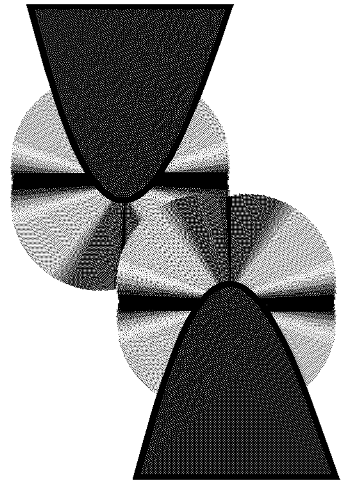
Radiative Exchange of Heat Between Nanostructures - a Quantum Story



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Radiative Exchange of Heat Between Nanostructures

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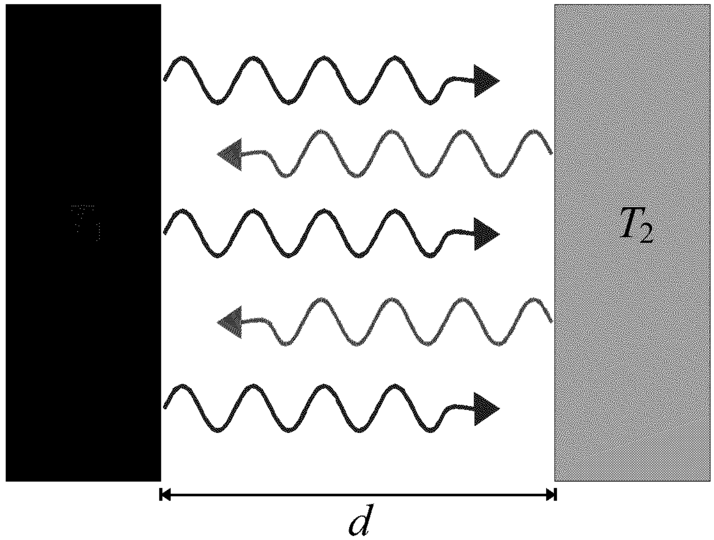
Stefan's Law

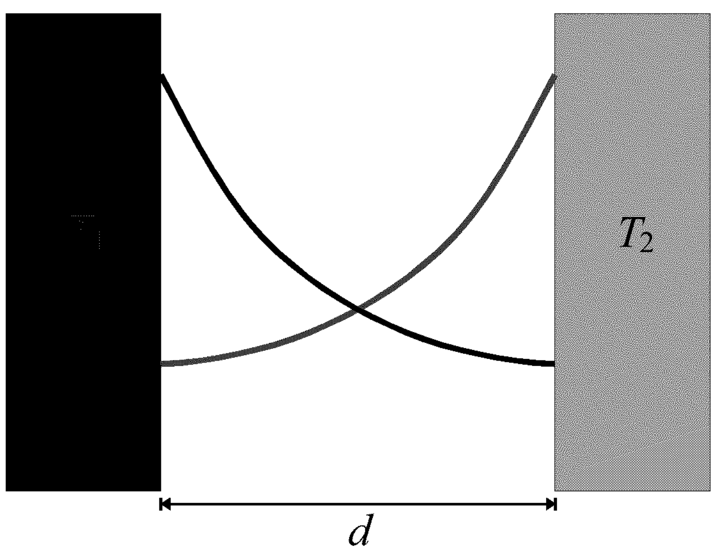


$$\dot{Q}_{BB} = \frac{\pi^2 k_B^4}{60\hbar^3 c_0^2}\left(T_1^4 - T_2^4\right), \qquad d >> \lambda_T = \frac{\hbar c_0}{k_B T}$$

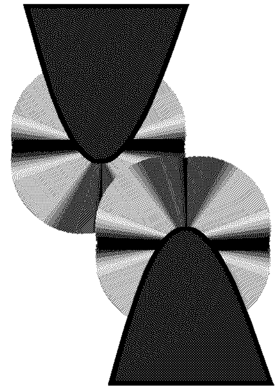
Stefan's Law neglects contributions from **evanescent photon states** which have large values of parallel momentum, **k**:

$$\mathbf{E}_p = E_{0p}\hat{\mathbf{K}}_p^+ \exp\left(i\mathbf{k}\cdot\mathbf{r}_{//} - \alpha z\right)$$

$$\alpha = +\sqrt{k^2 - \omega^2 c_0^{-2}}$$



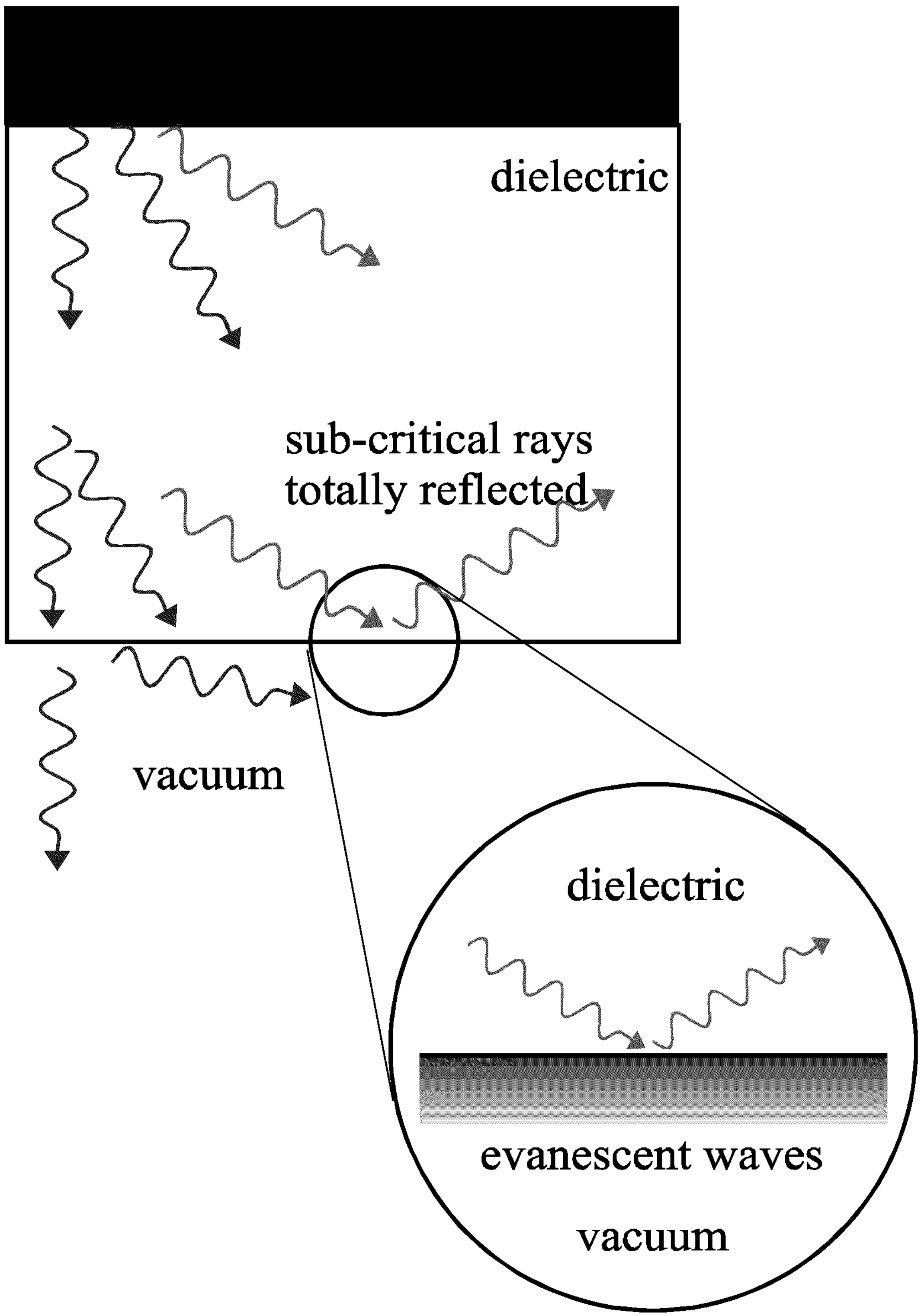
Critical Distance for Evanescent Waves to Dominate



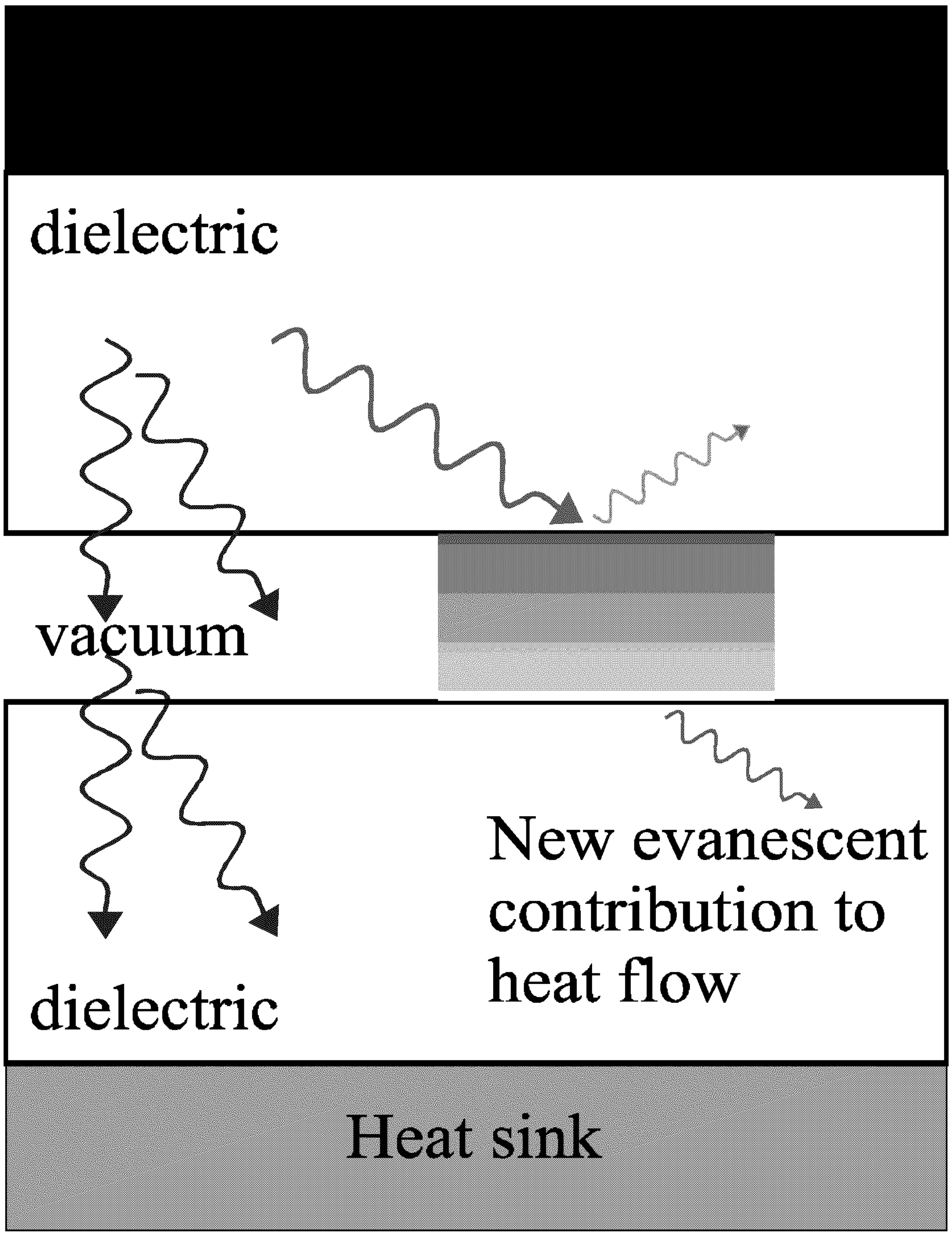
$T(K)$	λ_T(microns)
1	2289.8
4.2	545.2
100	22.9
273	8.4
1000	2.3

i.e. at distances of a few nanometres, radiative heat flow is almost entirely due to evanescent modes

Evanescent waves play no role in heat loss from a hot dielectric surface to vacuum ……

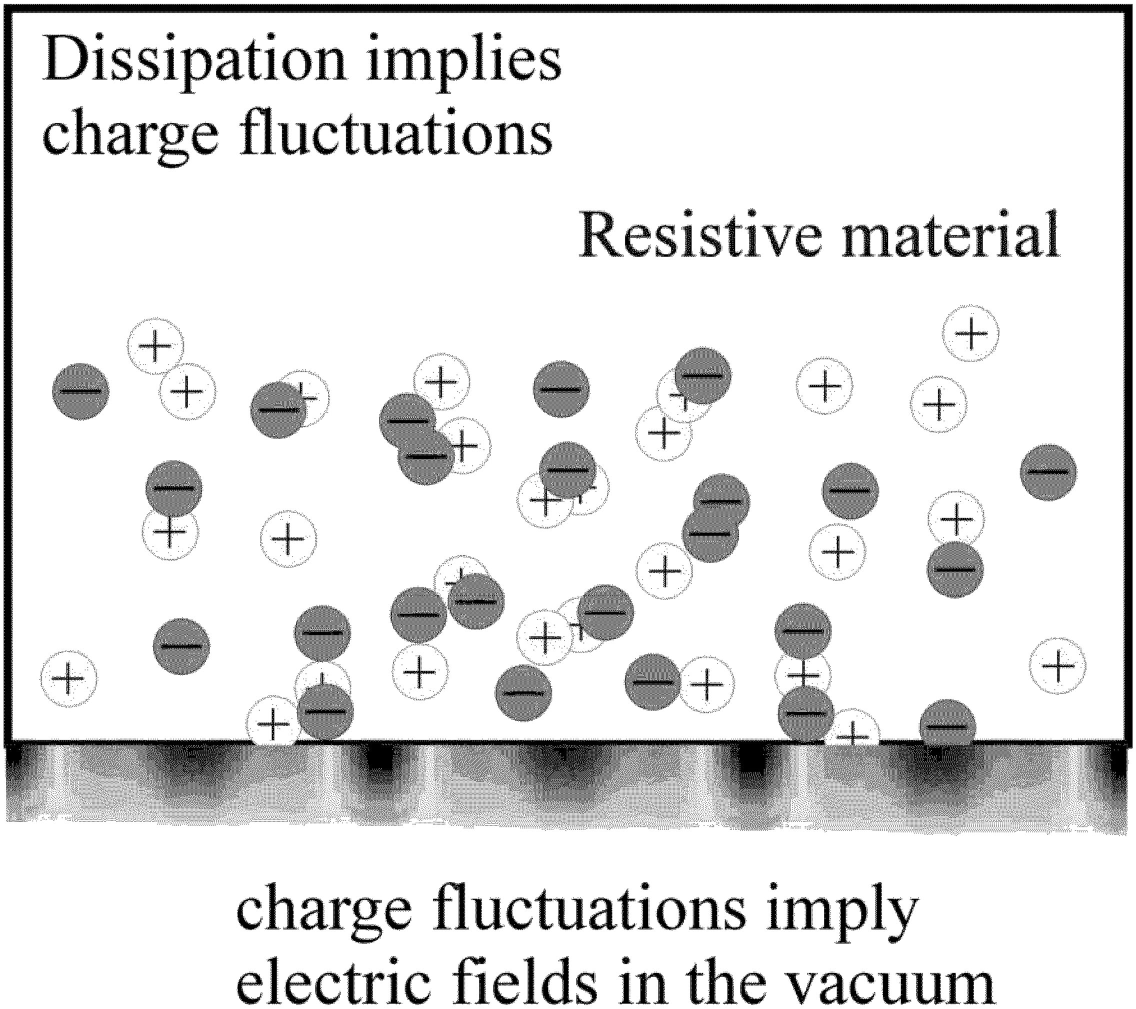


…… but evanescent waves can carry heat from a hot to a cold dielectric surface



Resistive Media and Evanescent States (1)

Resistive media also have evanescent waves outside their surfaces. In fact they are a much more potent source of evanescent waves because they support very short wavelength states not found near dielectrics.



These fluctuations are the nanoscopic equivalent of Johnson noise.

Resistive Media and Evanescent States (2)

The energy density at distance *d* is proportional to,

$$\boxed{\operatorname{Im} R_p(k,\omega)\exp(-kd)}$$

where $R_p(k,\omega)$ is the reflection coefficient of the surface at wave vector *k* and frequency ω, and ,

$$\operatorname{Im} R_p(k,\omega) \approx \operatorname{Im}\frac{\varepsilon(k,\omega)-1}{\varepsilon(k,\omega)+1}$$

$$= \frac{2\,\sigma/\omega\varepsilon_0}{4+(\sigma/\omega\varepsilon_0)^2}, \qquad k >> \frac{\omega}{c_0}$$

It is generally assumed that the conductivity is independent of (k,ω) over a wide range of values.

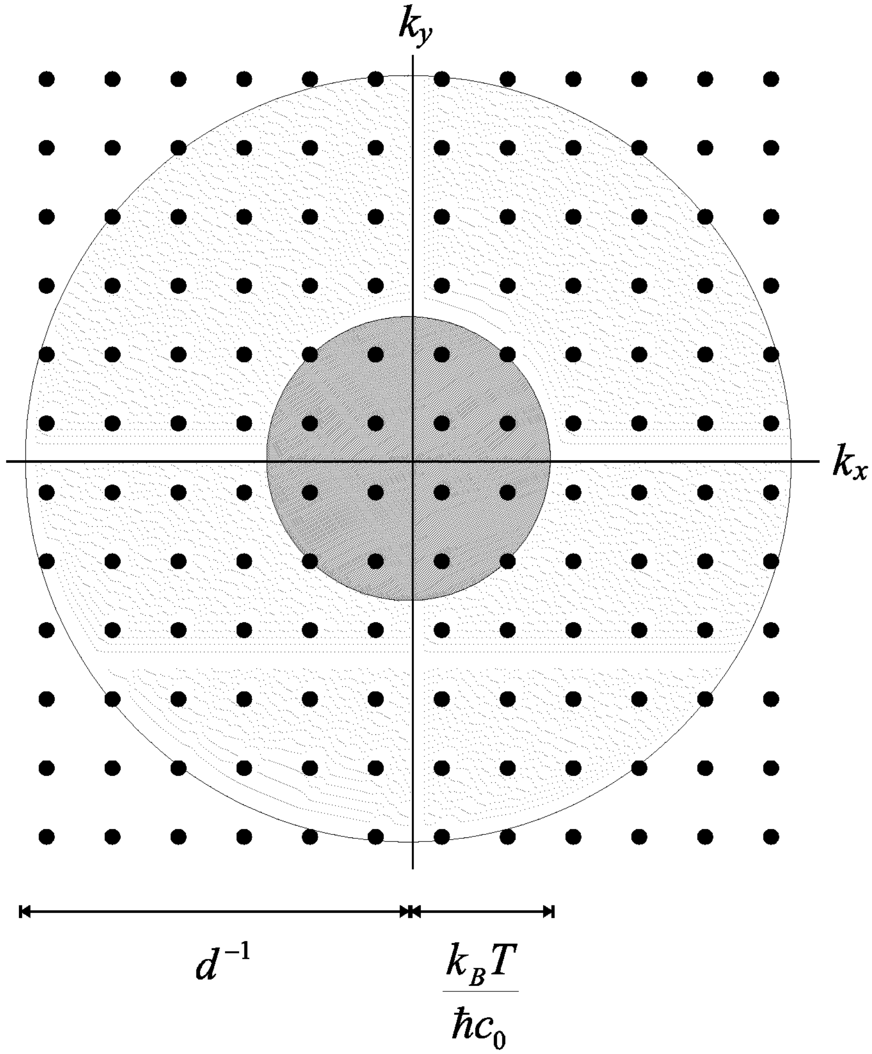
The energy density is a maximum when,

$$\boxed{\sigma_{\max} = 2\omega\varepsilon_0 \approx \frac{2k_B T\varepsilon_0}{\hbar} = 2.3T(\mathrm{m}\Omega)^{-1}}$$

where we have substituted a frequency typical of temperature *T*. At room temperature the optimum electrical conductivity is $690(\mathrm{m}\Omega)^{-1}$.

The importance of short wavelength fluctuations

At short distances, evanescent states dominate in phase space: propagating photon modes carry heat flux within the green circle, evanescent modes within the yellow circle.



optimal heat flow depends on how many k-points are active which is determined by,

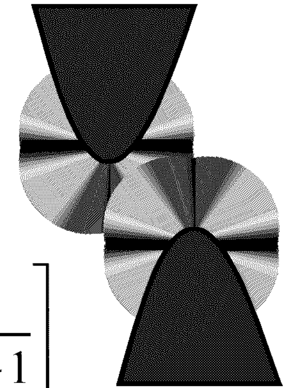
$$\alpha d \approx kd \approx 1$$

hence,

$$N_k \approx \frac{\pi k_{\max}^2}{(2\pi)^2} \approx \frac{1}{4\pi d^2}$$

provided that these modes exist in the solid.

Heat flux through evanescent modes



$$\dot{q}_p(k,\omega) = \frac{2\hbar|\omega|}{\pi}\left[\frac{1}{\exp(\hbar|\omega|/k_B T_1)-1} - \frac{1}{\exp(\hbar|\omega|/k_B T_2)-1}\right]$$

$$\times \exp(-2\alpha d)\frac{\operatorname{Im} R_{1p}(k,\omega)\operatorname{Im} R_{2p}(k,\omega)}{\left|1 - R_{1p}(k,\omega)R_{2p}(k,\omega)e^{-2\alpha d}\right|^2}d\omega$$

where p denotes the polarisation. The complete heat transfer from surface 1 to 2 via evanescent states is,

$$\dot{Q}_{EV}(d) = \sum_{\mathbf{k}}\int\left[\dot{q}_p(k,\omega) + \dot{q}_s(k,\omega)\right]d\omega$$

or most materials p-polarisation is dominant; s-polarisation is important only for magnetically active materials.

Maximising heat flux through evanescent modes at fixed T_1, T_2

Maximise, $$X = \frac{e^{-2\alpha d}(\operatorname{Im} R)^2}{\left|1 - R^2 e^{-2\alpha d}\right|^2}$$

i.e. $$R_i^2 + R_r^2 = e^{+2\alpha d}, \quad X = \frac{1}{4}$$

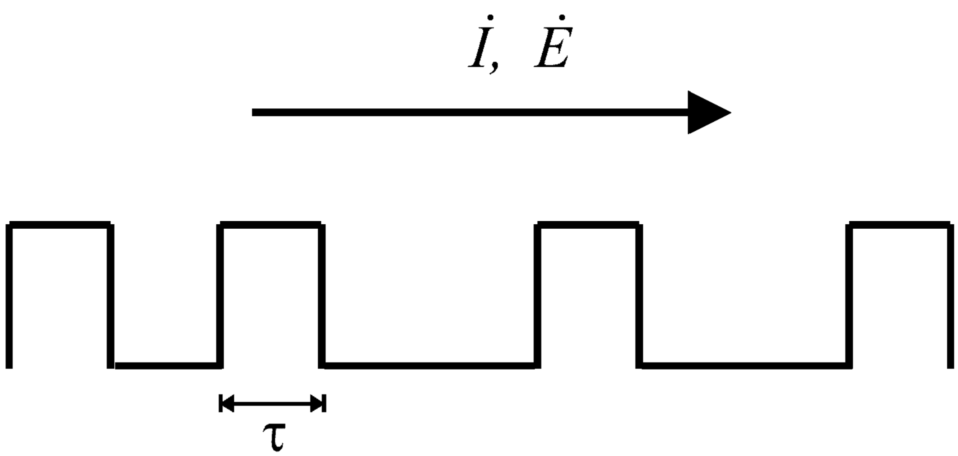
substituting,

$$\left[\dot{Q}_{EV}\right]_{\max} = \sum_{\mathbf{k}}\frac{\pi k_B^2}{3\hbar}T^2$$

assuming only p-polarised modes are active.

Connection to Quantum Information Theory (*qualitative* results)

Transmitting information by modulating a stream of particles (photons, phonons, electrons …..):



In each time slice, τ, one bit of information is provided by the presence or absence of a particle. Therefore the rate of flow of information is,

$$\dot{I} \approx \tau^{-1}$$

However in order to confine a particle in a time slice τ, energy is required of order,

$$\overline{E} \approx \hbar \tau^{-1}$$

Hence a flow of energy naturally and unavoidably accompanies the information,

$$\dot{E} \approx \tau^{-1}\overline{E} \approx \hbar \tau^{-2} \approx \hbar \dot{I}^{2}$$

In fact this is the minimum energy flow.

$$\boxed{\dot{E} > \approx \hbar \dot{I}^{2}}$$

Connection to Quantum Information Theory (*quantitative* results)

see amongst other references:

Quantum Limits to the Flow of Information and Entropy, JB Pendry, J. Phys. A. **16**, 2161-71 (1983).

From very general arguments the flow of information in a channel is limited by,

$$\dot{E} \geq \frac{3\hbar \ln^2 2}{\pi} \dot{I}^2$$

where $\dot{E}$ is the energy flow and $\dot{I}$ the information flow. Identifying the energy flow with heat flow, $\dot{Q}$,

$$\dot{E} = \dot{Q}, \qquad \dot{I} = \frac{\dot{Q}}{k_B T \ln 2}$$

we have,

$$\dot{Q} \leq \frac{\pi k_B^2 T^2}{3\hbar}$$

hence as above,

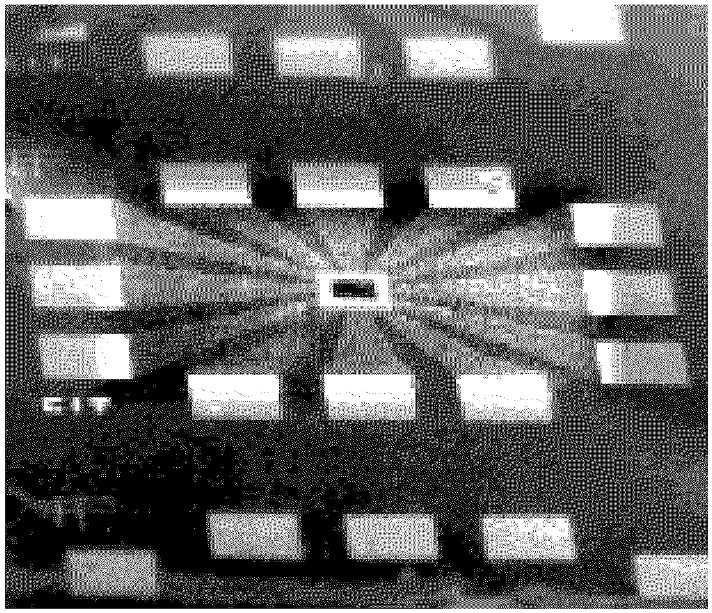
$$\boxed{\left[\dot{Q}_{EV}\right]_{\max} = \sum_{\mathbf{k}} \frac{\pi k_B^2}{3\hbar} T^2}$$

assuming only p-polarised modes are active.

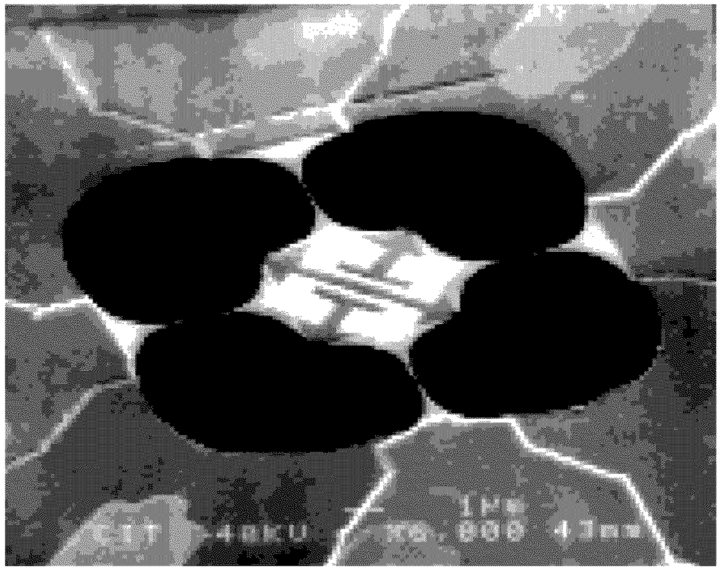
Measurement of the Quantum of Thermal Conductance

K. Schwab, E.A. Henriksen, J. M. Worlock, and M. L. Roukes

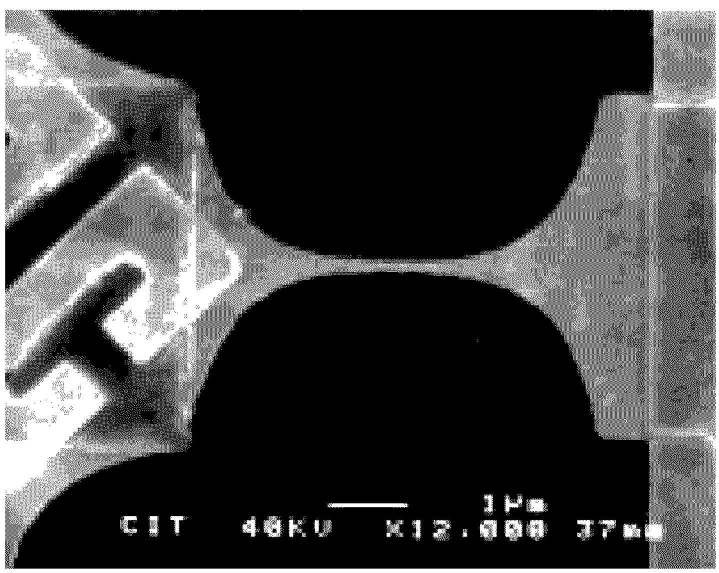
Suspended Mesoscopic Device - a series of progressive magnifications.



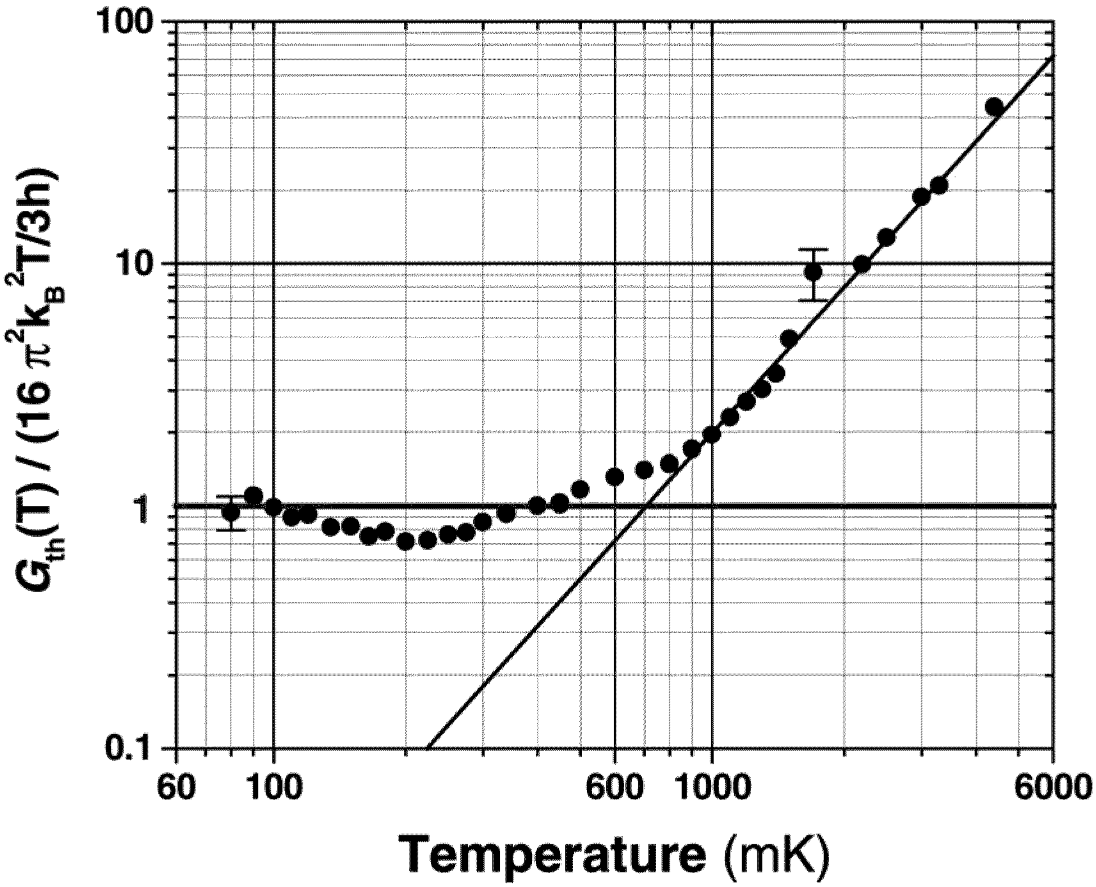
(a) Overall view of the ~1.0 x 0.8mm device, showing twelve wirebond pads that converge via thin film Nb leads into the center of the device. This central region is a 60nm thick silicon nitride membrane, which appears dark in the electron micrograph.



(b) View of the suspended device, which comprises a 4 x 4 mm "phonon cavity" (center) patterned from the membrane. In this view the bright "c" shaped objects on the cavity are thin film Au transducers, whereas in the dark regions the membrane has been completely removed. The transducers are connected to thin film Nb leads that run atop the "phonon waveguides"; these leads ultimately terminate at the wirebond pads.

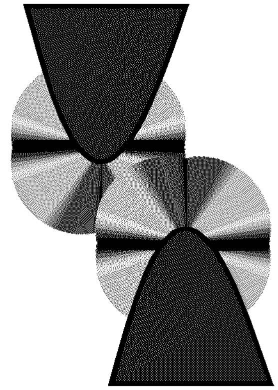


(c) Close up of one of the catenoidal waveguides, displaying the narrowest region which necks down to ~200nm width.



Thermal Conductance Data. We normalize the measured thermal conductance by the expected low temperature value for 16 occupied modes. For temperatures below T_{co} ~ 0.8K, we observe a dramatic saturation in G_{th} at a value near the expected quantum of thermal conductance. Measurement error is approximately the point size, except where indicated.

Thermal Conductivity of Vacuum



How close must two surfaces be to transmit heat as effectively as 1cm thickness of copper (thermal conductivity $400\,\mathrm{Wm^{-1}K^{-1}}$) at room temperature? Assume optimised surface reflectivity so that,

$$\left[\dot{Q}_{EV}\right]_{\max} = \sum_{\mathbf{k}} \frac{\pi k_B^2}{3\hbar} T^2 \approx N_k \frac{\pi k_B^2}{3\hbar} T^2$$

and,

$$N_k \approx \frac{\pi k_{\max}^2}{(2\pi)^2} \approx \frac{1}{4\pi d^2}$$

hence,

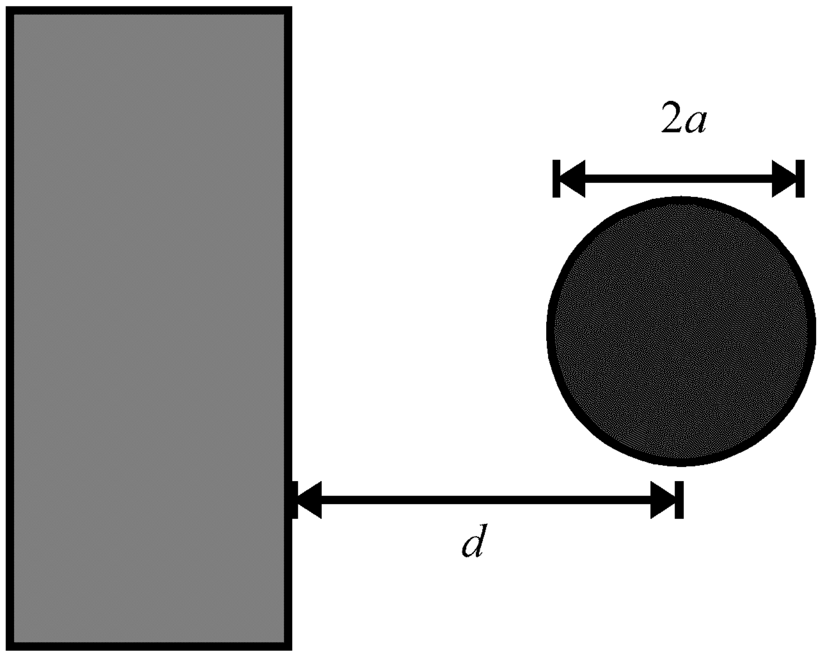
$$d \approx 336\text{Å}$$

Rate of Cooling of a Nanosphere

Rate of cooling of an isolated sphere in vacuo:

$$\dot{Q}_{BB} \approx \frac{24\pi^5 \varepsilon_0 k_B^6}{63 c_0^3 \hbar^5} \frac{a^3 T^6}{\sigma} \text{Watts}$$

where σ is the conductivity of the sphere, assumed large.



If we assume:

$$a << d, \quad d << \lambda_T$$

then the rate of cooling of a sphere outside a surface conductivity σ_s:

$$\dot{Q} \approx \frac{2\pi^3 a^3 k_B^4 T^4}{5 d^3 \hbar^3} \frac{\varepsilon_0^2}{\sigma_s \sigma}$$

Relative Efficiency of Black Body Versus Evanescent Modes

$$\frac{\dot{Q}}{\dot{Q}_{BB}} = \frac{63\varepsilon_0 c_0^3 \hbar^2}{60\pi^2 k_B^2} \frac{1}{T^2 d^3 \sigma_s}$$

substituting,

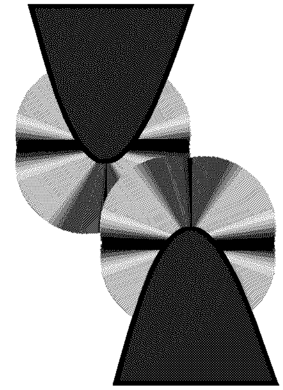
$$T = 300\text{K}$$

$$d = 10^{-8}\,\text{m}$$

$$\sigma_s = 10^5 (\text{m}\Omega)^{-1}$$

we have,

$$\frac{\dot{Q}}{\dot{Q}_{BB}} = 1.6 \times 10^5$$



Energy Density 1nm from a Hot Surface

$$\frac{1}{2}\varepsilon_0\overline{|\mathbf{E}|^2} = \frac{\varepsilon_0}{24\hbar}\frac{k_B^2 T^2}{d^3\sigma}$$

If we choose,

$$T = 300\text{K}, \quad d = 10^{-9}\,\text{m}, \quad \sigma = 10^4(\text{m}\Omega)^{-1}$$

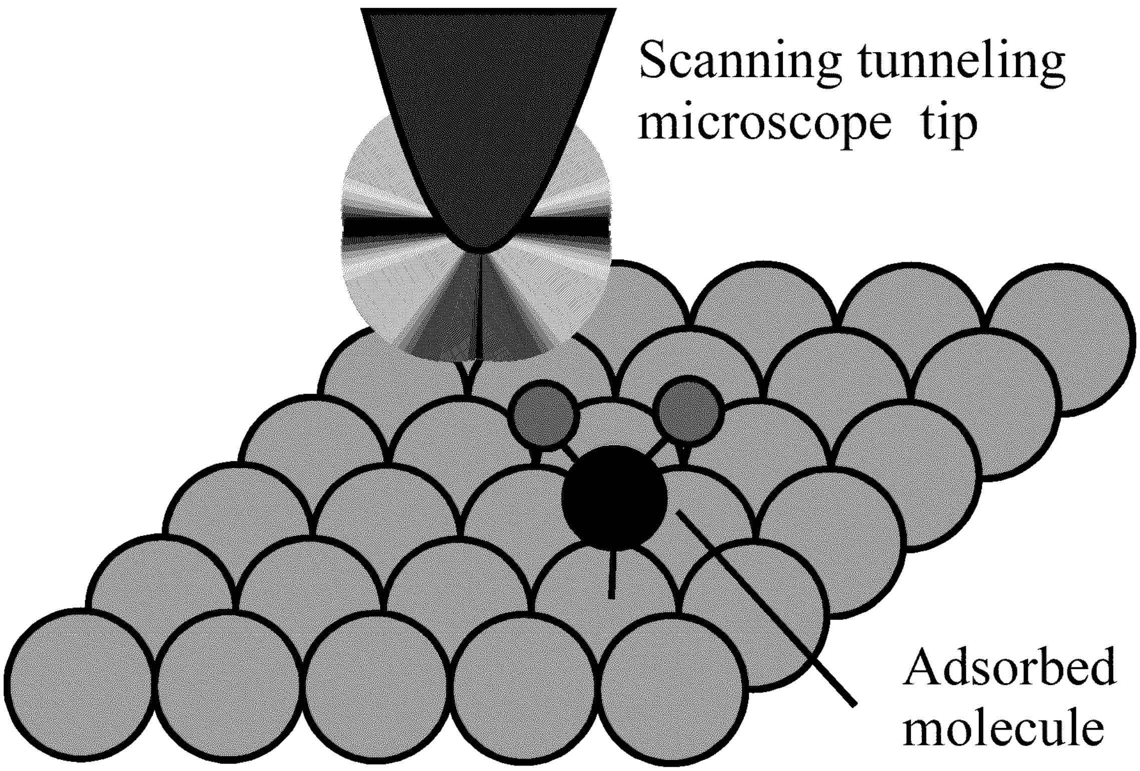
then,

$$\boxed{\frac{1}{2}\varepsilon_0\overline{|\mathbf{E}|^2} \approx 5.99\times 10^3\,\text{Jm}^{-3}}$$

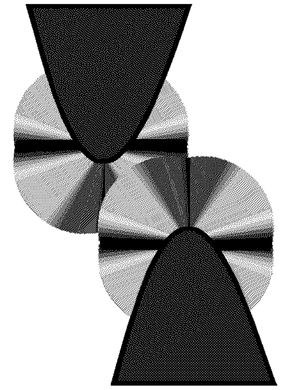
compare the energy density for black body radiation at 300K:

$$\frac{1}{2}\varepsilon_0\overline{|\mathbf{E}_{BB}|^2} = \frac{\pi^2 k_B^4}{60\hbar^3 c_0^3}T^4 = 1.53\times 10^{-6}\,\text{Jm}^{-3}$$

The STM as a Blowtorch
a 'Nanoheater'applying heat to individual molecules



NB it is vital that the tip is manufactured from a material of optimum resistivity.



Conclusions

- Photon tunnelling dominates radiative transfer between nanostructures.
- There is a simple formula for heat flow related to the dielectric properties of the nanostructures.
- Quantities entering the theory also occur in the theory of quantum friction.
- Photon tunnelling is a previously ignored source of heat flow and will have dramatic consequences for heat in nanostructures.

Questions

- What experiments should we do to confirm the effect?
- Can we exploit the effect to design 'nano-coolers' or 'nano-heaters'?
- In what other situations are evanescent states important, possibly at lower temperatures and on different length scales?