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Project Title: **ENHANCEMENT OF THE NUMERICAL MODEL SBEACH FOR SIMULATING BEACH PROFILE EVOLUTION IN THE OFFSHORE**

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ENHANCEMENT OF THE NUMERICAL MODEL SBEACH FOR SIMULATING BEACH PROFILE EVOLUTION IN THE OFFSHORE

I. SCIENTIFIC WORK ACCOMPLISHED

Modeling Coastal Morphology and Its Evolution

Introduction

Engineering structures and activities interfere with the sediment transport and morphological evolution in the coastal zone. Thus, it is necessary to understand, quantify, and predict the behavior of the morphology and how it interacts with such structures and activities when carrying out engineering projects on the coast (e.g., ports). The first step towards assessing the effects of engineering works on coastal morphology is to characterize the morphology itself and the mechanisms that control its evolution. Describing the morphology involves mean properties and trends as well as parameters quantifying a wide range of features that occur at many scales. The equilibrium shape of the beach profile or the shoreline, for a given configuration are typical examples of mean properties used to characterize the morphology, especially in engineering investigations. Morphological features in the coastal zone range from ripples to barrier islands, although it is typically only the feature at the scale of the project that has a potential for interaction with the engineering works and that needs to be studied. Here, the objective is to provide a brief overview of recent research in coastal morphology with the emphasis on quantifying the morphology of sandy beaches and its evolution. Most of the discussion is in the context of modeling, analytical or numerical, since engineering studies typically imply predictions about the future evolution of a particular coastal area and models are key tools in such studies.

First, different morphological features are discussed that may influence engineering works in the coastal zone, focusing on longshore bars and sand waves. Their typical properties are summarized and statistical techniques for analyzing and modeling such features are briefly described. Our present knowledge limits the possibility to model the evolution of these features and data-based techniques are often useful alternatives. Equilibrium beach profile (EBP) theory is used by engineers to derive a characteristic profile shape in analysis and design situations. Several different approaches have been put forward to arrive at such a shape, typically resulting in equations of power-type where the multiplicative coefficient depends on the grain size and wave parameters in the general case. A brief overview of past research work to establish EBPs is given as well as some recent studies focusing on deriving a composite profile that separates between the surf zone and offshore zone. Once the EBP has been established, cross-shore transport formulas may often be expressed in terms of a forcing function and the deviation from the equilibrium shape. Several such examples are presented here covering a wide range of scales.

Beach profile evolution models have been employed for simulating the response of the beach profile to severe storms for the last fifteen years. Also, the effects of various engineering structures and activities have been modeled including beach fills, placement of dredged material, and seawalls and similar hard structures. Analytical models of the profile response are restricted to simple initial and boundary conditions, whereas numerical models may be used for solving more general cases. Here a few analytical models are discussed with the emphasis on one particular model for predicting the evolution of dredged material placed in the offshore. The most common numerical models of beach profile evolution are then referred to after which a more detailed discussion is given for the SBEACH model that has been widely used in engineering projects since the beginning of the 1990s.

Shoreline evolution models belong to the oldest type of model for simulating certain properties of the coastal morphology. Both analytical and numerical approaches are commonly utilized to assess how engineering structures and activities change the shoreline. Such models make it possible to design engineering works in an optimal way to reduce erosion or limit undesired effects from these works (e.g., downdrift erosion in connection with harbor construction). After a brief summary of the theory underlying shoreline evolution modeling, a simple example is given where an analytical solution is obtained for the case of accumulation updrift a jetty with bypassing. Numerical shoreline evolution models are reviewed in general with particular examples provided regarding the widely used model GENESIS.

Coastal Morphological Features

Analysis of data on beach morphology and the variation in time provides important information on the processes governing sediment transport and beach evolution, and is a necessary first step in developing, calibrating, and validating models of these processes. In such analyses, a wide range of morphological features are encountered with characteristic length scales from cm (ripples) to many km (offshore sand banks). Besides being of scientific interest in their own right regarding, for example, formation and physical properties, some of these features may have important implications for human development and activities in the coastal zone. Thus, in carrying out engineering analyses and design of coastal projects, some of these features should be considered. Here, a brief discussion is provided on morphological features, their properties, and how to analyze and model these features using statistical methods. Concerning the morphological features, mainly longshore bars are treated since they are commonly occurring features on sandy beaches. However, longshore sand waves are also discussed since they may have impact on engineering structures and activities in the coastal zone.

Characterization and Properties of Features

Profile data collected 1981-1991 at the U.S. Army Field Research Facility (FRF), Duck, North Carolina, were analyzed by Larson and Kraus (1992, 1994) to determine the properties of longshore bars. Two bars were often found across the profile at Duck, namely an inner bar and an outer bar, and the analysis of the measured profiles yielded time series of several different bar properties. Beach profile data were analyzed to determine bar properties by comparing the surveyed profiles with a reference profile (see Fig. 1), which was taken as the modified equilibrium profile proposed by Larson (1991). Areas of deposition (the surveyed profile was above the reference profile) below mean sea level were identified as bars, and the crossing between the specific profile and the reference profile defined the start and end points of the bars. The selected bar properties were then calculated for each depositional area and statistically analyzed.

The following properties were calculated for every identified bar of each individual profile survey: depth to bar crest h_c; bar length L_b; bar height Z_m; bar volume V_b; location of bar mass center x_{cg}; and bar speed $\Delta x_{cg}/\Delta t$, where Δt is the time interval between profile surveys determining x_{cg}. Fig. 2 defines the various bar properties, where the inshore part of the modified equilibrium profile is shown together with a typical profile. The center of mass of the outer bar was typically located about 300 m from the shoreline, whereas the location of the center of mass of the inner bar varied more, with a characteristic distance of 100 m from the shoreline. The outer bar experiences breaking waves only during major storms, in contrast to the inner bar that is exposed to wave breaking most of the year, resulting in greater variability in its position. Thus, the inner and outer bars display significantly different temporal behavior.

Tables 1 and 2 summarize calculated morphodynamic parameters for the inner and outer bars, respectively. The average depth to crest for the inner bar was 1.6 m, average maximum bar height 0.9 m, and average bar volume 45 m^3/m based on 240 profiles where an inner bar was identified. The average speed of the inner bar was 1.4 m/day for onshore movement and 2.6 m/day for offshore movement, with maximum calculated speeds of 8.7 and 18 m/day, respectively. The average depth to crest for the outer bar was 3.8 m, average maximum bar height 0.4 m, and average bar volume 45 m^3/m based on 221 profiles where an outer bar was present. Although the outer bar on the average had a volume similar to the inner bar, the maximum height was considerably less, giving a much gentler bar shape. The average speed of the outer bar was 0.6 m/day for onshore movement and 1.1 m/day for offshore movement, with maximum speeds of 6.1 and 15.2 m/day, respectively. For storm-induced bar movement, these speeds tend to be underestimates because of the relatively long time interval between profile surveys. Birkemeier (1984), Sunamura and Takeda (1984), Sallenger *et al.* (1985), and Larson and Kraus (1992) have discussed bar movement in the field, and Sunamura and Maruyama (1987) and Larson and Kraus (1989) have examined bar movement generated in large wave tanks, where survey frequency is greater. Bar speed and dimensions are comparable between large wave tanks and the field.

Morphological features at many characteristic scales appear in the nearshore, ranging from ripples to large-scale longshore sand waves. At the upper end of this scale spectrum, features appear that evolve at the regional scale, having spatial wavelengths on the order of kilometers and existing over decades to centuries (Verhagen 1989, Thevenot and Kraus 1995, Gravens 1999). Typical amplitudes of these sand waves are 10-50 m for the ones responding at the decadal scale and 100-500 m for the ones changing over centuries. The sand waves can be stationary or move alongshore, but their amplitude typically varies with time. Propagation and amplitude variation induce changes that may have consequences for shoreline evolution and channel infilling.

Fig. 3 illustrates longshore sand waves occurring along Southampton Beach on the southern shore of Long Island. Thevenot and Kraus (1995) identified 11 sand waves along this stretch of coast, implying one sand wave per 1.5 m shoreline. The waves were observed to migrate south at typical speeds of 0.5-1.5 km/year, with greater speeds occurring in the winter. This speed is somewhat greater than the migration speed Verhagen (1989) determined for the larger sand waves appearing along the Dutch coast, where the waves moved 100-300 m/year. Sonu (1968) postulated that the longshore speed of sand bodies should approximately vary inversely with the wavelength of the body, which was confirmed by Thevenot and Kraus (1995).

Analysis and Modeling of Features Based on Data

The coastal areas constitute complex physical systems of high dimensionality where the forcing acts at many scales in time and space, often through complicated interactions and feedback mechanisms between the forcing and the system response (De Vriend 1991a). Thus, predicting sediment transport and evolution of beach morphology in these areas are difficult tasks, especially over longer time periods when the scale range is wide. Physically based prediction models have their inherent limitations due to insufficient knowledge of the governing processes as well as how these processes are described in equation form (De Vriend 1991b). In a more basic context there are limits to the predictability of morphological variables that are related to the issue of scale, but possibly also due to the nonlinearity of many coastal systems that may induce chaotic behavior.

A useful alternative to models of long-term beach evolution developed based on physics might be data-based models, where the components as well as the entire structure of the model could be constructed from analysis of available data on beach morphology. The fundamental assumption in data-based modeling is that the sampled data sufficiently well reflect the essential properties of the process under study in time and space. If this is not the case, model predictions will fail to capture the system behavior and prediction results will be poor. The strength of a data-based model is that it may reproduce the main features of the beach behavior observed at a particular site without any special physical insight. However, this property could also be the primary weakness of the model since the general applicability is not ensured. Using a data-based model developed for one site requires careful considerations before it is used at other sites.

A natural step prior to data-based model development is the analysis of the data to establish basic properties and the degree of association between these properties. Such analysis typically aims at detecting and quantifying dominant patterns in the data and their evolution in time and space, as well as how different patterns are related to each other. Thus, it is possible to obtain valuable information on the long-term behavior of beaches, which may be used not only for developing data-based models but also for increasing the understanding of the factors governing the morphological evolution. The use of a limited set of basic patterns to represent the data is often an effective way of distinguishing between signal and noise (Von Storch and Navarra 1995). The signal is associated with the morphological processes at the scale of interest, whereas the noise includes the effects of processes operating at smaller scales not sufficiently resolved by the data as well as inaccuracies in the measurements. Distinguishing between signal and noise is often non-trivial and depends on the specific application as well as the required accuracy of the data representation.

A majority of the advanced techniques utilized in analysis and modeling of morphological data are related to principal component analysis (PCA). These methods have shown great promise in terms of representing complex fields through a limited number of basic patterns in space (principal components or spatial EOFs) combined with multiplicative time functions (principal component scores or temporal EOFs). Even though the patterns do not necessarily have any physical relevance, it is often possible to give an interpretation that is physically based when beach topographies are analyzed. This is probably due to the fact that topographies are geometric constructs made up of different features, and the patterns extracted through PCA often happen to match these features. PCA has previously been employed in analysis of coastal data, typically to determine the shape of the EOFs for time series of beach profiles surveyed at a particular location (e.g., Winant *et al.* 1975; Aubrey 1979). Larson *et al.* (2002a) provided brief theoretical backgrounds and showed examples for some linear statistical techniques employed in beach morphology, including bulk statistics (various overall measures such as mean, standard deviation correlation as well as Fourier methods), empirical orthogonal functions, canonical correlation analysis, and principal oscillation pattern analysis. Southgate *et al.* (2002) undertook a similar review as Larson *et al.* (2002a) but focused on nonlinear techniques such as singular spectrum analysis, fractal analysis, and neural networks.

Equilibrium Beach Profile (EBP) Theory

Equilibrium Beach Profile Theory

The concept of an equilibrium beach profile (EBP) is of central importance to coastal engineers because it provides a basis for assessing a characteristic shape to a beach in design and analysis situations. A beach of a specific grain size, if exposed to constant forcing conditions (monochromatic waves or random waves with constant statistical properties), normally assumed to be short-period breaking waves, will develop a profile shape that displays no net change in time, although sediment will be in motion. The validity of this concept has been verified through a large number of laboratory experiments on beach-profile change (e.g., Waters 1939, Rector 1954, Saville 1957, Swart 1976, Kajima *et al.* 1982, Kraus and Smith 1994, Peters *et al.* 1996). On a natural beach, however, the forcing conditions are never constant and changes in the beach topography occur at all times. In spite of this, the beach profile in the field exhibits a remarkably persistent concave shape (Bruun 1954, Dean 1977), where changes may be taken as perturbations upon the main profile configuration. Such changes in beach profile shape can be regarded as adjustment of the profile from the course of one equilibrium state to another as the forcing conditions change (for example, during a storm). With this view the equilibrium concept is valid not only for the long-term average forcing conditions but for varying forcing conditions over different time scales (Larson *et al.* 1999b).

The conditions for equilibrium on a beach and associated slopes and profile shapes have been a topic of research since the 1950's. Bruun (1954) proposed a power law to describe the profile depth as a function of distance from the shoreline based on field data from the Danish West coast and from California (a power of 2/3 provided the best fit). To obtain a 2/3-power he assumed in the derivation that the bottom shear stress and wave energy dissipation were constant at equilibrium (no variation with the cross-shore distance). Dean (1977) analyzed an extensive data set consisting of beach profiles measured along the Atlantic and Gulf coasts of United States. He also found that a power law with a power of 2/3 provided the best overall fit to the measured profile shapes. Furthermore, Dean (1977) theoretically motivated this power law by assuming that equilibrium occurred for constant wave energy dissipation per unit water volume along the profile. This constant is known as the equilibrium energy dissipation and has been shown to be a function of grain size (Moore 1982) or fall speed (Dean 1987, Kriebel *et al.* 1991).

Bowen (1980) derived EBP shapes by analysis of the sediment transport formulas proposed by Bagnold (1963) for bedload and suspended sediment transport. Analytical expressions for the profile shape were obtained for the cases of a steady drift due to wave mass transport in the boundary layer and wave asymmetry; in both cases only suspended sediment transport was considered. For a steady drift a power of 2/3 was obtained, whereas wave asymmetry produced a power of 2/5. Larson and Kraus (1989) generalized the derivation by Dean (1977) to include the effect of gravity leading to a planar beach slope at the shoreline (and not an infinite slope as occurred in the original derivation by Dean). Dean (1990) also developed alternative forms of the equation governing the EBP shape that involved terms to account for the gravity effects. Larson (1991), Work and Dean (1991), and Horn (1992) investigated the effect of a varying grain size on the EBP shape. Bodge (1992) empirically fitted a power function and an exponential function to the data set employed by Dean (1977) and concluded that the exponential function provided an improved fit compared to the power function. Inman *et al.* (1993) divided the profile into two portions, an inner and outer region, which corresponded to the regions with breaking and non-breaking waves, respectively. Both portions were successfully approximated with power curves matched at the break point and the optimum values of the power were 0.4 for both curves. In fitting the power curve to the inner portion of the profile the height of the berm was employed as the base elevation; this differs from previous studies where typically the mean sea level (MSL) was used as vertical datum.

Composite Equilibrium Beach Profile. A number of previous studies have indicated the difficulties in fitting a single, simple expression, such as a power curve of 2/3-type, to measured beach profiles, both in the laboratory and the field (Larson *et al.* 1990, Bodge 1992, Inman *et al.* 1993, Peters *et al.* 1996). Observed profiles typically have a shape with a lower curvature in the offshore compared to the nearshore that a single power curve fails to describe. More complicated equations have been developed to remedy these discrepancies (Larson 1991, Work and Dean 1991, Bodge 1992) or several separate curves have been employed in the fitting process (Everts 1978, Inman *et al.* 1993). A varying grain size across the profile has been one explanation for more complicated EBP shapes; however, variation in the wave forcing is another factor. Ultimately, there is a coupling between the local grain size and the wave forcing, where the former will reflect the latter. Thus, a more advanced EBP model should predict the shape as well as the grain-size distribution from the forcing conditions.

Larson and Wise (1998) and Larson *et al.* (1999b) applied physically based theories to derive the EBP shape under both breaking and non-breaking waves that produced a realistic EBP shape over the entire active profile. In the following, a brief summary of these theories and the main results are presented. For a more comprehensive discussion and detailed derivations, Larson *et al* (1999b) should be consulted. It was assumed that the region where wave breaking prevails may be treated separately from the offshore zone where mainly non-breaking waves control the profile shape. This separation is conceptually justified because intense turbulence exists in the surf zone, making both bedload and suspended load significant, whereas bottom-boundary layer processes and bedload transport are expected to be dominant in the deeper and less turbulent water offshore under non-breaking waves. The complete EBP shape was obtained by combining the profile shapes derived for breaking and non-breaking waves, where the profiles are joined together in the area of incipient breaking.

EBP Equation for the Surf Zone. Although the theoretical models proposed by Bruun (1954) and Dean (1977) for EBPs produce shapes that are in agreement with field data, the physical justification for the equilibrium conditions are not clear and the assumptions made are rather ad hoc. Also, a profile that is close to equilibrium may still produce significant net sediment transport in the undertow that is difficult to explain within the framework of these models. Larson *et al.* (1999b) proposed an alternative model that relies on certain assumptions about the circulation of water and sediment in the surf zone. In this sense the model is non-local as opposed to the other models where the equilibrium conditions are established from a local criterion of zero transport.

A beach subject to breaking waves experiences a return flow across the profile that carries sediment stirred up by the waves offshore in the undertow. Even at equilibrium conditions, when no net change in the profile shape occurs, this transport should take place implying that material has to come onshore above the undertow layer to compensate for the offshore transport. When the undertow reaches the break point, the transported sediment has to be resuspended up into the water column and pushed onshore to ensure an equilibrium situation where no material moves offshore. Thus, such a simplified picture yields a surf zone with sediment moving, but with no net changes in time of the profile depths, and the break point acts almost as a singularity.

To arrive at an EBP shape Larson *et al.* (1999b) assumed that the change in the sediment transport rate in the undertow is balanced by sedimentation through the water column. This sedimentation represents the net effect from sediment being resuspended locally by the waves and the settling of the material. Thus, the stirring of the sediment at the bottom mobilizes grains that may be brought up into the water column by the turbulence from wave breaking. However, it is assumed that there are spatial differences in the total amount of material suspended that causes net sedimentation which balances the seaward increase in the transport by the undertow. Measurements by Barkaszi and Dally (1992) and Dally and Barkaszi (1994) partly confirm the simple picture of the sediment transport assumed here. Nielsen (1991) presented a general model for the development of sediment concentration profiles that provides a theoretical explanation for these measurement results.

A sediment balance equation may be written for an infinitesimal element in the cross-shore direction where the transport in the undertow, as well as that due to sedimentation, are included. In general, such a balance could also encompass transport due to beach slope and asymmetry of the velocity field; however, difficulties arise in simple parameterization of these terms. The assumption is made here that the transport in the undertow dominates the transport across the profile in comparison to the effects of beach slope and flow asymmetry. The sediment transport in the undertow is taken as the product between the flow and a characteristic concentration in the sediment layer that is transported offshore. The sediment conservation equation solved after appropriate parameterization of the flow in the undertow and the sediment concentration resulted in the following EBP,

$$h = \left(\frac{3}{5} \frac{\mu w}{\lambda_u \sqrt{g\gamma}} \right)^{2/3} x^{2/3}$$

where h is the water depth, x a cross-shore coordinate positive offshore, μ a sediment exchange coefficient, λ_u a coefficient quantifying the undertow flow, w the sediment fall speed, g the acceleration of gravity, and γ the ratio between wave height and water depth in the surf zone. Thus, the same EBP shape is obtained as predicted by Bruun's and Dean's equilibrium models. The functional dependence on w of the constant in front of $x^{2/3}$ (Dean's

A-parameter) agrees with what Kriebel *et al.* (1991) derived (also, compare with Bowen 1980).

EBP Equation for the Offshore Zone. Larson *et al.* (1999b) employed three different approaches to derive the EBP shape under non-breaking waves. The purpose of using several approaches was to investigate the sensitivity in the resulting EBP shape when different assumptions were utilized regarding the conditions of equilibrium and the governing mechanisms. It was shown that quite similar EBP shapes were obtained in all the three derivations; thus, the sensitivity was fairly low providing additional confidence in the results derived. The first approach was based on the heuristic assumption that the profile shape seaward of the break point at equilibrium is such that the waves dissipate a minimum of energy when traveling across the profile. In the second approach a detailed sediment transport formula proposed by Madsen (1991, 1993) was integrated over a wave period, and an equilibrium slope is determined that produces zero net transport. Finally, a conceptual model was formulated that assumed a balance at equilibrium between onshore transport due to wave asymmetry and offshore transport due to gravity. All these approaches produced EBPs following power functions with the value of the power being around 0.25.

Comparison With Laboratory and Field Data. In summary, the EBP shapes derived by Larson *et al.* (1999b) under breaking and non-breaking waves may be written, respectively,

$$h = Ax^m, \qquad 0 \le x \le x_b$$

$$h = \left(h_b^{1/n} + B(x - x_b)\right)^n, \qquad x \ge x_b$$

where A and B are shape parameters and b denotes the break point. The values of the powers were theoretically determined to m=2/3 and n=0.15-0.30. Parameter A has been related to grain size (or fall speed), whereas B is a function of the offshore wave conditions as well as the sediment characteristics.

The EBP equations were least-square fitted towards measured profiles in the laboratory and the field to evaluate how well they are able to characterize the profile shape in the surf zone and offshore zone. The distinction between these two zones was typically made based on the presence of a nearshore bar (compare Inman *et al.* 1993). Only A and B were optimized in the fitting procedure, whereas x_b and h_b were visually determined based on the observed profiles introducing an element of subjectivity in the calculations. The value of the power n was also varied initially, but n=0.3 provided the best overall fit. This value is somewhat larger than what was typically found from the theoretical analysis, although it still within the range of realistic values. The theoretical EBP models were essentially based on monochromatic (or representative) wave conditions and the effects of wave randomness were not explicitly addressed. In the validation towards the data, the least-square fit was carried out some distance away from the bar region, where the effects of randomness are expected to be most pronounced. Thus, seaward of the bar mostly non-breaking waves prevail and shoreward of the bar fully broken waves dominate, implying that a monochromatic wave description should be appropriate as a first approximation.

Data obtained in the German Large Wave Tank were employed to evaluate how well the derived EBPs could describe measured profile shapes at near-equilibrium conditions (Dette *et al.* 1998). The tests used for the comparison involved random waves according to a TMA spectrum with an H_{mo}=1.2 m and T_m=5 sec. In all cases, both in the surf zone and offshore zone, it was possible to obtain a close fit between the EBP equations and the data (one example is given in Fig. 4). Furthermore, profile measurements from several locations around the United States were employed to test the derived EBP shapes for field conditions. Data sets from Ocean City, Long Island (Fire Island, Westhampton Beach, and Ponds), Cocoa Beach, and Silver Strand were employed in this comparison representing a wide variety of wave and beach conditions. In most cases a more or less clear breakpoint bar was present along the profiles that provided a natural separation point between the surf zone and offshore zone. As an example, Figs. 5a and 5b display fits towards profiles measured at Fire Island and Westhampton Beach, respectively, along the south shore of Long Island.

The optimal value of the shape parameter B varied substantially between the studied beach profiles. In order to usefully employ the composite EBP for predictive purposes, a formula is needed that will yield B as a function of the governing factors (A is predicted from grain size according to Moore (1982) or Dean (1987)). A closer examination revealed a distinct correlation between B and h_b (see Fig. 6), implying that knowledge of the depth at the location where wave breaking typically occurs can be used to compute B. Several different empirical relationships were developed, encompassing both linear and power-type functions, each explaining about 70% of the variation in the data (in total 41 profiles from the above-mentioned sites were used in the fit). The following equation is dimensionally consistent and results if the EBPs are scaled with h_b (B was least-squared fitted towards $h_b^{7/3}$):

$$B = 0.142 h_b^{7/3}$$

The rational behind this equation is that the typical breaking wave height (and, thus, water depth at breaking) is the main quantity that scales profile behavior allowing for inter-comparison between profile shapes at different sites.

Transport Rates Based on EBP Theory

A common criterion for equilibrium on a beach profile is that the local net cross-shore transport rate taken over a certain time period is zero. Typically, several different mechanisms contribute to the cross-shore transport such as net currents (e.g., undertow, mass transport), wave orbital velocities (e.g., stirring, velocity asymmetry), and gravity. These mechanisms can often be quantified independently of each other or expressed in a form so that they mathematically become independent. Thus, the following sediment transport balance equation describes in a general form the conditions for equilibrium (Larson *et al* 1999a),

$$\sum_i q_{c,i} + \sum_i q_{cs,i} \frac{\partial h}{\partial x} = 0$$

where q_c and q_{cs} are cross-shore transport rates (taken with sign) produced by mechanisms not having and having a slope-dependence, respectively, and i an index referring to the different mechanisms included in the balance equation. The transport rates q_c and q_{cs} are both functions of wave height (H), period (T), and water depth (h), as well as sediment properties. Separating the transport mechanisms depending on whether the slope affects the transport rate or not is typically possible and convenient from the point of view of deriving the EBP shape. Thus, the above equation yields the following condition for the EBP shape (h_e):

$$\frac{dh_e}{dx} = -\frac{\sum_i q_{c,i}}{\sum_i q_{cs,i}}$$

In the case when the profile is not in equilibrium, a net transport (q_c) will arise according to,

$$q_c = \sum_i q_{c,i} + \sum_i q_{cs,i} \frac{\partial h}{\partial x} = \sum_i q_{cs,i} \left(\frac{\partial h}{\partial x} - \frac{dh_e}{dx} \right)$$

where the local slope at equilibrium was used to arrive at the expression on the right-hand side. This relationship gives the net transport rate in terms of a forcing function and the difference between the local slope and its equilibrium value at a specific water depth. The main advantage of using this equation is that empirical information may be used to derive the EBP shape, ensuring a more robust and reliable profile evolution, especially over longer time periods, than if purely theoretical formulations for h_e are employed. The EBP shape $h_e = f(x)$ typically displays monotonically increasing depth with distance offshore, implying that the inverse $x = f^{-1}(h_e)$ exists and that $dh_e/dx = df\{f^{-1}(h_e)\}/dx$. Thus, the EBP slope can often be directly specified in terms of the depth.

Examples of Cross-Shore Transport at Different Scales. In the following a few examples will be presented of cross-shore transport relationships expressed using EBP theory (Larson *et al.* 1999a). Three formulas are discussed that yield the net transport rate at different time scales ranging from a single wave period to seasons. Although each formula is valid over a limited scale range, the similarity of the functional form may provide a basis for simulating profile evolution where processes at widely different scales have to be included (e.g., including storm response as well as changes on the seasonal and annual scale).

Dean (1977) obtained an EBP shape for the surf zone (breaking waves) based on the assumption that zero net transport occurs when the energy dissipation (D) per unit water volume is constant ($=D_{eq}$), and Kriebel and Dean (1985) proposed a transport formula where the rate was proportional to $D-D_{eq}$. This type of formula has been successfully employed in modeling dune erosion during storm events, that is, the net transport rate represents averages over many wave periods (≈10-100). The formula may be expressed in a different form employing the derived EBP shape, giving the net transport rate q_c as,

$$q_c = K_1 \sqrt{h} \left[\frac{\partial h}{\partial x} - \frac{dh_e}{dx} \right]$$

where K_1 is a dimensional coefficient. The forcing given by $\sqrt{h}$ is related to the energy dissipation per unit water volume taken with respect to depth change.

Larson *et al.* (1999b) derived EBP shapes for the offshore zone (non-breaking waves) by considering wave asymmetry and gravity as the main mechanisms controlling the balance at equilibrium. When these mechanisms are not in balance a net transport results that is also possible to express in terms of the deviation from the EBP,

$$q_c = K_2 \frac{u_o^3}{g} \left[\frac{\partial h}{\partial x} - \frac{dh_e}{dx} \right]$$

where K_2 is a coefficient and u_o the bottom orbital velocity amplitude. This transport equation was developed to simulate profile changes in the offshore at the seasonal scale.

Madsen (1993) presented a formula for the instantaneous bedload transport under waves and currents that may be integrated over a wave cycle to yield an equilibrium slope, if the hydrodynamic conditions are calculated from boundary layer theory. The net transport rate during a cycle may be expressed in terms of the deviation from this slope,

$$q_c = K_3 \frac{I_s}{\tan^2 \phi_m - (\partial h / \partial x)^2} \left[\frac{\partial h}{\partial x} - \frac{dh_e}{dx} \right]$$

where K_3 is a dimensional coefficient, I_s an integral of the excess shear stress over the wave period, and ϕ_m the friction angle for a moving grain. Thus, all of the above cross-shore transport formulas, although representative of widely different scales, may be cast in a similar form containing a portion that is related to the fluid forcing and another portion related to the geometric deviation from the EBP.

Beach Profile Evolution Models

Storm events often cause significant cross-shore movement of sediment and associated change in the beach profile. During such events the gradients in the longshore direction for the forcing are typically small and most of the net changes in the profile shape are due to the cross-shore transport. Furthermore, wave-breaking in the nearshore is the most important process for mobilizing and transporting sediment under a storm. Breaking waves stir up the material and make it available for transport, as well as inducing currents in the nearshore that move the material. Viewed at a small scale the mechanics of sediment transport in the surf zone is extremely complicated involving a large number of particles moving in highly unsteady, strongly turbulent flow. Although the movement of individual particles display a random behavior, at a larger scale the evolution of the beach profile during a storm is smooth, following distinct cycles and exhibiting characteristic morphological features (e.g., longshore bars). At our present state of knowledge it is not possible to obtain a stable prediction of the profile response at a very detailed scale, but empirical formulations have to be introduced that reproduce the behavior at a larger scale. In the case of a storm the overall duration of the event could last a few days, and it might be necessary to resolve changes in the beach profile at a scale of 5-20 min to satisfactorily model the profile evolution.

Analytical Models of Profile Evolution

Analytical solutions, although describing idealized situations, are useful for determining combinations of parameters that govern the characteristic time and space scales of profile response. These quantities can be employed for first-order estimates of profile response and for preliminary design of structures and activities. Although the governing equations for cross-shore sediment transport and profile evolution typically are difficult to simplify enough to allow for analytical solutions, several examples exist where some aspect of cross-shore transport has been described through analytical models. Kobayashi (1987) estimated dune recession and eroded sediment volume during storm surge. An empirical transport formula was employed that yielded a diffusion equation after being combined with the sediment volume conservation equation. Kriebel and Dean (1993) used a convolution method to approximate the beach-profile response during a severe storm. An exponential response function was assumed to characterize the erosion rate, and various initial profile geometries and storm surge histories were investigated in terms of eroded volume and profile retreat.

Another example where analytical solutions may be applied is in the design of offshore mounds using dredged material (Larson *et al.* 1999a). A sediment transport relationship in combination with the sand volume conservation equation may, for certain conditions, be simplified so that a diffusion equation is obtained, implying that a variety of analytical solutions for the profile evolution are available. Larson *et al.* (1999a) showed that the following relationship may be derived for the cross-shore sediment transport in the offshore, assuming that transport due to wave asymmetry and gravity is predominant,

$$q_c = K_c \frac{u_{oc}^3}{g} \frac{\partial \Delta h}{\partial x}$$

in which u_{oc} is the bottom orbital velocity taken to be constant in the area of interest, $\Delta h = h - h_e$, representing the response of the mound with respect to the EBP (h_e denotes the EBP shape), and K_c a transport coefficient. To compute mound response to wave action, the transport equation is combined with the sand volume conservation equation given by,

$$\frac{\partial \Delta h}{\partial t} = \frac{\partial q_c}{\partial x}$$

where t is time. Combining the two equations above yields,

$$\frac{\partial \Delta h}{\partial t} = \varepsilon_d \frac{\partial^2 \Delta h}{\partial x^2}$$

where:

$$\varepsilon_d = \frac{K_c u_{oc}^3}{g}$$

This equation is formally identical to the diffusion equation, and analytical solutions are available that cover a large number of initial and boundary conditions. Larson *et al.* (1987) presented several such solutions for the one-line model of shoreline change, which reduces to the diffusion equation under certain assumptions. They discussed previously published and new solutions related to the shoreline evolution resulting from the placement of a beach fill in the nearshore such that the shoreline is initially out of equilibrium with the wave climate. Several of the solutions for shoreline change have direct analogies with mounds (or, alternatively, dredged linear trenches) in the offshore. Thus, the solutions presented by Larson *et al.* (1987) for various beach fill configurations are applicable and will describe the mound (trench) evolution (see also, Kobayashi 1982).

The following general solution of the one-dimensional diffusion equation (Crank 1975) describes the evolution of a mound (trench) in the offshore,

$$\Delta h(x,t) = \frac{1}{2\sqrt{\pi \varepsilon_d t}} \int_{-\infty}^{\infty} f(\xi) e^{-(x-\xi)^2/4\varepsilon_d t} d\xi$$

where f(x) is the initial shape of the mound (trench) and ξ a dummy integration variable. This integral may be explicitly solved for simple mound configurations. For example, the evolution of a rectangular mound is given by the following solution (Larson *et al.* 1987),

$$\Delta h(x,t) = \frac{1}{2} \Delta h_o \left(\operatorname{erf}\left(\frac{a-x}{2\sqrt{\varepsilon_d t}}\right) + \operatorname{erf}\left(\frac{a+x}{2\sqrt{\varepsilon_d t}}\right) \right)$$

where Δh_o is the initial mound height over sea bottom, a half the mound width, and erf denotes the error function. If the initial height of the mound is given with a negative sign, the solution will instead describe the filling by cross-shore sediment transport of a long trench dug in the offshore.

Characteristic Quantities for Mound Response. Leading quantities governing the response of an offshore mound or trench under cross-shore sediment transport can be identified by non-dimensionalizing analytical solutions, providing insight to the governing time and space scales. Also, these quantities allow comparison of the performance of different mound designs. The evolution of an offshore mound having an initial width a, will be governed by the non-dimensional time scale $t'=\varepsilon_d\, t/a^2$. Two mounds having the same configuration but differing in size will display the same non-dimensional evolution in time, if appropriately scaled. The control exerted by geometrical parameters on the evolution of a mound and or trench can be assessed by comparing the non-dimensional quantities. For example, the maximum non-dimensional height of two mounds with the same initial geometric shape will be the same after time t'. Translating this relationship into dimensional time yields,

$$\frac{t_1}{t_2}=\left(\frac{a_1}{a_2}\right)^2\frac{\varepsilon_{d2}}{\varepsilon_{d1}}$$

where the indices 1 and 2 refer to two different mounds. This equation shows that by doubling the width, a mound can withstand four times as long a period of the same wave action before experiencing the same relative decrease of the maximum height. The diffusion coefficient ε_d enters linearly, but inversely, so that a doubling of ε_d causes the time for the mound to experience a certain reduction to be halved. In a preliminary design situation, this equation is useful for examining the evolution of mounds with different geometric characteristics at a particular site (mounds exposed to the same wave climate). Dean (1991) reviews a similar relation for behavior of rectangular beach fill, in which the width of the fill has the same functional dependence as mound width in controlling evolution of the feature.

By expressing ε_d in terms of the local wave climate, the impact of the wave properties can be assessed. By linear wave theory,

$$\varepsilon_d=\frac{K_c}{8}\frac{H^3g^2T^3}{L^3}\frac{1}{\left(\cosh(2\pi h/L)\right)^3}$$

where H is the wave height, T wave period, and L wavelength. Again, comparing two cases and equating the non-dimensional time t' gives:

$$\frac{t_1}{t_2}=\left(\frac{a_1}{a_2}\right)^2\left(\frac{H_2}{H_1}\right)^3\left(\frac{T_2}{T_1}\right)^3\left(\frac{L_1}{L_2}\right)^3\left(\frac{\cosh(2\pi h_1/L_1)}{\cosh(2\pi h_2/L_2)}\right)^3$$

In the case of shallow water, this equation may be further simplified to yield:

$$\frac{t_1}{t_2}=\left(\frac{a_1}{a_2}\right)^2\left(\frac{H_2}{H_1}\right)^3\left(\frac{h_1}{h_2}\right)^{3/2}$$

Thus, the influence of the wave height and the water depth may be assessed by comparing the evolution of two mounds of identical initial shape. For example, doubling the characteristic wave height causes a mound to respond in 1/8 of the time as compared to the original conditions. Similarly, placement of an offshore mound in shallower water increases the response time as depth to the 3/2 power, as compared to a base condition.

Comparison with Field Data. Larson *et al.* (2002b) compared predictions of mound evolution by the analytical model to measurements from two field sites. An offshore mound was placed off Silver Strand State Park, California, in December of 1988 and detailed monitoring of the mound evolution was made during the following year (Andrassy 1991, Larson and Kraus 1992). During the first half year the wave climate was mild, and no major storms were recorded. Thus, these data constitute an excellent set for testing the analytical model developed to predict mound evolution in the offshore under non-breaking waves. An optimum value for the diffusion coefficient, ε_d=14 m^2/day, was determined through a least-square fit against the measured profiles. Fig. 7 illustrates the agreement between the measurements and analytical solution obtained by superimposing initially trapezoidal line segments (Larson *et al.* 1987). The surveys were carried out approximately 27, 55, and 119 days after the post-construction survey (used as the initial profile here). As seen in Fig. 7, the analytical solution produces symmetric diffusion of the mound, the result of specifying a constant diffusion coefficient (*i.e.*, u_{oc} constant). However, despite various simplifications the analytical solution captures the overall response of the mound, and it can be applied to obtain estimates of quantities such as the decrease in the maximum mound height and reduction in mound volume, within the original boundaries of the mound.

Cocoa Beach near Cape Canaveral served as a beneficial-use site for dredged material on three occasions between June 1992 and June 1994. The first placement was carried out in June 1992, whereas the second and third placements were conducted over longer time periods and broader areas. Data pertaining to the first disposal were considered here for further validation of the diffusion model. One survey was made immediately after construction of the mound followed by two surveys 136 and 291 days after the mound placement. Fig. 8 shows the agreement between the analytical model of mound evolution and the measured profiles. A diffusion coefficient value of 10 m^2/day produced satisfactory description of the mound response. As for the Silver Strand mound, the analytical model predicted some seaward diffusion not observed in the measurements, attributed to overestimation of ε_d in this region. However, the overall evolution of the mound is well described by the analytical solution, creating confidence in the simple diffusion model for first estimates of how the mounds placed in the offshore respond to the action of non-breaking waves.

Dependence of Mound Diffusion on Wave Conditions. In order to apply the analytical model for preliminary design of offshore mounds, it is necessary to estimate the diffusion coefficient. Although relative comparisons can be made based on the characteristic quantities presented, it is of practical value to quantify the absolute evolution of a mound. However, there are few data sets on mound evolution suitable for determining ε_d. In addition to Silver Strand and Cocoa Beach, Larson *et al.* (2002b) identified two other data sets for analysis of ε_d. These two mounds were located at Maunganui Beach off the coast of New Zealand (Foster *et al.* 1996) and at Perdido Key, Florida (Otay 1995; Work and Otay 1996). Analysis of data from these two sites produced ε_d=30 and 0.6 m^2/day for Maunganui Beach and Perdido Key, respectively. Thus, fortunately, a wide range of ε_d-values was obtained in the analysis, corresponding to a variety of conditions at the different sites.

The theoretical relationship between ε_d and u_{oc} was plotted in Fig. 9 together with the data from the four mounds. Although the scatter is considerable, the clear trend indicates that reasonable predictions for ε_d may be obtained based on the mean local wave conditions. The bottom orbital velocity employed was calculated from the mean significant wave height at the peak of the initial mound during the measurement period. A least-square fit of the theoretical equation to the data points yielded K_c=0.00285. Efforts were made to estimate K_c individually for each case and to relate these K_c-values to various non-dimensional parameters including grain size, but no clear relationship could be established. Presently available data are limited and do not support adoption of expressions for ε_d that are more complicated.

Numerical Models of Profile Evolution

During the past fifteen years, engineering need has focused considerable effort on developing numerical models for calculating beach profile change due to cross-shore sediment transport (Kriebel and Dean 1985, Watanabe and Dibajnia 1988, Larson and Kraus 1989, Roelvink and Stive 1989, Nairn 1990, Steetzel 1990, Brøker Hedegaard *et al.* 1991). These models have mainly been developed to determine the profile response on a time scale of days to describe dune and foreshore erosion during storms, and longshore bar formation and movement. Although some of the models could be used for predicting profile evolution at longer time scales, sufficient validation is often lacking to prove the model capabilities for such long-term simulations. The necessity of specifying the input forcing with a resolution that corresponds to the description of the physical processes makes long-term predictions difficult in most models (Larson and Kraus 1995).

Numerical models of beach profile change may be classified according to the characteristic scale employed in resolving the fluid and sediment motion. Models which attempt to describe scales of motion in time (t) and space (s) compatible with individual waves belong to the class of microscale models (e.g., t=10^{-1}-$10^{0}T$ and s=10^{-4}-$10^{-3}L$, where T is the wave period and L the wavelength), whereas mesoscale models (e.g., t=10^{1}-$10^{2}T$ and s=10^{-2}-$10^{-1}L$) focus on resolving scales of motion that are the result of many waves (Larson and Kraus 1995). The numerical models by Kriebel and Dean (1985) and Larson and Kraus (1989) are examples of models that mainly adopt a mesoscale process description; the local cross-shore flow pattern is not computed, and the net transport rate is derived directly from the variation in wave properties across shore. Most other numerical cross-shore models (Roelvink and Stive 1989, Nairn 1990, Steetzel 1990, Brøker Hedegaard *et al.* 1991) involve both elements of micro- and mesoscale descriptions. These models typically calculate local flow velocities and transport rates characteristic for a microscale approach but employ input wave conditions that are averaged at the mesoscale.

The main advantage of modeling processes at the mesoscale is that a more robust model behavior is obtained than if processes are described at the microscale. A microscale model employs transport equations that are closer to a first-principles approach than a mesoscale model; however, because profile change events of interest to model typically encompass time periods in the mesoscale range, a microscale model often involves spatial and temporal averaging. This averaging produces net values of a small magnitude that are difficult to calculate with a sufficient accuracy to ensure robust model behavior at larger scales. In practice, numerical profile change models that mainly employ a microscale process description utilize different techniques to increase model stability at larger scales such as filtering, smoothing, and the use of representative wave quantities. Simultaneously, mesoscale models often involve components that are derived through a microscale description. Thus, most existing profile change models include elements that are characteristic both for the micro- and mesoscale, and any classification into a specific category has to refer to the overall modeling approach.

The SBEACH Profile Evolution Model. The development of the numerical beach profile response model SBEACH was a joint research effort by the Department of Water Resources Engineering, Lund University, Sweden, and the Coastal and Hydraulics Laboratory, US Army Engineer Reseacrh and Development Center, USA. SBEACH was primarily developed to calculate the profile response during a storm, although the model can qualitatively reproduce post-storm recovery and seasonal changes in the profile shape. The model also describes the formation and movement of longshore bars, at least the bar associated with the main break point of the waves. SBEACH has been widely used both as an analysis tool in studies on coastal processes and as an instrument for design of beach nourishment operations to protect against erosion and flooding. The model is under constant development and model components are being replaced and introduced as research on nearshore processes and beach profile change progresses. In the following a brief summary is given of the model components and the governing equations, after which some model applications are briefly discussed. A list of references is provided that refer to works which present SBEACH and various applications in detail.

SBEACH Model Structure and Main Governing Equations. SBEACH is a two-dimensional model that computes the wave transformation in the surf zone, including shoaling, refraction, breaking, and reforming, the cross-shore transport rate, and the change in the beach profile. All these calculations are done at each time step and the profile shape is continuously updated to accurately represent the feedback from the profile on the waves. SBEACH has the options of handling both monochromatic and random waves, where the latter is typically employed for field conditions. In case of random waves, the following wave energy flux conservation equation is used to determine cross-shore wave properties (Larson 1995),

$$\frac{d}{dx}(F_{rms}\cos\theta) = \frac{\kappa}{h}(F_{rms} - F_{stab})$$

where,

$$F_{rms} = \frac{1}{8}\rho g H_{rms}^2 C_g$$

$$F_{stab} = \frac{1}{8}\rho g\left[(1-\alpha)H_n^2 + \alpha\Gamma^2 h^2\right]C_g$$

and H_{rms} is the root-mean-square (rms) wave height, C_g the wave group speed, θ the wave angle, h the total water depth (including wave setup/setdown), α the ratio of breaking waves, H_n the rms wave height for non-breaking waves, and κ and Γ empirical constants. The wave transformation equation is valid for wave conditions that are narrow-banded in frequency and direction; however, any type of offshore probability density function (pdf) may be used for the wave height, although a Rayleigh pdf is normally assumed. The cross-shore momentum equation and the wave number conservation equation are solved in parallel to obtain the mean water elevation and the wave angle, respectively.

Based on the work by Kriebel and Dean (1985) and Larson and Kraus (1989) on the net cross-shore transport rate for monochromatic waves a predictive equation for the transport under random waves was developed by Larson (1996),

$$q_c = K_T \zeta \left[D - \alpha \left(D_{eq} - \frac{\varepsilon_T}{K_T} \frac{\partial h}{\partial x} \right) \right]$$

where q_c is the average net cross-shore transport rate, D the average energy dissipation per unit water volume and D_{eq} its equilibrium value as defined by Dean (1977), K_T and ε_T empirical transport coefficients, and $\partial h/\partial x$ the bottom slope. The function ζ gives the relative weight of erosional and accretionary waves in the selected pdf, where the distinction between erosion and accretion is based on a criterion by Kraus *et al.* (1991). The equation above is used to calculate the transport rate under breaking waves, which give the main contribution to the profile change. Other transport formulas are employed to compute the transport under non-breaking waves and in the swash (Larson 1996). After the transport rate has been calculated at all cross-shore locations the equation for sediment volume conservation is employed to compute the depth changes and the new depths.

As input SBEACH needs time series of waves (height, period, and direction) and water level (and wind, if available), the initial beach profile shape, sediment characteristics (grain size, specific weight etc), and the value on several coefficients (Rosati *et al.* 1993). The coefficient K_T is the main one to be estimated (its value should preferably be determined through calibration against data), whereas the other coefficients may be assigned standard values. The main output from the model is the beach profile evolution in time, but local wave properties and sediment transport rates could also be obtained at specified times. SBEACH may be used to estimate the maximum contour recession and eroded volume during a storm.

Model Applications. SBEACH was originally developed and validated using data from large wave tank experiments (prototype-size; see Larson and Kraus 1989). Then, extensive model testing has been performed against field data, primarily from the US East Coast (Larson and Kraus 1989, Larson *et al.* 1990, Wise *et al.* 1996). High-quality field data sets involving the effect of severe storms, such as hurricanes or northeasters, on several different beaches have been used in the model testing. SBEACH has been shown to produce reliable predictions of storm impacts in terms of the nearshore profile shape, eroded sediment volume, and contour recession. The model has often been employed in studies on coastal processes where the impact of large storms needs to be assessed. Another principal application of SBEACH has been in the design of beach fills and the response of the beach to hard structures such as seawalls and revetments (Larson and Kraus 2000). SBEACH is used to calculate beach profile response of alternative design configurations to storms of varying intensity. Model predictions of beach erosion are used to estimate with- and without-project storm damages over the project design life. In the design process, storm erosion, flooding, and wave damage estimates are utilized in economic analyses to compare total project costs and total project benefits for each design alternative.

Shoreline Evolution Models

Mathematical modeling of shoreline change is a powerful and flexible engineering technique for understanding and predicting the long-term evolution of the plan shape of sandy beaches. For example, Hanson *et al.* (1988) give several applications. Mathematical models provide a concise, quantitative means of describing systematic trends in shoreline evolution commonly observed at groins, jetties, and detached breakwaters. Pelnard-Considere (1956) originated the mathematical theory of shoreline response to wave action under the assumption that the beach profile moves parallel to itself, *i.e.*, that it translates shoreward and seaward without changing shape in the course of eroding and accreting. He also verified his mathematical model by comparison to beach change produced by waves obliquely incident to a beach with a groin installed in a movable-bed physical model.

If the profile shape does not change, any point on it is sufficient to specify the location of the entire profile with respect to a baseline. Thus, one contour line can be used to describe change in the beach plan shape and volume as the beach erodes and accretes. This contour line is conveniently taken as the readily observed shoreline, and the model is therefore called the "shoreline change" or "shoreline response" model. Sometimes the terminology "one-line" model, a shortening of the phrase "one-contour line" model is used with reference to the single contour line.

Qualitative and quantitative understanding of idealized shoreline response to the governing processes is necessary for investigating the response of the beach to engineering actions. By developing analytical or closed-form solutions originating from mathematical models that describe the basic physics involved, essential features of beach response may be derived, isolated, and more readily comprehended than in complex approaches such as numerical and physical modeling. Also, with an analytical solution as a starting point, direct estimates can be readily made of characteristic parameters associated with a phenomenon, such as the time elapsed before bypassing of a groin occurs, percentage of volume lost from a beach fill, and growth of a salient behind a detached breakwater. In addition, analytical models avoid inherent numerical stability and numerical diffusion problems encountered in mathematical models.

Another useful property of analytic solutions is the capability of obtaining equilibrium condition from asymptotic behavior. However, the complexity of beach change implies that results obtained from an analytic model should be interpreted with care and with awareness of the underlying assumptions. Analytic solutions of mathematical models cannot be expected to provide quantitatively accurate results in situations involving complex boundary conditions and complex wave inputs. In engineering design, a numerical model of shoreline evolution would usually be more appropriate. Also, the equations describing shoreline evolution quickly become excessively complicated to permit analytical treatment, making it necessary to use numerical techniques. Therefore, to obtain a closed-form solution to shoreline change, a simple mathematical formulation has to be used, but one which still preserves the important mechanisms involved.

Analytical Models of Shoreline Evolution

Larson *et al.* (1987, 1997) presented analytical solutions for the shoreline evolution in the vicinity of coastal structures, including detached breakwaters, seawalls, and jetties with and without wave diffraction. Also, solutions for the response of beach fills with different initial configurations were presented. The solution for a detached breakwater produced the growth of a salient with time behind the breakwater and the associated initial shoreline retreat at locations across from the breakwater tips. A simple solution describing flanking of a seawall was obtained by using two different solution areas where the longshore sand transport rate and breaking wave angle vary. A similar technique was used to model diffraction downdrift of a groin or jetty by allowing the wave angle to vary with the distance alongshore according to a specified function or by employing a large number of solution areas. Cases were also presented for the accumulation updrift a groin and the shoreline response in a groin compartment with a breaking wave angle that varies sinusoidally in time. The solution for a single groin illustrated the impact of bypassing on the updrift accumulation.

In one-line modeling of shoreline change, the beach profile is assumed to move in parallel with itself, maintaining a fixed shape, implying that the sand volume conservation equation can be written,

$$\frac{\partial Q}{\partial x} + D_C \frac{\partial y}{\partial t} = 0$$

where Q is the longshore sand transport rate, y the shoreline position, and D_C the depth of closure. A general expression for Q is (compare with the CERC formula; SPM 1894),

$$Q = Q_o \sin 2\alpha_b$$

where Q_o is the sand transport rate amplitude and α_b the wave angle at breaking given by,

$$\alpha_b = \alpha_o - \arctan\left(\frac{\partial y}{\partial x}\right)$$

in which α_o is incident wave angle. Combining the transport equation and the continuity equation, assuming small wave angles and shoreline orientations, yields,

$$\frac{\partial y}{\partial t} = \varepsilon_s \frac{\partial^2 y}{\partial x^2}$$

where:

$$\varepsilon_s = \frac{2Q_o}{D_C}$$

Thus, the governing equation is formally identical to the one-dimensional equation describing conduction of heat in solids. Thus, many analytical solutions can be generated by applying the proper analogies between initial and boundary conditions for shoreline evolution and the processes of heat conduction and diffusion. Carslaw and Jaeger (1959) provide many solutions to the heat conduction equation, and Crank (1975) gives solutions to the diffusion equation. The coefficient ε_s, having the dimensions of length squared over time, is interpreted as a diffusion coefficient expressing the time scale of shoreline change following a disturbance.

As an illustration, the analytical solution to the case of bypassing around a jetty is discussed here (Larson *et al.* 1997). For other solutions, Larson *et al.* (1987, 1997) should be consulted. The following expression was used to estimate the bypassing rate, Q_b, at a jetty or groin,

$$Q_b = 2Q_o\alpha_o \frac{y}{L_G}$$

where L_G is the groin length. Combined with the linearized transport relationship, the following boundary condition is obtained at the jetty:

$$\frac{\partial y}{\partial t} = \alpha_o \left(1 - \frac{y}{L_G}\right)$$

Thus, the bypassing rate is proportional to the distance between the shoreline at the groin and the groin tip; this model is physically more appealing than simply prescribing the variation in Q_b with time as done by Larson *et al.* (1987).

The analytical solution to describe shoreline advance updrift of a groin, with Q_b given at $x = 0$, is (c.f Carslaw and Jaeger 1959),

$$\frac{y(x,t)}{L_G} = erfc\left(\frac{x}{2\sqrt{\varepsilon_s t}}\right) - e^{\alpha_o x/L_G + \alpha_o^2 \varepsilon_s t/L_G^2} erfc\left(\alpha_o \frac{\sqrt{\varepsilon_s t}}{L_G} + \frac{x}{2\sqrt{\varepsilon_s t}}\right)$$

where α_o is defined as positive when it produces transport towards the groin (in the negative x-direction). Fig. 10 illustrates the shoreline updrift of a groin for $\alpha_o = 12$ deg and selected dimensionless times t' $(=\varepsilon t/L_G^2)$, together with the solution for the case without bypassing (*i.e.*, total blocking by the jetty; see Larson *et al.* 1987). For small values of t' bypassing has little effect on the shoreline evolution and the two solutions are almost identical. However, with elapsed time the bypassing markedly restricts the accumulation.

Numerical Models of Shoreline Evolution

The one-line theory was numerically first implemented by Price *et al.* (1972), followed by many others. Willis (1977) applied a one-line model to prototype conditions, evaluating different formulas for the longshore transport, and Perlin (1979) simulated the shoreline evolution around detached breakwaters. Kraus *et al.* (1984) and Hanson and Kraus (1986) developed a one-line model with the overall aim to arrive at a model which could be used as an engineering tool. This was the first step towards a general shoreline evolution model that was named GENESIS (Hanson 1989), discussed below.

The equations governing shoreline change in numerical models are similar to the ones used in analytical models, but much more general in terms of being able to handle varying forcing, initial and boundary conditions as well as source and sink strength. For example, conservation of sand volume requires that,

$$\frac{\partial y}{\partial t} + \frac{1}{D_B + D_C}\left(\frac{\partial Q}{\partial x} - q\right) = 0$$

where D_B is the berm height and q a source term. In order to solve this equation, the initial shoreline position over the full reach to be modeled, boundary conditions on each end of the beach, and values for Q, q, D_B, and D_C must be given.

The GENESIS Shoreline Evolution Model. The GENESIS shoreline evolution model was a joint research effort by the Department of Water Resources Engineering, Lund University, Sweden, and the Coastal and Hydraulics Laboratory, US Army Engineer Reseacrh and Development Center, USA. The empirical predictive formula for the longshore sand transport rate used in GENESIS is,

$$Q = \left(H^2 C_g\right)_b \left(a_1 \sin 2\alpha_b - a_2 \cos\alpha_b \frac{\partial H}{\partial x}\right)_b$$

in which H is the wave height and C_g the wave group speed given by linear wave theory. The non-dimensional parameters a_1 and a_2 are given by,

$$a_1 = \frac{K_1}{16(s-1)(1-p)}$$

and,

$$a_2 = \frac{K_2}{8(s-1)(1-p)\tan\beta}$$

in which K_1 and K_2 are empirical coefficients, treated as a calibration parameters, and $\tan\beta$ the average bottom slope from the shoreline to the depth of active longshore sand transport. The first term in the transport equation accounts for longshore sand transport produced by obliquely incident breaking waves, and the second term describes the effect of the longshore gradient in breaking wave height on the sand transport rate. On open coasts, the first term is much greater than the second. However, in the vicinity of structures, where diffraction produces a substantial change in breaking wave height, inclusion of the second term provides an improved shoreline change modeling result.

GENESIS Applications. Most GENESIS applications involve analysis and design of structures or beach nourishments. There are at present more than 500 GENESIS installations world wide. The exact number of GENESIS field applications is not known, but should be in the hundreds. A lot of experience from these applications has been fed back into GENESIS that has, thus, undergone a continuous development over the years. For example of model applications the following references may be consulted: Hanson *et al.* (1988), Hanson *et al.* (1989), Hanson and Kraus (1991), and Kraus *et al.* (1994).

Concluding Remarks

Designing engineering structures and activities on sandy coasts require understanding of the beach morphology and its evolution. The morphology affects and is affected by engineering works through a complex interaction that must be considered for optimal functioning of such works. Thus, it is necessary to quantify morphological properties characterizing features and processes at the scale of interest. The EBP is one such property that is widely used by engineers to describe the average shape of the beach over time scales of decades. Recent progress to compute the EBP involves schematization of the profile into two distinct regions. One region, the surf zone, is shaped mainly by breaking waves, whereas the offshore zone, located seaward of the break point, is determined by non-breaking waves. The EBP in both regions is a power function, but the curvature of the EBP in the offshore zone is lower than in the surf zone. Shape parameters, being functions of grain size and wave properties, uniquely define the EBP in the two regions. Various morphological features may often be interpreted as perturbations on the EBP, for example, longshore bars. These features are typically difficult to model based on description of the governing processes but statistical techniques might be employed to quantify the features and their evolution. Similarly to EBP and associated morphological features in the cross-shore direction, equilibrium shapes occur in the longshore direction together with different types of rhythmic features. However, alongshore equilibrium is often dependent on fixed boundaries through structural or geological controls.

Models of coastal morphology are important tools in analysis and design of engineering structures and activities. Both analytical and numerical models are being utilized, where the former type of model is restricted to situations with highly schematized initial, boundary, and forcing conditions. Analytical models provide approximate solutions to be used in the initial stage of a project where a wide range of alternatives is considered. In situations where detailed simulations are required for complex conditions, numerical models must be employed. The two most common types of models aim at simulating the evolution of the beach profile and the shoreline. Analytical models of beach profile evolution may be employed to calculate dune erosion and contour retreat during storms as well as the response of mounds from dredged material placed in the offshore. Numerical models are able to handle most aspects of profile evolution, although it is still difficult to produce robust and reliable long-term simulations. SBEACH is one of the most widely used numerical models of beach profile evolution in engineering projects.

Analytical models of shoreline evolution are common tools to assess the impact of structures or beach fills on the shoreline. For example, accumulation updrift a groin, bypassing of a jetty, and alongshore spreading of a beach fill represent situations that may be described by analytical models. Again, more complex conditions require numerical models such as the GENESIS model, which is one of the earliest and most applied engineering models of shoreline evolution. GENESIS is able to handle most types of engineering structures and activities and it is under constant development.

Acknowledgement

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II: SIGNIFICANT ADMINISTRATIVE ACTIONS AND OTHER INFORMATION

No significant administrative actions were taken.

III. FUNDS REMAINING AND LIST OF PROPERTY ACQUIRED

The total contract is for $39,995. Payment upon receipt of this report is scheduled at $9,998.75. Therefore, the amount of funds remaining under the contract at the end of this report period is $0.

TABLE 1

Statistics for Inner Bar Properties at Duck, North Carolina

Property	Mean	Minimum	Maximum	Q_{25}*	Q_{75}*
Depth to crest, m	1.6	0.6	2.5	1.3	1.9
Bar height, m	0.9	0.2	1.7	0.7	1.1
Bar volume, m^3/m	45	6	102	29	57
Bar length, m	95	35	280	70	105
Bar mass center, m	215	150	330	200	235

* The quantities Q_{25} and Q_{75} denote the limits for which 25 and 75 percent of the values are below, respectively.

TABLE 2

Statistics for Outer Bar Properties at Duck, North Carolina

Property	Mean	Minimum	Maximum	Q_{25}*	Q_{75}*
Depth to crest, m	3.8	1.3	5.1	3.4	4.1
Bar height, m	0.4	0.0	1.4	0.27	0.6
Bar volume, m^3/m	45	0	120	20	67
Bar length, m	170	25	280	150	200
Bar mass center, m	410	200	520	390	440

*The quantities Q_{25} and Q_{75} denote the limits for which 25 and 75 percent of the values are below, respectively.

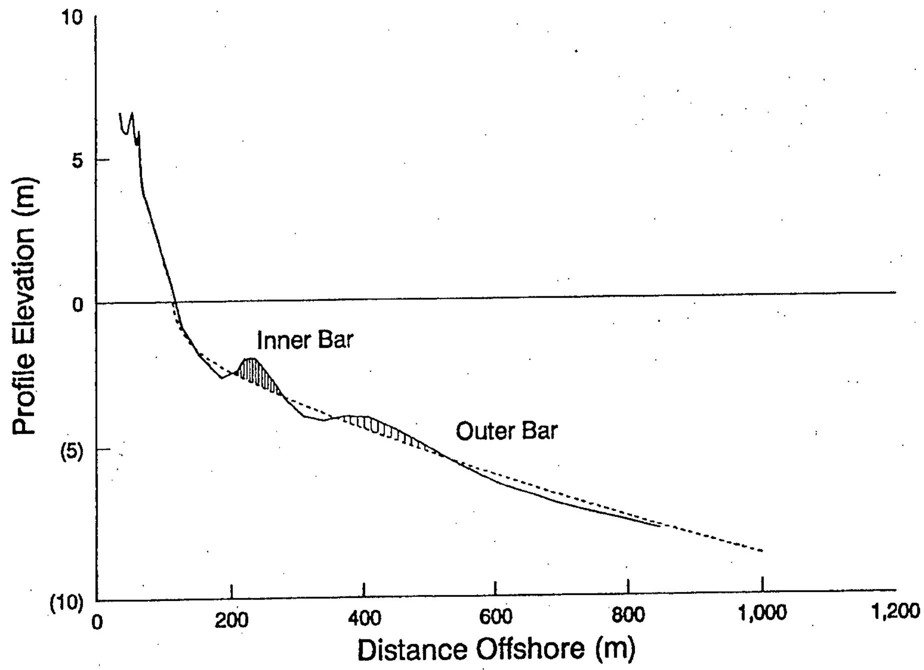

Figure 1. Definition of longshore bar extent using the modified equilibrium profile equation (hatched areas represent bars; from Larson and Kraus 1992).

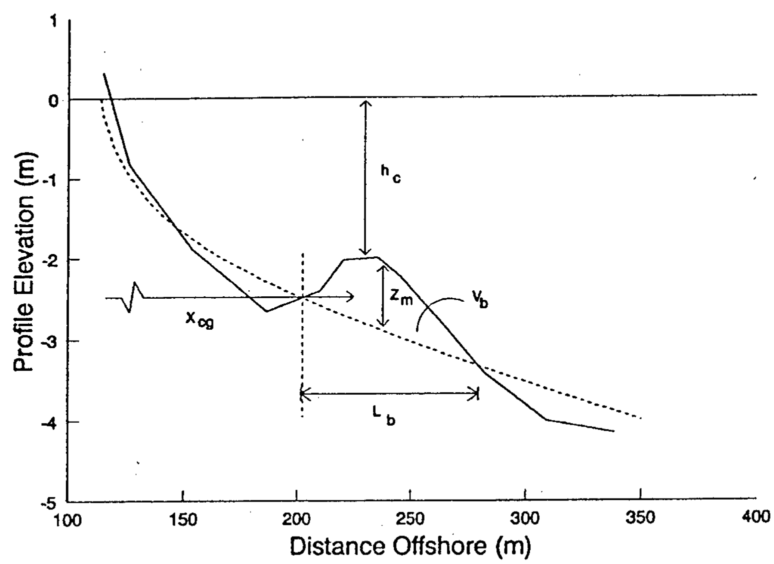

Figure 2. Definition sketch of bar properties calculated for individual profile surveys (from Larson and Kraus 1992).

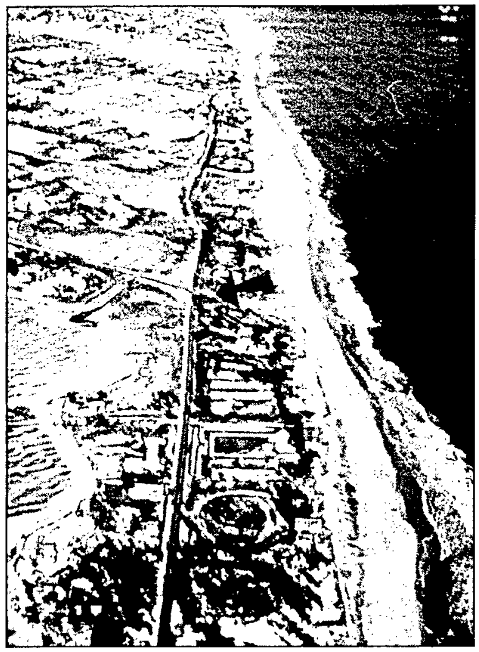

Figure 3. Alongshore sand waves at Southampton Beach, Long Island (from Larson *et al.* 2002c).

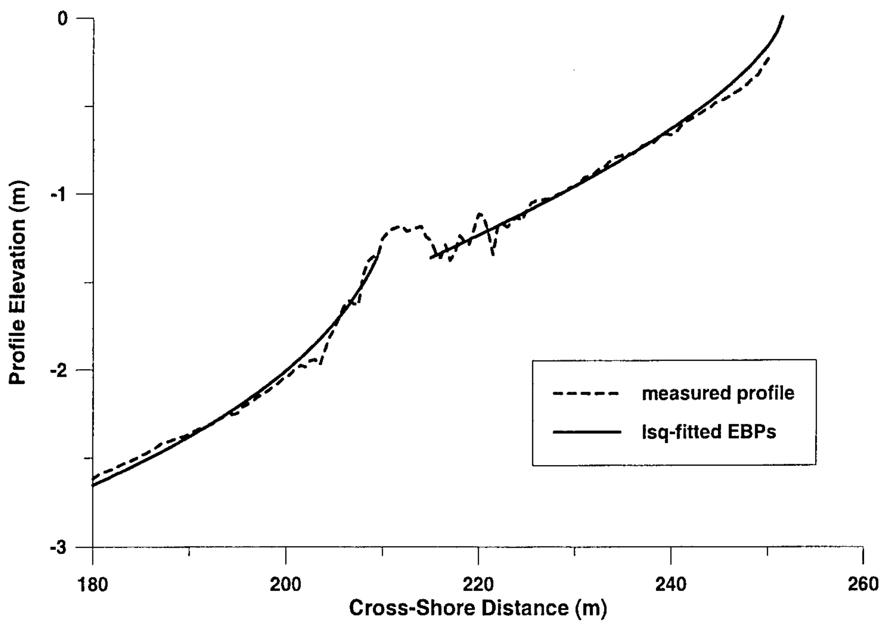

Figure 4. Comparison between theoretical EBPs and the measured final profile from Test C2 in the German Large Wave Tank (Dette *et al.* 1998).

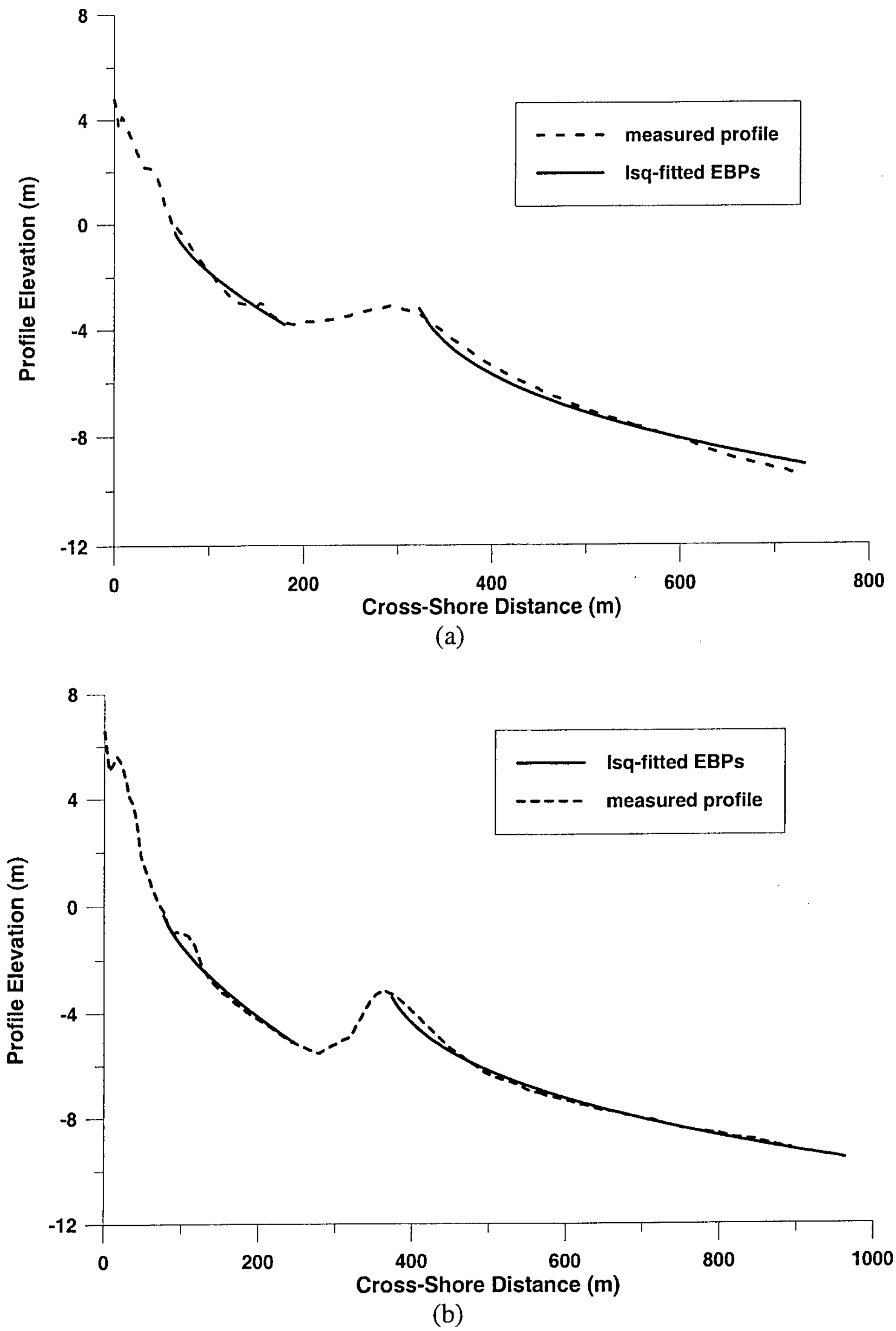

Figure. 5 Comparison between theoretical EBPs and measured profile from (a) Fire Island, Long Island, and (b) Westhampton Beach, Long Island.

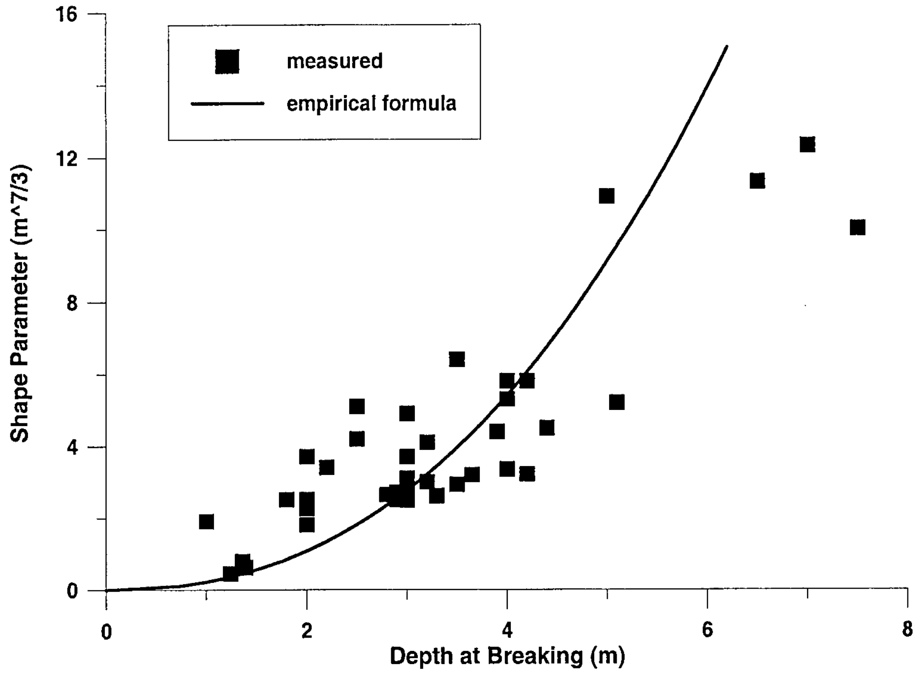

Figure 6. Relationship between shape parameter for the EBP in the offshore and the representative depth at breaking.

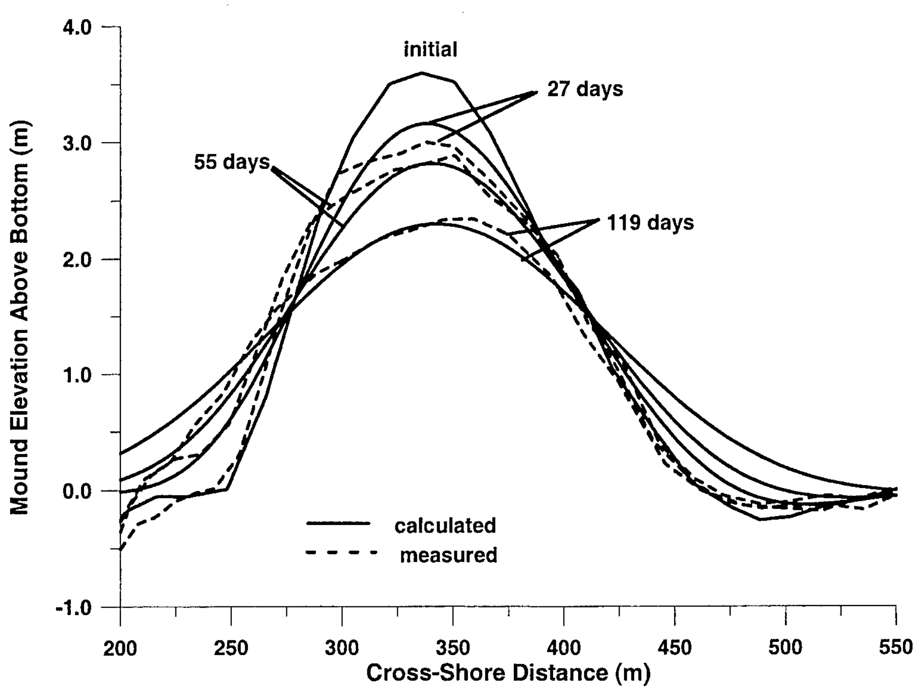

Figure 7. Comparison between analytical model of mound evolution in the offshore and measurements at Silver Strand, CA.

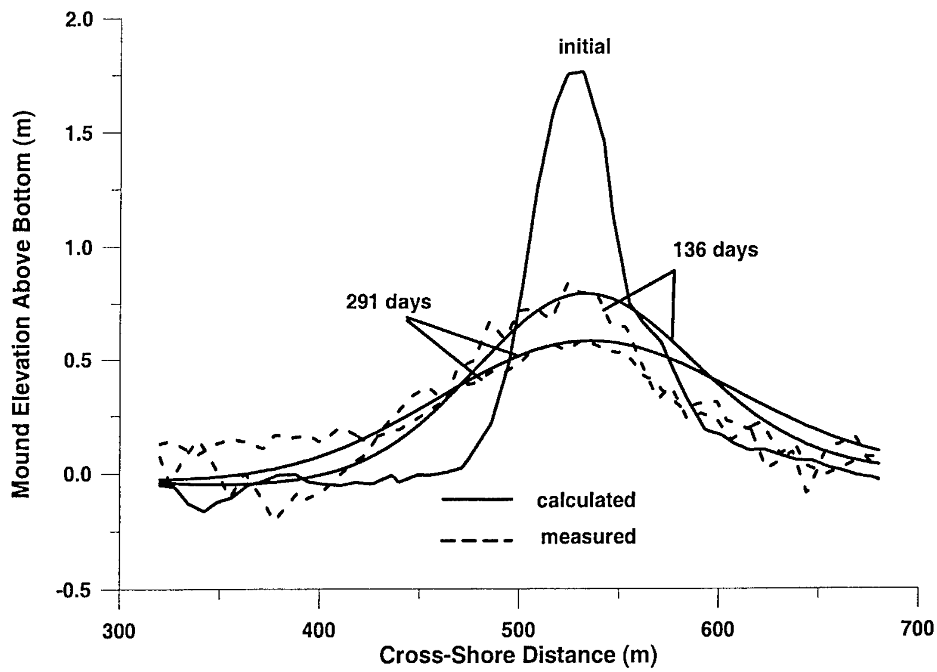

Figure 8. Comparison between analytical model of mound evolution in the offshore and measurements at Cocoa Beach, FL.

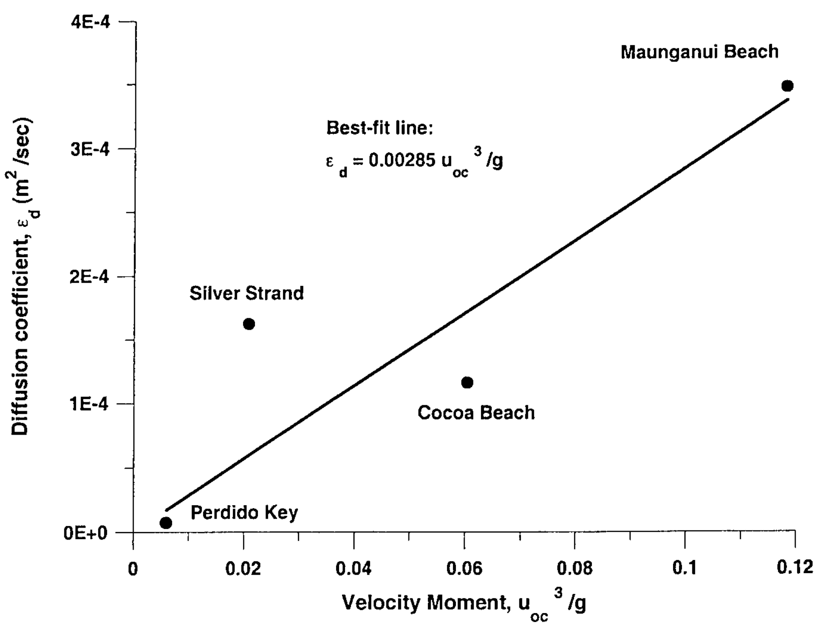

Figure 9. Estimated diffusion coefficients for different mounds based on field data and an empirical relationship.

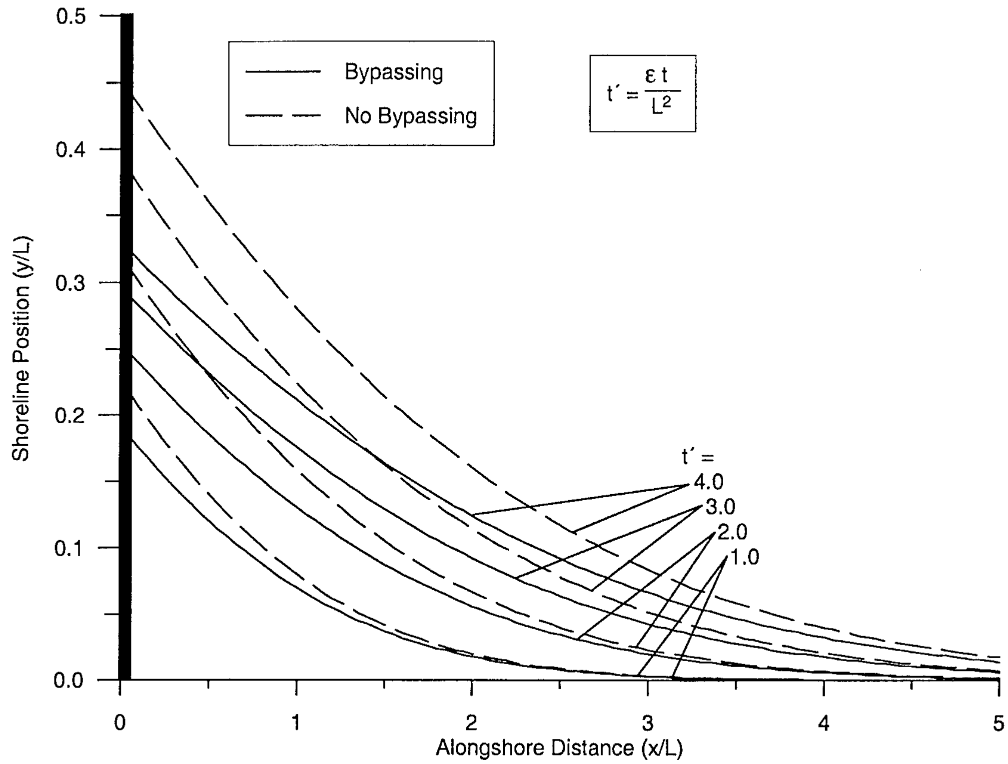

Figure 10. Shoreline evolution updrift a groin with and without bypassing (incident wave angle 12 deg)

AD NUMBER	DATE	DTIC ACCESSION NOTICE

1. REPORT IDENTIFYING INFORMATION

A. ORIGINATING AGENCY
Univ. of Lund, Sweden

B. REPORT TITLE AND/OR NUMBER
Enhancement of the Numerical Model Sbeach for Simulating Beach Profile

C. MONITOR REPORT NUMBER
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N68171-99-M-5815

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