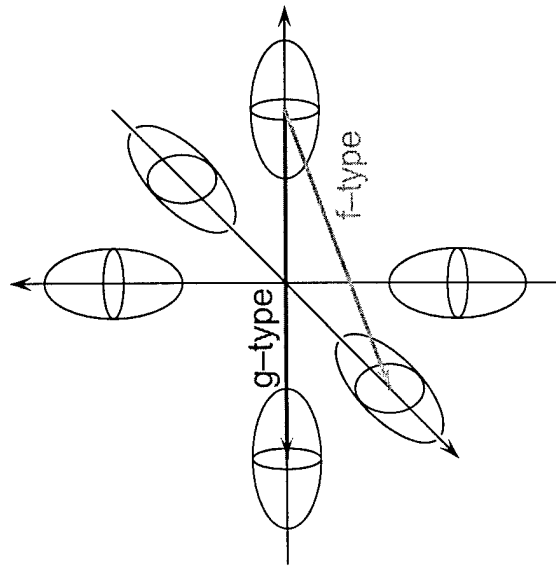
- Variational ground-state wavefunction:

$$\zeta_0(z) \approx \left(\frac{b^3}{2} \right)^{1/2} z e^{-bz/2}$$

with

$$b = \frac{3}{z_0} \propto n_s^{1/3}$$

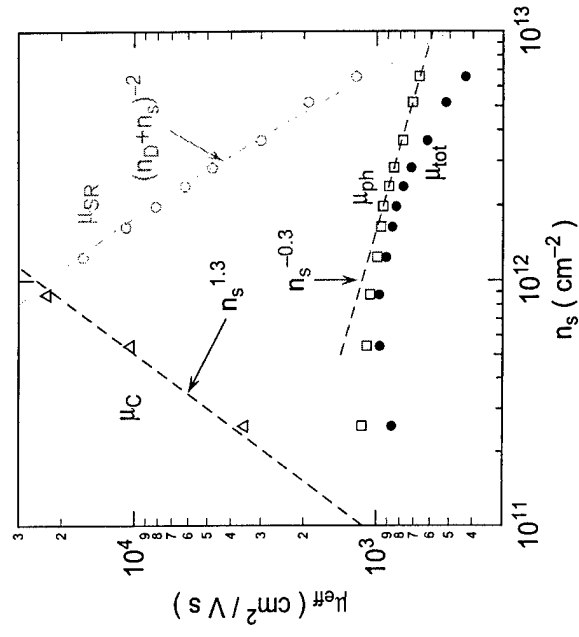
Phonon-assisted intervalley scattering



MVF

June 01 29

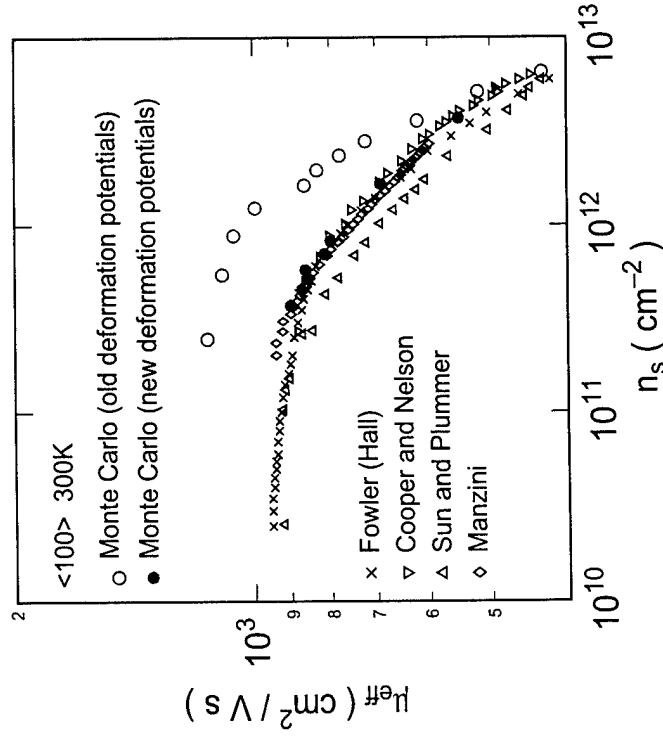
Mobility split into various components



MVF

June 01 30

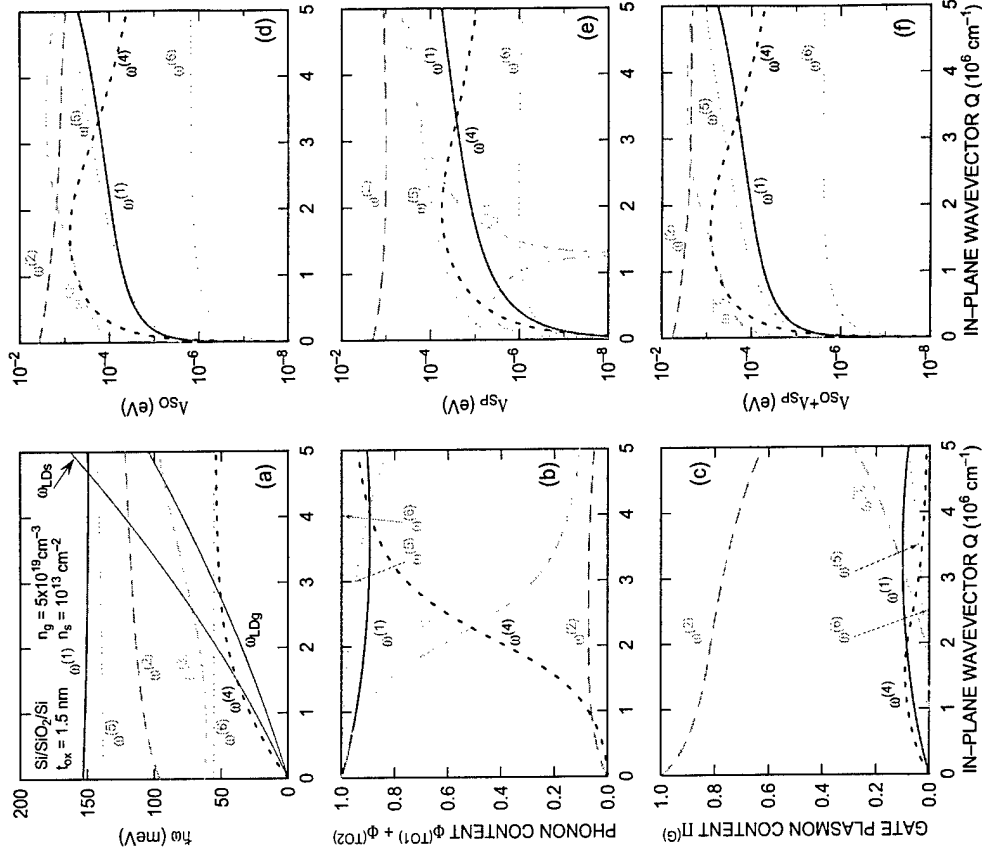
The fight for the right phonon-limited mobility



MVF

June 01 31

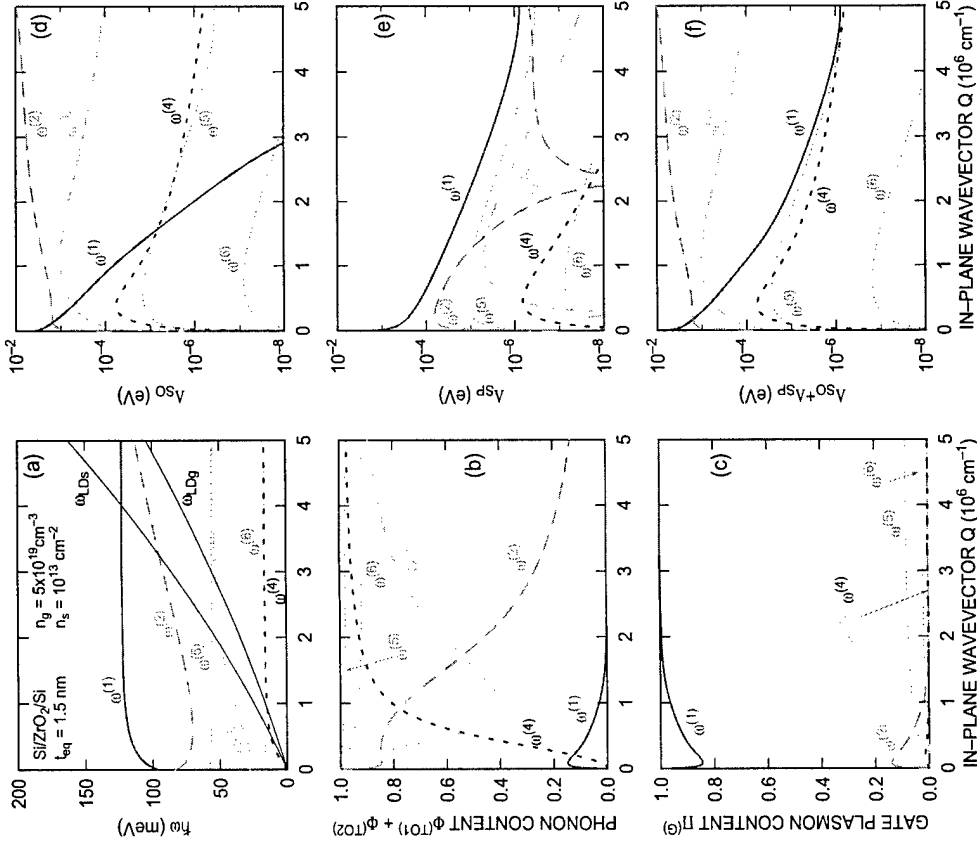
The Si/SiO₂/Si system



MVF

June 01 32

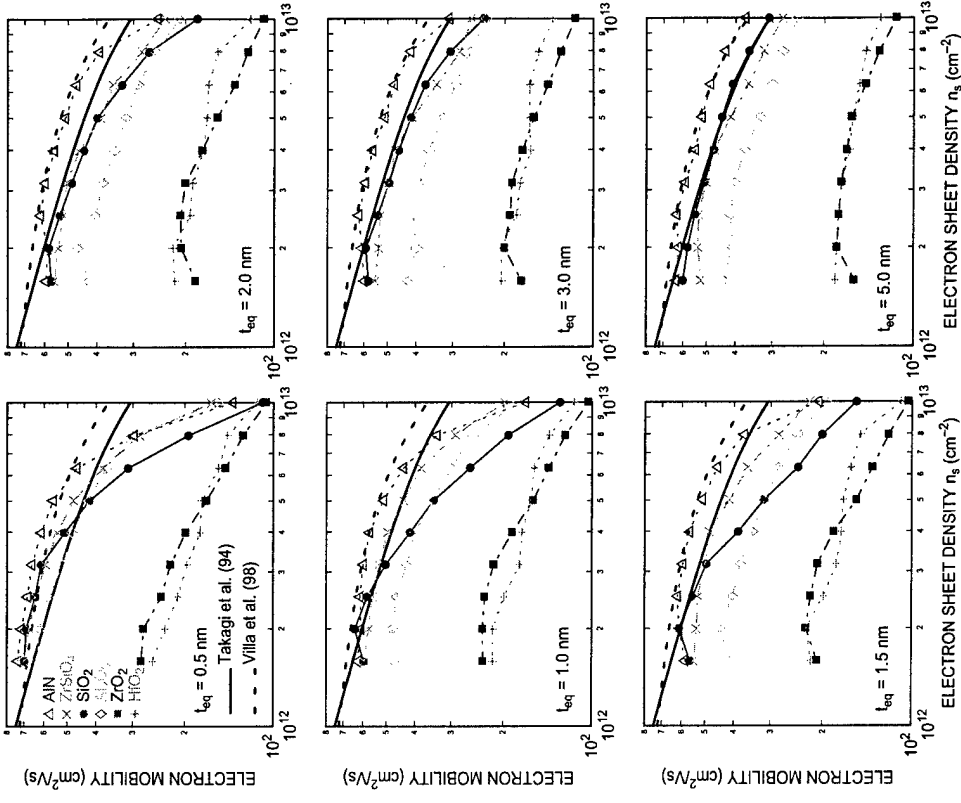
The Si/ZrO₂/Si system



MVF

June 01 33

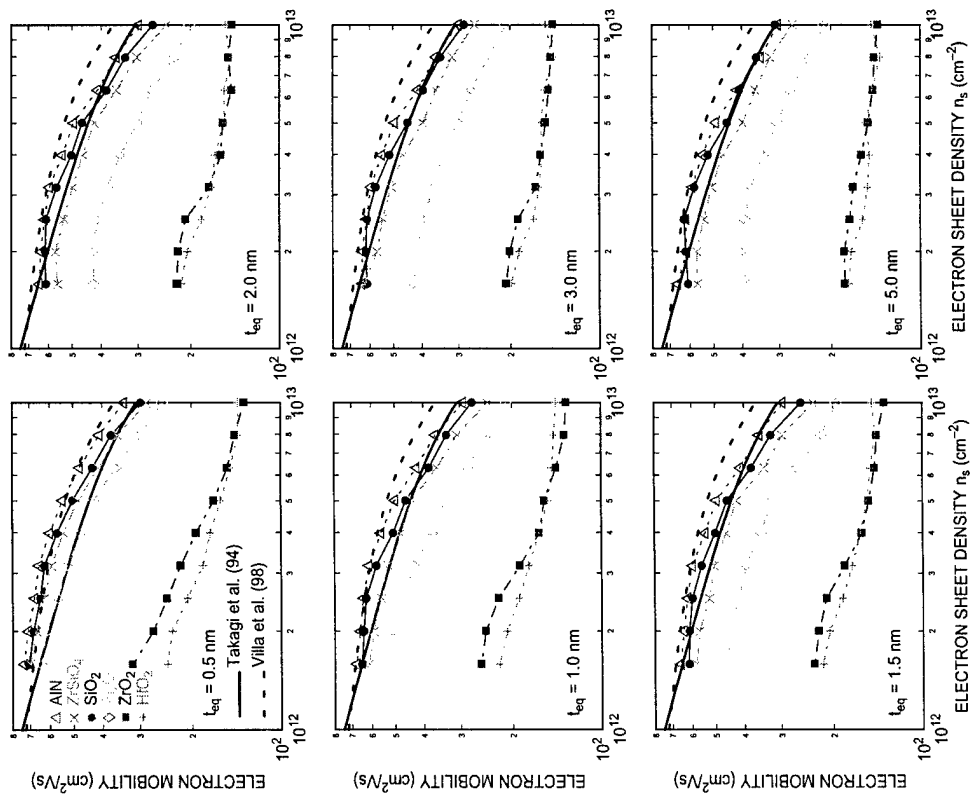
Effective mobility I (average n_g)



MVF

June 01 34

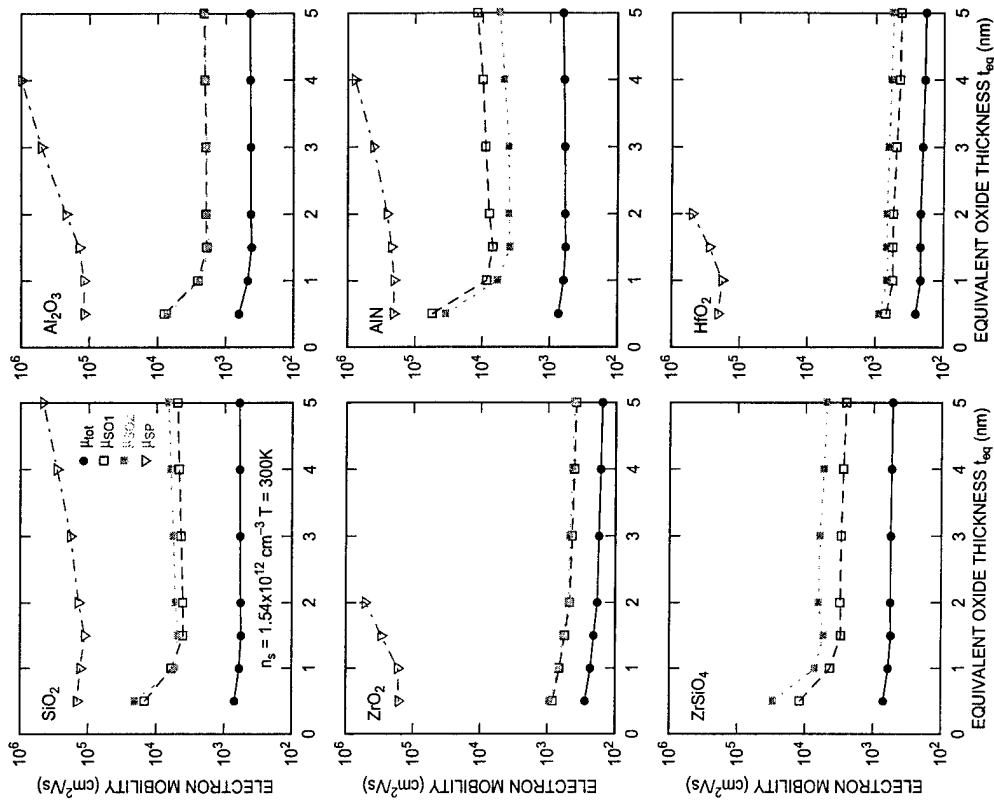
Effective mobility μ (Q -dependent n_g)



MVF

June 01 35

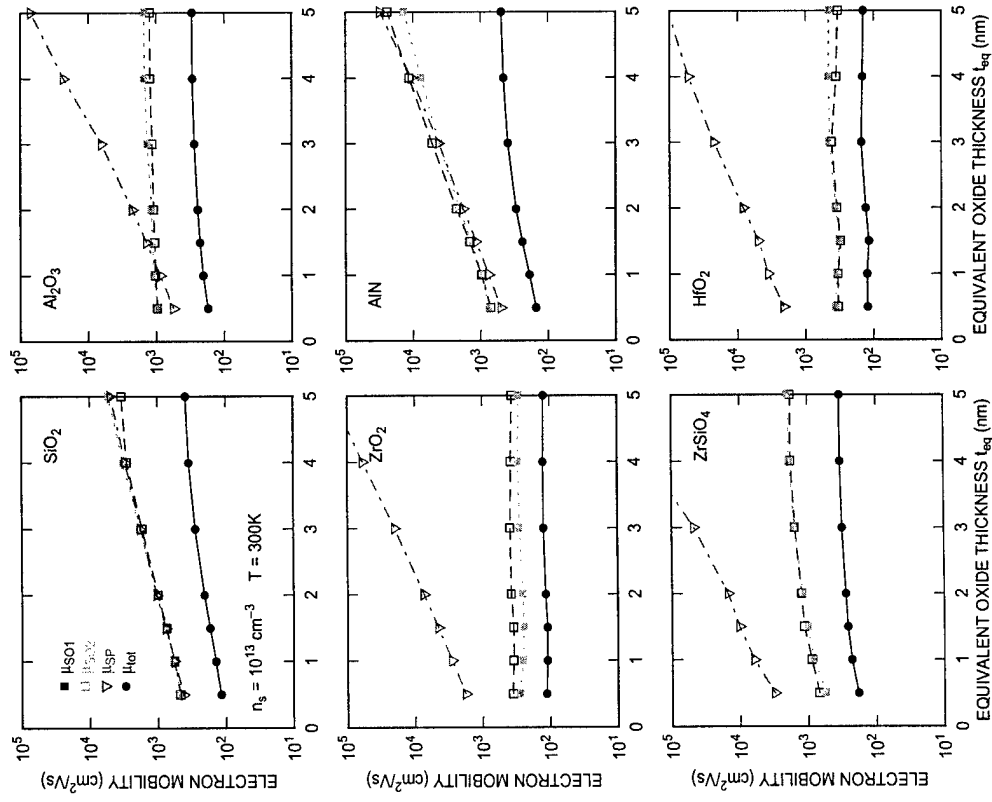
Components of the mobility (low density)



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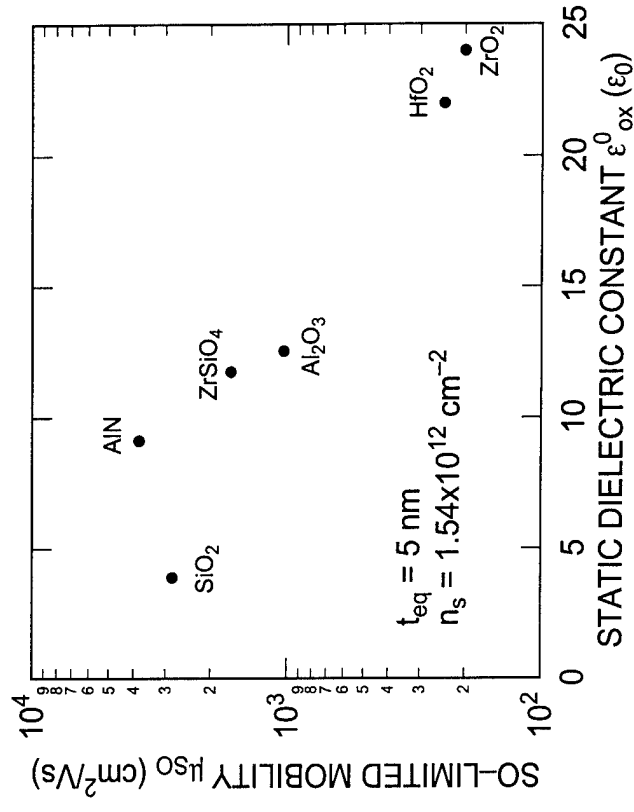
Components of the mobility (high density)



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Mobility vs static dielectric constant



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June 01 38

Effect of SiO₂ interfacial layer

- Interfacial layer probably unavoidable, possibly desirable (with moderation...)
- Full system Si/high- κ /SiO₂/Si too cumbersome... secular equation

$$\begin{aligned}
 &(\epsilon_g + \epsilon_\kappa)(\epsilon_s + \epsilon_{ox})(\epsilon_{ox} + \epsilon_\kappa) + (\epsilon_g - \epsilon_\kappa)(\epsilon_s + \epsilon_{ox})(\epsilon_{ox} - \epsilon_\kappa) e^{-2Qt_\kappa} \\
 &\quad - (\epsilon_g - \epsilon_\kappa)(\epsilon_s - \epsilon_{ox})(\epsilon_{ox} + \epsilon_\kappa) e^{-2Q(t_\kappa + t_{ox})} \\
 &\quad - (\epsilon_g + \epsilon_\kappa)(\epsilon_s - \epsilon_{ox})(\epsilon_{ox} - \epsilon_\kappa) e^{-2Qt_{ox}} = 0
 \end{aligned}$$

(16 coupled modes!)

- Consider instead 'unscreened' high- κ /SiO₂/Si system and only one TO-mode in each insulator.

3 modes with high- Q -frequencies:

1. SiO₂ mode Ω_{TO} localized at the Si/SiO₂ interface:

$$\omega_Q^{(-)} \approx \Omega_{TO} \left[\frac{\epsilon_{ox}^0 + \epsilon_{Si}^\infty}{\epsilon_{ox}^i + \epsilon_{Si}^\infty} \right]^{1/2}$$

2. SiO₂ mode Ω_{TO} localized at the high- κ /SiO₂ interface:

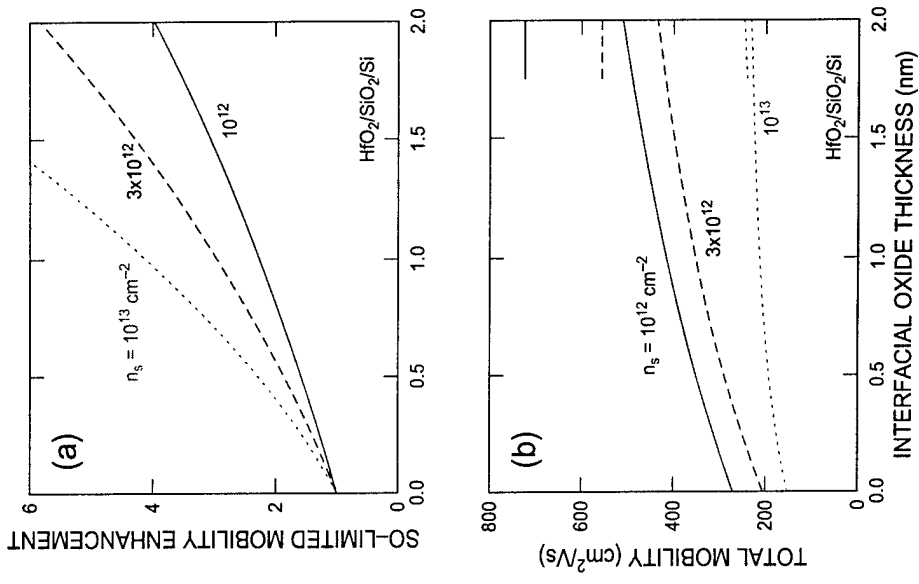
$$\omega_Q^{(+)} \approx \Omega_{TO} \left[\frac{\epsilon_{ox}^0 + \epsilon_\kappa^\infty}{\epsilon_{ox}^i + \epsilon_\kappa^\infty} \right]^{1/2}$$

(negligible, as for modes 5 and 6 before)

3. remote) high- κ mode ω_{TO} localized at the Si/SiO₂ interface:

$$\omega_Q^{(\kappa)} \approx \omega_{TO} \left[\frac{\epsilon_\kappa^0 + \epsilon_{Si}^\infty}{\epsilon_\kappa^i + \epsilon_{Si}^\infty} \right]^{1/2}$$

Effect of the SiO₂ interfacial layer



Conclusions

- Experimental confirmation?
 - Poor mobility observed almost always, but mainly due to poor material quality (traps, interface states, non-stoichiometry, interfacial layers). Must solve these problems first! n_s hard to measure accurately.
 - Parameters: what material do we really have? How far from 'ideal'?
 - Still...trends show HfO_2 and ZrO_2 yield worst mobility (Ragnarsson, Callegari, 00-01), ZrSiO_4 somewhat better than ZrO_2 (Q_i , 00), Al_2O_3 in-between, AlN very poor (but Si_3N_4 interfacial layer present)
- If theory is right:
 - Interfacial and many-body effects paramount in small devices:
 - * Coulomb effects make 'ballistic limit' only a theoretical dream (metal contacts?)
 - * there's no more 'bulk', only 'interfaces', in small devices
 - Must pay for high κ with poor mobility
 - Metal-oxides worst
 - Silicates promising
 - AlN promising (if we only could get rid of the Si_3N_4 interfacial layer!)
 - Thin SiO_2 interfacial layer desirable
 - Assessing whether this low mobility is a fatal flaw or not is up to the circuit designers. Alternatives to high- κ are even riskier propositions.

Full Bandstructure Calculations for Transport in Wide-Band-Gap Semiconductors*

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Outline

1. Wide-Bandgap Materials
2. Electronic Structure Calculations
3. Impact Ionization Rate
4. Electron-Phonon Interaction
5. Full-band Monte Carlo Simulation
6. Experimental Results

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Collaborators

- Richard Akis, Joy Barker, Manfred Dür*, David Ferry, Trevor Thornton, Arizona State University
- Shankar Pennathur*, John Wager, Oregon State University
- Marco Saraniti, Illinois Inst. Tech.
- Niels Fitzer, Ronald Redmer, Martin Reigrotzki*, Universität Rostock
- Martin Städele* and Peter Vogl, Technische Universität München
- Randy Shul, Sandia National Labs
- Daniel Koleske, Naval Research Labs

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Wide-Bandgap Semiconductors

Column IV

• SiC

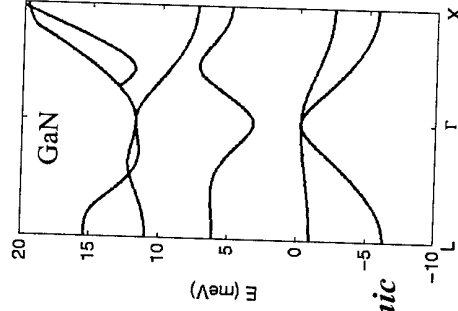
III-V Nitrides

• GaN, AlN, InN

II-VI Compounds

• ZnS, ZnSe, MnS, SrS, CoS

Compound Semiconductors: ionic
zincblende $\rightarrow$ wurzite $\rightarrow$ rocksalt
 $E_G \sim 3-5$ eV
Breakdown field ~ 1 MV/cm



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Wide-Bandgap Semiconductor Applications

High-Frequency Power Amplifiers

- GaN MESFETs
- AlGaIn/InGaIn HEMTs

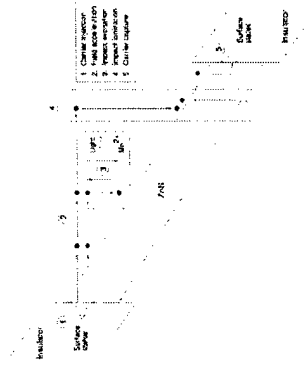
Optical Sources

- ZnS/ZnSe, AlGaIn/InGaIn LEDs/Lasers (Green-Blue-UV)
- Phosphor materials (ZnS:Mn, SrS:Ag, CeS:Mn)-LED, TFEL

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High Field Transport in TFEL Devices



High field phenomena affecting TFEL performance:

- Impact excitation of luminescent impurities
- Band to band impact ionization of electrons and holes

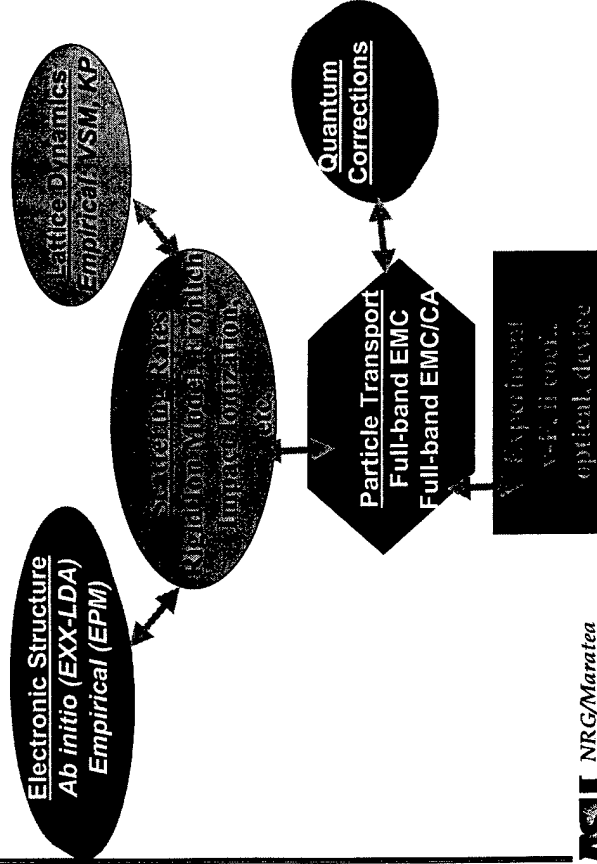
Issues

- Average carrier energies at 1-2 MV/cm are 2-3 eV, several conduction bands occupied: Full bandstructure must be considered
- Band to band impact ionization process in wide bandgap materials not well understood
- Electron-phonon interaction at high fields is unknown experimentally in phosphor materials

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General Modeling Approach



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Bandstructure for Cubic GaN, ZnS, and SrS

Empirical Pseudopotential Method

Local and nonlocal bandstructures for cubic GaN, ZnS and SrS are calculated either from formfactors in literature, or derived using least squares fit to existing critical point data.

- ZnS
 - Local: Walter and Cohen, *Phys. Rev.* 183, 763 (1969)
 - Nonlocal: M. Dür *et al.*, *JAP* 83, 3176 (1998)
- SrS
 - Nonlocal- Reigrotski *et al*, *JAP* 86, 4458, (1999)
- GaN
 - Oguzman *et al.*, *JAP* 80,4429 (1996)
 - Xia *et al.*, *Phys. B* 59,10119 (1999)

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Ab Initio Bandstructure

EXX-LDA (exact exchange-local density approximation) Method¹

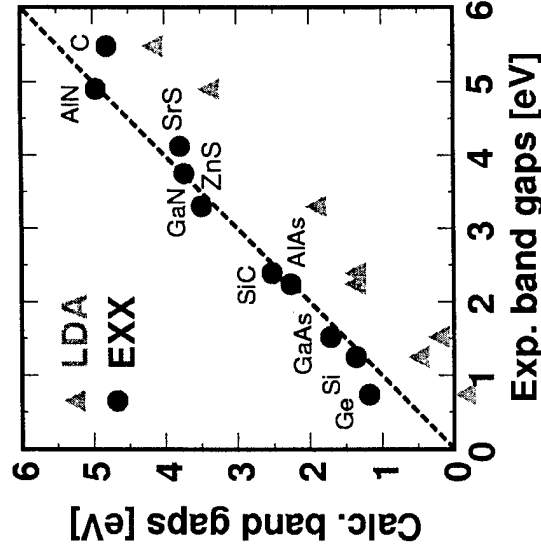
- Kohn-Sham method is particular, exact realization of density functional theory which maps interacting system onto non-interacting system of the same density
- LDA approximates BOTH exchange and correlation potential within the Kohn-Sham model
- Good ground state properties but poor band gaps
- EXX-LDA is a systematic step beyond LDA which treats the exchange potential EXACTLY
- EXX+LDA yields excellent ground state properties AND band gaps

¹M. Städele, J. A. Majewski, P. Vogl, and A. Görling, *Phys. Rev. Lett.* 79, 2089 (1997)

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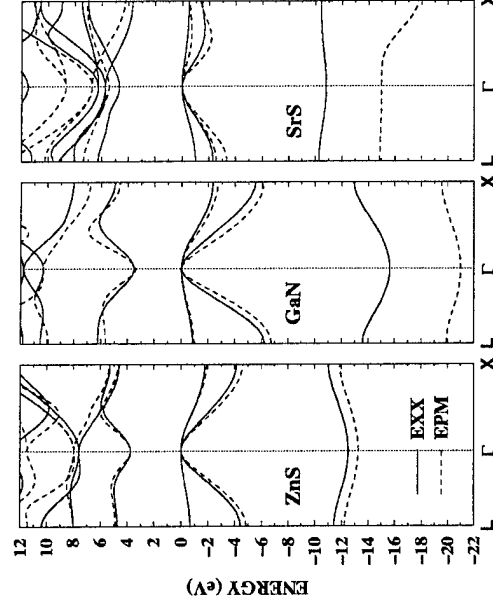
EXX Results for Cubic Semiconductors



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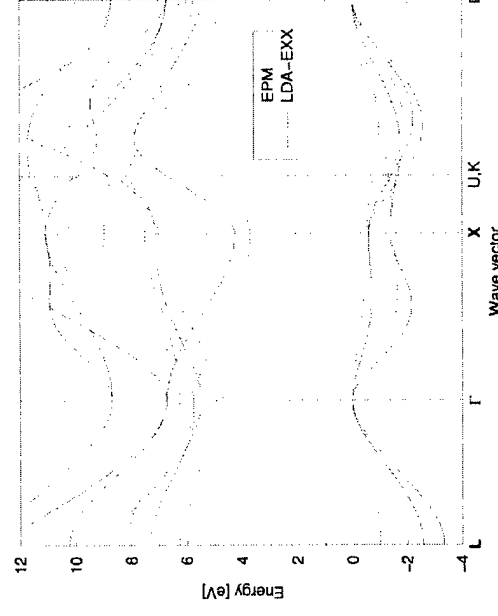
Band Structure Based on EXX and Empirical Pseudopotential Method (EPM)



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SrS Band Structure Based on EXX and Empirical Pseudopotential Method (EPM)



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Comparison of EXX and EPM Results for SrS and ZnS to Experimental Data (in eV)

SrS: indirect

SrS	EXP	EXX	EPM
$E(\Gamma_v-X_c)$	4.32	3.69	4.26
$E(\Gamma_v-\Gamma_c)$	5.33	4.67	5.45
$E(X_v-X_c)$	4.83	4.21	4.86

EXP: Kaneko and Koda, *J. Cryst. Growth* **86**, 72 (1988)

ZnS: direct

ZnS	EXP	EXX	EPM
$E(\Gamma_v-\Gamma_c)$	3.68	3.73	3.73
$E(L_v-L_c)$	5.73	5.45	5.59
$E(X_v-X_c)$	6.31	6.43	6.58

EXP: Walter and Cohen, *Phys. Rev.* **183**, 763 (1969)

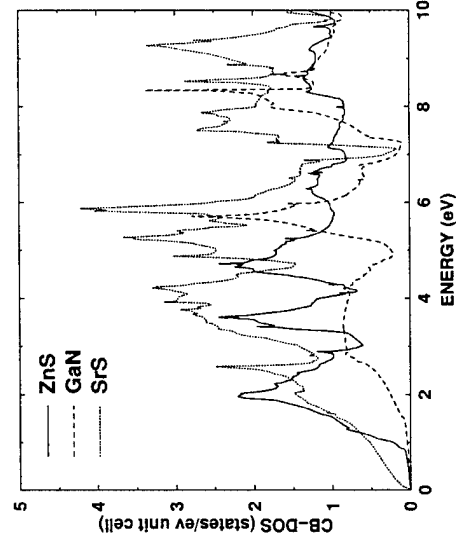
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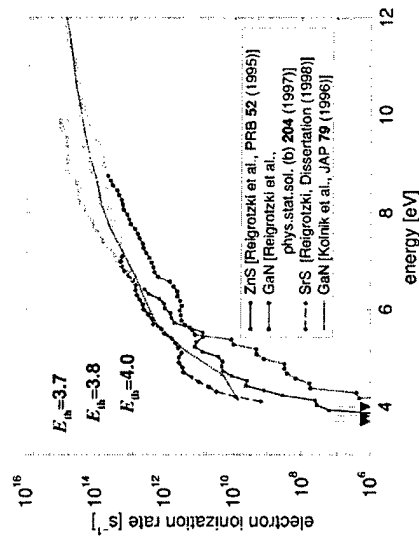
EXX Density of States for SrS, ZnS, and GaN

EXX



Impact Ionization Rate in Wide-Bandgap Materials

- Band to band impact ionization rate calculated directly from pseudopotential bandstructure (M. Reigrotzki *et al.*, *Phys. Rev. B* **52**, 1456 [1995]).
- Energy dependent scattering rate is fit to $R(E) = P (E-E_{th})^a$

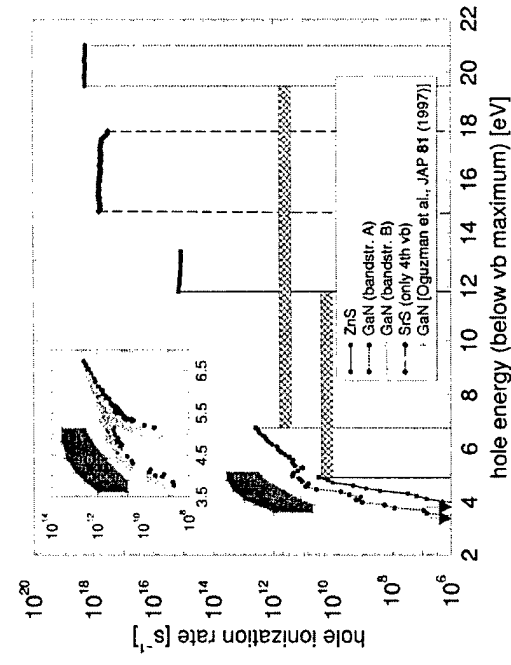


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Impact Ionization Rate for Holes Cut-off in hole rate due to finite VB width



Intercollisional Field Effect in Impact Ionization

- Quade et al. (PRB 50, 7398, 1994) ICFF in parabolic band approximation based on density matrix approach
- Redmer et al (JAP 87, 781, 2000) Extension of Quade et al. to full bandstructure, widebandgap materials using Zubarev nonequilibrium statistical operator approach.

$$\frac{\partial}{\partial t} f_{v,k}(t) + e\bar{E}_0(t) \bullet \frac{\partial}{\partial k} f_{v,k}(t) = J_e(v, k, t)$$

- Treating only electron-electron interactions,

$$r_{ii}(\varepsilon_1, E_0) = C \int_0^{\infty} dE \left(\frac{E}{E_{th}} \right)^a \frac{1}{E_F^{ii}} Ai \left(\frac{E_{th} - \varepsilon_1 + E}{E_F^{ii}} \right)$$

$$E_F^{ii} = \left[\frac{(1 + \alpha)(eE_0)^2}{8m_e \hbar} \right]^{1/3}$$

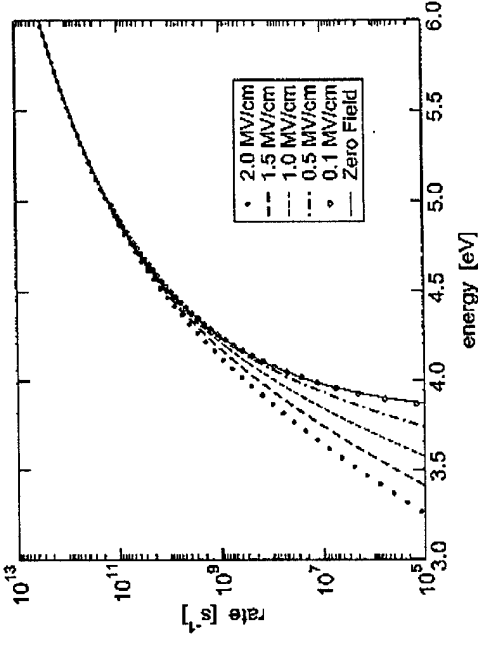
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Intercollisional Field Effect in Impact Ionization

- Softening of threshold field is main effect, little effect at high energy



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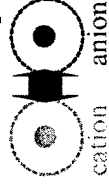
Direct Calculation of Electron-Phonon Scattering Rate

MOTIVATION

- The electron-phonon coupling is the dominant scattering process controlling the high field distribution
- Deformation potential model relies on constants which are not known experimentally for phosphor materials of interest
- A method is required to obtain the electron-phonon coupling directly from the electronic and vibrational properties of the crystal

METHOD

- Rigid-ion model (RIM) used² in which the rigid displacement of the atomic pseudo-potential (from EPM calculation) due to lattice vibrations gives the interaction potential

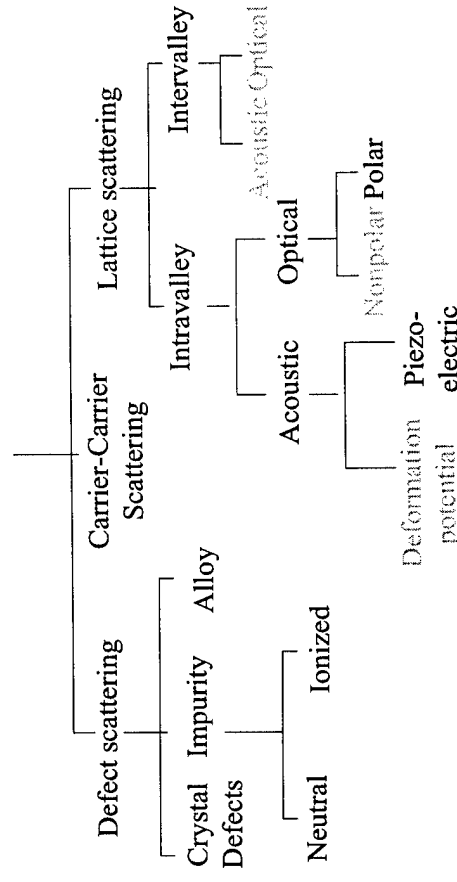


²M. V. Fischetti and J. M. Hignman, 1991

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Scattering Mechanisms



ASU

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RIM electron-phonon coupling constant that enters into the scattering rate

$$\Delta_j(n\mathbf{k}; n'\mathbf{k}') = -\frac{i}{2} (M_l + M_z)^{1/2} \times \sum_{\mathbf{G}\mathbf{G}'} [C_{\mathbf{G}}^n(\mathbf{k}) C_{\mathbf{G}'}^{n'}(\mathbf{k}')^* (Q - \mathbf{G} + \mathbf{G}' - \mathbf{G}_u)] \times \sum_{\alpha} e_{\alpha}^{\alpha} M_{\alpha}^{-1/2} V_{\alpha} (Q - \mathbf{G} + \mathbf{G}' - \mathbf{G}_u) \longrightarrow \times e^{-i(\mathbf{G} + \mathbf{G}' - \mathbf{G}_u) \cdot \mathbf{r}_{\alpha}} \delta_{\mathbf{k} - \mathbf{k}' + \mathbf{Q}, \mathbf{G}_u}$$

Important complication:
require the electron potential for
arbitrary $\mathbf{q}$ values, not just
the reciprocal lattice vectors

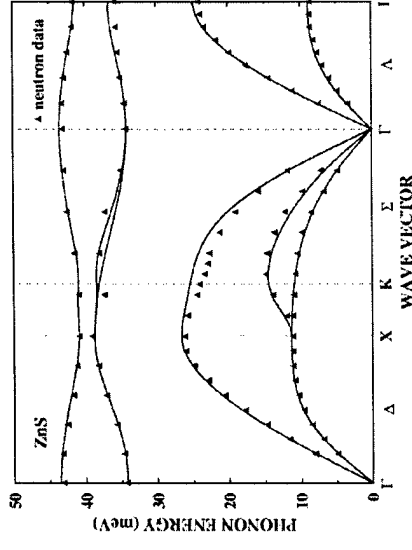
To calculate this quantity, you need the band structure, the pseudo wavefunctions and the phonon dispersion relations

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Phonon Spectrum of ZnS

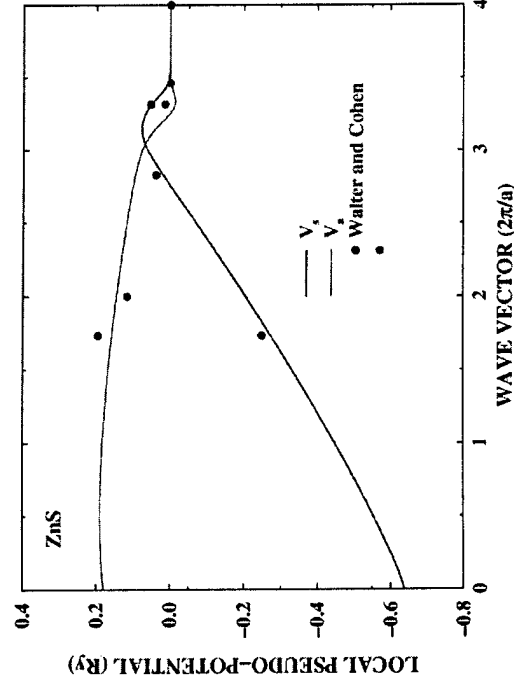
Phonon modes calculated using an empirical valence-shell model for the lattice dynamics with parameters fit to measured phonon dispersion (Vagelatos *et al.*, *J. Phys. Chem.* 60, 3613 [1974]).



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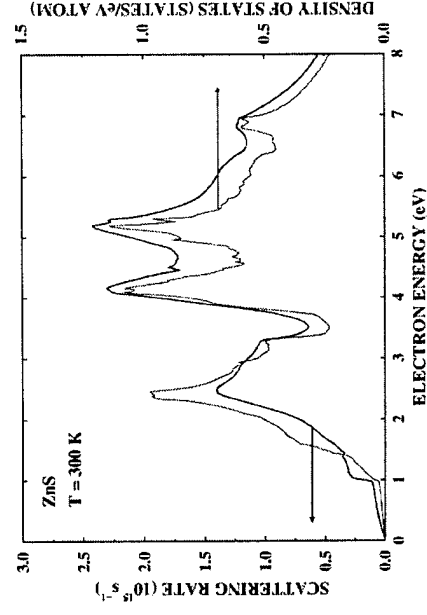
Interpolation of Local Pseudopotentials for ZnS



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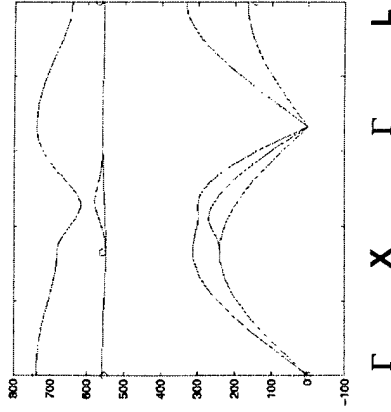
Calculated average electron-phonon scattering rate versus energy for ZnS using the rigid-ion model compared to density of states.



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Phonon dispersion for zincblende GaN obtained using a ten parameter empirical Shell model¹



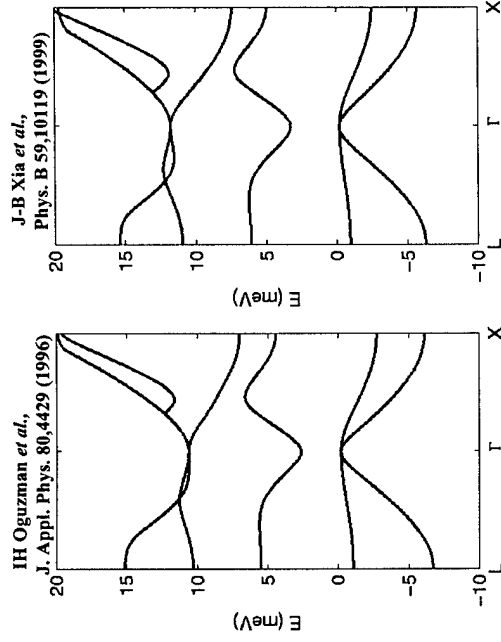
To fit the parameters of the model, experimentally determined phonon frequencies², as well as several theoretically values calculated using a Keating model³ were used

¹ K Kunc, OH Neilson, Comput. Phys. Comm. 17, 413 (1979)

² T. Azuhata *et al.* J. Phys. Condens. Matter 7, L129 (1995)

³ J. Zi *et al.* J. Phys. Condens. Matter 8, 6323 (1996)

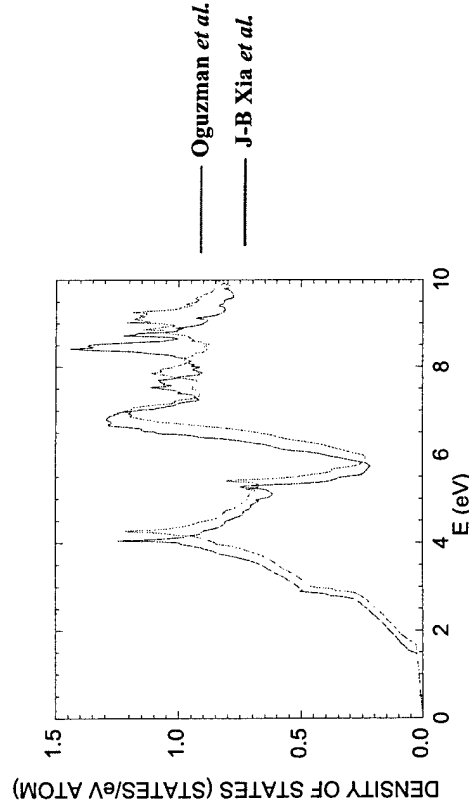
GaN band structures used based on the Empirical Pseudopotential Method (EPM)



The EPM uses fitted form factors to represent the atomic potentials $V_{Ga}(G)$, $V_N(G)$, for a limited set of reciprocal lattice points, G

Only local potentials were used in both cases

Density of states for the two cases:



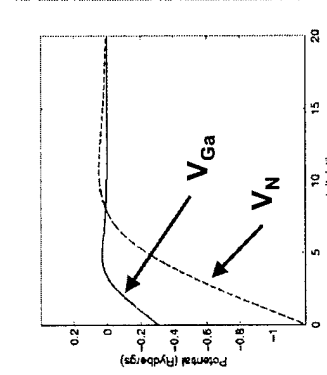
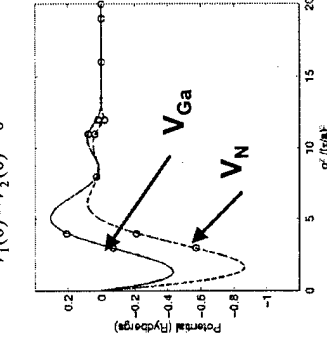
— Oguzman *et al.*
— J-B Xia *et al.*

Interpolation schemes for the pseudopotentials

Model 1: use polynomial interpolation to fit the form factors of Oguzman *et al.* (J. Appl Phys. 80, 4429, 1996)

Model 2: Use the same functional form and fits used by Xia *et al.* (Phys. Rev. B 59, 10119, 1999) :

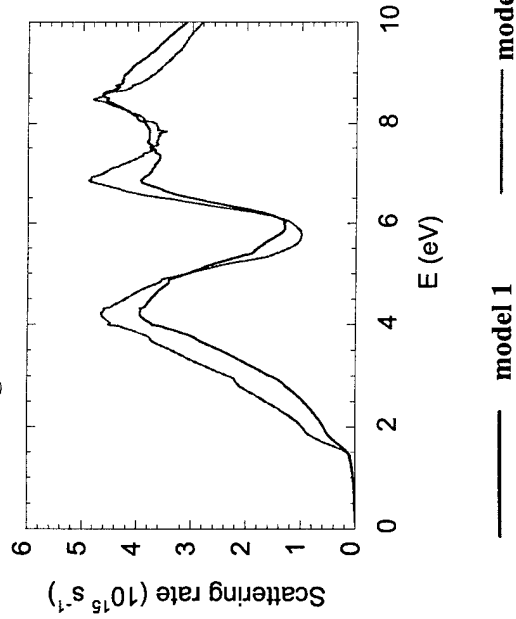
$$V_i(g) = a_1(g^2 - a_2)/[1 + \exp(a_3(g^2 - a_2))] \\ V_1(0) = V_2(0) = 0$$



Circles represent actual factors used by Oguzman *et al.*

Note that model 2 takes on more extreme values near zero

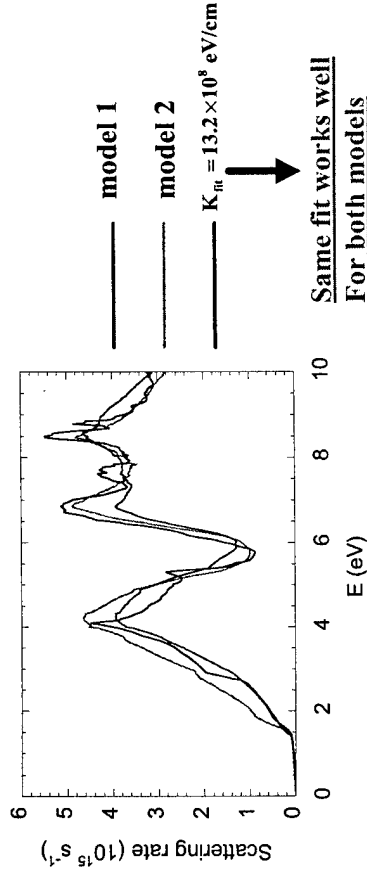
Phonon scattering rate results for the two models



Previous work on ZnS has suggested that the high field scattering rate in wide band gap materials is dominated by optical phonon deformation potential scattering: $R(E) \sim K_{fit}^2 N(E')$

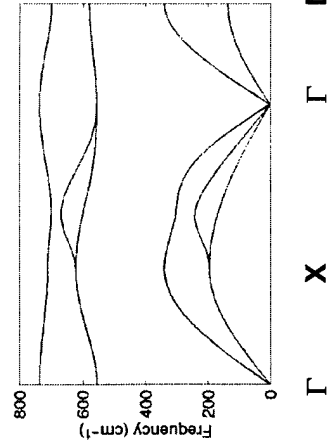
K_{fit} is a fitted deformation potential constant and $N(E')$ the final density of states

Comparison between fit and full calculations:



Second phonon dispersion model for zincblende GaN was also considered

valence shell fit to the data of
K. Karch, J.-M. Wagner, and F. Bechstedt, Phys. Rev. B 57, 7043 (1998)



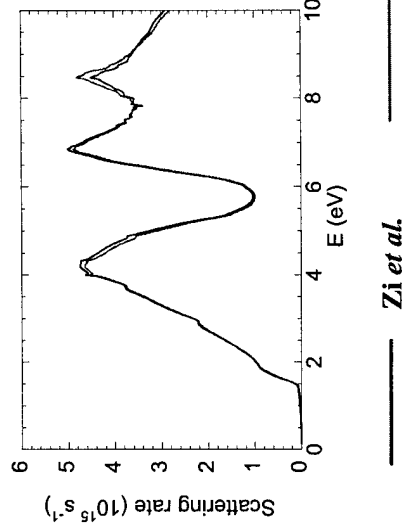
The data of Karch *et al.* was calculated within the framework of self-consistent density functional perturbation theory.

differences:

More dispersion shown TO branches than with the Zi *et al.* fit.

The frequencies attained by the TA branches are significantly lower than for the Zi *et al.* case.

Phonon scattering rate results for the two phonon models



using Oguzman *et al.* pseudopotential parameters

Full Band Ensemble Monte Carlo Model

- Full band dispersion used for particle dynamics
- Scattering treated a) pseudo isotropic, energy averaged rates from full band model b) full anisotropic rates in CA model
- Deformation potential Ansatz assumed, with values chosen either from fit to experimental data or extracted from RIM calculation
- Polar optical assuming Fröhlich coupling
- Ionized impurity, other elastic mechanisms included for low energies

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Impact Ionization Coefficient for Electrons

- Thompson and Allen (J. Phys. C 1989) measured the impact ionization coefficient in ZnS from multiplication measurements on photoinjected carriers in reverse biased Schottky barriers
- They assumed the e and h impact ionization rates were equal in order to relate multiplication to the ii coefficient

- Assuming the hole ionization rate is negligible, the expression relating carrier multiplication to the ionization coefficient is given by

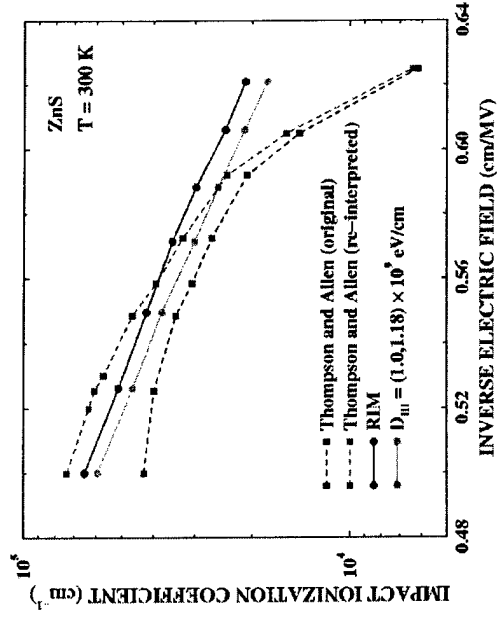
$$\ln M_n = \int_0^W \alpha_n dx = \alpha_n(E_m) W_{eff}$$

- Using this relation, we re-evaluated the impact ionization coefficient derived by Thompson and Allen assuming equal electron and hole coefficients

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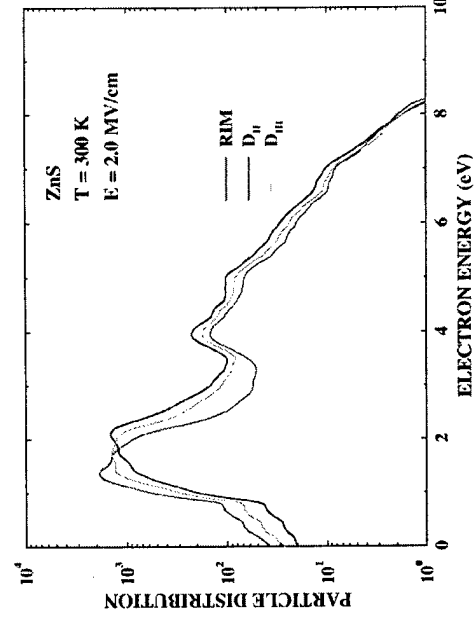
Re-Evaluation of Experimental Impact ionization coefficient assuming only electrons



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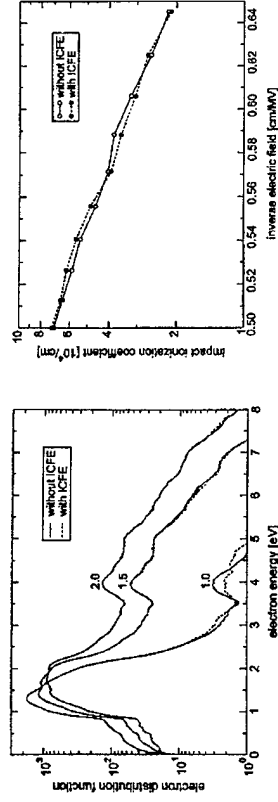
Comparison of steady-state particle distributions in ZnS at 2 MV/cm and 300 K using rigid-ion model and deformation potential sets D_{II} and D_{III}



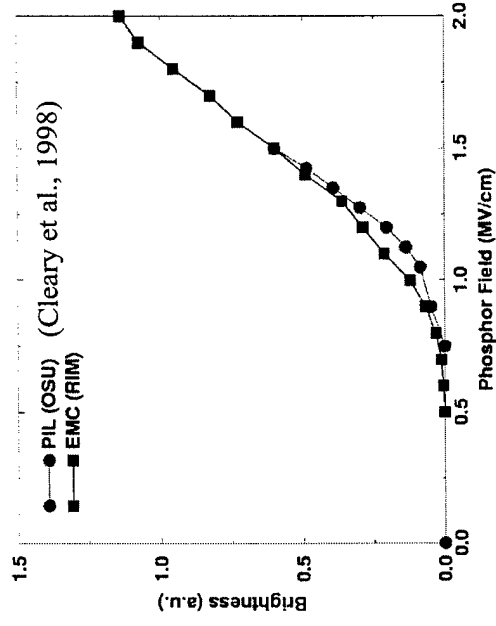
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Effect of ICFE on Distribution Function and Impact Ionization Coefficient

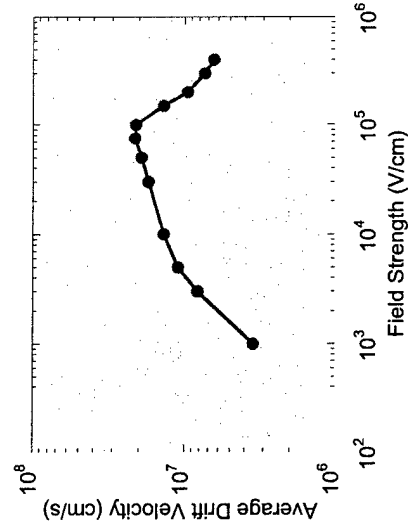


Calculated Brightness-Field Characteristics for ZnS:Mn²⁺ assuming zero space charge

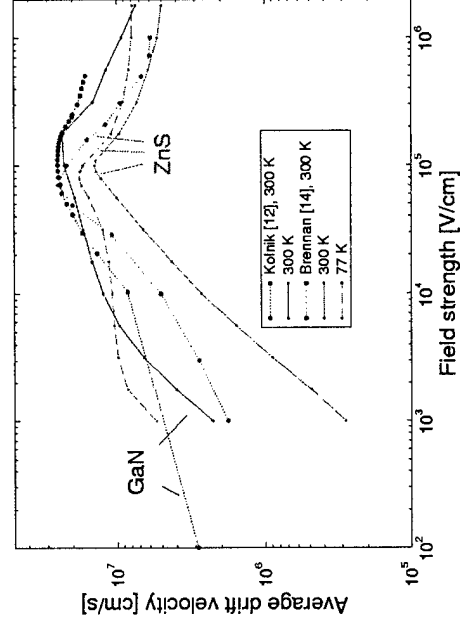


Calculated Velocity Field Characteristics

Drift velocity
results of full-band
Monte Carlo
simulations using
the fitted
deformation
potential at 300 K

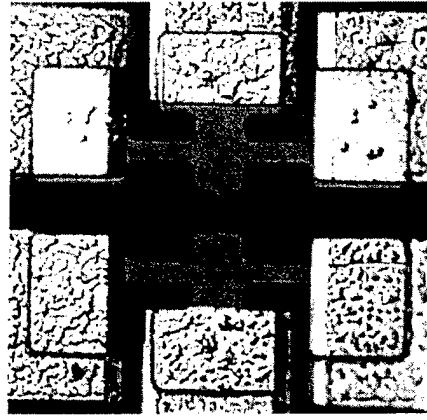


Average Electron Drift Velocity



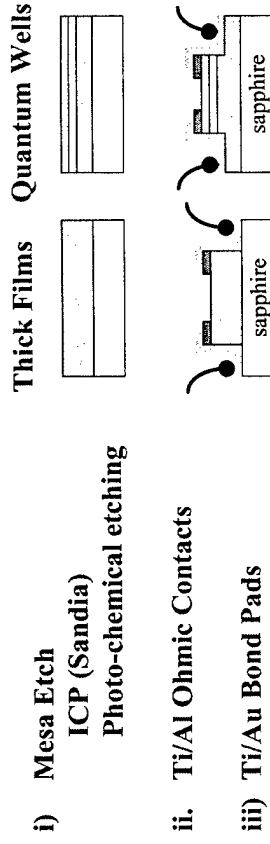
High Field Transport in GaN: Experimental

Pulse I-V measurements of velocity field characteristics



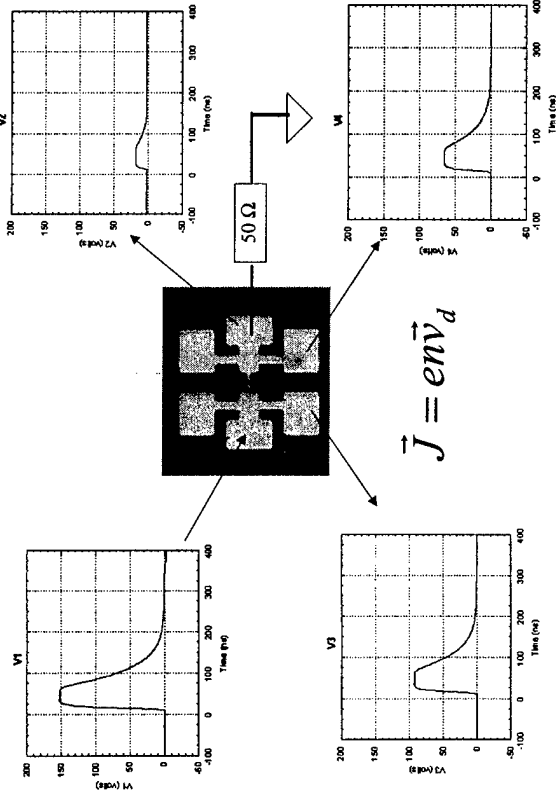
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GaN Test Structure Process Flow



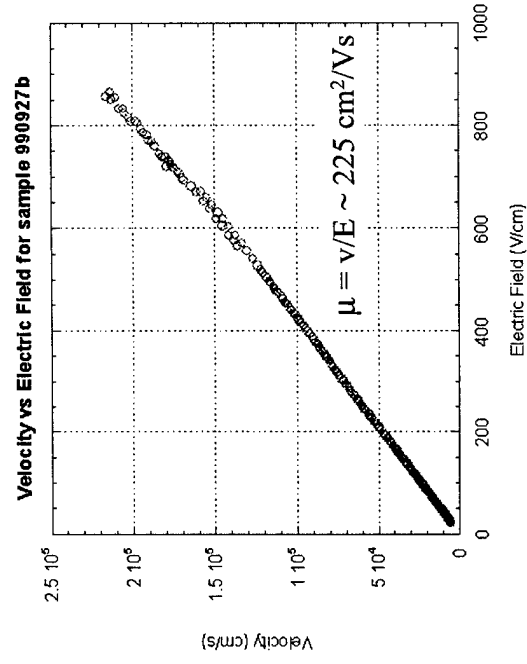
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Pulsed Measurements



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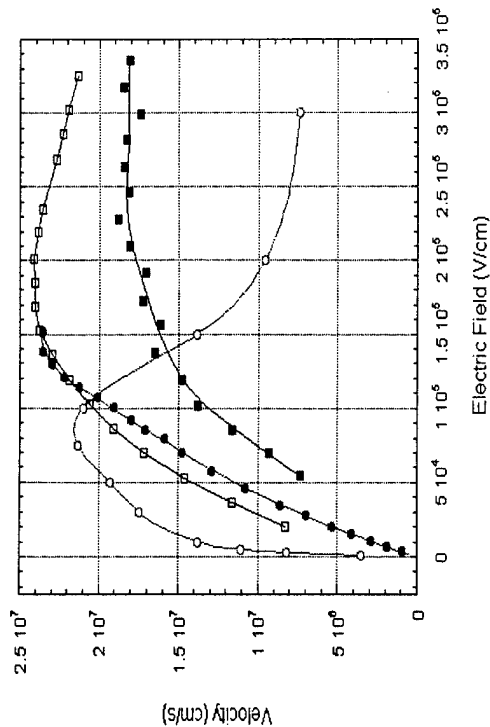
Results at low electric fields from GaN thick films



2.5 μm GaN Si-doped	Sapphire
-----------------------------------	----------

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High Field Pulse Measurements in Bulk GaN



—○— ASU Theory —●— ASU Experiment —□— ARO Theory —■— ARO Experiment

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Summary

Bandstructure Calculations

- Ab initio techniques more accurate, but not necessarily better than empirical methods for transport

Impact Ionization Rate for Holes in Phosphors

- Narrow valence band structure for ZnS and SrS implies impact ionization due to holes should be weak process
- Re-evaluation of Thompson-Allen data, comparable to theory
- No effect of ICFE on impact ionization

Velocity Field Characteristics

- Deformation potential of 1.3×10^9 eV/cm fits RIM calculation well: dominant mechanism above intervalley threshold
- Comparison to experimental measurement by different techniques

Semiclassical modeling of small semiconductor devices

Max Fischetti
IBM Research Division
T. J. Watson Research Center
Yorktown Heights, NY 10598

June 2001

Semiclassical modeling of small semiconductor devices

- Asen Asenov, Glasgow University,
Discrete impurities and quantum potentials in MOSFET modeling
- Richard Akis, Arizona State University,
Effective potentials for quantum effects in MOSFETs

MVF

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June 01 1

Semiclassical modeling

- Various flavors:
 - Strict: may use the **bulk (3D)** Boltzmann Transport Equation (BTE), à la Kohn-Luttinger (Liouville-von Neumann with infinitely-many randomly-distributed impurities), even if Quantum Mechanics is pervasive (bands, collisions, etc.)
 - Somewhat looser: may use a lower-dimensional BTE (quantum confinement), or any Master equation (dots). No transport 'in the quantized direction'
 - Very loose ('psychologically' classical): Approximate quantum effects so to allow the concept of 'particle' or 'trajectory' (Bohm trajectories, quantum potentials, effective potentials, Wigner paths/trajectories...)
- Anticipated failures:
 - Quantum confinement (strict): channels, wires, dots
 - Tunneling (somewhat looser): band-to-band, across barrier, resonant, ...
 - Interference effects (somewhat looser): Bohm-Aharonov, current oscillations in FN-tunneling and channel current (?), ...
 - 2D/3D confinement-effects (somewhat-looser/very-loose?): narrow channels in double-gate, point contacts, strongly-coupled dots
 - High-energy, high-field, short-time effects (Somewhat/very loose): Collisional broadening, intracollisional-field, coherent phenomena, ...
- Virtues of semiclassical modeling:
 - We have survived so far...
 - 'Intuitive', leading to deeper understanding
 - Simpler (or just...doable?), allowing more accurate description of effects which may be dominant in many real-life experiments/devices
 - Determination of 'bulk quantum properties' (band-structures, strain, electron-phonon coupling constants, Auger/impact processes)
 - Collective/many-body effects (Coulomb effects, quantum or MD)

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June 01 2

Dopant fluctuations

- Important (threshold control a crucial element in VLSI)...
- but complicated:
 - BTE suspect
 - Scatterers closer than mean-free-path:
 - * no rigorous use of mobility possible
 - * multiple-scattering, interference, non phase-breaking
 - What's the scattering potential?
- So far limited to drift-diffusion and to electrostatics (probably good enough)

MVF

June 01 3

Quantum/effective potentials

- From Wigner or Bohm
- 'Psychologically easy', mathematically hard
- So far approximated (equilibrium) or of a heuristic nature
- Of high appeal because of their 'intuitive' nature

Session 4

The Metal-Insulator Transition

- Moderator: Chihiro Hamaguchi, Osaka University
- 4A: Guenther Bauer, University of Linz, “The metal-insulator transition in $d=2$ ”
- 4B: Jonathan Bird, Arizona State University, “The metal-insulator transition in open quantum dots and arrays”

Metal-insulator transition in two dimensions

Guenther Bauer, University of Linz

- No metallic state in two dimensions in $B = 0$ (Abrahams et al. in 1979: Scaling theory).
 - With decreasing temperature the resistance should rise logarithmically (weak localization) or even exponentially (strong localization) with $R \rightarrow \infty$ for $T \rightarrow 0$.
 - In the limit of weak localization R should grow logarithmically for $T \rightarrow 0$.
-
- A critical density was found:
 - Below the system is insulating
 - Above the system shows metallic behaviors
 - Many explanations were put forward
 - Si-MOS
 - Experiments: weak localization
 - onset of metal-insulator transition: classical effect (screening, ionized impurity scattering.....)

- For strong interactions it should end up as a Wigner crystal (or pinned Wigner crystal).
- Therefore, neither in the limit of weak nor in the limit of strong interactions, 2D systems should conduct for $T \rightarrow 0$.
- Early experiments confirmed these predictions.
- However, experiments with very clean high mobility samples contradict it.
 - A non zero conductivity was found.
 - Evidence for a metal-insulator transition

The metal-insulator transition in open quantum dots and arrays

Jonathan Bird, Arizona State University

- Much recent interest in the observation of a Kondo effect in Coulomb blockaded quantum dots.
- In open quantum dots the Coulomb blockade is suppressed.
- Transport is found to be mediated by strongly scarred wavefunction states.
- The scars correspond to quasi-bound resonant states of the open system which are characterized by long trapping times at specific energies.

- The quasi-bound nature is expected to give rise to novel signatures in transport due to electron interactions.
- The behavior of the quasi-bound states will be explored in this presentation.




Intrinsic Fluctuations and Quantum Potentials in MOSFET Modelling

"The tyranny of the large numbers"

G. Timp

A. Asenov

A. R. Brown, S. Kaya, J. Watling, J. H. Davies, G. Slavcheva

Department of Electronics and Electrical Engineering

University of Glasgow

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Summary

- Introduction
- Quantum corrections
- Random Dopant Fluctuations
- Oxide Thickness Fluctuations
- Conclusions

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Scaling of MOSFETs to decanano dimensions

(International Roadmap for Semiconductors - 1999 Edition)

International Technology Roadmap

Year	1999	2001	2004	2008	2011	2014
MPU Gate Length (nm)	340	200	70			
Oxide thickness (nm)	10-25	1.5-1.9	1.2-1.5			
Drain extensions (nm)	40-70	30-50				

Intel Roadmap

Year	1998	2000	2003	2005	2007	2009
Technology node (nm)	250	180	130	90	65	45
Gate length (nm)	250	180	130	90	65	45

Solution exists
Solution Being Pursued
No Known Solutions

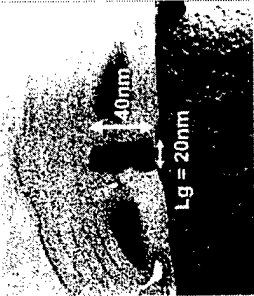
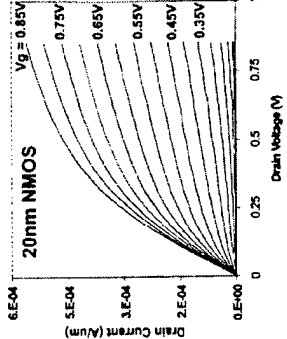
Vertical confinement
Direct oxide tunnelling
S-D tunneling

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Conventional MOSFET with 20 nm gate length

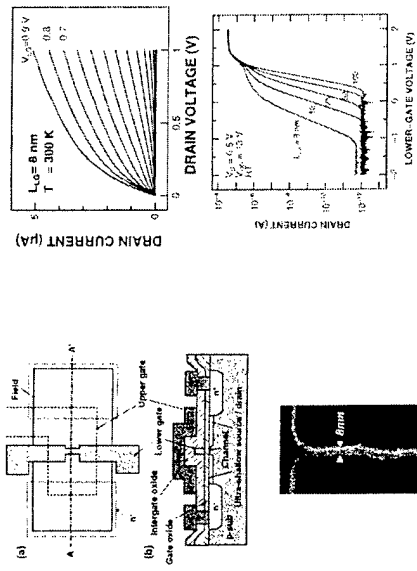



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Robert Chaw
Si Nanoel. Workshop 01

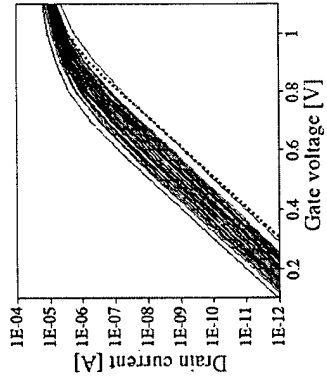
Double gate MOSFET with 8 nm gate length



H. Kawagura et al.
Si Nanoel. Workshop 99
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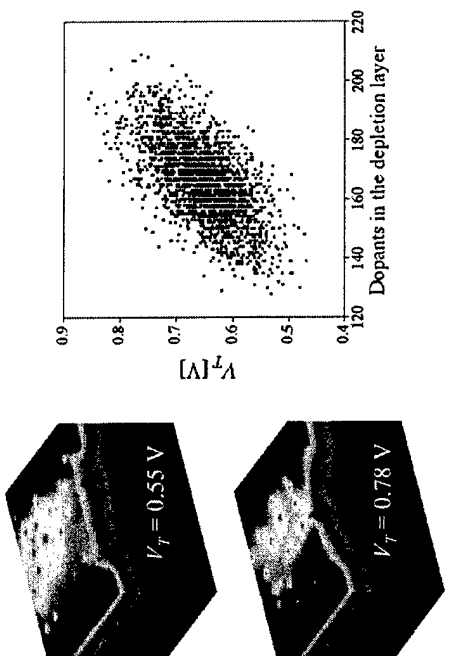
Random Dopants Fluctuations (RDF)



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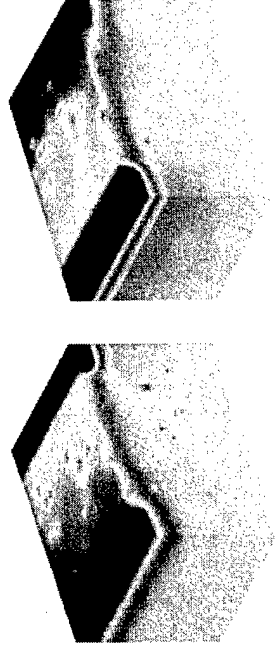
Numbers and position fluctuations



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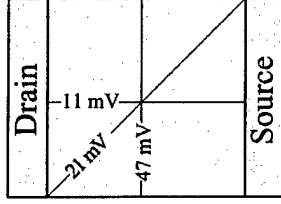
Threshold voltage asymmetry in a 50x50 nm MOSFET



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The poly-Si gate (A 50×50 nm MOSFET)

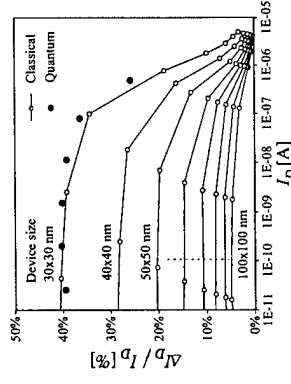
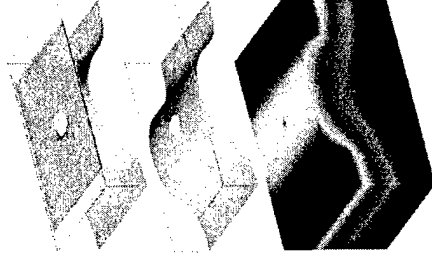


Grain boundaries

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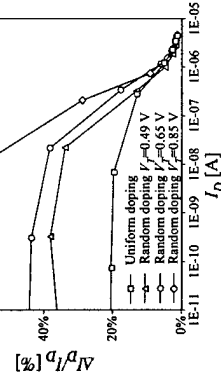
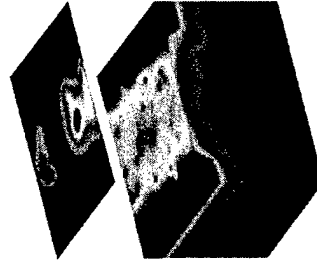
Single charge trapping



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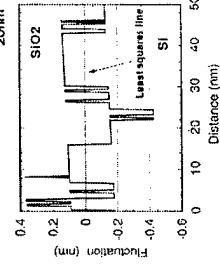
Single charge trapping (A 50×50 nm MOSFET with discrete dopants)



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Oxide Thickness Fluctuations (OTF)

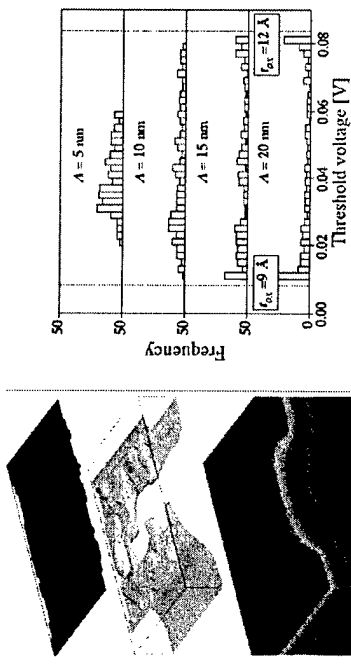


M. Niva et al.
Jpn. J. Appl. Phys.

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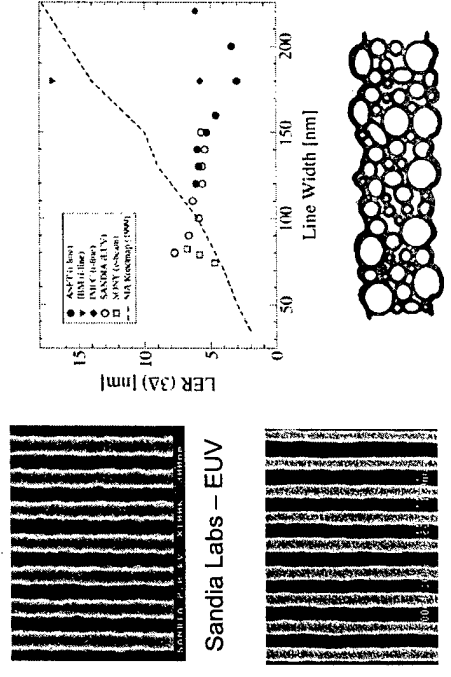
OTF induced MOSFET parameter fluctuations (A 30x30 nm MOSFET)



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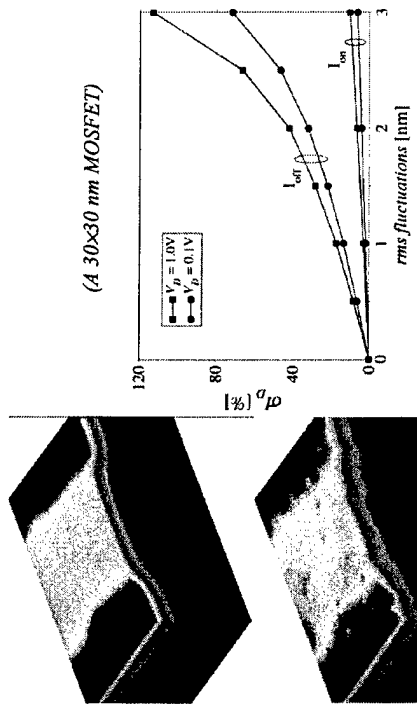
Line edge roughness (LER)



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Line edge roughness (LER)



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Summary

- ☐ Introduction
- ☐ Quantum corrections
- ☐ Random Dopant Fluctuations
- ☐ Oxide Thickness Fluctuations
- ☐ Conclusions

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Solution domain in 3D 'atomistic' simulation

- ☐ 3D DD simulations + quantum corrections
- ☐ Fine grain discretisation
- ☐ Statistical ensembles of microscopically different devices
- ☐ Estimation of averages and standard deviations

50×50 nm MOSFET

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The Density Gradient (DG) approach (C.S. Rafferty et al., SISPAD'98)

DG introduces QM corrections by introducing an extra term into carrier flux expression

$$F_n = n\mu_n \nabla \psi - D_n \nabla n + 2\mu_n \nabla \left(b_n \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right)$$

where

$$b_n = \hbar / (12qm_n^*)$$

Generalised quasi-Fermi potential

$$F_n = n\mu_n \nabla \phi_n$$

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DD with DG corrections for unipolar device (C.S. Rafferty et al., SISPAD'98)

The system

$$\nabla \cdot (\epsilon \nabla \psi) = -q(p - n + N_D^+ - N_A^-)$$

$$\nabla \cdot (n\mu_n \nabla \phi_n) = 0$$

$$2b_n \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} = \phi_n - \psi + \frac{kT}{q} \ln \frac{n}{n_i}$$

The variables

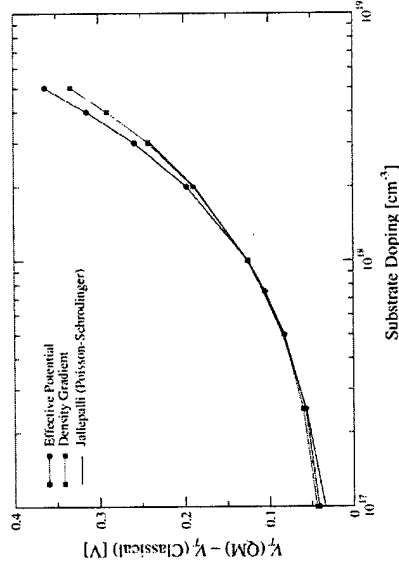
$$\psi, \phi_n, \sqrt{n}$$

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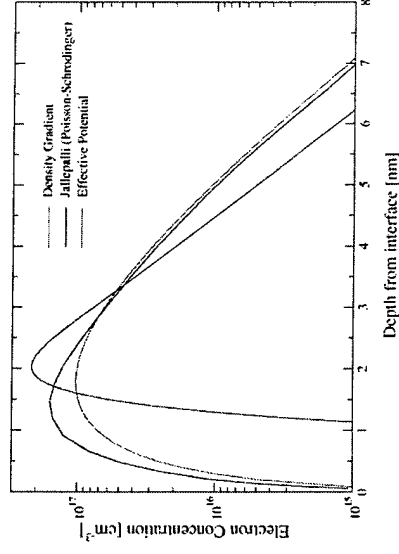
Quantum mechanical threshold voltage shift



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Quantum mechanical charge distribution

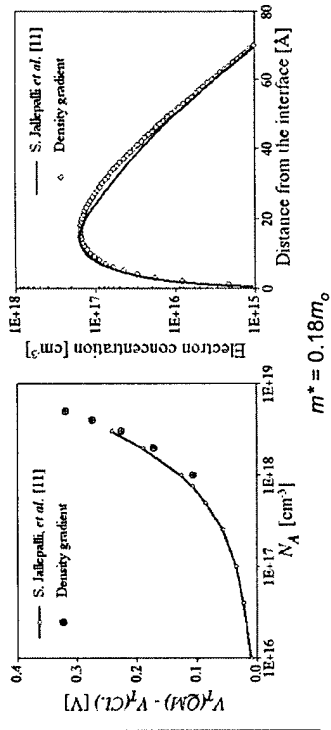


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m^* is a fitting parameter

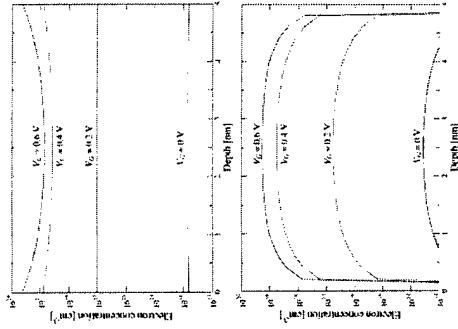


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Quantum Transport in Semiconductors
Maratea, June 2001

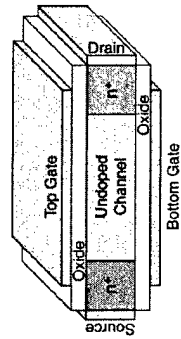


Double gate MOSFET

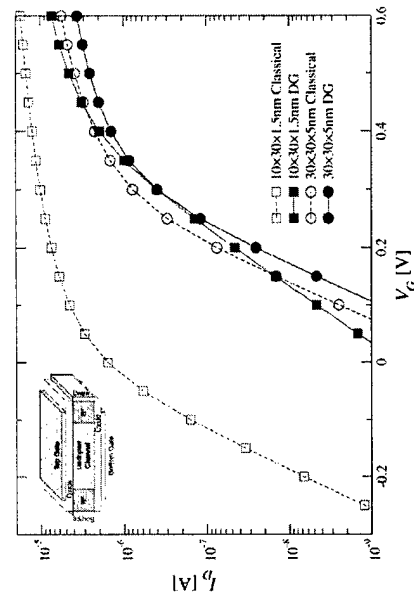


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Source-to-drain tunnelling



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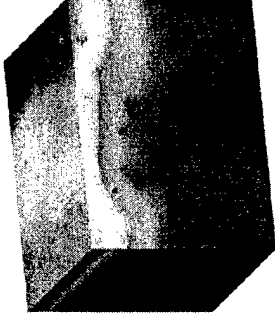
Summary

- ☐ Introduction
- ☐ Quantum corrections
- ☐ Random Dopant Fluctuations
- ☐ Oxide Thickness Fluctuations
- ☐ Conclusions

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Potential and electron distribution (A 50x50 nm MOSFET)



Potential

Electron concentration

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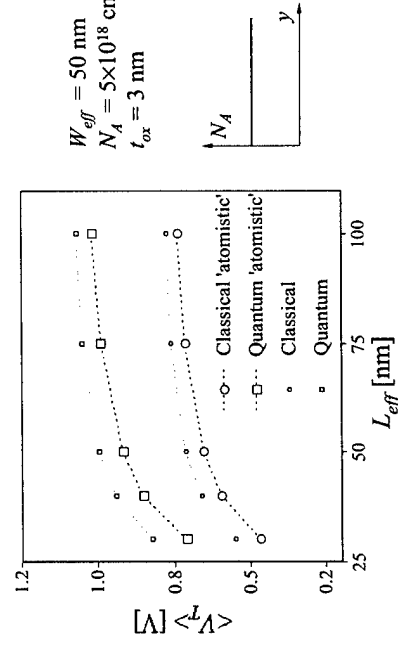
Electron equiconcentration distribution (A 50x50 nm MOSFET)

QuickTime™ and a
 decompressor
 are needed for this picture.

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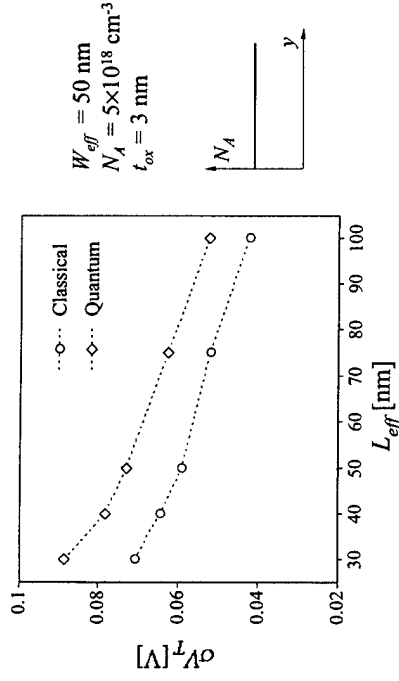
Threshold voltage as a function of the gate length



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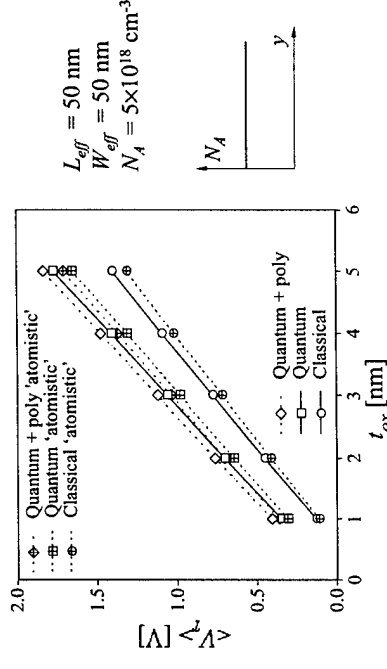
Threshold voltage standard deviation as a function of the gate length



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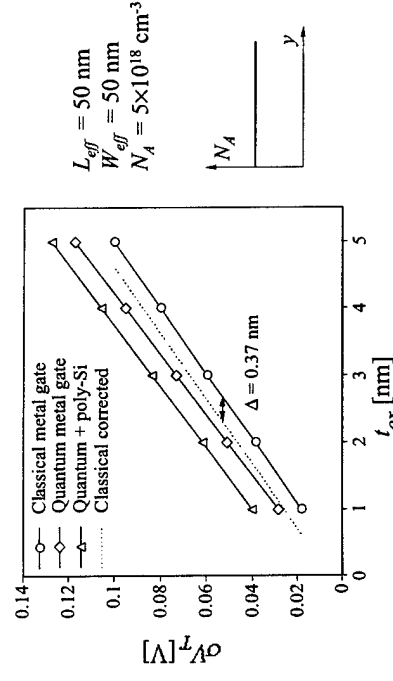
Threshold voltage as a function of the oxide thickness



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Threshold voltage standard deviation as a function of the oxide thickness



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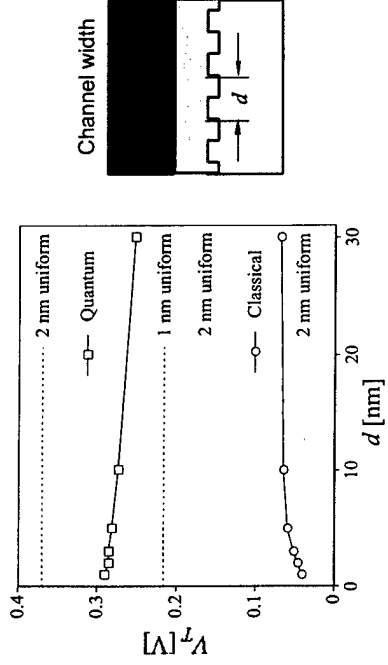
Summary

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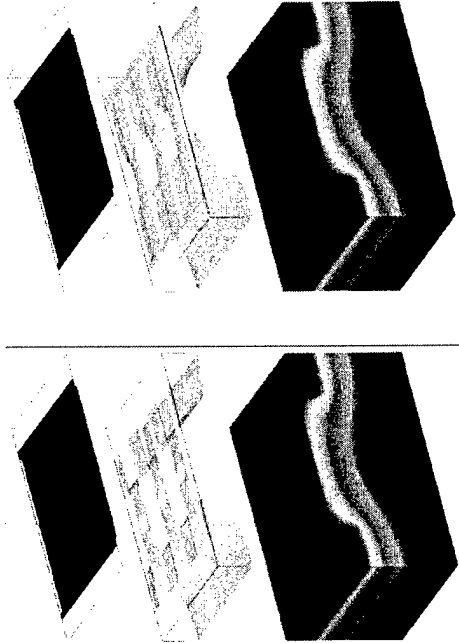
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Threshold voltage as a function of the period of oxide thickness superlattice



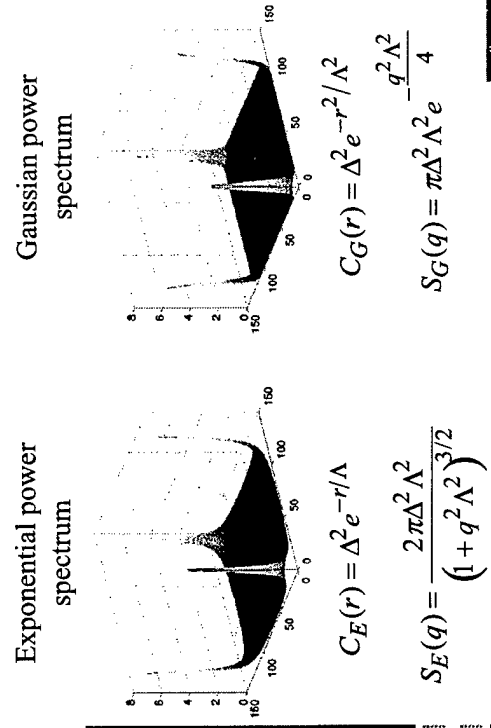
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Classical and quantum mechanical charge distribution in the oxide superlattice



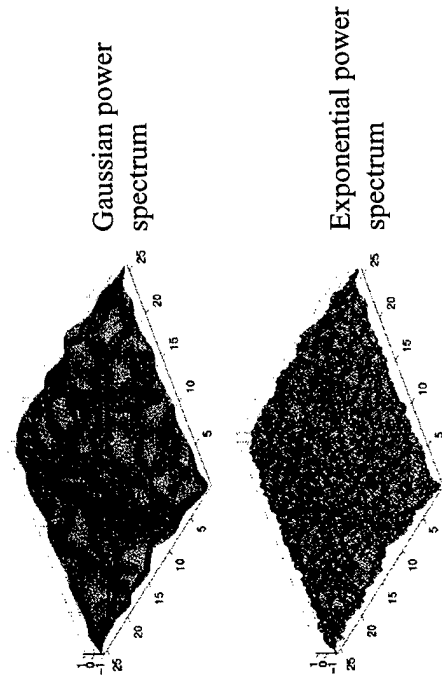
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Reconstruction of the Si/SiO₂ interface



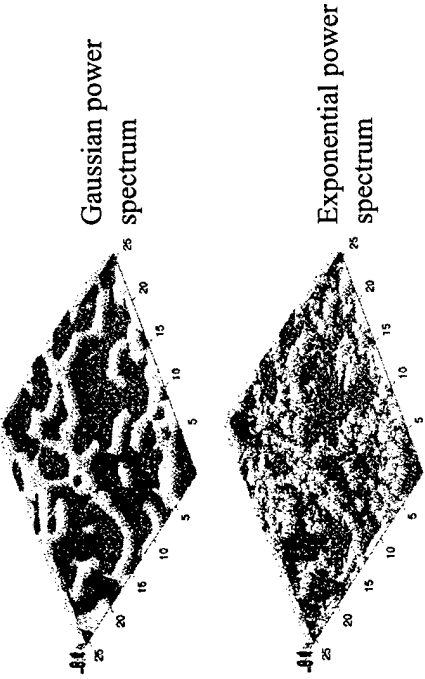
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Reconstruction of the Si/SiO₂ interface



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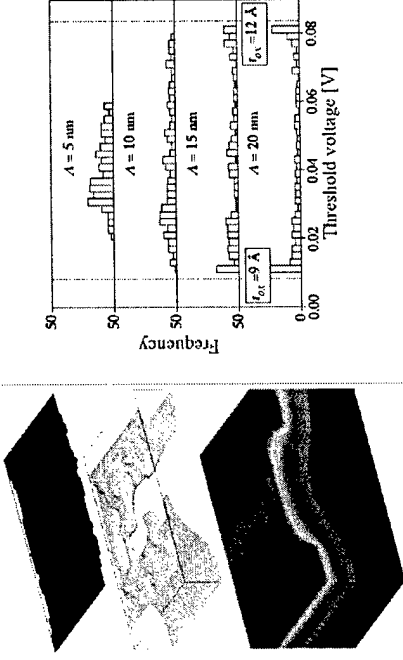
Digitisation of the Si/SiO₂ interface



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OTF induced MOSFET parameter fluctuations (A 30x30 nm MOSFET)

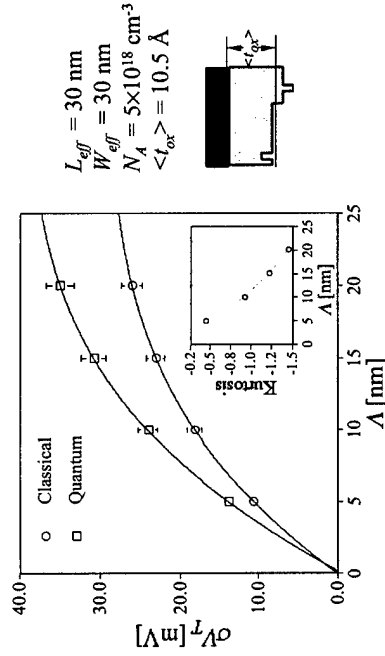


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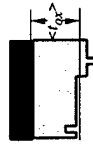
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Threshold voltage standard deviation as a function of the correlation length



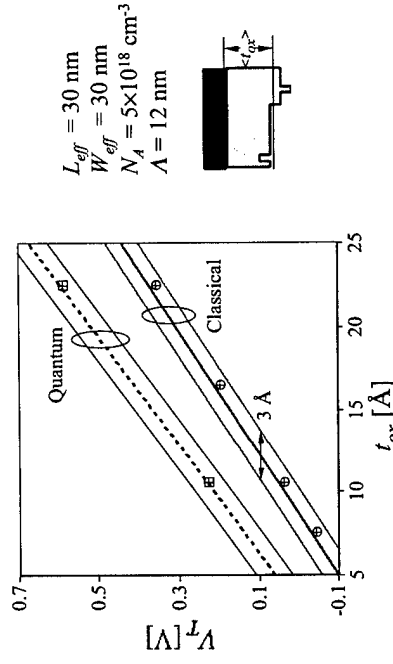
$L_{eff} = 30 \text{ nm}$
 $W_{eff} = 30 \text{ nm}$
 $N_A = 5 \times 10^{18} \text{ cm}^{-3}$
 $\langle t_{ox} \rangle = 10.5 \text{ \AA}$



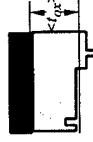
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Threshold voltage as a function of the average oxide thickness

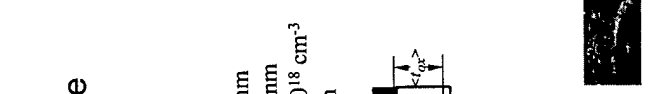


$L_{eff} = 30 \text{ nm}$
 $W_{eff} = 30 \text{ nm}$
 $N_A = 5 \times 10^{18} \text{ cm}^{-3}$
 $A = 12 \text{ nm}$

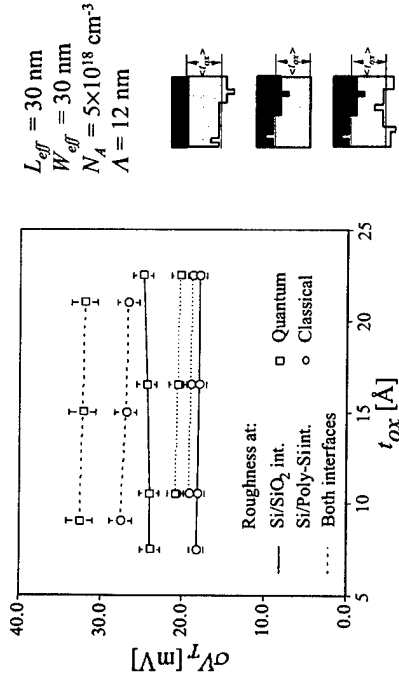


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Threshold voltage standard deviation as a function of the oxide thickness



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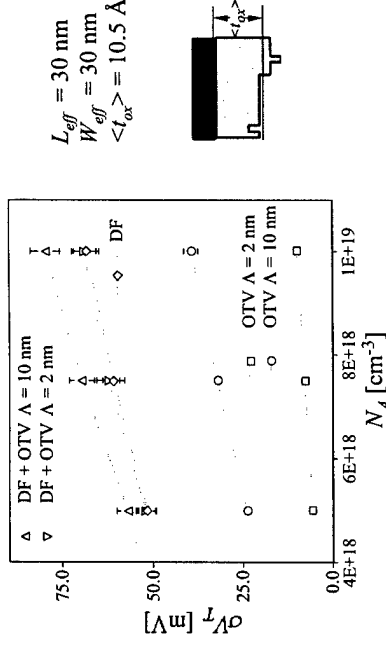
Conclusions

- The DG approach provides relatively simple means to include quantum correction in the 3D DD 'atomistic' MOSFET simulation.
- The inclusion of the QM corrections increases the intrinsic parameter fluctuations associated with discrete random dopants and oxide thickness variation.

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Threshold voltage standard deviation as a function of the doping concentration



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Effective Potentials for Quantum Effects in MOSFETs

Dr. Richard Akis
Department of Electrical Engineering
Arizona State University

Who else is to blame:

L. Shifren
S. Ramey
S. N. Milicic
D. Vasileska
and, in particular
D. K. Ferry

*Work supported by ONR, NSF, and SRC

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Outline

Introduction

the effective potential- what is it and why should you care

Including the effective potential in device simulations

1D simulation of a MOSFET channel

2D simulation of a "standard" MOSFET

simulation of SOI devices

simulation of a quantum point contact

Conclusions

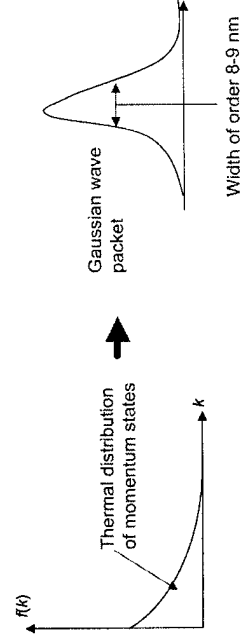
A Major Concern for Future ICs

While current production devices are only at 0.18 μm , predictions are that they will be at 50 nm by year 2012. At these sizes, we should begin to see **quantum effects**, as $\lambda_D \sim 3\text{-}5\text{ nm}$ at 300 K. The questions are:

- ⊙ In quantum mechanics, electrons no longer behave as point particles. How "large" is the electron wave packet?
- ⊙ How can we include space quantization effects efficiently into what are otherwise classical device simulators?



The smallest size will be determined by the effective size of the electron wave packet.

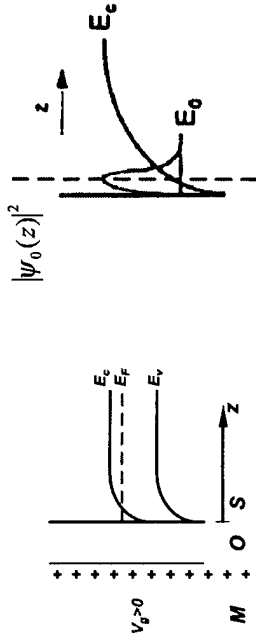


$$\psi(r) = \frac{\pi}{(\sqrt{2}\lambda_D)^{3/2}} \exp\left(-\frac{\pi r^2}{\lambda_D^2}\right)$$

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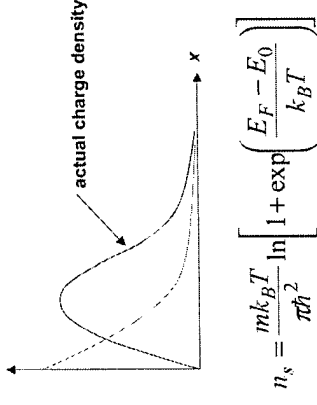
MOSFET conduction band profile along the growth direction

inversion layer formed
carriers at interface trapped in
a triangular well → creation of 1D subbands
that leads to charge set back

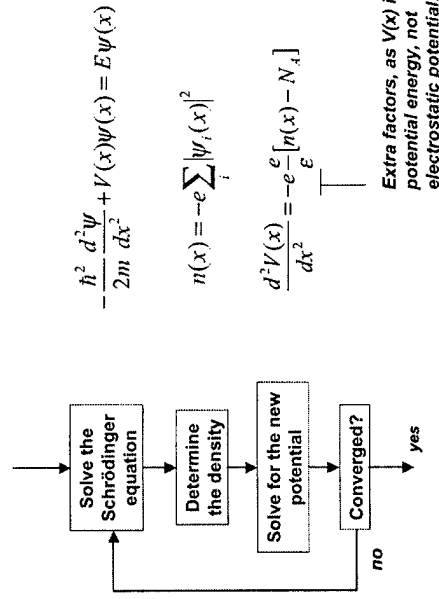


Quantization in the Channel of the MOSFET

The electrons in the inversion layer are quantized—their motion normal to the interface is quantized by the potential well between the oxide and the conduction band. The actual width of the inversion layer is found from the Schrödinger equation.



Quantum method of finding the wave functions and energy levels



✓ The concept of the effective potential was introduced by Feynman, and has been extended by several authors.

Classically, we use $e^{\beta V}$; thus, the idea of the effective potential is to seek a modified potential V_{eff} by which we can write the density as

$$n = n_0 e^{-\beta V_{\text{eff}}}$$

→ Sharp potentials no longer appear in the problem. Rather, these are “smoothed” by a Gaussian function, here derived from the effective minimal size of the electron wave packet in the system under study.

By replacing the sharp potentials by smoothed potentials, we can return to using quasi-classical point charges in our simulations, confident that the results will be quite appropriate.

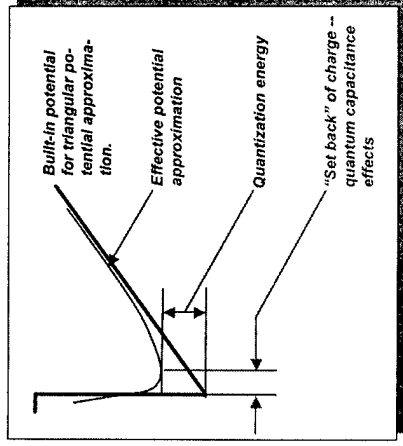
$$V_{\text{eff}}(x) = \frac{1}{\sqrt{2\pi}a_0} \int_{-\infty}^{\infty} V(x') \exp\left(-\frac{(x-x')^2}{2a_0^2}\right) dx'$$

Effective Potential Approach

In principle, the effective role of the potential can be rewritten in terms of the non-local density as (Ferry et al.¹⁾:

$$\begin{aligned}\vec{V} &= \int d\vec{r}' V(\vec{r}) \sum_i n_i(\vec{r}) \\ &\sim \int d\vec{r}' V(\vec{r}) \sum_i \int d\vec{r} \exp\left(-\frac{|\vec{r}-\vec{r}'|^2}{\alpha^2}\right) \delta(\vec{r}-\vec{r}_i) \\ &\sim \sum_i \int d\vec{r} \delta(\vec{r}-\vec{r}_i) \int d\vec{r}' V(\vec{r}') \exp\left(-\frac{|\vec{r}-\vec{r}'|^2}{\alpha^2}\right) \\ &\sim \sum_i \int d\vec{r} \delta(\vec{r}-\vec{r}_i) V_{eff}(\vec{r})\end{aligned}$$

Classical density
Smoothed, effective potential

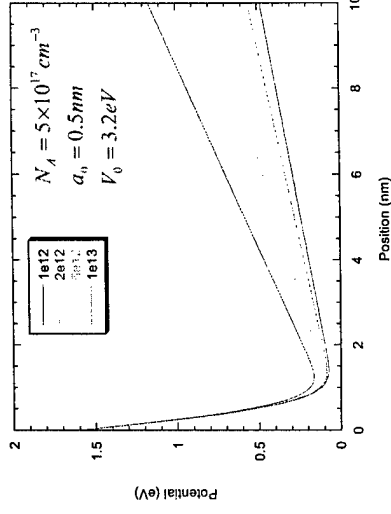


¹ D. K. Ferry, *Superlatt. Microstruct.* 27, 59 (2000); VLSI Design, in press.

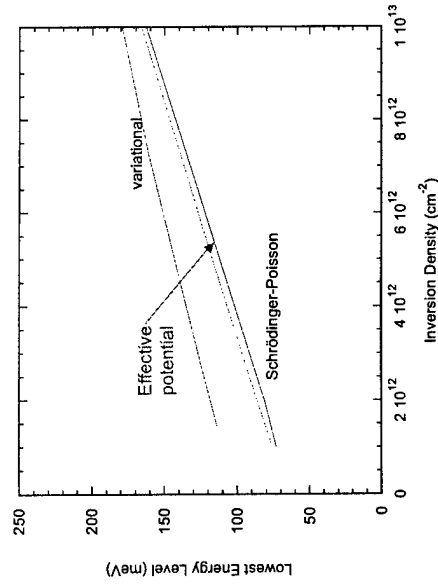
Use of the Effective Potential

$$V_{eff}(x) = \frac{V_0}{2} \left[1 - \exp\left(-\frac{x}{\sqrt{2}a_0}\right) \right] + \frac{F_0}{2} \left[1 + \exp\left(-\frac{x}{\sqrt{2}a_0}\right) \right] + \frac{F_0 a_0}{\sqrt{2}\pi} \exp\left(-\frac{x^2}{2a_0^2}\right)$$

$$F = \frac{e}{\epsilon} (N_A d_A + \frac{n_s}{2})$$

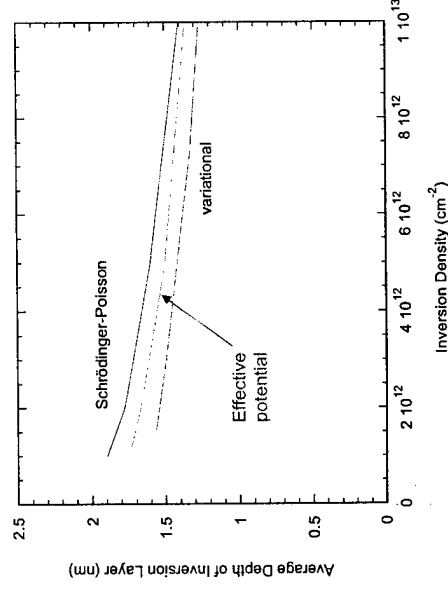


Use of the Effective Potential



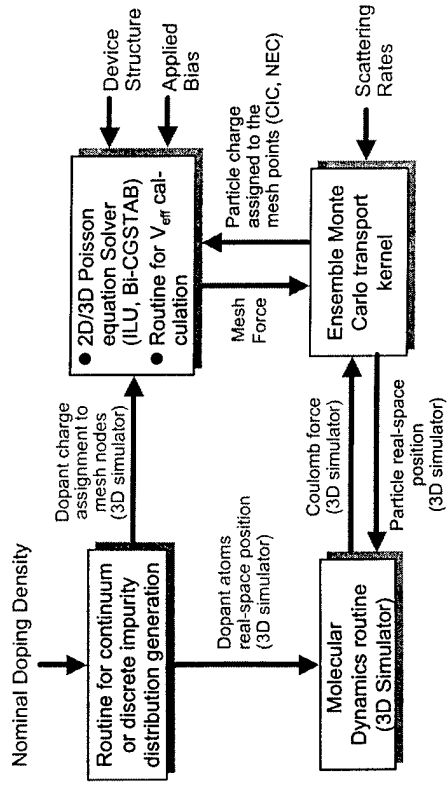
$$N_A = 5 \times 10^{17} \text{ cm}^{-3}$$

Use of the Effective Potential

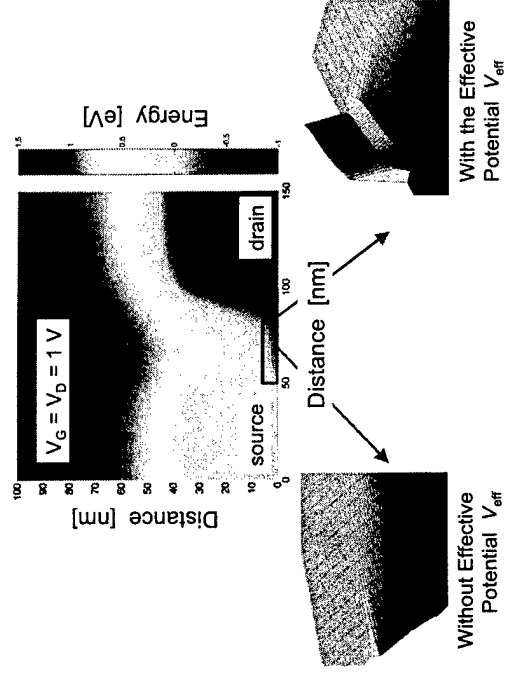


$$N_A = 5 \times 10^{17} \text{ cm}^{-3}$$

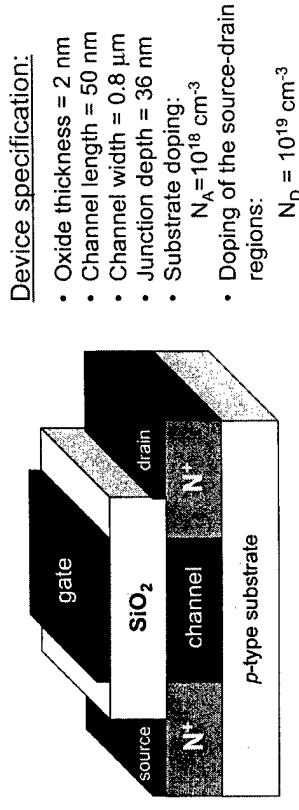
Description of the Existing 2D/3D Device Simulators



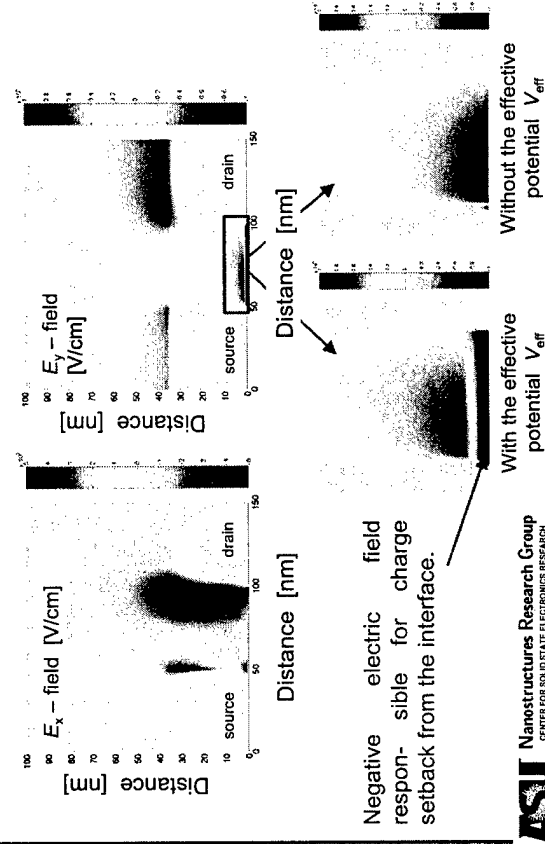
Conduction Band Profile



Simulation of a 50 nm MOSFET Device



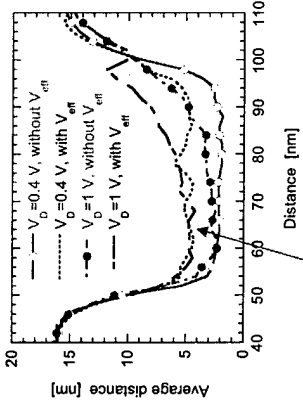
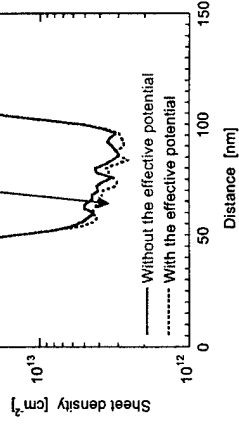
Electric Field Profile



Carrier Density and Average Distance

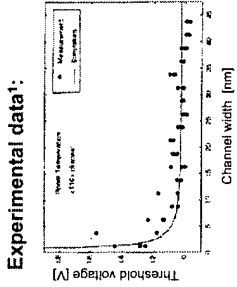
Applied bias: $V_G = 1\text{ V}$

Electron sheet density reduction due to quantum-mechanical band-gap widening effect

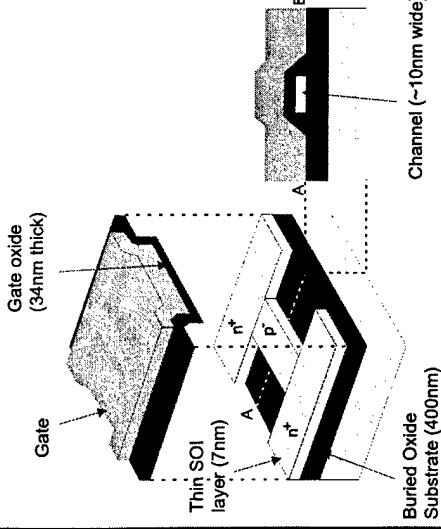


Additional 2-3 nm charge set-back from the interface due to quantum-mechanical space-quantization effect

SOI Device Structure

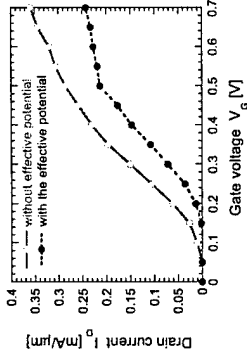


- Simulate this structure using full 3D Poisson solver coupled with 2D Schrödinger solver.
- Do same calculation with effective potential



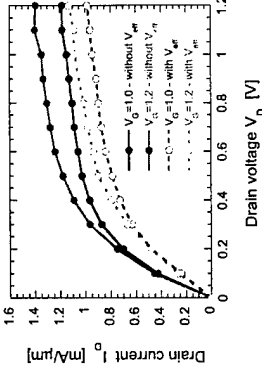
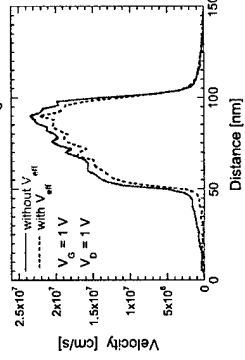
1. Majima, Ishikuro, Hiranoto, IEEE Et. Dev. Lett. 21, 386 (2000).

Transfer and Output Characteristics



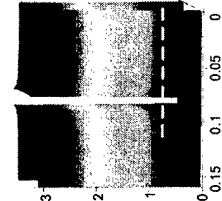
⊙ The observed threshold voltage shift is mainly due to the degradation of the device transconductance and is not due to mobility degradation in the channel.

⊙ The shift in the threshold voltage leads to a reduction in the on-state current.

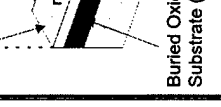
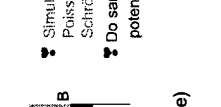
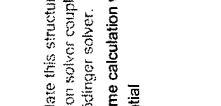
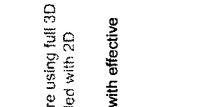
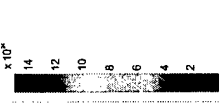
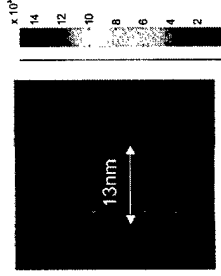
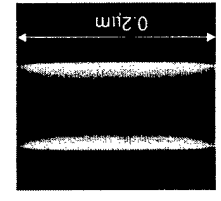


Conduction Band Edge and Electron Density

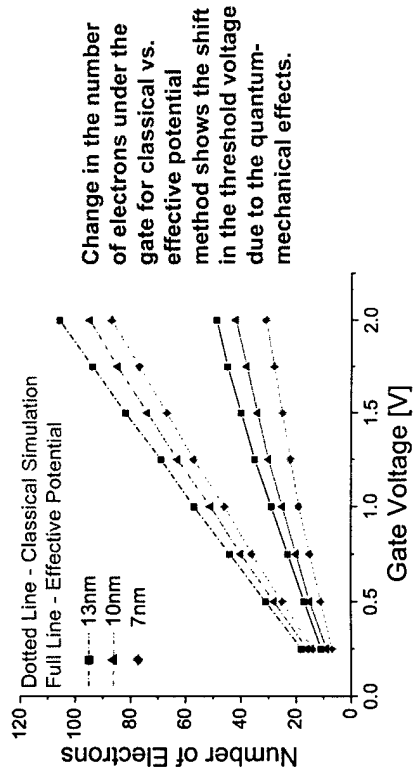
Classical Simulation



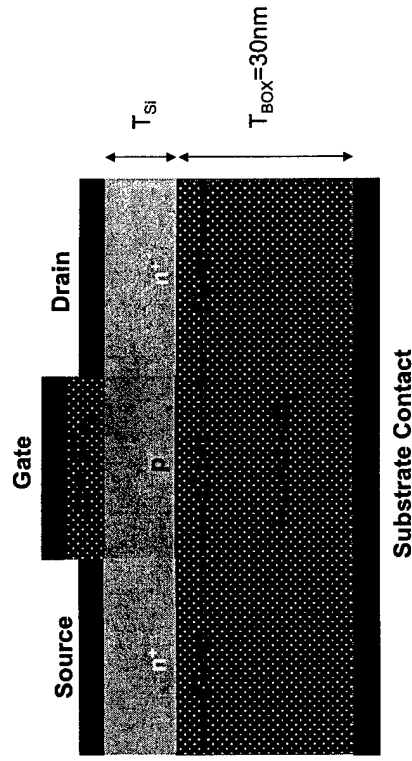
Smoothering of the potential rises its value in the channel and this influences electron density.



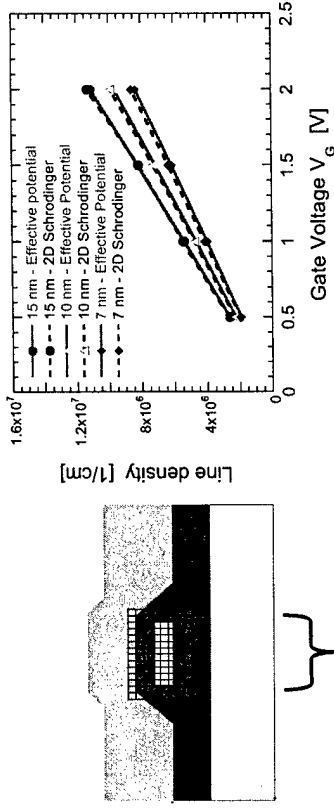
Number of Electrons in the Channel



SOI MOSFET Structure



Effective potential vs. 2D Schrodinger

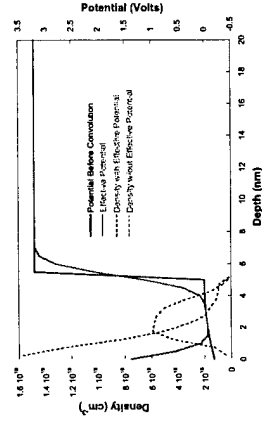


2D Schrodinger equation solved in cross section

effective potential results match full calculation rather well

Potential Profile Under Gate

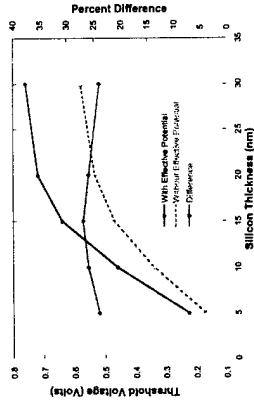
$$T_{OX}=2nm, V_D=0.1V, V_{anb}=0.0V, N_A=5 \times 10^{18} cm^{-3}$$



Results of Effective Potential on V_T

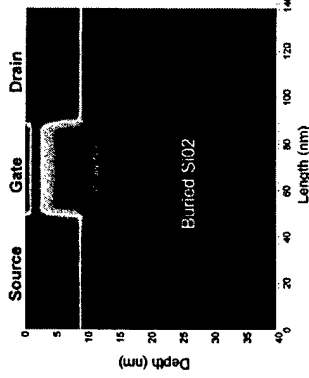
$$N_A=5 \times 10^{18} cm^{-3}, V_D=0.6V, V_P=0.1V, T_{OX}=2nm$$

Neglecting quantum underestimates V_T and therefore overestimates drive current.

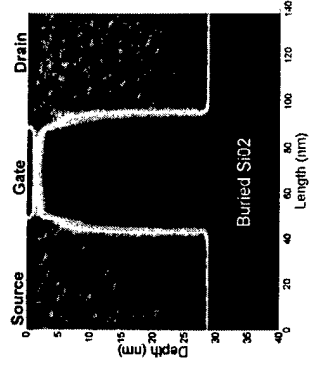


Electron Density Distribution

Fully Depleted



Partially Depleted

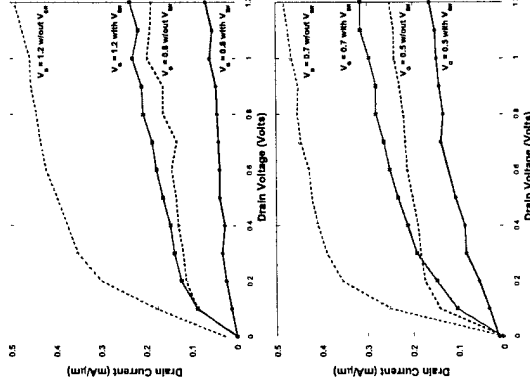


Gate Voltage = 1.0 Volt Drain Voltage = 0.1 Volts $N_A = 5 \times 10^{18} \text{ cm}^{-3}$

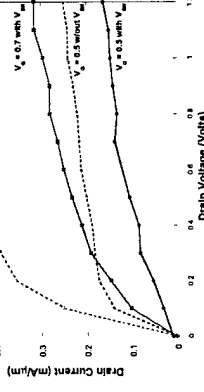
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I_D - V_D Characteristics

$T_{Si} = 30 \text{ nm}$



$T_{Si} = 10 \text{ nm}$



Decreasing silicon film thickness increases drive current and reduces V_T .

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Madelung and Bohm's reformulation of quantum mechanics

$$\text{Insert } \psi = Re^{iS/\hbar}, \quad R = \sqrt{n} = |\psi| \quad \rightarrow \quad -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\text{real part yields the quantum Hamilton-Jacobi equation:} \quad \frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

$$\text{where } S \rightarrow \text{action, and } Q = -\frac{\hbar^2}{2m^*} \frac{\nabla^2 |\psi|}{|\psi|} \text{ is the "quantum potential"}$$

$$\text{equations of motion:} \quad v(x, t) = \frac{\nabla S(x, t)}{m} = \frac{\hbar}{m} \frac{\text{Im}(\psi^* \nabla \psi)}{|\psi|^2}$$

$$a(x, t) = -\frac{\nabla(V + Q)}{m}$$

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Connection with the effective potential

$$V_{\text{eff}}(x) = \frac{1}{\sqrt{2\pi\alpha}} \int_{-\infty}^{\infty} V(x + \xi) e^{-\xi^2/2\alpha^2} d\xi$$

$$\equiv \frac{1}{\sqrt{2\pi\alpha}} \int_{-\infty}^{\infty} \left[V(x) + \xi \frac{\partial V}{\partial x} + \frac{\xi^2}{2} \frac{\partial^2 V}{\partial x^2} + \dots \right] e^{-\xi^2/2\alpha^2} d\xi$$

$$V_{\text{eff}}(x) = V(x) + \alpha^2 \frac{\partial^2 V}{\partial x^2} + \dots$$

In semiconductors, typically the dependence of the density upon the potential is as a factor $\exp(-\beta V)$

$$V_{\text{eff}}(x) = V(x) - \frac{2\alpha^2}{\beta} \frac{\partial^2 \ln(\sqrt{n/n_0})}{\partial x^2} + \dots$$

$$= V(x) - \frac{2\alpha^2}{\beta \sqrt{n}} \frac{\partial^2 \sqrt{n}}{\partial x^2} + \dots$$

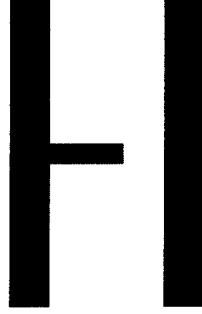
Within a factor of 2, the second term is now recognized as the density gradient term, but is more commonly known as the Bohm potential

$$Q = \frac{\hbar^2}{2m^*} \frac{\nabla^2 |\psi|}{|\psi|}$$

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quantum point contact

1. We start by consider a quantum wire with constriction
2. We solve the system with and without self-consistently
3. We smooth the self-consistent solution to obtain the effective potential
4. We use the effective potential to generate classical trajectories and the self-consistent solution to generate quantum trajectories



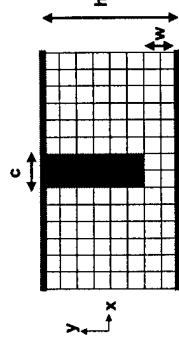
Self-Consistent Solutions

Solving Schrödinger's Equation

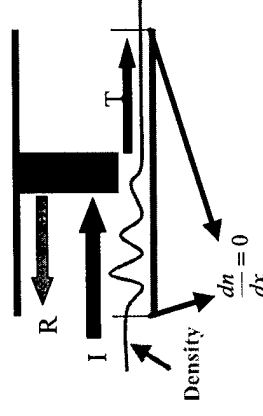
- Solution obtained via finite difference scattering matrix cascade

We assume:

1. that the wire extends to $\pm\infty$
2. the reflected waves do not interfere with the boundary



- Therefore, solutions have continuous boundary conditions
- This is our criteria for boundary conditions



Self-Consistent Solutions

1. Boundary conditions for the Poisson solver and the Schrödinger solver
2. Calculating the Hartree potential (via the Poisson solver) and included the exchange and correlation potentials
3. Maintaining a constant density in the system. A build up of density occurs due to the inclusion of the correlation potential which drops the conduction band edge
4. Using the correct electron density in the Poisson solver. (Taking into account the contributions of the positive ions in the system)

Self-Consistent Solutions

Dealing with the dynamic density

- Solving the self-consistent problem introduces a potential profile into the device
- The correlation energy lowers the conduction band edge, and therefore allows a build up of charge which increases the number of propagating modes and density in the device
- To keep density constant, the Fermi energy is adjusted with the lowering conduction band edge during iteration to maintain a constant density (propagating modes)



$$N = \frac{2}{\pi} \sum_{E_l < E_F} \left[\frac{2m^*(E_F - E_l)}{\hbar^2} \right]^{\frac{1}{2}}$$

When solving the Poisson equation, we need to make sure we do not over-count free electron charges in the system

$$\nabla^2 \phi(\mathbf{r}) = -\frac{e}{\epsilon} \rho(\mathbf{r})$$

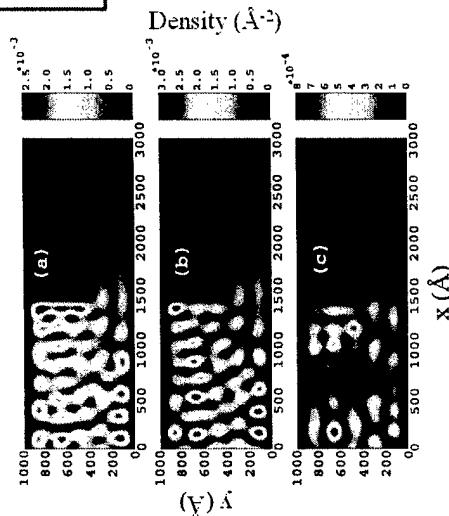
$$\rho(\mathbf{r}) = n(\mathbf{r}) - n_{eq}$$

Constriction in a quantum wire



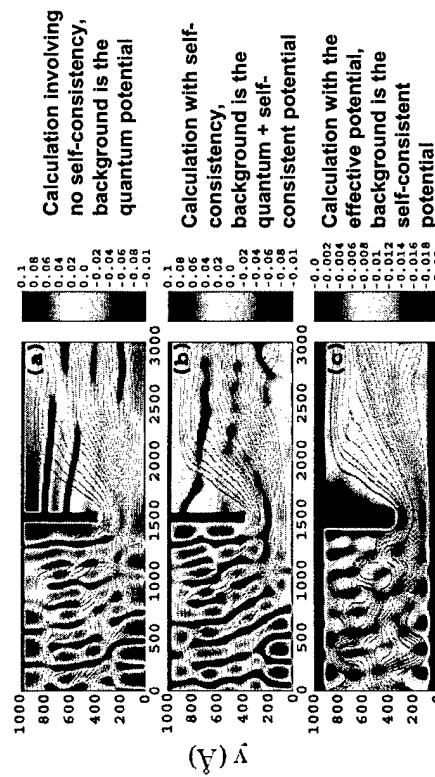
- We know there is a potential drop after the constriction
- We can interpret the drop in the Hartree potential to be the expected drop in the Fermi level

- (a) No potential
(b) Self-consistent potential
(c) Effective Potential



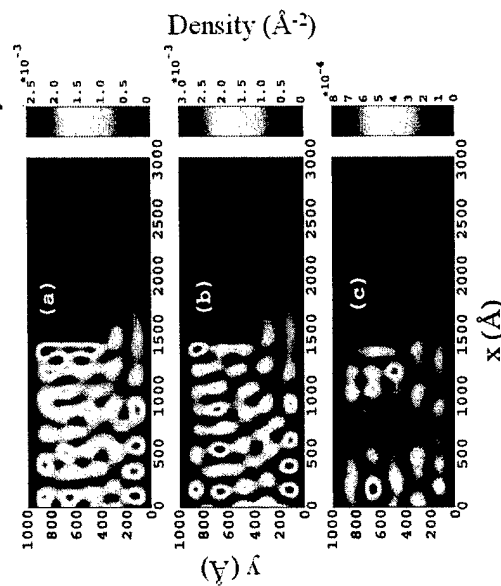
- As we begin to include higher levels of potential into the system [from (a)-(c)], we see the density move further from the walls of the device

Bohm trajectories flowing through the QPC. The background is the self-consistent potential.



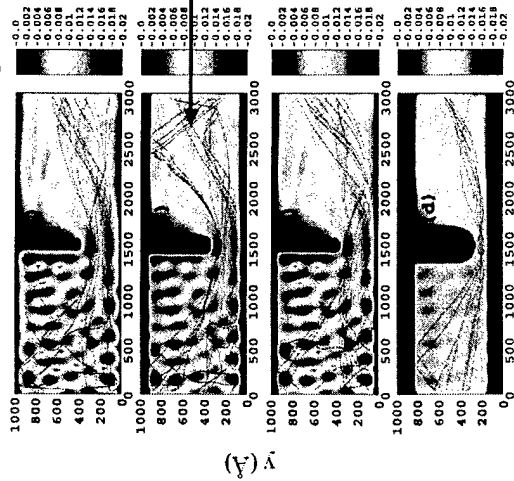
x (Å)

Electron density



x (Å)

Classical particles moving in the effective potential



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Conclusions

• The effective potential approach in many cases allows for a quite accurate approximation of the quantization effects in real semiconductor devices

• The numerical cost of including the effective potential is low – "more bang for the buck"

Some challenges remain....

△ Model valid only within the random phase approximation.

△ It is still "quasi-local", so it does not allow proper treatment phase interference effects

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Wave function approaches for self-consistent computations of transport in quantum dots and arrays

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Outline

- ❖ Discretization of the Schrödinger equation
- ❖ The Usuki mode matching technique
- ❖ Application to quantum dots
- ❖ Incorporation of Poisson's equation
- ❖ Further application to quantum dots

Special thanks to R. Akis, L. Shifren, J. Harris for their contributions, without which there would be no content to this talk. Also, discussions with C. Jacoboni on quantum computing have been most helpful.

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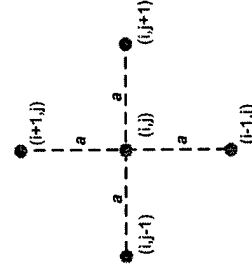
- ◆ Throughout this discussion, we will consider the region of interest to be the quasi-two-dimensional electron gas at the interface of a GaAs/AlGaAs heterojunction: all parameters will be for electrons in GaAs and confining potentials will be imposed upon this 2D gas.

The Schrödinger equation, in 2D, can be written as

$$-\frac{\hbar^2}{2m^*} \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} \right) \psi(x, y) + V(x, y) \psi(x, y) = E \psi(x, y)$$

where $V(x, y)$ is the local "site potential." This potential will be the self-consistent potential in later usage.

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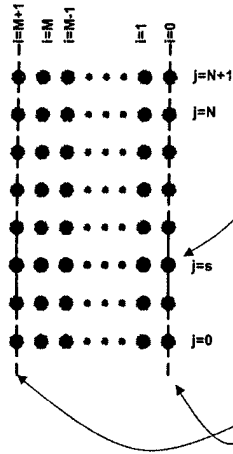
We now discretize the Schrödinger equation, using the 5-point scheme shown at left. This is a form of central differencing, that will introduce some unfortunate byproducts that will have to be addressed.

The Schrödinger equation, in its discretized form, now takes the shape of

$$-t[\psi(i+1, j) + \psi(i-1, j) + \psi(i, j+1) + \psi(i, j-1)] + [V(i, j) + 4t]\psi(i, j) = E\psi(i, j)$$

$$t = \frac{\hbar^2}{2m^* a^2} \text{ is the "hopping" energy.}$$

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On slice s , we may write the wave function as

$$\vec{\psi}(s) = \begin{bmatrix} \psi(M, s) \\ \psi(M-1, s) \\ \vdots \\ \psi(1, s) \end{bmatrix}$$

$$\psi(0, j) = \psi(M+1, j) = 0$$

There is an automatic assumption that

In general, this discretized Schrödinger equation can now be written in the form of coupled "slices" (columns j) as

$$H_0(s)\vec{\psi}(s) - t\mathbf{I}\vec{\psi}(s+1) - t\mathbf{I}\vec{\psi}(s-1) = E\vec{\psi}(s)$$

with

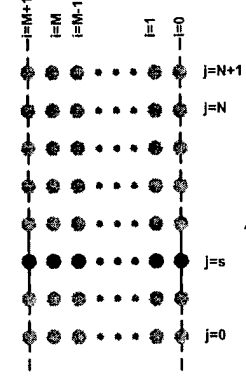
$$H_0(s) = \begin{bmatrix} [V(M, s) + 4t] & -t & 0 & \dots & \dots \\ -t & [V(M-1, s) + 4t] & -t & \dots & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \dots & \dots & -t & [V(2, s) + 4t] & -t \\ \dots & \dots & 0 & -t & [V(1, s) + 4t] \end{bmatrix}$$

There are two approaches that can be developed now to "iterate" the solutions from one end to the other. These are:

- ❖ The "recursive" Green's function, in which a Green's function is developed for each slice, and coupling from one slice to the next is carried out by use of Dyson's equation.
- ❖ We use the wave functions directly, matching the wave and its derivative from one slice to the next--unstable, but can be stabilized by use of the scattering matrix.

These two approaches are very nearly identical in procedure and in central iterations. They differ somewhat in the end-point processing and in extraction of the density and wave function.

We use the latter approach, since it allows us to actually follow individual modes.



The general approach is to begin with slice "0" and propagate the excited modes toward the end of the device, as slice "N+2"

We then back propagate the density at slice "N+1" to the beginning at slice "0" in order to determine the wave function and the density in the structure.

Matrix of the modes (columns) and their amplitude at each site (rows)

$$\begin{bmatrix} 0 & 1 \\ -H_{0,1}^{-1}H_{0,-1} & H_{0,1}^{-1}[E_F I - H_0(0)] \end{bmatrix} \begin{bmatrix} U(\pm) \\ ?U(\pm) \end{bmatrix} = \begin{bmatrix} U(\pm) \\ ?U(\pm) \end{bmatrix}$$

At slice "0", we must first determine the 2M modes (left and right propagating) by solving an eigenvalue equation:

hopping matrix $(H_{s,s-1})_{rk} = -te^{ir\phi} \delta_{rk}$

$H_{s,s+1} = H_{s,s-1}^+$

eigenvalues $? = \lambda_m \delta_{min}$

$\phi = \frac{eBa^2}{\hbar}$

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This creates the first "problem" as the discretization introduces an artificial band structure:

The energy diverges from the expected parabolic band

The artificial band has a width of $4t$ and here $t = 21.5$ meV.

As a result, it is imperative that all relevant energies in the problem be $< t$ in magnitude.

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The general propagation scheme for the s^{th} slice follows:

The LHS condition leads to:

$$P_2^s = [T_2^s C_2^s + T_{22}^s]^{-1}$$

$$P_1^s = -P_2^s T_2^s C_1^s$$

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We have to initialize at slice "0" with a different scheme which converts from the mode basis to the site basis:

The LHS condition leads to:

$$P_2^0 = [T_2^0 C_2^0 + T_{22}^0]^{-1}$$

$$P_1^0 = -P_2^0 T_2^0 C_1^0$$

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Finally, we have to convert back to the mode basis at the "N+2" slice:

$$\begin{bmatrix} C_1^{N+2} & C_2^{N+2} \end{bmatrix} = T_{N+1} \begin{bmatrix} C_1^{N+1} & C_2^{N+1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_1^{N+1} & P_2^{N+1} \end{bmatrix}$$

The LHS condition leads to:

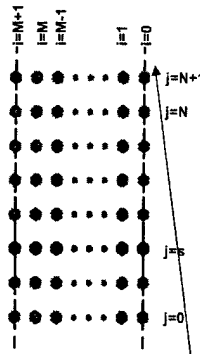
$$T_{N+1} = \begin{bmatrix} 0 & [U(+)?(+)]^{-1} \\ 1 & -U(+)[U(+)?(+)]^{-1} \end{bmatrix}$$

$$t = C_1^{N+2}$$

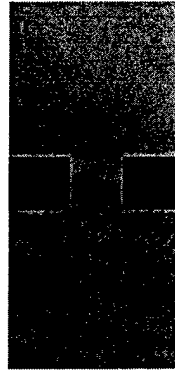
This part remains unchanged.

$$P_2^s = [T_{21}^s C_2^s + T_{22}^s]^{-1}$$

$$P_1^s = -P_2^s T_{21}^s C_1^s$$



Let us consider a simple example--that of a quantum point contact embedded in a wider wire:
 $a = 5 \text{ nm}$
width = 64 sites
QPC opening = 12 sites



Because we initialize at one end of the structure, the result is *not* symmetric in the long axis. Thus, there is a definite input side and a definite output side, which is not often recognized.

We then back propagate from a density near the "N+2" slice in order to get the density at each point in the structure.

Once we know the wave function at slice s , near the end, we can back propagate, using:

$$F^{(N+2,s)}(i, w) = P_1^s + P_2^s F^{(N+2,s+1)}(i, w)$$

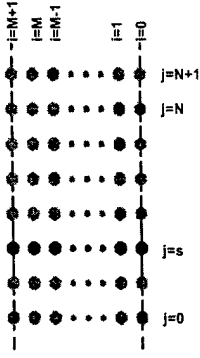
The "propagated" wave function connecting slice " s " to the output modes

Site " i " in the slice

Mode " w " in the density.

$$n_w(i, s) = |\Phi^{(N+2,s)}(i, w)|^2$$

$$n(i, s) = \sum_w |\Phi^{(N+2,s)}(i, w)|^2$$



Input

Output

Ripples caused by side wall reflections (mode structure) of hard wall boundary conditions



An Interesting Experiment:

STM/AFM tip with bias



We raster the biased tip across the surface. The bias affects the 2DEG and changes the transmission through the QPC. We plot the transmission change as a function of the tip position.

simulation



experiment

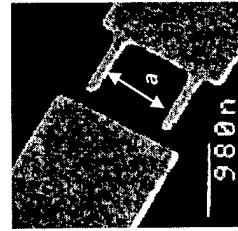


M. A. Topinka *et al.*, *Science* 289, 2323 (2000).

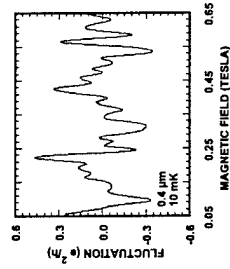
The three-fold pattern of the conduction change is seen only on the *input* side of the QPC (experiment is done in a.c. fashion rather in d.c. fashion assumed in the calculations).

As a second example, we consider an open quantum dot.

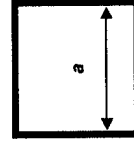
Dots can be of many shapes, even multiple dots:



Typical fluctuations as B or V_G is varied:



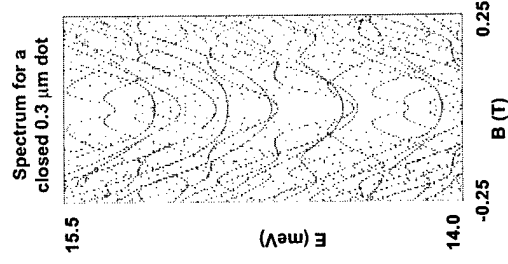
THE CLOSED DOT



$$\psi_{nm}(x, y) = \frac{2}{a} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{a}\right)$$

$$n, m = 1, 2, \dots$$

A magnetic field mixes the states and produces an *entangled* state.



The Open Dot

Opening the dot does NOT destroy the quantization within the dot!

The dot states hybridize (entangle) with the environment states.

Dot: states $\{\alpha_d\}$
parameters (ops., var.)

Environment: states $\{\alpha_e\}$
parameters (ops., var.)

$$H = H_d + H_e + H_{ed}$$

Define the system density matrix ρ_s
and the reduced, device density matrix $\rho_d = \text{Tr}_e\{\rho_s\}$

$$H = H_d + H_e + H_{ed}$$

Use a projection super-operator $\hat{P}$ to remove the environment variables

This leads to the new Liouville equation for the device density matrix:

$$i\hbar \frac{\partial \rho_d}{\partial t} = (\hat{H}_d + \hat{H}_{ed} + \hat{\Sigma}(t \rightarrow \infty)) \rho_d$$

Where (in Laplace transform notation):

$$\hat{H}_{ed} \rho_d(s) \equiv \text{Tr}\{\hat{P} \hat{H}_{ed} \hat{P} \rho_s(s)\}$$

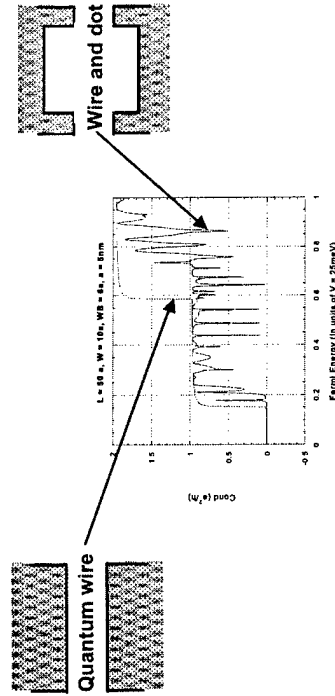
Modifications of spectrum by the environment

$$\hat{\Sigma}(s) = \hat{P} \hat{H} \hat{Q} (i\hbar s - \hat{Q} \hat{H} \hat{Q})^{-1} \hat{Q} \hat{H} \hat{P}$$

Superlattice effects from device array

Environmental Effects

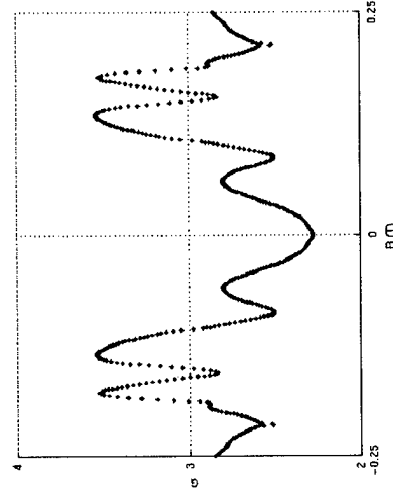
We can understand how the QPCs and the dot work together to provide the "spectrum" of conductance.



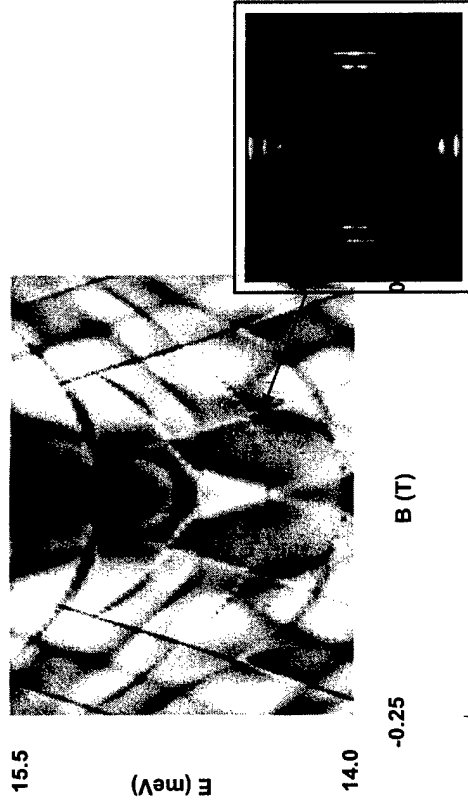
Dot modes, which are excited, work to provide modulation of the overall transmission.

The Open Dot

⇒ We determine the "spectrum" of the open dot from the conductance through the dot:
 $G \sim$ density of states
DOS $\sim$ energy levels



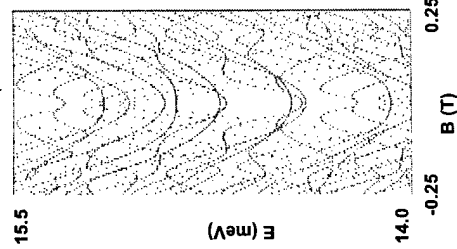
Decoration by "Scars"



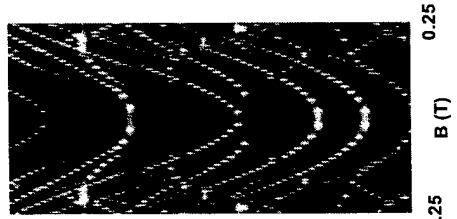
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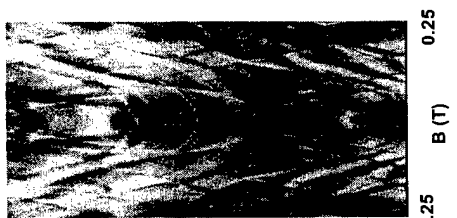
Spectrum for a closed 0.3 μm dot



Tunneling conductance of closed dot



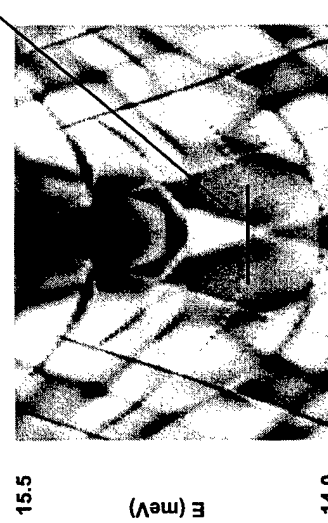
Conductance with one mode in QPC



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Four modes propagating through the quantum point contact:

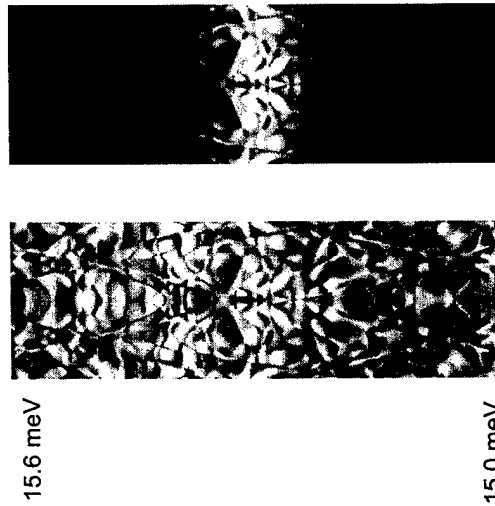


Basic structure arises from density of states: back-scattered trajectories (the various trace formulas)

The "back-scattered" trajectories are the FUNDAMENTAL trajectories, not additional structure!

Hence, there is nothing like weak localization in these dots!

Role of Averaging on the Spectra



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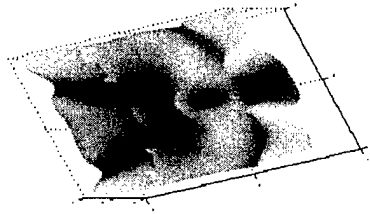
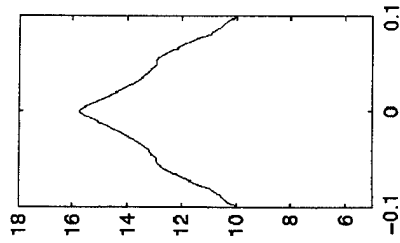
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Role of Averaging on the Spectra

Moving average



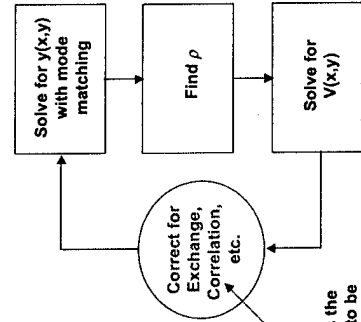
Let us now turn to the problem of self-consistency of the potential in the calculation--

The potential that is included in the site energy $V(i,j)$ can be included as a self-consistent quantity. Here, the charge density is

$$\rho(x, y) = -\frac{e}{\epsilon_s} \left(|\psi(x, y)|^2 - 1 \right) n_0$$

Uniform electron density set by remote dopants in AlGaAs

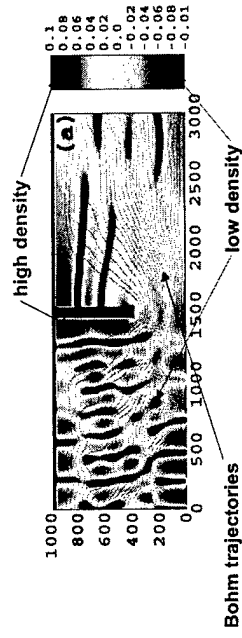
For simple systems, we can generate a relatively straight-forward loop to find the solutions:



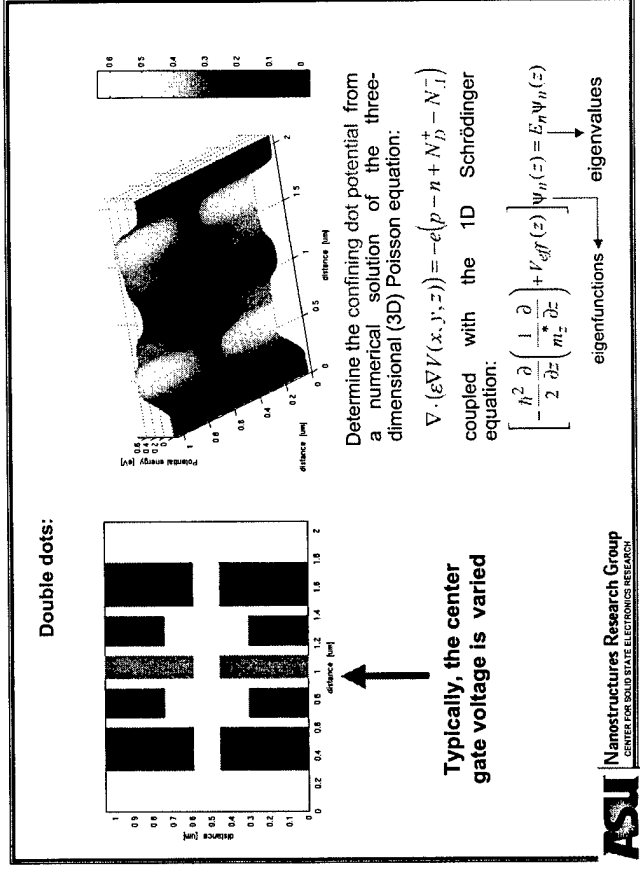
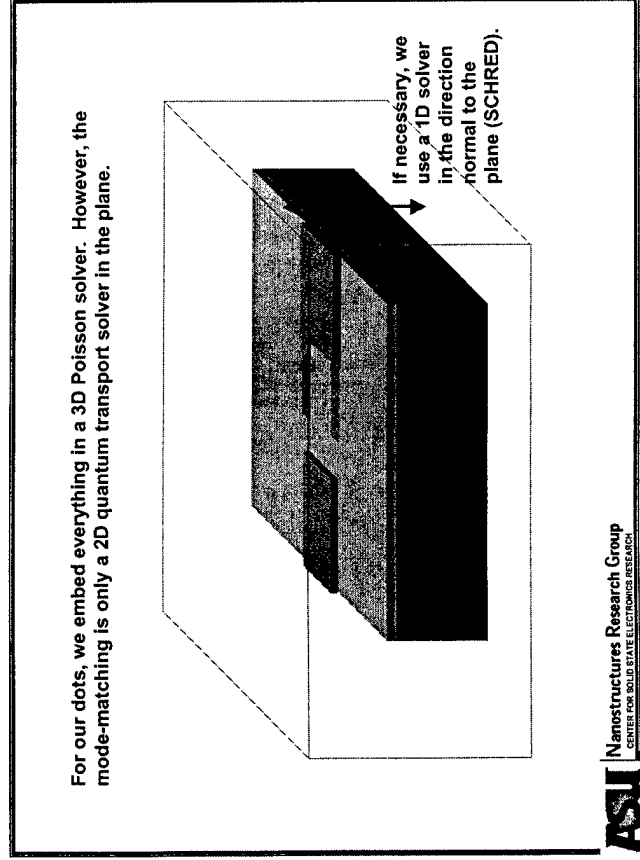
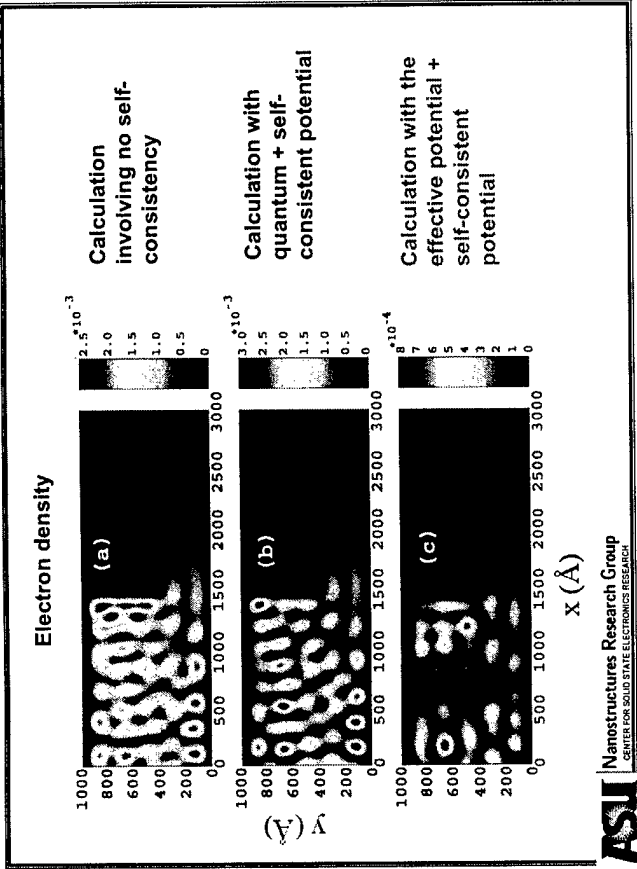
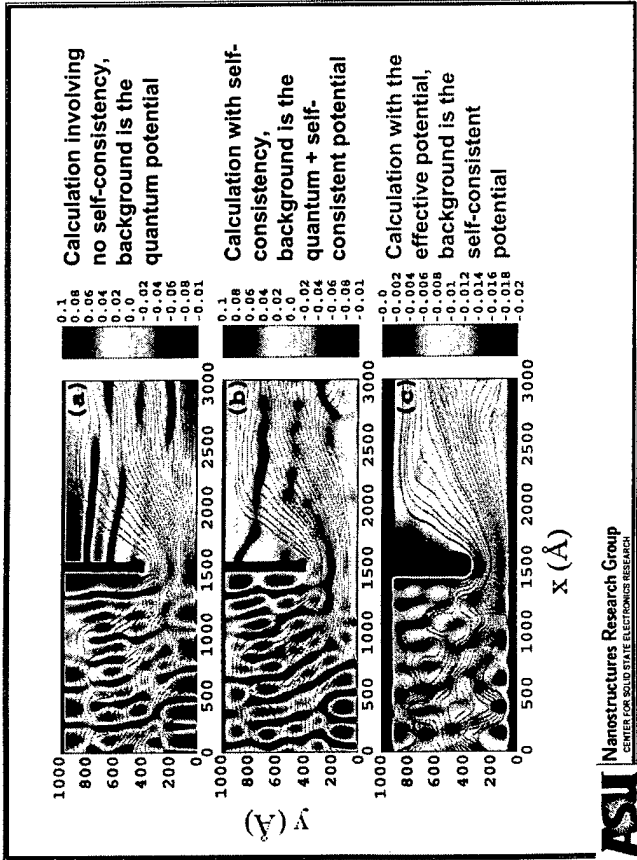
Often requires the Fermi energy to be adjusted to keep density constant.

Consider a simple quantum wire with an embedded QPC. Here, we move the QPC to one side of the wire:

barriers



Bohm trajectories

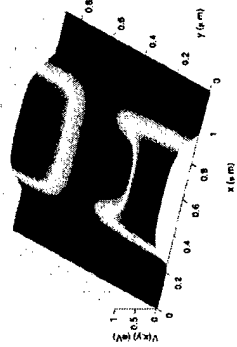


Problems with simple mode matching when:

- ❖ $V(x,y)$ is not a simple function
- ❖ A magnetic field is applied

Then the modes can no longer be expressed as well characterized analytical functions.

Example: a realistic quantum dot potential obtained by a self-consistent calculation



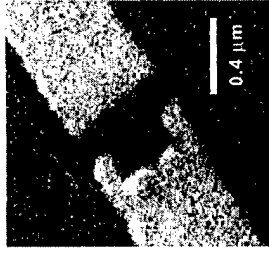
For better flexibility and accuracy it is advantageous to do the problem on a discrete lattice.

Our use of a solver for the modes, with a lattice representation solves this problem.

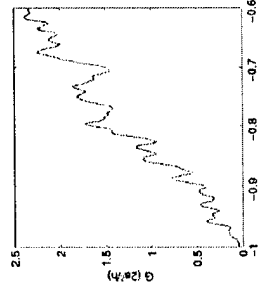
Confining potential has the form of a catalan parabolas

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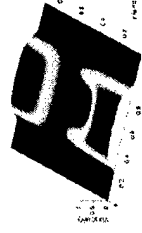
Split gate pattern



Measured conductance



In the normal direction, the Exchange and correlation corrections to the ground state energy of the are included by using the Local Density Approximation.



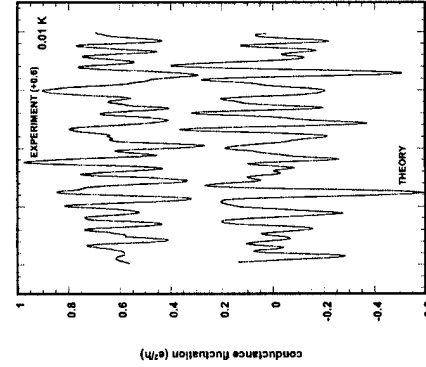
$V_g = -1.0 \text{ V}$

$V_g = -0.9 \text{ V}$

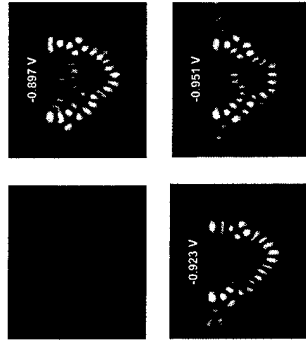
$V_g = -0.7 \text{ V}$

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Gate voltage comparison revisited



gate voltage (volts)

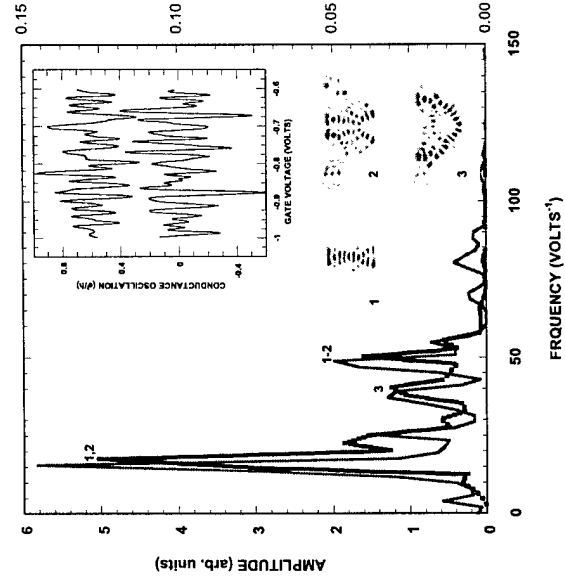


Simulations also reveal that certain scars may RECUR as gate voltage is varied. The resulting periodicity agrees WELL with that of the conductance oscillations

Persistence of the scarring at zero magnetic field indicates its INTRINSIC nature

The scarring is NOT induced by the application of the magnetic field

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- ❖ Mode matching approaches are a viable method of computing the conduction in mesoscopic systems.
- ❖ This can easily be incorporated with the Poisson equation for self-consistent solutions.
- ❖ In addition, the energy dependence can be converted to a temperature dependence for near-equilibrium systems. It is also possible to do non-equilibrium approaches with only slight changes in the program (in principle).

Trajectories in quantum mechanics

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Department of Electronics and Electrical Engineering
University of Glasgow



Advanced Research Workshop
on
Quantum Transport in Semiconductors



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Conventional device modelling: semi-classical

Empirical
Drift Diffusion
Hydrodynamic
Monte Carlo

Velocity flows
Particle trajectories

Self-consistent via Poisson equation

Can we obtain a trajectory description of quantum transport?

In principle, the the surprising answer is yes

1. Without any extension of quantum theory we may
define velocity flows.

2. With special assumptions we may define trajectories

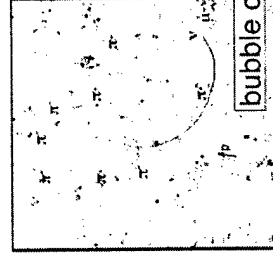
3. Bohm has also proposed deterministic extension to QM
which uses trajectories calculated in 1 or 2.

4. Others have defined stochastic trajectories in
extensions to QM

- Position- momentum uncertainty relations
- Non-locality of quantum mechanics



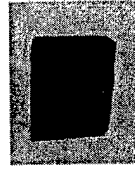
cloud chamber



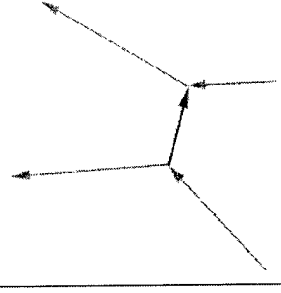
bubble chamber

Wigner trajectories

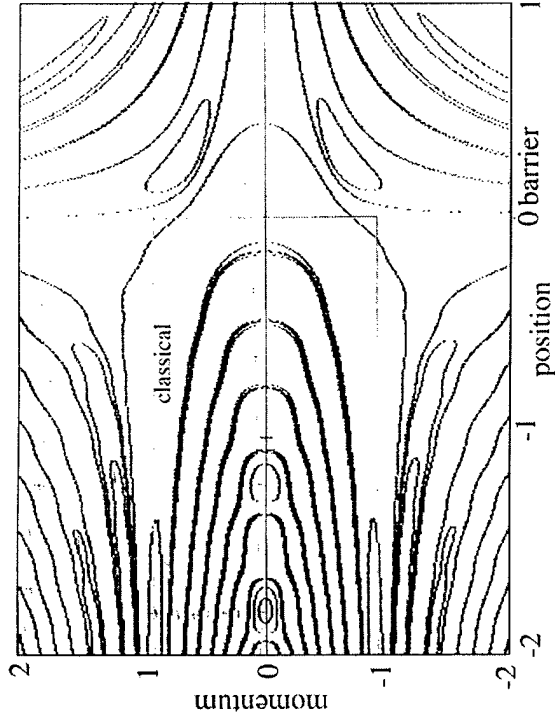
(stationary states only)
no compact support



Feynman paths



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A rigorous approach to trajectories

Velocity Flow Picture

- Probability density $n(\mathbf{x},t)$
 - Particle Current density $\mathbf{j}(\mathbf{x},t)$
 - Define a velocity field:
- $$\mathbf{v}(\mathbf{x},t) = \mathbf{j}(\mathbf{x},t)/n(\mathbf{x},t)$$

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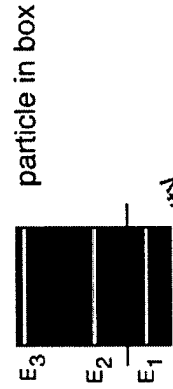
What do we want from a quantum transport theory?

A self-consistent treatment of the local charge and current density fields which describe

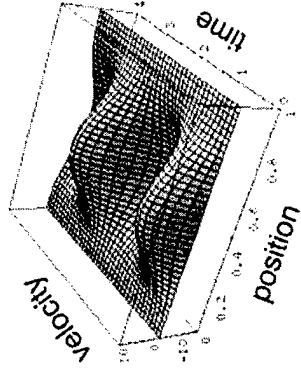
- tunnelling
- scattering
- interference effects
- size quantisation
- time-dependence
- de-coherence effects
- many-body issues

FOR FINITE REGIONS OF OPEN SYSTEMS

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$\psi(x,t) = \cos[\phi] \psi_1 + \sin[\phi] \psi_2$
$\langle x \rangle = (L/2)(1 - (32/9\pi^2) \sin[2\phi] \cos[\omega t])$
$\langle v \rangle = (16L/9\pi^2) \sin[2\phi] \sin[\omega t]$
$\langle E \rangle = \cos^2[\phi] E_1 + \sin^2[\phi] E_2$



Non-stationary states show trajectories

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Schrödinger Equation

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi(x,t)$$

$$H = -\frac{\hbar^2}{2m} \nabla^2 + \Phi$$

$$n(x,t) = \Psi^*(x,t)\Psi(x,t)$$

$$\mathbf{j}(x,t) = \frac{-i\hbar}{2m} \{ \Psi^* \nabla \Psi - \Psi \nabla \Psi^* \}$$

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Take the gradient of the phase equation

$$m \frac{\partial \mathbf{v}}{\partial t} + m \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla \Phi(\mathbf{x},t) - \nabla V_Q(\mathbf{x},t)$$

mdv/dt

Quantum Euler Eqn

$$\frac{\partial n}{\partial t} + \nabla \cdot n \mathbf{v} = 0$$

Continuity Equation

The first equation is not quite correct and we need further constraints as we shall see

$$V_Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \quad n = R^2 \quad m \mathbf{v} = \nabla S$$

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A posteriori quantum hydrodynamics

Express wave function in polar form: $\Psi = R \exp(iS/\hbar)$

$$\frac{\partial n}{\partial t} + \nabla \cdot n \mathbf{v} = 0$$

$$\frac{\partial S}{\partial t} + \frac{\nabla S \cdot \nabla S}{2m} + \Phi(\mathbf{x},t) + V_Q(\mathbf{x},t) = 0$$

$$V_Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \quad n = R^2 \quad m \mathbf{v} = \nabla S$$

Quantum potential

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Quantum corrected Drift - diffusion

$n, \mathbf{v}, \psi$

Newton's 2nd law for carriers:

$$m^* \frac{d\mathbf{v}}{dt} = q \nabla(\psi - \phi) - \frac{D}{n\mu} \nabla n - q \frac{\mathbf{v}}{\mu}$$

Density gradient model

$$\phi \rightarrow \phi - 2b \frac{\nabla^2 \sqrt{n}}{\sqrt{n}}$$

electrostatic potential

chemical potential

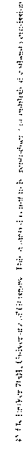
diffusion coefficient

mobility

effective mass

Poisson-Schrödinger equation

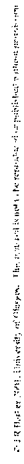
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A large, abstract, black and white image showing a dense, textured surface, possibly a close-up of a material or a microscopic view, with a central vertical crease and a horizontal band across the middle.

First reported by
Dewdney et al

On a trajectory



Uncertainty Relations

$$\Delta x \Delta p \geq \hbar / 2$$

$$\Delta x = \langle (x - \langle x \rangle)^2 \rangle^{1/2} = \left\{ \int x^2 n(x) dx - \left[\int x n(x) dx \right]^2 \right\}^{1/2}$$

$$\Delta p = \langle (p - \langle p \rangle)^2 \rangle^{1/2} =$$

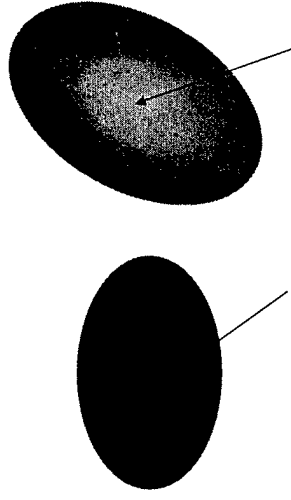
$$\left\{ \int (p^2(x) + 2mV_Q(x))n(x) dx - \left[\int p(x) n(x) dx \right]^2 \right\}^{1/2}$$

$$p = \partial S / \partial x$$

Barker (1992)

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Configuration Space



$$\Psi(r_1, r_2) = C[\Psi_\alpha(r_1)\Psi_\beta(r_2) + \Psi_\gamma(r_1)\Psi_\delta(r_2)]$$

No overlap: effective statistical mixture,
System point is in one or the other sub-state

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Entanglement

Factorisable wave-function $\rightarrow$ physical independence

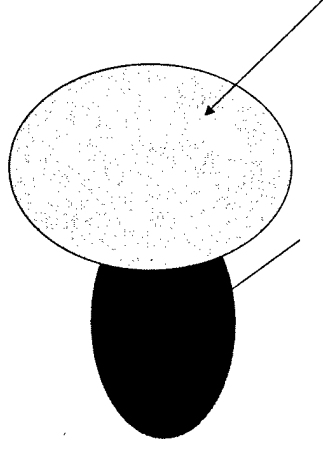
Sum of factorisable wave-functions :

1. Effective statistical mixture

2. Entangled state: correlation

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Configuration Space



$$\Psi(r_1, r_2) = C[\Psi_\alpha(r_1)\Psi_\beta(r_2) + \Psi_\gamma(r_1)\Psi_\delta(r_2)]$$

Overlap: entangled state,
Correlation

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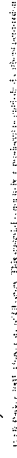
3



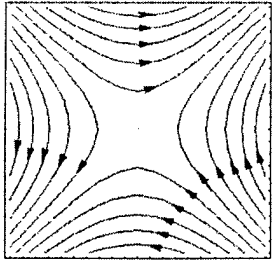
Effectively factorisable at long distances

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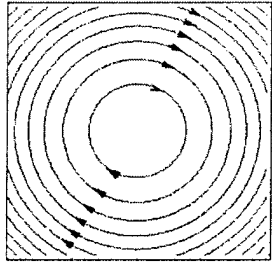
3

[illegible]

Flow near singularities



hyperbolic



vortex

At velocity nodes At density nodes

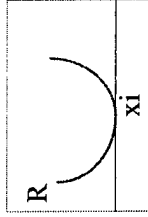
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Vortices occur at strong nodal points/lines (Barker 2001)

$$R \approx |\mathbf{r} - \mathbf{x}_i|^{N_i}$$

Or generally,

$$R \approx [(\mathbf{r} - \mathbf{x}_i) \cdot \mathbf{A} \cdot (\mathbf{r} - \mathbf{x}_i)]^{N_i/2}$$



$$V_Q = -(\hbar^2/2m)\nabla^2 R/R \approx -(\hbar^2/2m)N_i^2/|\mathbf{r} - \mathbf{x}_i|^2$$

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The quantisation of velocity circulation

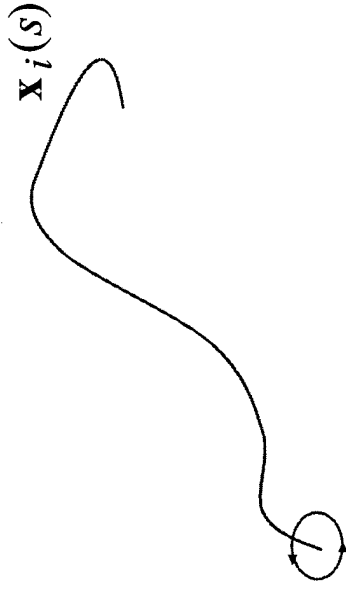
The *a posteriori* QHD equations automatically satisfy the velocity circulation theorem:

$$\oint \nabla S \cdot d\mathbf{r} = Nh \quad (N: \text{integer})$$

$$\oint \mathbf{v} \cdot d\mathbf{r} = Nh/m$$

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Vortex line



$$\nabla \times \mathbf{v} = N_i(h/m) \int \delta(\mathbf{r} - \mathbf{x}_i(s))(d\mathbf{x}_i/ds)ds$$

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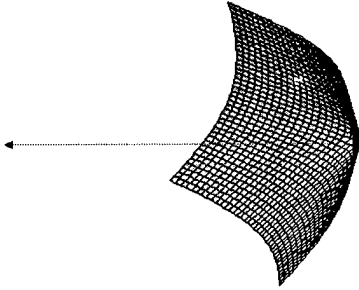
Near the strong nodal point:

$$\begin{aligned} \frac{1}{2}mv^2 &\approx -V_Q = (\hbar^2/2m)\nabla^2 R/R \\ &\approx (\hbar^2/2m)N_i^2/|\mathbf{r}-\mathbf{x}_i|^2 \end{aligned}$$

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Non-singular current density

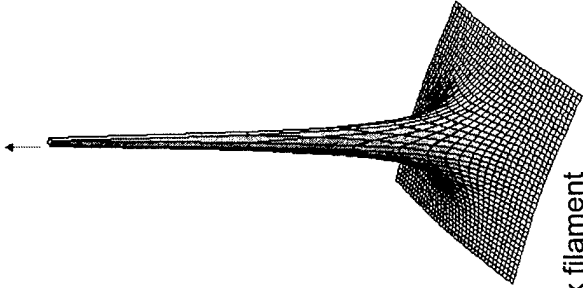
$$\mathbf{J} \propto |\mathbf{r}-\mathbf{x}_i|^{2N_i-1}$$



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Singular velocity

$$\mathbf{v} = \frac{N_i \hbar}{m |\mathbf{r}-\mathbf{x}_i|}$$



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Analogous to classical Eulerian vortex filament

$$\text{Phase } S = N_k \text{ArcTan}(y/x) \quad x, y \neq 0$$

$$\Psi \approx |\mathbf{r}-\mathbf{x}_k|^{N_k} \exp[iN_k \phi]$$

Re-constructed wave-function

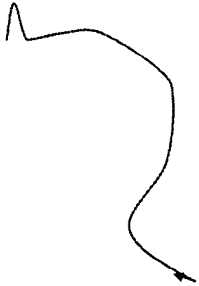
$$\oint \nabla S \cdot d\mathbf{r} = N_k h$$

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Traversal Time

requires Bohm interpretation

$$t = \int_{\eta}^{\eta_2} \frac{\mathbf{v} \cdot d\mathbf{r}}{v^2}$$



Kinetic energy appears as metric

On a vortex, one loop executed in time $t = Nh/2T$

Barker (1992)

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Ballistic flow
in open quantum dots

To describe vortex motion with *ab initio* quantum hydrodynamics we need to introduce a vector quantum potential $\mathbf{a}_Q(\mathbf{r}, t)$

$$\oint_C \mathbf{a}_Q \cdot d\mathbf{r} = nh$$

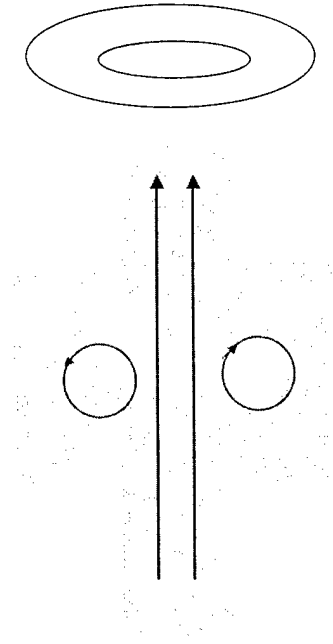
$$\begin{aligned} m \frac{\partial \mathbf{v}}{\partial t} + m \mathbf{v} \cdot \nabla \mathbf{v} &= -\nabla \Phi(\mathbf{x}, t) \\ -m \mathbf{v} \times \nabla \times \mathbf{a}_Q(\mathbf{x}, t) &- \nabla V_Q(\mathbf{x}, t) \end{aligned}$$

where

$$\nabla \times \mathbf{a}_Q = \sum_i N_i (h/m) \int \delta(\mathbf{r} - \mathbf{x}_i(s)) (d\mathbf{x}_i / ds) ds$$

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Quantum smoke ring



Compare Lord Kelvin's cigar box!

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Density evolution

QuickTime™ and a
Animation decompressor
are needed to see this picture.

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Travelling wave in single transverse mode

Density plot

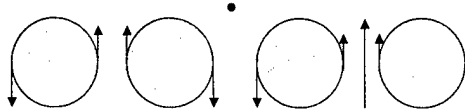
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Animation decompressor
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Current density evolution

4 vortices
formed

QuickTime™ and a
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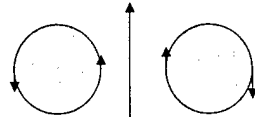
2 saddles

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Transmission Maximum

Current Density And Particle density

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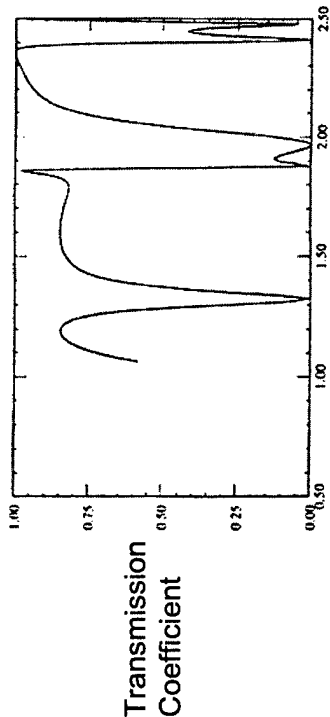
Smoke ring

2 vortices
formed

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Lower energy, transmission minimum (reflection)

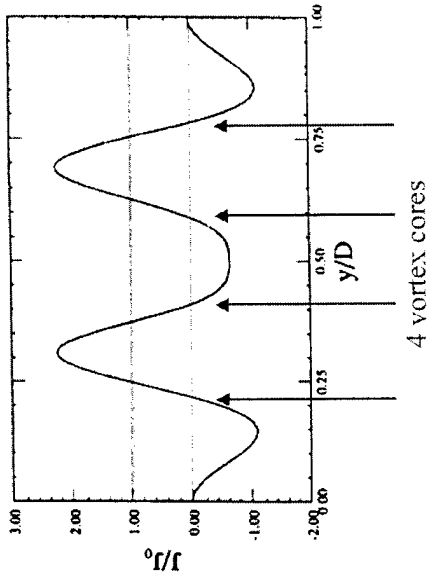
Current density magnitude transverse across the cavity



Compare Lent (1990)

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Total wave-vector



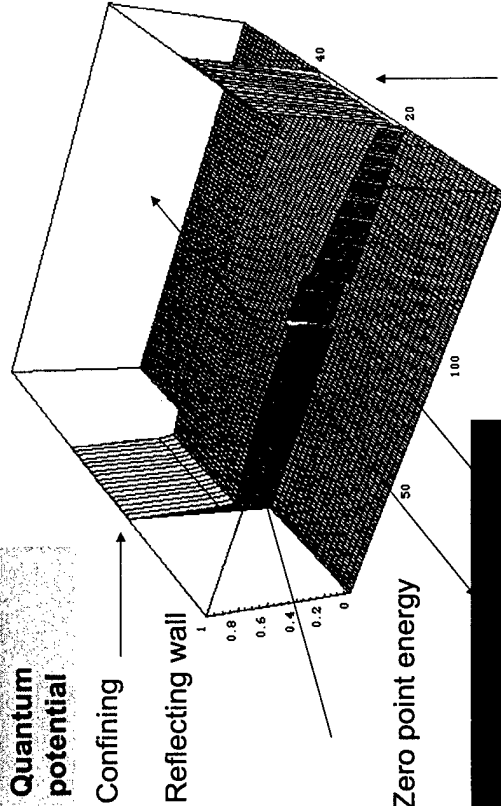
Compare Lent (1990)

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Quantum potential

Confining

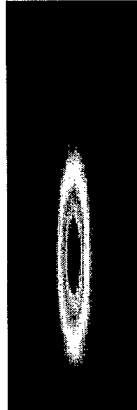
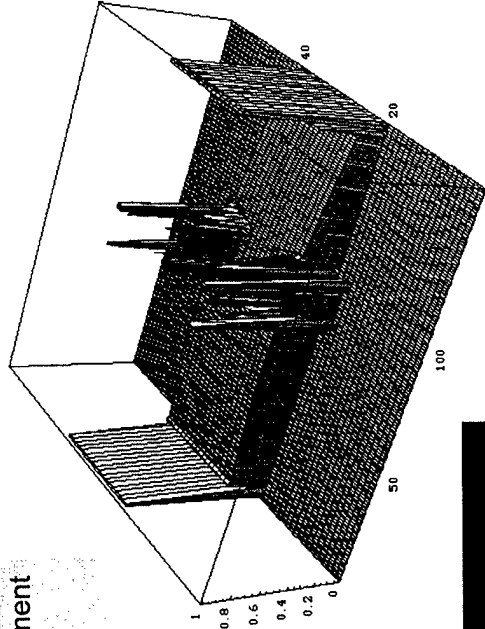
Reflecting wall



Note long range of V_Q

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Rapid development of V_Q with time



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Attractive V_Q heralds
start
of vortex motion

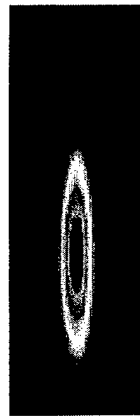
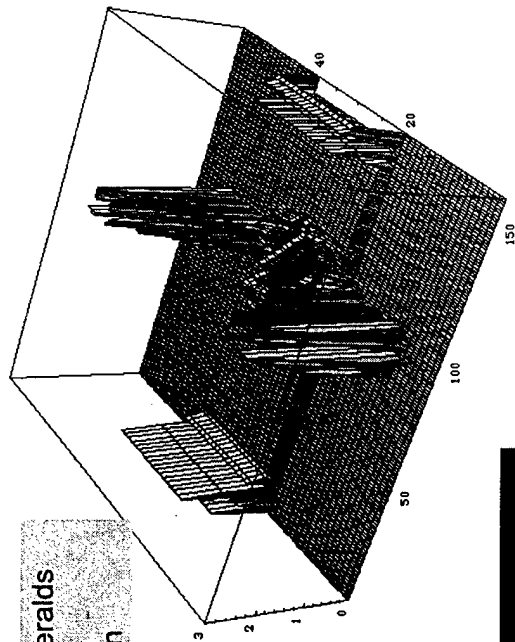


Fig. 1. Evolution of the system. The vertical axis is the magnitude of the order parameter.

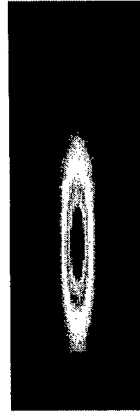
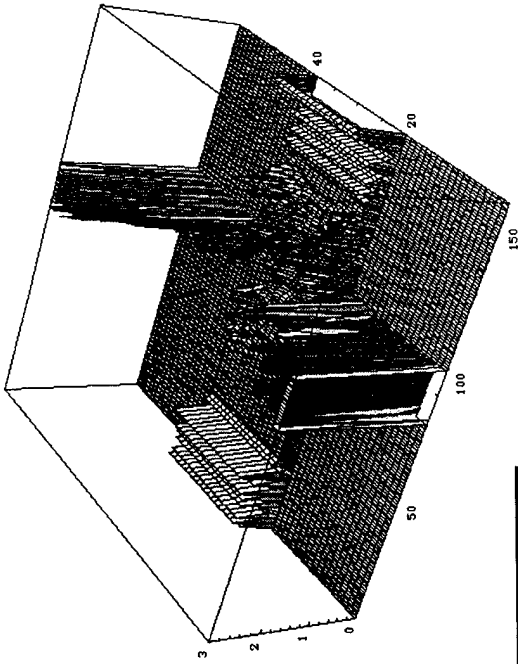
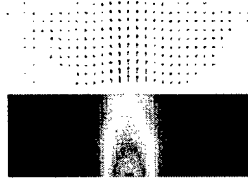


Fig. 2. Evolution of the system. The vertical axis is the magnitude of the order parameter.

Vortex flow
develops
around attractor
basins of V_Q .

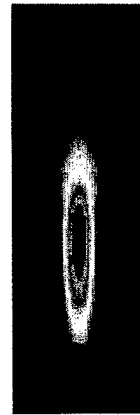
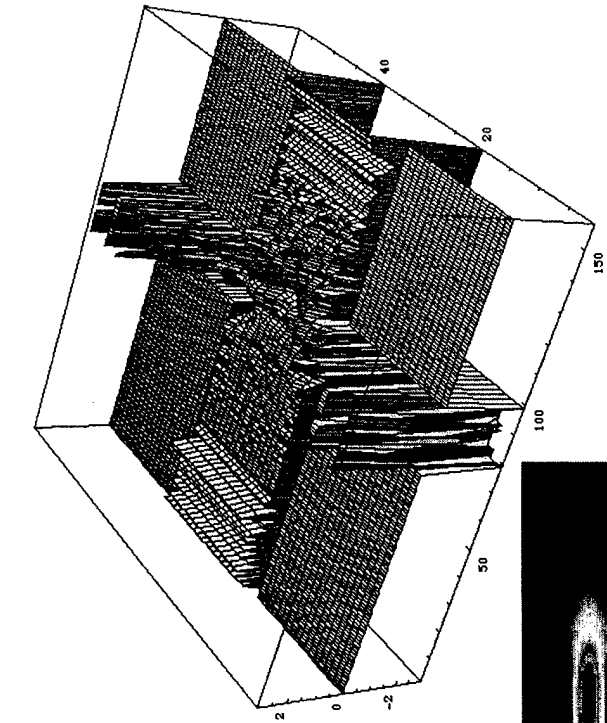
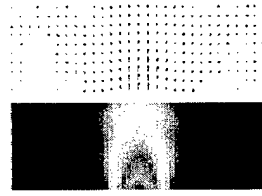


Fig. 3. Evolution of the system. The vertical axis is the magnitude of the order parameter.

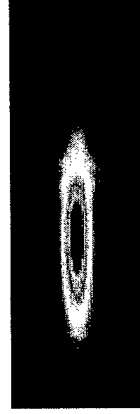
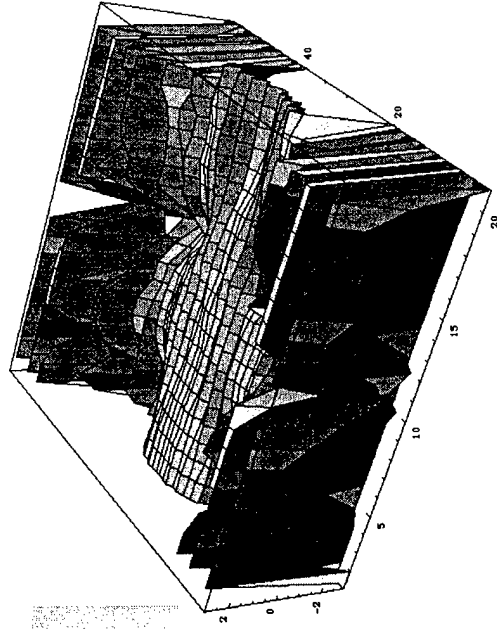


Fig. 4. Evolution of the system. The vertical axis is the magnitude of the order parameter.

Full vortices developed



The figure consists of three vertically stacked panels. The top panel is a 3D surface plot showing a complex, multi-lobed structure. The axes are labeled with values 0, 5, 10, 15, 20, 25, 30, 35, and 40. The middle panel is a 2D vector field plot showing a grid of small arrows representing flow direction and magnitude. The bottom panel is a 2D grayscale intensity plot showing several bright, irregular spots on a dark background.

$$r_k = \frac{1}{2} \left(\frac{1}{\lambda_k} + \frac{1}{\mu_k} \right) \quad \text{and} \quad \mu_k = \frac{1}{2} \left(\frac{1}{\lambda_k} - \frac{1}{\mu_k} \right) \quad \text{for } k = 1, 2, \dots, n.$$

150

$$\mathbf{l} = \mathbf{r} \times \nabla S$$

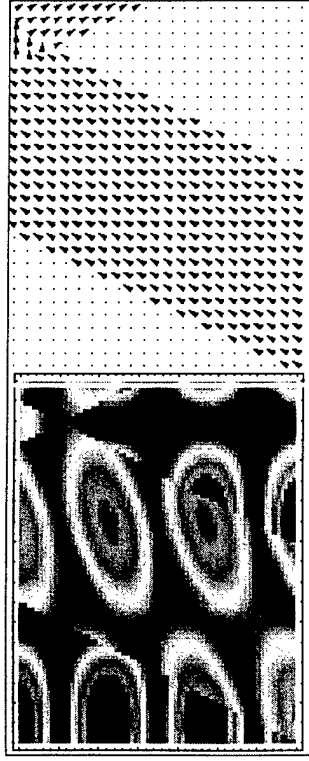
$$\mathbf{L}^2 = l^2 + L_Q^2$$

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S. J. R. Dooler (2001), University of Edinburgh. The work of is now to be regarded as a public good, a notion previously

Detail of upstream density and velocity flow



density

velocity field

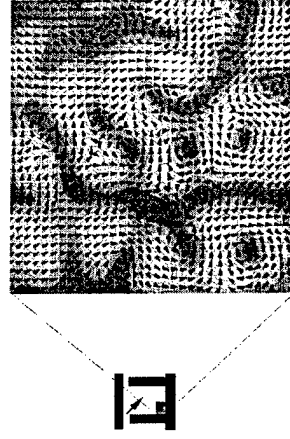
Scarred wave-function

Ray approximation

© P. A. Barker, 2001. Images are of 2D gases. The method is used to be applied to the problem of electron transport in

Quantum Flows

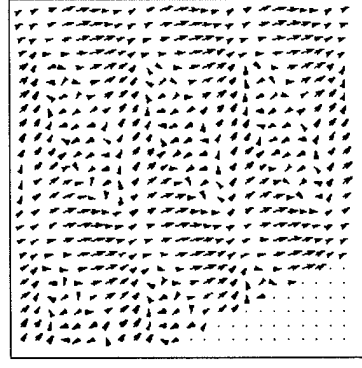
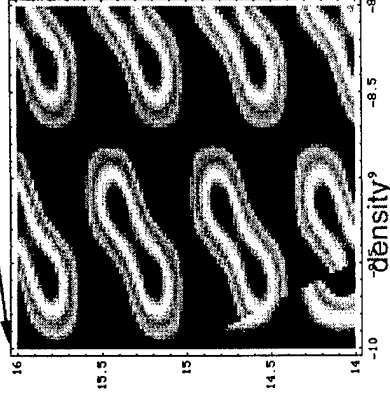
Barker
Ferry & Akis (2000)



- Velocity field in quantum dot
- Meandering open orbits
- Trapped flows - vortices

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Upstream flow: full form

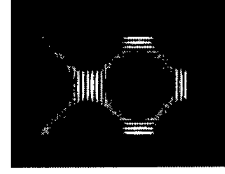
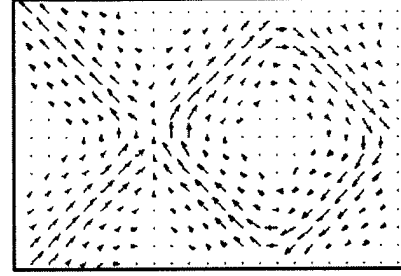
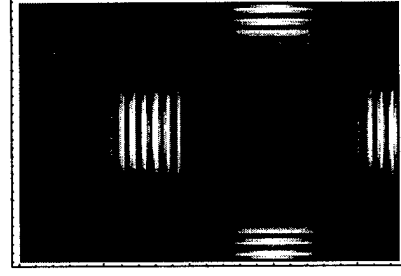


velocity field

vortices

© P. A. Barker, 2001. Images are of 2D gases. The method is used to be applied to the problem of electron transport in

Flow of an extended wave packet in a double throttle = open quantum dot



A posteriori quantum hydrodynamics

Calculate $\mathbf{J}$ and n , form $\mathbf{v}$

Quantum Euler equations automatically satisfied, including quantization of circulation.

Ab initio quantum hydrodynamics

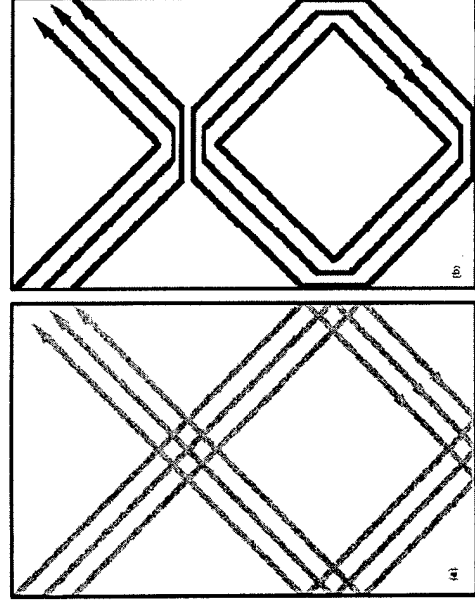
To be of predictive power rather than purely explanatory the velocity flow approach must become *ab initio*.

Quantum Euler equations, + scalar & vector quantum potentials
A neat way to put QM in Monte Carlo, Hydrodynamic, DD models.

Difficult to solve in general.

© 1994, Elsevier B.V. (Subseries of JHEP). This article is part of the proceedings of the 1994 conference on Quantum Hydrodynamics.

Classical and quantum ray paths



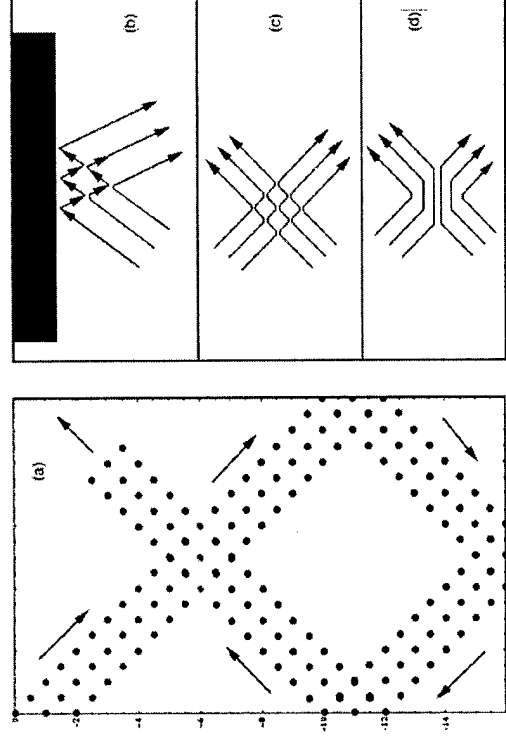
1994

conjecture

The open quantum dot analysis suggests:
Can we construct the quantum flow by first constructing the classical flow and then replacing any trajectory crossing points by flow separation?

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Spatially correlated virtual particles



1994

Dissipative quantum hydrodynamics

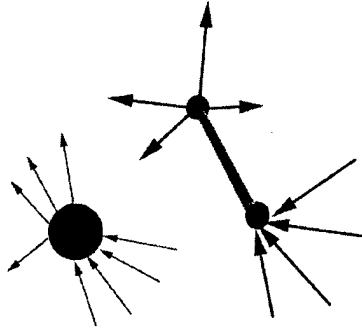
Conventional QHD has used classical HD model for scattering processes.

Less empirical approach

Use transient scattered wave or use non-hermitian Hamiltonian:

leads to description of trapping/de-trapping, elastic and inelastic scattering with extended collision zones and intra-collisional field effect

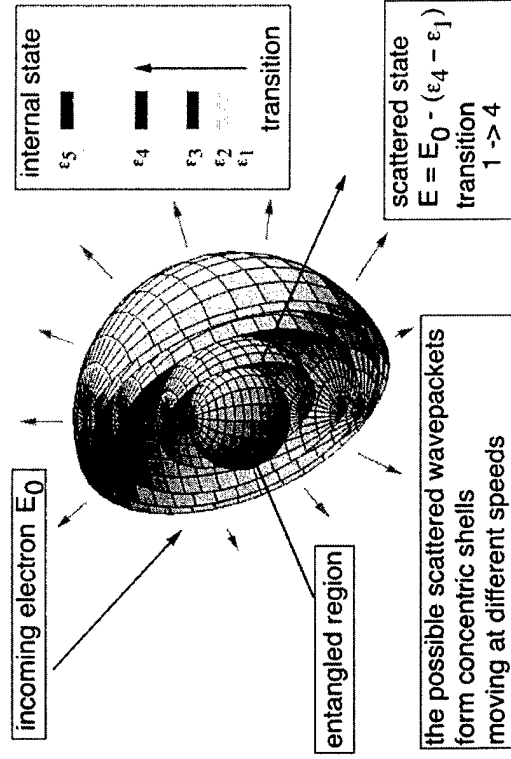
Barker and Ferry, APL 74 582 (1999),
Barker and Watling, Superlattices and microstructures, 27, 347 (2000)



New Topological features

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Inelastic Scattering



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Inelastic Scattering

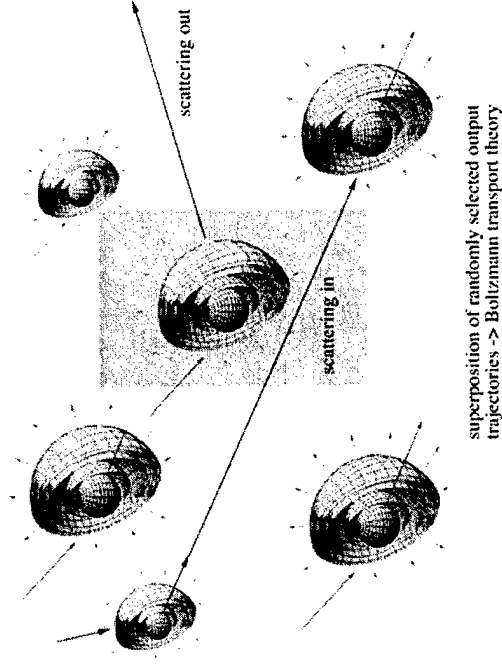
Flow occurs in configuration space
Electron coordinate $\mathbf{r}$
Scatterer generalised coordinate(s)
 $X, \dots$

Trajectories cannot cross in configuration space.

But trajectories may now cross in real space
PROVIDED.... Scatterer coordinates are different - a new result.

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Inelastic Scattering



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Dissipation may be shown to leads to crossings of quantum trajectories in the carrier space.

The trajectories in the full phase space do not cross.

Pure state, coherent

$$f(x, p; t) = n(x, t) \delta(p - \nabla S)$$

Pure state developing incoherence

f -> solution of a BTE-like transport eqn.

corresponds to evolution of mixed states

Recall the temperature tensor: the covariance matrix of the velocity

$$kT = m \langle (\mathbf{v} - \mathbf{V})(\mathbf{v} - \mathbf{V}) \rangle$$

Taking quantum mechanical averages this is just:

$$kT_Q = m \langle \mathbf{v} \mathbf{v} \rangle - m \langle \mathbf{v} \rangle \langle \mathbf{v} \rangle$$

$$NkT_Q = mV^2 + 2V_Q - mV^2 = 2V_Q$$

$$(N = 1, 2, 3)$$

Comparison between finite temperature mixed state quantum hydrodynamics and pure state QHD (vortex free)

$$\frac{\partial(nP)}{\partial t} + \frac{\partial(nU)}{\partial x} + n \frac{\partial}{\partial x} \left(\varphi + \frac{1}{3} V_Q \right) = 0 \quad U = mV^2 + kT$$

$$\frac{\partial(nP)}{\partial t} + \frac{\partial(nU_0)}{\partial x} + n \frac{\partial}{\partial x} (\varphi + V_Q) = 0 \quad U_0 = mV^2$$

Paradox: Pure state is not zero temperature limit?

Choosing N = 3 we get

$$\frac{\partial(nP)}{\partial t} + \frac{\partial(n[U_0 + kT_Q])}{\partial x} + n \frac{\partial}{\partial x} \left(\varphi + \frac{1}{3} V_Q \right) = 0$$

$$U = U_0 + kT_Q$$

which is essentially the finite temperature QHD result

A more detailed examination of the closure relations indicates further reconciliation of the two pictures although the finite temperature case involves coupling to a full hierarchy of moment equations.

Mixed states

For mixed states, existing quantum hydrodynamic and density gradient models are not complete.

If vortex formation may be ignored then empirical quantum potential is the most pragmatic approach.

However, if coherent flow occurs with scattering off more than one obstacle we expect vortex flows and QHD and DD needs to be solved under the constraint of a generalised velocity circulation theorem. By solving for nodal lines and computing the Vector quantum potential.

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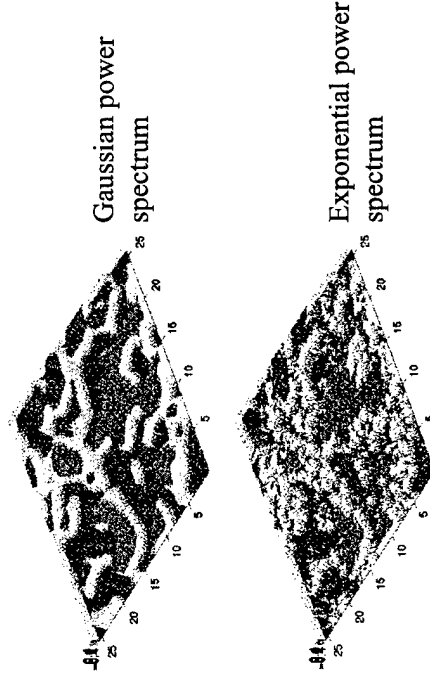
Application to MOS device modelling

At Glasgow, we have begun a systematic inclusion of quantum effects into finite MOSFET models at decanano dimensions.

The first level uses a phenomenological quantum potential model.

The effect is to keep charge away from interfaces and to induce some tunnelling effects.

Digitisation of the Si/SiO₂ interface topology

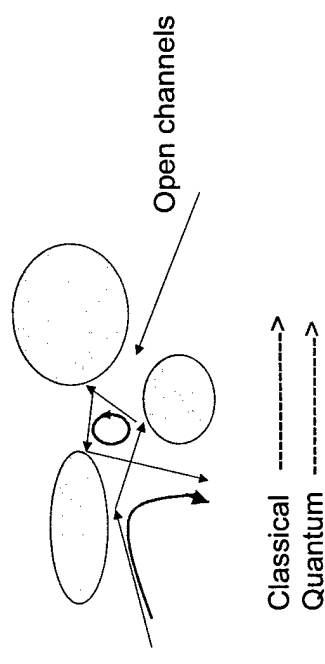


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The fluctuation landscape admits flows which involve multiply-connected paths:

Possibility of

Diffraction, interference effects-> vortex flows
And meandering orbits.



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Conclusions

- Quantum flow formalism is close to spirit of drift diffusion and hydrodynamic models.
- Vortex motion is ubiquitous
- In the immediate future, density gradient QHD may be parameterised from deeper quantum simulations in analogy with using BTE to re-parameterise Drift Diffusion and HD models beyond their normal range of validity.

Coulomb correlations in semiconductors and transient phenomena. Nonequilibrium Greens functions and beyond

Michael Bonitz, Universität Rostock

Workshop „Quantum Transport in Semiconductors“

Maratea, June 17-22 2004

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Outline

1. Introduction: Coulomb correlations in semiconductors
2. Short-time phenomena and Initial correlations
3. Nonequilibrium Greens functions:
 - main ideas
 - application to optics and high field transport
4. Strong correlations in semiconductors:
 - excitons, biexcitons, electron-hole liquid
 - Wigner crystal
5. Summary and Outlook

In collaboration with

1. Dirk Semkat (Rostock), Nai Kwong, Sigurd Köhler, Rolf Binder (Tueson)
2. Antigoni Alexandrou (Paliserou)
3. Hartmut Haug (Frankfurt), Ronald Redmer, Justino Madureira (Rostock)
4. Vladimir Filinov (Moscow), Stephan Koch, Walter Hoyer (Marburg)
5. Alexei Filinov, Yuri Lozovik (Moscow)

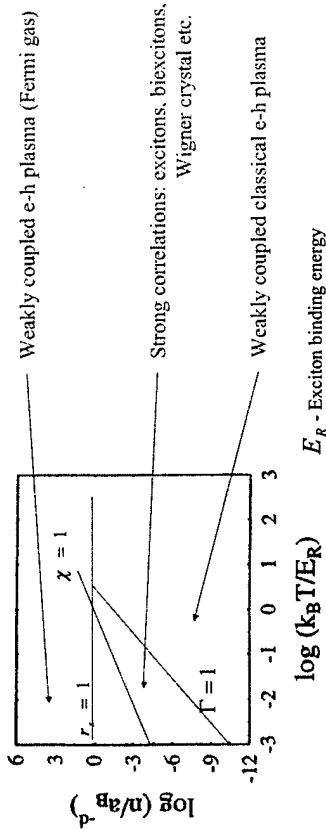
1. Introduction: Coulomb correlations in semiconductors

Coulomb correlations in semiconductors

Coulomb interaction between electrons / holes $U(r) = \pm e^2 / r$
Coupling strength: Ratio of Interaction energy/kinetic energy

$$\Gamma \equiv \langle U(r) \rangle / k_B T \quad r_s \equiv \langle U \rangle / E_F \propto \langle r \rangle / a_B$$

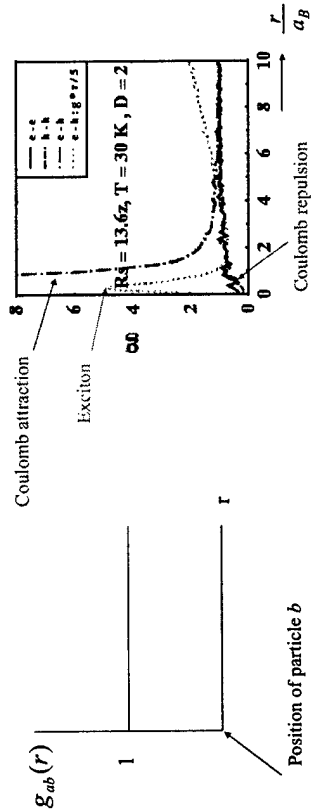
E_F - Fermi energy a_B - exciton Bohr radius
 λ - electron quantum wave length, d - dimension



Statistical treatment of correlations

➔ Pair distribution function $g_{ab}(r)$; a,b – electrons, holes
= probability of finding particle a at distance r from particle b
Normalization: $\int g_{ab}(r) dV = 1$ Interaction energy: $\langle U \rangle = \int U(r) g_{ab}(r) dV$

Non-interacting particles Interacting 2D e-h plasma

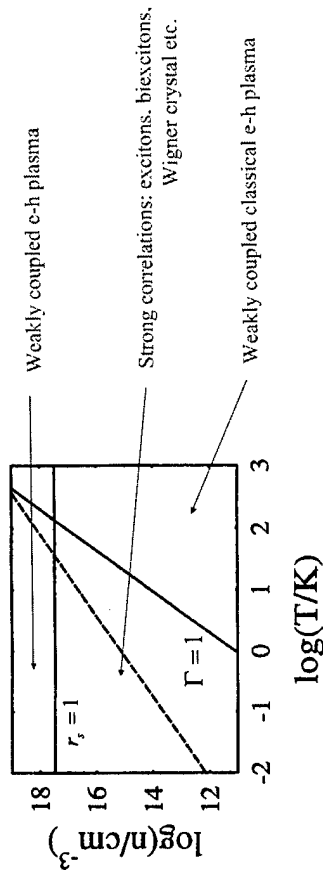


Pair correlation function: $c_{ab}(r) \equiv g_{ab}(r) - 1$

Coulomb correlations in semiconductors

$$\Gamma \equiv \langle U(r) \rangle / k_B T \quad r_s \equiv \langle U \rangle / E_F \propto \langle r \rangle / a_B$$

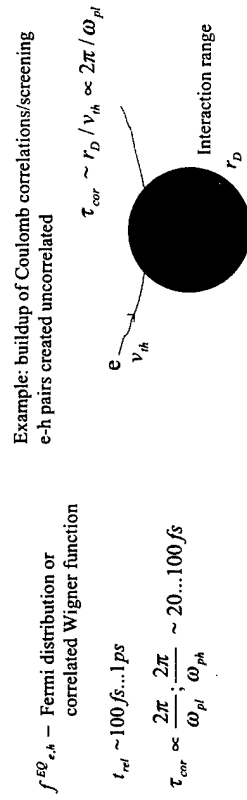
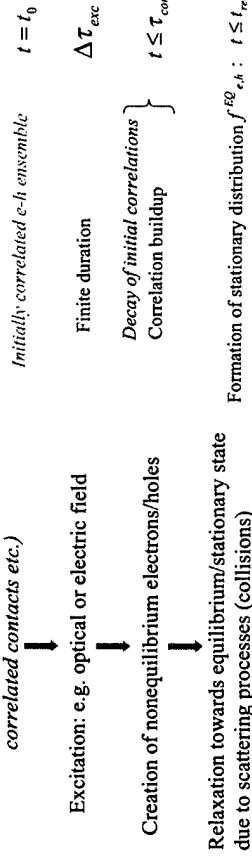
E_F - Fermi energy a_B - exciton Bohr radius
 λ - electron quantum wave length



2. Short-time phenomena and initial correlations

Relaxation processes in electron-hole systems

Pre-excited e-h plasma (doping/prepulse, correlated contacts etc.)



3. Nonequilibrium Greens functions:

- main ideas

When are initial correlations important?

→ Transient processes before stationary state

1. Short times: $t_0 \leq t \leq \tau_{cor}$ (Or high-frequency switching: $\omega_s \geq 2\pi / \tau_{cor}$)
2. Strongly correlated initial state: $|\langle U_{cor} \rangle|(t_0) \geq \langle E_{kin} \rangle(t_0)$
 - materials with strong electron-phonon coupling
 - materials with strong Coulomb interaction

In this case also: $\tau_{cor} \geq t_{rel}$
3. „bottleneck“ situations, threshold processes:
 - carrier excitation below phonon energy, impact ionization etc.
 - modified plasmon spectrum in p-doped semiconductors
4. Long-living initial correlations: e.g. bound states (excitons, impurities etc.), Wigner crystal, Bose condensate etc.
 - weak damping (slow decay) of initial correlations

Idea of Green's functions

1. Classical N-body system: statistical properties given by distribution function $f(r, p, t)$

$$\int \frac{d^3 p}{(2\pi \hbar)^3} f(r, p, t) = n(r, t), \quad \int d^3 r n(r, t) = N(t)$$
2. Quantum N-body system:
 - Wigner („quasi“)-distribution function $f(r, p, t)$
 - Wave properties, energy „spectrum“ $A(E)$

Idea: combine into „generalized distribution“ $g(r, p, t; E)$ E: independent variable

→ Green's function

Equilibrium: $g(p; E) = i f(E) A(p, E)$, $f(E) = 1 / (e^{\beta(E - \mu)} + 1)$

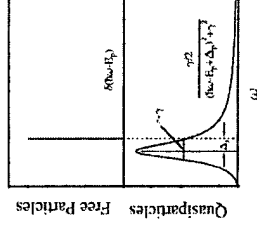
- non-interacting particles: $A(p, E) = 2\pi \delta(E - p^2 / 2m)$

- interacting (correlated) particles: $A(p, E) \propto \frac{\gamma / 2}{(E - p^2 / 2m + \Delta)^2 + \gamma^2}$

Correlated Wigner distribution $g(p)$

Finite line width (finite life time), related to pair correlations $c(r)$

→ Correlations included into single-particle properties (quasi-particle)



Nonequilibrium Green's functions

$g(r, p, t; E)$ Equivalent to function of two-times:

I. Fourier transform: $\int dE g(r, p, t; E) e^{iEt/\hbar} \rightarrow g(r, p, t, \tau)$

II. Define $t_{1,2} = t \pm \tau/2 \rightarrow g(r, p, t_1, t_2)$

III. Build in spin: relate g to fermion field operators: $\psi^*(r, t), \psi(r, t)$

Anticommutation: $\psi^*(r_1, t)\psi(r_2, t) + \psi(r_2, t)\psi^*(r_1, t) = \delta(r_1 - r_2)$

IV. Two-operator averages – two possibilities:
(two independent functions)
 $\langle \psi^*(2)\psi(1) \rangle \rightarrow g^<(1,2); 1 \equiv r_1, t_1, s_1$
 $\langle \psi(1)\psi^*(2) \rangle \rightarrow g^>(1,2)$

V. Physical contents: *on time diagonal*: $g^<(r_1, t, r_2, t) \approx f(r_1, r_2, t)$ Density matrix, Wigner distribution etc.
Across diagonal: $g^>(1,2) - g^<(1,2) \propto A(1,2)$ Spectral function, Density of states etc.

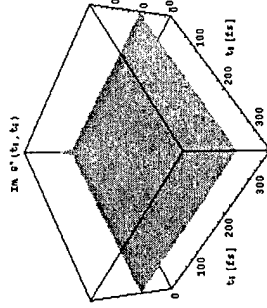
VI. Heisenberg equations for $\psi^*, \psi \rightarrow$ (Kadanoff-Baym/Keldysh) equations of motion for $g^>, g^<$

- ➡ Fully selfconsistent treatment of relaxation, correlations and energy spectrum
- ➡ Clear recipe for treatment of complex processes and systematic approximations

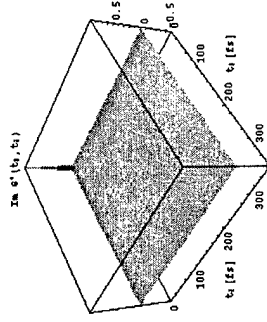
Direct Solution of the Kadanoff-Baym equations

Imaginary part of $g^<(k, t_1, t_2)$ for fixed momentum k

1. Relaxation of nonequilibrium electron distribution (peaked at $k=3.9/a_0$)
2. Build up of electron spectrum (of Coulomb correlations) across time diagonal



$k=2.7/a_0$



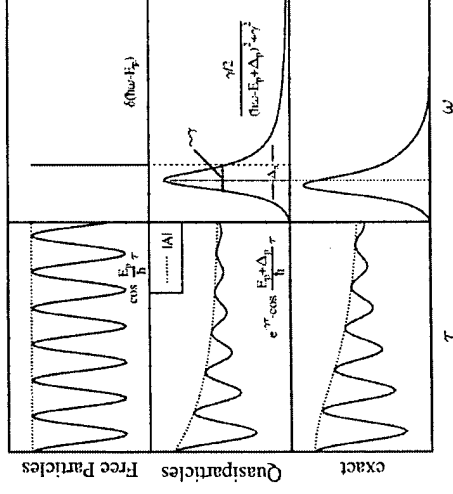
$k=3.9/a_0$

Homogeneous bulk GaAs $n = 10^{17} \text{ cm}^{-3}$

Dirk Semkat/Michael Bonitz

Spectral function of interacting electrons

in time space vs. frequency



E_p - Kinetic (quasiparticle) energy

γ, Δ_p imaginary and real part of selfenergy

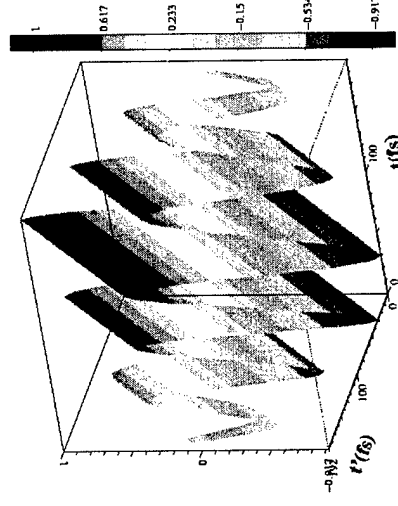
Exponential damping
Lorentzian power-law tail, wrong at large frequency!

Solution of the two-time Kadanoff-Baym equations. Zero slope at $\tau=0!$

Exact spectral function with Coulomb scattering

$$A(t, t') = i\hbar[g^>(t, t') - g^<(t, t')]$$

Spektralfunktion



Semkat/Kwong/Köhler/Binder/MB

Kadanoff-Baym equations with initial correlations

N-particle system, binary interaction $V(r)$, external potential $U(r)$

Non-equilibrium: Greens functions on time contour C (Keldysh, Schwinger)

$G. Baym (*)$

Complex time plane

$$g_{1\dots s}(1\dots s, 1'\dots s') = \left(\frac{1}{i}\right)^s \langle T_C [\Psi(1)\dots\Psi(s)\Psi^\dagger(s')\dots\Psi^\dagger(1')] \rangle$$

Equation of motion for $g_i(t_i, t_i')$: (Martin/Schwinger)

$$\int_C d\bar{1} \left\{ g_1^{0-1}(\bar{1}, 1) - U(\bar{1}, 1) \right\} g_1(\bar{1}, 1'; U)$$

$$= \delta(1 - 1') \pm i \int d2 V(1 - 2) g_{12}(12, 1'2^+)$$

with $g_1^{0-1}(1, 1') = \left(i \frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right) \delta(1 - 1')$

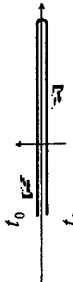
General boundary condition:

$$\lim_{t_1 \rightarrow t_0 \rightarrow -\infty} g_{12}(12, 1'2')|_{t_0} = [g_1(1, 1')g_1(2, 2') \pm g_1(1, 2')g_1(2, 1')]|_{t_0}$$

$$+ c(r_1 r_2, r'_1 r'_2; t_0)$$

* In: „Progress in Nonequilibrium Green's functions“, M. Bonitz (Ed.), 2000
Semkat, Kremp, Bonitz 1999, 2000
Haug/Jauho „Quantum Kinetics...“

Correlated initial state: e.g. Fujita, Hill, Danielewicz, Wagner



Kadanoff-Baym equations with initial correlations, contd.

Formal closure of hierarchy (generalized*): introduce selfenergy

$$\int_C d\bar{1} \Sigma(1, \bar{1}) g_1(\bar{1}, 1') = \pm i \int d2 V(1 - 2) g_{12}(12, 1'2^+)$$

$$= \pm i \int d2 V(1 - 2) \left\{ \pm \frac{\delta g_1(1, 1'; U)}{\delta U(2^+, 2)} + g_1(1, 1') g_1(2, 2^+) \right\}$$

Boundary condition:

$$\lim_{t_1 \rightarrow t_0 \rightarrow t_0^+} \int_C d\bar{1} \Sigma(1, \bar{1}) g_1(\bar{1}, 1') = \pm i \int d2 V(r_1 - r_2) [c(r_1 r_2, r'_1 r'_2; t_0)$$

$$+ g_1(r_1 r'_1, t_0) g_1(r_2 r'_2, t_0) \pm g_1(r_1 r'_2, t_0) g_1(r_2 r'_1, t_0)]$$

Binary correlations at $t=t_0$

Result: additional selfenergy contribution

$$\Sigma(1, 1') = \Sigma^{HF}(1, 1') + \Sigma^c(1, 1') + \Sigma^{in}(1, 1')$$

$$\Sigma^{in}(1, 1') = \Sigma^{in}(1, r'_1 t_0) \delta(t'_1 - t_0)$$

*Semkat, Kremp, MB, J. Math. Phys., 41, 7458 (2000)

Generalized Kadanoff-Baym equations

Result:

$$\int_C d\bar{1} \left[g_1^{0-1}(\bar{1}\bar{1}) - U(\bar{1}\bar{1}) - \Sigma^{HF}(\bar{1}\bar{1}) \right] g_1(\bar{1}\bar{1}')$$

$$= \delta(1 - 1') + \int_C d\bar{1} [\Sigma^c(\bar{1}\bar{1}) + \Sigma^{in}(\bar{1}\bar{1})] g_1(\bar{1}\bar{1}')$$

Matrix equations on the time contour

Result on the physical (real) time axis:

$$\left(i \frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right) g_1^{\pm}(11') - \int d\bar{1} U(\bar{1}\bar{1}) g_1^{\pm}(\bar{1}\bar{1}') - \int d\bar{r}_1 \Sigma^{HF}(\bar{1}\bar{1}) g_1^{\pm}(\bar{1}\bar{1}')$$

$$= \int_{t_0}^{\infty} d\bar{1} \Sigma^B(\bar{1}\bar{1}) g_1^{\pm}(\bar{1}\bar{1}') + \int_{t_0}^{\infty} d\bar{1} [\Sigma^{\pm}(\bar{1}\bar{1}) + \Sigma^{in}(\bar{1}\bar{1})] g_1^{\pm}(\bar{1}\bar{1}')$$

Result for selfenergy:

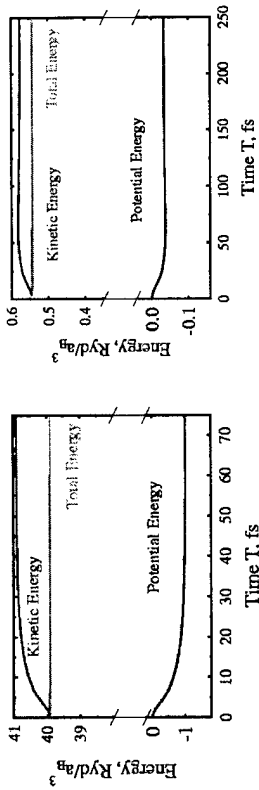
$$\Sigma^{in}(11') = \pm i \int d2 V(1 - 2) \int d\bar{r}_1 d\bar{r}_2 d\bar{r}_1 d\bar{r}_2$$

$$\times g_{12}^B(12, \bar{r}_1 t_0, \bar{r}_2 t_0) c(\bar{r}_1 t_0, \bar{r}_2 t_0, 1', \bar{r}_2 t_0) g_1^A(\bar{r}_2 t_0, 2^+) \delta(t'_1 - t_0)$$

Selfconsistent evolution and decay of initial correlations

$n = 2.77 a_B^{-3}$ (weak coupling)

$n = 0.277 a_B^{-3}$ (intermediate coupling)



Short-time relaxation with initial correlations

Solution of KB-equations with initial correlation selfenergy

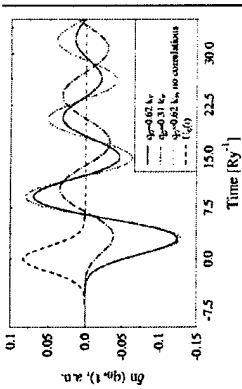
Example: bulk GaAs, *uncorrelated* vs. *over-correlated* initial state

- **Correct conservation** of total energy (kinetic + correlation)
- Energy relaxation reflects correlation build up/decay for $t \leq \tau_{cor}$

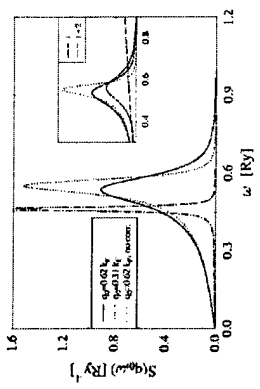
Dynamic Structure factor of interacting electrons

Solution of inhomogeneous Kadanoff-Baym Equations

Start from correlated initial state
At $t=0$ monochromatic perturbation
Density fluctuation
(Landau plus collisional damping)



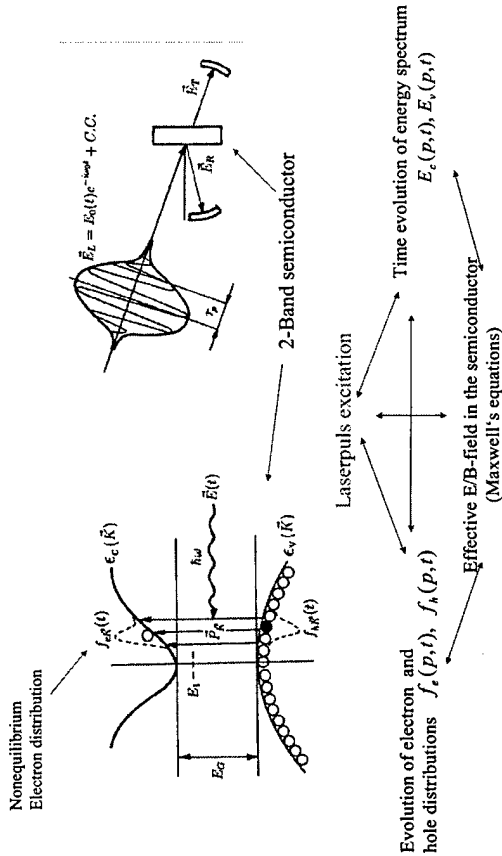
Selfconsistent dynamic
Structure factor with
Correlations
(Born approximation plus
vertex corrections)



Kwong, Bonitz, Phys. Rev. Lett. 84, 1768 (2000)

Electron dynamics in semiconductors

Example: Optical Excitation



3. Nonequilibrium Greens functions:
- application to optics and high field transport

Interband-Kadanoff-Baym equations

μ_1, μ_2 – Band indices, $E_\mu(p)$ – Semiconductor band structure

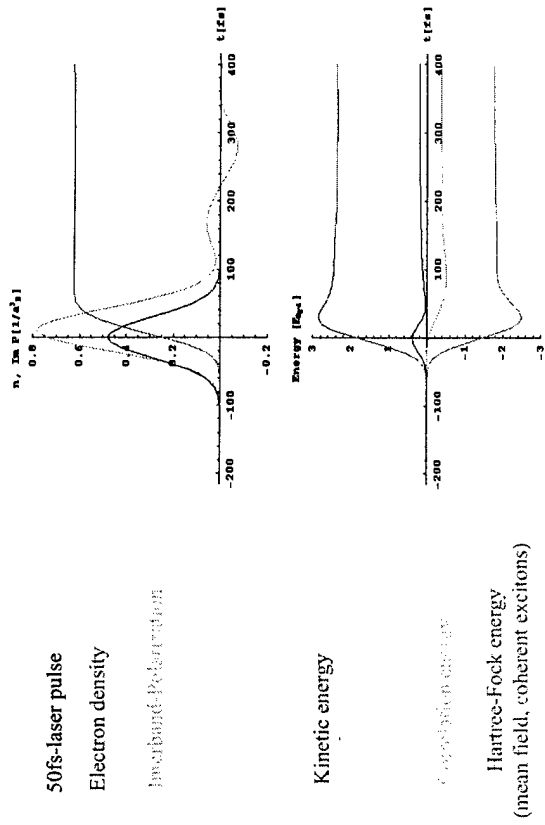
$$\begin{cases} \left\{ i\hbar \frac{\partial}{\partial t_1} - \epsilon_{\mu_1} \right\} g_{\mu_1 \mu_2}^<(t_1 t_2) - \sum_{\mu} \hbar \Delta_{\mu \mu_1} (t_1) g_{\mu \mu_2}^<(t_1 t_2) = I_{\mu_1 \mu_2}^<(t_1 t_2) \\ \left\{ -i\hbar \frac{\partial}{\partial t_2} - \epsilon_{\mu_2} \right\} g_{\mu_1 \mu_2}^<(t_1 t_2) - \sum_{\mu} g_{\mu_1 \mu}^<(t_1 t_2) \hbar \Delta_{\mu \mu_2} (t_2) = -I_{\mu_2 \mu_1}^<(t_2 t_1) \\ \hbar \Omega_{\mu_1 \mu_2}(t) = -\mathbf{d}_{\mu_1 \mu_2} \mathcal{E}(t) (1 - \delta_{\mu_1 \mu_2}) + i\hbar \sum_{\mathbf{k}} g_{\mu_1 \mu_2}^<(\mathbf{k}' t t) V_{\mu_1 \mu_2}(\mathbf{k} - \mathbf{k}') \end{cases}$$

Dipole moment Laser field Hartree-Fock mean field

$$\begin{aligned} \text{Correlations (collisions)} \quad I_{\mu_1 \mu_2}^<(t_1 t_2) &= \sum_{\mu} \int_{t_0}^{t_1} d\tilde{t} [\sigma_{\mu \mu}^<(\tilde{t} t_1 \tilde{t}) - \sigma_{\mu \mu}^<(\tilde{t} t_1 \tilde{t})] g_{\mu \mu}^<(\tilde{t} t_2) \\ \text{(with phonons, electrons, defects etc.)} \\ \sigma &= \text{self energies} \\ &= \sum_{\mu} \int_{t_0}^{t_2} d\tilde{t} \sigma_{\mu \mu}^<(\tilde{t} t_1 \tilde{t}) [g_{\mu \mu}^<(\tilde{t} t_2) - g_{\mu \mu}^<(\tilde{t} t_2)] \end{aligned}$$

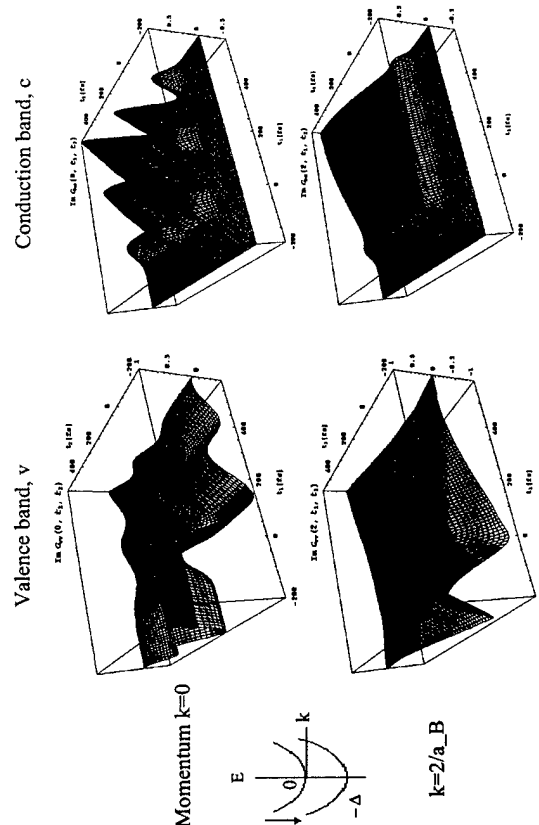
Kwong, Bonitz, Binder, Köhler, phys. stat. sol. (b) 206, 197 (1998)

Laser pulse excitation of a s.c. quantum well



50fs-laser pulse
Electron density
Kinetic energy
Hartree-Fock energy
(mean field, coherent excitons)

Damping of the Electrons and Renormalization of the energy bands

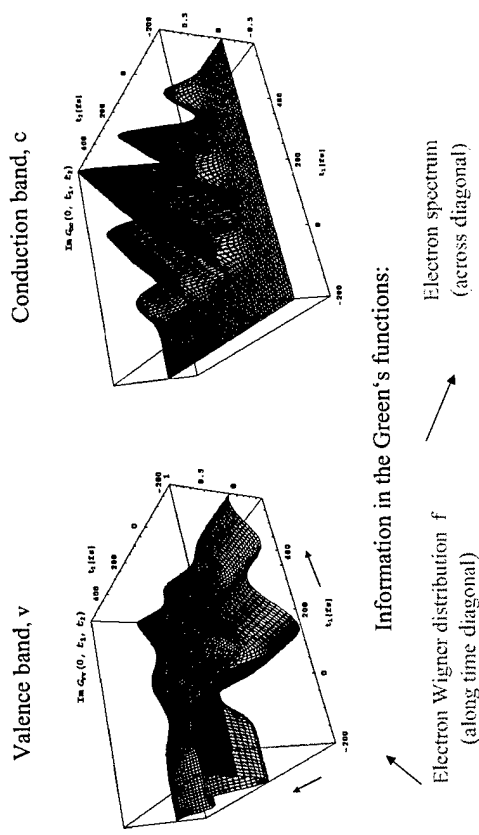


Momentum $k=0$
 $k=2/a_B$

Electron dynamics in a semiconductor

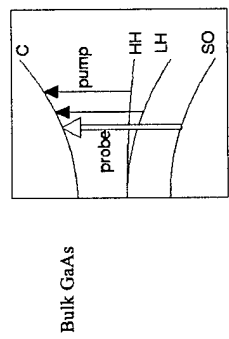
Laser „lifts“ Electrons from valence band to conduction band

(50fs-Pulse, Maximum at $t=0$)



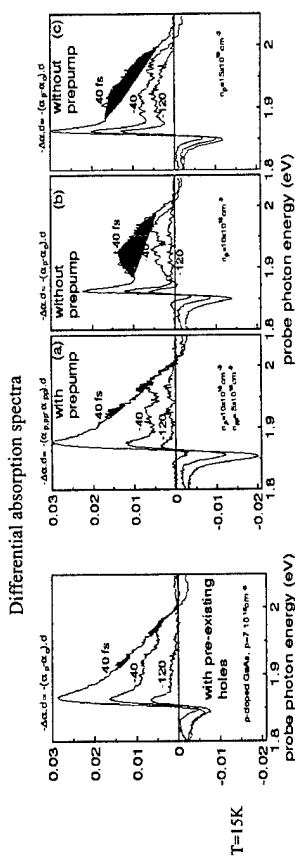
Information in the Green's functions:
Electron Wigner distribution f
(along time diagonal)
Electron spectrum
(across diagonal)

Relaxation in the presence of preexcited electrons/holes

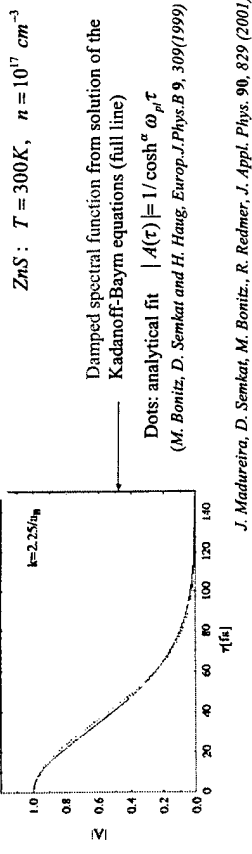
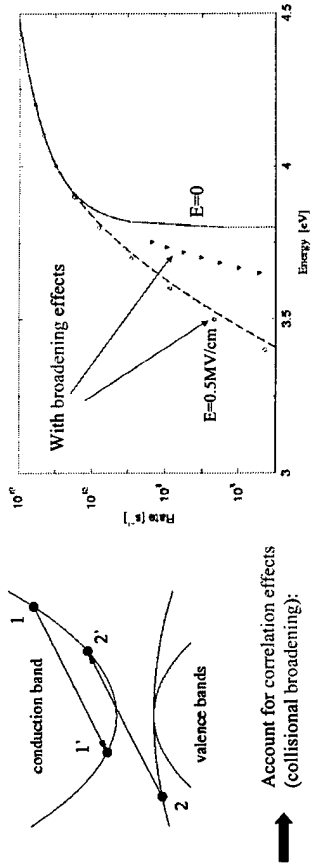


Drastically accelerated thermalization
In the presence of prepump or p-doping
Reason: pre-existing carriers
qualitatively modify plasmon
spectrum and e-e-h scattering rates
-> Pre-existing Coulomb correlations

Lampin, Alexandrou et al. 1999, Bonitz et al. PRB 62, 15724 (2000)



Impact Ionization rates in semiconductors

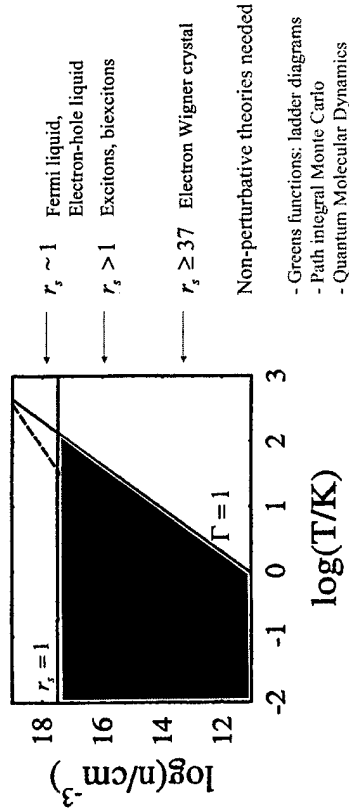


Strong Coulomb correlations

$$\Gamma \equiv \langle U \rangle / k_B T \quad r_s \equiv \langle U \rangle / E_F \propto r > a_B$$

E_F - Fermi energy a_B - exciton Bohr radius

Expected behavior:



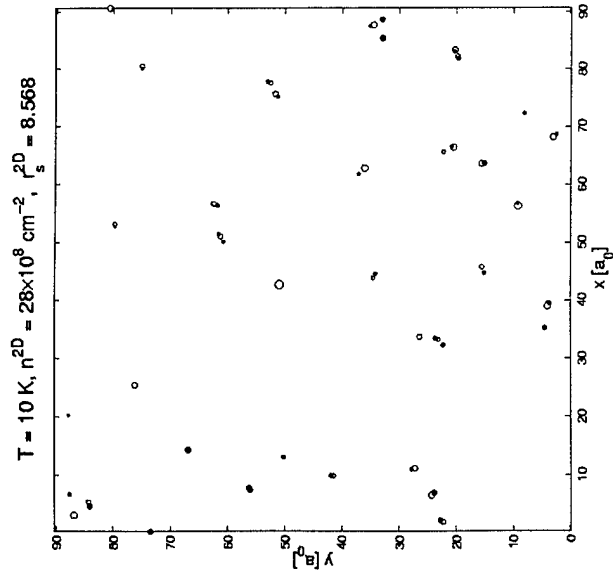
Greens functions treatment of Strong Coulomb correlations

Problem: strong Coulomb correlations, bound states, exciton formation etc. not possible in Born approximation

→ Need selfenergies in T-matrix (ladder) approximation
Electron spectrum: continuum and bound states

Bound state (peak) modified by surrounding e-h-plasma (Mott Effekt)

Screening parameter $\kappa = 4\pi e^2 / kT$

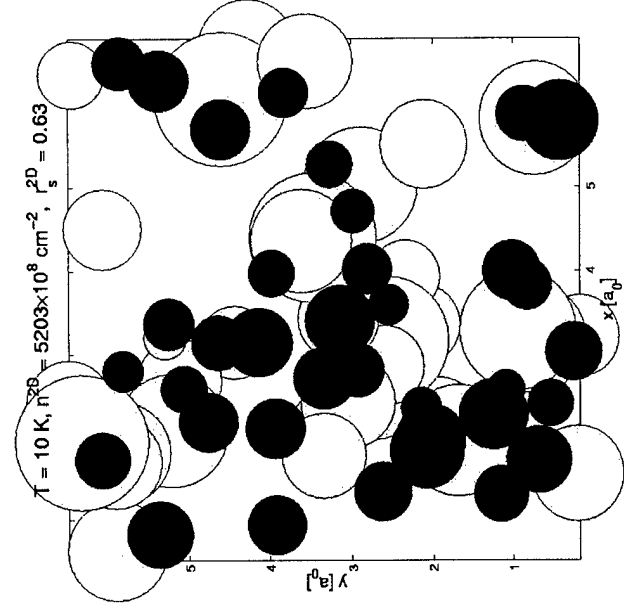
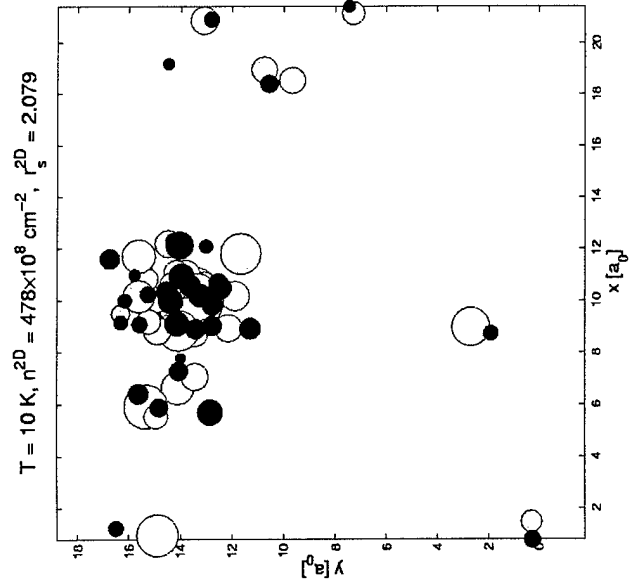
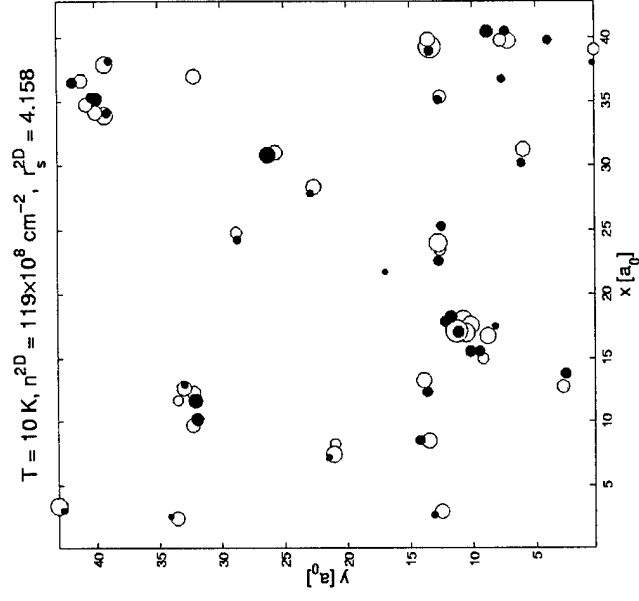


Rigorous Path integral
Monte Carlo simulations

Start with electrons and holes,
Full inclusion of
- Coulomb interaction
- Quantum effects
- Spin

Dots: $\psi(x, y)$ and hole
Location (wave function
squared) in quantum well

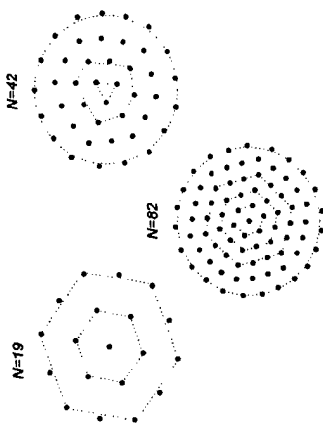
V. Filinov, W. Hoyer,
S.W. Koch, and M. Bonitz 2001



Mesoscopic electron clusters

Easy realization of strong correlations
 Question: possibility of Wigner crystallization?
 Is there a metal-insulator transition?

Problem: simultaneous account of correlations, quantum and spin effects
 many-body approaches (including NGF) not applicable
 → Path integral Monte Carlo

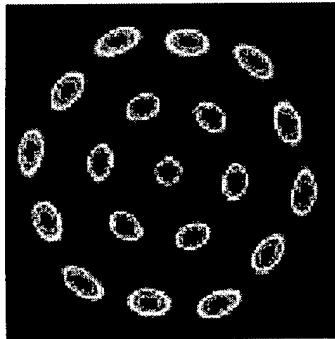


Model: 2D electron clusters
 in spherical trap, $N=1...100$
 $T=\text{const}$, $B=0$

Results: - Shell structure,
 - pronounced symmetry effects
 - Strong N-dependence

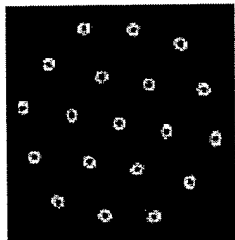
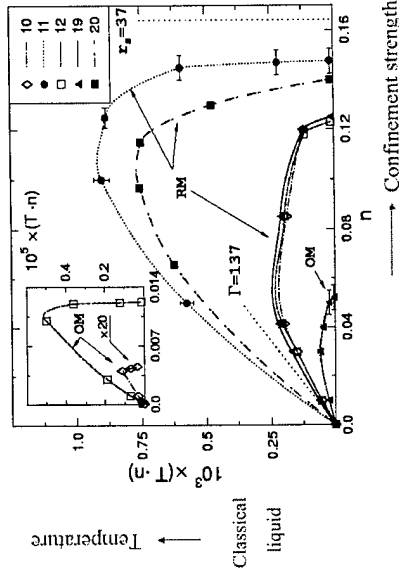
Wigner crystallization of electron clusters

Variation of temperature or density (confinement)
 Existence of two crystal phases: *intra*-shell and *inter*-shell ordering



A. Filinov, M. Bonitz, and Yu. Lozovik, Phys. Rev. Lett. **86**, 3851 (2001)

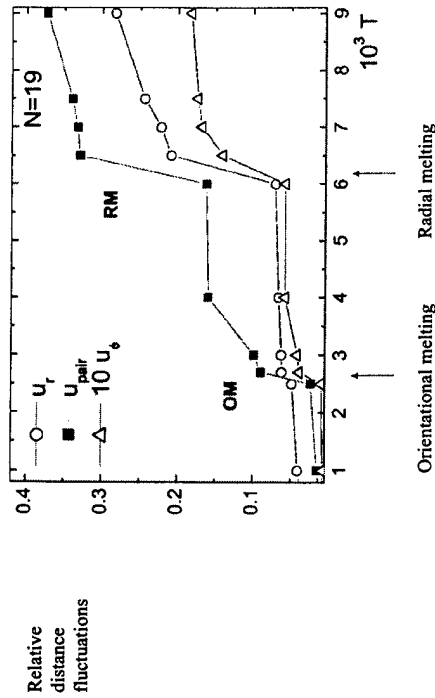
Phase diagram of the mesoscopic Wigner crystal



RM - Radial melting, OM - Orientational melting

Crystal melting and mobility increase

At the melting point: drastic increase of angular and radial distance fluctuations
 → Mobility and conductivity increase



Summary and Outlook

I. Nonequilibrium Greens functions powerful tool

- extended to arbitrary initial correlations
- well applicable to ultrafast/transient phenomena
- useful for high field transport

II. Strong correlations in semiconductors

- excitons, droplets, Wigner crystal
- quantum Monte Carlo and molecular dynamics

III. Future: combination of NGF and quantum simulations

References

1. Quantum kinetic theory with density operators and Greens functions, see M. Bonitz „Quantum Kinetic Theory“, B.G. Teubner, Stuttgart Leipzig 1998
2. An overview (review articles) on Kadanoff-Baym equations can be found in „Progress in Nonequilibrium Greens Functions“, M. Bonitz (ed.), World Scientific, Singapore 2000
3. Kadanoff-Baym equations with initial correlations, see D. Semkat, D. Kremp, and M. Bonitz, J. Math. Phys. **41**, 7458 (2000) and references therein
4. Greens functions applications to high-field transport, see e.g. Madhura, Semkat, Bonitz, Redmer, JAP **90**, 829 (2001)
5. Greens functions applications to semiconductor optics, see e.g. Kwong, Bonitz, Binder, Köhler, phys.stat.sol. (b) **206**, 197 (1998)
6. Fermionic path integral Monte Carlo (bound state formation etc.), see V.S. Filinov, M. Bonitz, W. Theding, and V.L. Tortov, Plasma Phys. Contr. Fusion **43**, 743 (2001) and references therein
7. Wigner crystallization in microscopic electron clusters (open quantum dots): A. Filinov, M. Bonitz, and Yu. Lozovik, Phys. Rev. Lett. **86**, 3851 (2001); Physical Review Focus April 19 (2001)

For updated references, please look at <http://cdcc.mpa.uni-rostock.de/nfb>

Content

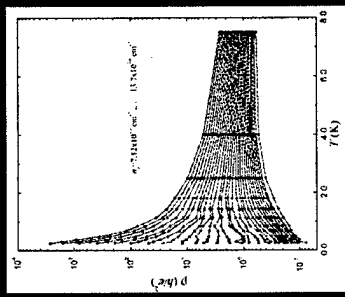
- Motivation
 - The observation of the 'metal-insulator transition' in 2D
- Scaling theory of localization
 - Scaling Function and phase coherence length
 - Importance electron-electron interaction
- Experiments
 - Measurement technique and Si-MOSFET devices
 - Standard transport measurements with and without magnetic field
- Weak localization
 - The role of quantum interference on the MIT
- The trap model
 - The capability and the limits
- Conclusions



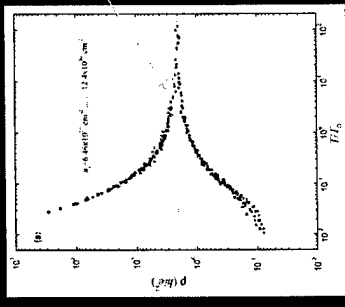
Motivation

The observation of the 'metal-insulator transition' in 2D

Temperature dependence of the resistivity

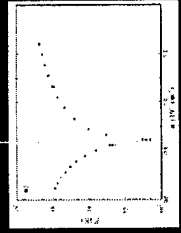


Resistivity vs T/T_0



mirror-reflection symmetry

Scaling parameter T_0

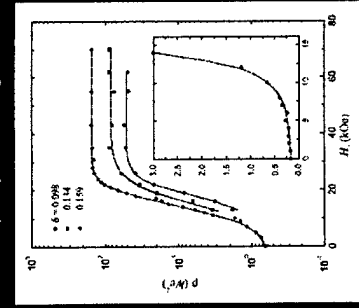


Kravchenko, Pudalov et al. PRB51 (1995)

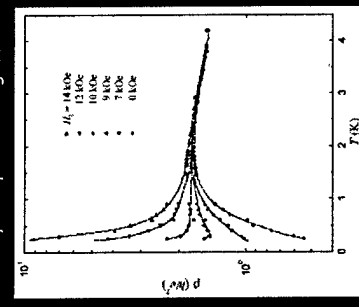
Motivation

Suppression of the conducting phase by a parallel magnetic field

Resistivity vs parallel magnetic field



Temperature dependence of the resistivity in a parallel magnetic field



Simonian et al., PRL79, 2304 (1997)

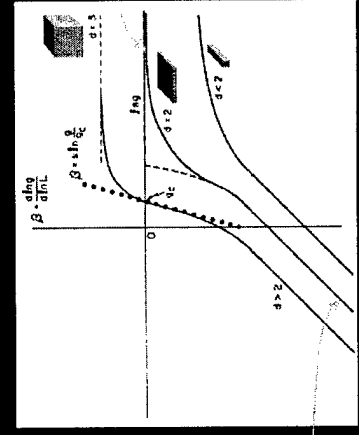


Scaling function beta(g)

for non-interacting electrons and for T=0K

$$\beta(g) = \frac{d \ln g}{d \ln L} = \frac{L}{g} \frac{dg}{dL}$$

g ... conductance in e^2/h
L ... system size



$$\beta(g) \sim \exp(-L/L_c)$$

Fig.1, PRL 42, 573 (1979)

Importance Electron-Electron Interaction

Coulomb energy versus Fermi energy:

$$r_s = \frac{E_{\text{Coul}}}{E_F} \propto \frac{1}{E_F} \propto \frac{1}{n^{1/3}} \propto \frac{1}{m^{1/3}}$$

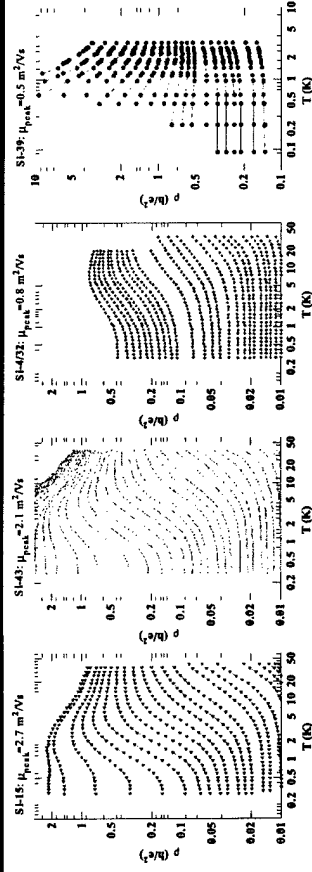
Material	n^*	r_s	n (10 ¹⁸ cm ⁻³)
n-Si-MOS	0.19	7.8	6.7
p-SiGe/Si	0.3	12.6	6.4
n-Si/SiGe	0.19	11.7	3.0
p-GaAs/AlGaAs	0.32	10.9	9.8
n-GaAs/AlGaAs	0.067	10.9	0.43

$r_s \approx 10$

Transport measurements in zero B-field

Temperature dependence of the resistivity

Comparison of different samples



The exponential drop increases with the peak mobility !

Possible explanations for the phenomenon

Failure of the quasi-particle picture of Landau due to EEI



new groundstate: Non-Fermi liquid



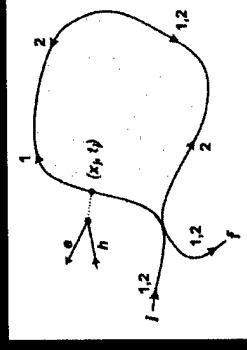
Superconductivity

or just a 'classical' effect ?

Weak Localization

Overview

Quantum mechanical interference enhances back scattering



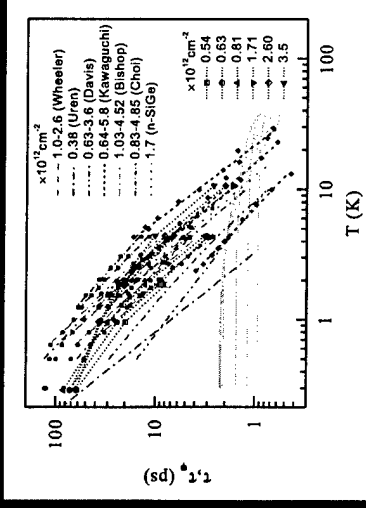
2 features of weak localization are important :

- (1) logarithmic temperature dependence of the conductivity
- (2) negative magnetoresistance

Weak Localization

electron density regime ($n > 10^{12} \text{ cm}^{-2}$)

Phase coherence time & momentum relaxation time vs T

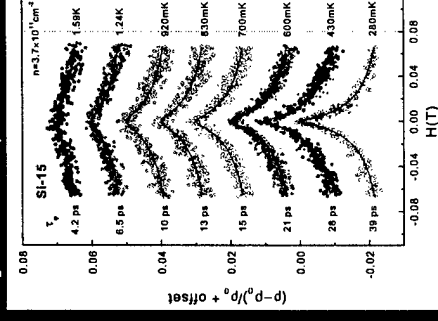


Si-43

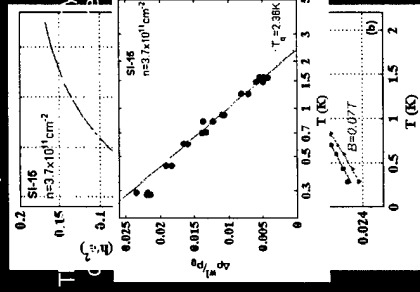
Weak Localization

Low electron density regime ($n < 10^{12} \text{ cm}^{-2}$)

Magnetoresistivity for $n = 0.37 \times 10^{12} \text{ cm}^{-2}$



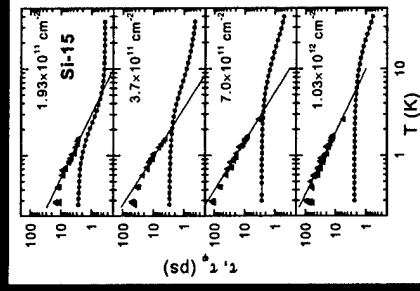
Temperature dependence of resistivity for $n = 0.37 \times 10^{12} \text{ cm}^{-2}$



Weak Localization

Low electron density regime ($n < 10^{12} \text{ cm}^{-2}$)

Temperature dependence of the phase coherence time

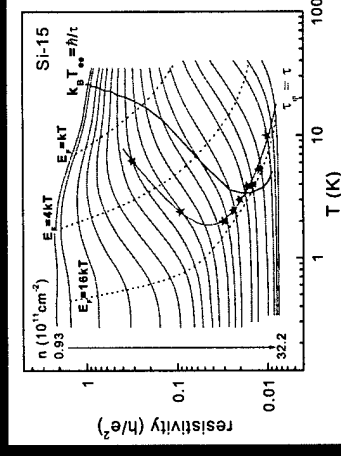


Single particle interference cease to effect resistivity for $\tau_\phi < \tau$

Weak Localization

Result of the weak localization analysis

Temperature dependence of resistivity with 'critical' borders



RED CURVE ... disappearance of the weak localization ... border for the EEI



Trap Model

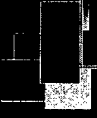
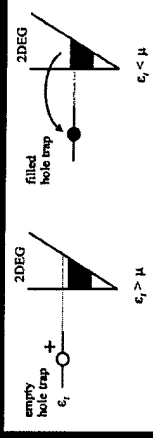
after B.L. Altshuler and D.L. Maslow

Overview

Assumptions:

- traps are distributed homogeneously in the oxide
- all the traps have the same energy ϵ_t
- traps are due to weak Si-Si bonds (occupied by $1e^-$ or $2e^-$)
- traps can be easily charged and discharged

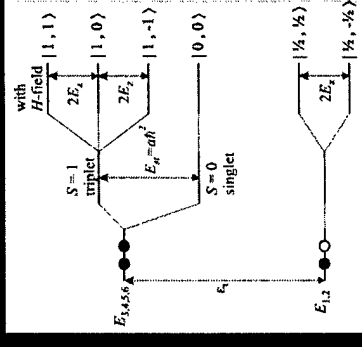
Charge state of the trap depends on the Fermi energy of the 2D electron gas



Trap Model

The effect of the magnetic field

Splitting of the trap energy levels in a magnetic field



Conclusions

- The experimentally determined **phase coherence time** is about 10 smaller than theoretically predicted, but is in agreement with prior works on low-mobility samples
- From analysis of the weak localization, **quantum interference** can be excluded as an origin for the metallic state and the MIT
- There is no indication of a non-Fermi liquid in the metallic regime → **no quantum phase**
- The trap model can explain qualitatively certain feature of the MIT, but fails to give quantitative results

The "Metal-Insulator Transition" in Open-Quantum Dots & Dot Arrays



Jonathan P. Bird

Department of Electrical Engineering &
Center for Solid State Electronics Research
Arizona State University
Tempe, AZ 85287-5706

S4700 15.0kV 12.3mm x11.0k SE(U) 5/31/00 15:31 5.00um

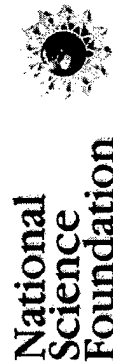


NANOSTRUCTURES RESEARCH GROUP



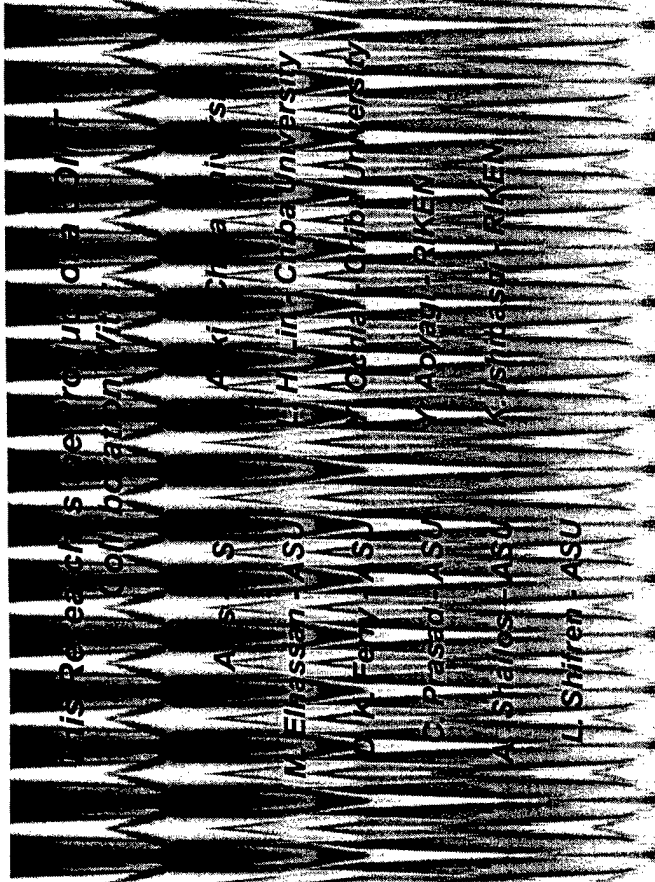
NANOSTRUCTURES RESEARCH GROUP

Work at ASU is Supported By:



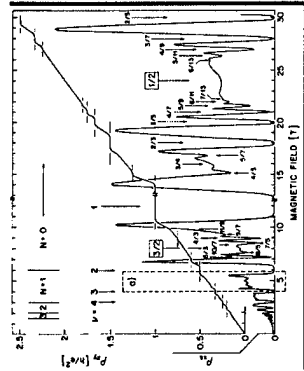
- INTRODUCTION
- TRANSPORT IN QUANTUM DOTS
- DEVICE FABRICATION & BASIC CHARACTERIZATION
- EXPERIMENTAL RESULTS
- CONCLUSIONS

OUTLINE

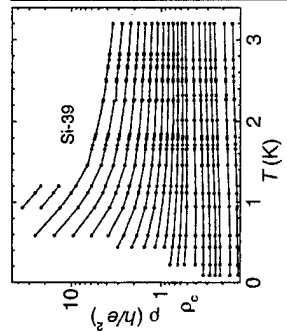


- The study of the transport properties of LOW-DIMENSIONAL systems has lead to the discovery of a number of novel MANY-BODY effects in recent years

* Striking examples include the recent discovery of COMPOSITE FERMIONS in the fractional quantum-Hall effect and the METAL-INSULATOR TRANSITION in two dimensions



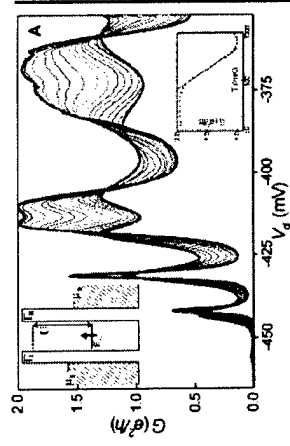
R. Willet et al.
Phys. Rev. Lett. **58**, 1776 (1987)



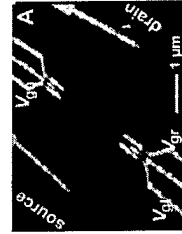
V. M. Pudalov et al.
Physica E **3**, 79 (1998)

- When the dots are coupled to their external reservoirs by means of WEAK tunnel barriers many-body effects have been argued to give rise to KONDO-LIKE behavior

* Here electrons confined WITHIN the dot play the role of the magnetic impurity in the more conventional Kondo effect

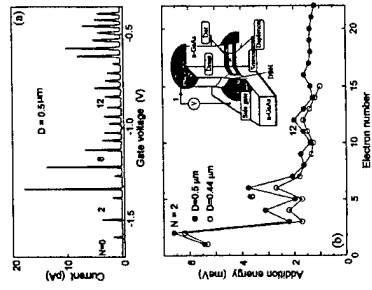


D. Goldhaber-Gordon et al.
Nature **391**, 156 (1998)
Science **282**, 1551 (1998)
Phys. Rev. Lett. **84**, 5824 (2000)
W. G. van der Wiel
Science **289**, 2105 (2000)



- In mesoscopic QUANTUM DOTS electron interactions are known to play an important role when the dots are coupled to their reservoirs by means of weakly transmitting TUNNEL BARRIERS

* In this regime the COULOMB BLOCKADE of transport gives rise to SINGLE-ELECTRON behavior which can be used to demonstrate SHELL-LIKE filling of electron states

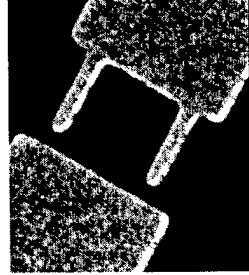


S. Tarucha et al.
Phys. Rev. Lett. **77**, 3613 (1996)

- In contrast to these studies our interest will lie in a discussion of transport in OPEN quantum dots

* These dots are coupled to their external reservoirs by means of quantum-point-contact LEADS that are configured to support a SMALL number of PROPAGATING modes

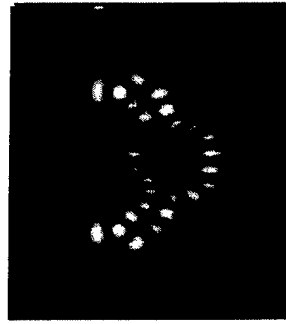
* Since transport through these structures is typically BALLISTIC in nature their electrical properties strongly reflect the details of the electron BOUNDARY scattering they generate



- While the Coulomb blockade is QUENCHED in open dots electrons may still be confined for LONG times in such structures

- We might therefore expect that the transport properties of open dots should exhibit NOVEL signatures that arise due to the MANY-BODY interactions of carriers trapped within them

- FEW studies to date appear to have explored this possibility however



R. Akis et al.
Physica E 2, 745 (2000)

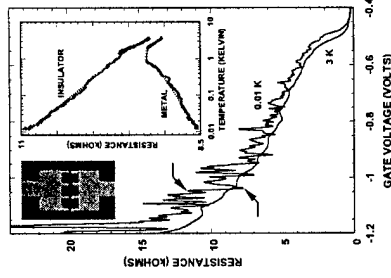
A. Shalios et al.
Phys. Rev. B 63, 241302 (2001)

OUTLINE

- INTRODUCTION
- TRANSPORT IN QUANTUM DOTS
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- EXPERIMENTAL RESULTS
- CONCLUSIONS

- We investigate evidence for MANY-BODY interactions in open quantum dots and quantum-dot arrays by studying the variation of their resistance with TEMPERATURE

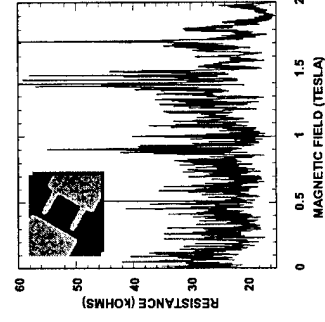
- We find evidence for novel LOCALIZATION behavior that we speculate arises due to an ENHANCED electron-electron interaction in these mesoscopic structures



- It is well known that at low temperatures the DISCRETE level spectrum of ISOLATED quantum dots can be RESOLVED in experiment

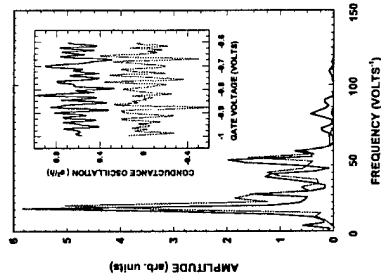
- When the dot is OPENED to the outside world the coupling between to the reservoirs gives rise to a NON-UNIFORM broadening of the dot states

- Discrete states that SURVIVE the coupling can be driven past the Fermi level by applying a magnetic field or gate voltage which gives rise to OSCILLATIONS in the conductance



J. P. Bird et al.
Europhys. Lett. 35, 529 (1996)

- The particular states that persist in the open dots are found to correspond to strongly SCARRED wavefunctions
- The conductance oscillations observed in experiment therefore represent MEASURABLE signatures associated with these wavefunction scars

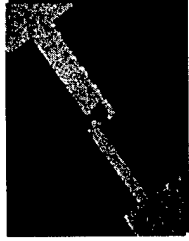
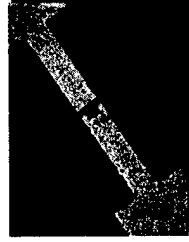


R. Akis et al.
Phys. Rev. Lett. **79**, 123 (1997)
J. P. Bird et al.
Phys. Rev. Lett. **82**, 4631 (1999)
R. Akis et al.
Physica E **7**, 745 (2000)

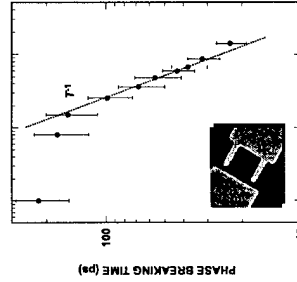


- By changing the ORIENTATION of the leads that couple the dot to the reservoirs DIFFERENT groups of states can be selected to mediate the transport behavior
- Evidence for this process of SCAR SELECTION can be observed directly in experiment

VOLUME 82, NUMBER 23 PHYSICAL REVIEW LETTERS 7 JAN 1999
Lead-Orientation-Dependent Wave Function Scarring in Open Quantum Dots
J. P. Bird,¹ R. Akis,¹ D. K. Ferry,¹ D. Vasilakis,¹ J. C. Coppi,² Y. Araya,² and T. Sargent,²
¹Center for Solid State Electronics Research & Department of Electrical Engineering,
University of California, San Diego, La Jolla, California 92037
²Manufacturing Materials Laboratory, Frontier Research Program, RIKEN,
2-1 Hirosawa, Wako, Saitama 351-0195, Japan

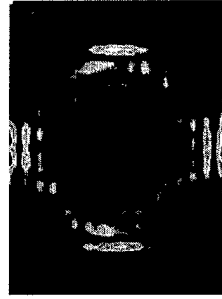


- From an analysis of the magneto-conductance fluctuations we can determine the electron PHASE-BREAKING time which is a measure of the time over which the coherent WAVE-LIKE nature of the electron is PRESERVED within the dot
- At low temperatures this time scale is several ORDERS of magnitude longer than the time required to traverse the dot thus allowing confinement effects to be strongly RESOLVED



J. P. Bird et al.
Phys. Rev. B **51**, 18037 (1995)
D. P. Pivko, Jr. et al.
Phys. Rev. Lett. **82**, 4687 (1999)

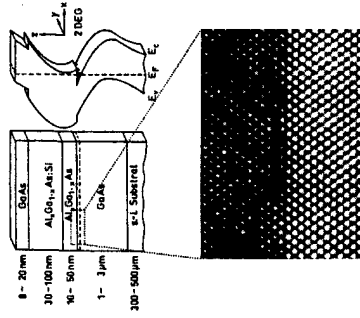
R. Akis et al.
Phys. Rev. B **54**, 17705 (1996)
Phys. Rev. Lett. **79**, 123 (1997)



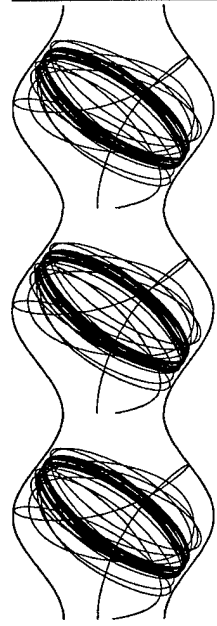
OUTLINE

- INTRODUCTION
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- DEVICE FABRICATION & BASIC CHARACTERIZATION
- EXPERIMENTAL RESULTS
- CONCLUSIONS

- In this report we discuss the transport properties of multiply-coupled QUANTUM-DOT ARRAYS
- The arrays are realized using the SPLIT-GATE technique in which a depleting voltage is applied to lithographically-defined Schottky gates to induce LATERAL confinement of the high-mobility two-dimensional electron gas formed at the heterointerface



- Some further COMMENTS on the transport characteristics of the arrays
- The carrier density within the arrays is found to remain UNCHANGED with gate voltage
- The transport mean free path (>10 μm) is very much LONGER than the size of the arrays so that transport within them is expected to be highly BALLISTIC in nature
- At low temperatures the electron phase-breaking length is of order 100 ps corresponding to a TOTAL coherent path length in EXCESS of 50 μm



- Measurements of the carrier density of the underlying two-dimensional electron gas indicate a value for the INTERACTION PARAMETER r_s that is close to UNITY
- Electron transport within the two-dimensional layer is therefore expected to exhibit WEAKLY-INTERACTING behavior

$$E_{ce} = \frac{e^2}{4\pi\epsilon\tilde{r}} = \frac{e^2}{4\pi\epsilon} \sqrt{\frac{m_s}{m}}$$

$$E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{m_s \hbar^2}{2m}$$

$$r_s = \frac{E_{ce}}{E_F} = \frac{m \cdot e^2}{4\pi\epsilon\hbar^2} \sqrt{\frac{m_s}{m}}$$

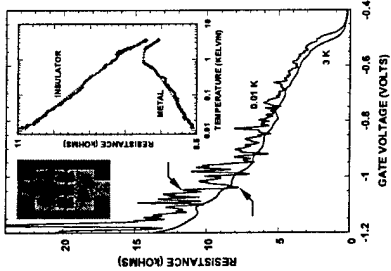
$$r_s(GaAs) = \frac{0.067 \times 9.1 \times 10^{-31} \times 2.6 \times 10^{-18}}{12.6 \times (12.4 \times 8.854 \times 10^{-12}) \times 1.1 \times 10^{-68}} \frac{1}{\sqrt{3.1 \times 5 \times 10^{15}}} \approx 0.9$$

OUTLINE

- INTRODUCTION
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- EXPERIMENTAL RESULTS
- CONCLUSIONS

- The details of quantum transport in the arrays are investigated by studying the TEMPERATURE dependence of their resistance

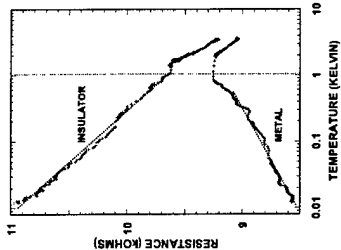
* These studies reveal evidence for behavior reminiscent of a METAL-INSULATOR transition



A. Andressen *et al.*
Phys. Rev. B **60**, 16050 (1999)
A. Shalios *et al.*
Phys. Rev. B **63**, 241302 (2001)

- The main characteristics of the metal-insulator behavior are an EXPONENTIAL increase of the resistance at intermediate temperatures and for ALL gate voltages

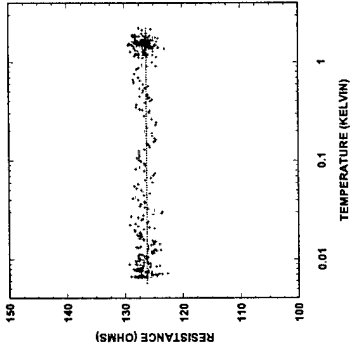
* This gives way to a LOGARITHMIC variation of the resistance at lower temperatures that may be either METALLIC or INSULATING in nature



A. Andressen *et al.*
Phys. Rev. B **60**, 16050 (1999)
A. Shalios *et al.*
Phys. Rev. B **63**, 241302 (2001)

- NO evidence for the temperature-dependent variations found in the arrays is seen in studies performed with the gates GROUNDED

* This indicates that these variations are INTRINSIC to the dot ARRAYS rather than being a property of the two-dimensional electron-gas layer

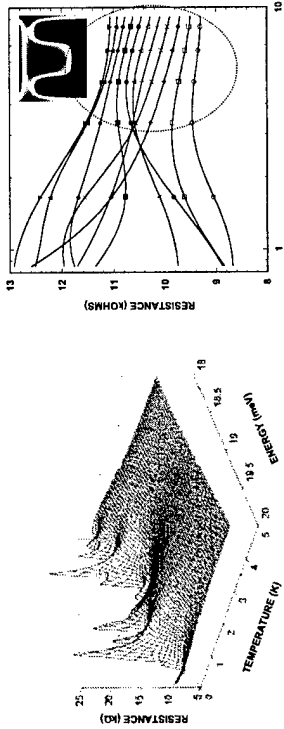


A. Andressen *et al.*
Phys. Rev. B **60**, 16050 (1999)
A. Shalios *et al.*
Phys. Rev. B **60**, E16239 (1999)
A. Shalios *et al.*
Phys. Rev. B **63**, 241302 (2001)

- An Important clue as to the origin of the exponential variation is provided by numerical studies where the conductance is CONVOLVED with the derivative of the Fermi function

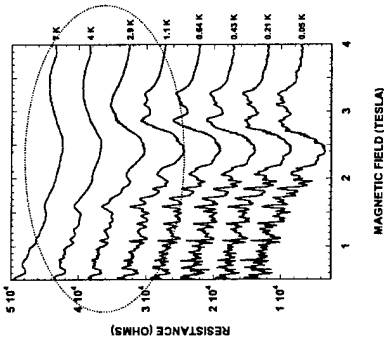
* This reveals a regime of increasing resistance with decreasing temperature PRIOR to the emergence of conductance oscillations

$$G(T, E_F) = \int G(E) \left[-\frac{df(T, E - E_F)}{dE} \right] dE$$



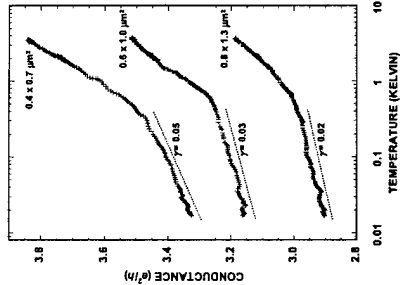
A. Shalios *et al.*
submitted for publication

- We suggest the exponential regime indicates a transition from THERMALLY-BROADENED to ENERGETICALLY-DISCRETE levels in the dots with decreasing temperature
- This assertion is further supported by studies of the temperature dependence of the MAGNETO-RESISTANCE which show a large change in amplitude in the exponential regime



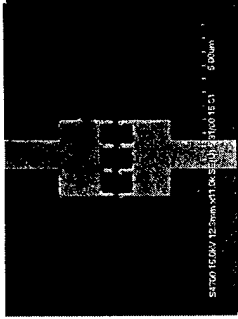
A. Shaloe et al.
submitted for publication

- The amplitude of the logarithmic term is found to be LARGER in the arrays composed of smaller dots
- We quantify this effect in terms of the SLOPE of the logarithmic variation (γ) in this regime



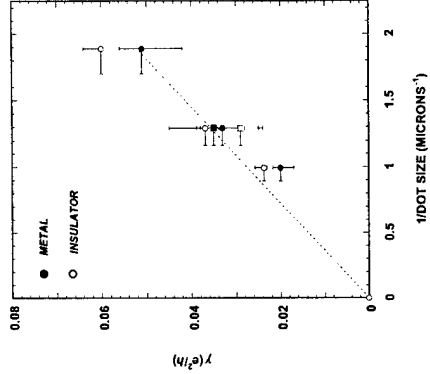
A. Shaloe et al.
Phys. Rev. B **63**, 241302 (2001)

- While our simulations show features reminiscent of the exponential regime found in experiment they are UNABLE to reproduce the LOGARITHMIC behavior seen at much lower temperatures
- To further investigate the properties of this term we have fabricated quantum dot arrays of DIFFERENT size



ARRAY	DOT SIZE (μm^2)
A	0.8 x 1.3
B	0.6 x 1.0
C	0.4 x 0.7

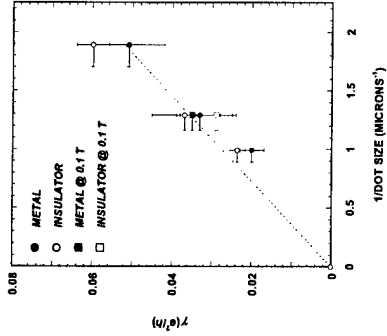
- Averaging the results of measurements performed at a NUMBER of different gate voltages we find that the logarithmic terms scales INVERSELY with dot size



A. Shaloe et al.
Phys. Rev. B **63**, 241302 (2001)

- We also find that the magnitude of the logarithmic term is **INSENSITIVE** to the application of a magnetic field sufficient to **BREAK** time-reversal symmetry

- This indicates that this term is **NOT** associated with a weak-localization effect in the dots

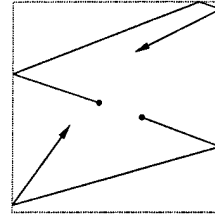
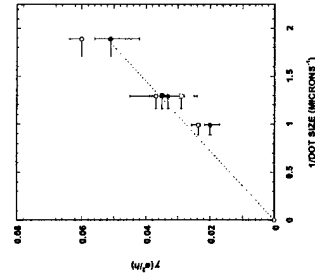


A. Shalosh *et al.*
Phys. Rev. B **63**, 241302 (2001)

- Based on experiment we have suggested that the logarithmic term in the conductance results from an **ENHANCEMENT** of the electron-electron interaction inside the dots

- The basic idea is that confining electrons within the ballistic dots should **AMPLIFY** the Coulomb many-body interaction by **SUPPRESSING** the tendency for charge separation

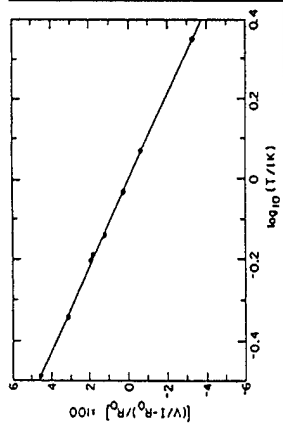
- In fact the **SCALING** of the logarithmic term with **INVERSE** dot size is consistent with an interaction that scales in inverse proportion to the **PERIMETER LENGTH**



A. Shalosh *et al.*
Phys. Rev. B **63**, 241302 (2001)

- Logarithmic conductance variations are well known from studies of **DISORDERED** mesoscopic systems in which they can result from **WEAK LOCALIZATION** or **ELECTRON INTERACTIONS**

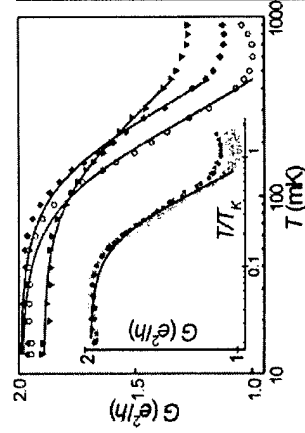
- In order to distinguish between these two effects it is necessary to apply a sufficient **MAGNETIC FIELD** to break time-reversal symmetry and so quench weak localization



G. J. Dolan and D. C. Osheroff
Phys. Rev. Lett. **53**, 721 (1979)

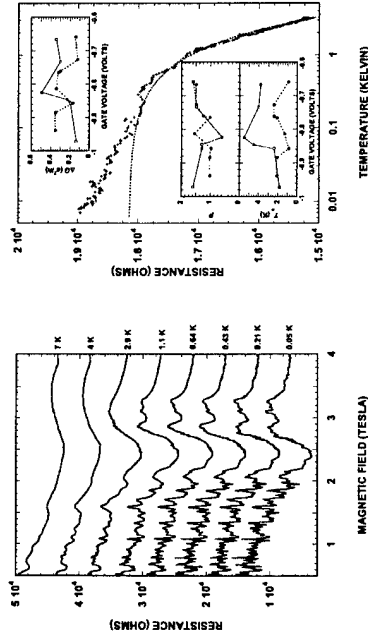
- In the absence of any formal theory we consider it **INTRIGUING** that a logarithmic variation of the conductance has also been found in studies of the **KONDO EFFECT** in tunnel-coupled dots

- While there are important **DIFFERENCES** with the behavior we observe in open dots this comparison appears to **SUPPORT** our assertion that the logarithmic term we observe does result from **MANY-BODY** effects



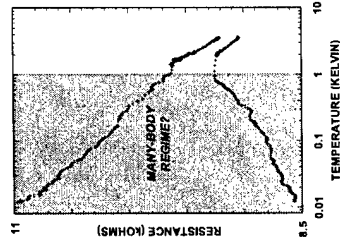
W. G. van der Wiel
Science **289**, 2105 (2000)

- We therefore arrive at the following MODEL to account for the different temperature variations
- At temperatures in EXCESS of a few Kelvin the discrete dot states are thermally OBSCURED
- Lowering the temperature in this regime allows the dot states to become RESOLVED and it is this transition that is reflected in the EXPONENTIAL resistance



A. Shaloe et al.
submitted for publication

- As the temperature is lowered below a KELVIN we enter the regime of strongly-resolved dot states and it is here that the LOGARITHMIC variation of resistance is observed
- An INTRIGUING issue concerns whether this variation signals the emergence of some NOVEL many-body state once the quantum nature of these structures is well resolved?



CONCLUSIONS

- WE HAVE STUDIED THE TEMPERATURE-DEPENDENT TRANSPORT IN COUPLED QUANTUM-DOT ARRAYS AND FIND VERY DIFFERENT BEHAVIOR COMPARED TO THAT OF THE UNDERLYING TWO-DIMENSIONAL ELECTRON GAS
- THE REGIME OF EXPONENTIAL RESISTANCE VARIATION IS SUGGESTED TO SIGNIFY THE EMERGENCE FROM THERMALLY-BROADENED TO ENERGETICALLY-RESOLVED DOT LEVELS WITH DECREASING TEMPERATURE
- THE LOGARITHMIC VARIATION SEEN AT EVEN LOWER TEMPERATURES CANNOT BE ACCOUNTED FOR BY EXISTING THEORIES AND MAY SIGNIFY THE EMERGENCE OF A NOVEL MANY-BODY STATE AT LOW TEMPERATURES THAT RESULTS FROM AN ENHANCEMENT OF ELECTRON INTERACTIONS IN THE CONFINED DOTS
- IT IS OUR HOPE THAT THIS WORK WILL STIMULATE RENEWED THEORETICAL INTEREST IN THE STUDY OF ELECTRON INTERACTIONS IN OPEN MESOSCOPIC SYSTEMS

Wednesday, June 20 Session 5:

1. David Awschalom, "Spin coherence and optical measurements"
2. Daniel Loss, "Quantum Computing in semiconductor systems"
3. Sankar Das Sarma, "Few electron (and few impurity) systems"

Also related posters Thursday: 3-1 Baumeid et al.
3-2 Puller et al.
3-3 Privman
3-9 Wagner
3-10 Filinov et al.

Workshop "Quantum Transport in Semiconductors"
Maratea, June 17-22 2001

C. Michael Bonitz

Introduction to Spins and few particle problems in semiconductors

Michael Bonitz, Universität Rostock

Workshop "Quantum Transport in Semiconductors"
Maratea, June 17-22 2001

Spintronics (alternative medicine?)

Idea: control devices by controlling the electron spin (instead of the charge)

Spin already in use: MRAMs, read heads of hard drives etc.

But: metal based. Challenge: semiconductors (integrability)

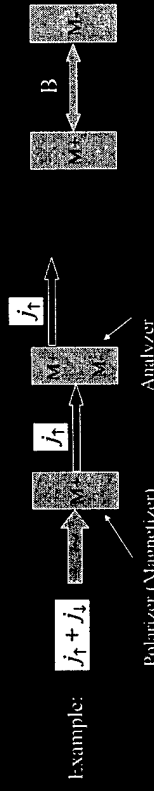
I. Improve classical devices (computers)

Many-electron problem

Replace current $j \rightarrow j_\uparrow$

voltage $U = \phi_+ - \phi_- \rightarrow N_\uparrow - N_\downarrow$

Gate field $E \rightarrow B$



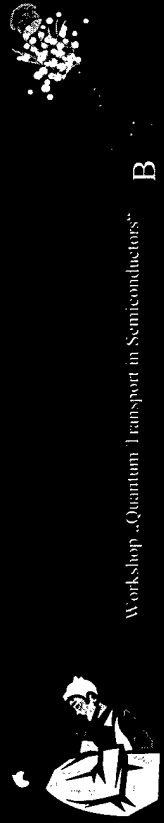
Problems: creation/injection/transport/storage..... (possible side effects?)

Welcome to the spin doctors!

I. What disease?

II. Who is sick?

III. What is the cure?



Workshop "Quantum Transport in Semiconductors"
Maratea, June 17-22 2001

B

Spintronics II.

11. Create new quantum computers (few-body problem)

1. Lack of classical theories for spin

Use quantum states (qubits) $|\uparrow\rangle, |\downarrow\rangle, \dots$
And their linear combinations $C = a|\uparrow\rangle + b|\downarrow\rangle + \dots$

2. Build a linear computer and perform 1 vector operation (C) on scalar (classical) operations at once

3. Build qubits from



4. Mix qubits by designing

→ True quantum-mechanical approach (use amplitude)

→ Optical analogue: holography

Semiconductor electrons in E/B fields

E-Field (couples to charge)

$$e: e\vec{E} \rightarrow \vec{v} \rightarrow \vec{j} \text{ current}$$

→ Transport (intra-band)

B-Field (couples to mag. moment)

$$\vec{s} \rightarrow \vec{m}: \vec{m}\vec{B} \rightarrow \omega_L = g\mu_B B / \hbar$$

→ Larmor precession

2. Interband transport (optics)

$$e \rightarrow d_{cv} \sim e \int dV \psi_c^*(r) r \psi_v(r)$$

Interband current (polarization) P

$$\left[\frac{\partial}{\partial t} + i \frac{\tilde{\epsilon}_v(k)}{\hbar} \right] P_k = -i \tilde{d}_{cv} E(t) [f_v(t) - f_c(t)]$$

- Coherent oscillations of P, Rabi flops, photon echo etc., coherent excitons, biexcitons

- Dephasing due to scattering $\sim -\frac{P}{T_2}$

$$\tilde{\epsilon}_{\pm k}$$

1D-renormalized gap

Zeeman splitting

- Coherent oscillations of M, "Larmor flops" spin echos?

- Very long dephasing times

- Bound states (cooper pairs)??

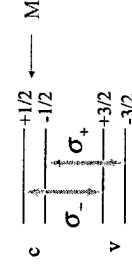


M-magnetization

$$\left[\frac{\partial}{\partial t} + i \frac{\tilde{\epsilon}_{\pm}(k)}{\hbar} \right] M_{\pm} = i g \tilde{\mu}_B B(t) [f_{\pm}(t) - f_{\mp}(t)]$$

Electrons in E/B-fields, contd.

3. Spin meets optics



- spin-sensitive absorption/emission of circular-polarized light (old)

- spin-modulated polarization of laser emission [Hallstein et al. PRB 56, R7076 (1997)]

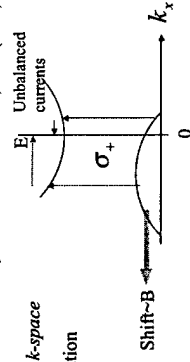
$$\left[\frac{\partial}{\partial t} + i \frac{\tilde{\epsilon}_{\pm}(k)}{\hbar} \right] M_{\pm} = i g \tilde{\mu}_B B(t) [f_{\pm}(t) - f_{\mp}(t)]$$

4. Spin meets intraband transport

- "Conversion of spin into directed electric current in Quantum Wells", Ganichev et al. PRL 86, 4356 (2001)

- B-field induced separation of +3/2 and -3/2 hh bands in k-space

- current direction determined by helicity of light polarization



Outlook

Number of quantum computing papers (title and abstract):
from 1994-2001 (June 14): 1, 6, 10, 15, 38, 59, 81, 122

→ On track with Moore's law

Number of spintronics papers:
from 1999-2001 (June 14 11:32 a.m. ET): 2, 8, 10

→ Way ahead of Moore's law



True reason for Intel's rush ahead of the road map

Workshop "Quantum Transport in Semiconductors"

Maratea, June 17-22 2001

Listen to (and interrupt!)
the spin doctors!



Workshop „Quantum Transport in Semiconductors“

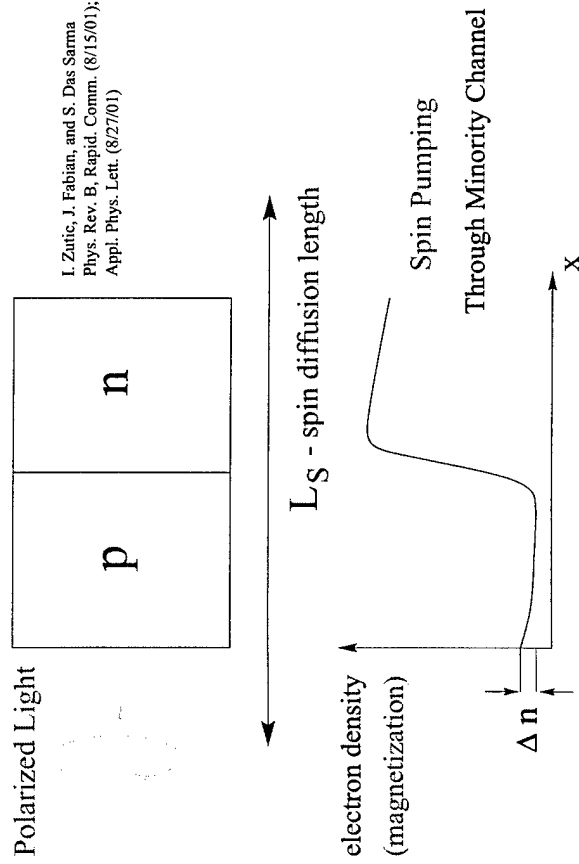
Maratea, June 17-22 2001

SPIN ELECTRONICS

Group of Sankar Das Sarma, University of Maryland
Igor Žutić, Jaroslav Fabian, Xuedong Hu, et al.

- theory of bipolar spin-polarized transport in inhomogeneous semiconductors (magnetic and nonmagnetic) and their heterostructures
- spin devices: proposal and modeling of spin-polarized and magnetic p - n junctions and solar cells, magnetic field sensors, spin transistors, ...
- dynamical mean field theory for critical temperature in ferromagnetic semiconductors (e.g., GaMnAs)
- spin entanglement and interactions in quantum dots: assessment of feasibility for spin-based quantum computing

Spin - Polarized p - n Junction



Group Publications (partial list):

- Theory of spin-polarized transport in inhomogeneous magnetic semiconductors, LANL preprint cond-mat/0106085
- Spin Electronics and Spin Computation, to appear in Solid State Commun., LANL preprint cond-mat/0105247
- A proposal for spin-polarized solar battery, to appear in Appl. Phys. Lett. (8/27/01); LANL preprint cond-mat/0104416
- Spin transport in inhomogeneous magnetic fields: a proposal for Stern-Gerlach-like experiments with conduction electrons, LANL preprint cond-mat/0104146
- Spin injection through the depletion layer: a theory of spin-polarized p-n junctions and solar cells, to appear in Phys. Rev. B, Rapid Commun. (8/15/01); LANL preprint cond-mat/0103473
- Issues, Concepts, and Challenges in Spintronics, IEEE 58th DRC Device Research Conference Digest, p. 95 (2000)
- Theoretical Perspectives on Spintronics and Spin-Polarized Transport, IEEE Trans. Magn. 36, 2821 (2000)
- Issues Concepts, and Challenges in Spintronics, Superlattice Microsc. 27, 289 (2000)
- Spin-polarized transport and Andreev reflection in semiconductor/superconductor hybrid structures, Phys. Rev. B 60 Rapid Commun., R16 322 (1999).

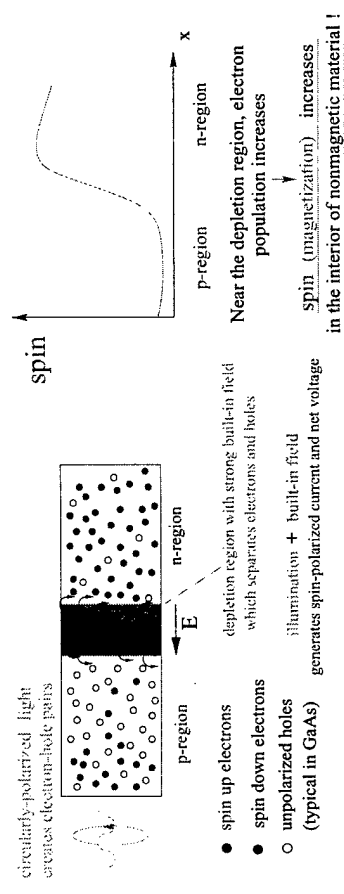
publications also available at :
<http://www.physics.umd.edu/rgroups/spin/papers.html>

Spin-Polarized and Magnetic p - n Junctions and Solar Batteries

spin-polarized p - n junction -- building block for semiconductor spin electronics , feasible with the current technology and available materials (e.g., GaAs with the appropriate doping)

spin polarization can be created by circularly polarized light, electrical spin injection, applied magnetic field. ...

Example: Spin-Polarized Solar Battery



Spin - Polarized Drift - Diffusion Equations

$$\nabla^2 \phi = - \frac{e}{\epsilon} (p - n + N_A - N_D)$$

and

4 Continuity equations for n_t , n_v , p_t and p_v

Generation - Recombination (τ_n, τ_p)

Spin Relaxation (T_1)

Results based on realistic GaAs parameters

$$p_t = p_v$$

$(n_t - n_v) / n \leftarrow$ max polarization 50%
(with circularly-polarized light)

$$T_1 \sim 0.2 \text{ ns}$$

Continuity Equation for n_t

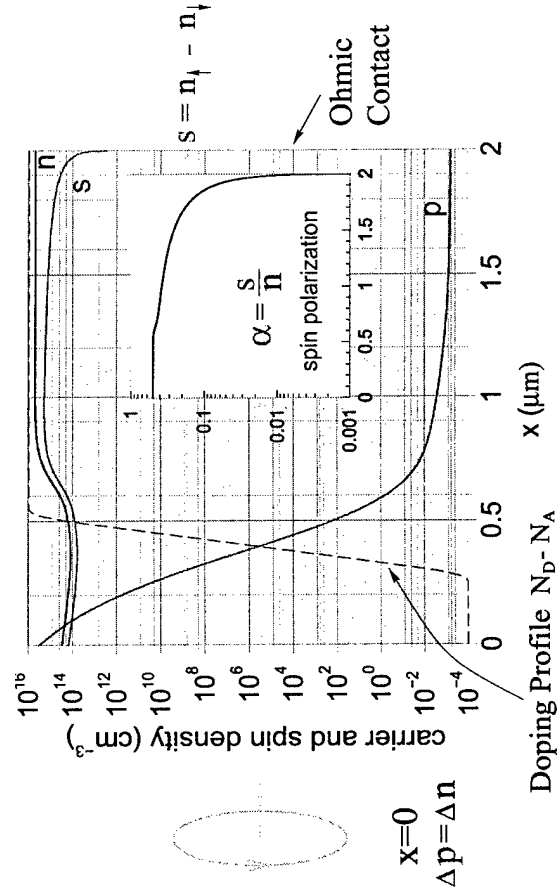
$$J_{n_t} = - n_t \mu_{n_t} E - D_{n_t} \nabla n_t \quad \text{particle current (for nonmagnetic } p - n \text{ junction)}$$

$$\frac{dn_t}{dt} + \nabla \cdot J_{n_t} = - w_{n_t} (n_t p - \frac{n_0 p_0}{2}) - \frac{n_t - n_v}{2T_1} + G_t$$

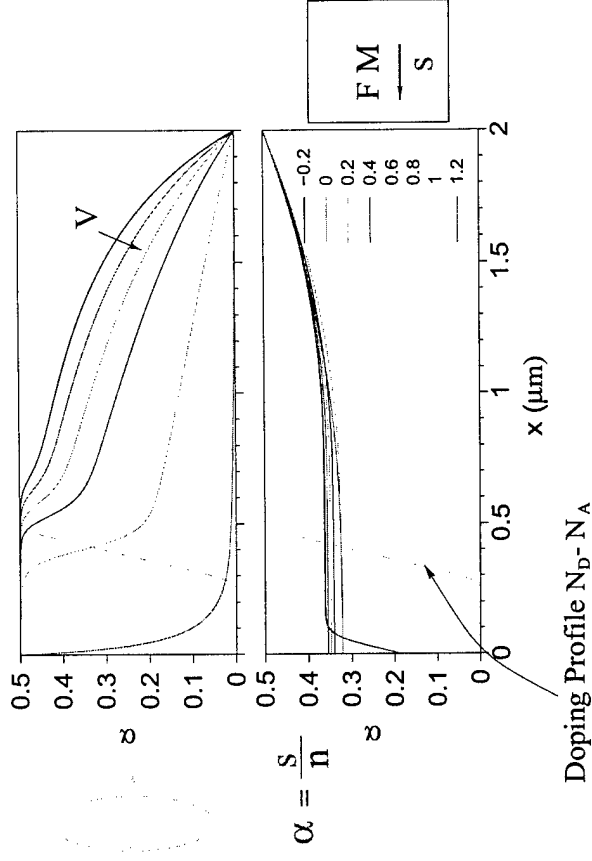
w_{n_t} - Generation - Recombination rate

G_t - Photo-excitation rate for n_t

Spin Injection through the $p - n$ junction

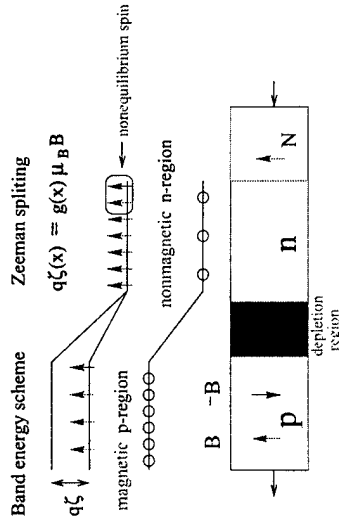


Spin Polarization Profiles for different voltages



Magnetic / Nonmagnetic p - n Junctions

I. Zutic, J. Fabian, and S. Das Sarma, LANL preprint cond-mat/0106085



p-region: $n_0 \approx (n_1^2 / N_A) \cosh(q\zeta / kT)$

at low bias: current ~ minority carriers -> exponentially large magnetoresistance with the increase in |B| (i.e. | ζ |)

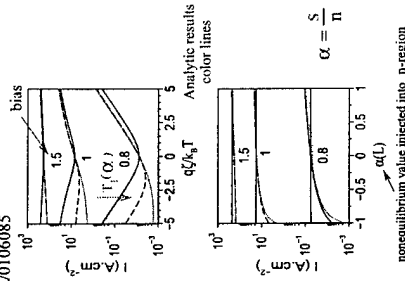
$J = J_n + J_p + J_n^* \rightarrow$ nonequilibrium spin-induced current

$J_{n,p} \sim (e^{(qV/kT)} - 1)$

$J_n^* \propto \alpha \sinh(q\zeta / kT) e^{(qV/kT)}$

next to depletion region

zero bias current changes sign with the change of α and / or B



I (V,B) characteristics electrical probe for B, α , T_1

current can flow at zero bias! spinvoltaic effect (spin analogue of photovoltaic effect)

Applications of Spin-Polarized and Magnetic p - n Junctions and Solar Batteries

- Generation of spin and charge currents
- Spin injection through the depletion layer
- Electronically tunable spin polarization
- Amplification of spin density
- Extension of the spin-diffusion range
- Exponential magnetoresistance and spinvoltaic effect
- Magnetic field sensors
- Electrical detection of spin polarization and spin relaxation

Applications of quantum transport in devices

Gerhard Klimeck

Jet Propulsion Laboratory,
California Institute of Technology

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http://hpc.jpl.nasa.gov/PEP/gekco

Gerhard Klimeck

Application of Quantum Transport in Devices

• What is the focus of the research?

• Quantum Transport

⇒ Devices/Structures are a tool to explore the needed theory

- Relevant Theories:
Green Functions, Wigner Functions, Rate Equations
- Relevant Structures:
quantum dots/wires, molecules, RTDs (for time dependence only)

• Devices / Applications

⇒ Quantum transport is a tool to design/optimize devices

- Relevant devices: super-scaled FETs, RTDs, Esaki diodes
- Need quantitative agreement between experiment and theory
 - DC, high bias performance
 - AC / time-dependent high bias performance
- Need realistically sized devices - contacts/reservoirs.
- Need realistic electron interactions with environment: phonons, light, bandstructure.

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Quantitative Modeling of Devices

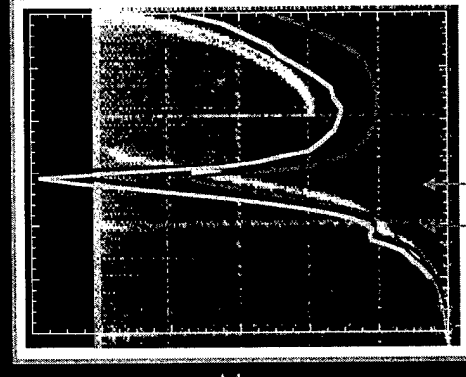
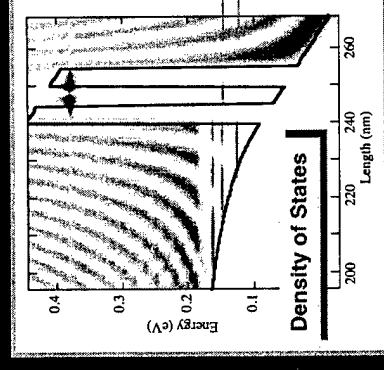
Quick review of DC transport simulations in RTDs - NEMO 1-D

- Realistic contacts:
 - Quantized and continuous states in the emitter
- Realistic bandstructure:
 - Band-non parabolicity - emitter states and RTD state alignment
- Putting it together:
 - Valley current at high temperatures due to bandstructure effects (thermionic emission)
 - Bistability (in symmetric structures) a numerical problem due to limited device models
 - Test matrix - comparison to experiment

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Work on this slide performed by NEMO team at Texas Instruments / Raytheon 1995-1997

Realistic Devices have Extended Contacts



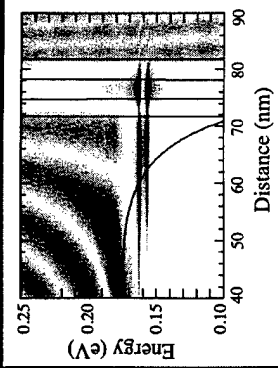
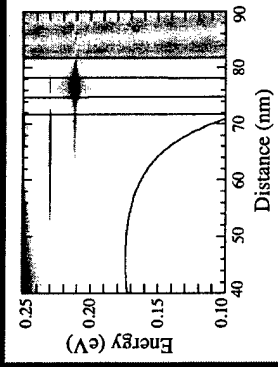
Potential Current
— 1 Band 1 Band
--- 2 Bands 2 Bands
--- 1 Band 1 Band

Quantum self-consistent potential

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Work on this slide performed by NEMO team at Texas Instruments / Raytheon 1995-1997

Band non-parabolicity modifies momentum dependence in emitter-RTD coupling

Density of States ($k_x=0.00$)Density of States ($k_x=0.03$)

Resonance coupling depends on the transverse momentum

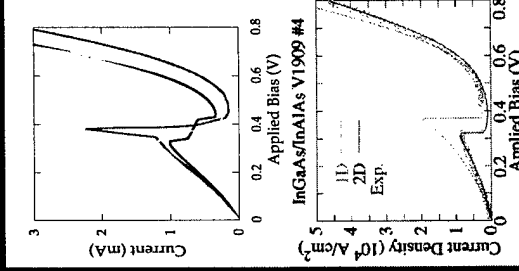
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Full Band Simulation of Electron Transport

- 1D integration assuming parabolic subbands can lead to unphysical current overshoots.
- 2 Examples on InGaAs/InAlAs simulations:
 - Sp3s* simulation with partial charge self-consistency -> sharp spike at turn-off
 - Parameterized single band simulation which incorporates the band-non-parabolicity -> overall current overshoot.

-> 2D integration with good bandstructure fixes these unphysical results.

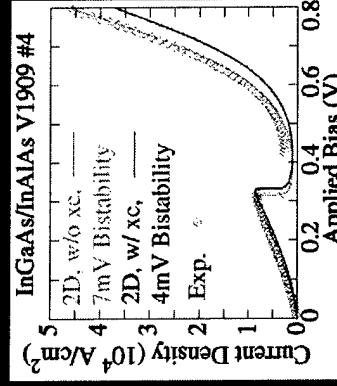
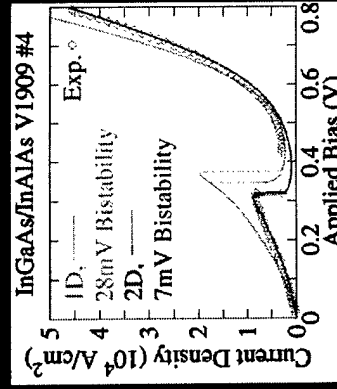


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Spurious Bistability: More Physics -> Better results

Full band integration + Exchange & Correlation



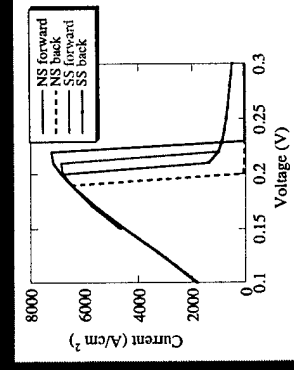
- Calculate the exchange and correlation potential in LDA.
- Exchange and correlation energy does not eliminate (in general) the bistability, it does reduce it however.
- Inclusion of scattering in the simulation reduces the bistability region as well.

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Work on this slide performed by NEMO team at Texas Instruments / Raytheon 1995-1997

Scattering also reduces the numerical bi-stability

- **Current Model:**
 - self-consistent elastic and single triagonal POP scattering
- **Potential Models:**
 - Hartree self-consistency
 - no scattering
 - self-consistent elastic and triagonal POP scattering
- **Compare forward to reverse bias sweep:**
 - Scattering reduces the width of the bistability region.
 - not shown: inclusion of exchange correlation does not change the width of the bistability in this device.



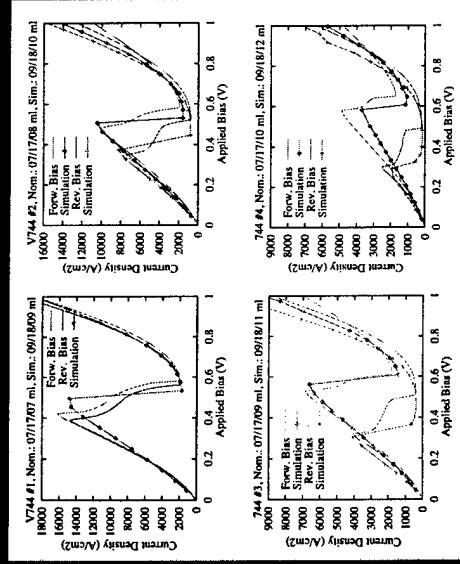
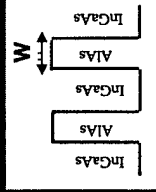
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Testmatrix-Based Verification (room temperature)

Strained InGaAs/AlAs 4 Stack RTD with Asymmetric Barrier Variation

Vary One Barrier Thickness



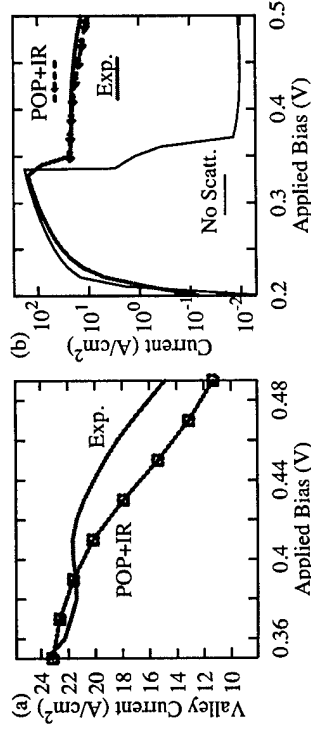
Four increasingly asymmetric devices:
 20/50/20 Angstrom
 20/50/23 Angstrom
 20/50/25 Angstrom
 20/50/27 Angstrom

Presented at IEEE DRC 1997

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Work on this slide performed by NEMO team at Texas Instruments / Raytheon 1995-1997

Tow Temperature: Polar Optical Phonon and Interface Roughness Scattering



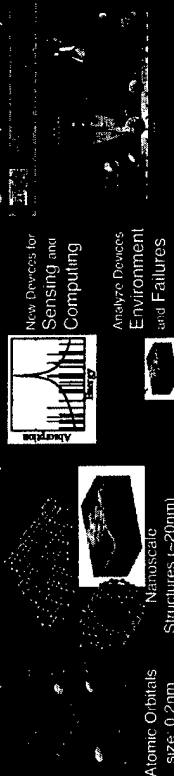
scattering arises valley current by several orders of magnitude

Gerhard Klimeck

Work on this slide performed by NEMO team at Texas Instruments / Raytheon 1995-1997

Nano-scale Device Analysis / Synthesis

Development of a Bottom-Up Nanoelectronic Modeling Tool (NEMO-3D)



Assertions / Problems:

- Nanoscale structures are built today!
- The design space is huge: choice of materials, compositions, doping, size, shape
- Radiation on today's sub-micron devices modifies the electronics on a nanoscale.

Approach:

- Deliver a 3-D atomistic simulation tool
- Enable analysis of arbitrary crystal structures, particles, atom compositions and bond/structure at arbitrary temperatures and ambient electric and magnetic fields.

Collaborators:

- U. of Alabama, Ames, Purdue, Ohio State, NIST

NASA Relevance:

- Enable new devices needed for NASA missions beyond existing industry roadmap:
 - Water detection -> 2-3µm Lasers and detectors.
 - Avionics -> High density, low power computing.
- Analyze state-of-the-art devices for non-commercial environments:
 - Europe -> Radiation and low temperature effects. Aging and failure modes.
 - Jovian system -> Magnetic field effects
 - Venus -> high temperature materials: SiGe

Impact:

- Low cost development of revolutionary techn.

Gerhard Klimeck

Modeling will narrow the empirical search space!

Speakers in the Program

- Carlo Jacoboni,
Modena University,
"The Wigner function and quantum transport"
- Harold Grubin,
SRA, Inc.,
"Modeling resonant tunneling diodes with Wigner functions and density matrices"
- Dejan Jovanovic,
Motorola,
"Non-equilibrium Green's functions for MOSFET modeling"

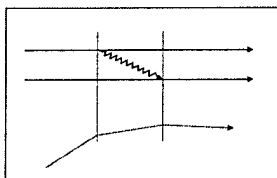
Gerhard Klimeck

**WIGNER-FUNCTION AND QUANTUM TRANSPORT
 IN SEMICONDUCTORS**

Carlo Jacoboni

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 Andrea Bertoni
 Stefano Monstra

Antonio Abramo
 Marco Pascoli
 Fausto Rossi

- Spectral density

Contents

Elementary definition and
 properties

Dynamical equations

- Classical force
- Infinite potential barrier

Electron - phonon
 interaction

Wigner paths and MC
 simulation

- Paths and diagrams
- Multiplicity of Wigner paths
- Quantum self scattering

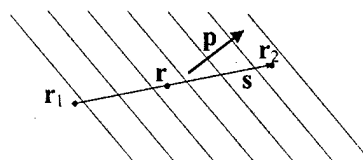
Two-time Wigner function
 $f_w(p, \omega)$

THE WIGNER FUNCTION

The Wigner-function approach is a phase-space formulation of quantum mechanics that allows to establish many analogies with the semiclassical theory based on the concept of distribution function

Elementary definition:

$$\rho(r_1, r_2)$$



$$f_w(r, p, t) = \int ds e^{-ips/\hbar} \Psi(r+s/2, t) \Psi^*(r-s/2, t)$$

Properties of the Wigner function

$$\frac{1}{h^3} \int f_w(r, p, t) dp = |\Psi(r, t)|^2$$

$$\frac{1}{h^3} \int f_w(r, p, t) dr = |\Phi(p, t)|^2$$

$$\langle A \rangle = \frac{1}{h^3} \int dr dp f_w(r, p, t) A_w(r, p)$$

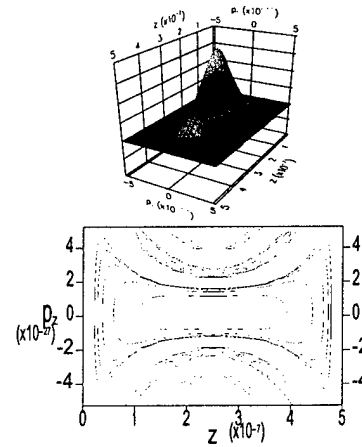
$$A_w \hat{A} p \hat{I} = \hat{A} s e^{-ips/\hbar} \langle r+s/2 | \hat{A} | r-s/2 \rangle$$

Linear in f

NOT positive definite
Strong oscillations
 $\neq 0$ also where $\Psi = 0$

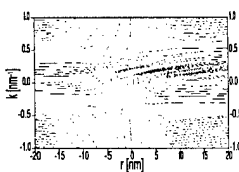
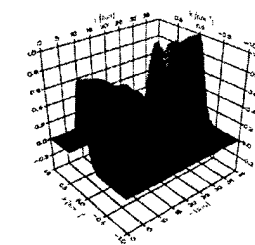
WIGNER FUNCTION

Ground state in a box:

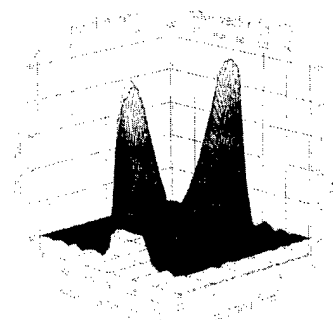


WIGNER FUNCTION

Potential step with scattering states:



WIGNER FUNCTION FOR A DOUBLE BARRIER



COHERENT PROPAGATION OF THE W.F.

If

$$\mathbf{H}|\varphi_n\rangle = \hbar\omega_n|\varphi_n\rangle$$

then

$$f_w(\mathbf{r}, \mathbf{p}, t) = \sum_{n,m} f_{n,m}(\mathbf{r}, \mathbf{p}) e^{-i(\omega_n - \omega_m)(t-t_0)} \frac{1}{h^3} \int d\mathbf{r}' \int d\mathbf{p}' f_{n,m}^*(\mathbf{r}', \mathbf{p}') f_w(\mathbf{r}', \mathbf{p}', t_0)$$

Linear

where

$$f_{n,m}(\mathbf{r}, \mathbf{p}) = \int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2 | \varphi_n \rangle \langle \varphi_m | \mathbf{r} - \mathbf{s}/2 \rangle$$

DYNAMICAL EQUATION OF THE W.F. (1)

From the Liouville – Von Neumann

$$i\hbar \frac{d}{dt} \rho = [\mathbf{H}, \rho]$$

with

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{V}(\mathbf{r})$$

$$\mathbf{H}_0 = -\frac{\hbar^2}{2m} \nabla^2$$

Obtain

$$\frac{\partial}{\partial t} f_w(\mathbf{r}, \mathbf{p}, t) = \int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \times \langle \mathbf{r} + \mathbf{s}/2 | [\mathbf{H}_0 + \mathbf{V}(\mathbf{r}), \rho] | \mathbf{r} - \mathbf{s}/2 \rangle$$

With standard elaboration:

$$\int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2 | \frac{1}{i\hbar} [\mathbf{H}_0, \rho] | \mathbf{r} - \mathbf{s}/2 \rangle = -\frac{\mathbf{p}}{m} \nabla f_w$$

and

$$\begin{aligned} \int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2 | \frac{1}{i\hbar} [\mathbf{V}(\mathbf{r}), \rho] | \mathbf{r} - \mathbf{s}/2 \rangle \\ = \int d\mathbf{p}' V_w(\mathbf{r}, \mathbf{p} - \mathbf{p}') f_w(\mathbf{r}, \mathbf{p}', t) \end{aligned}$$

where

$$V_w(\mathbf{r}, \mathbf{p}) = \frac{1}{i\hbar} \frac{1}{h^3} \int d\mathbf{s} e^{i\mathbf{p}\mathbf{s}/\hbar} [V(\mathbf{r} - \mathbf{s}/2) - V(\mathbf{r} + \mathbf{s}/2)]$$

DYNAMICAL EQUATION OF THE W.F. (2)

Collecting the above:

$$\frac{\partial}{\partial t} f_w(\mathbf{r}, \mathbf{p}, t) + \frac{\mathbf{p}}{m} \frac{\partial}{\partial \mathbf{r}} f_w = \frac{\partial f_w}{\partial t} \Big|_{\text{cl}}$$

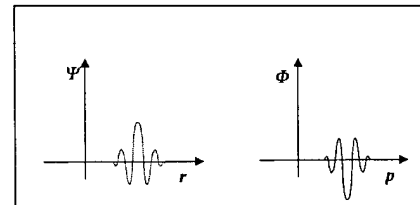
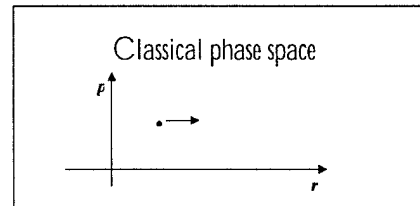
For "regular" potential (up to quadratic):

$$\frac{\partial}{\partial t} f_w(\mathbf{r}, \mathbf{p}, t) + \frac{\mathbf{p}}{m} \frac{\partial}{\partial \mathbf{r}} f_w + \mathbf{F} \frac{\partial}{\partial \mathbf{p}} f_w = 0$$

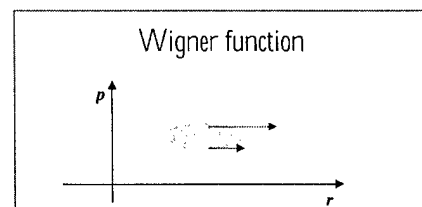
as in Liouville theorem.

WIGNER PATHS:

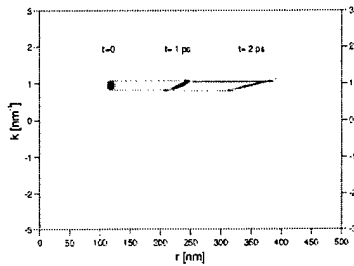
The Wigner function of electrons in presence of potentials up to quadratic evolves as an ensemble of classical particles: each point follows a classical path in the Wigner phase space (Wigner path)



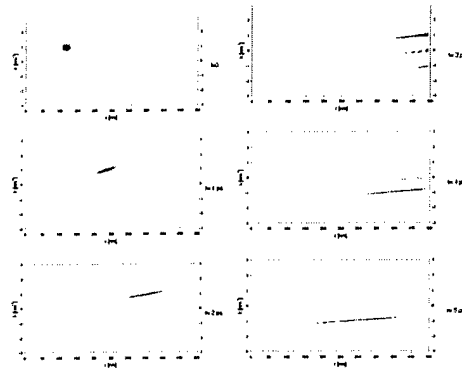
Quantum mechanics



Time evolution of the Wigner Function of a wave packet



Time evolution of the Wigner Function reflected by an infinite potential barrier



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WIGNER PATHS WITH POTENTIAL SCATTERING

$$V = V_0 + V' \quad F = -\nabla V_0$$

V_0 linear or quadratic - Integro-differential equation

$$\frac{\partial}{\partial t} f_w(r, p, t) + \frac{p}{m} \frac{\partial}{\partial r} f_w + F \frac{\partial}{\partial p} f_w = \int dp' V_w(r, p - p') f_w(r, p', t)$$

where

$$V_w(r, p) = \frac{1}{h^3} \int ds e^{-ips/\hbar} [V'(r + s/2) - V'(r - s/2)]$$

path variables:

$$r(\tilde{t}) = r - \int_{t_0}^{\tilde{t}} p(t') / m \quad p(\tilde{t}) = p - \int_{t_0}^{\tilde{t}} F(r(t')) dt'$$

time integration:

$$f_w(r, p, t)$$

$$= f_w(r(t_0), p(t_0), t_0) + \int_{t_0}^t \int dp' V_w(r(t'), p - p(t')) f_w(r(t'), p', t')$$

as in Chambers kinetic integral equation, without scattering "out", with similar interpretation:

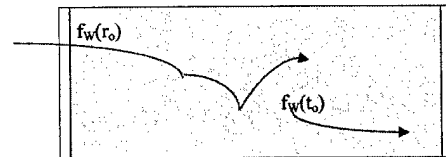
$$f(t_0) \rightarrow f(t) + \int_{t_0}^t f(t') \rightarrow f(t)$$

Neumann expansion

$$\text{Wigner "paths":}$$

$$f(t_0) \rightarrow f(t) + \int_{t_0}^t f(t') \rightarrow f(t) + \int_{t_0}^t \int_{t'}^t f(t'') \rightarrow f(t) + \dots$$

Boundary conditions



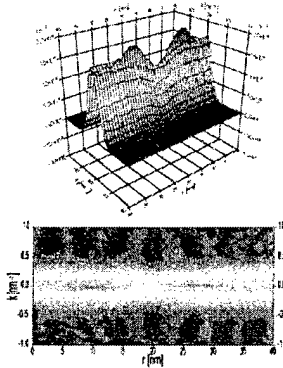
Phys.Rev.B 58, 3503 (1998)

Phys.Rev.B 59, 3060 (1999)

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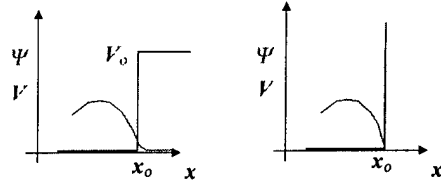
Single barrier (4nm x .05eV)



Obtained by means of W . Paths with potential scattering in a Monte Carlo approach

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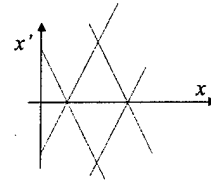
Wigner Function confined by an infinite potential barrier



For the infinite barrier the Schroed. eq. holds only inside.
For a box (a,b)

$$f_w(x, p, t) = \int_{-\xi(x)}^{\xi(x)} ds e^{-ips/\hbar} \Psi(x+s/2, t) \Psi^*(x-s/2, t)$$

where



$$\xi(x) = V(r+s/2) \begin{cases} 2(x-a) & \text{if } x < (a+b)/2 \\ 2(b-x) & \text{if } x \geq (a+b)/2 \end{cases}$$

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Wigner Function confined by an infinite potential barrier(ii)

In the derivation of the dynamical equation, the derivatives at the boundaries do not vanish, and we have an extra integral term

$$\frac{\partial f_w}{\partial t} + \frac{p}{m} \frac{\partial f_w}{\partial x} = \frac{1}{\hbar} \int dp' V_w(x, p-p') f_w(x, p', t) + \frac{1}{\hbar} \int dp' B(x, p-p') \frac{\partial}{\partial x} f_w(x, p', t)$$

where

$$B(x, p) = \frac{2\hbar}{m} \sin \left[\frac{p}{\hbar} \xi(x) \right]$$

simulation ...

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SEPARATION BETWEEN CLASSICAL FORCE AND QUANTUM EFFECTS

$$\frac{\partial f_w}{\partial t} + \frac{p}{m} \frac{\partial f_w}{\partial r} = \frac{1}{\hbar^3} \int dp' V_w(r, p-p') f_w(r, p', t)$$

where

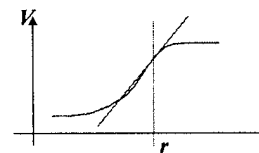
$$V_w(r, p) = \frac{1}{i\hbar} \int ds e^{-ips/\hbar} \beta(r, s) \\ \beta(r, s) = V(r+s/2) - V(r-s/2)$$

or

$$\frac{\partial f_w}{\partial t} + \frac{p}{m} \frac{\partial f_w}{\partial r} + F \frac{\partial f_w}{\partial p} = \frac{1}{\hbar^3} \int dp' \tilde{V}_w(r, p-p') f_w(r, p', t)$$

where

$$F = -\frac{\partial V}{\partial r} \\ \tilde{V}_w(r, p) = \frac{1}{i\hbar} \int ds e^{-ips/\hbar} \tilde{\beta}(r, s) \\ \tilde{\beta}(r, s) = \left[V(r+s/2) - \frac{\partial V}{\partial r} s/2 \right] - \left[V(r-s/2) + \frac{\partial V}{\partial r} s/2 \right]$$



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ELECTRON-PHONON SCATTERING WITH THE WIGNER FUNCTION

Extend the definition of Wigner function:

$$f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n'_q\}, t) = \int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2, \{n_q\} | \rho(t) | \mathbf{r} - \mathbf{s}/2, \{n'_q\} \rangle$$

Hamiltonian:

$$\mathbf{H} = \mathbf{H}_e + \mathbf{H}_{ph} + \mathbf{H}_{e-p} + \mathbf{V}_e(\mathbf{r}) + \mathbf{V}'(\mathbf{r})$$

where

$$\mathbf{H}_e = -\frac{\hbar^2}{2m} \nabla^2 \quad \mathbf{H}_{ph} = \sum_q \mathbf{b}_q^\dagger \mathbf{b}_q \hbar \omega_q$$

$$\mathbf{H}_{e-p} = \sum_q i\hbar F(q) (\mathbf{b}_q e^{i\mathbf{q}\cdot\mathbf{r}} - \mathbf{b}_q^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}}) \quad \mathbf{V}_e(\mathbf{r}) = e\mathbf{E} \cdot \mathbf{r}$$

$\mathbf{V}'(\mathbf{r}) = \text{potential profile}$

Differentiate the definition above and use

$$i\hbar \frac{d}{dt} \rho = [\mathbf{H}, \rho]$$

Obtain

$$\frac{\partial}{\partial t} f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n'_q\}, t) = \int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \times \langle \mathbf{r} + \mathbf{s}/2, \{n_q\} | \frac{1}{i\hbar} [\mathbf{H}_e + \mathbf{H}_{ph} + \mathbf{H}_{e-p} + \mathbf{V}_e(\mathbf{r}) + \mathbf{V}'(\mathbf{r}), \rho] | \mathbf{r} - \mathbf{s}/2, \{n'_q\} \rangle$$

ELECTRON-PHONON SCATTERING WITH THE WIGNER FUNCTION- II

Several terms:

$$\int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2, \{n_q\} | \mathbf{H}_e | \mathbf{r} - \mathbf{s}/2, \{n'_q\} \rangle = -\frac{\hbar}{m} \nabla \cdot \mathbf{p}$$

$$\int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2, \{n_q\} | \mathbf{H}_{ph} | \mathbf{r} - \mathbf{s}/2, \{n'_q\} \rangle = -\frac{1}{2} [\varepsilon(\{n_q\}) - \varepsilon(\{n'_q\})] f_w$$

$$\int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2, \{n_q\} | \mathbf{H}_{e-p} | \mathbf{r} - \mathbf{s}/2, \{n'_q\} \rangle = -F \frac{\partial}{\partial \mathbf{p}} f_w$$

$$\int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2, \{n_q\} | \mathbf{V}'(\mathbf{r}) | \mathbf{r} - \mathbf{s}/2, \{n'_q\} \rangle = \int d\mathbf{p}' V_w(\mathbf{r}, \mathbf{p} - \mathbf{p}') f_w(\mathbf{r}, \mathbf{p}', \{n_q\}, \{n'_q\}, t)$$

Electron-phonon term:

$$\begin{aligned} \int d\mathbf{s} e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \mathbf{r} + \mathbf{s}/2, \{n_q\} | \mathbf{H}_{e-p} | \mathbf{r} - \mathbf{s}/2, \{n'_q\} \rangle = \\ \sum_{q'} F(q') \{ e^{i\mathbf{q}'\cdot\mathbf{r}} \sqrt{n_{q'}+1} f_w(\mathbf{r}, \mathbf{p} - \frac{\hbar\mathbf{q}'}{2}, \{n_1 \dots n_{q'}+1, \dots, \{n'_q\}, t) \\ - e^{-i\mathbf{q}'\cdot\mathbf{r}} \sqrt{n_{q'}} f_w(\mathbf{r}, \mathbf{p} + \frac{\hbar\mathbf{q}'}{2}, \{n_1 \dots n_{q'}-1, \dots, \{n'_q\}, t) \\ - e^{i\mathbf{q}'\cdot\mathbf{r}} \sqrt{n'_{q'}+1} f_w(\mathbf{r}, \mathbf{p} + \frac{\hbar\mathbf{q}'}{2}, \{n_q\}, \{n'_1 \dots n'_{q'}+1, \dots, t) \\ + e^{-i\mathbf{q}'\cdot\mathbf{r}} \sqrt{n'_{q'}} f_w(\mathbf{r}, \mathbf{p} - \frac{\hbar\mathbf{q}'}{2}, \{n_q\}, \{n'_1 \dots n'_{q'}-1, \dots, t) \} \end{aligned}$$

Path variables, time integration, Neumann expansion ...

PHONON AVERAGE

Reduction to electron Wigner function

Trace over phonon variables:

$$f_w(\mathbf{r}, \mathbf{p}, t) = \sum_{\{n_q\}} f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n_q\}, t)$$

Assume separate equilibrium initial density matrix:

$$f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n_q\}, t=0) \propto \prod_q P_{eq} \mathcal{G}_q$$

For final diagonal terms, with initial diagonal W.f., each Monte Carlo term contains a phonon mode twice and has factors like

$$\Delta f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n_q\}, t) \propto n_{q_1} \dots n_{q_{i-1}} + 1 \dots f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n_q\}, t)$$

The simulation must be thought of as repeated a 'large number' of times for each $\{n_q\}$. Then, if hot-phonon effects are ignored,

$\Rightarrow$ for absent q :

$$\sum_{\{n_q\}} P_{eq} \mathcal{G}_q \hbar$$

$\Rightarrow$ for real or virtual absorptions:

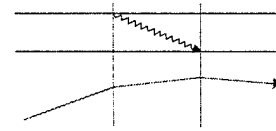
$$\sum_{n_q} \mathcal{G}_q + 1 \hbar \mathcal{G}_q + 1 \hbar \langle n_q \rangle \sum_{n_q} \mathcal{G}_q \hbar \mathcal{G}_q \hbar \langle n_q \rangle$$

$\Rightarrow$ for real or virtual emissions:

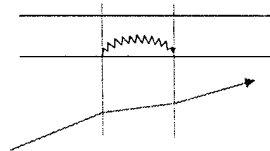
$$\sum_{n_q} P_{eq} \mathcal{G}_q - 1 \hbar \langle n_q + 1 \rangle \sum_{n_q} \mathcal{G}_q + 1 \hbar \mathcal{G}_q \hbar \langle n_q + 1 \rangle$$

Phonon scattering

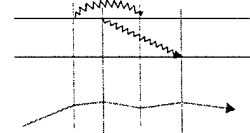
Real emission ("in")



Virtual emission ("out")



Multiple scattering

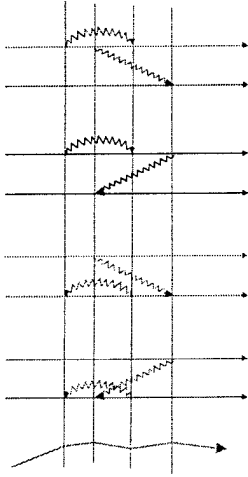


Intracollisional field effect, interference with potential scattering, ...

MONTE CARLO

Multiplicity of Wigner paths

Each multiple path can be obtained with different diagrams



Each graph contributes with a factor

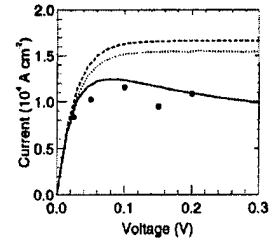
$$e^{i(qr_{ij} - \omega t_{ij})}$$

Summing up all terms, the contribution becomes

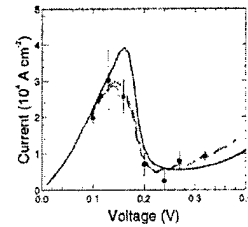
$$2 \cos(qr_{ij} - \omega t_{ij}) 2 \cos(q'r_u - \omega' t_u) \rightarrow 2^n \prod \cos$$

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Potential step – one phonon scattering



Double barrier – one phonon scattering



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Quantum self-scattering

IN STRICT FORMAL ANALOGY WITH SELF SCATTERING IN SEMICLASSICAL TRANSPORT SIMULATION

Define:

$$\tilde{f}_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n'_q\}, t) = e^{\Gamma(t-t_0)} f_w$$

By substitution into the dynamical equation, an extra term is added to the interaction, proportional to Γ .

Each “free flight” in the Wigner paths is multiplied by a factor

$$e^{-\Gamma(t_i - t_j)}$$

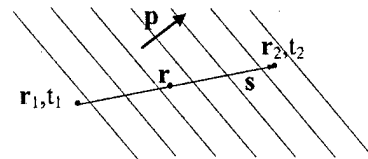
corresponding to an approximate constant life-time of the electron state (imaginary part of the self energy).

At each “scattering event” a choice is made between self or actual scattering, thus correcting, at each order, the approximate self energy with the exact one.

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TWO-TIME WIGNER FUNCTION

$$\rho(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2)$$



$$\mathbf{r} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \quad \mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1$$

$$t = \frac{t_1 + t_2}{2} \quad \tau = t_2 - t_1$$

$$f_w[\mathbf{r}, \mathbf{p}, \omega, t, \mathbf{k}] = \int \tau e^{i\omega\tau} \int s e^{-i\mathbf{p}\mathbf{s}/\hbar} \langle \Psi_{\mathbf{d}+\tau/2} | \Psi_{\mathbf{d}-\tau/2} \rangle \langle \Psi_{\mathbf{d}-\tau/2} | \Psi_{\mathbf{d}+\tau/2} \rangle$$

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TWO-TIME WIGNER F. WITH PHONONS (1)

$$G^<(r_1, t_1; r_2, t_2) = \frac{i}{\hbar} \langle \Psi^\dagger(r_2, t_2) \Psi(r_1, t_1) \rangle$$

$$= \frac{i}{\hbar} \langle \Phi(t_2) | \Psi^\dagger(r_2) U(t_2, t_1) \Psi(r_1) | \Phi(t_1) \rangle$$

$$\Phi(r, \{n_q\}, t) = \langle r, \{n_q\} | \Phi(t) \rangle$$

$$\mathbb{H} = \mathbb{H}_e + \mathbb{H}_p + \mathbb{H}_{ep}$$

$$G^< = \frac{i}{\hbar} \sum_{\{n_q\}} \sum_{\{n'_q\}} \langle 0, \{n_q\} | e^{-iH_p(t_2-t_1)/\hbar} | 0, \{n'_q\} \rangle \Phi^*(r_2, \{n_q\}, t_2) \Phi(r_1, \{n'_q\}, t_1)$$

$$= \frac{i}{\hbar} \sum_{\{n_q\}} e^{-i\omega_p(t_2-t_1)} \Phi^*(r_2, \{n_q\}, t_2) \Phi(r_1, \{n_q\}, t_1)$$

Put

$$g(r, \{n_q\}, t) = e^{i\omega(\{n_q\})t} \Phi(r, \{n_q\}, t)$$

Then

$$G^<(r_1, t_1; r_2, t_2) = \frac{i}{\hbar} \sum_{\{n_q\}} \overline{g^*(r_2, \{n_q\}, t_2)} g(r_1, \{n_q\}, t_1)$$

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TWO-TIME WIGNER F. WITH PHONONS (3)

Dynamical equation

$$\frac{\partial}{\partial t} f_w(r, p, \{n_q\}, \{n'_q\}, t, \omega) = \int e^{ipr'/\hbar} dr' \int e^{-i\omega t} d\tau \left[\frac{\partial}{\partial t} g(r-r'/2, \{n_q\}, t-\tau/2) g^*(r+r'/2, \{n'_q\}, t+\tau/2) + g \frac{\partial}{\partial t} g^* \right]$$

$$\frac{\partial}{\partial t} g(r, \{n_q\}, t) = i\omega(\{n_q\})g + \frac{1}{i\hbar} \mathbb{H}g$$

If

$$\mathbb{H} = \mathbb{H}_e + \mathbb{H}_p + \mathbb{H}_{ep}$$

with

$$\mathbb{H}_p = \sum_q \mathbf{b}_q^\dagger \mathbf{b}_q \hbar \omega_q$$

then

$$\begin{aligned} \frac{\partial}{\partial t} g(r, \{n_q\}, t) &= i\omega(\{n_q\})g + \frac{1}{i\hbar} (\mathbb{H}_e + \mathbb{H}_p + \mathbb{H}_{ep})g \\ &= i\omega(\{n_q\})g + \frac{1}{i\hbar} (\mathbb{H}_e + \mathbb{H}_{ep})g + \frac{1}{i\hbar} \hbar \omega(\{n_q\})g \end{aligned}$$

Thus the free-phonon dynamics is eliminated

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TWO-TIME WIGNER F. WITH PHONONS (2)

Performe traditional transformation:

$$t = \frac{t_1 + t_2}{2}, \quad \tau = t_2 - t_1, \quad r = \frac{r_1 + r_2}{2}, \quad r' = r_2 - r_1$$

and Fourier transform:

$$\begin{aligned} f_w(r, p, t, \omega) &= -i\hbar G^<(r, p, t, \omega) = \sum_{\{n_q\}} \int e^{ipr'/\hbar} dr' \\ &\int e^{-i\omega t} d\tau g(r-r'/2, \{n_q\}, t-\tau/2) g^*(r+r'/2, \{n'_q\}, t+\tau/2) \end{aligned}$$

For dynamical equation generalize to non diagonal phonon states:

$$\begin{aligned} f_w(r, p, \{n_q\}, \{n'_q\}, t, \omega) &= \int e^{ipr'/\hbar} dr' \\ &\int e^{-i\omega t} d\tau g(r-r'/2, \{n_q\}, t-\tau/2) g^*(r+r'/2, \{n'_q\}, t+\tau/2) \end{aligned}$$

and

$$f_w(r, p, t, \omega) = \sum_{\{n_q\}} f_w(r, p, \{n_q\}, \{n'_q\}, t, \omega)$$

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TWO-TIME WIGNER F. WITH PHONONS (4)

The various terms in the dynamical equations are treated as in the standard case:

$$\mathbb{H}_p = -\frac{\hbar^2}{2m} \nabla^2$$

yields

$$\frac{\partial}{\partial t} \bigg|_{\omega} f_w(r, p, \{n_q\}, \{n'_q\}, t, \omega) = -\frac{p}{m} \nabla_r f_w$$

$$\mathbb{V} = V(r)$$

yields

$$\begin{aligned} \frac{\partial}{\partial t} \bigg|_{\omega} f_w(r, p, \{n_q\}, \{n'_q\}, t, \omega) \\ = \int_{-\infty}^{\infty} dp' V_w(r, p-p') f_w(r, p', \{n_q\}, \{n'_q\}, t, \omega) \end{aligned}$$

where

$$V_w(r, p) = \frac{1}{i\hbar (2\pi\hbar)^3} \int dr' e^{ipr'/\hbar} [V(r-r'/2) - V(r+r'/2)]$$

For a constant or harmonic force F

$$\frac{\partial}{\partial t} \bigg|_{\omega} f_w(r, p, \{n_q\}, \{n'_q\}, t, \omega) = -F \frac{\partial}{\partial p} f_w$$

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TWO-TIME WIGNER F. WITH PHONONS (5)

Taking into account that

$$\begin{aligned} & \mathbf{b}_q g(\mathbf{r}-\mathbf{r}'/2, \{n_q\}, t-\tau/2) \\ &= e^{i\omega(\{n_q\})(t-\tau/2)} \langle \mathbf{r}-\mathbf{r}'/2, \{n_q\} | \mathbf{b}_q | \Psi(t-\tau/2) \rangle \\ &= e^{-i\omega_q(t-\tau/2)} \sqrt{n_q+1} g(\mathbf{r}-\mathbf{r}'/2, \{n_1, \dots, n_{q-1}, n_q+1, \dots\}, t-\tau/2) \end{aligned}$$

and similar,

the electron-phonon interaction

$$H_{ep} = i\hbar \sum_{q'} F(q') (\mathbf{b}_q e^{iq'r} - \mathbf{b}_q^\dagger e^{-iq'r})$$

yields

$$\begin{aligned} & \frac{\partial}{\partial t} \bigg|_{ep} f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n'_q\}, t, \omega) = \sum_{q'} F(q') \{ \\ & e^{i(q'r-\omega_q t)} \sqrt{n_q+1} f_w(\mathbf{r}, \mathbf{p}-\hbar\mathbf{q}'/2, \{n_1, \dots, n_{q-1}, n_q+1, \dots\}, \{n'_q\}, t, \omega-\omega_q/2) \\ & - e^{-i(q'r-\omega_q t)} \sqrt{n_q} f_w(\mathbf{r}, \mathbf{p}+\hbar\mathbf{q}'/2, \{n_1, \dots, n_{q-1}, n_q-1, \dots\}, \{n'_q\}, t, \omega+\omega_q/2) \\ & + e^{-i(q'r-\omega_q t)} \sqrt{n'_q+1} f_w(\mathbf{r}, \mathbf{p}-\hbar\mathbf{q}'/2, \{n_q\}, \{n'_1, \dots, n'_{q-1}, n'_q+1, \dots\}, t, \omega-\omega_q/2) \\ & - e^{i(q'r-\omega_q t)} \sqrt{n'_q} f_w(\mathbf{r}, \mathbf{p}+\hbar\mathbf{q}'/2, \{n_q\}, \{n'_1, \dots, n'_{q-1}, n'_q-1, \dots\}, t, \omega+\omega_q/2) \} \end{aligned}$$

TWO-TIME WIGNER F. WITH PHONONS (6)

The general equation

$$\frac{\partial}{\partial t} f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n'_q\}, t, \omega) + \frac{\mathbf{p}}{m} \frac{\partial}{\partial \mathbf{r}} f_w + F \frac{\partial}{\partial \mathbf{p}} f_w = \frac{\partial f_w}{\partial t} \bigg|_{ep} + \frac{\partial f_w}{\partial t} \bigg|_{ph}$$

here

$$\begin{aligned} & \frac{\partial}{\partial t} \bigg|_{ph} f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n'_q\}, t, \omega) = \int d\mathbf{p}' V_w(\mathbf{r}, \mathbf{p}-\mathbf{p}') f_w(\mathbf{r}, \mathbf{p}', \{n_q\}, \{n'_q\}, t, \omega) \\ & V_w(\mathbf{r}, \mathbf{p}) = \frac{1}{i\hbar} \frac{1}{(2\pi\hbar)^3} \int d\mathbf{r}' e^{i\mathbf{p}'\mathbf{r}'} [V(\mathbf{r}-\mathbf{r}'/2) - V(\mathbf{r}+\mathbf{r}'/2)] \end{aligned}$$

and

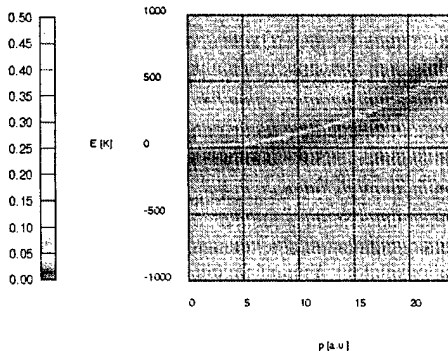
$$\begin{aligned} & \frac{\partial}{\partial t} \bigg|_{ep} f_w(\mathbf{r}, \mathbf{p}, \{n_q\}, \{n'_q\}, t, \omega) = \sum_{q'} F(q') \{ \\ & e^{i(q'r-\omega_q t)} \sqrt{n_q+1} f_w(\mathbf{r}, \mathbf{p}-\hbar\mathbf{q}'/2, \{n_1, \dots, n_{q-1}, n_q+1, \dots\}, \{n'_q\}, t, \omega-\omega_q/2) \\ & - e^{-i(q'r-\omega_q t)} \sqrt{n_q} f_w(\mathbf{r}, \mathbf{p}+\hbar\mathbf{q}'/2, \{n_1, \dots, n_{q-1}, n_q-1, \dots\}, \{n'_q\}, t, \omega+\omega_q/2) \\ & + e^{-i(q'r-\omega_q t)} \sqrt{n'_q+1} f_w(\mathbf{r}, \mathbf{p}-\hbar\mathbf{q}'/2, \{n_q\}, \{n'_1, \dots, n'_{q-1}, n'_q+1, \dots\}, t, \omega-\omega_q/2) \\ & - e^{i(q'r-\omega_q t)} \sqrt{n'_q} f_w(\mathbf{r}, \mathbf{p}+\hbar\mathbf{q}'/2, \{n_q\}, \{n'_1, \dots, n'_{q-1}, n'_q-1, \dots\}, t, \omega+\omega_q/2) \} \end{aligned}$$

- The left hand side of the above equation is the same Liouvillian as in the Boltzmann equation
- Path variables and integral form of the transport equation
- Neumann expansion
- Wigner Paths
- Monte Carlo

• At each interaction “vertex” half phonon momentum and half phonon energy is transferred to or taken from the electron

Spectral Density
A(p, ω) = fw(p, ω)/fwp

GaAs
Polar optical phonons
T=300 K
up to 3 scattering
t=150 fs



Conclusions

Wigner paths can be defined as ballistic “flights” separated by scattering processes

The Wigner-function approach allows a description of quantum transport in terms of phase-space analogous to the semiclassical case

A Monte Carlo algorithm can be implemented that accounts for given initial and boundary conditions

Momentum and frequency can be considered as independent variables in a two-time Green function approach, maintaining the path Monte Carlo approach

PROBLEMS ...

Modeling Resonant Tunneling Devices With Wigner Functions and Density Matrices

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- Collaborators:
R.C. Buggeln and J. P. Kreskovsky
- Supported by Office of Naval Research

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Quantum Transport RTD Studies

- Use Wigner Function (primarily) and Density Matrix to describe *transient* transport in nanoscale devices
- Go beyond using switching speed as a primary characteristic of a device. Instead we also determine the *recovery* time.
- We do this within the framework of an RTD as a self-excited sustained relaxation oscillator (RTD-RO) and seek the highest frequency of oscillation.

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Simulation Requirements

- Suitable transient quantum transport equation—Wigner equation/Density Matrix
- Poisson's equation + flat band boundary conditions
- Circuit equations/transmission line equations—treated as boundary conditions
- Suitable description of reservoirs and dissipation

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WF/DM Equation

- Can be obtained in simple cases directly from Schrodinger's equation or formally as follows:

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Density Operator

$$\rho_{op}(t) = \sum_i |i(t)\rangle P(i) \langle i(t)|$$

Single time
Density Operator

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Density Operator

$$\rho_{op}(t) = \sum_i |i(t)\rangle P(i) \langle i(t)|$$

Coordinate representation

$$\begin{aligned} \langle \mathbf{x} | \rho_{op}(t) | \mathbf{x}' \rangle &= \sum_i \langle \mathbf{x} | i(t) \rangle P(i) \langle i(t) | \mathbf{x}' \rangle \\ &= \sum_i P(i) \Psi_i^*(\mathbf{x}', t) \Psi_i(\mathbf{x}, t) \end{aligned}$$

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Time Dependence/Liouville Eq.

$$i\hbar \frac{d\rho_{op}(t)}{dt} = [H(t), \rho_{op}(t)]$$

$$i\hbar \frac{\partial \langle \mathbf{x} | \rho(t) | \mathbf{x}' \rangle}{\partial t}$$

$$= \left\{ -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial \mathbf{x}^2} - \frac{\partial^2}{\partial \mathbf{x}'^2} \right) + V(\mathbf{x}) - V(\mathbf{x}') \right\} \langle \mathbf{x} | \rho(t) | \mathbf{x}' \rangle$$

$$\text{dissipation} \longrightarrow +i\hbar \left\{ \frac{\partial \langle \mathbf{x} | \rho | \mathbf{x}' \rangle}{\partial t} \right\}_{\text{scattering}}$$

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Density & Current Density Matrix

$$\rho(\mathbf{x}, \mathbf{x}') = \langle \mathbf{x} | \rho_{op}(t) | \mathbf{x}' \rangle$$

Density $\rho(\mathbf{x}) = \langle \mathbf{x} | \rho_{op}(t) | \mathbf{x} \rangle$

$$\mathbf{j}(\mathbf{x}, \mathbf{x}') = \frac{\hbar}{2mi} (\nabla_{\mathbf{x}} - \nabla_{\mathbf{x}'}) \langle \mathbf{x} | \rho_{op}(t) | \mathbf{x}' \rangle$$

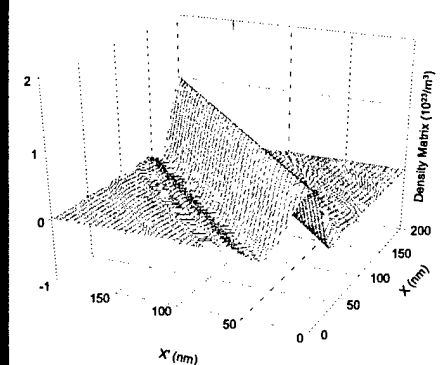
Current Density $\mathbf{j}(\mathbf{x}) = \lim_{\mathbf{x}'' \rightarrow \mathbf{x}'} \mathbf{j}(\mathbf{x}, \mathbf{x}'')$

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Density Matrix Flat Profile

Real Part of the Density Matrix, T=0.0 K

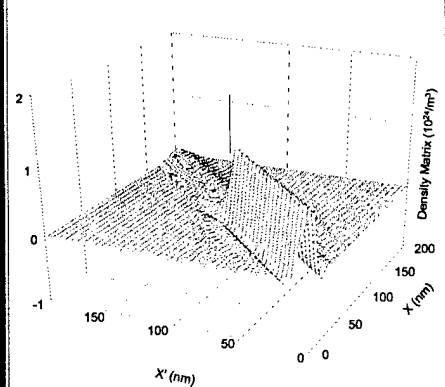


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Density Matrix Hetero-structure

Real Part of the Density Matrix

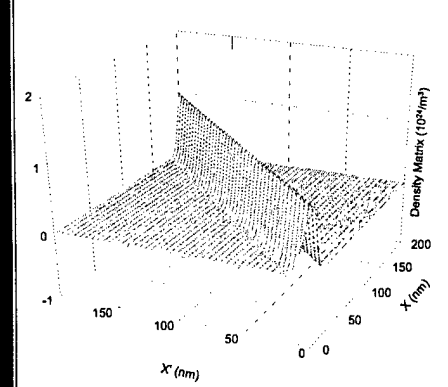


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Real Part Flat Profile, 300K

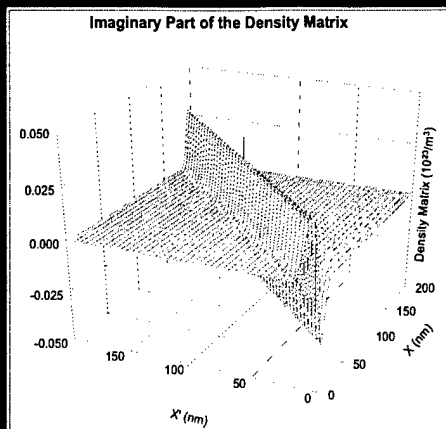
Real Part of the Density Matrix



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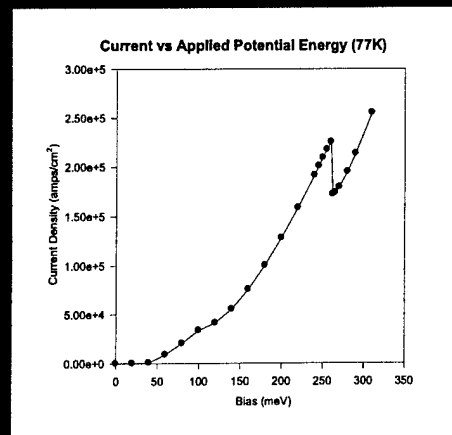
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Imag. Part Flat Profile, 300K



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IV Double Barrier RTD



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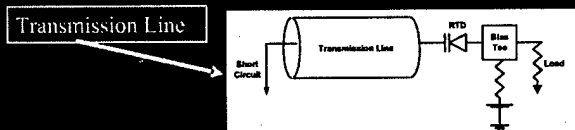
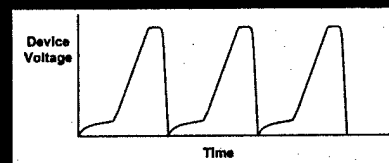
What do we do with the RTD?

- Switching device!
 - There are many of these
- Logic device-particularly multiple value logic elements
- Quantum Van der Pol oscillator
 - Negative differential resistance can (sometimes) yield sustained oscillations.

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RTD as a Relaxation Oscillator:

(Verghese, C. D. Parker, E.R. Brown APL 1998)



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Quantum Van der Pol Oscillator

- It should be of interest to physicists because it is capable of providing a practical measure of the speed and response time of resonant tunneling diode.
 - How?
 - Determine the maximum frequency of a controlled oscillation that can *drive or affect* the total system
 - Use the RTD/RO as a clock.

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There are non-Van der Pol Oscillations

- We have observed weak (above threshold) voltage oscillations at frequencies near 700 GHz!

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We will concentrate on the Wigner Function

- More success than with using the DM.
- Not as fast as the DM.

Wigner Function/Equation from the Density Matrix through the Weyl Transformation

$$f_w(\mathbf{k}, \mathbf{r}) = \frac{1}{2} \int d\mathbf{s} \left\langle \mathbf{r} + \frac{\mathbf{s}}{2} \right| \rho_{eq}(t) \left| \mathbf{r} - \frac{\mathbf{s}}{2} \right\rangle \exp(-i\mathbf{k} \cdot \mathbf{s})$$

The Wigner Equation

Transient Dissipation Diffusion

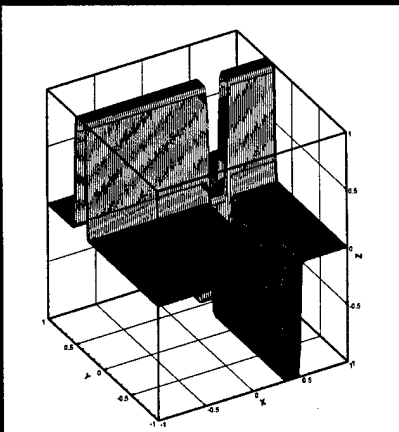
$$0 = \frac{\partial f_w(\mathbf{k}, x)}{\partial t} + \frac{f_w(\mathbf{k}, x) - f_{eq}(\mathbf{k}, x)}{\tau(x)} + \frac{\hbar k_x}{m} \frac{\partial f_w(\mathbf{k}, x)}{\partial x} - \frac{1}{\pi \hbar} \lim_{\xi \rightarrow 0} \int_{-\xi}^{\xi} d\zeta \left[\frac{V(x+\zeta)}{-V(x-\zeta)} \right] \int dk_x' f_w(k_x', k_y, k_z, \eta_x) \sin[2(k_x' - k_x)\xi]$$

Quantum Mechanics: The Wigner integral represents a correlation of states in the coordinate representation.

The Wigner Integral Treated Analytically-Highlights Correlations

$$\lim_{\xi \rightarrow 0} \int_{-\xi}^{\xi} d\zeta \left[\frac{V(x+\zeta)}{-V(x-\zeta)} \right] \sin[2(k_x' - k_x)\xi] = \frac{2V_0 \sin[2(k_x' - k_x)(x_0 - x)] \sin[(k_x' - k_x)\Delta]}{i(k_x' - k_x)}$$

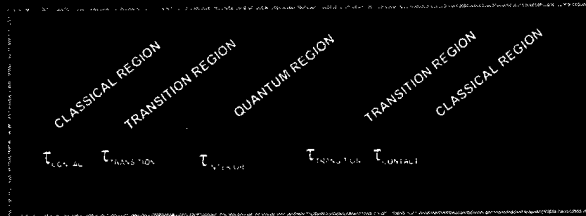
$V(x) - V(x')$ for a single asymmetrically placed barrier



The Five Zone Structure

- Five zone structure:
 - The end regions (two) represent the contact regions;
 - The central region (one)-- incorporates the quantum barriers and wells;
 - The transition region (two)
- Variable scattering time, smallest in contact region.

Device Geometry



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Structure Emphasized *The Baseline Studies*

- 200 nm, DBRTD, 250/300meV-5nm-5nm-5nm, low doped central region
- DC and transient studies
- Hysteresis studies
- (Note: In the absence of barriers can compute IV of an NIN structure as well as linear resistor)

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Structure Emphasized *The Baseline Study, cont.*

- New derivative boundary conditions on Wigner function show flat-band and qualitative displacement momentum contributions.
- Do not need to assert displaced Fermi boundary conditions.
- Equilibrium distribution obtained first.

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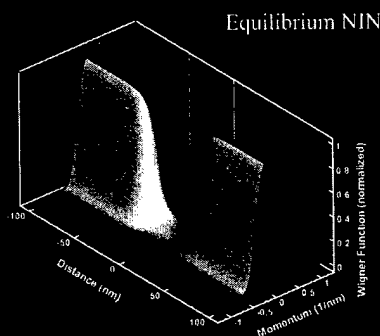
Structure Emphasized *The Baseline Study, cont.*

- Converged dc solutions via small bias increments show *no hysteresis!*
- Large signal transients show hysteresis.
- Relaxation to steady state occurs within a pico-sec and is bias dependent
- Devices sustain steady relaxation oscillations!

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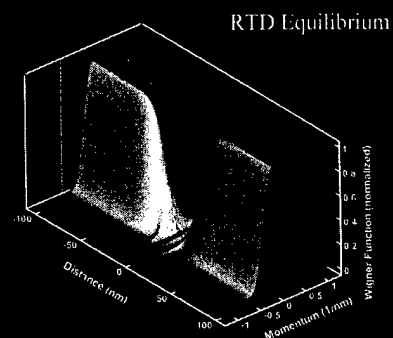
Wigner Function NIN



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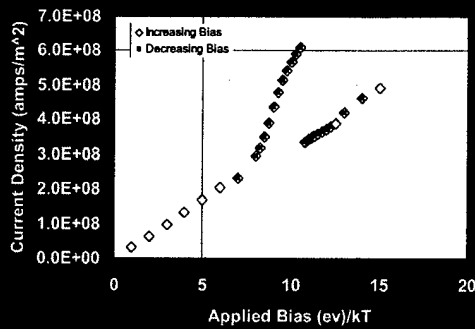
Wigner Function RTD



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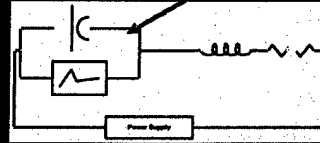
Current Density Versus Applied Bias



RTD as a Relaxation Oscillator (Lumped Circuit Elements, RTD Represented by Wigner Equation)

Wigner Equation
Represents the
RTD

Realistic Device
Response



RO Equations

$$\frac{dv_D}{dt} = \frac{2\pi Z_0}{R_D} \{i(t) - i_D(v_D)\}$$

$$\frac{di}{dt} = \frac{R_D}{2\pi Z_0} \left\{ v_{APPLIED}(t) - v_D(t) + i(t) \frac{R_{LOAD}}{R_D} \right\}$$

RO Equations

$$\frac{dv_D}{dt} = \frac{2\pi Z_0}{R_D} \{i(t) - i_D(v_D)\}$$

$$\frac{di}{dt} = \frac{R_D}{2\pi Z_0} \left\{ v_{APPLIED}(t) - v_D(t) + i(t) \frac{R_{LOAD}}{R_D} \right\}$$

$$v_D = V_D / V_P, \quad i = I / I_P, \quad t = \tau / T_{ref}$$

RO Equations

$$\frac{dv_D}{dt} = \frac{2\pi Z_0}{R_D} \{i(t) - i_D(v_D)\}$$

$$\frac{di}{dt} = \frac{R_D}{2\pi Z_0} \left\{ v_{APPLIED}(t) - v_D(t) + i(t) \frac{R_{LOAD}}{R_D} \right\}$$

$$v_D = V_D / V_P, \quad i = I / I_P, \quad t = \tau / T_{ref}$$

$$Z_0 = \sqrt{L / C_D}, \quad T_{ref} = 2\pi \sqrt{LC_D}, \quad R_D = V_P / I_P$$

RO Equation Parameters

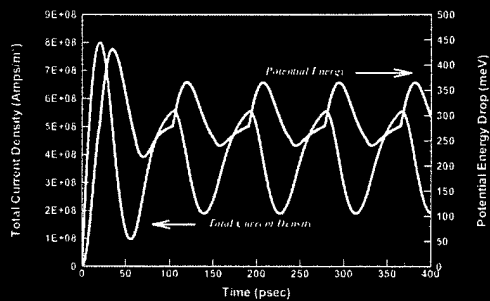
$$Z_0 = \sqrt{L / C_D}$$

$$T_{ref} = 2\pi \sqrt{LC_D}$$

$$R_D = V_P / I_P$$

Observation: For a fixed Z, oscillation characteristics are the same for all oscillatory periods provided the NDR is not dynamic.

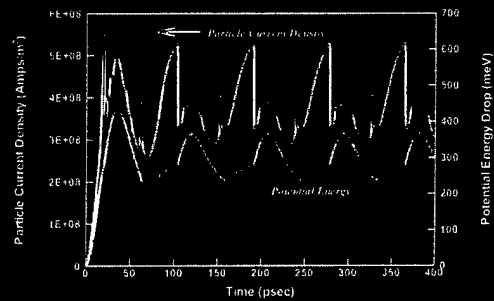
RO at 11.5 GHz: Total Current Density and Potential Energy Drop



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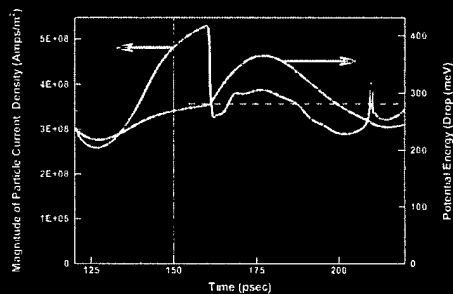
RO at 11.5 GHz: Particle Current Density and Potential Energy Drop



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Details of the RO at 11.5 GHz: Particle Current Density and Potential Energy Drop



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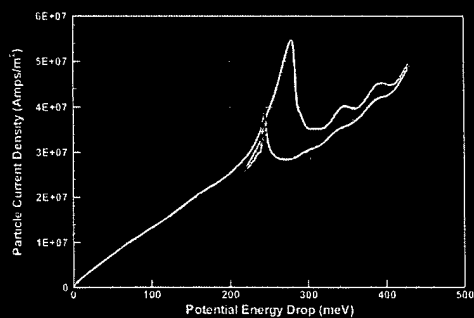
Details of the RO at 11.5 GHz: Particle Current Density and Potential Energy Drop

- First part of the oscillation
Linear I versus V, L-R rise time
- Second part of the oscillation
Drop in particle current and increase in potential drop
- Third part of the oscillation
– Decrease in potential drop, particle current remains below values associated with first part of the oscillation

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RO at 11.5 GHz: Dynamic 'IV'



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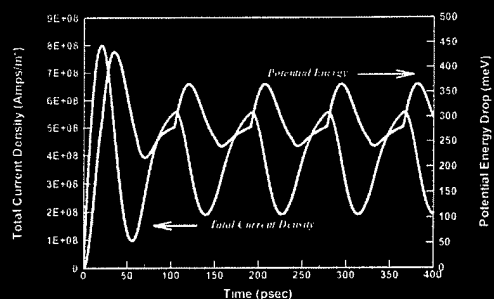
'Temporal Invariance'

When the device oscillates, its characteristics are independent of oscillatory period!

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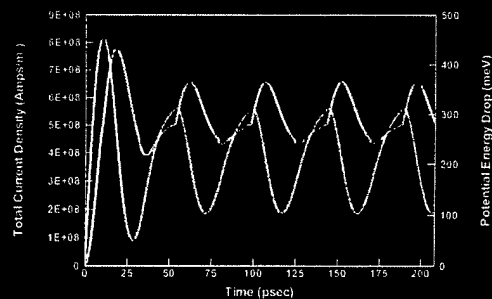
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RO at 11.5 GHz: Total Current Density and Potential Energy Drop



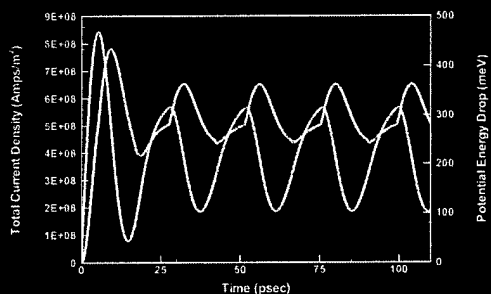
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RO at 22.0 GHz: Total Current Density and Potential Energy Drop



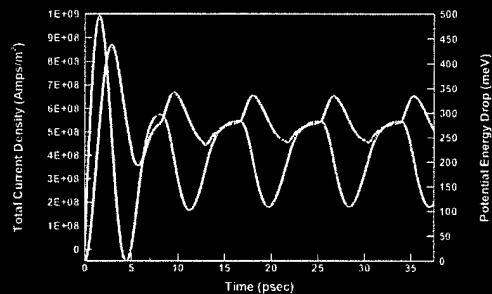
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RO at 41.8 GHz: Total Current Density and Potential Energy Drop



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RO at 113.7 GHz: Total Current Density and Potential Energy Drop

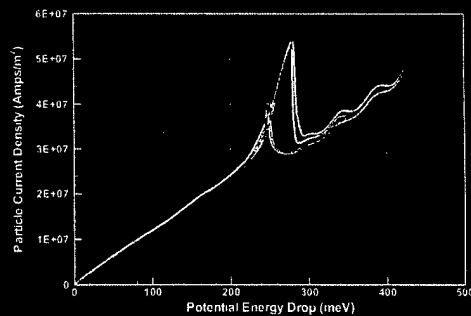


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Dynamic 'IV'

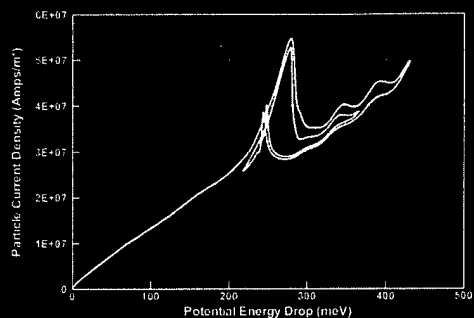
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RO at 6 GHz: Dynamic 'IV'



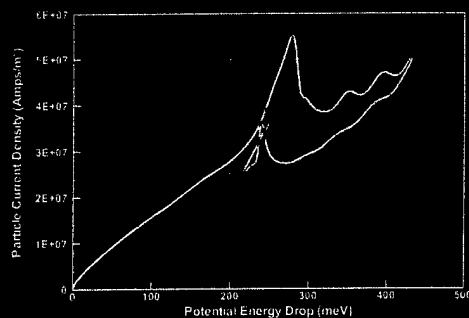
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RO at 11.5 GHz: Dynamic 'IV'



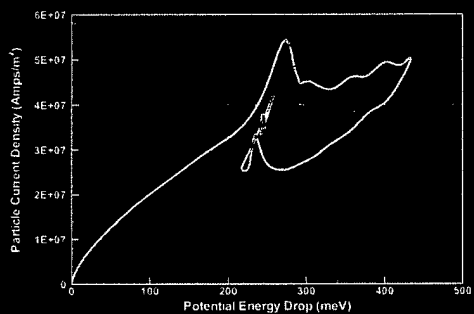
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RO at 22.0 GHz: Dynamic 'IV'



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RO at 41.8 GHz: Dynamic 'IV'

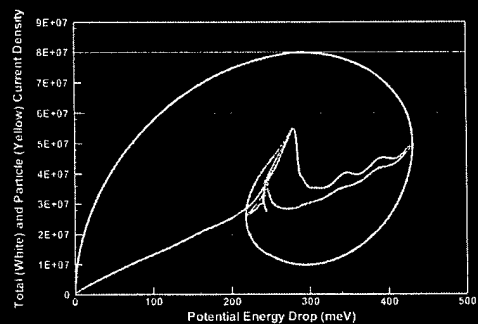


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Total and Dynamic 'IV'

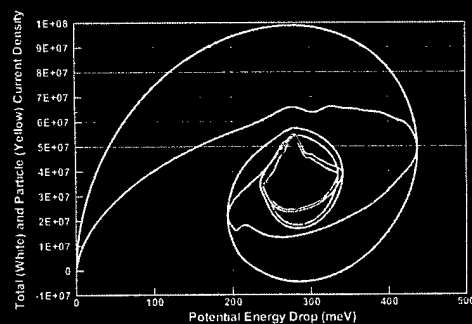
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RO at 6 GHz: Total and Dynamic 'IV'



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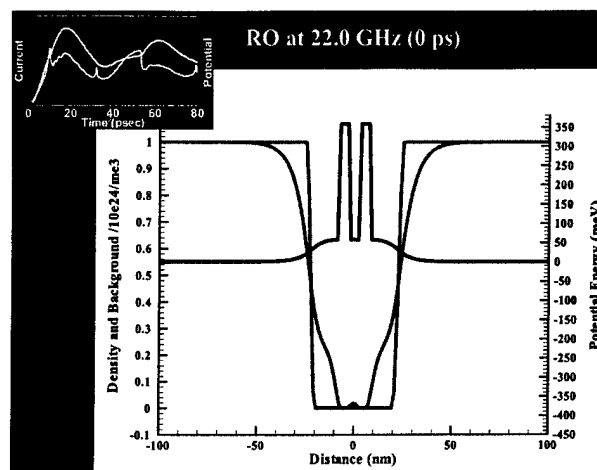
RO at 113.72 GHz: Total and Dynamic 'IV'



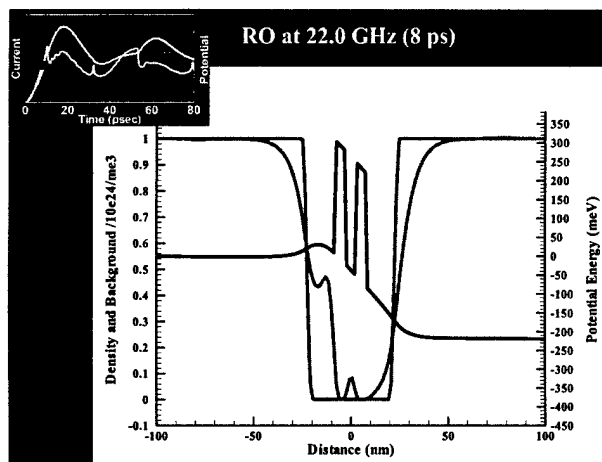
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Summary of Charge and Current for the 22.0 GHz RO

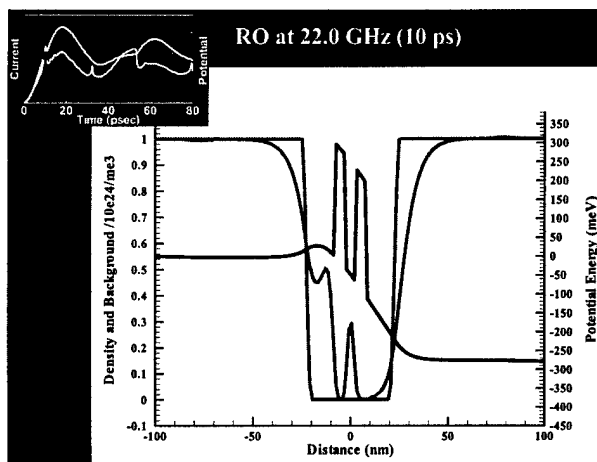
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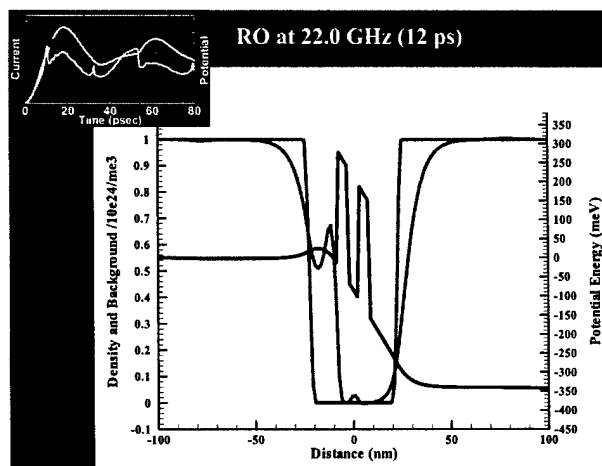
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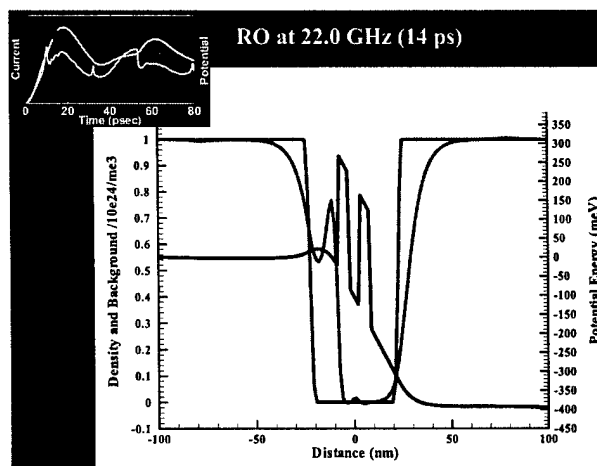
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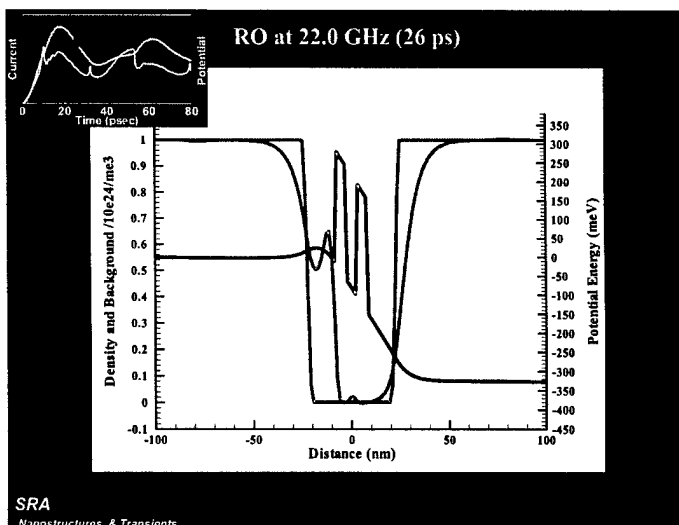
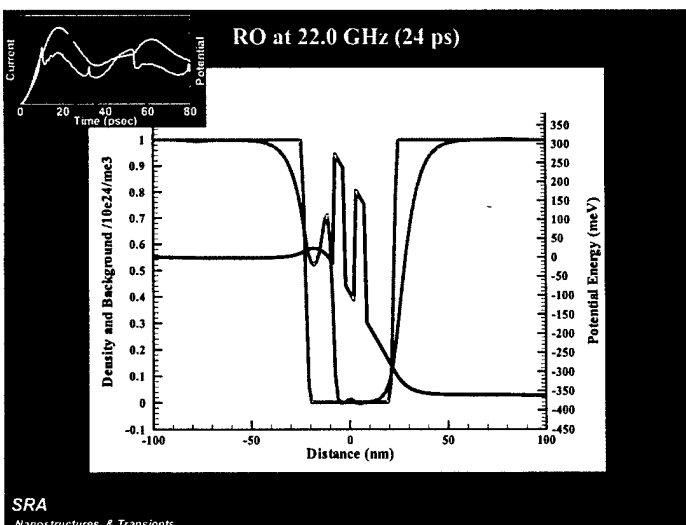
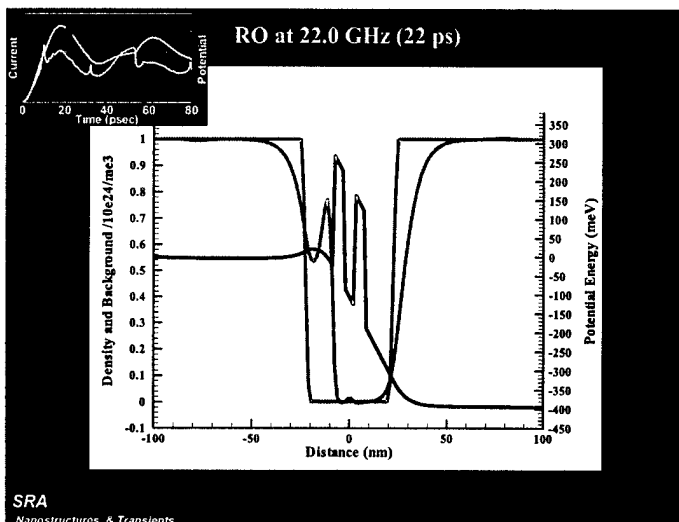
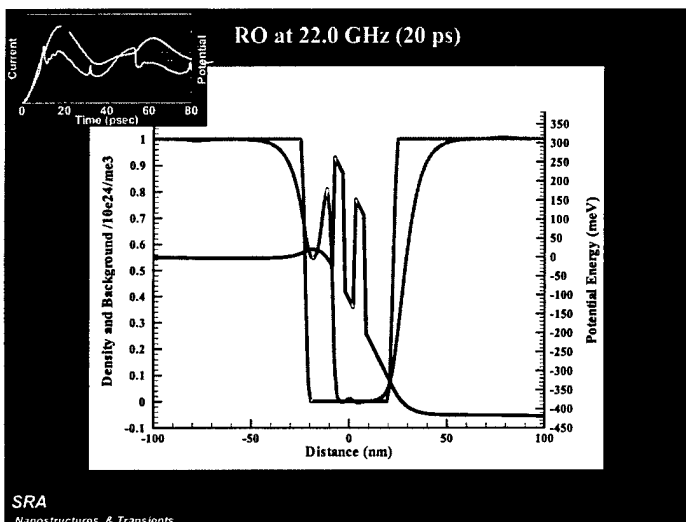
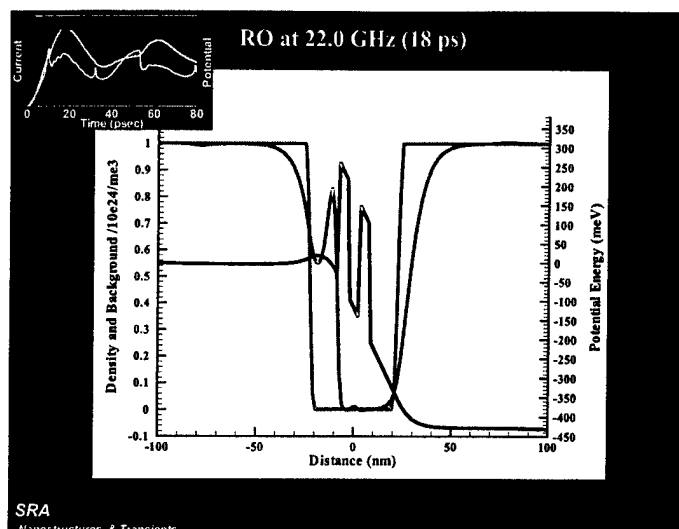
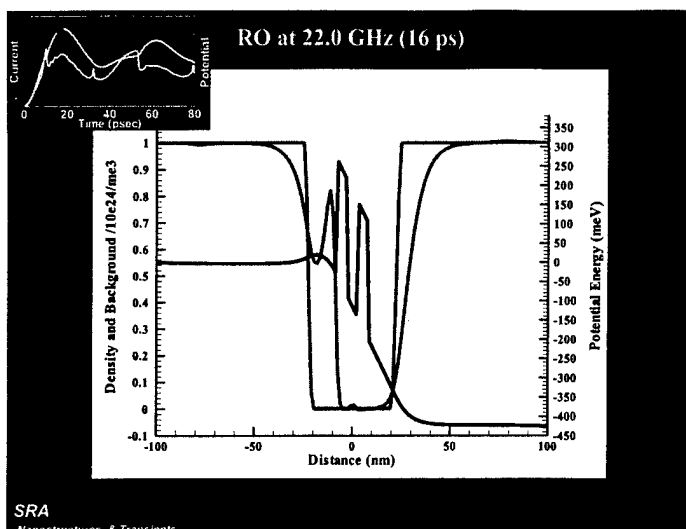
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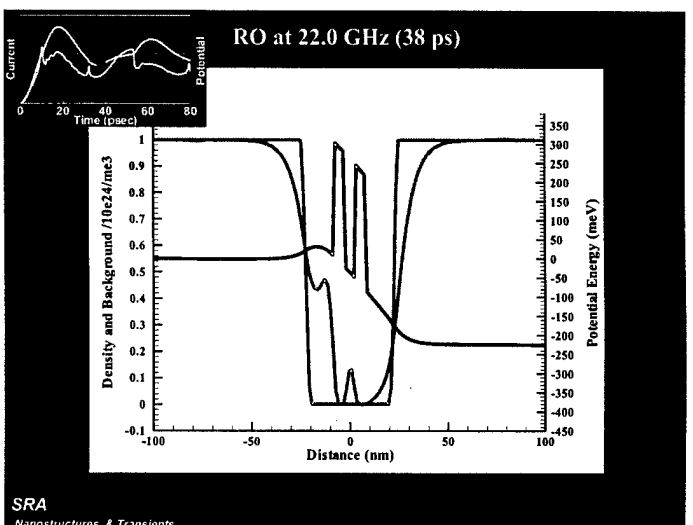
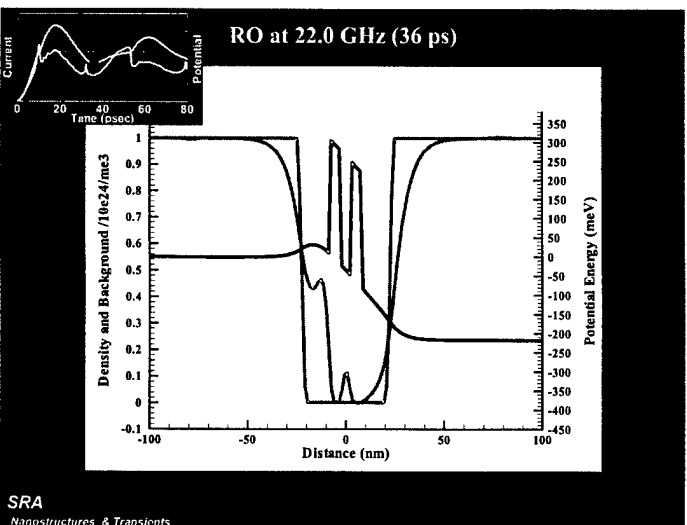
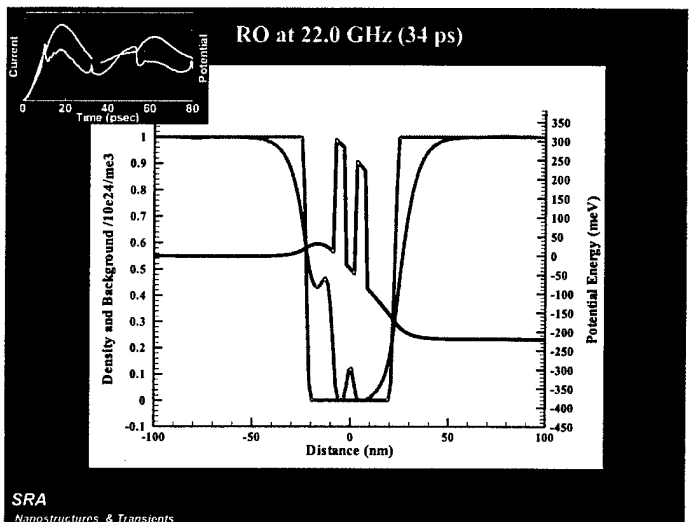
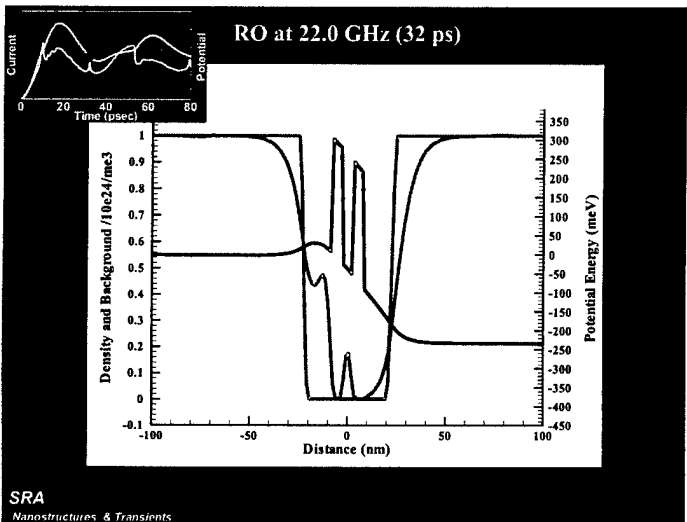
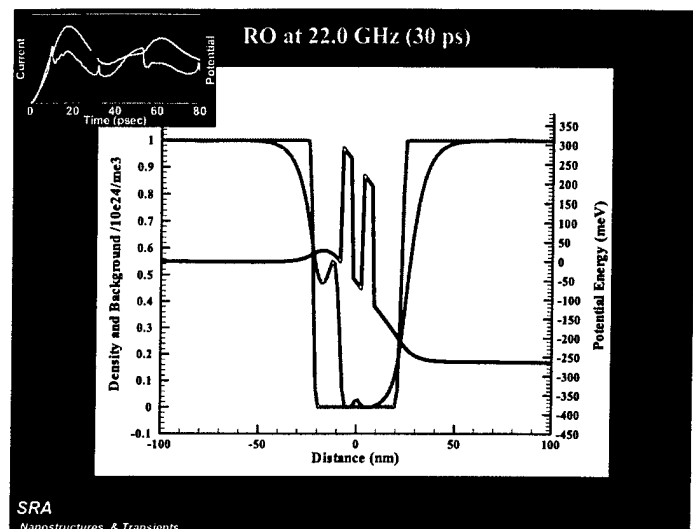
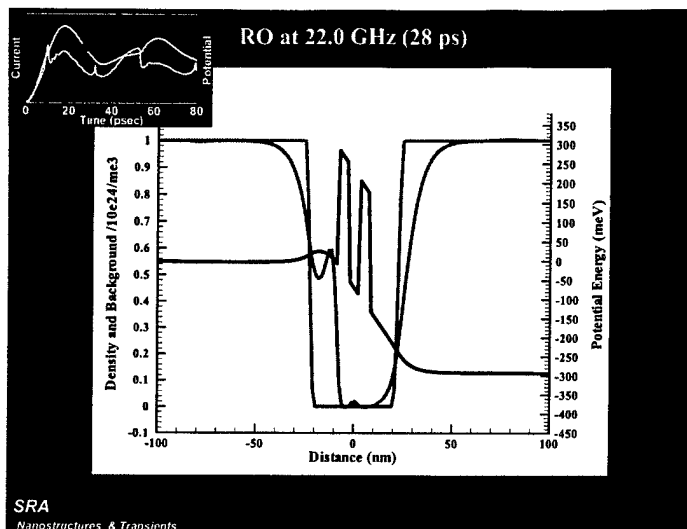


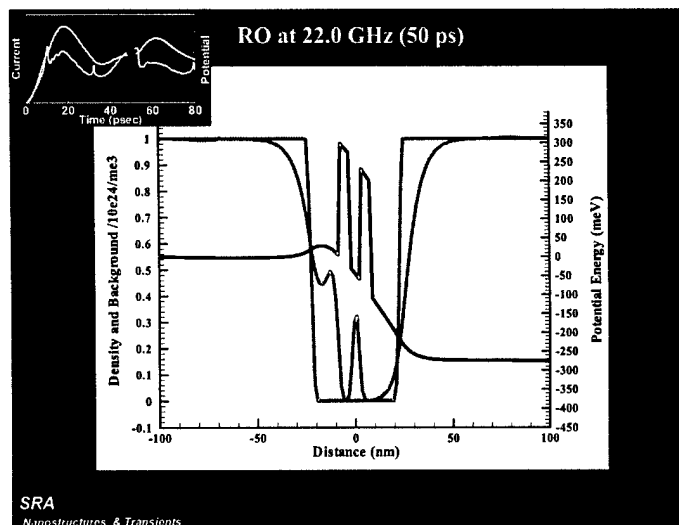
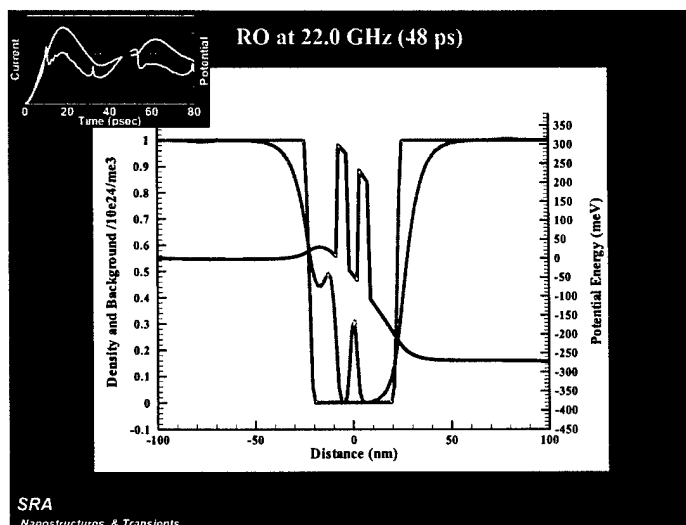
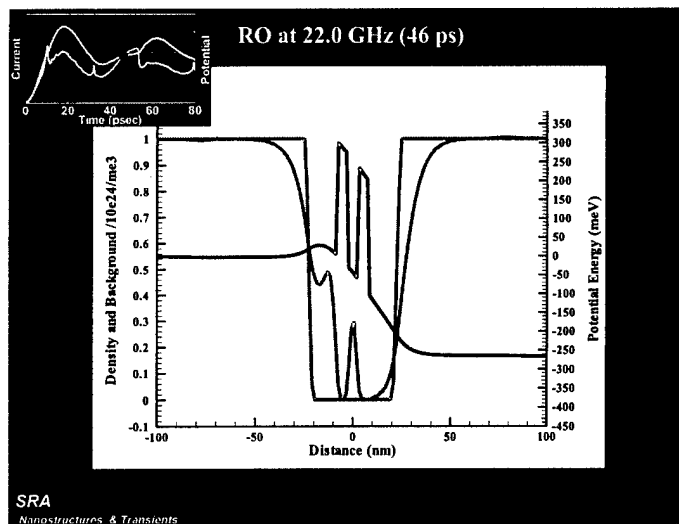
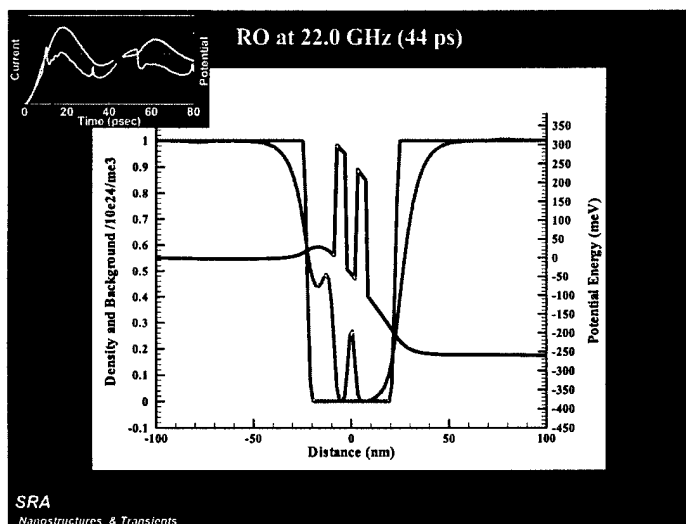
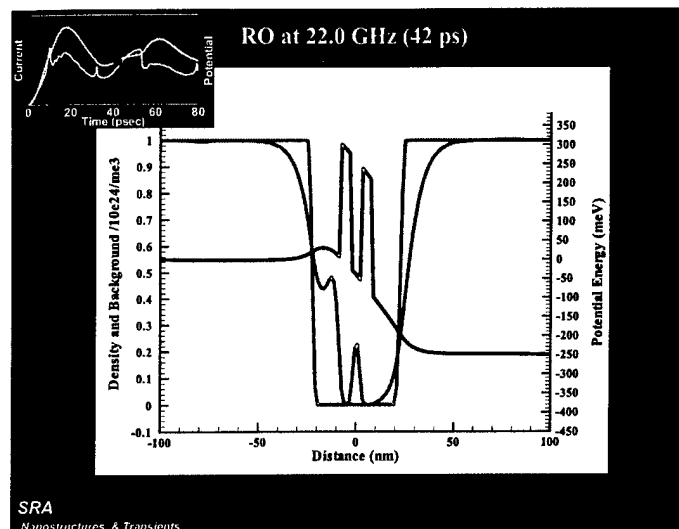
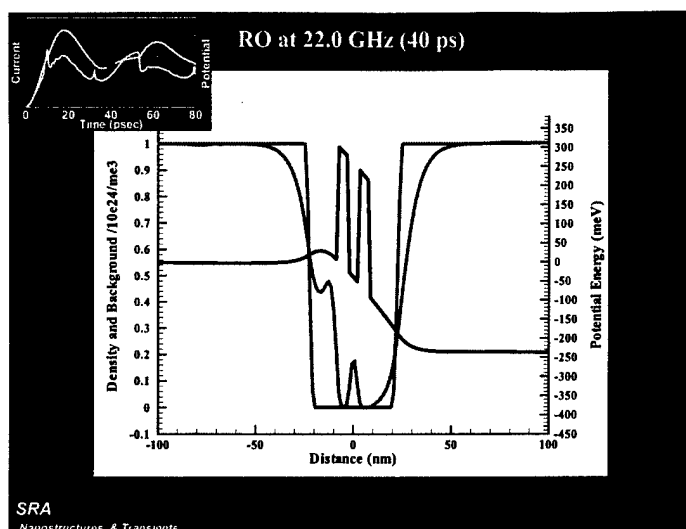
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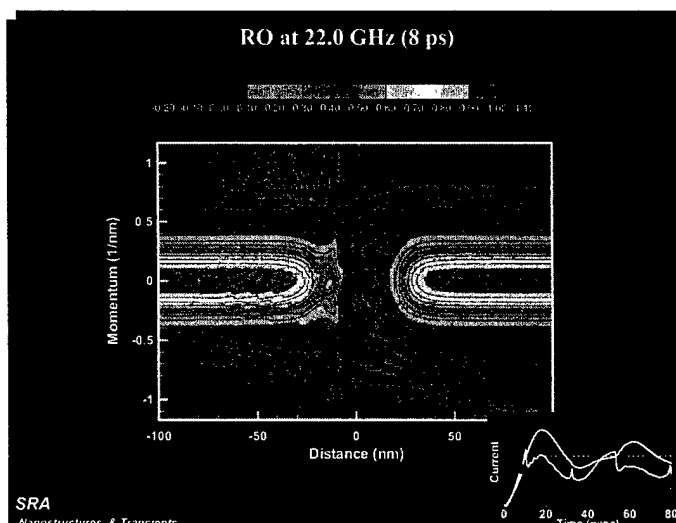
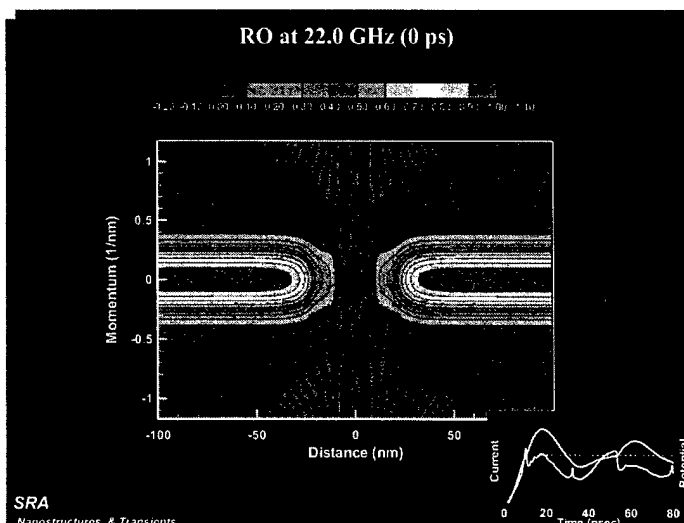
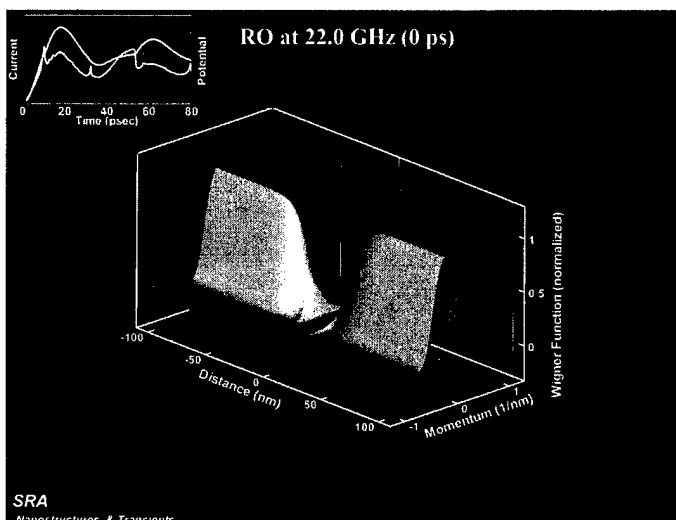
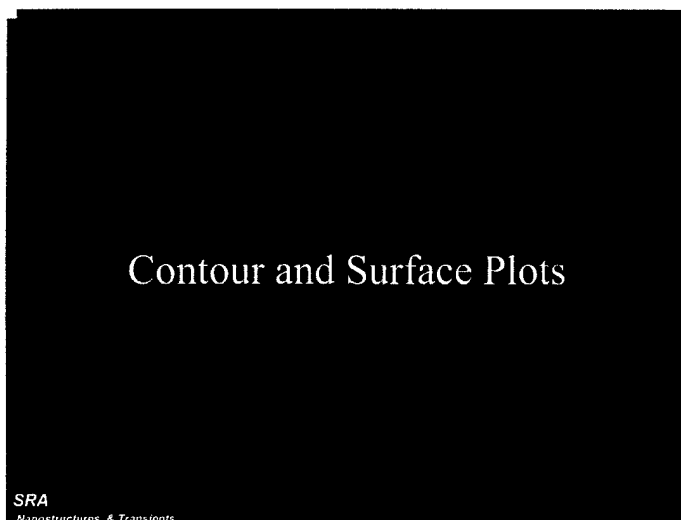
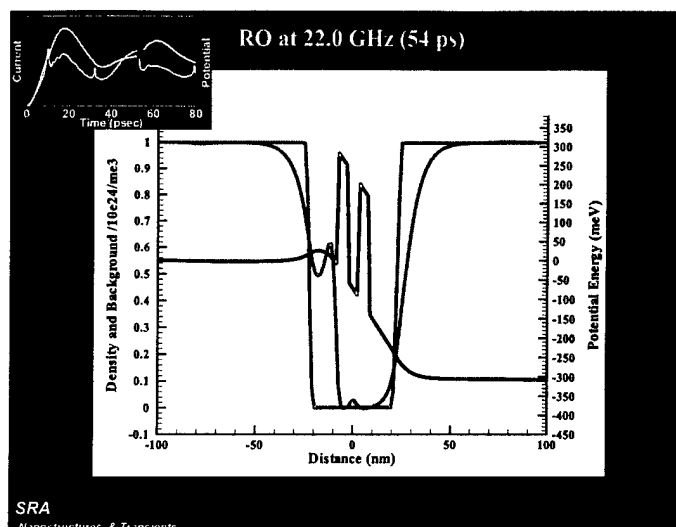
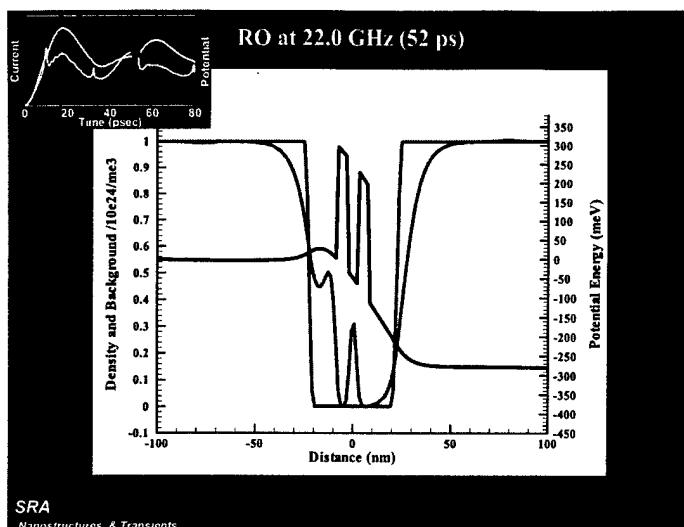


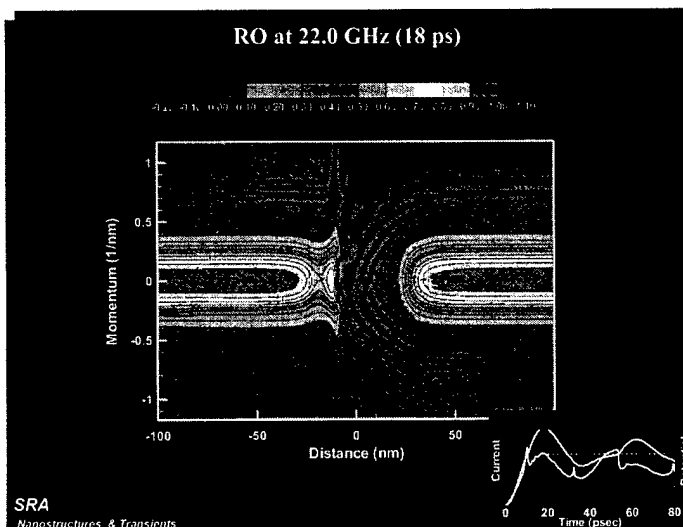
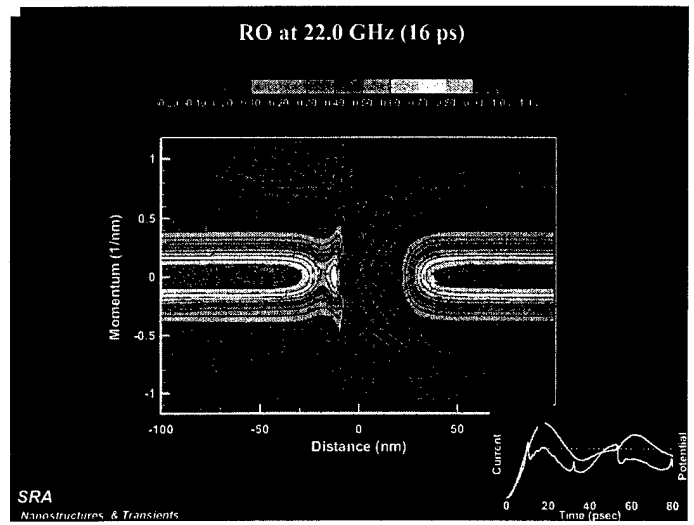
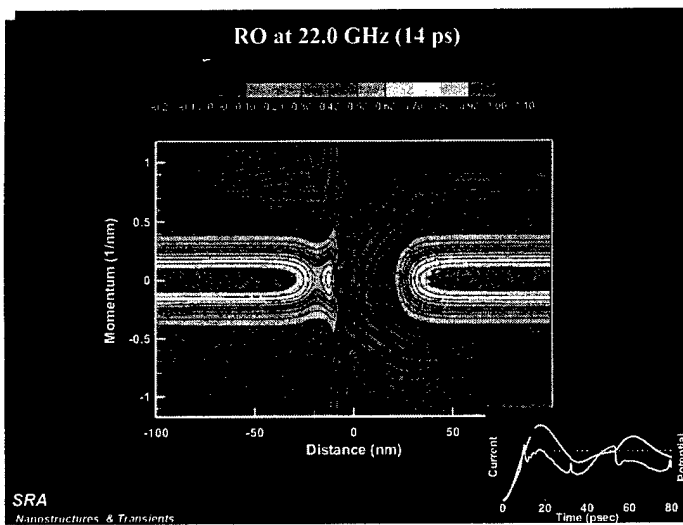
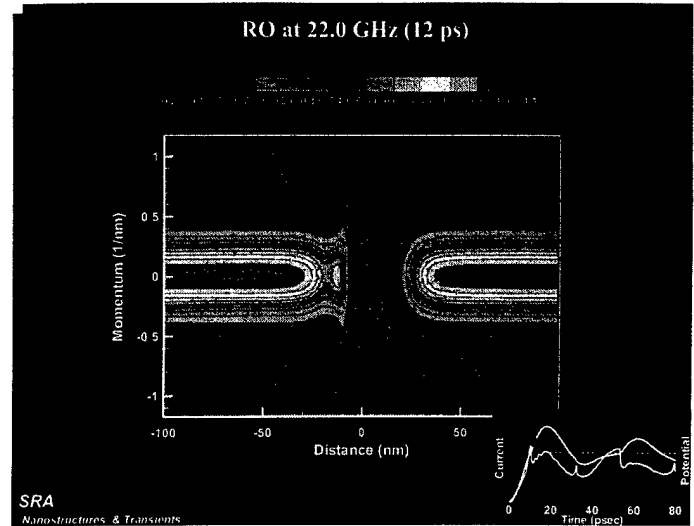
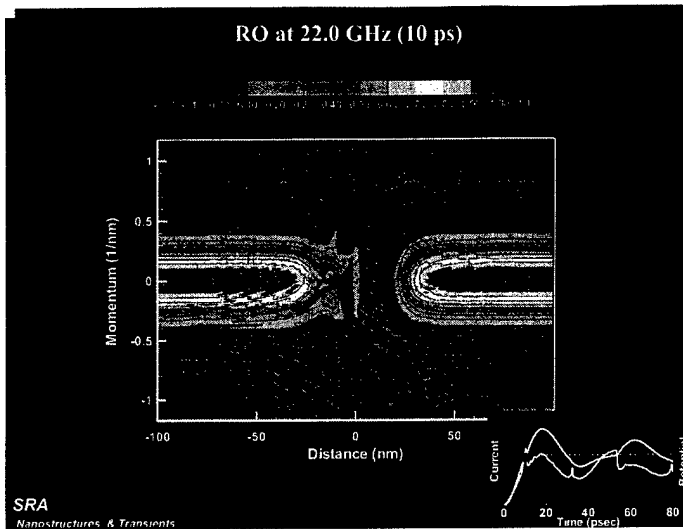
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Observations

- Increasing circuit frequency results in increased looping of IV.
- Oscillations cease at approximately 120GHz for these device parameters.
- There is an invariant quantity prior to the current drop-back. This quantity is the "Bohm Quantum Potential".
- Oscillations are stable to circuit noise and device fluctuations.
- Noise characteristics can probably be explained by early discussions of M. Lax.

The Bohm Quantum Potential and the Steady State Schrodinger Equation

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi = E \psi$$

$$\psi(x) = R \exp[iS(x)/\hbar]$$

Quantum Potential

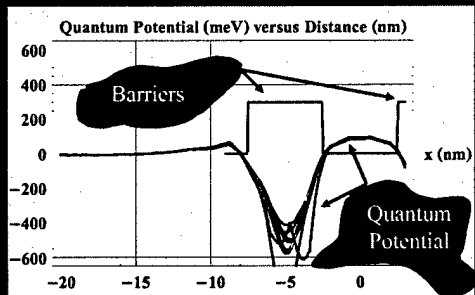
$$a. \frac{(\partial_x S)^2}{2m} + V + Q = E$$

$$b. \frac{\partial}{\partial x} \left(R^2 \frac{\partial_x S}{m} \right) = 0$$

$$c. Q = -\frac{\hbar^2}{2m} \frac{\partial_{xx} R}{R}$$

Q represents the ground state for simple double barrier problems

From Simulations (Post Processed): Quantum Potential, Within and to the Left of the Barrier—For Different Values of Bias



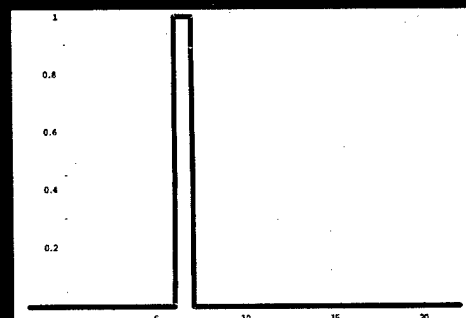
Conclusions Suggested by the Quantum Potential

- Simulations are consistent with the filling of a well defined quantum state prior to the drop-back in current.
- Quantum potential is not well defined after the drop-back.
 - Certainly there are computational difficulties with small values of charge. But we may be dealing with a mixture of a large number of quantum states, scattering states and dissipation.

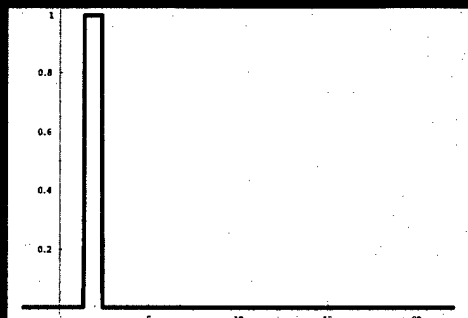
Noise Considerations

- For nonlinear systems amplitude noise is not an issue (M. Lax and our studies)
- Phase noise is additive.
- Amount of phase noise depends on time of disturbance.
- Recovery from disturbance occurs within one cycle.
- Illustrate with ODE and Quantum RO

Injected Current Source at a Particular Instant of Time

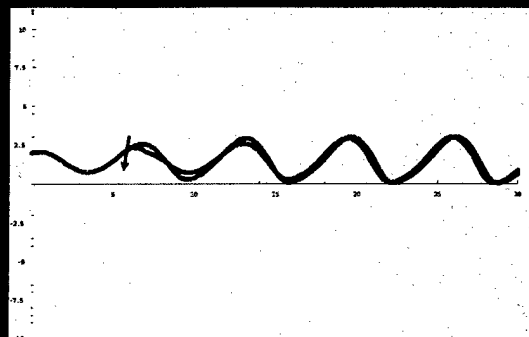


Same type of current source:
different time



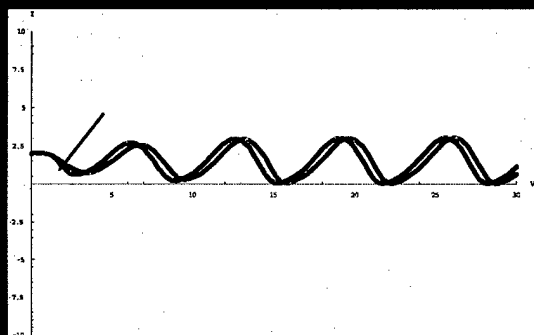
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ODE/RO Circuit Response



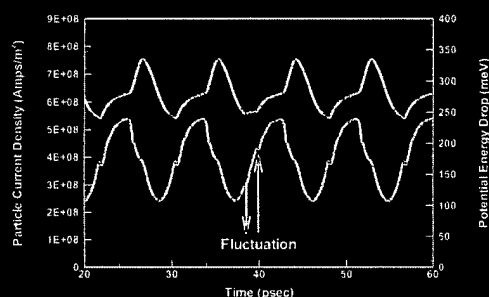
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ODE/RO Circuit Response



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Quantum RO at 113.7 GHz: Fluctuation Response of Particle Current and Potential Energy



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Conclusions

- Quantum Van der Pol oscillator provides quantitative values for the large signal switching and recovery times of RTD.
- Large signal frequencies near 120 GHz for non-optimized structure.
- Small signal oscillations sustained at higher frequencies. ($f > 670$ GHz).
- Quantum Van der Pol oscillators recover from fluctuations within one cycle.
- Phase noise dominates.

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Speculations

- Periodic invariance is probably due to time increments. If use was made of femtosecond increments would probably see high frequency contributions.
- Quantum potential interpretation not as simple for higher (> 300 meV) barriers.

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Non-Equilibrium Green's Functions for MOSFET Modeling

Dejan Jovanovic
Benjamin Liu
Roland Stumpf
Keith Beardmore
Ramesh Venugopa (Purdue)
Mark Lundstrom (Purdue)
Supriyo Datta (Purdue)

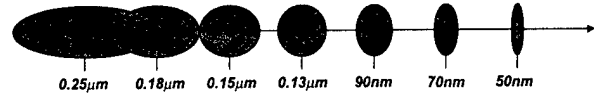
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Conventional TCAD

- Process-Device simulation (TCAD) is used by the semiconductor industry to expedite and optimize process development
- TCAD enables the efficient use of experimental cycles and provides major cost/time savings during early design iterations
- TCAD device models are based on semi-classical transport theory which relies on approximations to facilitate performance efficiency
- Semiclassical transport models will become increasingly inaccurate with decreasing L_{gate} due to the onset of quantum effects

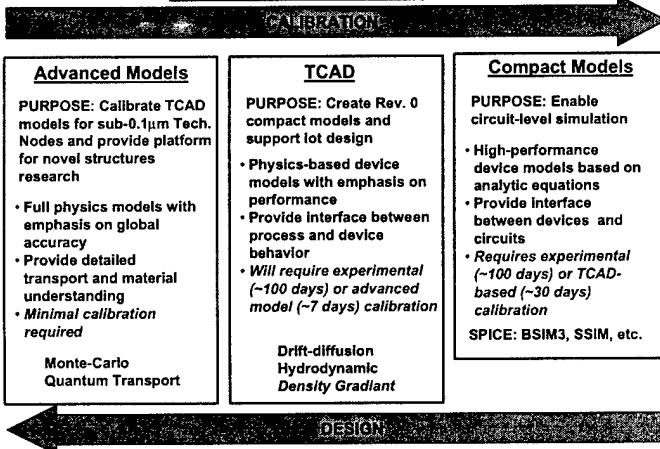


Experimental calibration will be increasingly required to calibrate TCAD device simulators which will undermine their usefulness

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Device Model Deployment



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Comparison of Contemporary Device Simulators

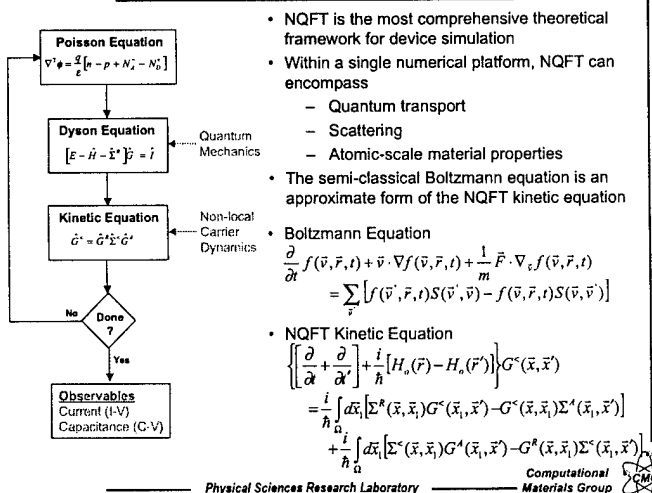
Device Simulator	Transport Model	Scattering	Quantum Effects	Leakage	Material Properties	Biasing		Performance
						$V_G < V_T$	$V_G > V_T$	
Drift-Diffusion	Semi-Classical	Mobility based		Sub-threshold				1 hr/bias
Hydro-Dynamic	Semi-Classical	Mobility based		Sub-threshold				1 hr/bias
Density Gradient	Semi-Classical	Mobility based	Quantum Potentials	Sub-threshold				1 hr/bias
Monte Carlo	Semi-Classical	Discrete	Sub-threshold		Advanced			~1 hr/bias
Quantum Transport	Quantum Mechanical	Discrete	Quantum Effects					

Good
Average
Poor

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Nonequilibrium Quantum Field Theory



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Principal Equations

$$[E - \hat{H}_0(\vec{k}_1) - \hat{\Sigma}^R(E, \vec{k}_1)] \hat{G}^R(E, \vec{k}_1) = \hat{I} \quad \leftarrow \text{Impulse response}$$

$$G_{ij}^c(E, \vec{k}_1) = \sum_{i,j} G_{i,j}^R(E, \vec{k}_1) \Sigma_{i,j}^c(E, \vec{k}_1) G_{i,j}^A(E, \vec{k}_1) + g_{i,j}^c(E, \vec{k}_1) \quad \leftarrow \text{Convolution Integral}$$

Scattering

$$\Sigma_{ij}^R(E, \vec{k}_1) = \frac{1}{(2\pi)^3} \sum_{\vec{k}_2} \int dE' \int d\vec{r} d\vec{r}' V_{i,j}(\vec{r}, \vec{r}') G_{i,j}^R(E, \vec{k}_1, E', \vec{k}_2') G_{i,j}^R(E', \vec{k}_2, E, \vec{k}_1)$$

$$\Sigma_{ij}^c(E, \vec{k}_1) = \frac{1}{(2\pi)^3} \sum_{\vec{k}_2} \int dE' \int d\vec{r} d\vec{r}' V_{i,j}(\vec{r}, \vec{r}') G_{i,j}^R(E, \vec{k}_1, E', \vec{k}_2') G_{i,j}^c(E', \vec{k}_2, E, \vec{k}_1)$$

$$\frac{1}{\tau_{ij}^R(E)} \Leftrightarrow -\frac{2}{\hbar} \text{Im}[\Sigma_{ij}^R(E, \vec{k}_1)]$$

Scattering is both the key and impediment to numerical quantum transport simulation

Definitions

- $\hat{H}_0(\vec{k}_1)$ - Tight-binding Hamiltonian
- $\hat{\Sigma}^R(E, \vec{k}_1)$ - Retarded self-energy (out-scattering)
- $\hat{G}^R(E, \vec{k}_1)$ - Retarded Green Function (Propagator)
- $\hat{G}^c(E, \vec{k}_1)$ - Zero-order correlation function
- $\Sigma_{ij}^c(E, \vec{k}_1)$ - Causal Self-Energy (in-scattering)
- $G_{ij}^c(E, \vec{k}_1)$ - Full correlation function
- $V_{ij}(\vec{r}, \vec{r}', E, \vec{k}_1)$ - Interaction Potential (Green Function)

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Nonequilibrium Quantum Field Theory Physical Observables

Density of States

$$A_q(E, \vec{k}_\perp) = -2 \operatorname{Im} [G^R_q(E, \vec{k}_\perp)] \quad (\text{quantum mechanical DOS})$$

Electrons

$$G^R_q(E, \vec{k}_\perp) = i f(E, \phi_F) A_q(E, \vec{k}_\perp) \quad (\text{equilibrium})$$

$$= i f(E, \phi_F^L) A_q^L(E, \vec{k}_\perp) + i f(E, \phi_F^R) A_q^R(E, \vec{k}_\perp) \quad (\text{non-equilibrium})$$

$$n_i = -2i \int \frac{dE}{2\pi} \int \frac{d\vec{k}_\perp}{(2\pi)^2} G^R_i(E, \vec{k}_\perp) \quad (\text{electron density})$$

$$J_i = \frac{2e}{\hbar} \int \frac{dE}{2\pi} \int \frac{d\vec{k}_\perp}{(2\pi)^2} [H_{i,i+1}(\vec{k}_\perp) G^R_{i+1,i}(E, \vec{k}_\perp) - G^R_{i,i+1}(E, \vec{k}_\perp) H_{i+1,i}(\vec{k}_\perp)] \quad (\text{electron current})$$

Holes

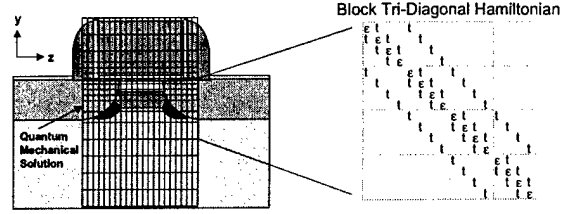
$$G^R_h(E, \vec{k}_\perp) = -i [1 - f(E, \phi_F)] A_h(E, \vec{k}_\perp) \quad (\text{equilibrium})$$

$$= -i [1 - f(E, \phi_F^L)] A_h^L(E, \vec{k}_\perp) - i [1 - f(E, \phi_F^R)] A_h^R(E, \vec{k}_\perp) \quad (\text{non-equilibrium})$$

$$p_i = -2i \int \frac{dE}{2\pi} \int \frac{d\vec{k}_\perp}{(2\pi)^2} G^R_i(E, \vec{k}_\perp) \quad (\text{hole density})$$

$$J_i = \frac{2e}{\hbar} \int \frac{dE}{2\pi} \int \frac{d\vec{k}_\perp}{(2\pi)^2} [H_{i,i+1}(\vec{k}_\perp) G^R_{i+1,i}(E, \vec{k}_\perp) - G^R_{i,i+1}(E, \vec{k}_\perp) H_{i+1,i}(\vec{k}_\perp)] \quad (\text{hole current})$$

Nonequilibrium Quantum Field Theory Spatial Discretization



- Presently using multi-valley effective-mass based discretization
- Simulated bandstructure is intrinsically linked to grid morphology in lateral (z) direction

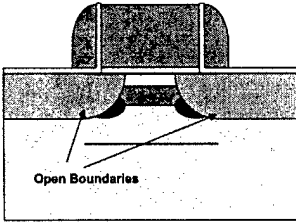
$$E(k_z) = \frac{\hbar^2}{m_z^* \Delta_z} [1 - \cos(k_z \Delta_z)]$$

- Uniform grid is required for the lateral (z) direction to avoid spurious reflections
- Non-uniform grid can be used in vertical (y) direction

Nonequilibrium Quantum Field Theory Boundary Conditions for Green's and Correlation Functions

Boundary condition assumptions

- Contacts are modeled as translationally invariant open boundaries and are assumed to be in local equilibrium
- Closed boundaries are set where G^R and $G^<$ can be safely assumed to be negligible



Boundary eigenvalue problem

$$\left[Z_m + (Z_m)^{-1} - \frac{E - \epsilon_j}{t_j} \right] \phi_{j,m} = 0$$

Boundary Green's function

$$\tilde{g}_{0,1}^R = Q Z Q^{-1} \tilde{g}_{0,0}^R$$

$$\tilde{g}_{0,0}^R = [E - \epsilon - i Q Z Q^{-1}]^{-1}$$

$$\tilde{g}_{0,0}^< = -i 2 f(\phi_F^L) \operatorname{Im} [\tilde{g}_{0,0}^R]$$

Nonequilibrium Quantum Field Theory Recursive Solution of Dyson and Kinetic Equations

General Dyson equations for G^R and $G^<$

$$\hat{G}_{i,j}^R = \hat{g}_{i,j}^R + \sum_{j,k} \hat{g}_{i,j}^R \hat{\Sigma}_{j,k}^R \hat{G}_{k,j}^R$$

$$\hat{G}_{i,j}^< = \hat{g}_{i,j}^< + \sum_{j,k} \hat{g}_{i,j}^< \hat{\Sigma}_{j,k}^A \hat{G}_{k,j}^A + \sum_{j,k} \hat{g}_{i,j}^< \hat{\Sigma}_{j,k}^R \hat{G}_{k,j}^R + \sum_{j,k} \hat{g}_{i,j}^< \hat{\Sigma}_{j,k}^< \hat{G}_{k,j}^<$$

Recursive Greens Function (RGF) Technique

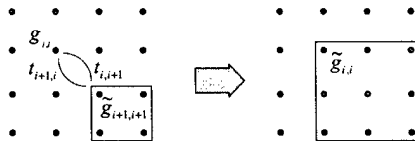
- Direct solution of the Green's functions is computationally prohibited ($N_z N_y^3$)
- Physical observables are clustered along the Green's function diagonals
- Dyson equations can be used to recursively construct Green and correlation function diagonals in a two-step manner $N_z N_y^3$
- Perturbative elements are the off-diagonal elements of the Hamiltonian

$$\tilde{g}_{i,j}^R = g_{i,j}^R + \sum_{j,k} g_{i,j}^R t_{j,k} \tilde{g}_{k,j}^R$$

$$\tilde{g}_{i,j}^< = g_{i,j}^< + \sum_{j,k} g_{i,j}^< t_{j,k} \tilde{g}_{k,j}^A + \sum_{j,k} g_{i,j}^< t_{j,k} \tilde{g}_{k,j}^<$$

- Self-consistent scattering can be efficiently incorporated into the RGF approach

Nonequilibrium Quantum Field Theory Recursive Solution of Semi-infinite Green/Correlation Functions



$$\tilde{g}_{i,j}^R = g_{i,j}^R + g_{i,j}^R t_{j,i+1} \tilde{g}_{i+1,j}^R$$

$$= g_{i,j}^R + g_{i,j}^R t_{j,i+1} \tilde{g}_{i+1,j+1}^R t_{i+1,j} \tilde{g}_{i,j}^R$$

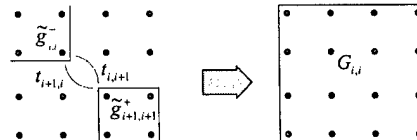
$$\tilde{g}_{i,j}^< = g_{i,j}^< + g_{i,j}^< t_{j,i+1} \tilde{g}_{i+1,j}^A + g_{i,j}^< t_{j,i+1} \tilde{g}_{i+1,j}^<$$

$$= g_{i,j}^< + g_{i,j}^< t_{j,i+1} \tilde{g}_{i+1,j+1}^A t_{i+1,j} \tilde{g}_{i,j}^A + g_{i,j}^< t_{j,i+1} \tilde{g}_{i+1,j+1}^< t_{i+1,j} \tilde{g}_{i,j}^<$$

$$\tilde{g}_{i,j}^R = [g_{i,j}^R]^{-1} - t_{j,i+1} \tilde{g}_{i+1,j+1}^R t_{i+1,j}$$

$$\tilde{g}_{i,j}^< = \tilde{g}_{i,j}^R [g_{i,j}^< + t_{j,i+1} \tilde{g}_{i+1,j+1}^< t_{i+1,j}] g_{i,j}^A$$

Nonequilibrium Quantum Field Theory Recursive Solution of Full Green/Correlation Functions



$$G_{i,j}^R = \tilde{g}_{i,j}^R + \tilde{g}_{i,j}^R t_{j,i+1} G_{i+1,j+1}^R t_{i+1,j} \tilde{g}_{i,j}^R$$

$$= [\tilde{g}_{i,j}^R]^{-1} - t_{j,i+1} \tilde{g}_{i+1,j+1}^R t_{i+1,j}$$

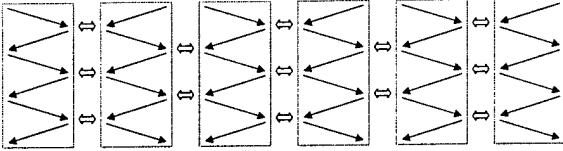
$$G_{i,j}^< = \tilde{g}_{i,j}^< + \tilde{g}_{i,j}^< t_{j,i+1} G_{i+1,j+1}^A t_{i+1,j} \tilde{g}_{i,j}^< + \tilde{g}_{i,j}^< t_{j,i+1} G_{i+1,j+1}^< t_{i+1,j} \tilde{g}_{i,j}^<$$

$$= G_{i,j}^R [g_{i,j}^< + t_{j,i+1} \tilde{g}_{i+1,j+1}^< t_{i+1,j}] G_{i,j}^A$$

Nonequilibrium Quantum Field Theory

Parallel Implementation of the Recursive Green's Function Method

- Parallel energy integration**
 - Green's function calculations at each energy are distributed to slave processors
 - Found to be highly inefficient for scattering simulations due to varying execution time per node
- Parallel Recursive Green's function algorithm**
 - Spatial grid topology is distributed across processors
 - Cross element coupling terms are calculated in parallel
 - Boundary Green's functions are calculated and distributed to enable parallel calculation of G^R
 - Correlation functions are then iteratively calculated until scattering self-consistency is achieved



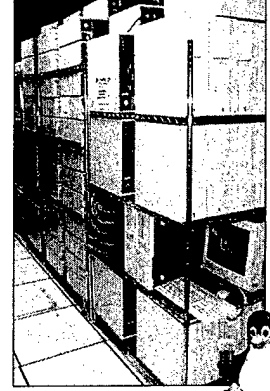
- 80% parallel efficiency is exhibited for 60 processor simulations

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Quantum Mechanical Simulation of MOSFETs Numerical Issues and Computational Platforms

- Algorithms**
 - Numerical NQFT solutions rely heavily on matrix inversion and multiplication
- Computing Platforms**
 - Commodity clusters are the enabling platforms for 2D NQFT simulation
 - All runs were performed on a 110 node 2xPIII 450MHz Linux cluster which cost under \$250,000
- Performance**
 - 90nm Ballistic simulations: ~1 hour/bias point
 - 90nm Scattering simulations: ~8 hour/bias point



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Scattering Model

- Kinetic Equation**

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \vec{v} \right] G^r(\vec{r}, \vec{r}') = \frac{i}{\hbar} [H_s(\vec{r}) - H_s(\vec{r}')] G^r(\vec{r}, \vec{r}') + \frac{i}{\hbar} \int d\vec{r}_1 [\Sigma^A(\vec{r}, \vec{r}_1) G^r(\vec{r}_1, \vec{r}') - G^r(\vec{r}, \vec{r}_1) \Sigma^A(\vec{r}_1, \vec{r}') + \Sigma^A(\vec{r}, \vec{r}_1) G^A(\vec{r}_1, \vec{r}') - G^A(\vec{r}, \vec{r}_1) \Sigma^A(\vec{r}_1, \vec{r}')] G^r(\vec{r}, \vec{r}')$$
- Current Continuity**

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \vec{v} \right] G^r(\vec{r}, \vec{r}') = \frac{\partial}{\partial t} n(\vec{r}, t) + \vec{\nabla} \cdot \vec{J}(\vec{r}, t) = 0$$

$$\int d\vec{r}_1 [\Gamma(\vec{r}, \vec{r}_1) G^r(\vec{r}_1, \vec{r}') - \Sigma^A(\vec{r}, \vec{r}_1) A(\vec{r}_1, \vec{r}')] = 0$$
- Rate-Based Scattering Model**

$$\Sigma_{i,j}^A(E, \vec{k}) \equiv i\delta_{i,j} \delta_{j,i} \int \frac{dE'}{2\pi} \int \frac{d\vec{k}'}{2\pi} \lambda_{i,j}(E, \vec{k}, E', \vec{k}') \equiv -i \frac{\Gamma_{i,j}(E, \vec{k})}{2}$$

$$\Sigma_{i,j}^r(E, \vec{k}) \equiv -2\delta_{i,j} \delta_{j,i} \int \frac{dE'}{2\pi} \int \frac{d\vec{k}'}{2\pi} \lambda_{i,j}(E, \vec{k}, E', \vec{k}') \equiv \frac{\tilde{\Gamma}_{i,j}(E, \vec{k})}{A_{i,j}(E, \vec{k})} G_{i,j}^r(E, \vec{k})$$

Approximation:

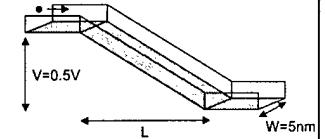
$$\Gamma_{i,j}(E, \vec{k}) \equiv \tilde{\Gamma}_{i,j}(E, \vec{k}) \equiv \Gamma_{i,j}^0(E) \delta_{i,j}$$
 - Enables efficient self-consistent scattering calculation
 - Simple calibration
 - Accounts for resistive effects (e.g. L_{off} scaling)

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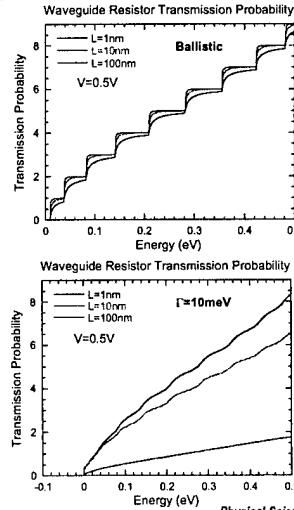
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Empirical Scattering Model

- Resistive electron wave-guide



- Ballistic Case**
 - Subband structure is apparent and retained up to 100nm
 - Variation in L causes subband smearing but no change in transmission envelope
 - Resistive effects are not accounted for
- Constant-Rate model**
 - Accounts for incoherence effects and back-scattering
 - Subband structure disappears for L=100nm
 - Attenuation of transmission envelope occurs for increasing L
 - Can account for resistive effects with proper calibration



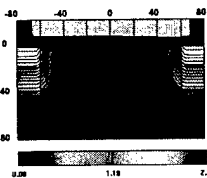
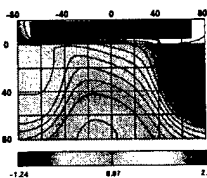
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MIT Reference MOSFETs

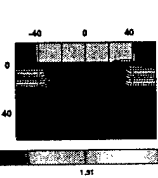
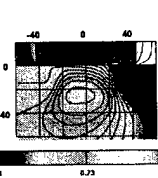
MIT 90nm MOSFET

- $L_{poly} = 0.13\mu m$
- $t_{ox} = 4.5nm$



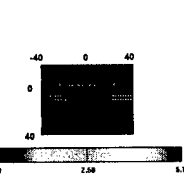
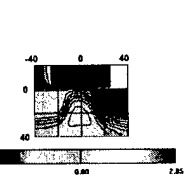
MIT 50nm MOSFET

- $L_{poly} = 85nm$
- $t_{ox} = 2.0nm$



MIT 25nm MOSFET

- $L_{poly} = 50nm$
- $t_{ox} = 1.5nm$

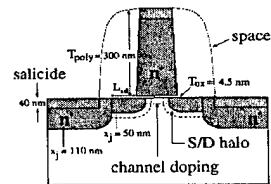


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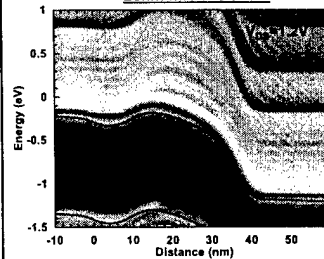
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MOSFET Quantum Mechanical Effects Sub-bands

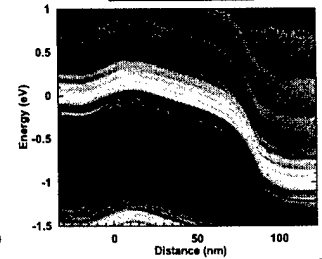
- Quantum mechanical DOS (spectral function) data taken at Si-SiO₂ interface
- Striations in DOS plots are sub-bands
- Spectral shift evident near source barrier
- Multiple sub-bands are required for accurate scattering calculations



25nm MIT MOSFET

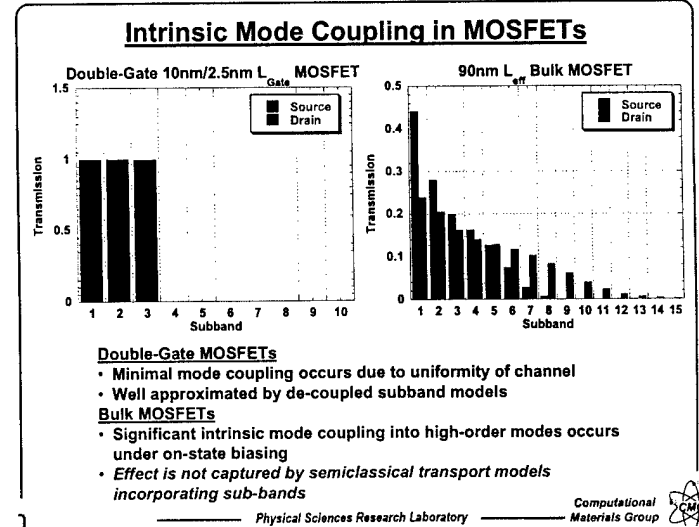
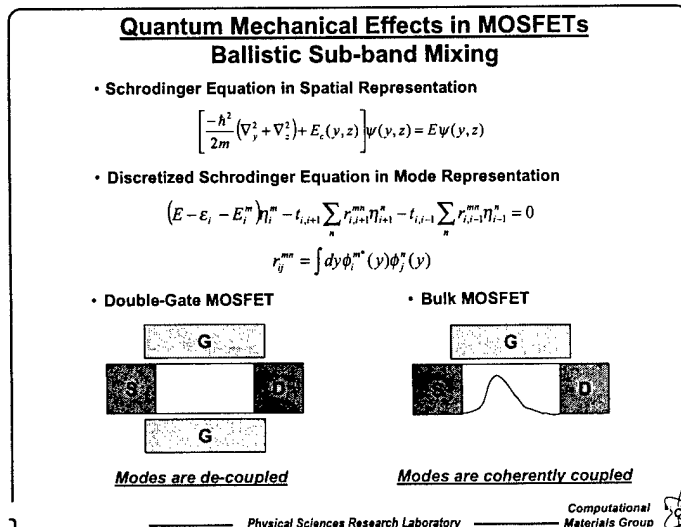
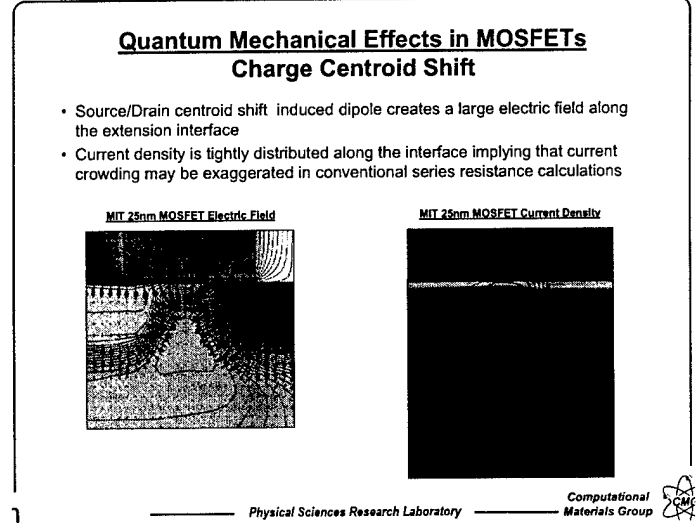
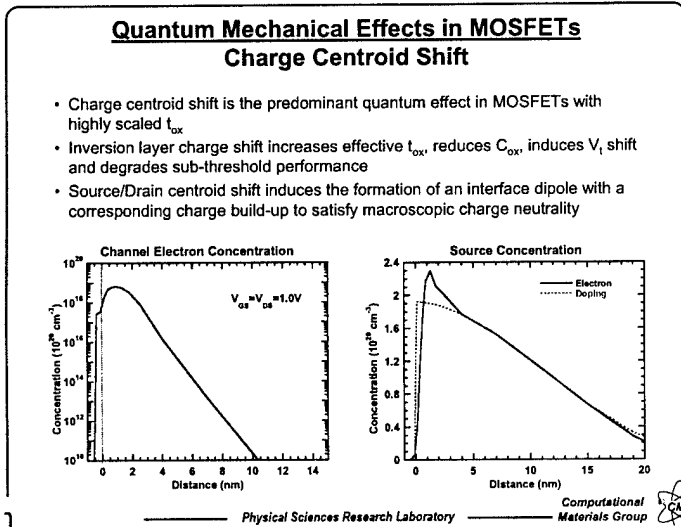
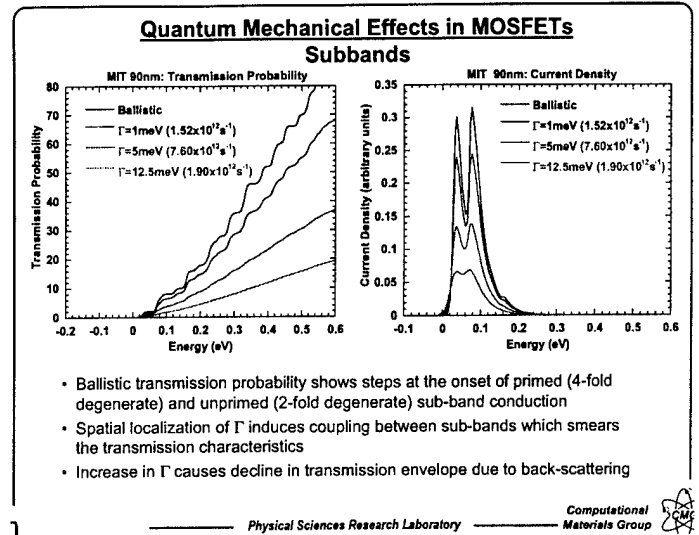
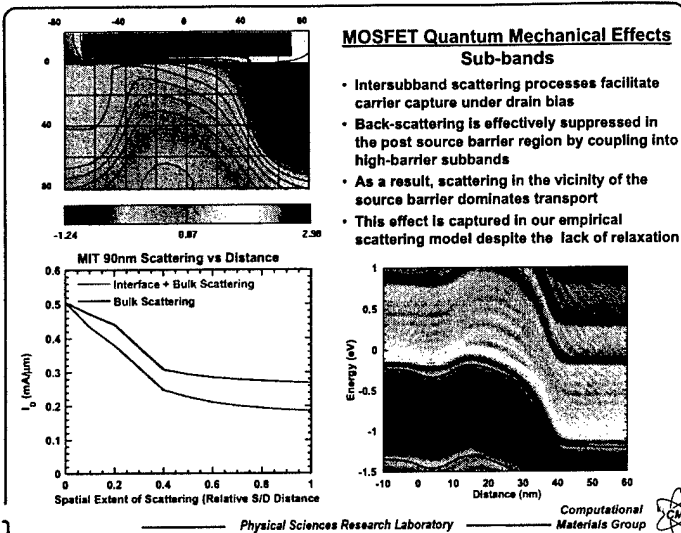


90nm MIT MOSFET



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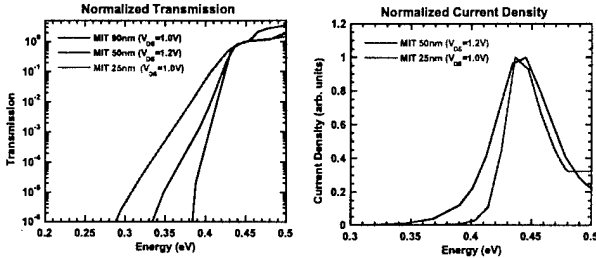
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Quantum Mechanical Effects in MOSFETs

Sub-Threshold Tunneling

- Sub-threshold tunneling through the source barrier enhances off-state leakage and compromises the I_{on}/I_{off} trade-off
- 50nm L_{eff} MOSFET exhibits a ~25% I_{off} increase
- 25nm L_{eff} MOSFET exhibits a ~40% I_{off} increase
- Further complications may arise from trap-assisted tunneling due to S/D extension lateral dopant diffusion into the channel

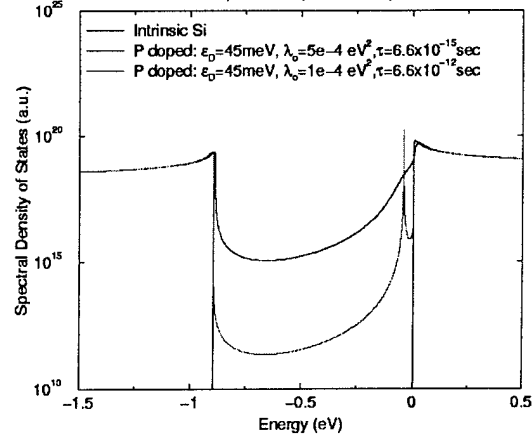


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Si Spectral DOS

Phosphorus-doped vs undoped



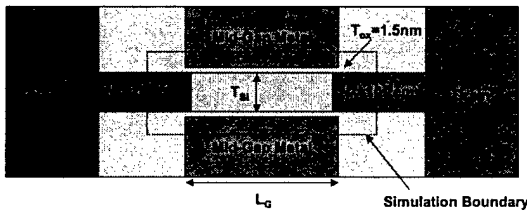
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Quantum Mechanical Effects in MOSFETs

I_D - V_{GS} Saturation

Gate



- Double-Gate MOSFETs are predicted to display optimal scaling properties and carry the potential of realizing (sub?) 10nm L_g technologies
- However, electrostatics dictates that L_g/T_{si} remain roughly constant to suppress short-channel effects and achieve target I_{on}/I_{off}

Scaling Study

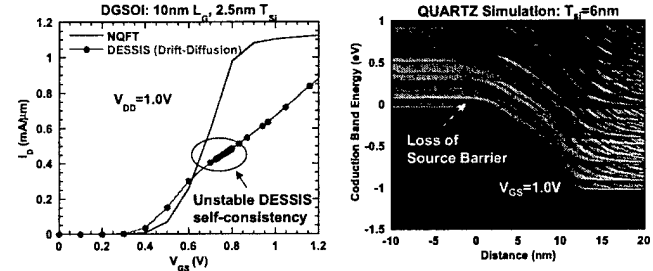
- Set $L_g/T_{si} = 4$ to control SCE ... keeping $t_{ox} = 1.5nm$ constant
- Evaluate device performance for $10nm < L_g < 80nm$

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Double-Gate SOI Simulation Study

Saturation of Gate Control



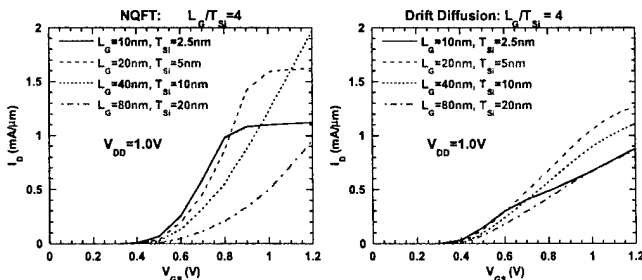
- Drift-diffusion simulations show an instability in self-consistency for $V_{GS} > 0.8V$ due to breakdown of macroscopic charge neutrality at the source boundary
- Boundary charge neutrality is maintained for quantum transport simulations by invoking the equilibrium approximation in the S/D extensions
- Quantum transport simulation predicts current saturation with $V_{GS} = 0.8V$ due to elimination of source barrier

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Double-Gate SOI Simulation Study

Impact of Current Saturation on Device Scaling



- Cases studied have $L_g/T_{si} = 4$ to maintain control of SCE with L_g scaling
- QUARTZ predicts I_{dsat} @ $V_{DD} = 1.0V$ increases with decreasing L_g down to ~30-40nm
- Subsequent reductions in L_g lead to reduced I_{dsat} @ $V_{DD} = 1.0V$ due to elimination of source barrier and current bottleneck with decreasing T_{si}
- A technology with $V_{DD} = 0.6V$ show increasing I_{dsat} with L_g for all cases studied
- DESSIS shows reduction of I_{dsat} at $L_g = 10-20nm$

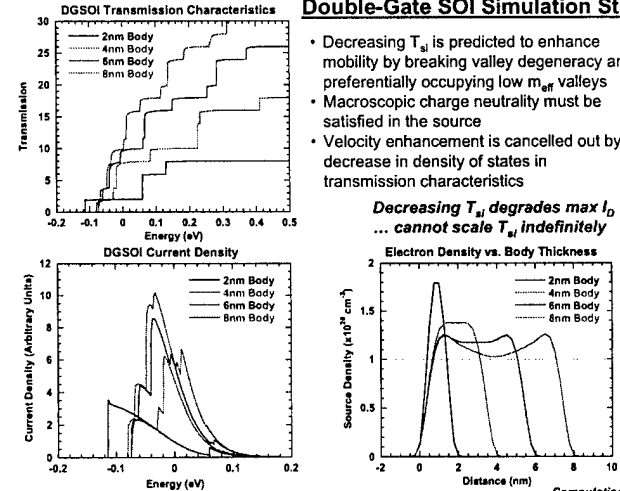
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Double-Gate SOI Simulation Study

- Decreasing T_{si} is predicted to enhance mobility by breaking valley degeneracy and preferentially occupying low m_{eff} valleys
- Macroscopic charge neutrality must be satisfied in the source
- Velocity enhancement is cancelled out by decrease in density of states in transmission characteristics

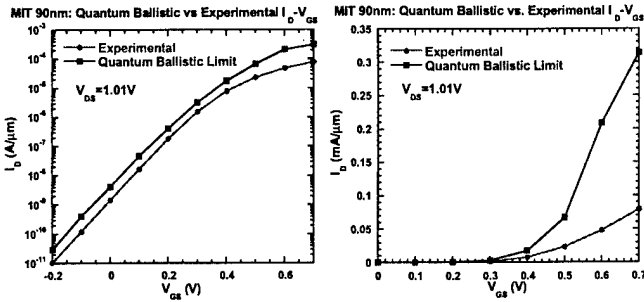
Decreasing T_{si} degrades max I_D
... cannot scale T_{si} indefinitely



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Quantum Transport Simulations of MOSFETs Scattering Model Calibration

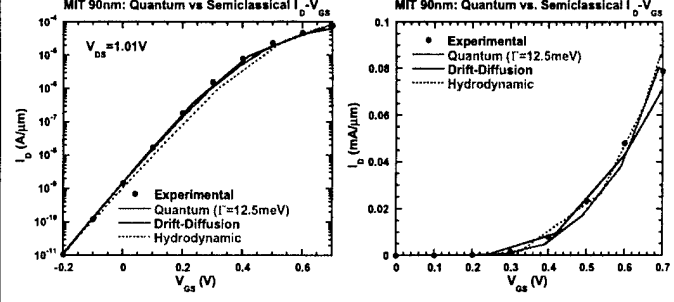


- Discrepancy in quantum mechanical sub-threshold current is due to non-locality of self-consistent charge
- Device drive current is 25% of theoretical limit

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MIT 90nm Device Quantum Mechanical vs Semiclassical Transport

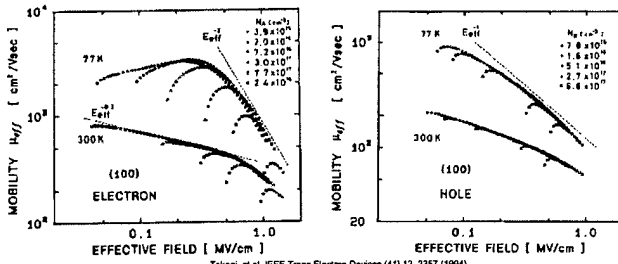


- Hydrodynamic model shows minor current discrepancy in sub-threshold regime
- Drift-diffusion model shows minor current discrepancy for saturation biases
- Quantum-mechanical simulation using single parameter rate-based scattering model shows good overall fit with experimental data

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CMOS Effective Mobility



- Effective mobility is a measure of the local mobility averaged over the vertical carrier density in the channel
- Both nMOS and pMOS show a decrease in effective mobility with increasing effective field (inversion charge density)
- Roll-off of mobility has been generally attributed to interface roughness
- Atomistic interpretation has been lacking

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Non-Equilibrium Quantum Field Theory Calibration Mobility Model

- Linear Response Conductivity

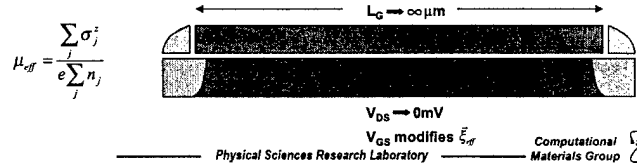
$$\langle \sigma_z \rangle = \lim_{\omega \rightarrow 0} \text{Im} \left\{ \frac{\Pi(\vec{x}, \vec{x}^*)}{\omega} \right\} \quad (\text{Current-Current Response Function})$$

$$= \frac{e^2 \hbar^3}{8m^2} \lim_{\vec{k}_i \rightarrow \vec{k}_f} [\nabla_z - \nabla_z^*] \int \frac{dE}{2\pi} \frac{\partial f(E)}{\partial E} \int \frac{dk_z}{2\pi} \int d\vec{r}_{11} \lim_{\vec{r}_{12} \rightarrow \vec{r}_{21}} [\nabla_{z_1} - \nabla_{z_2}] A(\vec{r}_{11}, \vec{r}_{12}, k_z, E) A(\vec{r}_{12}, \vec{r}_{21}, k_z, E)$$

- Conductivity: Final Form

$$\sigma_j^z = \frac{2e^2}{h} \int dE \frac{\partial f(E)}{\partial E} \int \frac{dk_z}{2\pi} \sum_m \phi_m \phi_m^* a \text{Re} \left\{ \frac{(e^{i\lambda_m} - e^{-i\lambda_m})^2}{v_m \Gamma_m(E)} + \frac{(e^{i2\lambda_m} - e^{-i2\lambda_m})}{v_m^2} \right\}$$

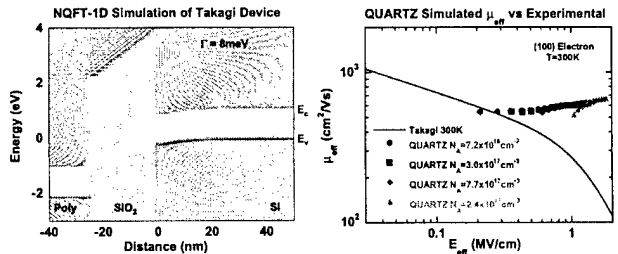
- Effective Mobility and Corresponding MOSFET Test Structure



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Non-Equilibrium Quantum Field Theory Mobility Model Single Parameter Scattering



- Single-parameter scattering model brings mobility into empirical range for $\Gamma=8\text{meV}$
- Universal mobility behavior is exhibited with the NQFT mobility model
- Strong disagreement with empirical universal mobility curve begins to occur for high effective fields ($E_{\text{eff}} > 0.3 \text{ MV/cm}$)
- Single-parameter scattering model alone is insufficient for capturing effective mobility behavior

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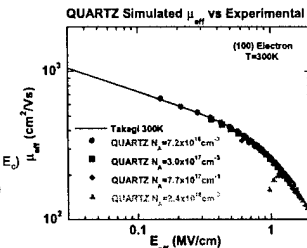
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Quantum Transport Simulated Mobility: Bulk + Interface Scattering Model

$$\Sigma_g^r(E, \vec{k}_\perp) = \frac{\lambda_g}{E - \epsilon_g + i\gamma} \delta_{g_i}$$

Parameters:

- $\lambda_g = 1.5\text{eV}$ (coupling strength)
- $\epsilon_g = 0.42$ (defect energy above E_c)
- $\gamma = \frac{\hbar}{\tau} = 90\text{meV}$ linewidth (lifetime)



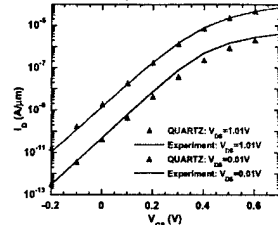
- Interface model assumes a strong peak of localized states exists in the Si conduction band due to atoms in the Si-SiO₂ transition layer
- Qualitative view of the interface suggests that there should be a transition layer that averages Si and SiO₂ conduction/valence band properties
- Bulk + interface scattering model produces universal mobility behavior

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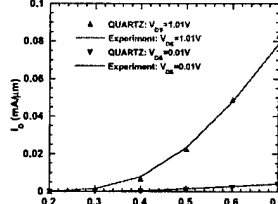
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90nm MIT MOSFET Comparison

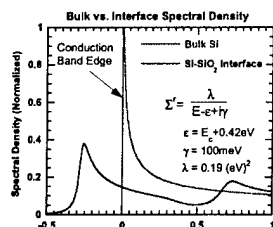
QUARTZ vs Experiment: MIT 90nm nMOS



QUARTZ vs Experiment: MIT 90nm nMOS



- Calibrated bulk+interface scattering model shows good agreement between theory and experiment
- Band-gap re-normalization leads to states extending into the Si band-gap
- Model agrees with recent gap-state measurements by Lacaita, Pacelli, et al.

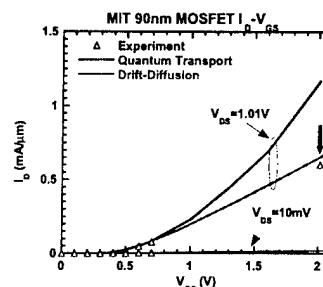


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90nm MIT MOSFET Comparison

Calibrated localized interface scattering model shows poor agreement with both experimental data and drift-diffusion simulation for high V_{GS}



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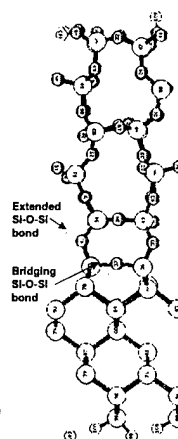
Si-SiO₂ Mobility Model Validation

Validation:

- Direct experimental validation is difficult due to mixing of interface and bulk states
- Density Functional Theory (DFT) electronic structure calculations can provide information on structure and density of states
- Existing DFT work has not focused on the detailed interface properties

DFT Calculations (Liu and Stumpff)

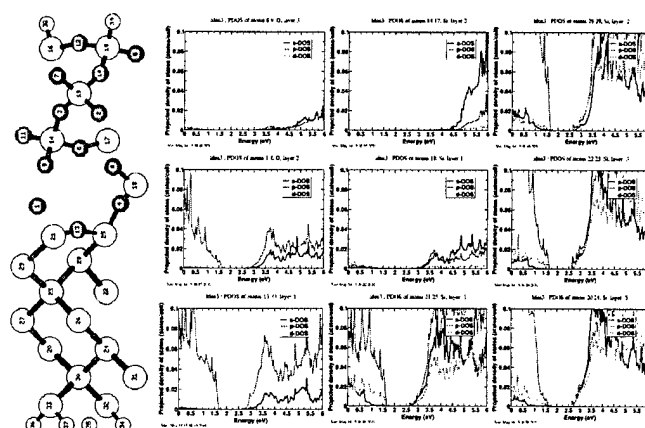
- Performed using VASP on CMG Linux cluster
- Examined three interface structures:
 - Si - SiO₂ (β-quartz)
 - Si - SiO₂ (tridymite)
 - Si - SiO₂ (β-cristobalite)
- Two basic interface Si-O-Si bonding structures observed:
 - Si-O-Si Bridging bonds
 - Si-O-Si Extended bonds
- The bridging and extended interface bonds are present in all SiO₂ structures and are periodically distributed along the interface



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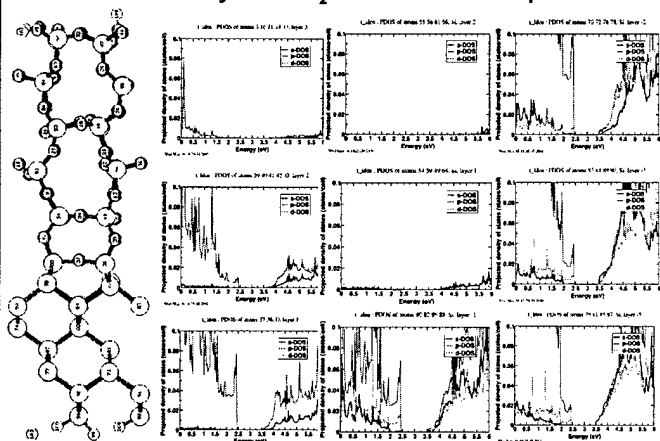
β Quartz SiO₂ / Si interface: Idos plots



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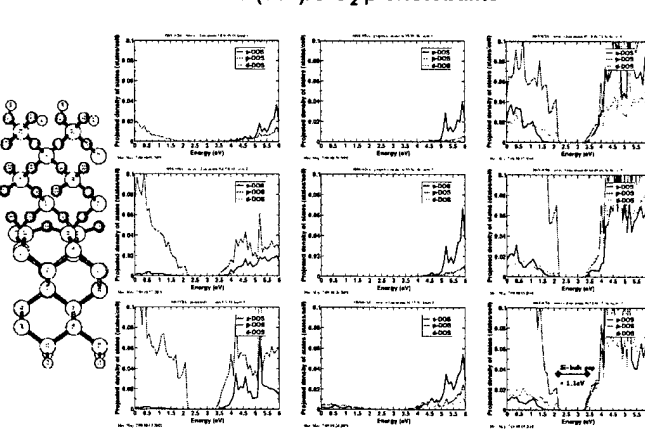
Tridymite SiO₂ / Si interface: Idos plots



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Si(001)/SiO₂ β-cristobalite

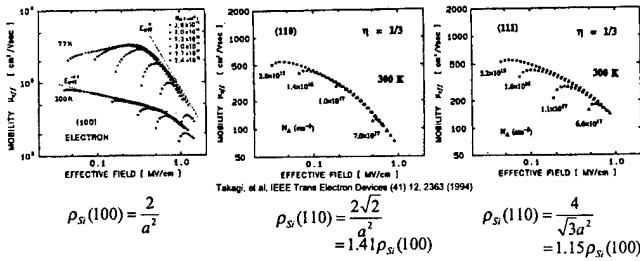


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Effective Mobility Dependence on Si Orientation

- Interface scattering may depend on
 - Non-tetrahedral atomic arrangement of bridging and extended interfacial O atoms
 - Distortion of Si atomic positions below the Si-SiO₂ interface
- If interfacial O and/or distorted Si atomic positions are responsible for high-field μ_{eff} behavior, there should be some correlation to Si areal density (ρ_{Si})
- Takagi's data shows that (110) Si orientation should have the strongest interface scattering followed by (111) then (100)



Scattering from interfacial O and/or distorted Si lattice as the cause of interfacial μ_{eff} degradation is supported by experimental data for orientation dependence

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Proposed Model for Effective Mobility Degradation due to Atomic Interface Scattering

The Coherent Picture

- As V_{GS} increases, carriers are increasingly driven into the interfacial O layers where there is a modification to the crystalline periodicity and DOS leading to coherent back-scattering
- As V_{GS} increases, carriers are also increasingly driven into the region with interfacial Si distortion where there is additional modification to the crystalline periodicity and DOS leading to coherent back-scattering
- Long range distortion of the interfacial atomic positions corresponds to interface roughness which may also play a role ...

The Incoherent Picture

- Phonon-assisted trapping in the interfacial O atoms de-phases electrons and leads to incoherent backscattering
- Other mechanisms ??

- Model is supported by Si orientation studies on μ_{eff} behavior
- Anomalous enhancement of nMOS mobility in strained Si devices may be explained by modification of interfacial atomic position leading to μ_{eff} enhancement

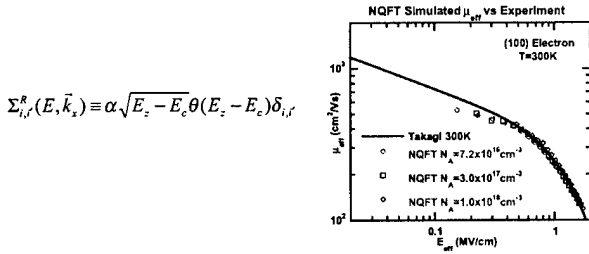
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Quantum Transport Simulated Mobility: Atomistic Interface Scattering Model Calibration

Interface scattering model calibration:

- A monotonically increasing Σ^R is used to account for the increasing interfacial O and distorted Si DOS
- The interface model is spatially extended 0.5nm into both the SiO₂ and Si to account for the spatial extent of the interfacial O and distorted Si
- Model is calibrated to effective mobility data

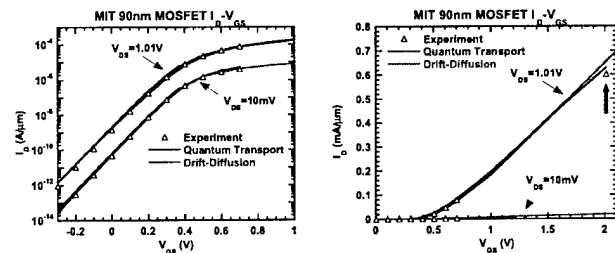


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Quantum Transport Simulated Mobility: Atomistic Interface Scattering Model Calibration

- Incorporation of the effective mobility calibrated interface model leads to excellent theory-experiment agreement for 2D MOSFET quantum transport simulation
- Quantum transport MOSFET simulation can now serve as the basis for TCAD device simulator calibration and device engineering studies

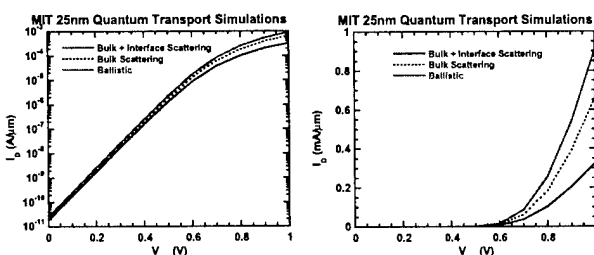


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Quantum Transport Simulated Mobility: Ballistic Transport vs Scattering

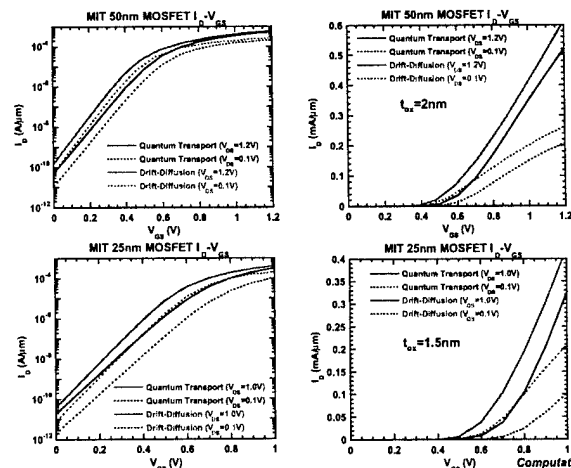
- Ballistic quantum transport simulations reveal that the MIT 25nm device is operating at ~30% of the ballistic limit
- Interface scattering dominates 25nm MIT device I_{Dsat}
- If the atomic interface scattering model is to be believed, there is little hope of ever achieving ballistic performance in MOSFETs



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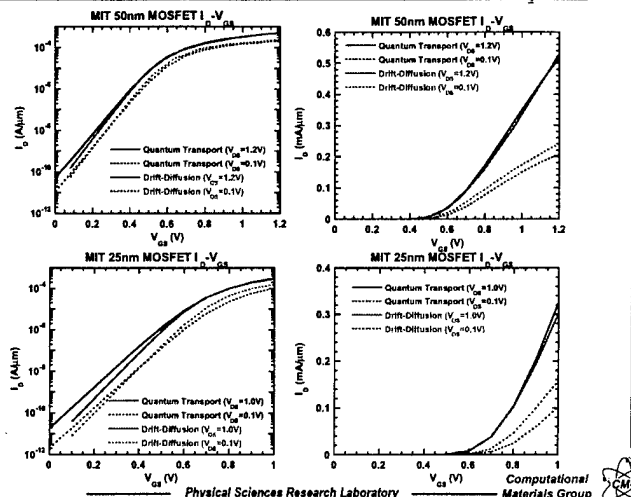
MIT MOSFET Simulation Comparison



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MIT MOSFET Simulation Comparison Eliminating V_t Shift



MIT MOSFET Simulation Comparison

Comparison Summary

- Centroid induced V_t shift is approximately 50meV for the 50nm MIT MOSFET ($t_{ox}=2nm$) and 100meV for the 25nm MIT MOSFET ($t_{ox}=1.5nm$) as compared to drift diffusion data
- Increasing DIBL, sub-threshold slope, and I_{on} are observed with decreasing L_{poly} (and t_{ox}) indicating the centroid shift may be compromising resistance to short channel effects
- Sub-threshold slope and I_{on} increase are also influenced by increasing sub-threshold tunneling with decreasing L_{poly} ... further study into relative effects of sub-threshold tunneling and centroid shift is required
- Drift-diffusion model show excellent agreement with calibrated quantum transport theory in predicting post-threshold behavior

Conclusion

Appropriately calibrated effective potential corrections applied to drift diffusion models will provide all the essential physics required for TCAD device simulation for the foreseeable future

Quantum kinetics and the femtosecond time scale in optical excitation of semiconductors

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ARW Quantum Transport in Semiconductors Maratea, 17-22 June 2001

Quantum kinetics in semiconductors overview

Introduction

Phonon quantum kinetics in homogeneous systems
density-matrix theory
relation to NGFs
energy-time uncertainty
nonequilibrium phonons
scattering between renormalized states
phonon quantum beats
intracollisional field effect

Phonon quantum kinetics in inhomogeneous systems
different representations
wave-packet dynamics
coherent phonons
carrier-trapping dynamics

Conclusions

Quantum kinetics in semiconductors introduction

Semiclassical transport and kinetics

Free flights

electrons accelerated by local electric field ($\sim \text{grad } V$)

Scattering processes

instantaneous in time
local in space
between well-defined k-states
different mechanisms additive

Dynamical equation

Boltzmann equation

Carrier-phonon interaction 1-band model

Hamiltonian

$$H = \sum_k \epsilon_k^{\dagger} c_k + \sum_q \hbar \omega_q b_q^{\dagger} b_q + \sum_{k,q} (g_q^{\dagger} c_{k+q}^{\dagger} b_q c_k + h.c.)$$

dynamical variables

distribution functions

$$f_k^{\dagger} = \langle c_k^{\dagger} c_k \rangle \quad n_q = \langle b_q^{\dagger} b_q \rangle$$

phonon-assisted density matrix

$$S_{k+q,k}^{\dagger} = \frac{i}{\hbar} g_q^{\dagger} \langle c_{k+q}^{\dagger} b_q c_k \rangle$$

$\Rightarrow$ correlations between electrons and phonons

equations of motion

$$\frac{d}{dt} f_k^{\dagger} = \sum_q [2\text{Re} \{ S_{k+q,k}^{\dagger} \} - 2\text{Re} \{ S_{k,k-q}^{\dagger} \}]$$

$$\frac{d}{dt} n_q = \sum_k 2\text{Re} \{ S_{k+q,k}^{\dagger} \}$$

Quantum kinetics in semiconductors introduction

Quantum effects on short time and length scales

Energy-time uncertainty
single-particle energies not conserved on short time-scales

Momentum-position uncertainty
no local transitions between well-defined momentum states

Memory effects

dynamics of distribution functions and polarizations non-local in time and space

Quantum-mechanical correlations

scattering between renormalized states
phonon-assisted transitions
intracollisional field effect

Electron-phonon quantum kinetics hierarchy

equation of motion for S

involves expectation values of 4 operators

$$\langle c_{k-q}^{\dagger} c_k^{\dagger} c_q c_k \rangle \quad \langle c_{k-q}^{\dagger} c_k^{\dagger} b_q^{\dagger} b_q \rangle$$

$\Rightarrow$ hierarchy of equations of motion

truncation by factorization

$$\langle c_{k-q}^{\dagger} c_k^{\dagger} b_q^{\dagger} b_q \rangle \approx f_k^{\dagger} n_q b_{k,k-q}^{\dagger} b_q$$

$\Rightarrow$ neglecting higher order correlations

factorized equation of motion

$$\frac{d}{dt} S_{k,k}^{\dagger} = \frac{i}{\hbar} (\epsilon_k^{\dagger} - \epsilon_k - \hbar \omega_q) S_{k,k}^{\dagger}$$

$$+ \frac{1}{\hbar^2} |g_q|^2 \left[f_k^{\dagger} (1 - f_k^{\dagger}) (n_q + 1) - (1 - f_k^{\dagger}) f_k^{\dagger} n_q \right]$$

$\Rightarrow$ quantum kinetic second Born approximation

Electron-phonon quantum kinetics memory effects and Markov limit

formal solution

$$S_{k,k}^{\epsilon}(t) = S_{k,k}^{\epsilon} \exp \left[\frac{i}{\hbar} (\epsilon_k^{\epsilon} - \epsilon_k^{\epsilon} - \hbar\omega_q) t \right] + \frac{1}{\hbar^2} |\rho_q|^2 \int_0^t dr \exp \left[\frac{i}{\hbar} (\epsilon_k^{\epsilon} - \epsilon_k^{\epsilon} - \hbar\omega_q) r \right] \times [f_k(1-f_k)(n_q+1) - (1-f_k)f_k n_q]_{(t-r)}$$

⇒ Dynamics with memory

Markov limit

$$\begin{aligned} \frac{d}{dt} f_k &= \frac{2\pi}{\hbar} \sum_{q \neq k} |\rho_q|^2 \left\{ \delta(\epsilon_k^{\epsilon} - \epsilon_k^{\epsilon} - \hbar\omega_q) \right. \\ &\quad \times \left[(1-f_k) \left(n_q + \frac{1}{2} \pm \frac{1}{2} \right) f_{k-q} - f_k (1-f_{k-q}) \left(n_q + \frac{1}{2} \pm \frac{1}{2} \right) \right] \\ \frac{d}{dt} n_q &= \frac{2\pi}{\hbar} \sum_k |\rho_q|^2 \delta(\epsilon_k^{\epsilon} - \epsilon_{k-q}^{\epsilon} - \hbar\omega_q) \\ &\quad \times \left[(1-f_k) n_q f_{k-q} - f_k (1-f_{k-q}) (n_q + 1) \right] \\ &\rightarrow \text{Boltzmann equations} \end{aligned}$$

Electron-phonon quantum kinetics diagonal approximation

Classification of higher-order contributions

equation of motion for $S_{k,k'}$ involves terms

(a) $\sim S_{k,k'}$ (b) $\sim \sum_{q,q'} S_{k+q,k'+q'}$
assumption: (b) small due to random phases of S
⇒ diagonal approximation: k -components decoupled

Markov approximation for $T^{(1)}$

$$\begin{aligned} \frac{d}{dt} S_{k,k}^{\epsilon} &= \left[\frac{i}{\hbar} (\epsilon_k^{\epsilon} - \epsilon_k^{\epsilon} - \hbar\omega_q) - \Gamma_k - \Gamma_1 \right] S_{k,k}^{\epsilon} \\ &\quad + \frac{1}{\hbar^2} |\rho_q|^2 [f_k(1-f_k)(n_q+1) - (1-f_k)f_k n_q] \\ \text{with} \quad \Gamma_k &= \frac{\pi}{\hbar} \sum_{q \neq k} |\rho_q|^2 \delta(\epsilon_k^{\epsilon} - \epsilon_{k+q}^{\epsilon} \pm \hbar\omega_q) \\ &\quad \times \left[(n_q + \frac{1}{2} \pm \frac{1}{2}) f_{k+q} + (n_q + \frac{1}{2} \mp \frac{1}{2}) (1-f_{k+q}) \right] \end{aligned}$$

⇒ polaron self energy (real part neglected)
⇒ quantum kinetics with damped memory

Electron-phonon quantum kinetics two-particle correlations

including deviations from factorization:

$$\begin{aligned} T_{k,k,q}^{(1)} &= \frac{g_0^2}{\hbar^2} \left[(c_{k+q}^{\dagger} b_{k-q}^{\dagger} c_k c_q) - \delta_{k-k,q} f_k^{\epsilon} n_q \right] \\ T_{k,k,q}^{(2)} &= \frac{g_0^2}{\hbar^2} (c_{k+q}^{\dagger} b_{k-q}^{\dagger} b_k c_q) \\ T_{k,k,q}^{(3)} &= \frac{|\rho_q|^2}{\hbar^2} \left[(c_{k+q}^{\dagger} c_k c_{k+q} c_q) + \delta_{k,k} f_k^{\epsilon} f_{k+q}^{\epsilon} \right] \end{aligned}$$

exact equation of motion for S

$$\begin{aligned} \frac{d}{dt} S_{k,k}^{\epsilon} &= \frac{i}{\hbar} (\epsilon_k^{\epsilon} - \epsilon_k^{\epsilon} - \hbar\omega_q) S_{k,k}^{\epsilon} \\ &\quad + \frac{1}{\hbar^2} |\rho_q|^2 [f_k(1-f_k)(n_q+1) - (1-f_k)f_k n_q] \\ &\quad + \sum_{k'} \left[T_{k,k',q}^{(1)} - T_{k,k',-q}^{(1)} + T_{k,k',q}^{(2)} - T_{k,k',-q}^{(2)} + T_{k,k',q}^{(3)} \right] \\ &\quad \text{(with } q = k - k') \end{aligned}$$

required: equations of motion for $T^{(i)}$

Electron-phonon quantum kinetics density matrices and Greens functions

Generalized Kadanoff-Baym Equation

$$\begin{aligned} \frac{d}{dt} f_k^{\epsilon}(t) &= -i \frac{d}{dt} G_k^{\epsilon}(t, t) \\ &= - \int_{-\infty}^t dt' [\Sigma_k^{\epsilon}(t, t') G_k^{\epsilon}(t', t) - \Sigma_k^{\epsilon}(t, t') G_k^{\epsilon}(t', t)] \\ &\quad - G_k^{\epsilon}(t, t') \Sigma_k^{\epsilon}(t', t) + G_k^{\epsilon}(t, t') \Sigma_k^{\epsilon}(t', t) \end{aligned}$$

self-energy

$$\Sigma_k^{\epsilon}(t, t') = \frac{i}{\hbar} \sum_q |\rho_q|^2 D_k^{\epsilon}(t, t') G_{k-q}^{\epsilon}(t, t')$$

Generalized Kadanoff-Baym Ansatz (GKBA)

$$G_k^{\epsilon}(t, t') = -G_k^{\epsilon}(t, t') f_k^{\epsilon}(t') + f_k^{\epsilon}(t) G_k^{\epsilon}(t, t')$$

second Born approximation:

GKBA with $G^{r(0)}, G^{a(0)}$ (no Σ)

fourth Born approximation, diagonal terms:

GKBA with G^{r^*}, G^a including Σ

Electron-phonon quantum kinetics two-particle correlations

truncation by factorization on 5-point level

$$(c_k^{\dagger} b_k^{\dagger} b_q c_q) \approx \delta_{k,q} n_q (c_k^{\dagger} b_{k-q} c_k) + \delta_{k,q} n_q (c_k^{\dagger} b_q c_k)$$

factorized equation of motion for T

$$\begin{aligned} \frac{d}{dt} T_{k,k,q}^{(1)} &= \frac{i}{\hbar} (\epsilon_{k+q}^{\epsilon} - \epsilon_k^{\epsilon} + \hbar\omega_q - \hbar\omega_q) T_{k,k,q}^{(1)} \\ &\quad + \frac{1}{\hbar^2} |\rho_q|^2 \left[(1 + n_q - f_{k+q}^{\epsilon}) S_{k+q,k}^{\epsilon} - (n_q + f_k^{\epsilon}) S_{k+q,k}^{\epsilon} \right] \\ &\quad + \frac{1}{\hbar^2} |\rho_q|^2 \left[(1 + n_q - f_k^{\epsilon}) S_{k,k+q}^{\epsilon} - (n_q + f_{k+q}^{\epsilon}) S_{k,k+q}^{\epsilon} \right] \\ &\quad \text{(with } q = k - k') \end{aligned}$$

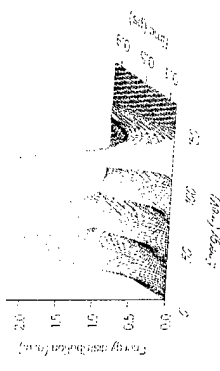
formal solution yields $T_{k,k,q}^{(1)}(t) = T_{k,k,q}^{(1)} [f_k, n_q, S_{k,k}^{\epsilon}]_{(t-\tau)}$

⇒ closed integro-differential equation for S

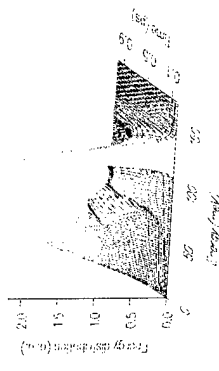
⇒ quantum kinetic fourth Born approximation

Carrier-phonon interaction 1-band model

Boltzmann

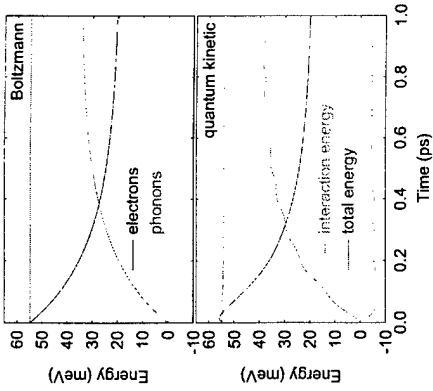


quantum kinetic



relaxation from given initial distribution
⇒ time-dependent broadening

Carrier-phonon interaction 1-band model



mean energy per electron
 $\Rightarrow$ energy conservation with interaction

Equations of motion phonon-assisted density matrices

$$\begin{aligned} \frac{d}{dt} S_{k,k}^{\pm} &= \frac{i}{\hbar} (\tilde{\epsilon}_k - \tilde{\epsilon}_k - \hbar\omega_{qp}) S_{k,k}^{\pm} \\ &+ i\tilde{\mu}_k T_{k,k}^{(+)} - i\tilde{\mu}_k T_{k,k}^{(-)} \\ &+ \frac{1}{\hbar} |g_q|^2 [(n_q + 1)(1 - f_k^{\pm}) f_k^{\mp} - n_q f_k^{\pm} (1 - f_k^{\mp})] \\ &- \frac{1}{\hbar} g_q^{\dagger} g_q \tilde{\epsilon}_k^{\pm} \tilde{\mu}_k \end{aligned}$$

$$\begin{aligned} \text{with } \tilde{\mu}_k &= M_k \cdot E_0(t) e^{-i\omega t} - \sum_q V_q p_{k+q} \\ \tilde{\epsilon}_k^{\pm} &= \epsilon_k^{\pm} - \sum_q V_q f_{k+q}^{\pm} \end{aligned}$$

cross terms (renormalizations)
 "Boltzmann" terms (scattering processes)
 polarization scattering (eh-coherence)

Carrier-phonon interaction 2-band model

dynamical variables

distribution functions

$$f_k^{\pm} = \langle c_k^{\dagger} c_k \rangle \quad n_q = \langle b_q^{\dagger} b_q \rangle$$

interband polarization

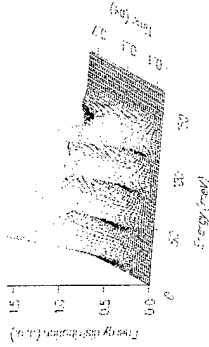
$$p_k = \langle d_{-k} c_k \rangle$$

phonon-assisted density matrices

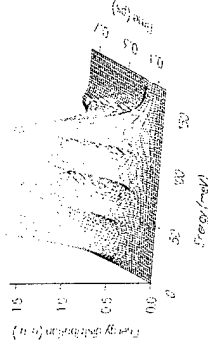
$$S_{k,k}^{\pm} = \frac{i}{\hbar} g_q^{\dagger} \langle c_k^{\dagger} b_q c_k \rangle \quad S_{-k,-k}^{\pm} = \frac{i}{\hbar} g_q^{\dagger} \langle d_{-k}^{\dagger} b_q d_{-k} \rangle$$

$$T_{-k,k}^{(+)} = \frac{i}{\hbar} g_q^{\dagger} \langle d_{-k} b_q c_k \rangle \quad T_{-k,k}^{(-)} = \frac{i}{\hbar} g_q^{\dagger} \langle c_k^{\dagger} b_q d_{-k} \rangle$$

Boltzmann generation and scattering



coherent generation, Boltzmann scattering



excitation with 100 fs pulse
 $\Rightarrow$ broadening of generation

Equations of motion distribution functions and polarization

$$\begin{aligned} \frac{d}{dt} f_k^{\pm} &= 2\Re \{ i\tilde{\mu}_k p_k \} \\ &+ \sum_q [2\Re \{ S_{k+q,k}^{\pm} \} - 2\Re \{ S_{k-k-q}^{\pm} \}] \\ \frac{d}{dt} f_{-k}^{\pm} &= 2\Re \{ i\tilde{\mu}_k^* p_k \} \\ &+ \sum_q [2\Re \{ S_{k+q,-k}^{\pm} \} - 2\Re \{ S_{-k-k-q}^{\pm} \}] \\ \frac{d}{dt} n_q &= \sum_k [2\Re \{ S_{k+q,k}^{\pm} \} + 2\Re \{ S_{k-k-q,-k}^{\pm} \}] \\ \frac{d}{dt} p_k &= -i\Omega_q^0 p_k - i\tilde{\mu}_k (1 - f_k^+ - f_k^-) \\ &+ \sum_q [T_{-k+q,k}^{(+)} - T_{-k+q,k}^{(-)} - T_{-k-k-q}^{(+)} + T_{-k-k-q}^{(-)}] \end{aligned}$$

$\Rightarrow$ semiconductor Bloch equations

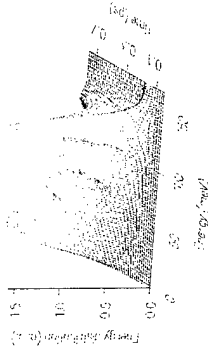
$$\text{with } \tilde{\mu}_k = M_k \cdot E_0(t) e^{-i\omega t} - \sum_q V_q p_{k+q}$$

$$\hbar\Omega_q^0 = \tilde{\epsilon}_k^+ + \tilde{\epsilon}_k^-$$

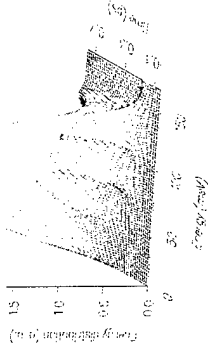
$$\tilde{\epsilon}_k^{\pm} = \epsilon_k^{\pm} - \sum_q V_q f_{k+q}^{\pm}$$

Carrier-phonon interaction 2-band model

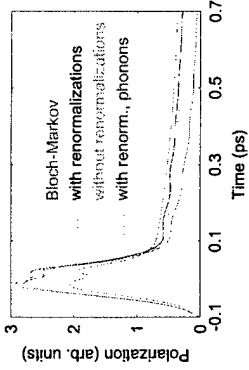
quantum kinetic



quantum kinetic with renormalizations



excitation with 100 fs pulse
 $\Rightarrow$ time-dependent broadening



decay of incoherently summed polarization
($P = \sum |p_k|$)
⇒ phonon quantum beats

Electron-phonon quantum kinetics spatially inhomogeneous system

density matrix theory: dynamical variables

distribution functions

$$f_{k\sigma} = \langle c_{k\sigma}^\dagger c_{k\sigma} \rangle \quad f_{k\sigma}^h = \langle d_{k\sigma}^\dagger d_{k\sigma} \rangle$$

interband polarization

$$p_{k\sigma} = \langle d_{-k\sigma} c_{k\sigma} \rangle \quad p_{k\sigma}^h = \langle c_{k\sigma}^\dagger d_{-k\sigma}^\dagger \rangle$$

coherent phonon amplitudes

$$B_q = \langle b_q \rangle \quad B_q^* = \langle b_q^\dagger \rangle$$

incoherent phonon distribution function

$$n_{q\sigma} = \langle (b_q^\dagger - B_q^*)(b_q - B_q) \rangle$$

phonon assisted density matrices

$$S_{k,q,\sigma}^h = \frac{i}{\hbar} g_q (c_{k-q,\sigma}^\dagger (b_q - B_q) c_{k,\sigma}) \quad S_{k,q,\sigma}^h = -\frac{i}{\hbar} g_q (d_{k,\sigma}^\dagger (b_q - B_q) d_{k-q,\sigma})$$

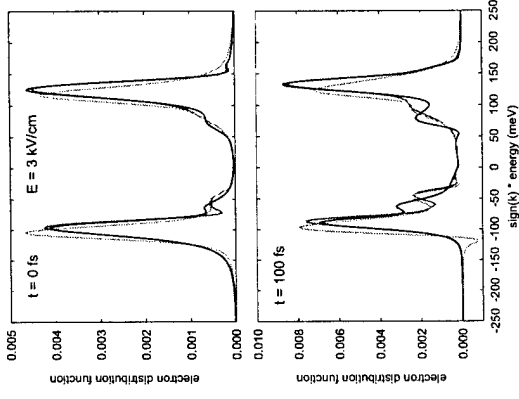
$$T_{k,q,\sigma}^{(+)} = \frac{i}{\hbar} g_q (d_{-k,\sigma} (b_q - B_q) c_{k,\sigma}) \quad T_{k,q,\sigma}^{(-)} = -\frac{i}{\hbar} g_q (c_{k,\sigma} (b_q^\dagger - B_q^*) d_{k-q,\sigma})$$

$$\begin{aligned} \frac{d}{dt} f_k^h &= \frac{eE}{\hbar} \frac{\partial}{\partial k_z} f_k^h + 2\text{Re} \{ i \tilde{u}_{k,q}^h p_k \} \\ &\quad + \sum_q [2\text{Re} \{ S_{k+q,k}^h \} - 2\text{Re} \{ S_{k,k-q}^h \}] \\ \frac{d}{dt} p_k &= \frac{eE}{\hbar} \frac{\partial}{\partial k_z} p_k - i(\tilde{\epsilon}_k^h + \tilde{\epsilon}_k^h) p_k - i \tilde{u}_{k,q}^h (1 - f_k^h - f_{-k}^h) \\ &\quad + \sum_q [T_{-k-q,k}^{(+)} - T_{-k+q,k}^{(+)} - T_{k,k-q}^{(+)} + T_{k,k+q}^{(+)}] \\ \frac{d}{dt} S_{k,k}^h &= \frac{eE}{\hbar} \left(\frac{\partial}{\partial k_z} + \frac{\partial}{\partial k_z} \right) S_{k,k}^h + \frac{i}{\hbar} (\tilde{\epsilon}_k^h - \tilde{\epsilon}_{-k}^h - \hbar \omega_{LO}) S_{k,k}^h \\ &\quad + i \tilde{u}_{k,q}^h T_{-k,k}^{(+)} - i \tilde{u}_{k,q}^h T_{k,k}^{(-)} - \frac{1}{\hbar^2} g_q^h p_k^h p_k \\ &\quad + \frac{1}{\hbar^2} |g_q^h|^2 [(n_q + 1)(1 - f_k^h) f_{-k}^h - n_q f_k^h (1 - f_{-k}^h)] \end{aligned}$$

$$\text{with } \hbar \tilde{u}_{k,q}^h = -M E - M_k \cdot E_0(t) e^{-i\omega_{LO} t} - \sum_q V_q p_{k+q}$$

$$\tilde{\epsilon}_k^h = \epsilon_k^h - \sum_q V_q f_{k+q}^h$$

acceleration terms
Zener term



drift term for f, p, S, T
drift term for f, p
drift term for f, S, T

Spatially inhomogeneous system equations of motion

phonon assisted density matrices

$$\begin{aligned} \frac{d}{dt} S_{k,q,k}^h &= \frac{i}{\hbar} \sum_k (\epsilon_{k-q,k}^h S_{k,q,k}^h - \epsilon_{k,k-q}^h S_{k,q,k}^h) - i \omega_{LO} S_{k,q,k}^h \\ &\quad + \frac{i}{\hbar} \sum_k (u_{k,q}^h T_{k,q,k}^{(+)} - u_{k,q}^h T_{k,q,k}^{(-)}) \\ &\quad + \frac{1}{\hbar^2} \sum_{q,k} g_q g_q^h [(n_{q,q} + \delta_{q,q}) f_{k,k+q}^h (\delta_{k,k-q}^h - f_{k,k-q}^h) \\ &\quad - n_{q,q} f_{k,k}^h (\delta_{k,k-q}^h - f_{k,k-q}^h)] \\ &\quad - \frac{1}{\hbar^2} |g_q|^2 p_{k,q}^h p_{k,q-k}^h \end{aligned}$$

renormalization of scattering dynamics due to:
external field (intracollisional field effect)
induced field
coherent phonons
exchange energy and Coulomb enhancement
spatial inhomogeneities

Spatially inhomogeneous system equations of motion

distribution functions

$$\begin{aligned} \frac{d}{dt} f_{k\sigma}^h &= \frac{i}{\hbar} \sum_q [\epsilon_{k-q,k}^h f_{k\sigma}^h - \epsilon_{k,k-q}^h f_{k\sigma}^h + u_{k,q}^h p_{k\sigma}^h - u_{k,q}^h p_{k\sigma}^h] \\ &\quad + \sum_q [S_{k+q,k}^h - S_{k,k-q}^h + S_{k,k+q}^h - S_{k,q,k-q}^h] \end{aligned}$$

coherent phonon amplitude

$$\frac{d}{dt} B_q = -i \omega_{LO} B_q + \frac{i}{\hbar} g_q \sum_k [f_{k,k+q}^h - f_{k,k+q}^h]$$

energy matrices:

$$\begin{aligned} \epsilon_{k\sigma}^{h,h} &= \left[\epsilon_k^h \pm \frac{icE}{2} \left(\frac{\partial}{\partial k_z} - \frac{\partial}{\partial k_z} \right) \right] \delta_{k\sigma}^h + V_{k,\sigma}^{h,h} \\ &\quad - \sum_q \left[V_{q,k+q,k+q}^h - V_{k,\sigma}^h (V_{q,k+q,k+q}^h - V_{q,k+q,k+q}^h) \right] \\ &\quad \pm g_{q,k+q}^h (p_{k,\sigma}^h - p_{k,\sigma}^h) \end{aligned}$$

effective field matrices:

$$u_{k\sigma}^h = E M_k \delta_{k,\sigma}^h + M_k (n_q + 1) E_0(t) - \sum_q V_q p_{k+q,k+q}^h$$

single-particle potential

external field (drift and Zener term)

exchange energy and Coulomb enhancement

induced field (Hartree term)

coherent phonons

dynamical variables:

$$f^r(\mathbf{k}, \mathbf{r}) = \sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} (c_{\mathbf{k}-\frac{1}{2}\mathbf{q}}^\dagger c_{\mathbf{k}+\frac{1}{2}\mathbf{q}})$$

$$p(\mathbf{k}, \mathbf{r}) = \sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} (d_{-\mathbf{k}+\frac{1}{2}\mathbf{q}}^\dagger c_{\mathbf{k}+\frac{1}{2}\mathbf{q}})$$

$$B(\mathbf{r}) = \sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} B_{\mathbf{q}}$$

spatially resolved variables:

electron density $n^e(\mathbf{r}) = \sum_{\mathbf{k}} f^r(\mathbf{k}, \mathbf{r})$

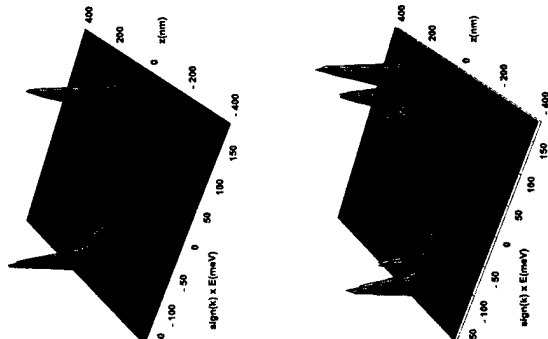
energy density $u^e(\mathbf{r}) = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} f^r(\mathbf{k}, \mathbf{r})$

mean kinetic energy per carrier $E(\mathbf{r}) = \frac{u^e(\mathbf{r})}{n^e(\mathbf{r})}$

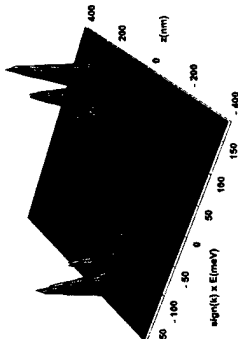
lattice polarization $\mathbf{P}(\mathbf{r}) = \frac{-i\epsilon_0}{e} \sum_{\mathbf{q}} \mathbf{q} g_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} (B_{\mathbf{q}} - B_{-\mathbf{q}}^*)$

Spatially resolved kinetics
Wigner function

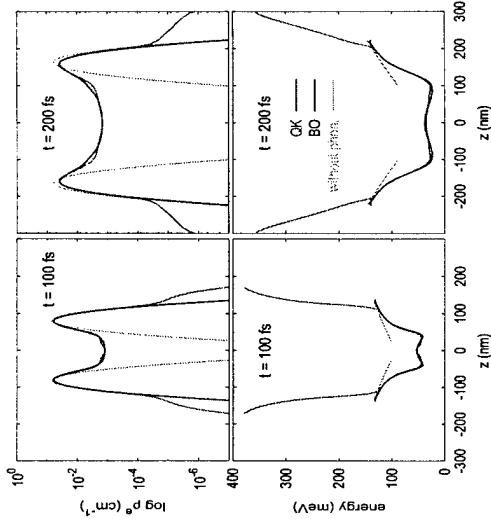
t = 100 fs



t = 300 fs

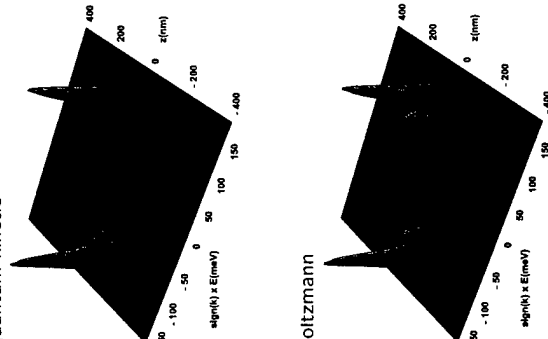


Spatially resolved kinetics
density and mean energy

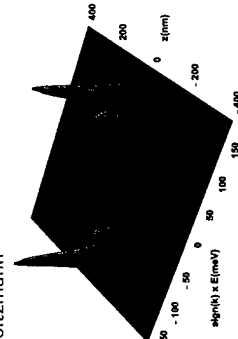


Spatially resolved kinetics
Wigner function

t = 100 fs, quantum kinetic

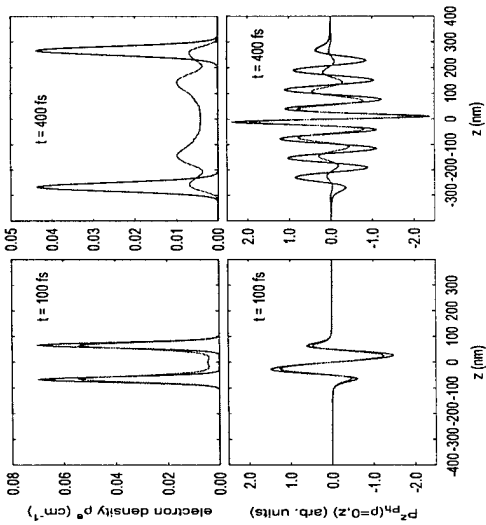


t = 100 fs Boltzmann



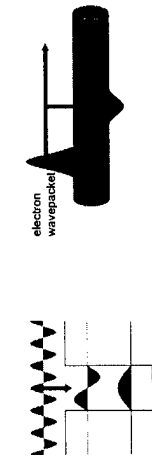
Kinetics of charge separation
coherent phonons

without/with incoherent phonons

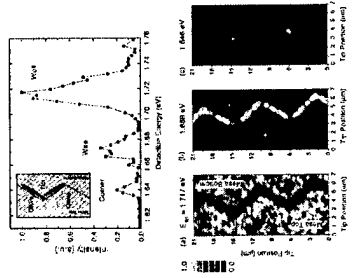


Dynamics with inhomogeneous potential
carrier trapping process

transitions between states of different dimensions

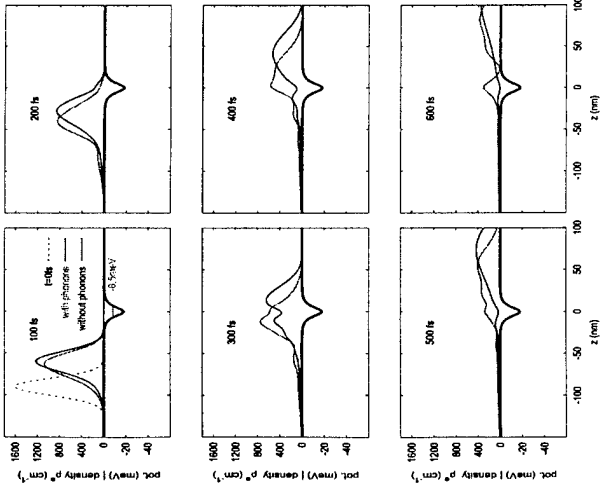


experimental realization & near-field spectroscopy

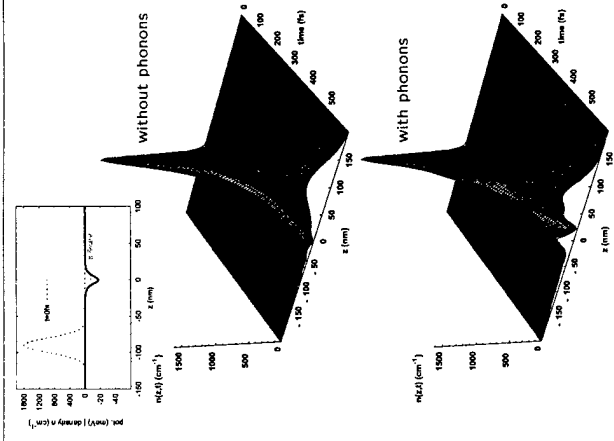


Carrier trapping in dot states shallow smooth potential

Initial energy: 18 meV, width: 7.5 meV

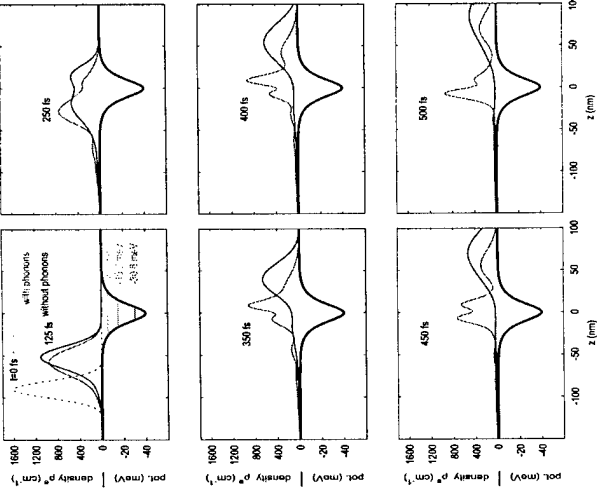


Carrier trapping in dot states shallow smooth potential

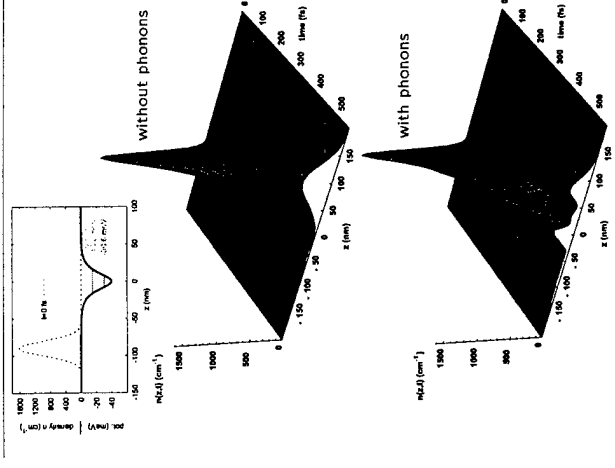


Carrier trapping in dot states resonant case

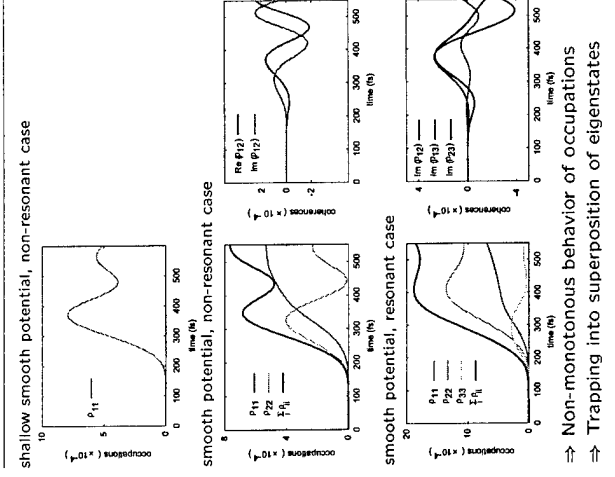
Initial energy: 18 meV, width: 7.5 meV



Carrier trapping in dot states smooth potential, resonant case

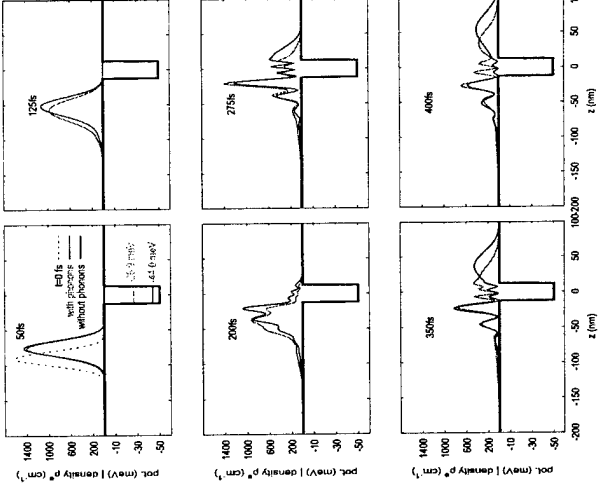


Carrier trapping in dot states density matrix of bound states



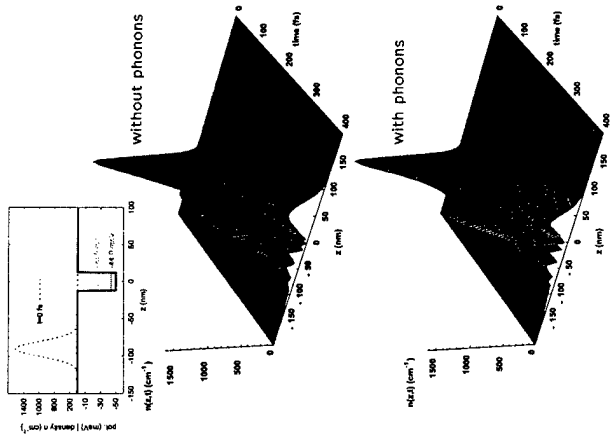
Carrier trapping in dot states square well potential

Initial energy: 18 meV, width: 7.5 meV



⇒ Non-monotonous behavior of occupations
⇒ Trapping into superposition of eigenstates

Carrier trapping in dot states square well potential



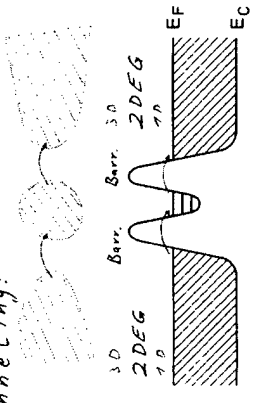
Quantum kinetics in semiconductors conclusions

Phonon quantum kinetics in homogeneous systems
density-matrix theory
relation to NGFs
energy-time uncertainty
nonequilibrium phonons
scattering between renormalized states
phonon quantum beats
intracollisional field effect

Phonon quantum kinetics in inhomogeneous systems
different representations
wave-packet dynamics
coherent phonons
carrier-trapping dynamics

Transport through a Quantum Dot:

Tunneling:



- Discrete Energy Levels (Confinement) ΔE
- Charging Energy $E_c = \frac{e^2}{2C}$ (Capacitance C)

Overview:

- Single-Electron Tunneling
- Transport through Quantum Dots
- Spectroscopy
- Influence of Emitter States
- Spin Effects
- Conclusions

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Single-Electron Tunneling through Quantum Dots (Single-Electron Charging in Quantum Dot Arrays)

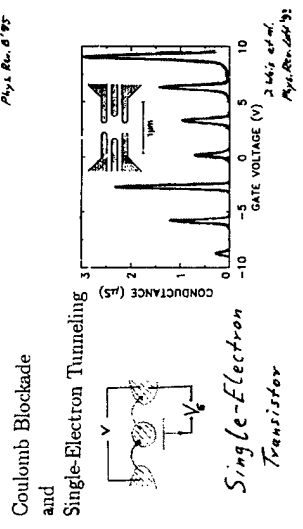
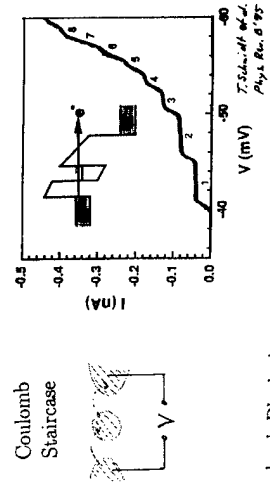
Rolf J. Haug

University of Hannover
Germany

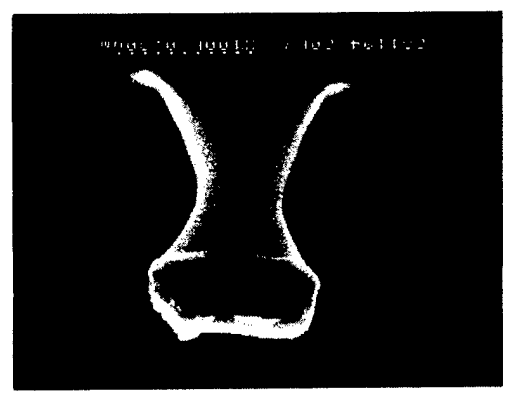
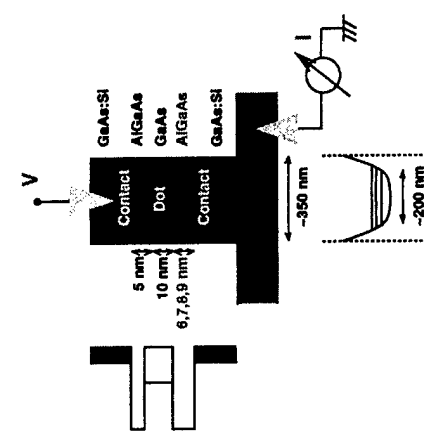
nanosstrukturen
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Advanced Research Workshop on Quantum Transport in Semiconductors, 21.6.2001

Charging Effects

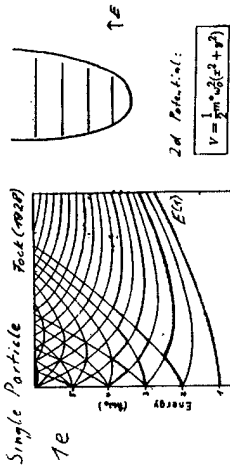


Double-barrier quantum-dot



Transport Spectroscopy of Artificial 'Atoms'?

in a magnetic field:



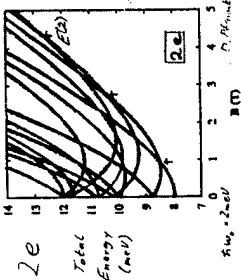
Many Particle - Ground State

Excited States

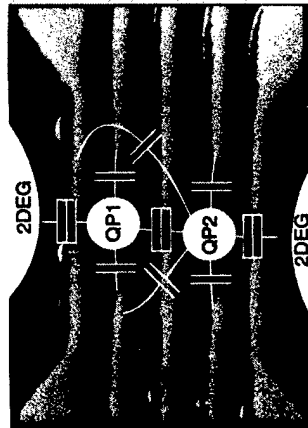
14
13
12
11
10

Total Energy

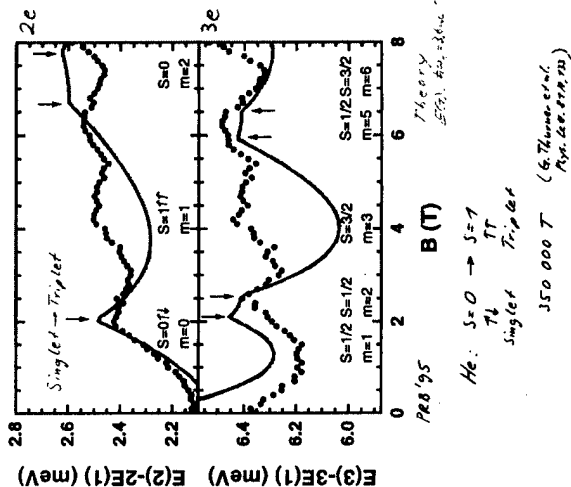
Ze



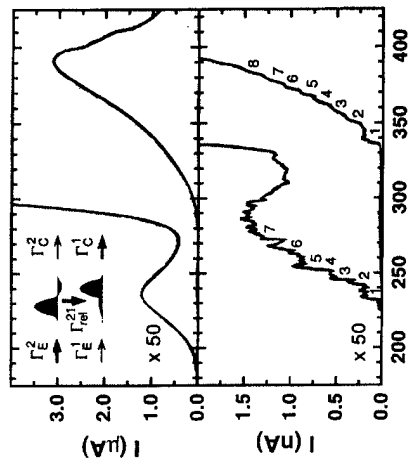
Lateral
Double Quantum Dot



Quantum-dot Helium and Lithium

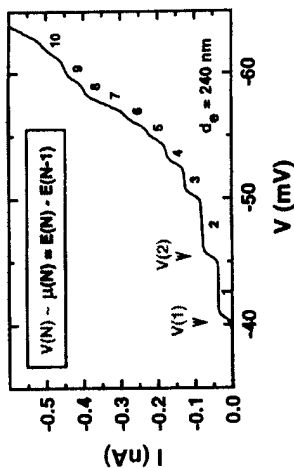


coupled
quantum wells
 $2\mu\text{m}\varnothing$

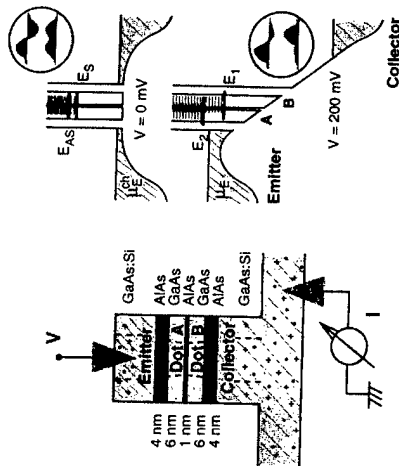


250 m ϕ V (mV)
coupled
quantum dots

Many-electron ground states $E(N)$

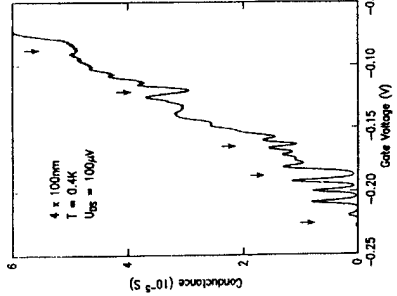
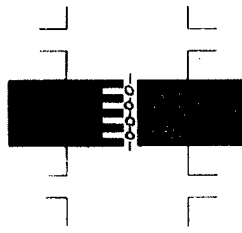
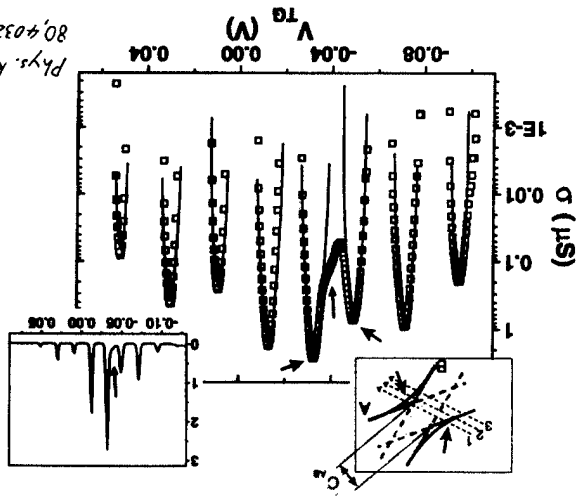


Coupled Quantum Dots



Phys. Rev. Lett.
71, 7546 (1993)

Phys. Rev. Lett.
80, 4032 (1998)



Theory: Stiefel and his group
Phys. Rev. Lett. 72, 3550 (1994)

R. Z. Huang
et al.
Proc. of 20th IAS,
3463 (1992)
Phil. Mag. Ser. B, 66-26,
25 (1992)
Surf. Sci. 251 (1992)
Europhys. Lett. 59

One-Dimensional Superfence
with a period of 150nm

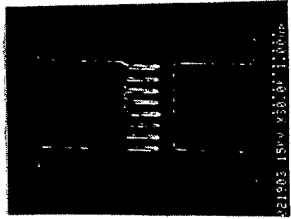
• Gate level



IBM

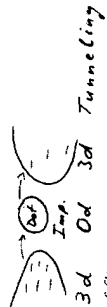
Transport through Several
Quantum Dots

Gate level



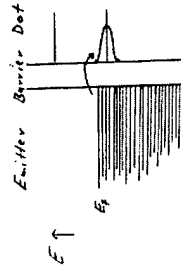
IBM

Influence of Contacts

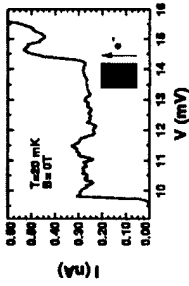


• Density-of-States

• Interaction Effects



Quantum Dot as Spectrometer

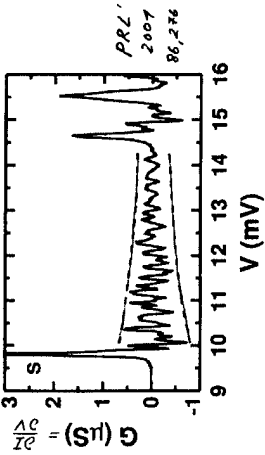
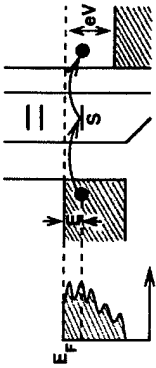


Spectrometer discrete and localized in space
(10nm)
→ local density of states fluctuations

nanoscale
all manner

Diff. Conductance

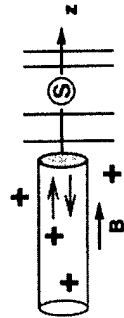
GaAs:Si
AlGaAs
GaAs
AlGaAs
GaAs:Si



Quantum Interference in high magnetic fields

- Magnetic field used for Landau quantization

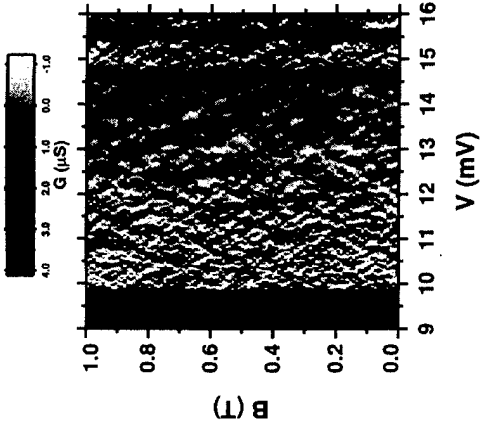
$$E = \hbar \omega_c \left(n + \frac{1}{2} \right) + E_{\text{spin}}(k_z)$$



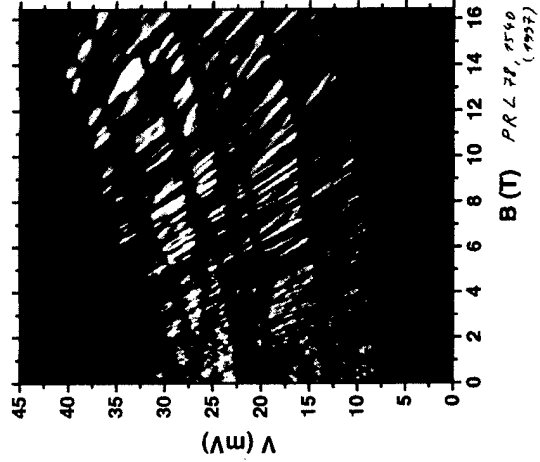
- Interference-induced 1D resonances α

$$E_{\text{min}}(k_z) = E^c$$

Magnetic Field $B \parallel z$

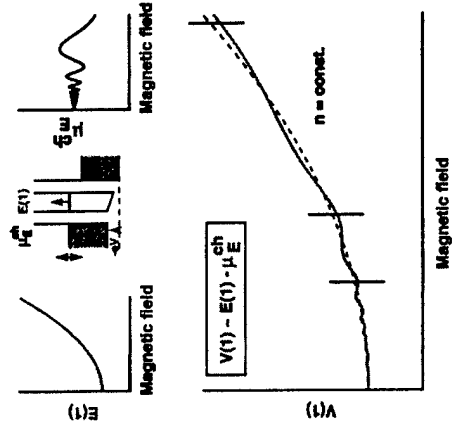


Relaxation $\sim E^2$
clean system $\sim E^2$
disordered system $\sim E^2$

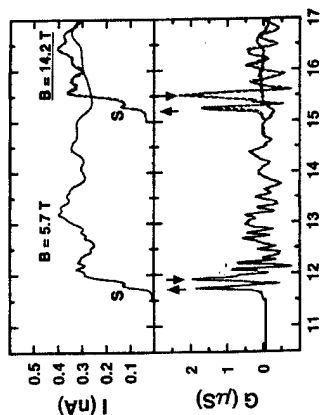
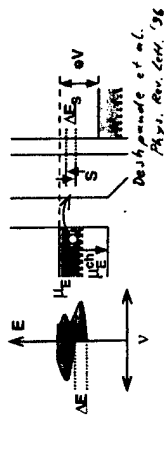


- $\omega_c \tau > 1$, Landau band formation relevant

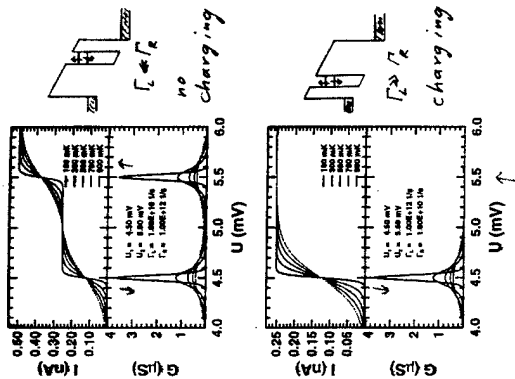
Magnetic-field dependence of the contact states



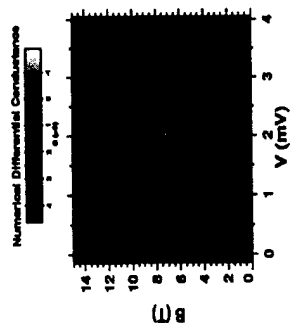
Two Spin-Levels in the Dot



Calculations of I and G for ↑ and ↓ in a magnetic field



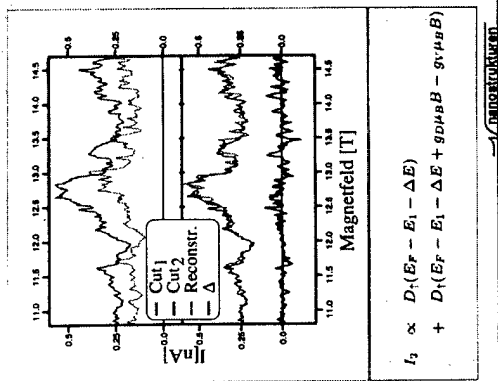
Zeeman Splitting



sign: $g_s = -0.44$ (later)
 $\Delta V \sim g_s \mu_B B$
 Kistner et al. PRB 58, 1635 (1998)

UNIVERSITÄT DUISBURG
 ESSEN

The Reconstructed Current



$$I_2 \propto D_1(E_F - E_1 - \Delta E) + D_2(E_F - E_1 - \Delta E + g\mu_B B - g_2\mu_B B)$$

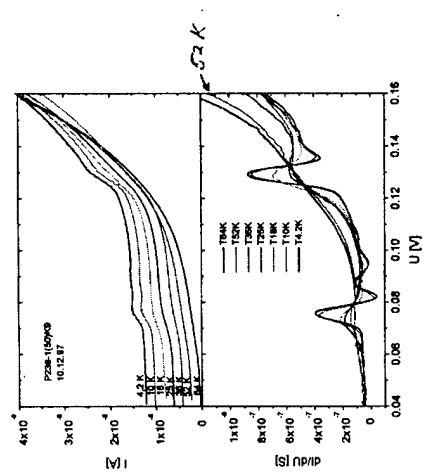
Spin effects beyond Zeeman splitting?

$\tau_s \approx 30$ ps
 Europhys. Lett. '2001
 54, 495

with $g_{\text{Bulk}} \approx -0.35$
 sign! (Literature value: $g_{\text{Bulk}} = -0.44$)
 see also PRB 64, 1635 (2001)
 7/16/2001

Other works:

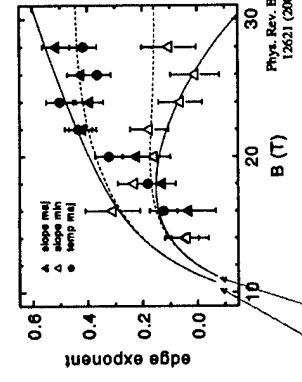
- Isheverich et al. PRB 54, 46407 (1996)
- Suzuki et al. Jpn. J. Appl. Phys. 36, 1993 (1997)
- Han, Liao et al. Appl. Phys. Lett. 70, 405 (1997)



In As dots $\rho = 14 \text{ nm}$
(single dots!) (from diameter of dot)

Exponents

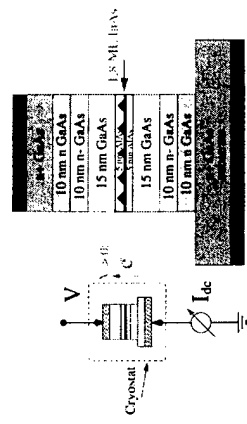
Temperature dependence $I_{\text{Peak}} \propto T^{-\alpha}$
Voltage dependence $I \propto (V - V_c)^\alpha$



Theory (H. Frahm) $\rightarrow$ FES

Phys. Rev. B 62, 12621 (2000)

Sample structure

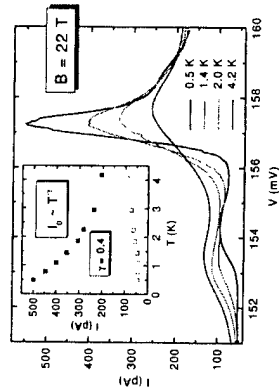


3D-Electrode (Si-doping):
 • undoped GaAs buffer layer
 • $n^- = 10^{16} \text{ cm}^{-3}$
 • $n^+ = 10^{17} \text{ cm}^{-3}$
 • $n^+ = 2 \cdot 10^{18} \text{ cm}^{-3}$
 Vertical tunneling diode: $A = 40 \times 40 \text{ (}\mu\text{m)}^2$
 $\Rightarrow 1.6 \cdot 10^8 \text{ InAs quantum dots}$

Semicon. Tech. 79, 299 (1999)

nanofabrikation

Temperature dependence



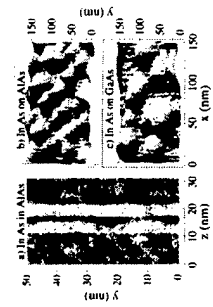
- decrease of majority spin peak
- increase of minority spin peak
- Peak height: $I_{\text{H}} \sim T^{-1}$ (at $V = V_c$)

Peak form $I \sim (V - V_c)^Y \Theta(V - V_c)$
($T=0$)

nanofabrikation

InAs Quantum dots

1.8 ML InAs on AlAs and GaAs



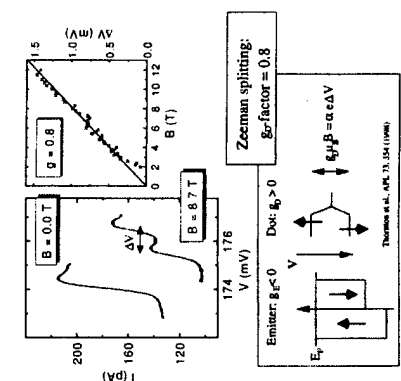
- Lattice mismatch $\Rightarrow$ Stranski-Krastanov growth
- In situ growth of small quantum dots

InAs on	height (nm)	ϕ (nm)	density (μm^{-2})
GaAs	4	30	250
AlAs	4	15	1000

nanofabrikation

Small magnetic fields $B \parallel I$

Positive bias



Zeeman splitting:
 g-factor = 0.8
 Emitter: $g < 0$ Dot: $g > 0$
 E_F
 $E_F \pm \beta = E_F \pm \alpha \Delta V$
 Thomas et al., PRL 73, 154 (1994)

nanofabrikation

Peaks:

Fermi-Edge Singularity

(Mahan 1967)

Interaction of Charge on Dot with States in the Emitter



FES in Tunneling:
see Geim et al. Phys. Rev. Lett. 72, 2061
Cobden et al. Phys. Rev. Lett. 75, 4274

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und Nanopore

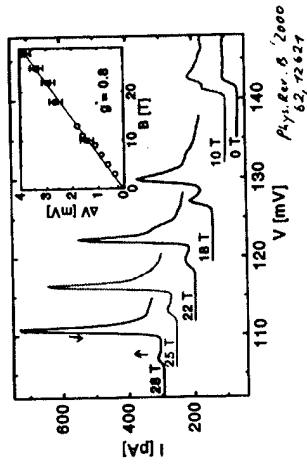
Conclusions

- Transport through Quantum Dot Arrays
- Single and Coupled Quantum Dots
- Lateral and Vertical Devices
- Spectroscopy
- Local Density of States of Emitter
- Relaxation
- Spin
- InAs-Quantum Dots
- Interaction Effects (FES)

nanosstrukturen
und Nanopore

InAs quantum dots

High magnetic fields $B \parallel I$



Phys. Rev. B 2000
62, 12627

nanosstrukturen
und Nanopore

Theory:

lowest Landau level $n=0$

wave function $\psi = \psi_n(s, \phi) \sim k_z z$

eq. 1.1.1. $m \neq 0$

$$\psi_n(s, \phi) \sim s^m \exp(-i m \phi - \frac{s^2}{4l_B^2})$$

$$l_B = \sqrt{\frac{\hbar}{eB}}$$

main effect:

FES for tunneling from

lowest n -state

(l_B = dot diameter)

Modeling Ballistic Current Flow in Carbon Nanotube Wires

M. P. Anantram
Nanoelectronics & Device Modeling Group
NASA Ames Research Center
Moffett Field, CA
U. S. A.

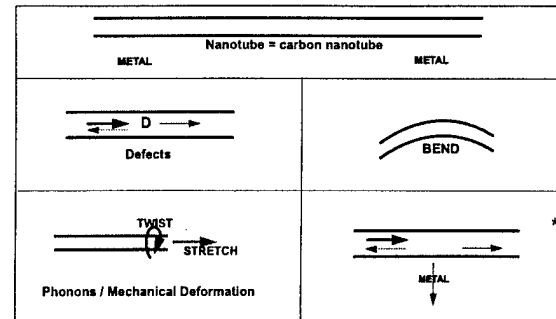
Acknowledgements:

T. R. Govindan
Jie Han
Supriyo Datta
Liu Yang
Natalio Mingo

Outline:

- * Role of Defects
- * Bragg reflection
- * Metal-nanotube coupling
 - wave vector conservation
 - Armchair versus Zigzag

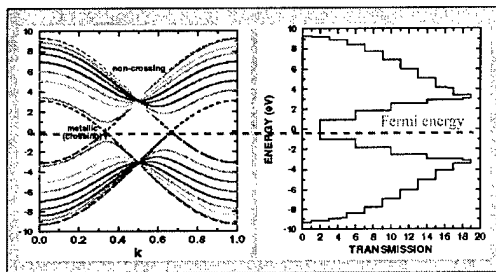
Topics Studied



* Bragg refl.: Intrinsic mechanism, which exists even in an ideal situation

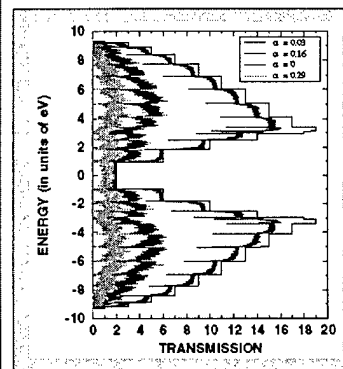
* = This Talk

Transmission vs Energy of a (10,10) Nanotube



- Close to $E=0$, Resistance = 6 k Ω ; At higher energies, less than 300 Ω
- Experimentally:
 - Max. small bias conductance ~ 12.5 k Ω
 - Max. large bias conductance ~ 4 k Ω

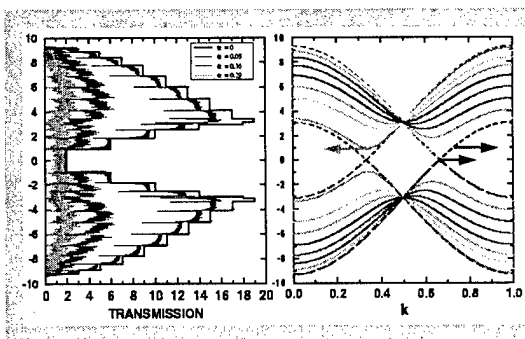
Transmission versus Energy with Defects



Transmission Probability:

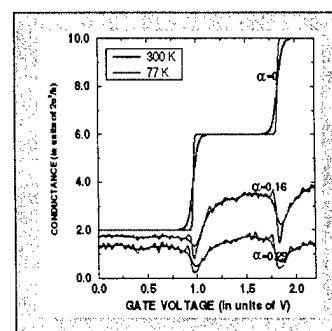
- Decreases with disorder
- Large Dips at some energies
- Very small decrease around $E=0$ (WIRES?)
- Localization length

Physical explanation of the transmission dips



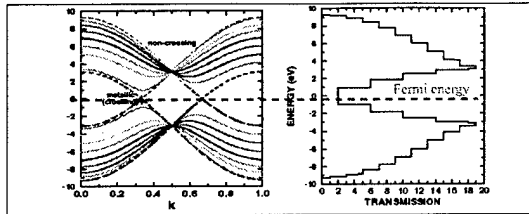
- $E=0$, Crossing bands: Large velocity (dE/dk)
- Large velocity states ($\rightarrow$) at higher energies are prone to REFLECTION as they couple to low velocity states ($\leftarrow$ and $\leftarrow$)

Conductance vs. Gate Voltage



- DIPS IN THE CONDUCTANCE WHEN THE FERMI ENERGY IS CLOSE TO THE SUB-BAND OPENINGS
- FERMI ENERGY AT THE BAND CENTER: GOOD WIRE

Current-carrying capacity of carbon nanotubes

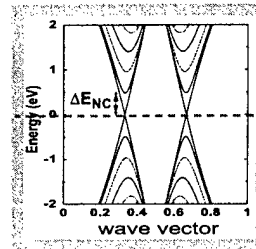


- Close to $E=0$, only two sub-bands, $Conductance = \frac{4e^2}{h}$ ($6 \text{ k}\Omega$)
- At higher energies, $Conductance = \frac{(20-30)e^2}{h}$ ($< 1 \text{ k}\Omega$)

An important question is if subbands at higher energies be accessed to drive large currents through these molecular wires?

Experiments by various groups have shown that the differential resistance of a nanotube decreases by small amounts with increase in applied voltage. i.e. the current carrying capacity does not increase better than an ordinary resistor with applied bias.

At what applied voltage are electrons injected into higher subbands?



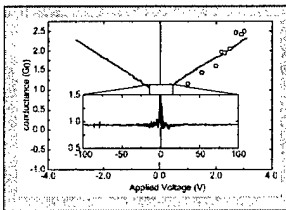
Bias at which electrons are injected into non crossing subbands is ΔE_{NC}

	7Å			56Å	
size	(5,5)	(10,10)	(20,20)	(40,40)	
ΔE_{NC} (eV)	1.9	0.98	0.5	0.25	

For example, in a (20,20) nanotube electrons are injected into over 20 subbands at an energy of 2.5eV.

The maximum conductance if the Fermi energy is at 2.5 eV is $= 40e^2/h$

One exception is the experiment by Frank et. al in Science 280 (1998) which showed a modest decrease in differential resistance with bias.

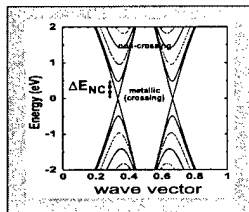


Main experimental features are:

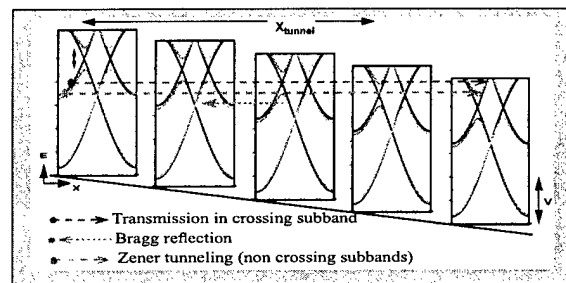
- $V_{APPLIED} < 200 \text{ mV}$, conductance $= G_0$
- $V_{APPLIED} > 200 \text{ mV}$, conductance increases modestly to about $1.75 G_0$

- For large diameter nanotubes such as used in the experiments, the non-crossing subbands open at about 100 meV.
- Further there are about 80 subbands at an energy of about 2 eV.

- Yet the conductance is only $\sim 1.75 G_0$



Semiclassical picture is useful

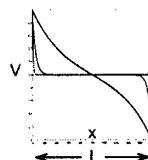


- The strength of the two processes are determined by:
 - Tunneling distance (X_{tunnel}) $\rightarrow$ Screening length
 - Barrier height, $2\Delta E_{NC}$
 - Scattering and Defects
- $\Delta E_{NC} \propto 1/\text{Diameter}$. So, the importance of Zener tunneling increases with increase in nanotube diameter.

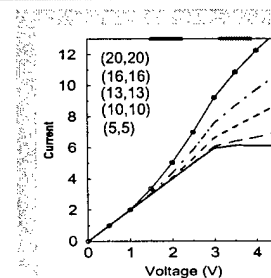
Model

- pi-orbital based tight binding calculation [Phys. Rev. B 58, 4882 (1998)]
- Ideal contacts - reflection less contacts
- Electrostatic potential drop
 - Linear
 - Exponential (Screening length, L_{sc})

$$V(x) = \frac{V_a}{2} \left(1 + \frac{1 + e^{-x/L_{sc}}}{e^{x/L_{sc}} - e^{-x/L_{sc}}} e^{-x/L_{sc}} - \frac{1 + e^{-x/L_{sc}}}{e^{x/L_{sc}} - e^{-x/L_{sc}}} e^{x/L_{sc}} \right)$$



- $L=2400 \text{ Å}$, $L_{sc}=6, 50, 500 \text{ Å}$
- e-e and e-p scattering are not included



- $L=10 \text{ Å}$
- $dI/dV \approx 4e^2/h$ for $V_a < 2\Delta E_{NC}$
- Threshold changes with diameter
- Barrier height (ΔE_{NC}) decreases with increase in diameter
- Total Current increases with increase in diameter
- $dI/dV > 0$ for $V_a > 3.1 \text{ V}$, except for the (5,5) nanotube
- (5,5) nanotube $\Delta E_{NC} = 1.9 \text{ eV}$

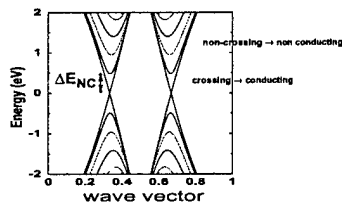
The differential conductance is NOT comparable to the increase in the number of subbands.

For a (20,20) nanotube, there are 35 subbands at $E = \pm 3.5 \text{ V}$.

Bragg reflection severely limits the current carrying capacity

The crossing metallic-type bands conduct current.

The non crossing semiconductor type bands do not conduct current



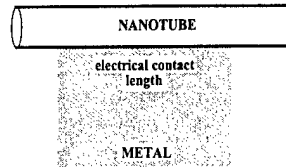
Large diameter nanotubes: non-crossing bands will partially conduct due to Zener-type tunneling.

Coupling of carbon nanotubes to metallic contacts

The electronic properties of nanotubes are closely related to chirality. The metallic versus semiconducting nature of nanotubes and the bandgap change with deformation depend on chirality.

Questions:

- Is there a preferable nanotube chirality to maximize current flow?
- Role of wave vector conservation?
- Explain experimentally observed scaling of conductance with contact length

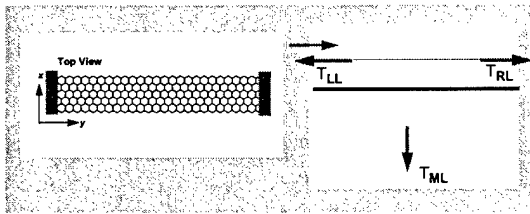


Parameters that influence current flow:

- Strength of coupling to metal
- Length of metal-nanotube contact
- Defects
- Metal Fermi wave vector

How do we model the system?

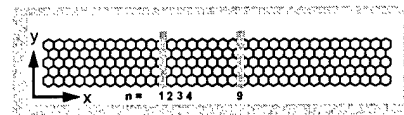
- Same code as in our MOSFET work
- π electron tight binding model
- Metal is modeled as a free electron gas (k_F)



- $T_{RL} + T_{ML} + T_{LL} = 2$.
- Phys. Rev. B, v.58, p. 4882 (1998) and v. 61, p. 14219 (2000)
- Compute self energy due to: (i) metal & (ii) semi-infinite CNT leads

Calculation of the total transmission from nanotube to metal, T_{ML} are presented for armchair and zigzag nanotubes:

Scattering rate



The wave function of a nanotube is $\Psi = e^{ink_x L} \phi$

n is an integer and ϕ is wave func. of atoms in a 1D unit cell

The scattering rate from metal to nanotube (Born approx.) is,

$$1/\tau \propto |\langle \Psi_{nt} | V_{m-nt} | \Phi_m \rangle|^2$$

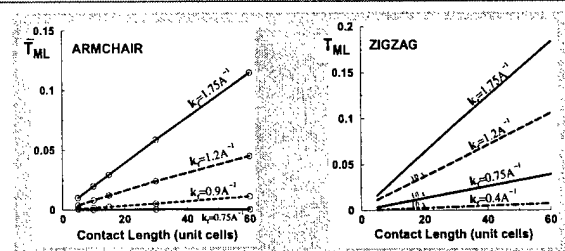
$$\delta(k_x - k_x^m) |\langle \phi | V_{m-nt} | \phi_m \rangle|^2$$

This implies that

- k_x is conserved
- k_y conservation is relaxed due to finite width of contact area

As a result of the difference in k_y corresponding to $E=0$ (Fermi energy), important differences should arise as a function of chirality.

ARMCHAIR	ZIGZAG
$E=0$ at $k_y = 2\pi/3a_0 = 0.85 \text{ \AA}^{-1}$	$E=0$ at $k_y = 0$
Metal with $k_{Fermi} < 0.85 \text{ \AA}^{-1}$ couples weakly	No threshold for k_{Fermi}

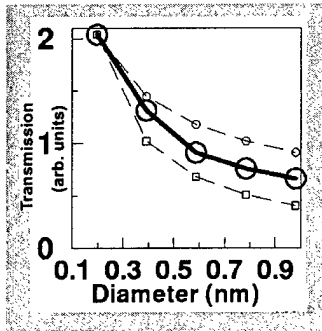


- Note that threshold value is the value of k_F below which the transmission does not scale with contact length

The main differences between the armchair and zigzag case are:

- Threshold value of k_F is $\frac{2\pi}{3a_0} = 0.85 \text{ \AA}^{-1}$ for armchair nanotubes (see $k_F = 0.75 \text{ \AA}^{-1}$) and is 0 for zigzag nanotubes (see $k_F = 0.4 \text{ \AA}^{-1}$).
- Beyond the threshold k_F , transmission *increases* with contact length as seen in experiment by Tans et. al., Nature, vol. 386, 474 (1997)
- For zigzag tubes, T_{LM} is small for $k_F \leq 1.2 \text{ \AA}^{-1}$ as a result of the large angular momentum in the circumferential direction.

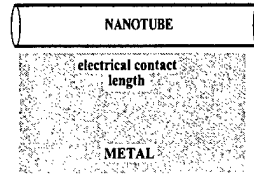
Diameter dependence of Transmission



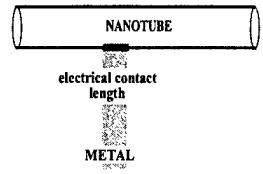
Transmission decreases as the diameter increases because the nanotube tends to a graphite sheet.

The previous values of T are small compared to the maximum possible value of 2. Two possible scenarios to increase T are:

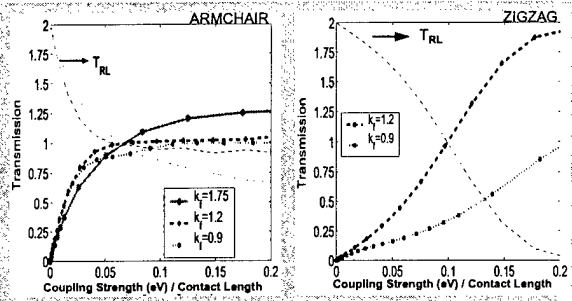
* Large contact length - Small coupling



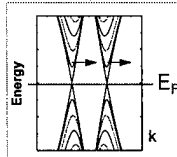
* Small contact length - Large coupling



- For nanoelectronics, the second option (right) is better.
- We model a contact length of 30 unit cells (72 Å for armchair and 125 Å for zigzag nanotubes), and vary the coupling strength. The main results of the calculation are rather surprising and are presented below:

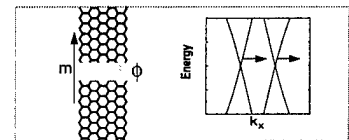


- Armchair: Transmission is pinned at values close to unity for metal k_F of 0.9 and 1.2 Å⁻¹
 - Zigzag: Transmission increases to two (maximum possible value)
 - At small coupling strengths, transmission increases more rapidly in the armchair case
- Metallic-zigzag are preferable from metal-nanotube coupling viewpoint



Nodes on the cylinder - Shape of NT wave function

$$\Psi = e^{imk_x a_0} \phi$$



$k_x \uparrow 0.85 \text{ Å}^{-1}$
 metal $k_F = 0.90 \text{ Å}^{-1} \rightarrow$ distance between nodes 6 Å
 Gold $k_F = 1.20 \text{ Å}^{-1} \rightarrow$ distance between nodes 3.4 Å
 Aluminum $k_F = 1.75 \text{ Å}^{-1} \rightarrow$ distance between nodes 2 Å

Scattering rate from Metal to Nanotube $\propto \langle \phi_{\text{nanotube}} | H_{\text{coupling}} | \Psi_{\text{metal}} \rangle$
 $\propto - | H_{\text{coupling}} | \Psi_{\text{metal}} \rangle \sim 0$. Hence, Transmission is pinned close to unity.

- Side-contacted: zigzag nanotube are more desirable (curvature)
- Larger metal Fermi wave vector helps.

Conclusions

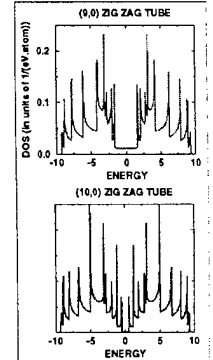
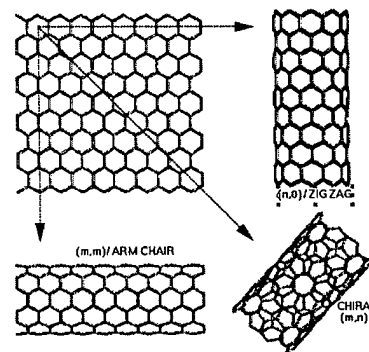
- dI/dV versus V does not increase in a manner commensurate with the increase in number of subbands.



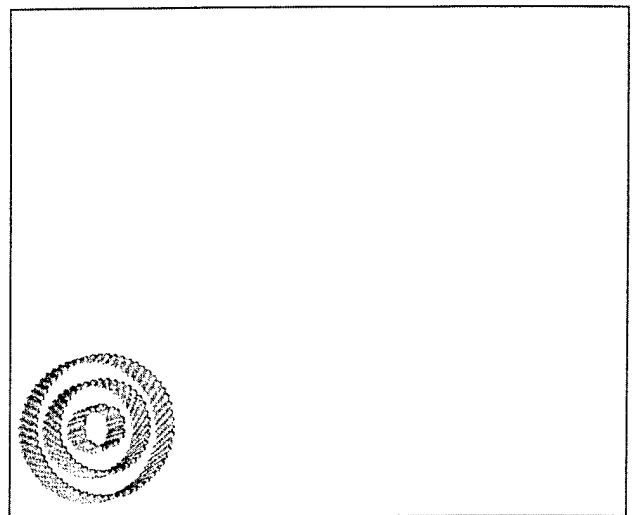
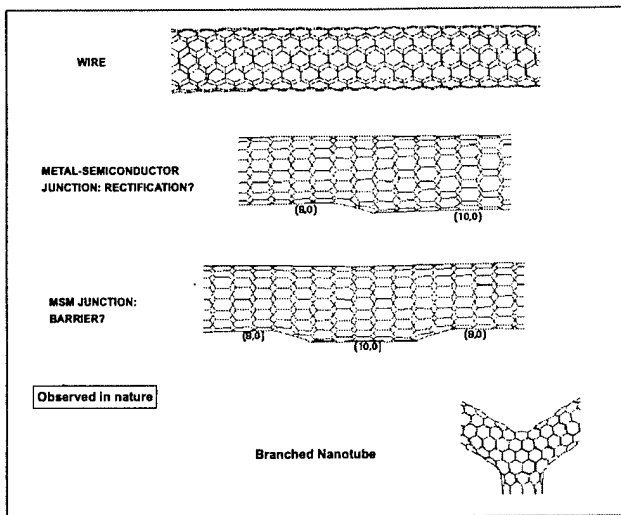
- The increase in dI/dV with bias is much smaller than the increase in the number of subbands - a consequence of bragg reflection
- Requirement for axial wave vector conservation:
 ARMCHAIR cut-off $k_{F\text{Fermi}} = 2\pi/3a_0 = 0.85 \text{ Å}^{-1}$
 ZIGZAG cut-off $k_{F\text{Fermi}} = 0$
- Our calculations show an increase in transmission with length of contact, as seen in experiments.
- It is desirable for molecular electronics applications to have a small contact area, yet large coupling. In this case, the circumferential dependence of the nanotube wave function dictates:
 - Transmission in armchair tubes saturates around unity
 - Transmission in zigzag tubes saturates at two

WHAT ARE CNT?

- STRIP OF A GRAPHENE SHEET ROLLED INTO A TUBE



- Varying the diameter and (m,n): metal, semiconductor
- $n-m = 3 \cdot \text{integer}$: metal



A POSSIBLE LOOPHOLE IN THE THEOREM OF BELL

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Abstract

The celebrated inequalities of Bell are based on the assumption that local hidden parameters exist. When combined with conflicting experimental results these inequalities appear to prove that local hidden parameters cannot exist. This suggests to many that only instantaneous action at a distance can explain Einstein, Podolsky, Rosen (EPR) type of experiments. The Bell inequalities are based on a simple mathematical model of the EPR- experiments. They have no experimental confirmation since they contradict the results of EPR-experiments. Furthermore, in addition to the assumption that hidden parameters exist, Bell tacitly makes a variety of other assumptions which could contribute to his being able to obtain the desired contradiction. For instance, Bell assumes that the hidden parameters are governed by a single probability measure independent of the analyzer settings. Moreover, we argue that the mathematical model of Bell excludes a large set of local hidden variables and a large variety of probability densities of these local (in the sense of Einstein) variables. These exclusions have neither a physical nor a mathematical basis but are based on Bell's translation of the concept of Einstein locality into the language of probability theory. Our additional set of local hidden variables includes time like correlated parameters and a generalized probability density. We prove that our extended space of local hidden variables does not permit Bell-type proofs to go forward. This opens the possibility to explain EPR-experiments within Einstein's space-time continuum, with a finite velocity of light and without recourse to spooky action at a distance.

1 Introduction

Einstein, Podolsky and Rosen (EPR) [1] challenged Bohr [2] and the completeness of quantum mechanics by designing a "Gedanken" experiment that suggested the existence of "hidden parameters" and of a theory that was more complete than quantum mechanics. The

EPR design was later realized in various implementations [3] with experimental results close to the quantum mechanical prediction. These experimental results by themselves do not have any bearing on the EPR claim that quantum mechanics was incomplete, nor on the existence of hidden parameters. However, inequalities derived by Bell [4] that are based on the assumption that local hidden parameters exist, taken together with the experimental results that happen to be inconsistent with the result of the Bell inequalities, do appear to prove that local hidden parameters cannot exist. This has been discussed in great detail in [5] and [6].

The Bell theorem is based on a mathematical model of the EPR- experiments. It has, by itself, no experimental confirmation since its result contradict the results of the EPR-experiments. The standing of the Bell theorem has therefore unique features in the history of modern physics: the mathematical model and the theorem of Bell are taken to be correct and seen by many as valid as the second law of thermodynamics is, while there exists no experimental confirmation. However, instead of discarding altogether a mathematical model that contradicts experiment, the contradiction to the experiment is used to prove that the basic assumption of the theorem, the existence of local hidden parameters is incorrect. The framework of research that has developed around the Bell theorem claims the necessity of "gross non-localities". In simple words, the correlated spins of the EPR-experiment are in some contact over arbitrary space like distances of our space-time continuum and if one spin is measured in one station, the correlated spin in another station is instantaneously influenced. This contradicts the locality conditions of Einstein and Einstein's very argument for the lack of completeness of quantum mechanics. Einstein called the instantaneous interaction of the spatially separated spins "spukhafte Fernwirkungen (spooky action at a distance)". He did not accept the possibility of such spooky action and since quantum mechanics appeared to demand it, it had to be at least incomplete. The Bell theorem and its standard interpretation has turned the logic around. Its supporters now claim that local hidden parameters do not exist and cannot explain the EPR-experiments. Quantum mechanics does agree with experiments and spooky action at a distance must be accepted as a fact of nature. However, it has been shown in a series of papers of which we cite only two of the more recent [7] [8], that Bell's theorem does contain more than self-evident locality assumptions. These are related to the role of time in the experiments and the admissibility of more general probability measures.

We show in this paper that the assumption of the existence of local hidden variables is not the only assumption in the proof of the Bell inequalities. We show that the mathematical model of Bell excludes a large set of hidden variables and a large variety of probability densities of these variables that all fulfill Einstein's locality conditions perfectly. This exclusion has neither a physical nor a mathematical basis but is based on Bell's mathematical interpretation of what Einstein locality means in terms of probability theory. Our additional set of hidden variables or, as we will call them, parameter random variables includes time like correlated parameters and a generalized probability density which is a sum of what we later define as setting dependent subspace product measures (SDSPM's). We demonstrate that Bell type proofs cannot go forward using our extended space of hidden variables.

The paper is organized as follows. We first review the theorem of Bell. We then analyze

the restrictions that Bell's proof puts on the parameter space and probability measure and show that a much larger space and a more general probability measure can be constructed without violation of Einstein locality conditions. We demonstrate that a variety of proofs of theorems similar to that of Bell cannot be performed in this larger parameter space and with the more general probability measure, and that these theorems and inequalities are therefore not valid in this space. We finally point toward a mathematical model that uses this larger space and permits the construction of a hidden parameter theory that does agree with EPR-experiments.

2 The theorem of Bell

We first present a short summary of the work of Bell. In EPR experiments, two particles having their spins in a singlet state are emitted from a source and are sent to spin analyzers (instruments) at two spatially separated stations, S_1 and S_2 . The spin analyzers are described by Bell using unit vectors $\mathbf{a}, \mathbf{b}$, etc. of three dimensional Euclidean space and functions $A = \pm 1$ (operating at station S_1) and $B = \pm 1$ (operating at station S_2): furthermore A does not depend on the settings $\mathbf{b}$ of station S_2 , nor B on the settings $\mathbf{a}$ of station S_1 (Einstein separability or locality). Bell permits particles emitted from the source to carry arbitrary hidden parameters λ of a set Ω that fully characterize the spins and are "attached" to the particles with a probability density ρ (we denote the corresponding probability measure by μ). Neither the parameters λ nor the probability measure μ are permitted to depend on the settings at the stations. Einstein separability is again cited as the reason for this restriction. The analyzer settings are changed rapidly in the experiments and do change after emission from the source. Therefore the source parameters and their probability measure must not depend on the settings at the time of measurement. Bell further assumes that the values of the functions A and B are determined by the spin analyzer settings and by the parameters such that:

$$A = A_{\mathbf{a}}(\lambda) = \pm 1 \text{ and } B = B_{\mathbf{b}}(\lambda) = \pm 1 \quad (1)$$

Thus $A_{\mathbf{a}}(\lambda)$ and $B_{\mathbf{b}}(\lambda)$ can be considered as stochastic processes on Ω , indexed by the unit vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Quantum theory and experiments show that, for a given time of measurement for which the settings are equal in both stations, we have for singlet state spins

$$A_{\mathbf{a}}(\lambda) = -B_{\mathbf{a}}(\lambda) \quad (2)$$

with probability one. Bell further defines the spin pair expectation value $P(\mathbf{a}, \mathbf{b})$ by

$$P(\mathbf{a}, \mathbf{b}) = \int_{\Omega} A_{\mathbf{a}}(\lambda) B_{\mathbf{b}}(\lambda) \rho(\lambda) d\lambda = - \int_{\Omega} A_{\mathbf{a}}(\lambda) A_{\mathbf{b}}(\lambda) \mu(d\lambda) \quad (3)$$

From Eqs.(1)-(3), Bell derives his celebrated inequality [4]

$$1 + P(\mathbf{b}, \mathbf{c}) \geq |P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c})| \quad (4)$$

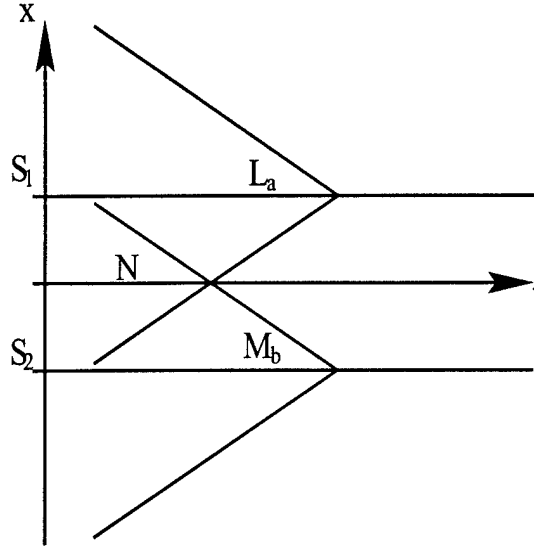


Figure 1: Light cone used by Bell. x denotes the space coordinate and t the time

and observes that this inequality is in contradiction with the result of Quantum Mechanics:

$$P(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b} \quad (5)$$

Here $\mathbf{a} \cdot \mathbf{b}$ is the scalar product of $\mathbf{a}$ and $\mathbf{b}$.

The proof of Bell's inequality is based on the obvious fact that for $x, y, z = \pm 1$ we have

$$|xz - yz| = |x - y| = 1 - xy \quad (6)$$

Substituting $x = A_b(\lambda)$, $y = A_c(\lambda)$, $z = A_a(\lambda)$ and integrating with respect to the measure μ one obtains Eq.(4) in view of Eq.(3). Thus, from the vantage point of mathematics, the Bell inequality is a straight-forward consequence of the set of hypotheses and assumptions that are imposed.

3 Extension of Bell's parameter space and probability measure

We are going to argue below that Bell's parameter space is not general enough and excludes without any necessity a manifold of parameters that has at least the cardinality of the continuum. Bell's probability measure is, correspondingly, not as general as the physics of relativity would permit. To show this we start with a discussion of the parameter space and corresponding probabilities out of Bell's book [9].

Bell [9] defines the following parameter sets that are in the backward light cone, as defined by relativity and as illustrated in Fig. 1. He lets N denote the specification of all entities that are represented by parameters and belong to the overlap of the backward light cones

of both space-like separated stations S_1 and S_2 . In addition he considers sets of parameters L_a (our notation) that are in the remainder of the backward light cone of S_1 and M_b for S_2 respectively. Bell (see p56 of ref. [9]) denotes the conditional probability that the function A_a assumes a certain value with $A_a = \pm 1$ by $\{A_a|L_a, N\}$ and similarly for $B_b = \pm 1$. Then he claims that in a local causal theory we have:

$$\{A_a|L_a, N, B_b\} = \{A_a|L_a, N\} \text{ and } \{B_b|M_b, N, A_a\} = \{B_b|M_b, N\} \quad (7)$$

Eq.(7) appears entirely plausible as a consequence of the finite speed of light: whatever happens at station S_1 to the result of A_a cannot be causally connected to the result of B_b in station S_2 within a local theory. When the switching of the settings is fast enough, the probability that A_a assumes a certain value must be the same no matter what value B_b might assume. While this conclusion is undoubtedly correct at a given instance of measurement, Bell's use of Eq.(7) as identical and valid for all times of measurement with a given setting is fatally flawed. The reason is the possible dependence of A_a and B_b on time-like correlated parameters that may be setting dependent. The mathematics of Bell-type proofs requires complete statistical independence of A_a and B_b for the whole set of measurements and not only at a given time. It also contains the assumption of identical L_a and M_b for all measurements of a run. This, however, cannot be guaranteed because physical phenomena other than the setting of the polarizer by the experimenter can occur in the stations and these can be correlated.

Consider, for example, two clocks, one in each station. These clocks may have different settings (e.g. pendulum length and/or starting time etc.). The time that one clock shows is certainly not the causal reason for the time of the other clock. It is the same physical law that is at work in both stations and that causes a correlation in the periodicity of the processes in the clocks or in some general periodic processes for that matter. It is, of course well known that two gyroscopes in the two stations could also be used as clocks as they may indicate the rotation of the earth. As mentioned, there may be also other periodic processes that cause correlations and these correlations may be influenced by the settings a and b . Although there are clear analogies of gyroscopes and spin properties, we do not wish to push this comparison too far. We do, however, wish to point out the dangers of using of Eq.(7) without proper caution. Bell's argument resulting in Eq.(7) does not include the vital fact that the experiments are made in a time sequence and that the backward light cones change and evolve with time. The situation is illustrated in Fig.2 which shows that for each instant of measurement there is a different light cone. Fig. 2 illustrates our point for measurements at two different times t_1 and t_2 .

The backward light cones contain sets of parameters L_{a,t_1} , M_{b,t_1} , N_{t_1} and L_{a,t_2} , M_{b,t_2} , N_{t_2} respectively. It is clear, that the set L_{a,t_1} and the set L_{a,t_2} may contain setting dependent parameters λ_a^* with different probability densities. It is also clear from the discussion with clocks that the sets L_{a,t_1} , L_{a,t_2} , M_{b,t_1} , M_{b,t_2} etc. need not be statistically independent. This has several consequences that do not permit Bell-type proofs to go forward. We outline below the most crucial problems.

Bell uses combinations of Eq.(7) for different settings in his proofs as follows (see e.g.

else assumed that the λ_a^* and λ_b^{**} be statistically independent. He argued this independence from the fact that the parameters are in different stations and he did not consider time-like correlations as described above. Bell assumes that setting dependent parameters in the stations analyzers (called instruments by Bell) must be statistically independent or do not exist. This is explicitly stated in his book ([9] p 38): "...it is necessary that the equality holds in (8) (which is equal to our Eq.(1)), i.e. for this case the possibility of the results depending on hidden variables in the instruments can be excluded from the beginning ..". Of course, in a run with all different settings that still would be true. However, $P(\mathbf{a}, \mathbf{b})$ is evaluated from measurements with fixed $\mathbf{a}$ and $\mathbf{b}$. Therefore a possible time dependence can cause statistical correlations. To visualize this, assume that the parameters λ_a^* and λ_b^{**} are identified with the the hour pointers of two clocks in the two stations. The clock in station S_1 is connected to a plane that is perpendicular to the setting $\mathbf{a}$ and the clock in station S_2 to a plane perpendicular to $\mathbf{b}$. Let the direction of the pointers be idealized by unit vector λ_a^* in station S_1 and λ_b^{**} in S_2 at each instant of measurement. Clearly, these parameters will exhibit time correlations. Note that it is of no concern that the measurements are taken at random times. It is the time correlation in the two stations at any given time that matters.

In addition to the generalization of Bell's probability densities shown in Eq.(10), one needs a further generalization and replace $\rho(\lambda)$ by

$$\sum_{t_i} \rho_{t_i}(\lambda, \lambda_a^*, \lambda_b^{**}) \quad (12)$$

Of course, to obey Einstein locality, λ_a^* and λ_b^{**} must be station specific and can only be correlated by time like correlations i.e. by some relation to local periodic processes. It is also important to note that the station parameters λ_a^* and λ_b^{**} cannot be emulated by the parameter pair $\mathbf{a}, \lambda$ or $\mathbf{b}, \lambda$, as always implied by Bell by use of his functions A and B . The source parameters λ of Bell appear with a given probability density. Since the parameters λ_a^* and λ_b^{**} can have different probability distributions for different $\mathbf{a}$ and $\mathbf{b}$, that are not related in any way to the parameters λ , it becomes clear that the joint density $\rho_{t_i}(\lambda, \lambda_a^*, \lambda_b^{**})$ can depend, a priori, on the setting vectors $\mathbf{a}$ and $\mathbf{b}$. It is irrelevant that by lucky coincidence, the triples $(\mathbf{a}, \lambda, \lambda_a^*)$ and $(\mathbf{b}, \lambda, \lambda_b^{**})$ could perhaps be written as $\mathbf{a}, \Lambda$ and $\mathbf{b}, \Lambda$ for some Λ incorporating λ and the station parameters. The probability density that must be considered, in general, for all these parameters is therefore also different from that of Bell and must exhibit a time dependence. This implies the necessity of a more general probability measure that includes time-like correlated parameters.

We have shown [8], that a properly chosen sum of what we call setting dependent sub-space product measures (SDSPM) does not violate Einstein-separability and does lead to the quantum result of Eq.(5) while still always fulfilling Eq.(2). By this we mean the following. The probability space Ω is partitioned into a finite number M of subspaces Ω_m

$$\Omega = \cup_{m=1}^M \Omega_m \quad (13)$$

For given $\mathbf{a}$ and $\mathbf{b}$, a setting dependent measure $(\mu_{\mathbf{ab}})_m$ is defined on each subspace Ω_m . This measure can be extended to the entire space Ω by setting

$$(\mu_{\mathbf{ab}})_m(\Omega_j) = 0 \text{ if } j \neq m \quad (14)$$

The final measure μ is then defined on the entire space Ω by

$$\mu = \sum_{m=1}^M (\mu_{\mathbf{ab}})_m \quad (15)$$

and the index m indicates the time correlations. In the above notation we would have $m = t_i$. We have shown in reference [8] that a product measure can be found such that

$$(\mu_{\mathbf{ab}})_m = (\mu_{\mathbf{a}} \times \mu_{\mathbf{b}})_m \quad (16)$$

This is of minor concern for the main argument presented here but does provide flexibility to completely avoid any hint of spooky action within our system of setting dependent subspace product measures (SDSPM's).

It is clear that Bell's proof does not go through with such a probability measure since integrating Eq.(6) to obtain Eq.(4) works only with a single setting independent probability measure. In addition, one can show in a rather intricate proof [8] that the quantum result of Eq.(5) can be obtained with a probability measure as in Eq.(15). In other words hidden parameters are possible if the parameter space is properly extended. We also have shown in [8] that the parameters that are considered that way show no trace of spooky action.

4 Conclusions

We have presented a mathematical framework that is more extensive than that of Bell and permits the possibility of describing the spin-pair correlation in EPR-type experiments by use of hidden parameters. A key-element of our approach is contained in the introduction of time-like correlated parameter random variables that also depend on the setting of the station in which they influence the measurements. This leads in a natural way to a setting dependent probability measure composed of subspace product measures (SDSPM's). Use of such SDSPM's does not permit the proof of Bell to go forward (nor any other proofs of similar theorems known to us as given e.g. in references [9] and [11]). We conclude that setting and time dependent parameter random variables present a possible loophole in theorems ala Bell.

5 Acknowledgement

The work was supported by the Office of Naval Research N00014-98-1-0604.

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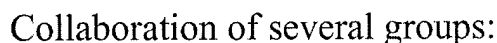
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Antti-Pekka Jauho

MIC, Technical University of Denmark

- 1) Motivation: Atomic/Molecular-scale Electronics
- 2) Modelling: Challenges
- 3) Nonequilibrium Green function formalism
- 4) The *TranSIESTA ab initio* transport program.
- 5) Results for atomic gold wires.
- 6) A toy molecular device.



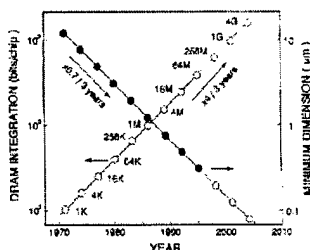
Mads Brandbyge, Kurt Stokbro, Jeremy Taylor, MIC, Denmark

Pablo Ordejón, Jose-Luis Mozos, UAB, Spain

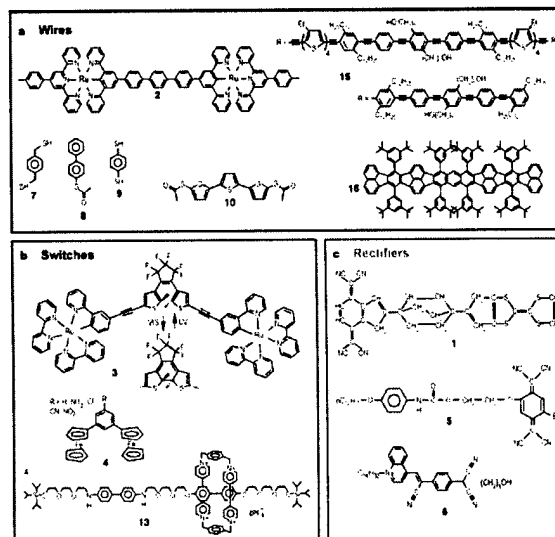


Smallest functional unit: The single molecule (Aviram & Ratner, 1974)

- “Top-down” silicon technology cannot continue to shrink in component size (dissipation, tunneling, cost of fabs...)
“End of the road map”



- “Bottom-up”: The single molecule = Smallest functional unit
- Chemists can synthesize organic molecules with a range of properties
- Use novel quantum effects





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(June 12, 2001)

We investigate electronic transport through two types of conjugated molecules. Mechanically controlled break-junctions are used to couple thiol endgroups of single molecules to two gold electrodes. Current-voltage characteristics (IV's) of the metal-molecule-metal system are observed. These IV's reproduce the spatial symmetry of the molecules with respect to the direction of current flow. We hereby unambiguously detect an intrinsic property of the molecule, and are able to distinguish the influence of both the molecule and the contact to the metal electrodes on the transport properties of the compound system.

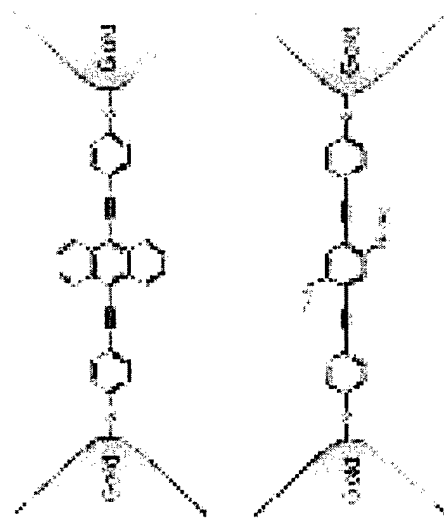


FIG. 1. Scheme of the experimental setup: a spatially symmetric
(9,10-Bis((2'-para-mercaptophenyl)-ethynyl)-anthracene) and
an asymmetric molecule (1,4-Bis((2'-para-mercaptophenyl)-
ethynyl)-2-acetyl-5-nitro-benzene) in between two gold
electrodes.

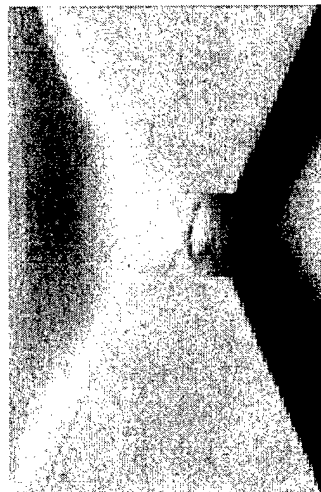


FIG. 2. Scanning electron microscope picture of the lithographically fabricated break junction. The setup consists of a metallic plate, covered by an insulating layer of polyimide. On top of this, a gold film with a small constriction (smallest diameter $50 \times 50 \text{ nm}^2$) is deposited, laterally structured by e-beam lithography. Two electrodes lead outside to connect the bridge electrically. The polyimide is partially etched away so that in the constriction region, the bridge is freely suspended over the polyimide substrate.

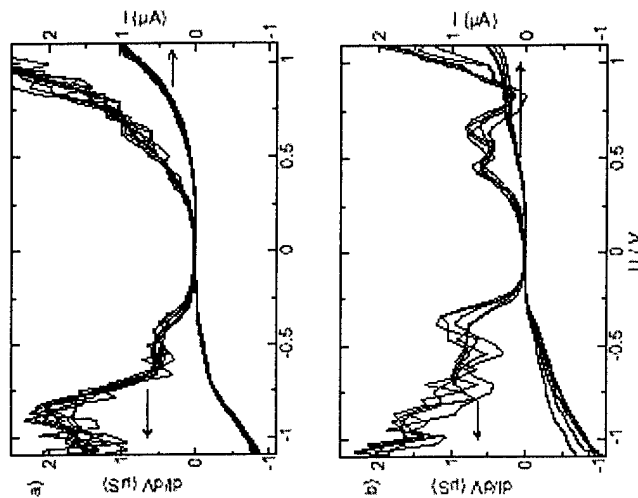
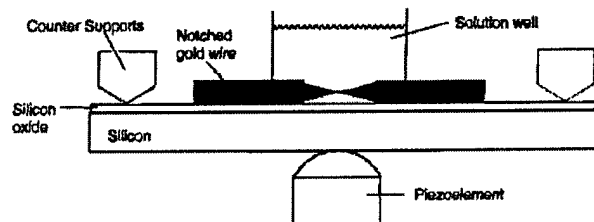
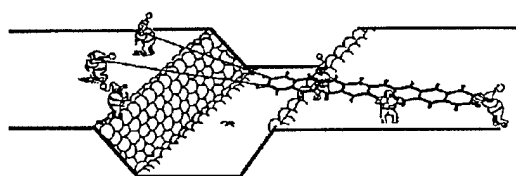


FIG. 3. Transport data of the asymmetric molecule. a) Current-voltage (IV) raw data (dashed lines, nine subsequent voltage sweeps) on a stable junction and the numerically differentiated data dI/dV (solid lines) from the above IV. b) Data from a subsequent junction.

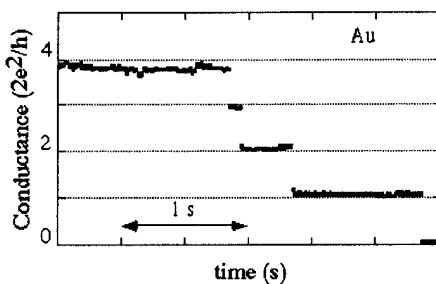


Mechanical Break Junction Experiments

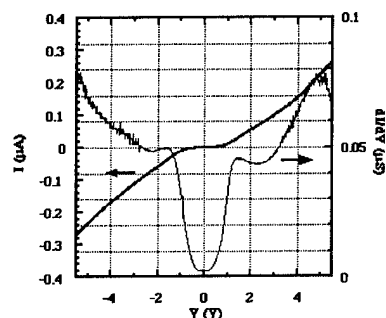


No Molecules/Solution

Conductance Quantization: Atomic short circuit



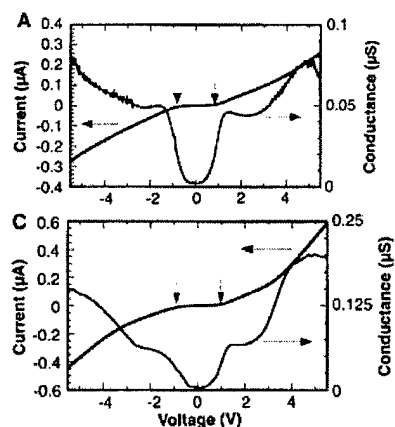
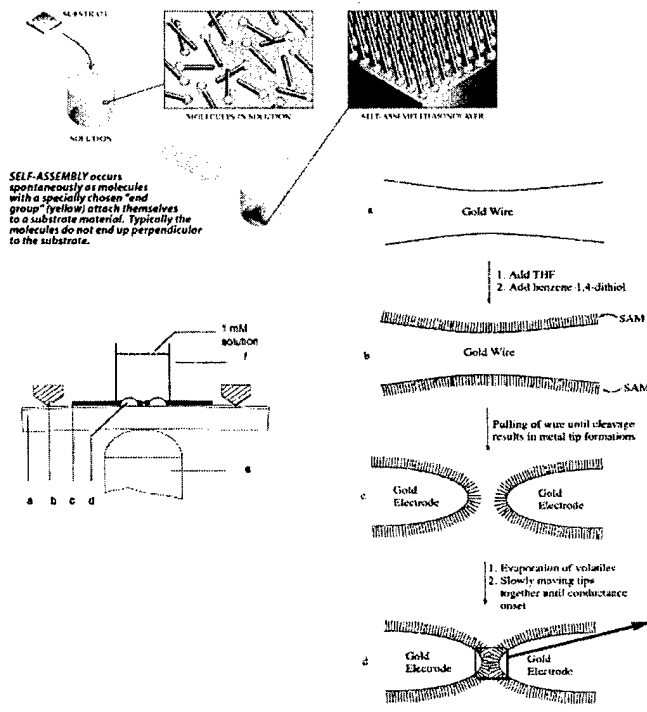
Molecules in solution



"Conductance of a Molecular Junction," M.A. Reed et al., Science, 278, 252 (1997).



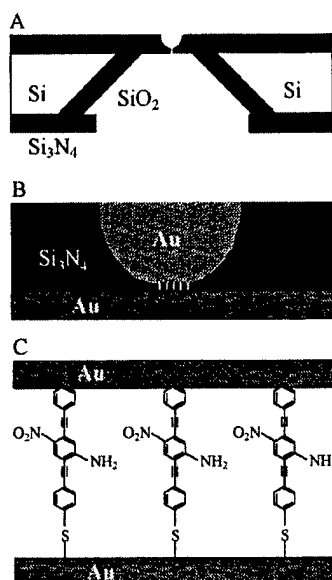
"Single" Molecule Experiments



"Conductance of a Molecular Junction," M.A. Reed et al., Science, 278, 252 (1997).



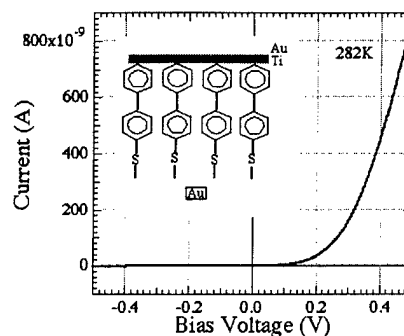
Nano-pore



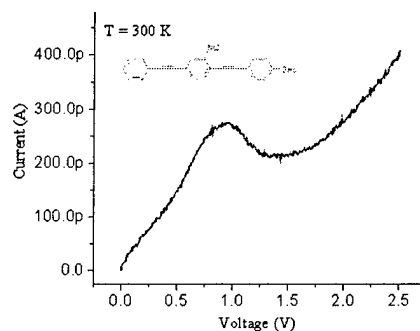
C. Zhou et al., "Nanoscale Metal/Self-Assembled Monolayer/Metal Heterostructures", Appl. Phys. Lett., 71, 611 (1997)

J. Chen et al., "Room-Temperature Negative Differential Resistance in Nanoscale Molecular Junctions", Appl. Phys. Lett., 21, 1224 (2000)

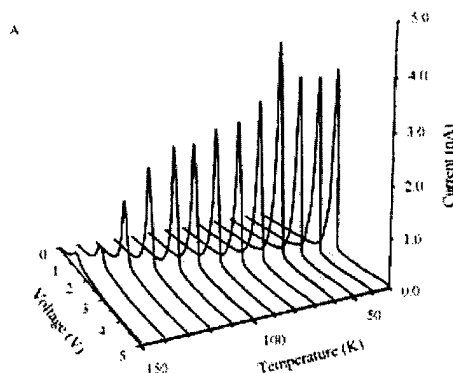
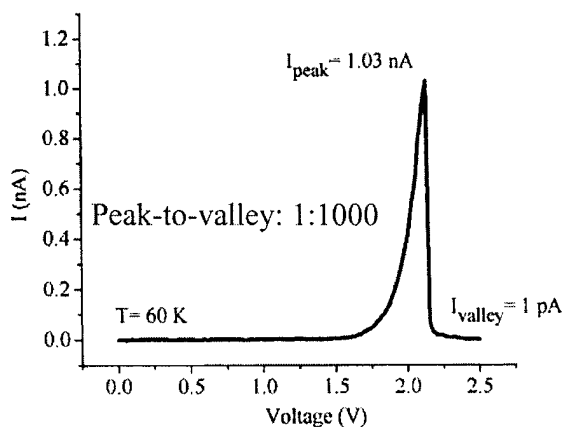
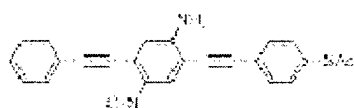
Diode behavior



Negative Differential Resistance



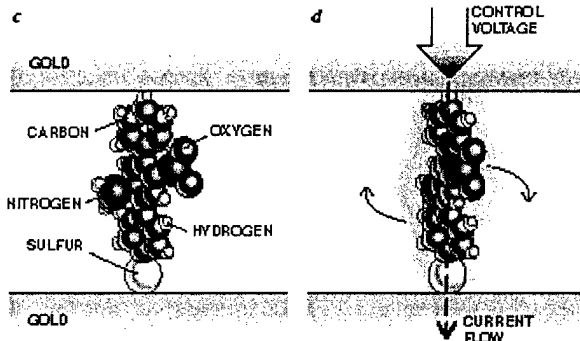
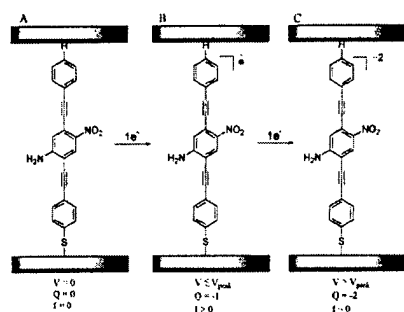
The Tour wire: "Giant" NDR



NDR has strange temperature dependence



The Tour wire: Mechanism ???



No explanation yet

From Mark A. Reed and James M. Tour, Scientific Am. June 2000

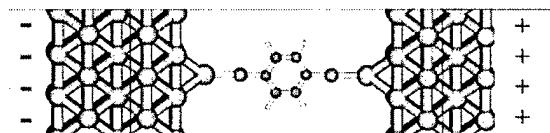
Possible factors:

- A) Change in molecular states
- B) Change in geometry
- C) Change in charge states
- D) Interaction of molecular dipole with external field

Only calculations for the *isolated* molecule has been published



Modelling atomic-scale conductors: Challenges

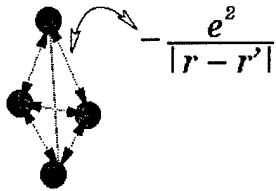


- Model a molecule coupled to bulk (infinite) electrodes
- Model the molecule-electrode system from “first principles”:
No parameters fitted to the particular system
- Include finite bias voltage/current
- Calculate the conductance (quantum transmission through the molecule)
- Determine geometry: Relax the atomic positions to an energy minimum



Density Functional Theory

Challenge: Many interacting electrons



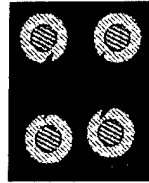
Interacting electrons
+ real potential

DFT gives a good description of energy and geometry of isolated molecules and perfect crystals

Way out: DFT



*If the electron density is correct so is the total energy,
W. Kohn, Nobel prize 1998.*



Non-interacting, fictitious particles + effective potential (V_{eff})
 $V_{eff}[n(r)]$, $n(r)$ is the electron density

Schrödinger's equation

$$H\Psi = E\Psi, \quad H = T + V_{eff}$$

instead of solving this we solve for G :

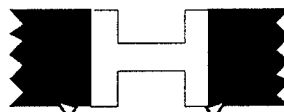
$$G(z) = (z - H)^{-1}$$

We can get the density from G

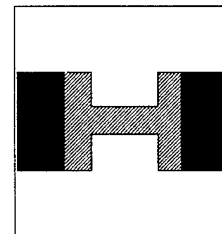


Open/infinite System

Challenge: Coupling to *infinite* electrodes



Semi-infinite bulk



$$G = \begin{array}{ccccc} & & 0 & & -1 \\ & & V_L & & 0 \\ 0 & V_L & z-H & V_R & 0 \\ & & V_R & & \\ & 0 & 0 & & \end{array}$$

We only want this part of G

$$\begin{array}{c} \text{[Shaded Box]} \\ \parallel \\ \begin{array}{c} z-H \\ -\Sigma \end{array} \end{array} \quad -1$$

The selfenergy: $\Sigma = \Sigma_L + \Sigma_R$
(can be calculated from $1/(z-H^{bulk})$)



Summary of important relations from Nonequilibrium Green Function Theory

We consider the Hamiltonian:

$$H = \sum_{k,\alpha} \epsilon_\alpha(t) c_{k,\alpha}^\dagger c_{k,\alpha} + \sum_{k,\alpha,n} \left[V_{k,\alpha,n}(t) c_{k,\alpha}^\dagger d_n + \text{h.c.} \right] + H_{\text{cen}}[\{d_n\}, \{d_n^\dagger\}, t]$$

Current is calculated as follows (Caroli et al.(70's), Meir and Wingreen (1992), Jauho et al. (1994)):

$$\begin{aligned} J_L(t) &= \langle (-e) \dot{N}_L(t) \rangle \\ &= \frac{2e}{h} \text{Re} \left\{ \sum_{k,\alpha,n} V_{k,\alpha,n}(t) G_{n,k\alpha}^<(t, t) \right\} \\ &= -\frac{2e}{h} \int_{-\infty}^t dt_1 \int d\epsilon \text{ImTr} \left\{ e^{-ie(t_1-t)} \mathbf{\Gamma}^L(\epsilon, t_1, t) \right. \\ &\quad \left. [\mathbf{G}^<(t, t_1) + f_L^0(\epsilon) \mathbf{G}^r(t, t_1)] \right\}. \end{aligned}$$

Here the Green functions are *matrices* in the indices (m, n) , and the linewidth functions $\mathbf{\Gamma}$ are defined as (here given for time-independent situation)

$$[\mathbf{\Gamma}^L(\epsilon_k)]_{mn} = 2\pi \sum_{\alpha \in L} \rho_\alpha(\epsilon_k) V_{\alpha,m}^*(\epsilon_k) V_{\alpha,n}(\epsilon_k)$$



Time-independent case (Meir-Wingreen):

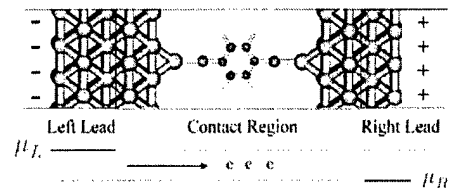
$$\begin{aligned} J &= \frac{ie}{2h} \int d\epsilon \text{Tr} \left\{ [\mathbf{\Gamma}^L(\epsilon) - \mathbf{\Gamma}^R(\epsilon)] \mathbf{G}^<(\epsilon) \right. \\ &\quad \left. + [f_L^0(\epsilon) \mathbf{\Gamma}^L(\epsilon) - f_R^0(\epsilon) \mathbf{\Gamma}^R(\epsilon)] [\mathbf{G}^r(\epsilon) - \mathbf{G}^a(\epsilon)] \right\} \\ &= \frac{e}{h} \int d\epsilon [f_L^0(\epsilon) - f_R^0(\epsilon)] \text{Tr} \left\{ \mathbf{G}^a \mathbf{\Gamma}^R \mathbf{G}^r \mathbf{\Gamma}^L \right\}, \end{aligned}$$

where the second line holds, if the central region can be described in terms of an *effective one-body potential* (such as in density-functional theory).

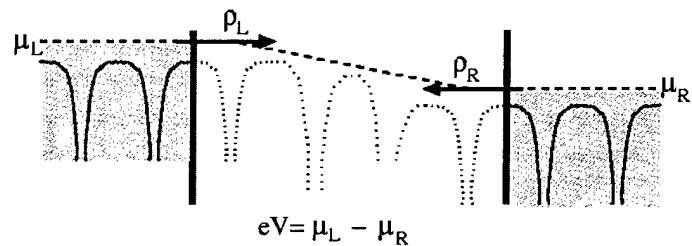


Finite Voltage Bias/Current

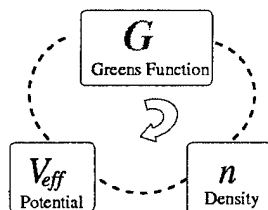
Challenge: non-equilibrium electron distribution



The Non-equilibrium Density Matrix



Selfconsistent solution

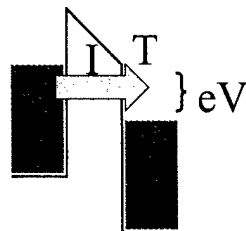


$$D_{\alpha,\beta} = \int_{-\infty}^{\infty} dE \rho_{\alpha,\beta}^L(E) n_F(E - \mu_L) + \rho_{\alpha,\beta}^R(E) n_F(E - \mu_R)$$

$$\rho_{\alpha,\beta}^L(E) = \frac{1}{\pi} (G(E + i\delta) \text{Im} [\Sigma_L(E + i\delta)] G^\dagger(E + i\delta))_{\alpha,\beta}$$



Conductance = Transmission



Conductance (Landauer-Büttiker)

$$I = \frac{2e^2}{h} \int dE (f_R(E) - f_L(E)) T_{\text{tot}}(E)$$

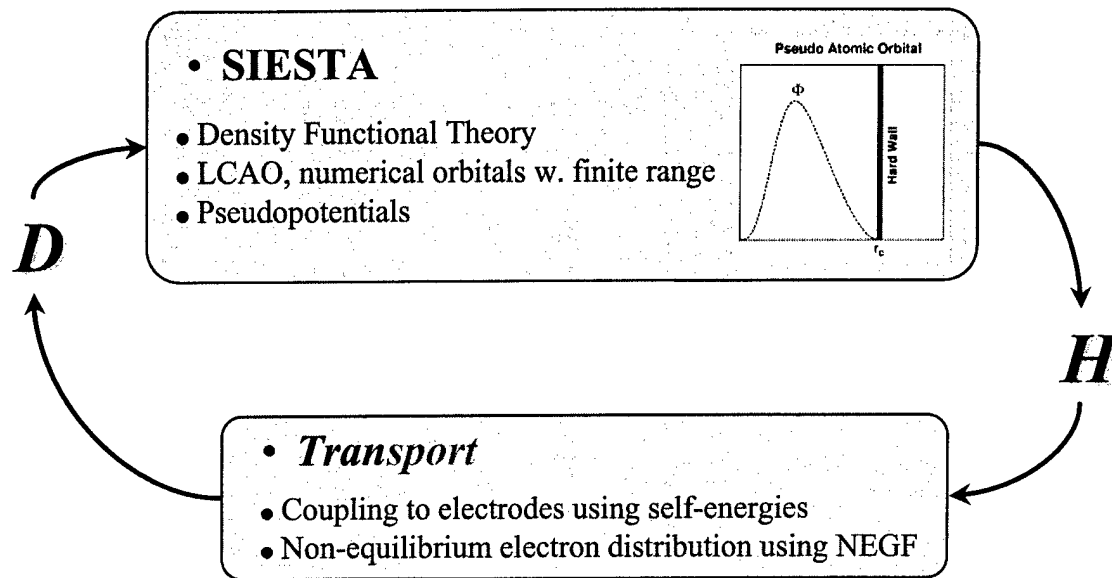
$T_{\text{tot}}(E)$: Total Quantum Transmission

$$T_{\text{tot}}(E) = \text{Tr}[t^\dagger t](E)$$

$$t(E) = (\text{Im} [\Sigma_R](E))^{1/2} G(E) (\text{Im} [\Sigma_L](E))^{1/2}$$

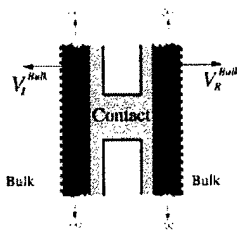


Implementation: TranSIESTA



Setup

Physical system:



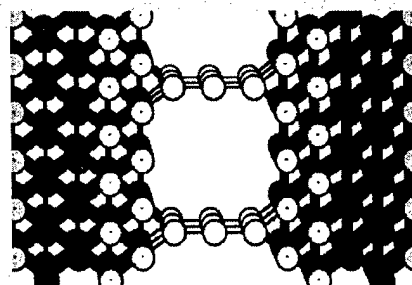
Representation (PBC in x,y):



Example:

3-atom Au chain connecting
Au(100) electrodes.

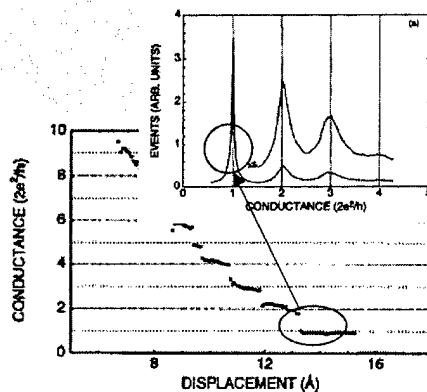
Periodic Boundary Conditions in (x,y)
3 × 3 unit cell, ϕ -point approximation





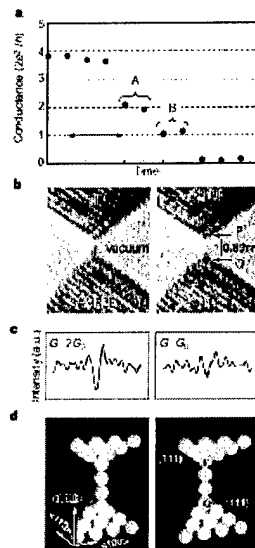
Benchmark atomic-scale conductors: Single-atom gold contacts

Good statistics:
1 atom = 1 conductance quantum



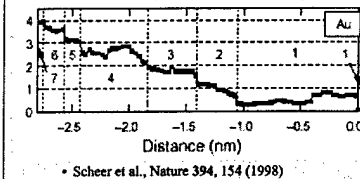
• Brandbyge et al., PRB 52, 8499 (1995)

Structures directly imaged



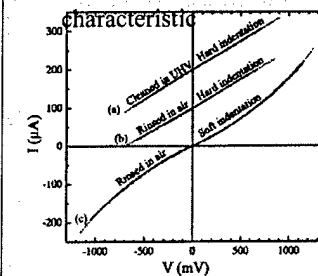
• Ohnishi et al., Nature 395, 780 (1998)

Single channel transport



• Scheer et al., Nature 394, 154 (1998)

Linear I-V

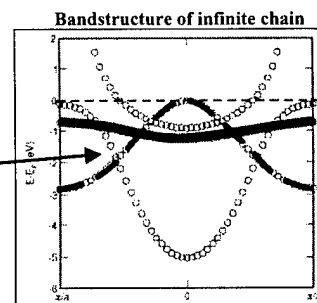
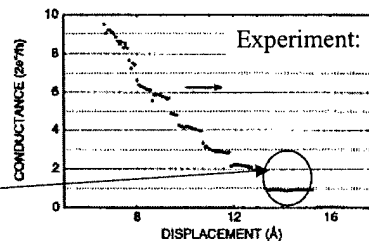
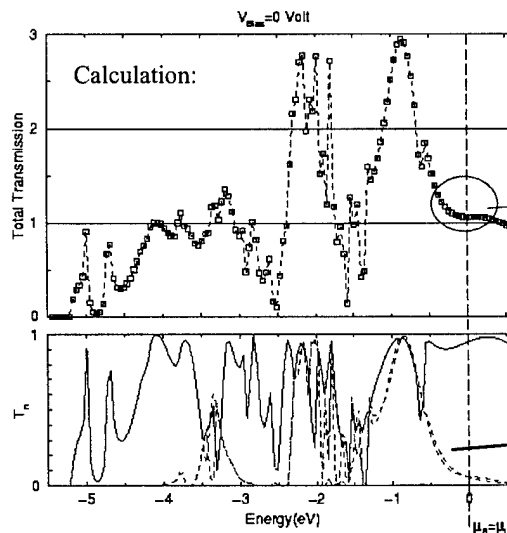
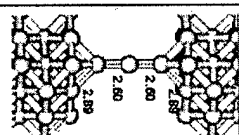


• Hansen et al., APL 77, 708 (2000)



Atomic short circuit: Single-atom contact

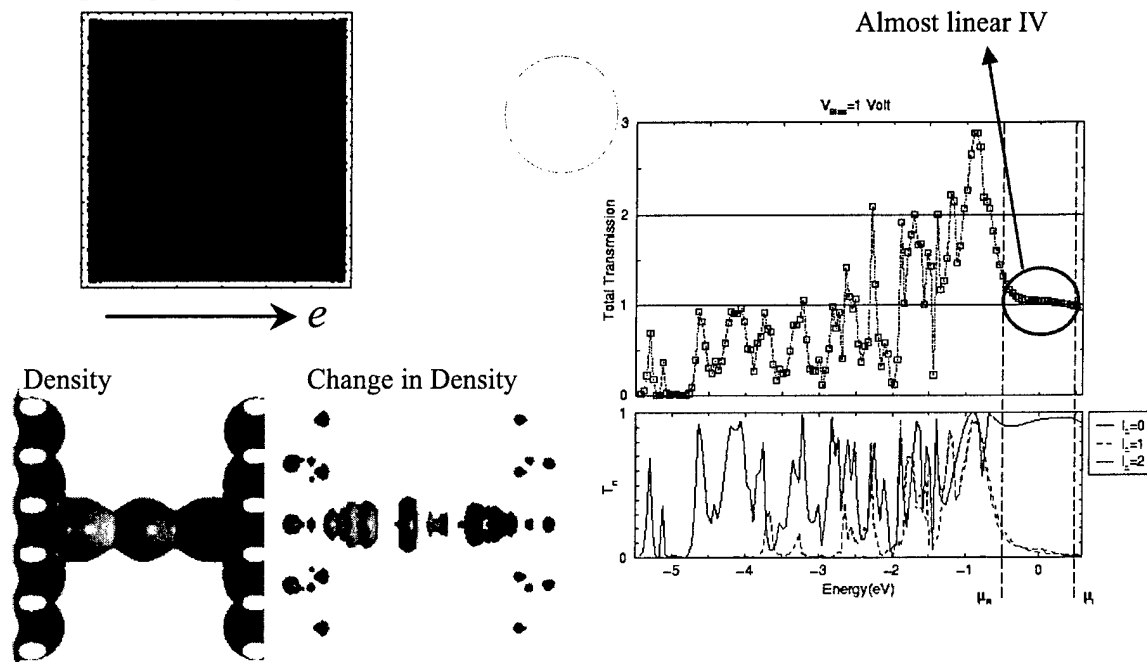
3-atom Au chain connecting
Au(100) electrodes.
 3×3 unit cell, δ -point approximation





Single atomic contact: 1 Volt Bias

Voltage drop through the contact



Zig-zag chains

VOLUME 83, NUMBER 19

PHYSICAL REVIEW LETTERS

8 NOVEMBER 1999

Stiff Monatomic Gold Wires with a Spinning Zigzag Geometry

Daniel Sanchez-Portal,¹ Emilio Artacho,² Javier Junquera,² Pablo Ordejón,¹ Alberto García,³ and José M. Soler²

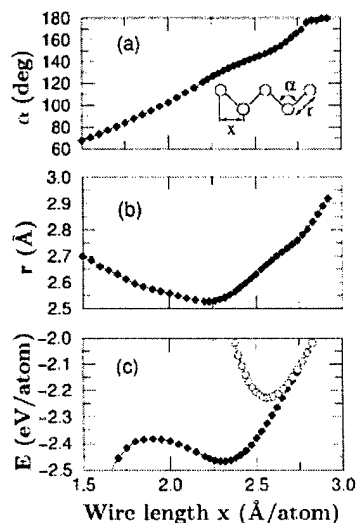


FIG. 1. First-principles, density-functional results for the bond angle α (a), and bond length r (b) in a monatomic gold wire with zigzag geometry, as a function of its length per atom. (c) Binding energy E in the zigzag (solid symbols) and linear wires (open symbols).

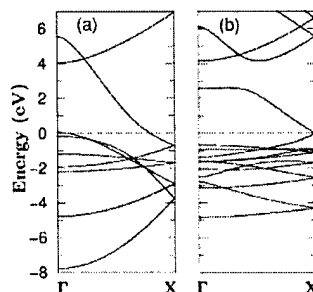
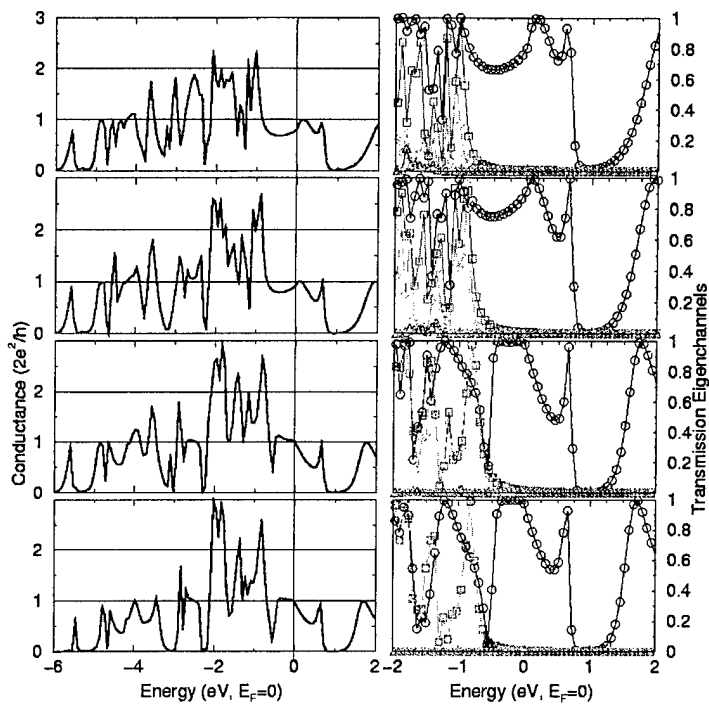
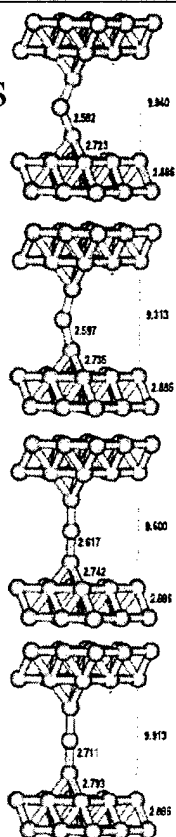


FIG. 2. Electronic band structure of the linear (a) and zigzag (b) wires for a length of 2.32 Å/atom. The linear-wire bands have been folded onto a two-atom Brillouin zone to facilitate the comparison. The energies are relative to the Fermi level.



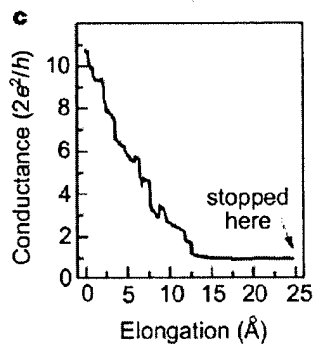
Zig-zag wires on Au(111)



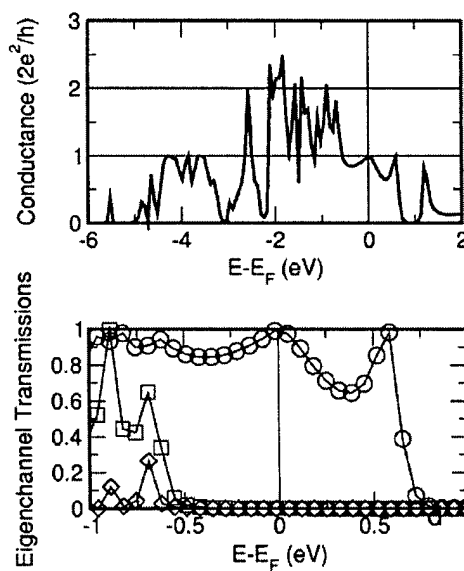
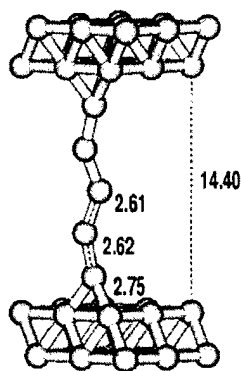
5 atom long gold chain

Chains of at least 4
gold atoms has been observed

Last plateau length = 12 Å



Yanson et al., Nature 395, 783 (1998)

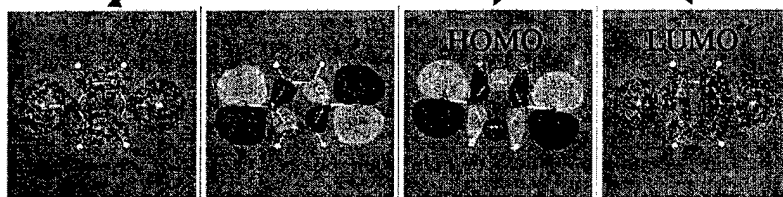
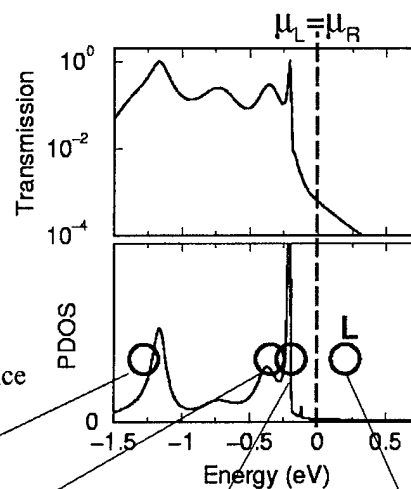
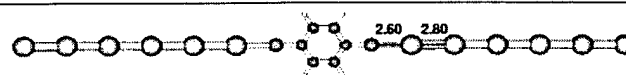




"Toy model"

Benzene-1,4-dithiolate
coupled to gold chains

- The molecular states have a node plane
- The chain band states are rot. symmetric at the Fermi energy

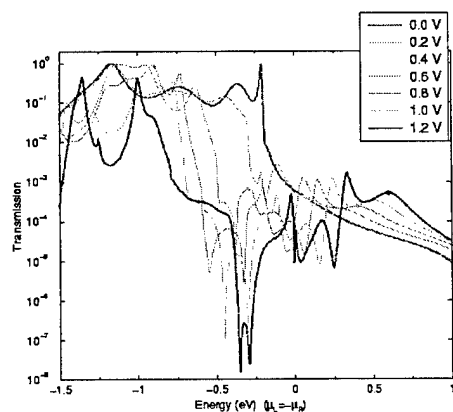
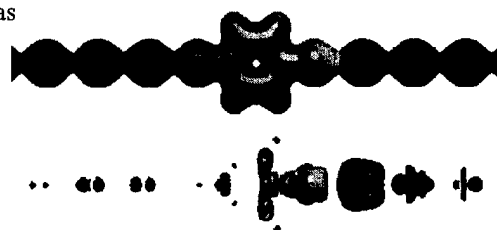
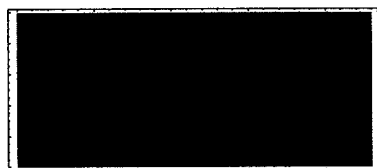


Brandbyge et al., Proceedings MRS fall meeting 2000, D9.25

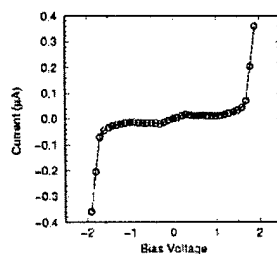


Benzene-1,4-dithiolate coupled to gold chains: Finite Bias

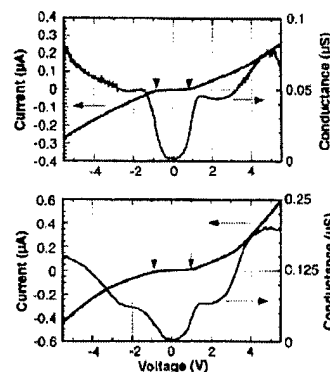
Change in effective potential and density for 1Volt bias



Calculated IV



Measured IV



From Reed et al., Science 278, 252 (1997)

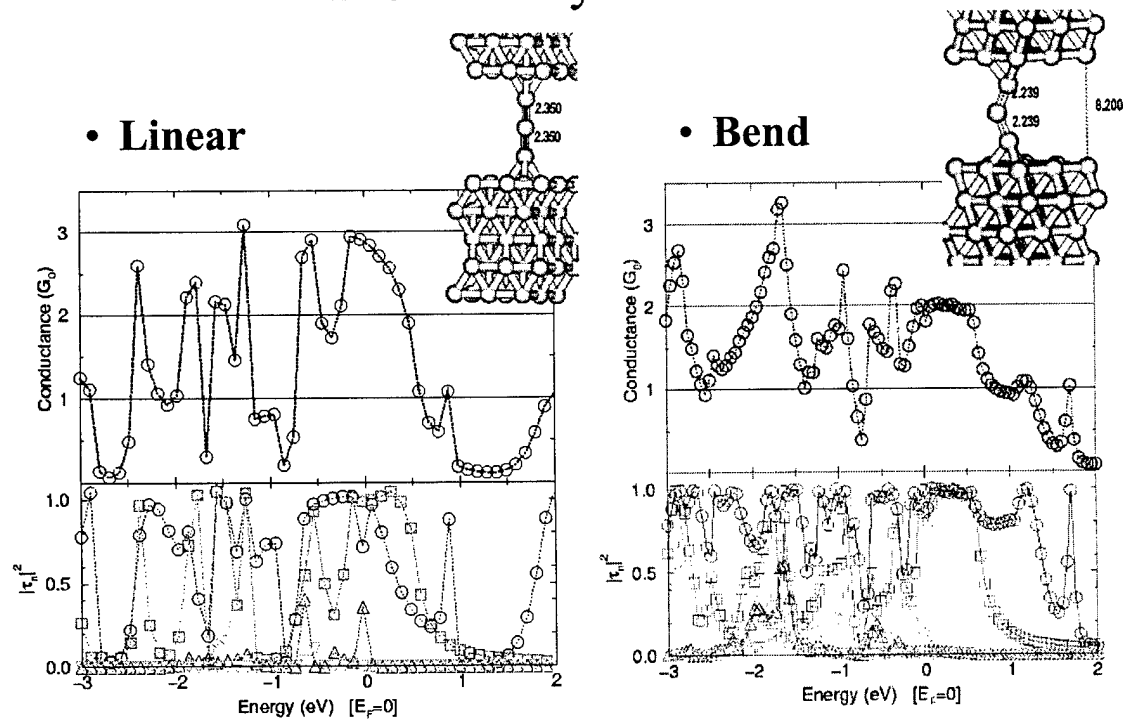


Summary

- ◆ Many exciting and promising experiments on atomic/molecular scale conductors.
- ♣ Many unknown factors in most experiments.
- ♥ We are developing a tool for first principles modelling (*TranSIESTA*)



Preliminary Platinum



Landauer / McKelvey

Approach to MOSFET Modeling

Mark Lundstrom
Electrical and Computer Engineering
Purdue University, West Lafayette, IN

1. Introduction
2. Landauer/McKelvey Theory of the MOSFET
3. The Ballistic MOSFET
4. Back-scattering in MOSFET's
5. Discussion
6. Summary

Lundstrom

additional information at: www.ece.purdue.edu/celab

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Zhibin Ren, Ramesh Venugopal, Jung-Hoon Rhew
Dave Rumsey, Anisur Rahman, Jing Guo, Sayed Hasan

sponsors:

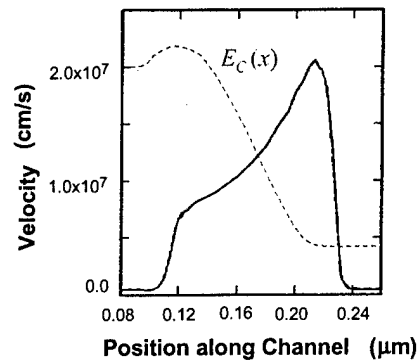
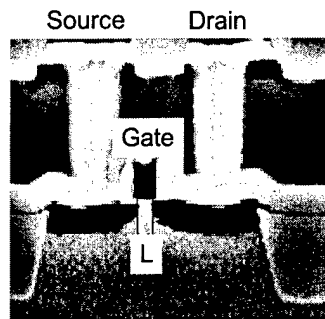
NSF
SRC
MARCO/DARPA
Indiana 21st Century Research and Technology Fund

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1. Introduction

nanoscale MOSFETs



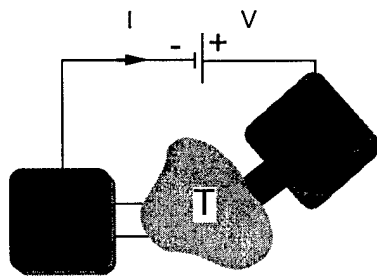
Lundstrom

Purdue

1. Introduction

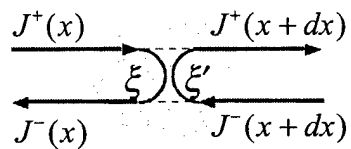
conceptual approach

Landauer, 1959



$$G = \frac{2e^2}{h} TM$$

McKelvey, 1961



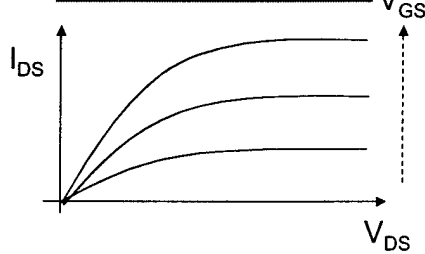
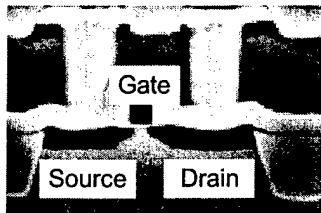
$$\frac{dJ^+}{dx} = -\xi J^+ + \xi' J^-$$

$$\frac{dJ^-}{dx} = -\xi J^- + \xi' J^+$$

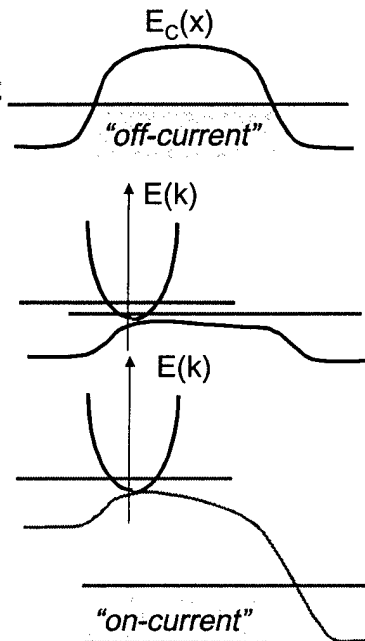
Lundstrom

Purdue

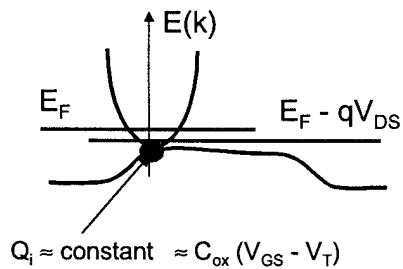
2. Landauer/McKelvey Theory of the MOSFET



Lundstrom



2. Landauer/McKelvey Theory of the MOSFET



$$I_{DS} = W Q_i(V_{GS}) \tilde{v}_T \times \left[\frac{1 - \frac{F_{1/2}(\eta_F - U_{DS})}{F_{1/2}(\eta_F)}}{1 + \frac{F_0(\eta_F - U_{DS})}{F_0(\eta_F)}} \right]$$

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$$J^+ = n^+ \tilde{v}_T^+$$

$$n^+ = \frac{m}{2\pi\hbar^2} \ln(1 + e^{\eta_F})$$

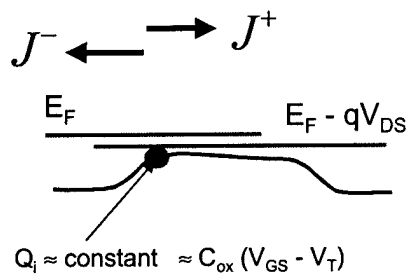
$$\tilde{v}_T^+ = \sqrt{\frac{2k_B T}{\pi m} \frac{F_{1/2}(\eta_F)}{F_0(\eta_F)}}$$

$$I_D = Wq(J^+ - J^-)$$

$$Q_i(V_{GS}) = q(n^+ + n^-)$$

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2. Landauer/McKelvey Theory of the MOSFET



$$J^+ = n^+ \tilde{v}_T^+$$

$$J^- = (1 - T)J^+ + TJ_{ball}^-$$

$$I_D = Wq(J^+ - J^-)$$

$$Q_i(V_{GS}) = \frac{q(J^+ + J^-)}{\tilde{v}_T^+}$$

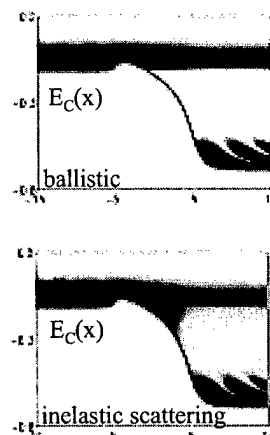
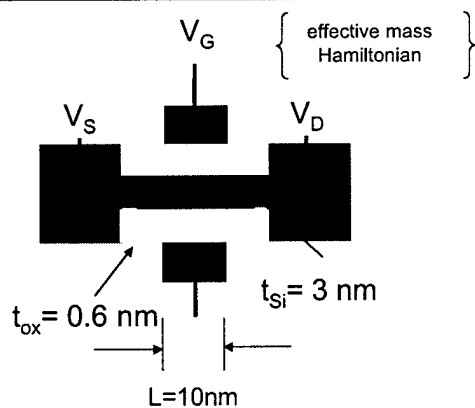
$$I_{DS} = WQ_i(V_{GS})\tilde{v}_T\left(\frac{T}{2-T}\right) \times \left\{ \frac{1 - \frac{F_{1/2}(\eta_F - U_{DS})}{F_{1/2}(\eta_F)}}{1 + \left(\frac{T}{2-T}\right)\frac{F_0(\eta_F - U_{DS})}{F_0(\eta_F)}} \right\}$$

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2. Landauer/McKelvey Theory of the MOSFET

NEGF Simulations



Zhibin Ren and Ramesh Venugopal (Purdue)
Dejan Jovanovic (Motorola, Los Alamos)

Lundstrom

Purdue

2. Landauer/McKelvey Theory of the MOSFET

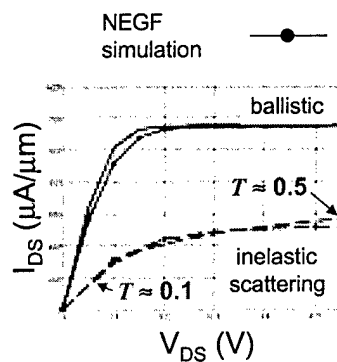
Landauer/McKelvey model

$$I_{DS} = WC_{ox}(V_{GS} - V_T) \tilde{v}_T \left(\frac{T}{2-T} \right) \times$$

$$\frac{F_{1/2}(\eta_F)}{\ln(1 + e^{\eta_F})} \times \left\{ \frac{1 - \frac{F_{1/2}(\eta_F - U_{DS})}{F_{1/2}(\eta_F)}}{1 + \left(\frac{T}{2-T} \right) \frac{\ln(1 + e^{\eta_F - U_{DS}})}{\ln(1 + e^{\eta_F})}} \right\}$$

$$T = \frac{\lambda_o}{\lambda_o + (k_B T / e V_{DS})^\alpha}$$

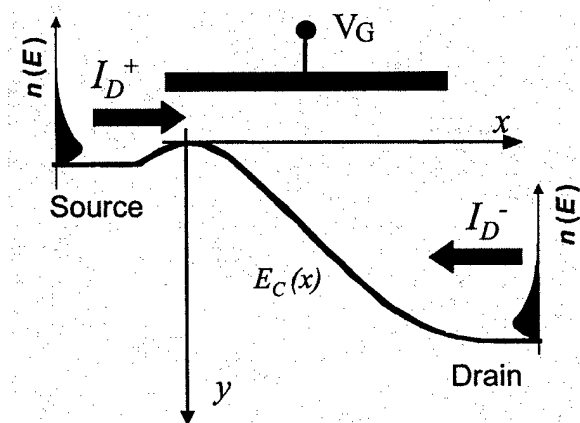
Lundstrom



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3. The Ballistic MOSFET.....

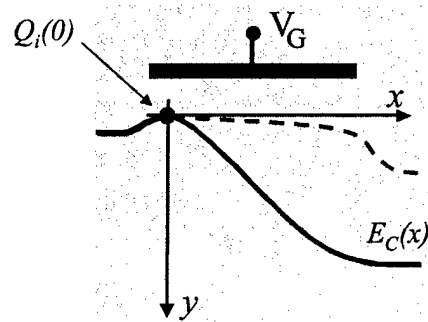
thermionic emission



Lundstrom

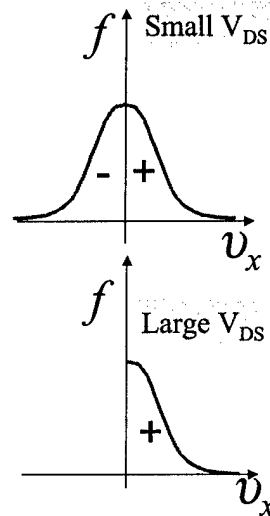
Purdue

3. The Ballistic MOSFET.....



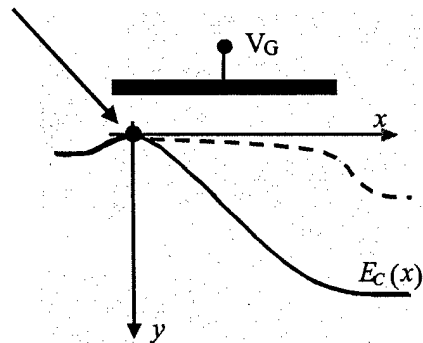
$$f_0 = e^{-E/k_B T} \sim e^{-m^* v_x^2 / 2k_B T}$$

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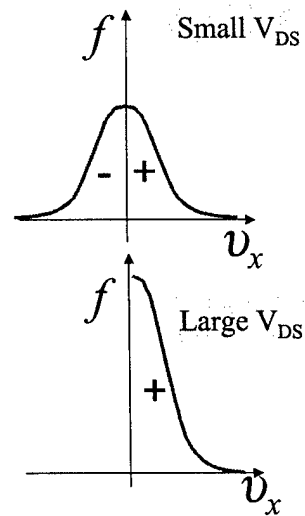


3. The Ballistic MOSFET.....

$$Q_i(0) \approx C_{ox} (V_{GS} - V_T)$$

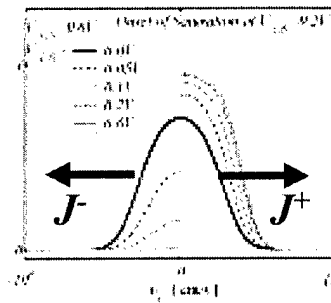
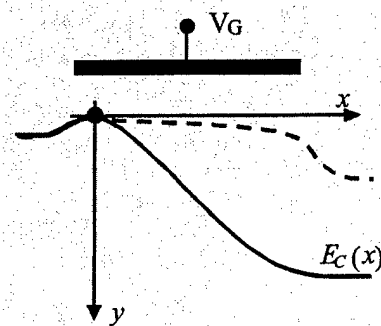


Lundstrom



3. The Ballistic MOSFET.....

numerical solution of the ballistic BTE



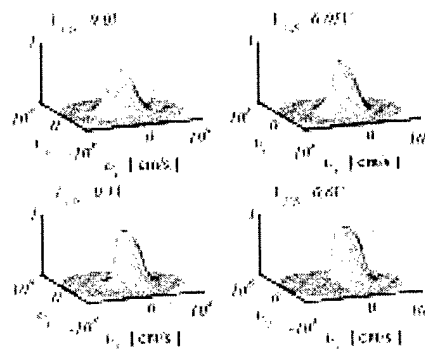
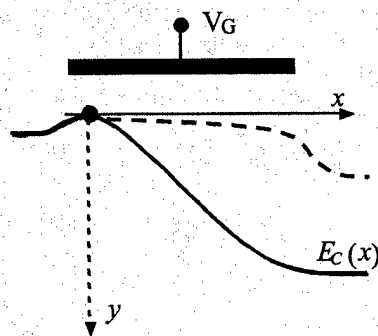
$$R \equiv J^- / J^+$$

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3. The Ballistic MOSFET.....

numerical solution of the ballistic BTE

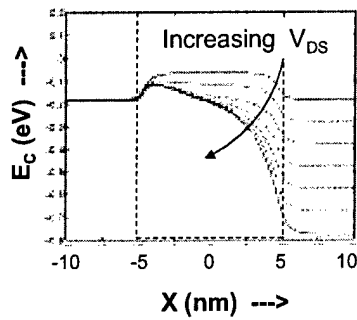


Lundstrom

Purdue

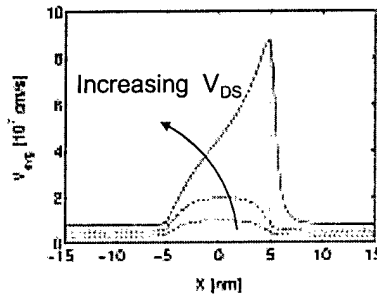
3. The Ballistic MOSFET.....

E_C vs. x for $V_{GS} = 0.5V$



Lundstrom

- i) $Q_i(0) \approx \text{constant}$
- ii) $\langle v(0) \rangle \rightarrow v_T$

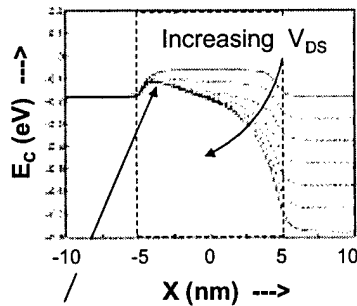


Purdue

3. The Ballistic MOSFET.....

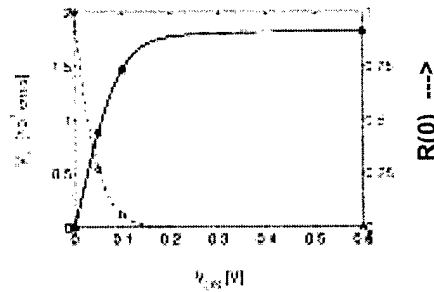
velocity saturation in a ballistic FET

E_C vs. x for $V_{GS} = 0.5V$



Lundstrom

- i) $Q_i(0) \approx \text{constant}$
- ii) $\langle v(0) \rangle \rightarrow v_T$

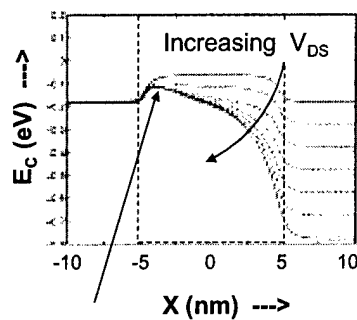


Purdue

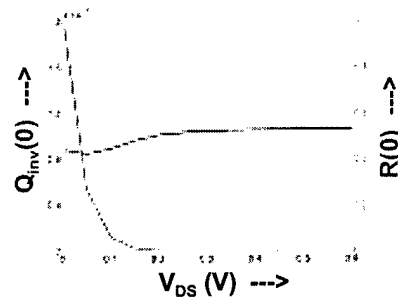
3. The Ballistic MOSFET.....

'(2-T) effect'

E_c vs. x for $V_{GS} = 0.5V$



- i) $Q_i(0) \approx \text{constant}$
- ii) $\langle v(0) \rangle \rightarrow v_T$



The (2-T) effect: $Q_{inv} \sim \text{constant}$

Lundstrom

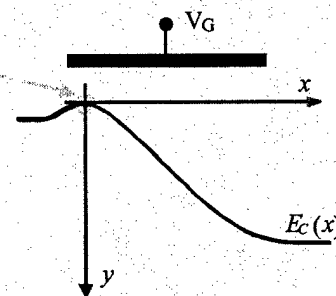
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3. The Ballistic MOSFET.....

quasi-equilibrium

$$n_s(V_{GS}) = C_{eff}(V_{GS} - V_T)$$

(Gradual Channel
Approximation
+ DIBL)



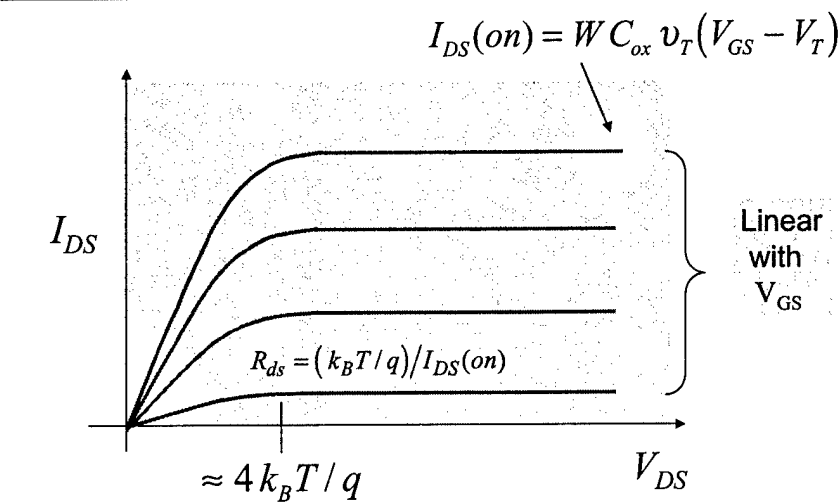
$$I_{DS} = W C_{ox} v_T (V_{GS} - V_T) \left[\frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}} \right] \quad v_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

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(Boltzmann statistics and one subband)

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3. The Ballistic MOSFET.....

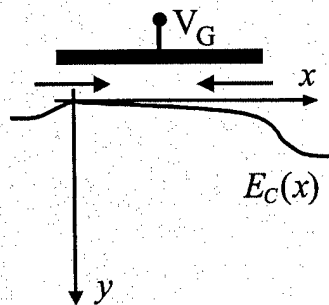


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3. The Ballistic MOSFET.....

Why does a ballistic MOSFET have a finite channel resistance?



$$I_D^- = I_D^+ e^{-qV_{DS}/k_B T}$$

$$I_{DS} = I_D^+ - I_D^-$$

$$I_{DS} = I_D^+ (1 - e^{-qV_{DS}/k_B T})$$

$$I_{DS} \approx \frac{I_D^+}{(k_B T / q)} V_{DS}$$

Lundstrom

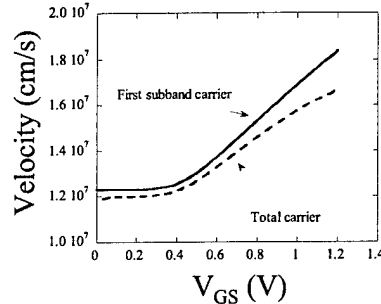
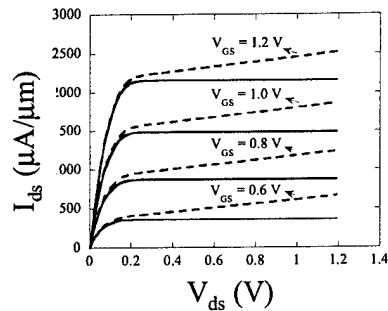
3. The Ballistic MOSFET.....

100 nm technology node....

$t_{ox} = 1.5 \text{ nm}$ $N_A = 2 \times 10^{18} \text{ cm}^{-3}$

$V_{DD} = 1V$

DIBL = 100 mV/V



$$I_{DS} \propto (V_{GS} - V_T)^\alpha$$

$$1 \leq \alpha \leq 1.5$$

$$I_{DS(on)} / W = q n_S v_T$$

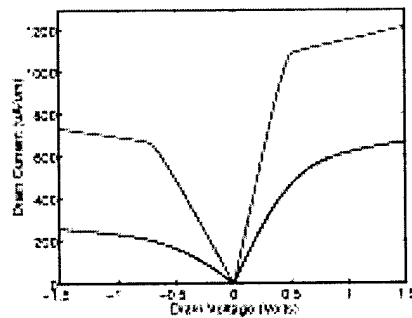
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3. The Ballistic MOSFET.....

comparison with measurements.....

$L_{eff} = 115/125 \text{ nm}$ technology



—————

ballistic
(with measured R_s)

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NMOS: ~ 55% of limit

PMOS: ~ 33% of limit

How can a device with L_{eff}
~ 6 mfps operate at ~50%
of the ballistic limit?

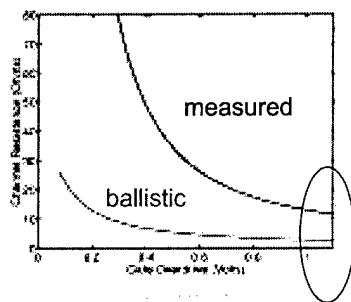
- Farzin Assad, et al. (1999 IEDM)
- Dave Rumsey
- G. Timp, J. Bude, et al., (1999 IEDM)
- A. Lochtefeld, D. Antoniadis (EDL, Feb 2001)

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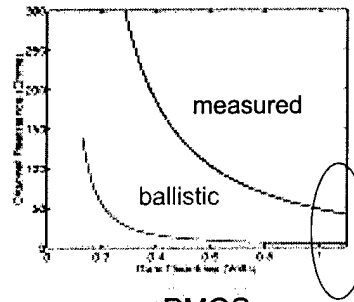
3. The Ballistic MOSFET.....

comparison with measurements.....

Channel resistance vs. V_{GS}



NMOS



PMOS

$$R_{SD} = R_{par} + R_{ballistic} + R_0 \frac{L}{W}$$

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3. The Ballistic MOSFET.....

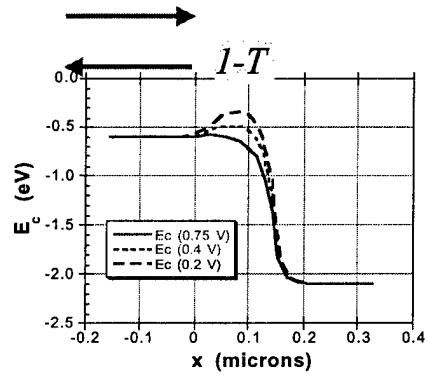
Summary

- the ballistic I-V is readily computed and the ballistic MOSFET is readily understood
- present day devices operate at ~ 50% of the ballistic limit
- future devices will have to operate even closer
- **backscattering** limits the performance of realistic devices

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4. Back-Scattering in MOSFETs.....

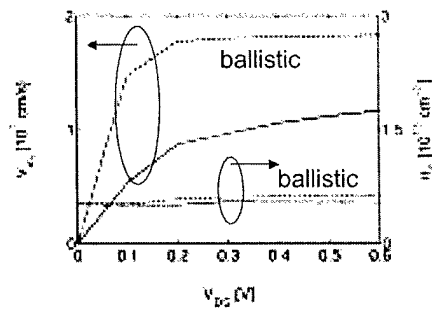
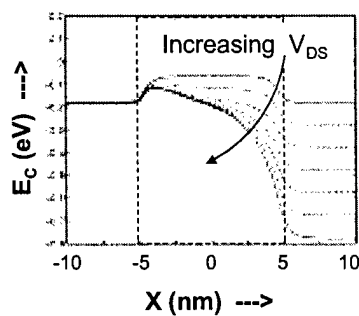


carriers at the top of the barrier: MOS electrostatics
 carriers that cross the channel: backscattering, $R=1-T$

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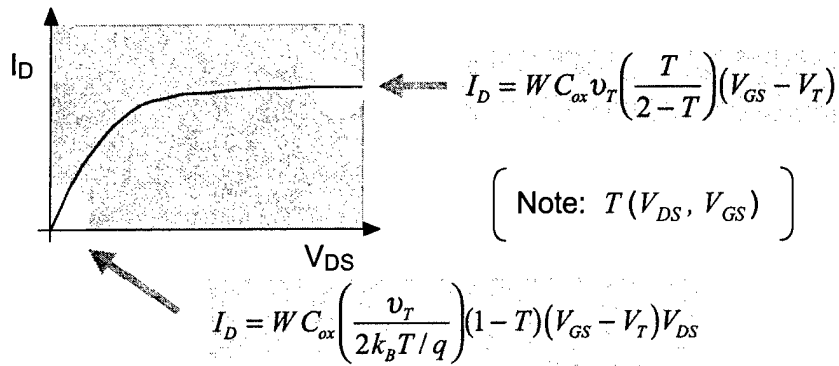
4. Back-Scattering in MOSFETs.....



Lundstrom

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4. Back-Scattering in MOSFETs.....



M.S. Lundstrom, "Elementary scattering theory of the MOSFET," *EDL*, **18**, 361, 1997

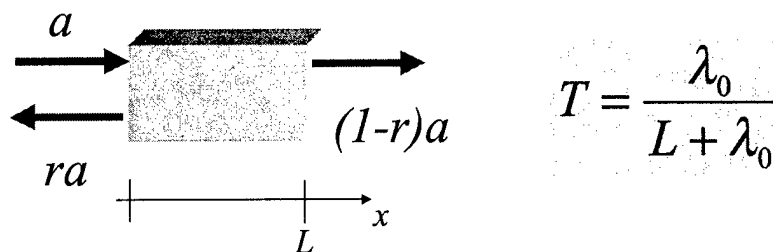
S. Datta, et. al, "The MOSFET from a Transmission Viewpoint," *Superlattices and Microstructures*, 1998

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4. Back-Scattering in MOSFETs.....

computing T : low V_{DS}



$$I_{DS} = W C_{ox} \left(\frac{v_T}{k_B T / q} \right) T (V_{GS} - V_T) V_{DS}$$

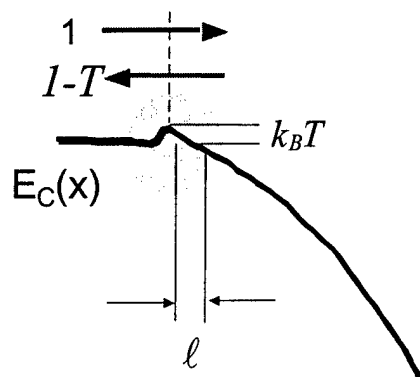
★
$$I_{DS} = \mu_{eff} C_{ox} \left(\frac{W}{L + \lambda_0} \right) (V_{GS} - V_T) V_{DS}$$

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4. Back-Scattering in MOSFETs.....

computing T : high V_{DS}



$$T = \frac{\lambda_o}{l + \lambda_o}$$

Bethe condition for a MOSFET:

ballistic current: $l < \lambda$

(not: $L < \lambda$)

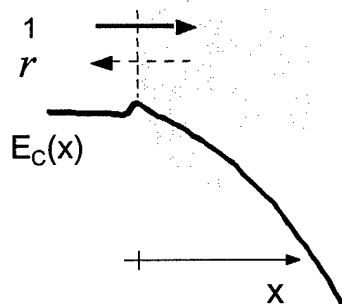


mobility is important for nanoscale FETs

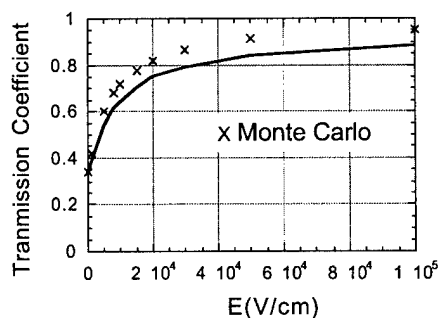
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4. Back-Scattering in MOSFETs.....



$$r = \frac{l}{l + \lambda_o}$$



See:
P.J. Price, *Semiconductors and Semimetals*,
14, 249-334, 1979
H. U. Baranger and J.W. Wilkins, *Phys. Rev. B*,
36, 1487-1502, 1987.

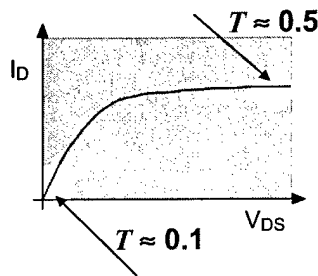
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4. Back-Scattering in MOSFETs.....

Landauer/McKelvey model

$$I_D = W C_{ox} v_T \left(\frac{T}{2-T} \right) (V_{GS} - V_T)$$



$$I_{DS} = W C_{ox} (V_{GS} - V_T) \tilde{v}_T \left(\frac{T}{2-T} \right) \times$$

$$\frac{F_{1/2}(\eta_F)}{\ln(1 + e^{\eta_F})} \times \left\{ \frac{1 - \frac{F_{1/2}(\eta_F - U_{DS})}{F_{1/2}(\eta_F)}}{1 + \left(\frac{T}{2-T} \right) \frac{\ln(1 + e^{\eta_F - U_{DS}})}{\ln(1 + e^{\eta_F})}} \right\}$$

$$I_D = W C_{ox} \left(\frac{v_T/2}{k_B T/q} \right) T (V_{GS} - V_T) V_{DS}$$

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Landauer / McKelvey Approach to MOSFET Modeling

Mark Lundstrom
Electrical and Computer Engineering
Purdue University, West Lafayette, IN

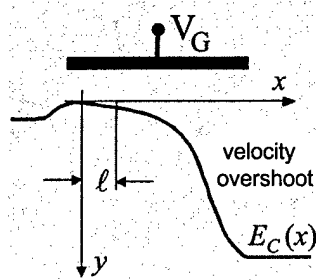
1. Introduction
2. Landauer/McKelvey Theory of the MOSFET
3. The Ballistic MOSFET
4. Back-scattering in MOSFET's
5. Discussion
6. Summary

Lundstrom

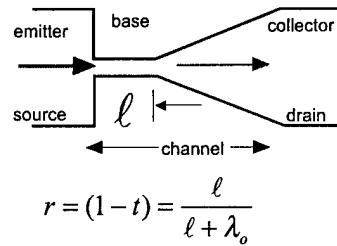
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5. Discussion....

essential physical picture....



See: "Essential physics of carrier Transport in nanoscale MOSFETs," M. Lundstrom, et al.



$$r \approx 50\% \Rightarrow l \approx \lambda_o$$

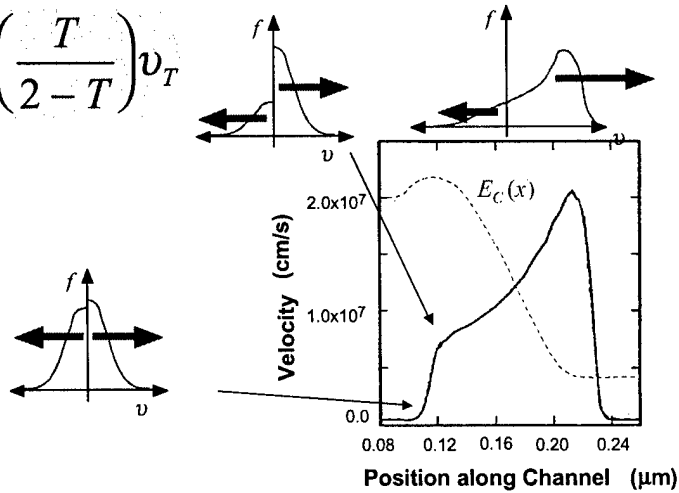
Lundstrom

Purdue

5. Discussion....

interpreting simulations

$$\langle v(0) \rangle = \left(\frac{T}{2 - T} \right) v_T$$



Lundstrom

Purdue

5. Discussion....

how does T scale?

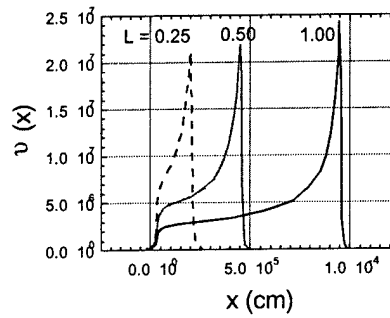
$$T = \frac{\lambda}{\ell + \lambda} \approx 50\%$$

➔ $\ell \approx \lambda$

How does T scale?

as $L \downarrow \dots \ell \downarrow$ and $\lambda \downarrow$

➔ Scaling maintains a constant T (at min L)



Lundstrom

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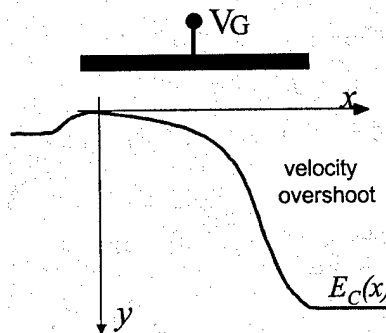
5. Discussion....

How to increase T ?

To reduce backscattering:

1) increase mfp λ (μ)

2) decrease ℓ



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5. Discussion....

The role of mobility

Backscattering is related to λ_o and λ_o is related to mobility

Scattering theory gives:

$$\left. \begin{aligned} \frac{\delta I_D}{I_D} &= \frac{\delta \mu}{\mu} (1 - B) \\ B &\equiv \frac{I_D}{I_D(\text{ballistic})} \end{aligned} \right\}$$

i) linear region: $B \approx 0.2$

$$\frac{\delta I_D}{I_D} \approx \frac{\delta \mu}{\mu}$$

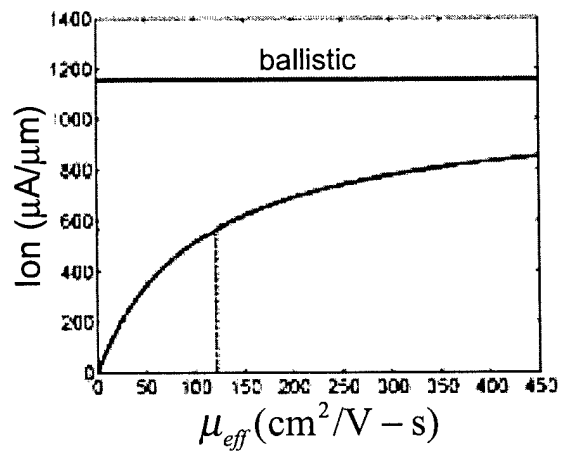
ii) saturated region: $B \approx 0.5$

$$\frac{\delta I_D}{I_D} \approx 0.5 \frac{\delta \mu}{\mu}$$

Lundstrom

Purdue

5. Discussion....

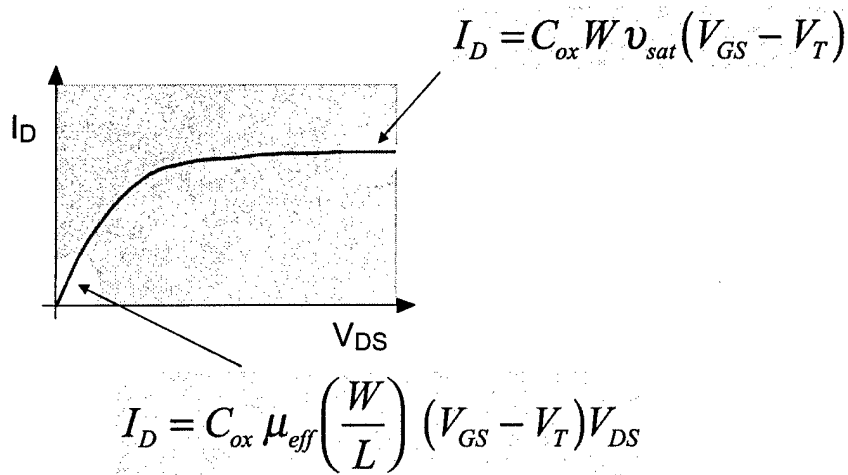


Lundstrom

Purdue

5. Discussion....

connections to traditional theory



Lundstrom

Purdue

5. Discussion....

connections to traditional theory

Scattering theory	Linear (low V_{DS}) drain current
$\left(\begin{aligned} I_D &= W C_{ox} \left(\frac{v_T}{2k_B T / q} \right) T (V_{GS} - V_T) V_{DS} \\ T &= \frac{\lambda_o}{L + \lambda_o} \end{aligned} \right)$	
	Conventional theory $\left(I_D = W \mu_{eff} C_{ox} \frac{W}{(L + \lambda_o)} (V_{GS} - V_T) V_{DS} \right)$

Lundstrom

5. Discussion....

connections to traditional theory

Scattering theory

$$\left(\begin{array}{l} I_D = W C_{ox} v_T \left(\frac{T}{2 - T} \right) (V_{GS} - V_T) \\ T = \frac{\lambda_o}{\ell + \lambda_o} \quad \ell = \left(\frac{k_B T / q}{V_{DS}} \right) L \end{array} \right)$$

on-current

Conventional theory

$$\left(\begin{array}{l} I_D = \frac{W C_{ox} v_T m (V_{GS} - V_T) V_{DS}}{1 + m V_{DS}} \\ m = \left(\frac{\mu}{2 L v_T} \right) \end{array} \right)$$

Lundstrom

Veeraraghavan and Fossum
IEEE TED, 35, p. 1866, 1988

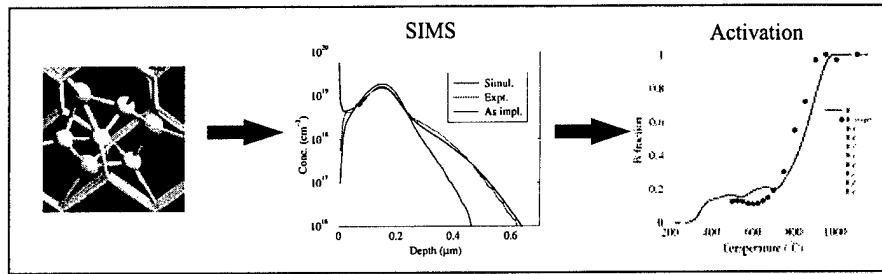
6. Summary

- Nanoscale transistor physics is simply understood by beginning at the ballistic limit
- Scattering theory provides a simple, physical view of nanotransistors and compact, analytical models
- Useful for interpreting simulations and guiding experiments
- A bridge to post-CMOS devices

Lundstrom

Purdue

Diffusion and Clustering of Impurities – “a Problem that Cannot be Ignored”



Wolfgang Windl

*Digital DNA™ Laboratories, Motorola, Inc.
Austin, TX*

in collaboration with

Benjamin Liu, Dejan Jovanovic, Mike Masquelier (Motorola Labs)
Blas Uberuaga, Hannes Jónsson, Scott Dunham (UW)

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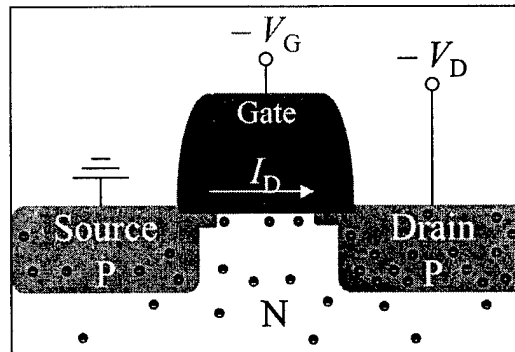
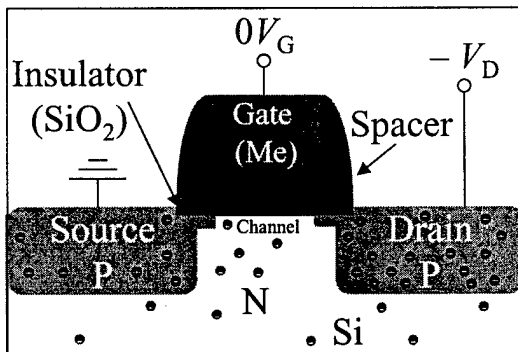
© 2001 Wolfgang Windl (Motorola, Inc.). All rights reserved.

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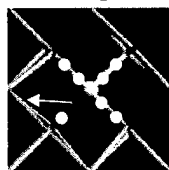
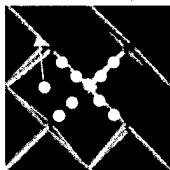
MOSFET Basics

Metal Oxide Semiconductor Field Effect Transistor



Doping:

N: e⁻, e.g. As (Donor) P: holes, e.g. B (Acceptor)



- Analog: Amplification
- Digital: Logic gates

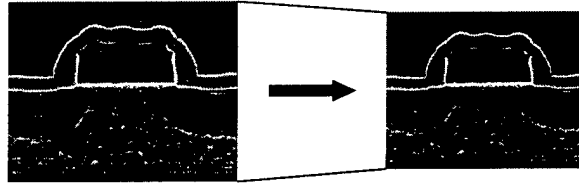
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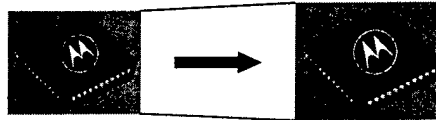


Semiconductor Technology Scaling

- Feature size shrinks on average by 12% p.a. ($f = 0.88$)



- Chip size increases on average by 2.3% p.a. ($d = 1.023$)



of transistors / area ($\sim d^2 / f^2$) $\uparrow$ by 35% p.a.

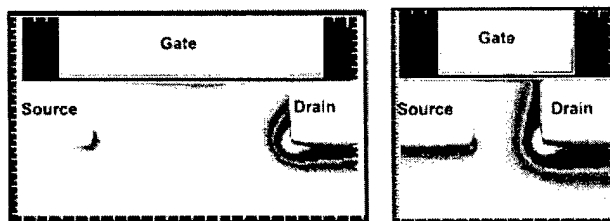
Switching speed ($\sim 1 / f$) $\uparrow$ by 14% p.a.

Overall performance: $\uparrow$ by $\sim 55\%$ p.a. or
 $\sim$ doubling every 18 months ("Moore's Law")



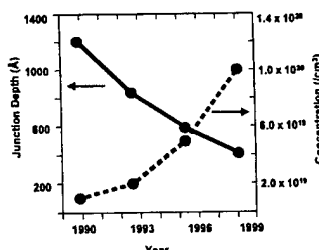
MOSFET Scaling Challenges

Shorter Channel



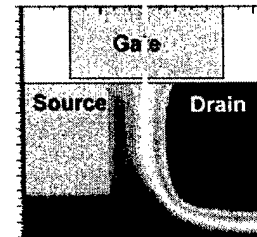
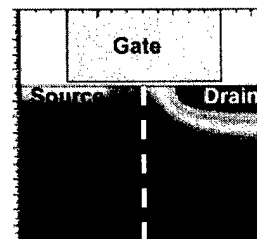
Potential across channel (on)

Higher Doping



To get enough current with shallow source & drain

Shallower Implant

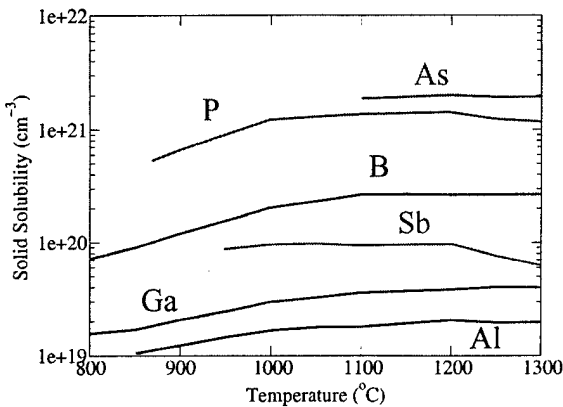


Better insulation (off)

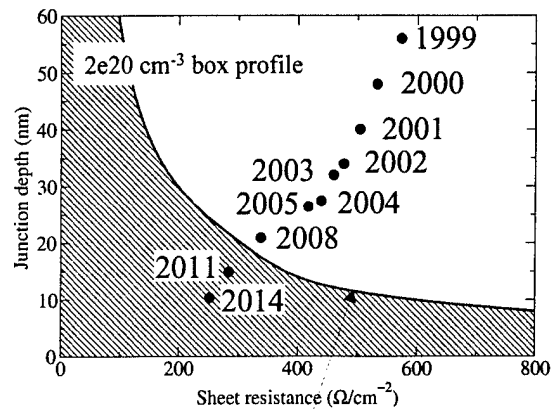


ITRS Requirements & Solubility Limit

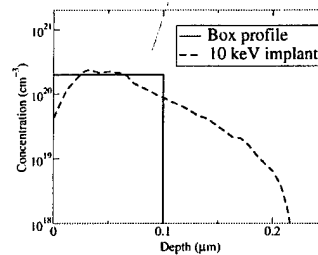
Solid Solubility of Si Dopants



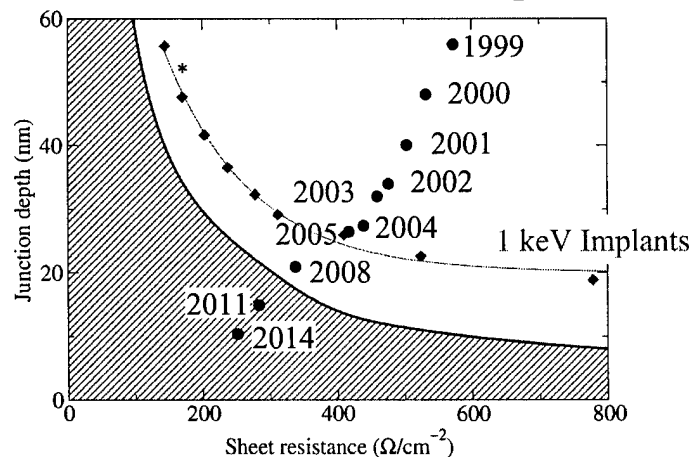
ITRS & Solubility Limit



- Electrically active concentrations may be much lower
- “Practical” solubility $\sim 2e20 \text{ cm}^{-3}$



How Are We Doing?

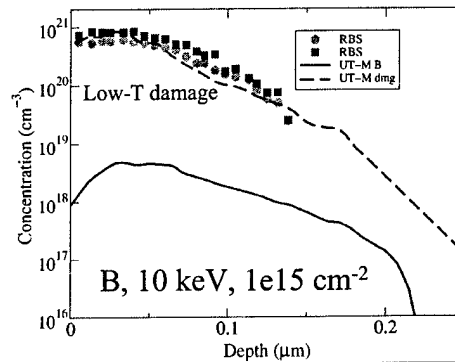
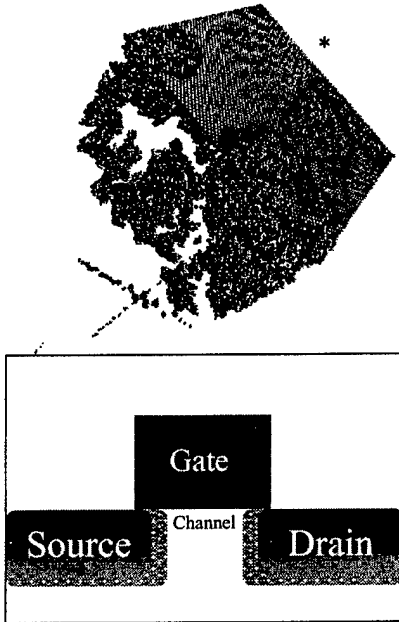


- 1 keV implants + optimum anneals can meet 2005 ITRS requirements (100 nm).
- Further reduction in implant energy can perhaps meet 70 nm (2008) needs, but cannot meet needs beyond this node.
- Need to find way to exceed $2e20 \text{ cm}^{-3}$ concentration limit for n and p dopants, or need new device structures (double gate, surround gate, superhalo, etc.).



Ion Implantation and Annealing

- Dopants inserted by ion implantation damage
- Damage healed by annealing
- During annealing, dopants diffuse fast (assisted by defects)
important to optimize anneal



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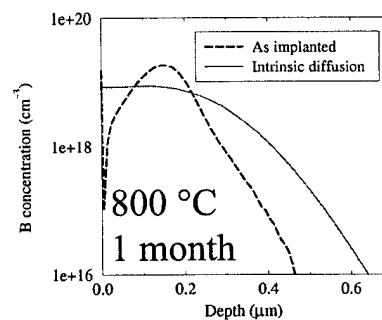
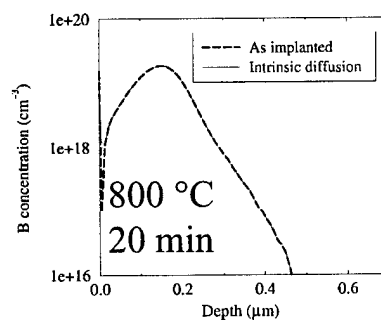
*S. Srinivasan, K. Beardmore, N. Jensen

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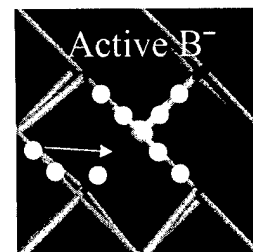
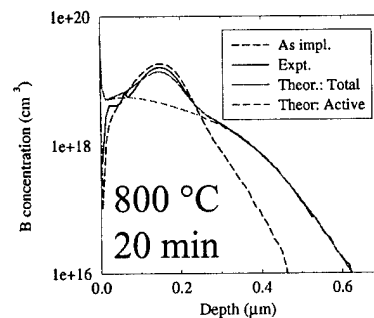


TED & Deactivation

What you expect:
Intrinsic diffusion



What you get:
TED:
•fast diffusion
•immob. peak



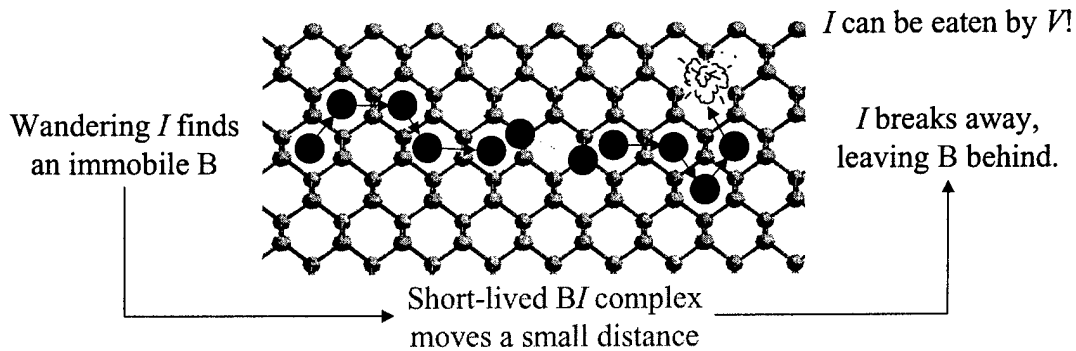
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B Diffusion in Bulk Si: Qualitative Picture

- B diffusion exclusively mediated by Si self-interstitials*
- B diffusion limited by self-interstitial diffusion**



*A. Ural, P. B. Griffin, and J. D. Plummer, J. Appl. Phys. **85**, 6440 (1999)

** W. Windl, M.M. Bunea, R. Stumpf, S.T. Dunham, and M.P. Masquelier, Proc. MSM99 (Cambridge, MA, 1999), p. 369; MRS Proc. **568**, 91 (1999); Phys. Rev. Lett. **83**, 4345 (1999).

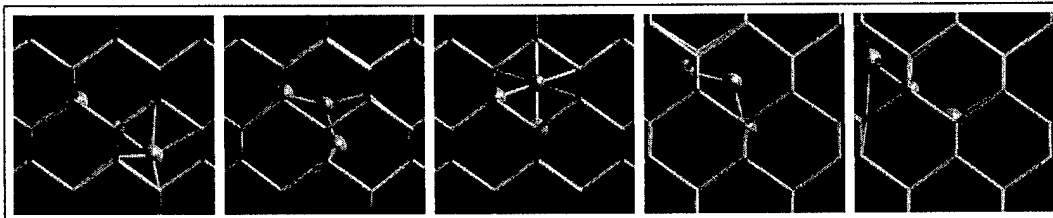
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Reason for TED: Implant Damage

Interstitial assisted two-step mechanism:



Intrinsic diffusion:

Create interstitial, B captures interstitial, diffuse together

$$\text{Diffusion barrier: } E_{\text{form}}(I) - E_{\text{bind}}(BI) + E_{\text{mig}}(BI)$$

$$4 \text{ eV} \quad 1 \text{ eV} \quad 0.6 \text{ eV} \sim 3.6 \text{ eV}$$

After implant:

Interstitials for “free” diffusion barrier $\sim 0.6 \text{ eV}$

W. Windl, M.M. Bunea, R. Stumpf, S.T. Dunham, and M.P. Masquelier,
Proc. MSM99 (Cambridge, MA, 1999), p. 369; MRS Proc. **568**, 91 (1999); Phys. Rev. Lett. **83**, 4345 (1999).

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Reason for Deactivation?

Phenomenological considerations:

- Boron is smaller than Si substitutional B strains Si lattice
- Boron crystal structure: Icosahedron (buckyball), threefold coord.
at higher B concentrations: New structures form,
bind & deactivate B

Experimental findings:

- Structures too small to be seen in EM only “few” atoms
- Clustering dependent on B concentration and I concentration
formation of $B_m I_n$ clusters postulated;
experimental estimate: $m / n \sim 1.5^*$

Approach:

Calculate clustering energies from first principles up to “max.” m, n
Build kinetic Monte Carlo / continuum model from it

*S. Solmi *et al.*, J. Appl. Phys. **88**, 4547 (2000).



Previous Work - More Motivation

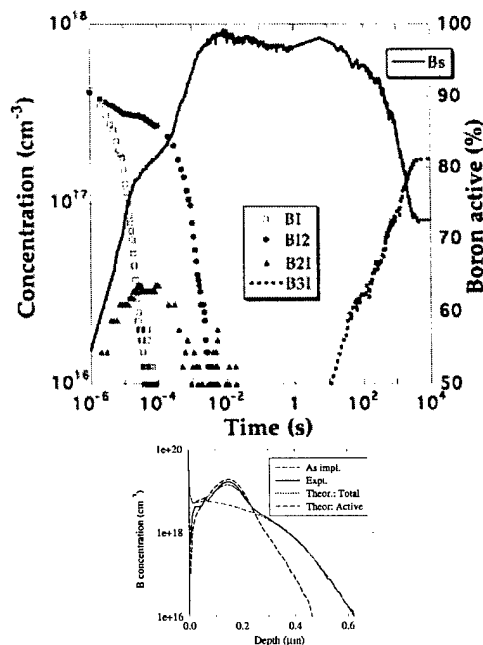
De la Rubia's group (LLNL)
predicts with *ab initio* based kMC
model* (up to $B_4 I_2$) a B
“activation window” for
annealing activation

*M. J. Caturla *et al*, APL **72**, (1998) p. 2736

However:

Experiment* finds **no** activation
window, once activated stays
activated.

*Mokhberi, Griffin, Plummer (Stanford).

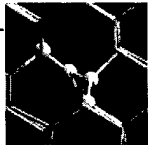
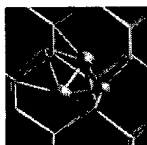
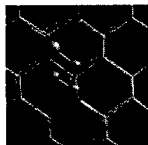
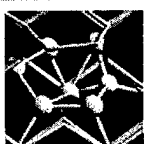
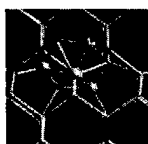


Calculation of the Clustering Energies

- DFT plane-wave code VASP (Technische Universität Wien), LDA and GGA
- Simulation cells of 64 atoms (converged ~7% vs. 216 atoms), $E_{\text{cut}} = 230 \text{ eV}$, 4^3 k-points
- Relax many different, “guessed” initial structures for each cluster; dangerous, can miss groundstate!

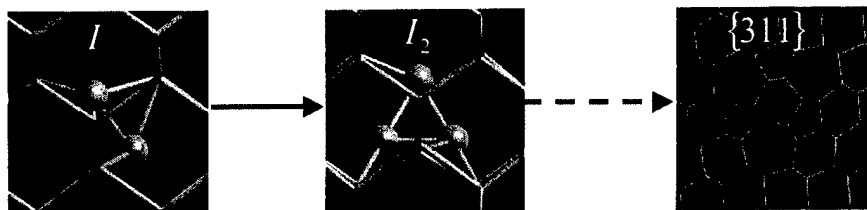


B-I Cluster Structures from *Ab Initio* Calculations

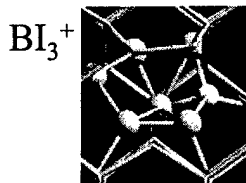
$B_x I$	BI^+		$B_2 I^0$		$B_3 I^-$		$B_4 I^{2-}$	
	LDA -0.5 GGA -0.4		-2.0 -1.2		-3.0 -2.2		-5.5 -4.3	
$B_x I_2$	BI_2^0		$B_2 I_2^0$		$B_3 I_2^0$		$B_4 I_2^0$	
	-2.5 -2.3		-3.0 -2.4		-4.1 -2.6		-5.5 -4.3	
$B_x I_3$	BI_3^+		$B_2 I_3^0$		$B_3 I_3^-$		$B_4 I_3^-$	
	-4.8 -4.5		-6.0 -5.3		-6.6 -5.6		-7.0 -5.9	
$I > 3$	$B_4 I_4^-$		X.-Y. Liu, W. Windl, and M. P. Masquelier, APL 77, 2018 (2000); similar to Lenosky <i>et al.</i> , APL 77, 1834 (2000).				$B_{12} I_7^{2-}$	
	-9.2 -7.8						-24.4 N/A	



Influence of B Clustering on I Clustering!?

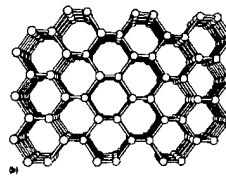
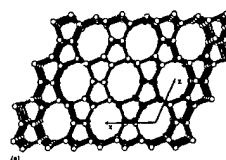


(e.g. Kim, Wilkins,... PRB 1997)



New ring structure for I_3 :

- *Without B:* Ring ~ 0.2 eV higher than more compact I_3 structures.
 - *With B:* Ring has lowest energies
- B promotes ring growth!?



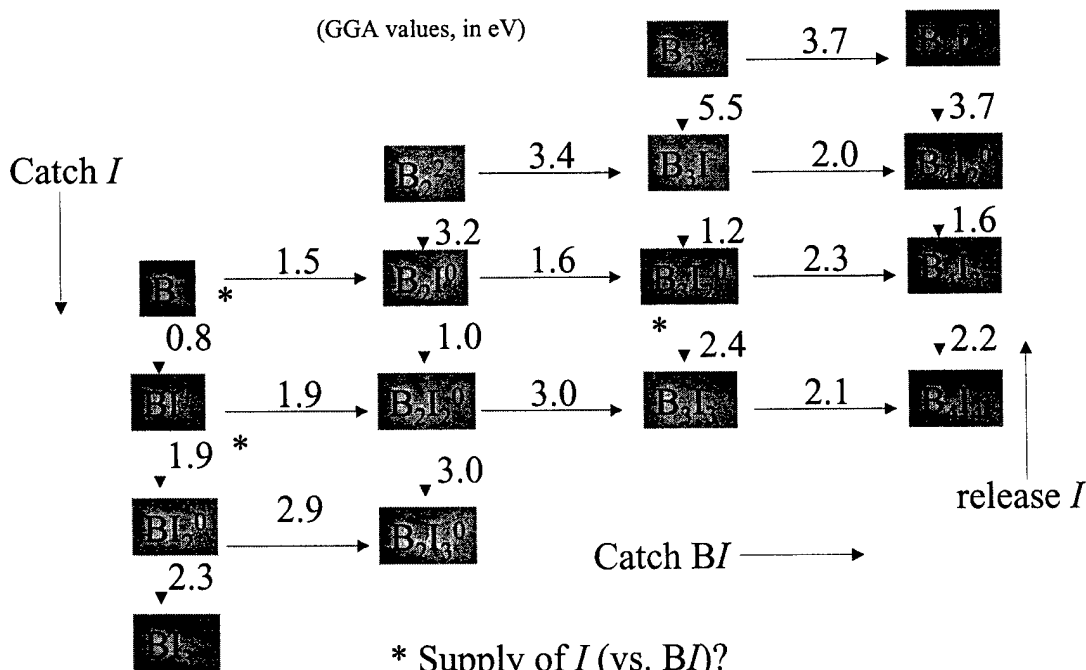
“Silisils”

(Demkov, Windl, Sankey, PRB 1996)

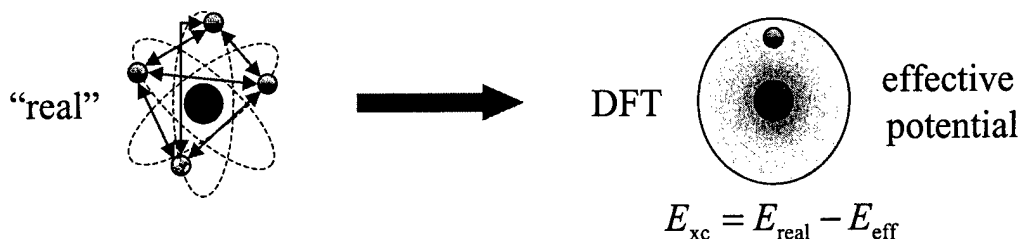


B-I Clustering Reaction Paths & Binding Energies

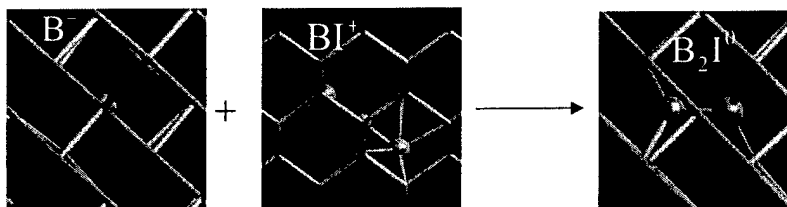
(GGA values, in eV)



DFT Flavors: GGA and LDA



For some cases, energetics differ considerably:

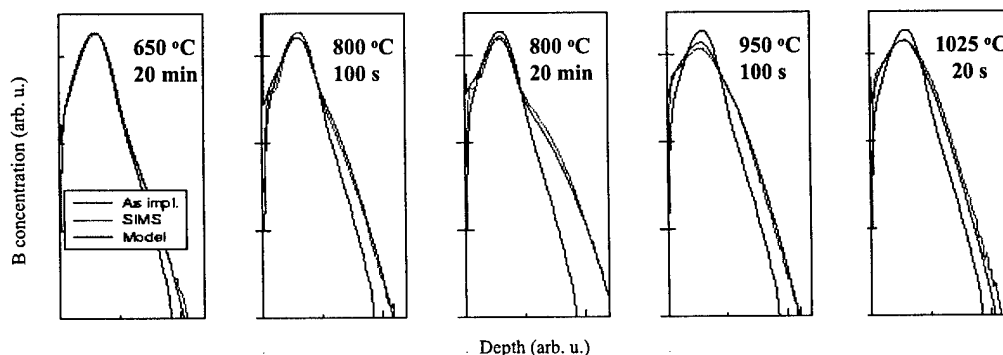


Reaction binding energy: 1.5 eV GGA
0.9 eV LDA.

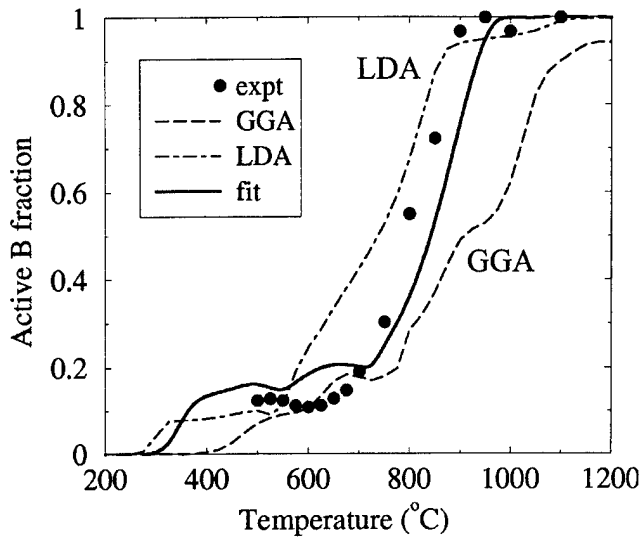


Calibration with SIMS Measurements

- Continuum model recalibration of ab initio numbers necessary
- Genetic Algorithm:
 - Start with many random parameter sets within boundaries
 - Select pairs of parameter sets (“parents”), biased by χ^2
 - Mate parents (mix parameters randomly), add child to population
- Nearly all fitted parameters between LDA and GGA values



Activation Results

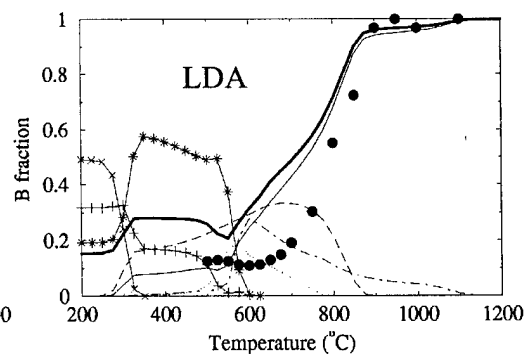
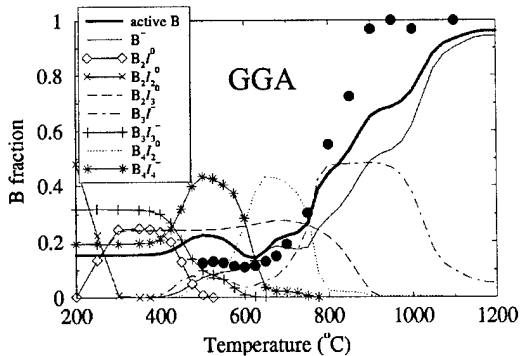


- No activation window in agreement with expt.
- SIMS calibration leads to good prediction of activation model (hopefully) reasonably physical.
- Assume only B_s active

Expt: A. Mokheri, P. B. Griffin, J. D. Plummer
fit from SIMS



Activation and Clusters

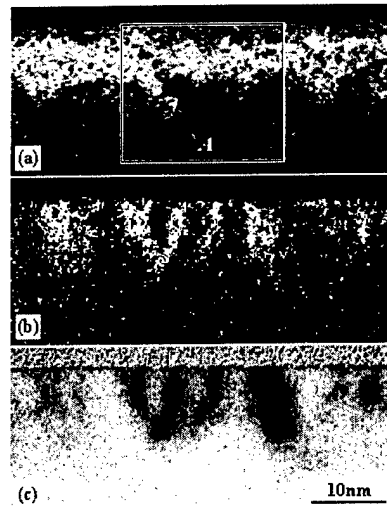
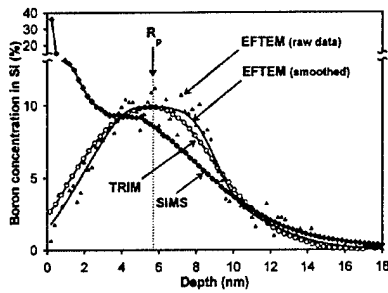
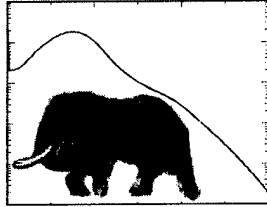


- LDA, GGA, fit do not give consistent clustering pattern:
- GGA: Dominated by $B_3I_3^-$ cluster (also $B_2I_3^0$)
- LDA: Dominated by $B_2I_3^0$ cluster (also $B_3I_3^-$)
- Fit: Dominated by $B_4I_2^0$



Uniqueness of Clustering Model!?

"You can fit an elephant with 21 parameters."
(Ulrich Schröder)

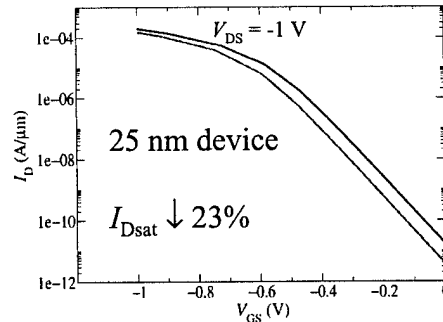
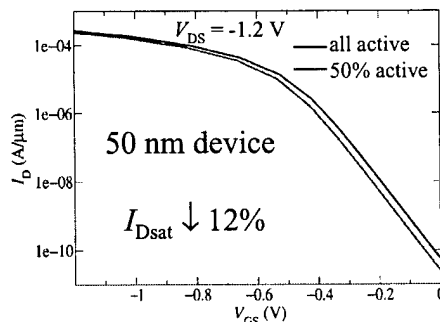
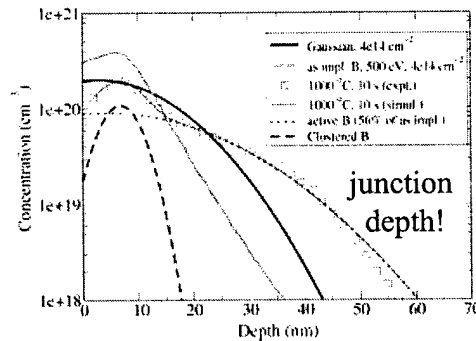
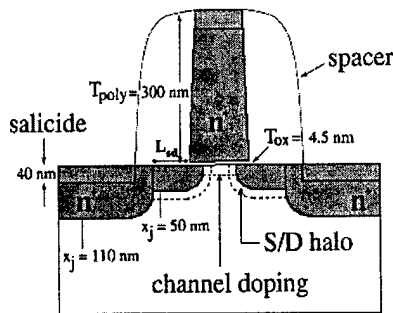


Energy filtered TEM B maps from (a) as-implanted (1 keV, $5 \times 10^{15} \text{ cm}^{-3}$) and (b) 900 °C annealed sample; (c) bright field image of (b). [Wang *et al.*, APL 77, 3586 (2000)].

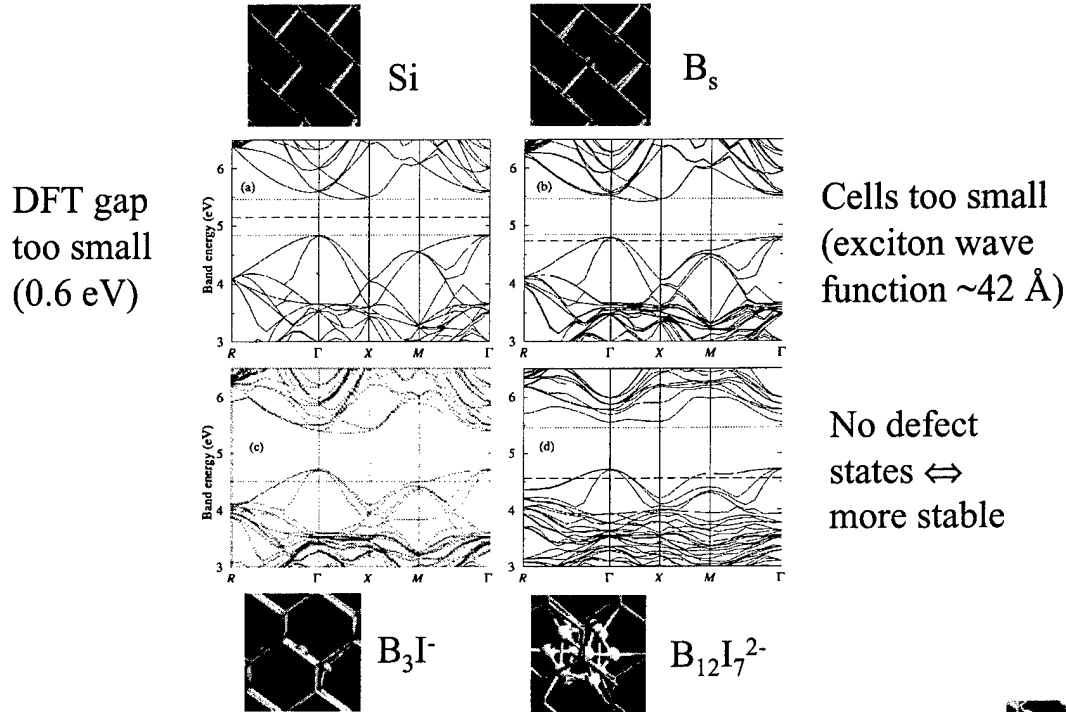


Application: Well Tempered MOSFET (MIT)

B profile



Cluster Energy and Band Structure

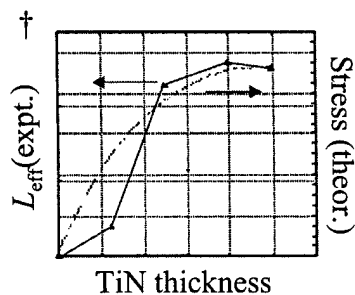
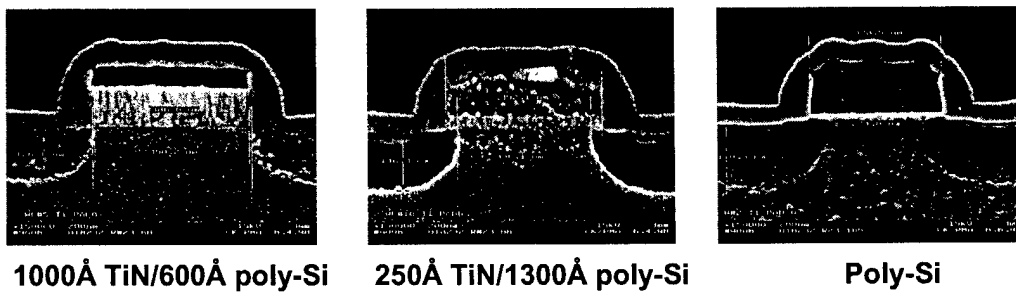


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Evidence of Stress Effect under Metal Gate



Experiments by Maiti *et al.*.*

- L_{eff} function of TiN thickness
 - Stress estimate $\sim L_{\text{eff}}$
- Stress effect

*B. Maiti *et al.*, IEDM, Session 29, 1998.

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†M. Laudon, N. N. Carlson, M. P. Masquelier,
M. S. Daw, and W. Windl, APL78 (2001).

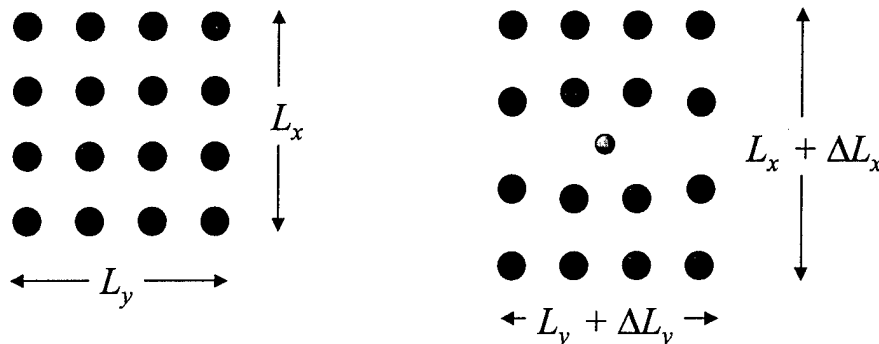
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Stress Dependence of Diffusivities

- Diffusivity: $D = D_0 \exp(-E/kT)$
- Under hydrostatic pressure: $E \rightarrow E + p \Delta V$

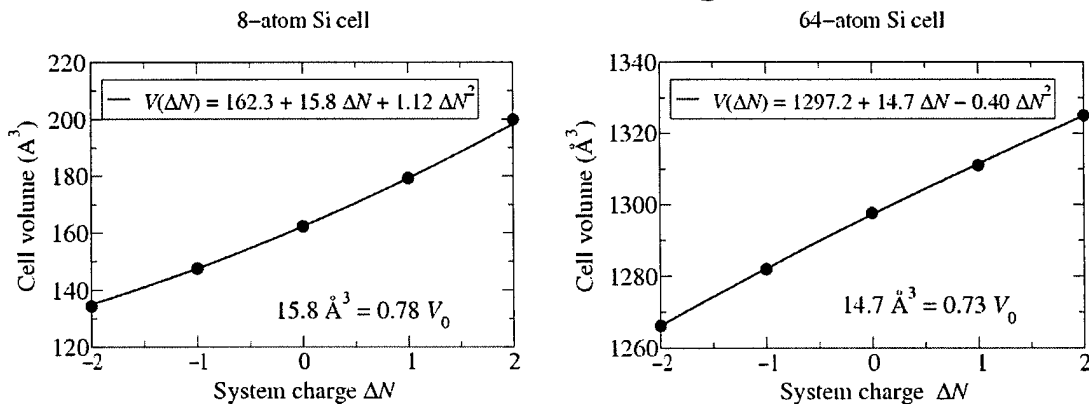
Perfect Si (or reference sys.): Create defect ($B_s, I, V, \dots$) ΔV :



*M. S. Daw, W. Windl, N. N. Carlson, M. Laudon, and M. P. Masquelier (PRB July 2001).



Cell Volume and Charge in Si



- For all defects, cell sizes, etc., every additional electron adds $\sim 15 \text{ \AA}^3$ to the volume
- Is this real or an artifact of the charged calculation?



Electron Volume and Maxwell Relations

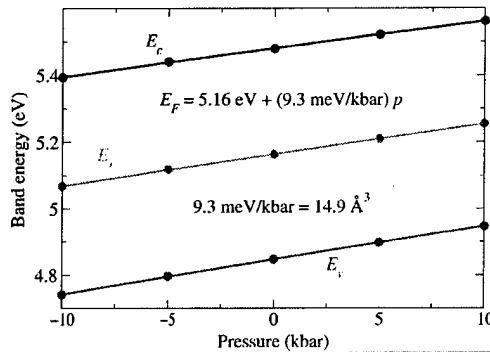
• Maxwell relation: $\frac{\partial V}{\partial N} = \frac{\partial \mu}{\partial p}$ $E_F(p) = E_F(0) + V_e p$

- Pressure dependence of bands can be calculated in good agreement with experiment.

TABLE IV. First- and second-order coefficients describing the dependence of the direct band gap at Γ (E_g) under hydrostatic pressure [$E_g(P) = E_g + aP + bP^2$] for Si, Ge, and GaAs. The experimental results are from Goñi, Syassen, and Cardona (Ref. 1).

Semiconductor	E_g		a (meV/GPa)		b (meV/GPa ²)	
	Theor.	Expt.	Theor.	Expt.	Theor.	Expt.
Si	3.273		100.8		0.05	
Ge	-0.084	0.795	125.4	121	0.2	0.2
GaAs	0.41	1.43	99.1	108	-0.1	-0.1

Alouani and Wills, PRB 54 (1996).



- We calculate again 15 Å^3 for the electron volume in Si

seems to be real

- Ferry* $\sim 5 \text{ nm}$ (de Broglie).

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*D. K. Ferry and H. L. Grubin, IEEE 1998.

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Conclusions & Outlook

- Ab-initio methods are being used to provide predictive capability for dopant/defect profile evolution.
- With recalibration, boron clustering model can predict quantitatively activation and SIMS over a wide range of temperatures and annealing times.
- Dopant deactivation can easily be 50%; concentrations $> 2 \times 10^{20} \text{ cm}^{-3}$ not achievable. Deactivation can easily change I_{Dsat} by 10s of percents.
- TED can change junction depth substantially (factor 2 even at “good” annealing conditions) short channel effects; resistance.
- We calculate the volume of an electron in Si to be 15 Å^3 .

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