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# FINAL REPORT
## *Assessing Student Understanding of Concepts in Electricity to inform Instructional Decisions*

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## 1. INTRODUCTION

We have been investigating students' knowledge and understanding of basic concepts in electricity and their application to solving electrical circuit problems. In previous work, we identified and characterized domain concepts that students had difficulty applying correctly to problem solving tasks mainly in the DC domain. We found that student knowledge was "in pieces," and their lack of understanding could be broadly classified into four different categories: (i) undifferentiated concepts, (ii) experiential impoverishment, i.e., the inability to link physical processes and parameters to abstract circuit models, (iii) incomplete metaphors, and (iv) simplifying assumptions of minimum causality [Biswas, et al., 1997]. Moreover, the "invisible" nature of electricity made it difficult to comprehend, and beginning students came into the domain with very few preconceptions (and, therefore, misconceptions). Most of what a student knew was picked up from instruction. We also discovered that the range of misconceptions and student learning styles were best handled by employing different perspectives and instructional resources. We developed a learning environment that provided resources for self-assessment along with learning, and pilot studies showed that approach to be quite useful in learning difficult concepts.

In this phase of the project, our initial focus was on characterizing the AC circuit domain, and analyzing student understanding and problem solving ability in this domain. Protocol analyses on beginning and more advanced undergraduate students in the Electrical Engineering (EE) program revealed that students have very little physical intuition of AC circuit concepts. Students' problem solving primarily involved the generation of mathematical formulations (equations), and manipulating these formulations to derive numerical solutions to problems. Appeal to everyday physical phenomena did not seem to clarify or improve the students' understanding of these concepts. For example, one of

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[1] The following belong to the ONR Research Group: Gautam Biswas, Daniel Schwartz, Bharat Bhuva, John Bransford, Doug Holton, Amit Verma, and Jay Pfaffman.

the most prevalent misconceptions among beginning EE students is that the sinusoidal waveform represents a spatial property of the voltage and current in a wire as opposed to a time-varying description of behavior that occurs simultaneously at every point in the wire. These students also have no notion of what it means for voltage and current to take on negative values. Some of this may be attributed to the students' lack of understanding of the physical nature of voltage and current. However, a more direct reason may be the natural mapping those students create from the visual representation of the sinusoidal waveform to the spatial dimension of a wire. Unlike the DC domain, our attempts to link AC waveforms to everyday phenomena, such as the operation of radio receivers, and the transmission of signals from different radio stations to a receiver, did not help in clarifying misconceptions. Students also had a lot of difficulty in understanding the behavior of components, such as capacitors and inductors, which exhibit time varying behavior in AC circuits. In such situations, most students could not correctly formulate and explain the equations for analyzing AC circuit behavior. We have developed a test in the AC domain to capture the primary misconceptions that students exhibit in understanding AC circuit behavior. This is discussed in greater detail in Section 3.

Our protocol analyses and misconceptions studies have established that students have very little understanding of AC circuit concepts. As a result, they exhibit a lot of difficulty in formulating and solving problems in this domain. Moreover, a number of the students' misconceptions and difficulties can be linked to instruction, as opposed to preconceived notions of domain concepts. These observations have led us to turn to dynamic assessment approaches (Feurestein, 1979; Campione and Brown, 1985 and 1987; Bransford, et al, 1987) and focus more on how to prepare students to learn through instruction. This methodology, called the *Assessment for Domain Learnability* is described in greater detail in Section 4.

In the last year, we have decided to adopt a systematic methodology for instruction, learning, and assessment in this domain. We adopt a generic framework for describing physical systems in terms of their *structure, behavior,* and *function*. We link this descriptive framework to three broad categories of problems that engineers and technicians encounter in their everyday work: *analysis, diagnosis,* and *design*. These three tasks can also be looked upon as mappings between the structure, behavior, and function of a circuit, and the formulation is used to develop sets of questions for students to assess their understanding of the various concepts in the domain.

To aid students in developing a systematic and well-structured problem-solving paradigm, we have adopted an instructional strategy that emulates expert problem solving behavior. An important component of this process is to get students to reason about phenomena using qualitative techniques, so that their focus is on the application of the laws that govern circuit behavior, and not on mathematical manipulations. We introduce the notion of *invariants* that capture the fundamental laws and concepts that govern electrical circuit behavior. We have developed a web-based software system for self-assessment and learning called Inductor (described in Section 5) that presents students with a set of multiple-choice questions about a variety of AC circuits. The sequence goes from simpler questions to progressively more difficult ones, and starts from purely resistive circuits, and then goes onto RC and RLC circuits linked to different real-world applications. Students are required to pick the relevant invariant relations and analyze them qualitatively to derive the solution to the problem. A detailed description of the system, and preliminary experiments that we have conducted with the system are described in Section 5.

The report ends with a summary of the current status of our work, and proposes directions for future research. A set of appendices provides details of the Inductor system.

## 2. AC DOMAIN DESCRIPTION

Like DC circuits, the fundamentals of the AC domain are represented in terms of voltage, current, and power. In AC circuits, these values are time varying, and described visually as waveforms, most typically *sinusoidal* waveforms. Two parameters of these waveforms, the *frequency* and the *phase,* play an important rȯle in characterizing the behavior of AC circuits. Typically beginning students are able to reproduce voltage and current values in mathematical (the sinusoidal equation) and visual forms (sine waves), but do not really understand the link between the waveforms and the voltage drops and current flows in a given circuit.

The time-varying nature of voltage and current is the basis for the differences in AC and DC circuit analysis. For purely resistive circuits, this difference is not significant because voltage and current remain in phase, and resistance values are not affected by frequency changes. Therefore, voltage and current computations are based on simple algebraic relations. Power computations in AC circuits have an equivalent DC expression when voltages and currents are expressed as root mean square (RMS) values.

Capacitor and inductor elements exhibit significantly different behaviors in AC circuits. Their *impedance* values (the equivalent of resistance) are a function of the frequency of the AC waveform, and this property is exploited in the design of a number of applications. Capacitor and inductor elements also cause a phase difference between voltage and current, and this is used in the design of applications like filters, oscillators, and signal generators.

Our approach to analyzing student understanding of DC and AC concepts is based on the observation that the two domains share a number of fundamental concepts. Our protocol studies on AC understanding were divided into two phases. The first phase focused on these basic concepts. The second phase looked at more advanced AC concepts in the context of applications. The primary applications of AC systems are in power transmission, broadcasting, and communication. AC is still the most effective way for power generation and transmission, but in the present day digital generation, most equipment, such as computers, convert the input AC voltage to a DC voltage before use. Communication systems use AC waveforms superimposed on DC signals for their operation. In keeping with our previous protocol studies (Biswas et al., 1997; Schwartz, Biswas, Bransford, Bhuva, Balac, & Brophy, 2000), where we studied DC concepts in the context of real-world devices, our study of student understanding of advanced AC concepts has been in the context of the applications discussed above.

# 3. MISCONCEPTION STUDIES

We briefly review previous work in analyzing misconceptions in the domain of electricity. Most of this work has been targeted to DC circuits. We extend the analysis of DC misconceptions to the AC circuit domain, and present the results of our protocol studies. To analyze misconceptions in a more systematic way, we have developed a misconceptions test for AC concepts. We briefly describe the test in this report. The complete set of test question can be accessed at http://relax.ltc.vanderbilt.edu/onr/ac-misconceptions.doc.

## Previous Work

The DC misconception literature lists the erroneous conceptions students have about the domain as well as the omissions of knowledge that they demonstrate. In our previous work (Biswas, et al, 1997; Schwartz, et al, 2000) we categorize and report most of the known misconceptions and omissions that students have about the notion of voltage, current, resistance, power and other electrical circuit concepts.

Cohen, Eylon, and Ganiel (1982) found that students think of current as the primary concept (potential difference is regarded as a consequence of current flow, and not as its cause), and that the battery is often regarded as a source of constant current rather than constant voltage. They also observed students' "difficulties in analyzing the effect that a change in one component has on the rest of the circuit" and dealing with a simultaneous change of several variables. These misconceptions cause major problems in students' reasoning about electrical circuits. Other literature in the field concentrated on student understanding using analogical models. For example, Gentner and Gentner (1993) dealt with two different analogical models: (i) the "flowing water model," where the flow of current through wires is analogical to the flow of water through pipes and (ii) the "teeming crowds model," where the analogy was made between current or the flow of charged particles and the movement of crowds through passageways. Magnusson, Temple, and Boyle (1997) discovered eight different students' models of the path of electric current in parallel circuits and adapted six different models of students' conceptions of current from work reported in Osborne (1983), Russell (1980), and Arnold and Millar (1987).

Hunt & Minstrell (1994) have generated a list of pre-scientific knowledge pieces, or facets, that students may have, including misconceptions about concepts in electricity. They developed a program (DIAGNOSER) that targets and assesses these misconceptions with carefully constructed test questions. Upon identifying a specific difficulty a student has, DIAGNOSER also provides some instruction and resources addressing this misconception.

## AC Misconception Studies

We have begun extending the work on student understanding and misconceptions in the DC circuit domain to the AC and DC domains. We generated a series of circuit questions relevant to the AC domain and interviewed students as they worked through these problems. As a result of these structured interviews, we identified specific areas in which students had misconceptions or lacked experience (listed later in this section). More recently we also constructed part of a misconceptions multiple choice test that targets these misconceptions, in cooperation with Steve Parchman and other researchers (also described later in this section).

### Experimental Setup

For the protocol analysis studies, we made up a number of AC problems, starting from the simple DC flashlight, but replacing the DC source with an AC source. The first set of problems were set up for students to analyze contrasting cases, such as what happens in the flashlight circuit when the DC source is replaced by an AC source, and where would you place fuses to protect a component in identical DC and AC circuits. In this study, we were specifically looking for misconceptions that students had exhibited in an earlier study on DC circuits, such as (i) the empty pipe and sequential flow misconceptions, (ii) the inability to recognize the differences between voltage and current, and (iii) the belief that current remained constant in a circuit, and what impact these misconceptions may have on their understanding of AC circuits. In addition, there were questions that asked students to analyze the effect of changing source frequency on power consumed in a circuit. In some cases, the students were asked to plot the voltage and current waveforms at different points in a circuit. The students involved in this study were beginning Electrical Engineering (EE) students at Vanderbilt University who had completed their first circuits course. We also interviewed students in the Navy training center at Memphis.

We also developed a second, more advanced AC problem set, where students were asked to explain how a particular device worked, and especially why it exhibited certain behaviors and functionality. The second set of problems tested student understanding of capacitors and inductors in AC circuits, and the use of RC and RLC circuits in practical applications. This set of problems was presented to senior undergraduate students and some graduate students. We also interviewed an electrical technician. The focus was on whether students could analyze the circuits and produce a qualitative explanation of the observed system functionality. Our last report [Biswas, et al., 1999] describes the problem sets in greater detail.

### AC Misconceptions

The analysis of student responses provided interesting results. We interviewed a total of 18 subjects at Vanderbilt University, and about 6 trainees in their first EE technician course at the Memphis naval center. All 12 Vanderbilt students in the first group were in the beginning electrical engineering course (EECE 112), and the 6 students in the second group were juniors, seniors, and graduate students. In our protocol analysis we found a variety of erroneous knowledge about basic AC concepts. They are summarized below. We divided the misconceptions into three categories:

1. Those directly related to characteristics of AC waveforms,
2. General classes of difficulties that are linked to cognitive difficulties, and
3. Lack of knowledge of general domain principles.

These are discussed in greater detail below.

*List of Misconceptions specific to AC waveforms.*

1. **Spatial AC misconception.** The sinusoidal AC voltage and current waveforms are not a representation of variation of these variables at a point in time. Rather they depict a variation of their magnitudes along the length of the wire in which the current is flowing. For example, students said that a string of identical light bulbs in series when connected to an AC source would light up in sequence, and some of the light bulbs may be on when others are off. At the same instant of time, the brightness of the bulbs would vary depending on their position in the circuit.

2. **Negative part of AC cycle is just a mathematical artifact.** No current flowing in circuit or power delivered during negative part of AC cycle. For example, a number of students said that a light bulb only lights up during the positive part of the sinusoidal cycle. Others said that there could be *"no such thing as negative current. That is just a mathematical artifact. If current reverses, the electrons would reverse direction too. They would then run into each other, stopping flow, which implies there could be no current."*

3. **Alternate form of this misconception.** The negative current "cancels" out the positive current. So bulb will never light up when you connect to true AC source.

4. **Empty pipe misconception.** During AC cycle electrons stop, turn around, and go the other way. In some cases when you have very long wires, they may never reach the light bulb connected to the end of the wire. Students thought that you would need two fuses to provide protection in an AC circuit, where you could do with one in a DC circuit.

5. **Incorrectly importing DC models to explain AC.**
   A. Students often surmised that the alternating current going through a resistor was constant in time.
   B. Students often hypothesized that a capacitor behaved the same in AC and DC circuits.

6. **Difficulties understanding circuit behavior when AC and DC signals are combined.** Students had difficulty "separating" or recognizing the AC and DC components of a signal in problems in which the midpoint of a sinusoidal voltage was not zero.

7. **More generally, difficulty thinking of circuit behavior when multiple waveforms, frequencies are combined.** Even advanced students stated that the number of channels you can got from cable TV was a function of the number of wires in the cable, or the thickness of the cable.

*General classes of difficulties that are not specific to AC.* [Schwartz, et al. 2000]

8. **Failure to differentiate among concepts**. Examples, voltage and current, series and parallel configurations, role of capacitor in DC versus AC circuits.

9. **Minimum causality error.** (Incorrect simplifying assumptions). Single change in outcome must be a result of single change in cause. (e.g., a 10W bulb must have greater resistance than a 5W bulb).

10. **Overly local reasoning.** Not thinking of global constraints, invariants.

11. **Bad framing.** Incorrect generalizations, trouble switching from equations to physical explanations to analogical models.

12. **Experiential impoverishment.** Electricity is invisible except for its end products.

*Lack of basic circuit knowledge.*

13. **Lack of Ohm's law** (how resistance affects current when voltage is constant)

14. **Lack of KCL** (current through all components of a loop must be equal).

15. **Lack of KVL** (the voltage drop across components of a loop must sum to zero).

*Note that 14 and 15 together represent the conservation laws: (i) charge cannot disappear, and (ii) energy must be conserved.*

16. **Lack of knowledge of the behavior of capacitors** (such as C=Q/V)

17. **Lack of knowledge of Capacitor and Inductor impedance as a function of frequency.**

18. **Topographic misunderstanding of the circuit (e.g. unable to differentiate series from parallel).**

### Misconceptions Test

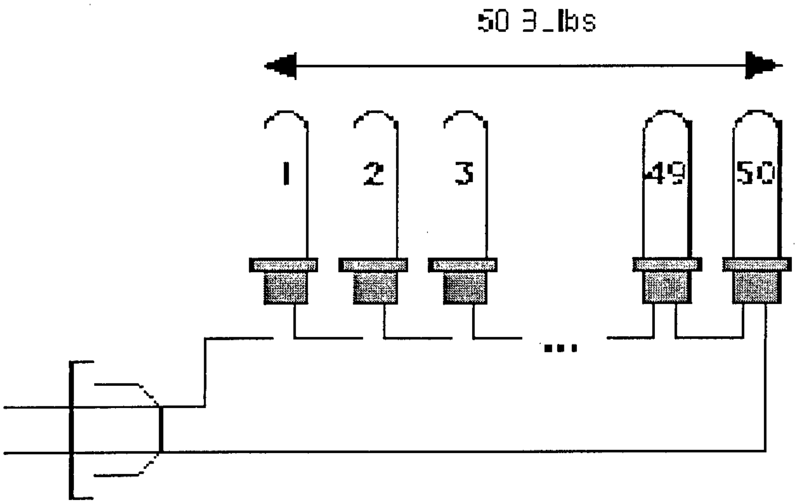
Using the above list of AC misconceptions, we developed a set a number of multiple choice test questions to target these misconceptions in cooperation with Steve Parchman's group in Florida, and other researchers. An example question is shown below in Figure 1.

This question focuses on the spatial misconception that students have regarding electricity (water-pipe model, and the spatial variation of AC signals). The question also addresses the notion of electricity as a substance, i.e., electricity gets consumed as it goes along the string of lights.

The misconceptions test has been conducted with naval students, and Steven Parchman's group is currently analyzing the results. The full test can retrieved from http://relax.ltc.vanderbilt.edu/onr/ac-misconceptions.doc.

## Discussion

Our preliminary study of student understanding in the AC domain has proven to be quite revealing. Beginning students seem to have very little understanding of the time-varying nature of AC voltage and current. This can be attributed to a combination of problems they exhibit in their basic understanding of concepts. The empty pipe misconception affects their understanding of current flow, and makes it especially difficult for them to reason about current that reverses direction periodically. The inability to differentiate between voltage and current and the lack of understanding in mapping from physical concepts to abstract circuit parameters compounds students' problems. They are often stuck with beliefs such as a source provides constant current, and a source cannot deliver power unless the current flows in one direction from one of its terminals to another. These misconceptions and lack of knowledge are not unique to the AC domain; in fact students exhibited the same problems when reasoning in the DC domain.



A Christmas light strand contains 50 identical light bulbs connected in series to form a light string. When it is plugged into a 110 volt AC power socket of frequency 60Hz, which light will burn the brightest?

a) The first bulb is always the brightest.
b) The 50th bulb always burns the brightest.
c) Since the current is alternating, each of the bulbs starting from the first to the 50th is the brightest in turn.
d) All of the bulbs are equally bright at all times.

Correct answer:

d) All of the bulbs are equally bright at all times.

**Figure 1: A Misconceptions Test question and accompanying figure**

From the point of view of instruction, these observations can be interpreted in many ways. On the one hand, one can make the argument that since DC instruction traditionally precedes AC instruction, it is very important to ensure that students do not develop misconceptions and omissions described above during DC instruction. Careful contrasts also need to be made when making the transition from the DC to the AC domain. On the other hand, one could say that the similarity of the basic concepts in the two domains imply that the most effective form of teaching should focus on the concepts and their implications in problem solving rather than spend a lot of effort in focusing on the differences. For resistive circuits, the time-varying nature of AC voltage and current has no strong implications on behavior. Students need to understand the concept of power delivered, and how to compute the power delivered. As discussed earlier, the time-varying nature of current and voltage has important implications in circuits with capacitors and inductors, and it may be best to introduce these concepts by demonstrating their use in real applications and devices. The latter approach may be further justified by the observation that a number of the misconceptions of the beginning students seemed to go away as they moved on to more advanced courses.

Another issue of importance that we have observed among students is their reliance on mathematical formulations and solving of equations to derive answers to problems. As discussed earlier, this implies the students lack understanding of the underlying physical phenomena, and therefore, do not develop a deep understanding of the basic concepts in the domain. This problem is even further compounded in the AC domain, especially when students have to deal with the more complex phenomena associated with real world devices and systems. When dealing with the questions in problem set 2, a number of students attempted to convert the given circuit or problem description into mathematical equations. However, the resultant differential equations were hard to analyze, and did not directly provide the information required to solve the problem. The implication here is that students need to develop a better qualitative understanding of phenomena, and how these phenomena combine to produce circuit and system functionality. In our protocol studies on the second problem set, a number of students had to be coached to reason about a problem qualitatively. Only then were they able to analyze the problem, and generate the desired solutions and explanations. Developing qualitative reasoning skills and function-level understanding may also contribute to the development of better troubleshooting skills, a long-term goal of this research.

In the next section of the report, we develop a methodology for instruction that combines learning with assessment. The goal is to exploit computer technology to provide students with an environment for selecting from a set of available resources depending on their self-identified needs.

# 4. FROM PROTOCOL ANALYSIS TO INSTRUCTION:

## The Assessment of Domain Learnability Framework

Our studies of student understanding in AC and DC circuit problem solving suggested that student misconceptions and difficulties could be linked to instruction as opposed to the preconceived notions of domain concepts. These observations led us to turn to dynamic assessment approaches (Feurestein, 1979; Campione and Brown, 1985 and 1987; Bransford, et al, 1987) and focus more on how to prepare students to learn through instruction. Our first step in this direction was to build a computer-based tools using the STAR.Legacy framework to help students self-assess their understanding of concepts linked to DC circuit problem solving, and to provide resources to help students learn these concepts they found difficult to learn.

## Assessing Domain Learnability

It appears that some electricity concepts may be more difficult to learn than others. With respect to the instruction in this domain, we believe that an important research task is to identify features and concepts that influence learnability of concepts that affect problem solving tasks. We will call this task "assessing domain learnability" or ADL for short. By trying to remediate people's misconceptions and missing conceptions, we may determine which are particularly difficult to remediate given our methods of instruction (e.g., Heller & Finley, 1992), and which type of understanding has the greatest impact on subsequent learning. The basic observation is that not all misconceptions are equally strong or equally relevant to future instruction. For example, although we have rarely seen it in the literature (Cooke & Breedin, 1994), it would be interesting to ask people to compare their confidence in answers where they exhibit misconceptions relative to those that they do not. We suspect that for many of the misconceptions that have been documented, people are reasonably aware that they do not know what they are talking about. For those misconceptions that are of low confidence, should we expect that people would be more likely to overcome their misconceptions and learn? Much of the research on misconceptions has no handle on this question. An ADL approach seems more likely to provide an answer.

There may be limitations to ADL as we have conceptualized it so far. One possible weakness of ADL is that it is particularly prone to the ways that we assess whether someone has learned a correct conception or not. For example, if we ask the exact same question that we taught, does this mean that people have learned in any meaningful sense? The problem of assessing and deciding upon ecologically satisfactory understanding, however, is a problem faced by much educational research. ADL actually fairs better than most in this regard. This is because the ultimate test for ADL is whether a given concept has implications for future learning. For example, consider the typical course sequence in electrical engineering where students begin with direct current (DC) circuits and then move to study alternating current (AC) circuits. Students start with many misconceptions about DC circuits. Are all the misconceptions and their correct counterparts equally important in shaping students' ability to learn AC circuits? This is the question that ADL is designed to answer.

A second potential weakness to ADL is that if our instruction fails to teach a correct conception of a domain, it is hard to determine whether this was a function of the domain's difficulty or a function of our teaching methods. On the one hand, we can never disentangle these two possibilities beyond a reasonable appraisal. On the other hand, it is the interactions of the instruction and the domain that constitute the important parameters of assessing domain learnability. The emphasis of ADL is not on domain learnability in the abstract, but rather domain learnability with respect to the state of the art in instruction. The next section describes a computer environment that captures many of our ideas about the state of the art.

## A Computer-Based Learning Environment for DC Problem Solving

A computer-based environment provides an integrated learning-assessment tool for pulling together different instructional techniques and resources that can be applied to a domain. A single instructional technique would be too restrictive for ADL. For example, one might use a dynamic tutoring system to teach the procedural knowledge of a domain, but there are other types of knowledge that are important to assess as well, like, do people have difficulty constructing a mental model of the domain (Lajoie & Lesgold, 1992). Similarly, one might create a system that matches an individual's misconceptions against a known "bug list" and teaches to those bugs directly, but this typically assumes that misconceptions are non-interacting.

Our software environment for assessing the learnability of DC concepts was created using the STAR.Legacy framework (Schwartz, et al. 2000). In line with the test-teach-retest model of dynamic assessment, students begin with a question in the *look ahead* problem and end with the same question when they *reflect back*. In this case, the *look ahead* and *reflect back* problem asks students to explain what happens in a simple flashlight circuit when a 5-watt bulb is replaced by a 10-watt bulb. The overall assessment and learning task is divided into three *challenges*, which were chosen on the basis of our protocol research described earlier (Biswas, et al, 1997). We found three problem situations that were particularly good at making students' thinking visible. *Challenge 1* asked students to reason about the possible causes of a dim bulb. This problem was intended to help students differentiate voltage and current, to help them overcome the minimum causality error, and to give them some increased experience in the domain and its analogies. *Challenge 2* asked students to design a battery operated drill that could run at different speeds. In this design problem, students progressively deepen their understanding of the topics raised by *challenge 1* while adding the issues of local reasoning and framing. Finally, *challenge 3* tried to bring the lessons together into a single problem. In this challenge, students were asked to reason about a flashlight that has two bulbs, one that points forward and one that points to the ground. They are told that somebody wants to change the forward bulb to a higher wattage. How will that effect the flashlight overall? These challenges are intended to bring forward the different classes of misconceptions that students may possess. At the same time, we expect the interaction of the challenges and instruction to reveal other conceptual hot spots. This is one of the attractive features of ADL -- it can reveal misconceptions in the context of instruction.

After reading a challenge, students try to generate their first thoughts about how to prepare for solving the challenge. These initial thoughts usually provide both instructors and the student with a sense of the strengths and weaknesses of the student, and it helps the student choose which of the multiple perspectives to listen to. Each perspective directly targets key learning difficulties with a 10-15 second comment by an expert. For example, one of the perspectives has an expert explain the minimum causality error, although not in those terms. The expert states, *"a common mistake that people make with these problems is that they often do not realize that when the power changes, two other things in the circuit must change."* Another perspective tries to tie the perceptual phenomena (a dim bulb) to relevant electrical concepts by pointing out that a dim bulb means less power is being consumed. Another perspective, under the assumption that the students have been taught some form of water analogy, tries to get students to think how voltage and current map into the water domain.

When students listen to the perspectives, instructors may ask the student to explain whether they understand what the experts are saying. This provides them with valuable knowledge about which aspects of the domain the student may be having trouble with. For example, some students do not know that *"two things must change,"* whereas others may not know how to draw the analogy between water and electricity. This becomes important when the interview proceeds to Research & Revise. The student and interviewer choose which resources to work with depending on the gaps in knowledge.

Figure 2 shows the resources that are available for challenge 1. A chalk talk on Ohm's law explains why two things must change if the power changes. There is also a set of multiple-choice problems that allow students to practice using Ohm's law. These problems include automated feedback that states the qualitative implications of the student's incorrect answers. For example, one feedback comment reads, "This answer implies that as you increase the voltage across the circuit, current will decrease! For example, if we used a more powerful battery, the current in the flashlight circuit would decrease. Does that make sense?" This form of feedback helps the students to think about qualitative relationships as opposed to simply making algebraic manipulations of numbers.

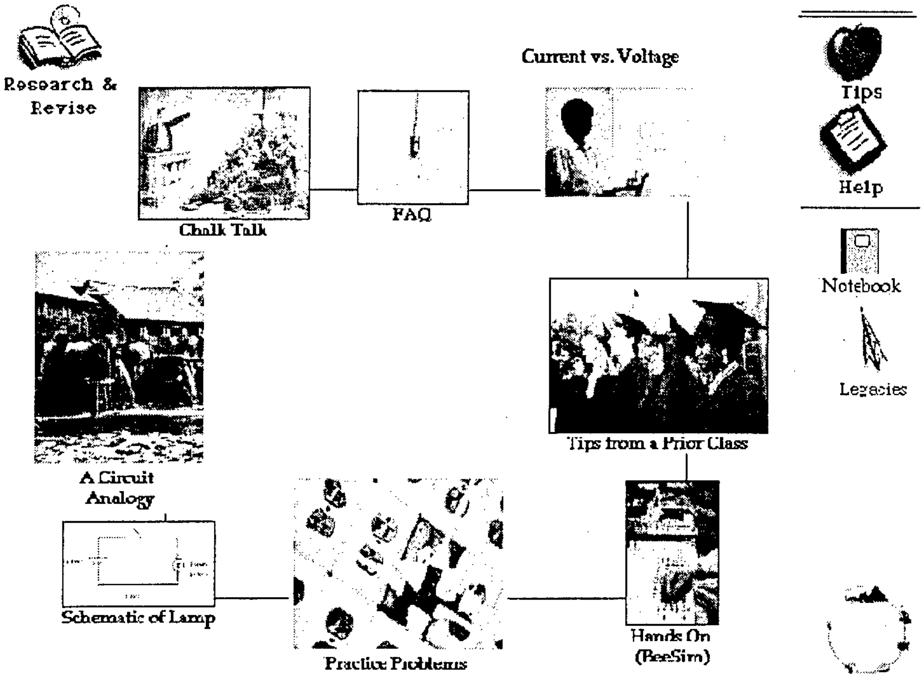


**Figure 2: Resources for challenge 1: by clicking on an image, a learner can gain access to its resources**

Another resource is a brief presentation of a mnemonic that helps students memorize that current is a "through" property whereas voltage is an "across" property. There are also pairings of simulations of a circuit and an analogous water system. The resource page also includes connections to web sites that we have found helpful, comments by students who have completed the process and offer their thoughts about key insights that helped their learning, and pointers to simulations and hands-on activities developed by others (e.g., Parchman, 1997).

Depending on how comfortable and confident students feel about the material, they can move between resources and perspectives to probe further and learn more about the relevant concepts. Once the students feel that they have made satisfactory learning progress, they move to *test your mettle* to test the strength of their knowledge. After students complete the learning cycle for challenge 1, they move to subsequent challenges, which require students to rethink concepts that they have already learnt, and also to deal with new concepts and misconceptions. Subsequent challenges are structured like the first challenge.

This dynamic assessment environment is different from other dynamic assessment models that are automated (Lajoie & Lesgold, 1992) because it keeps the instructor in the loop. In part this is because it makes it much easier for others to replicate our efforts as compared to the overhead of creating automated or self-contained systems (Bell, 1998; Murray, 1998). But in part, we have left the instructor in the loop because ADL requires a level of flexibility we cannot reasonably program into a machine. Our instructors try everything at their disposal to help students learn. They try to adapt to student needs and to the peculiar demands of the domain. DC-Legacy helps in this endeavor because it provides a flexible but pedagogically sound structure, multiple methods of instruction, and a single gathering of "at the ready" resources. There are two questions that come to mind now. One question is what aspects of the domain were generally difficult or impossible to remediate. A second question is whether certain conceptualizations facilitate the students' subsequent learning.

## Discussion

In this section, we described a theory that can help evaluate misconceptions in the context of instruction. To this end we proposed a dynamic assessment approach to assessing domain learnability. In this approach, researchers try their best to teach students. Those concepts that students still have difficulty with tell us something about the components of the domain that are particularly difficult to learn, at least with respect to the instruction that we can provide. The results help focus attention on those concepts that are particularly problematic, rather than simply making a list of possible misconceptions in a domain. Ideally, the important concepts are also identified with respect to their impact on future learning in the domain.

We described several classes of learning difficulties (misconceptions) that others and we have found for the domain of electricity. We tried to organize these misconceptions according to the way they fit into basic cognitive processes -- differentiation, simplifying assumptions, local reasoning, and the need for framing. Our underlying assumption is that domain learnability is best understood as the interaction of individuals' cognitive tendencies, the demands of the domain, and instruction. We constructed a DC-Legacy that targeted these different classes of learning difficulties to fulfill the instructional component of our assessment. Using DC-Legacy, several members of our group were able to add their own expertise to make a rich dynamic assessment environment for learning DC circuits.

We conducted a small study to see if there is merit to the approach. The preliminary results are promising. We found that some commonly cited misconceptions, like the difficulty of handling parallel resistors, are not problematic by the time students leave a typical college course in electricity. We found that other misconceptions, like local reasoning about the movement of current from point to point, are not treated by our courses. However, they are easily remediated and do not have to serve as blocks to learning the domain. And, we found that some aspects of the domain are difficult to learn even with special attention. In particular, it appeared that people have trouble integrating multiple causes and this is exacerbated by faulty intuitions that cause them to focus on singular causes. We suspect that most instruction does not sufficiently help students construct mental models that incorporate both the empirical reasoning of causal intuition and the helpful structure of mathematics (Schwartz & Moore, 1998). In electricity it seems particularly important to help students make sense of the mathematical formulas (qualitatively or quantitatively) so they may overcome the tendency towards minimum causality. In our protocols with college professors and field experts alike, we have found that when they come to an obstacle in their reasoning, they resort to equations to solve difficult conceptual problems. And, in our discussions of AC circuits, electrical engineering experts rely so heavily on mathematics that they often cannot even generate physical analogies. They are reasoning about representations, primarily mathematical; the empirical phenomena are far in the background.

The current theorizing, the computer environment that implements our theories, and the empirical results present our beginning efforts at creating dynamic assessment tools that can inform instruction in complex domains. As such, none of the three are ideal and more work is left to be done.

## 5. INDUCTOR: DESCRIPTION AND PILOT STUDIES

Inductor was designed as an online assessment tool by Jay Pfaffman to provide "a means to author, administer, grade, and learn from multiple-choice tests" (Pfaffman, Schwartz, & Martin, 2001). These tests are often criticized for being shallow and for promoting memorization, but they can be transformed into assessments for learning (Bransford, Brown, & Cocking, 1999). They may be redesigned to specifically target misconceptions and important knowledge principles, and when they are presented in a computer environment, they can be designed to provide immediate and elaborate feedback to students (Hunt & Minstrell, 1994). By using an online system, we may also provide access to outside resources for learning, such as instructional web sites and simulations or animations (see Appendix B for an annotated listing of some of the resources used in our tests with Inductor). This creates a learning environment that goes beyond the typical sequestered problem solving context for taking tests in which there is no access to tools or resources (Bransford & Schwartz, 1999).

Inductor is implemented as a web-based tool, through the use of PHP scripting to create a web browser interface that is connected to a MYSQL database backend. This allows instructors with a web browser to remotely create and author test questions, and students to work on assignments outside of the classroom environment. Test authors may include graphics, audio, and video with each question, and provide general and specific feedback for each multiple-choice answer (both the right and wrong ones) to a question.

Appendix A visually demonstrates how a student uses the online Inductor tool (see also: http://relax.ltc.vanderbilt.edu/onr/demo.php ). The student selects a test to take, and is presented with detailed instructions. Once they are done with the instructions and the preliminary screens, the student sees a grid on Inductor depicting all of the questions in the test, organized by topic or some other structure that the designer has chosen. For example, in Inductor we organized the test into four categories of questions: DC resistive circuits, AC resistive circuits, DC circuits with capacitors, and AC RC (filter) circuits. These were laid out row-wise. In addition, we identified three classes of problems: diagnosis, design, and analysis problems. An icon associated with a question indicated the type of question to the student. Inductor is set up to provide continual feedback to students on their performance via the "control your own destiny" (CYOD) grid interface. Students access a question by clicking on the question's icon. They may work on one question at a time, but after answering a question may revisit it any number of times to review its contents. After answering a question, the student returns to the grid. At all times, color-coding on the question icons (green for correct answers, red for wrong answers) tells the student how well he or she has performed thus far. At this point, students may review these previously answered questions, review resources, or move on to a new question.

All questions are described in text with an accompanying schematic or circuit diagram, and a set of multiple choice answers. The student is asked to pick the right answer to the question, and provide additional information, such as explanations in additional boxes that we provide on the screen. Every screen also has pointers to generic resources, as well as specific ones that may contain material relevant to that particular question. In this study, we required students to develop a problem solving methodology using *invariants*. Invariants come in two forms. First, they may pertain to laws of the domain that directly apply to the problem solution being sought. Second, they are also linked to the set of variables in the particular problem-solving situation that do not change. Foe example, if one is dealing with a flashlight problem, where the bulb is replace by a higher wattage bulb, once can reason that the source voltage (battery) does not change, but the resistance of the bulb does. Therefore, the source voltage is an invariant. Depending on the question asked, one can then invoke an invariant law, such as Ohm's Law, the Power laws, or Kirchoff's Voltage Law to answer the question. The list of invariant laws we have used in the study can be accessed at http://relax.ltc.vanderbilt.edu/onr/invariants/invariants.html.

To promote and teach expert-like reasoning, we made students select the invariants they felt were relevant to solving the given circuit problem. Another unique characteristic of our questions was that there were no numbers associated with the circuit parameters ad variables. Students were required to derive their answers using qualitative reasoning techniques (i.e., current will be higher, the resistor must be shorted, etc.). This was to get them to reason from first principles as opposed to putting in a lot of effort into numeric calculations. After answering a question, selecting the relevant invariants, and explaining their answer, students received feedback about their answer. If incorrect, students were shown what invariants an expert chose as relevant to the problem. In addition, students answering incorrectly received a hint and links to relevant outside resources to explore. Finally, regardless of the correctness of the answer to a question, students received an expert explanation for the circuit problem, emphasizing the invariant principles involved.

In a study that used related techniques, Leonard, Dufresne, and Mestre (1996) had physics students describe the principles involved in physics problems and write a justification for their answer. The professors also described problem solving strategies during their lectures, much like the invariant-based explanations and techniques for problem solving that we present through Inductor. They found that the students who were taught problem solving strategies generated more correct answers to problems, were less-dependent on surface features of problems alone for selecting important principles involved in solving problems, and were better in recalling the major principles covered in the course months later.

There are two major differences, however, between the Leonard, et al. study and the pilot studies we have run to date. One is that their study was conducted as a semester long course, and students were graded on their answers, their use of strategies, and their identification of principles they applied to solving problems. We have not yet integrated Inductor into an electrical engineering or physics courses, and thus student participation in these studies has been voluntary, and not directly connected to their course work. Also, their performance has had no direct impact on their class grades, and, as a result, a number of students have not put any significant effort in using or learning from our system. This has affected their motivation for participation and learning in the tasks. Secondly, the instructors in the Leonard, et al. study carefully reviewed all the students' writing and provided feedback to the students on their strategy use. This provided invaluable feedback and learning opportunities, but also undoubtedly represents a significant investment of time and effort on the part of the instructors. Inductor makes a trade-off by providing automatic feedback in the form of hints and expert explanations and learning resources to students. Our focus is more on self-assessment, and we hope students will be more motivated to learn by conventional instruction or through on line systems, once they realize what their particular deficiencies in understanding the material are.

## First Pilot Test of Inductor (November, 2000)

We ran a first pilot study of the Inductor tool using our DC and AC test questions in November 2000. We created two 16-item tests that involved simple DC, simple AC, DC capacitor, and AC RC circuits. We wrote expert explanations for each of the circuit problems that emphasized the invariant properties of the circuit. These were presented to each student after answering each question. We found and organized outside resources, such as web sites and Java simulations that we felt were valuable for learning the specific invariant principles our test questions targeted (see Appendix B).

In order to explore learning effects with the Inductor tool, we counter-balanced the order in which students took both of our tests, to provide some measure of control for any differences in difficulty between the two tests. Our goal was to find out how the students fared, with their answers, selection of invariants, and explanations as they progressed through Inductor. After completing both tests, we asked students to fill out a follow-up survey to get their feedback on the tool and on the use of invariants.

## Methods

Participants. Forty undergraduate students in a first year electrical engineering course volunteered to take part in the study.

Procedure and Design. Students used Inductor with the procedure described in Appendix A. Two 16-item test assignments (tests A & B) were given. Both tests had questions involving simple DC circuits, AC circuits, DC capacitor circuits, and AC RC circuits. The questions on tests A and B were different but similar in content.

The design for this initial study was for half of the students to take test A first and then test B, and the other half to take test B and then A, counterbalancing any differences in difficulty between the items on both tests. If students learnt while answering the questions and receiving feedback and resources, then performance should improve over time.

We recorded the answer each student selected for each multiple-choice question, the invariant principles they selected as having relevance to the problem, and any initial explanations for their answer, as well as revisions to their explanations after having received feedback. After completing both tests, students filled out the follow-up survey.

## Results of First Pilot Study

Sixteen of the forty students completed all questions on both of the tests. We attributed the lower participation rate to the fact that the tests were not required or connected to their current electrical engineering course, the length of the tests, and the approaching holidays and finals in late November and early December. Five of the students who finished both tests were in the group that took test A first and then test B second ("Class A"), and eleven students took test B first and test A second ("Class B").

Test Scores. An analysis of the total scores indicated that Test B was significantly easier than Test A (F(1,14)=6.95, p=.02). Test B averaged 66.8% correct, and Test A 60.1%. This difference was true for all categories of questions of the tests (DC, AC, DCC, ACRC) except for AC. Class A averaged 55% correct answers and Class B averaged 67% correct, however this was not a significant difference (F(1,14)=2.09, p=.171), due to the low number of students in Class A (5) and the high variance between individual scores. The interaction between class and test, graphed below in Fig. 3, is also non-significant (F(1,14)=2.83, p=.115). Four of five students in Class A improved from their first test to the second, but only three of eleven in Class B did. Class B participants first took the easier test (B) and then the harder one. We do not have data such as pre-test scores or student GPAs to control for individual and test differences to better reveal learning effects in the student scores using Inductor.

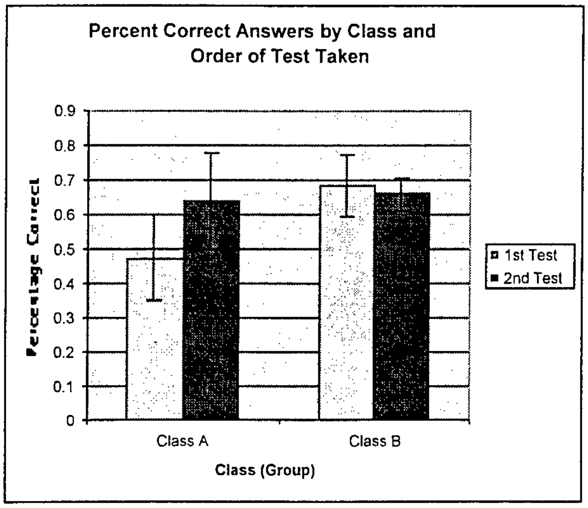


**Figure 3: Interactions between class and test**

Surprisingly, we found students answered more correctly for AC circuit questions than DC (F(1,14)=6.95, p=.023). Students also had difficulties with capacitor questions (F(1,14)=22.85, p<.0001). The three graphs in Fig. 4 reveal test scores by category of question and class.

<u>Background for Invariant Selection Analysis</u>. For each circuit problem, we used three experts to select the invariants that were most relevant and helpful for solving the problem. After initial selection they met to come to a consensus about the relevant invariants for each of the questions. They also identified the invariants that were clearly irrelevant. The impedance of an inductor, for example, is irrelevant for a RC circuit. The remaining invariants were placed in a third category. A link could be drawn between these invariants and the circuit behavior, but they were not particularly useful for solving the problem asked about the circuit. Ohm's law, for example, applies to all circuits with resistive components, but it is not always necessary for a particular question about a circuit.

We needed a way to quantify how well students selected invariants for a particular problem. In our case, a simple percent correct measure is insufficient for characterizing students' use of invariants, because it rewards students who select more invariants regardless of their importance to the problem. To control for such response biases, we utilized a nonparametric discrimination measure known as *Yule's Q*. Nelson (1984) contrasted simple percentage correct measures, *d'* measures and Yule's Q, and advocated Yule's Q over *d'* on the basis that it was thought to make weaker assumptions about the data and required fewer observations. For our purposes this measure rewards the selection of invariants that our experts agreed were appropriate for the problem, while controlling for the selection of clearly irrelevant invariants.

The Yule's Q measure was constructed first by calculating the percentage of invariants a student chose out of those invariants experts chose as relevant (h, or hit rate) as well as the percentage of invariants a student chose out of those invariants experts deemed clearly irrelevant (f, or false alarm rate). Invariants that are technically correct but less relevant to a problem were ignored in this computation. The Yule's Q score was then calculated by the formula: $\frac{h-f}{h-2fh+f}$. A Q of one implies perfect discrimination of the relevant from irrelevant invariants, and zero implies pure chance performance.

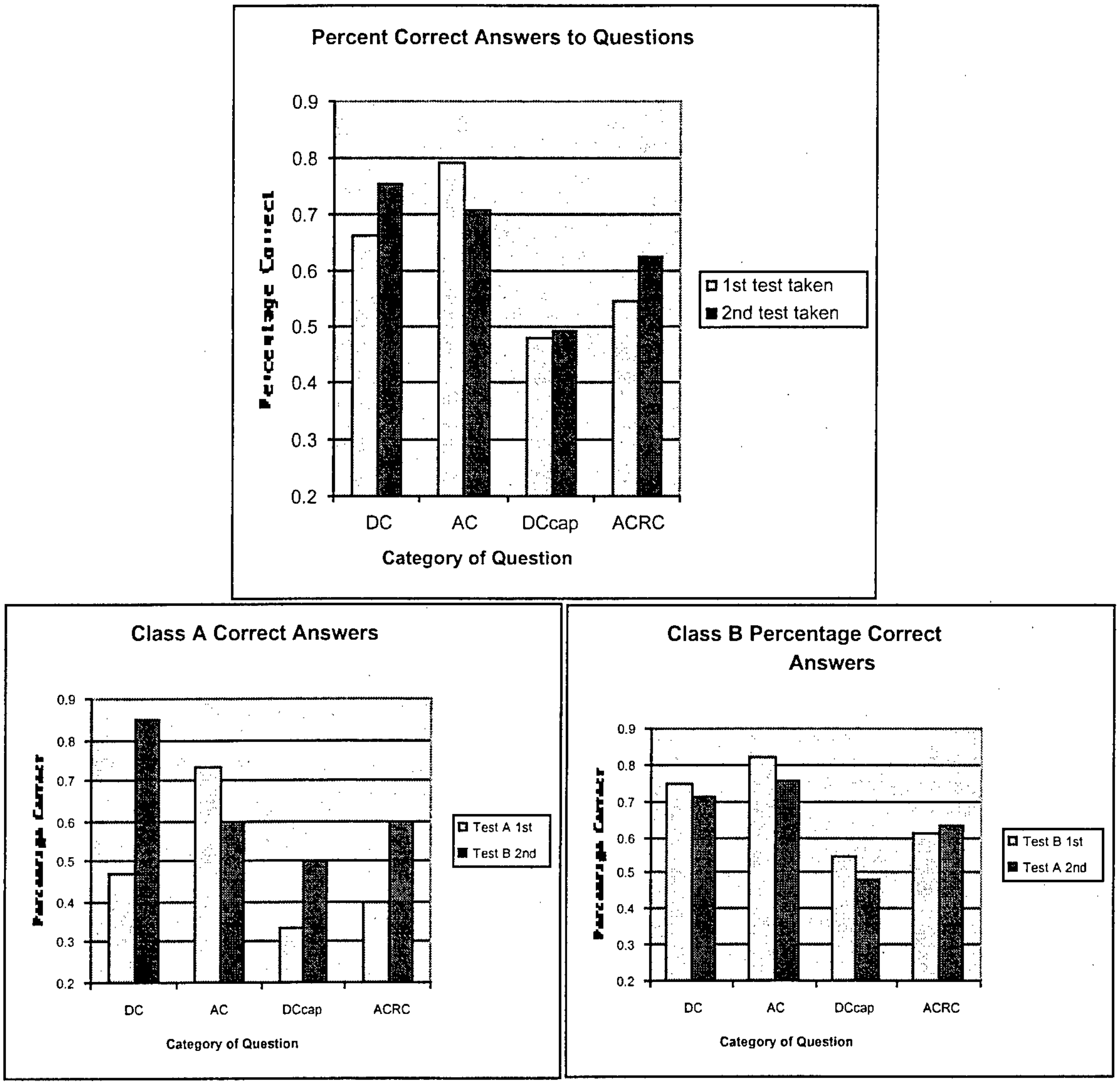


**Figure 4: Test scores by category for Class A and Class B**

Results for Invariant Selection and Discrimination. The average discrimination of invariants (as calculated by Yule's Q) when a correct answer to a circuit problem was chosen was 0.53. The average discrimination when an incorrect answer was chosen was 0.39. Thus students were more likely to select those invariants that experts deemed relevant on questions they answered correctly.

However, we found that students' selection of relevant invariants declined from the beginning of the tests to the end. The graphs in Fig. 5 reveal this pattern across each of the categories of questions and across both classes. Different explanations may be provided for this pattern of results. One is that the questions grow more difficult from the beginning to the end of a test. Another explanation is a fatigue or indifference factor. Students may have been concentrating only on getting the correct answer to questions, and gradually paid less attention to the invariant selection. In this study students actually selected the answer to the question first and then selected the invariants. In the second study, we changed this and also asked students to only select the primary invariant relevant to a circuit problem, not all the invariants that are relevant at all.

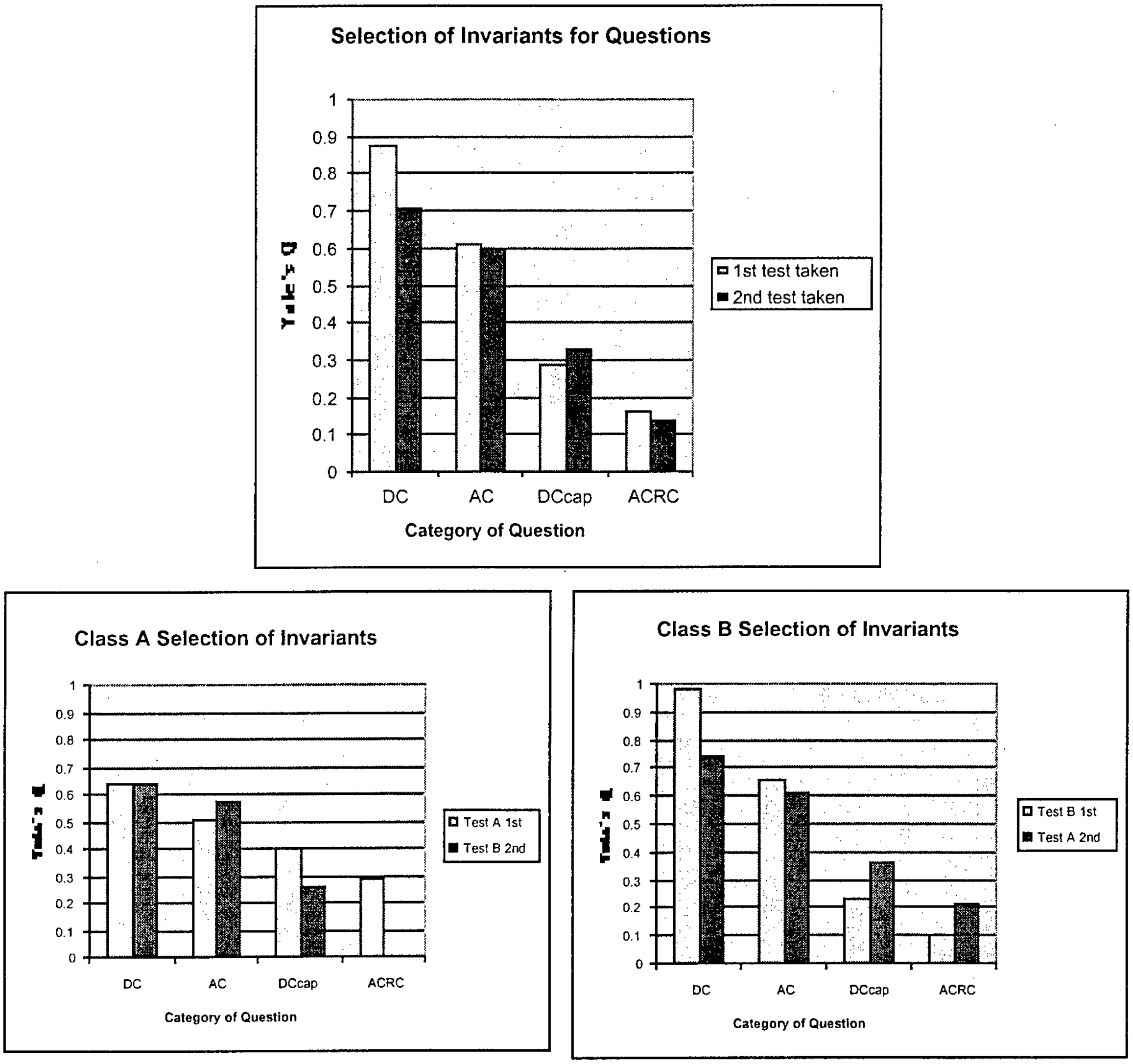


**Figure 5: Selection of Invariants across questions**

Student Survey Responses. Appendix C summarizes and counts the type of student responses to questions on a follow-up survey. In general, students liked the practical applicability of the concepts and liked the outside resources and hints we provided. Some of the students would have liked to see even more resources. However, some students felt the wording of some of the questions and explanations were confusing. We worked on rewriting and improving our test questions and expert explanations for the second study, and indeed, no students mentioned wording as a problem in the follow-up survey given after the second test study.

When asked what invariants are, students mentioned they were like rules or laws, and unchangeable properties of a circuit. They felt that considering the invariants that are involved in a circuit gave them a "baseline" or place to start solving a problem with a circuit.

## Second Pilot Test of Inductor (April 2001)

Before testing the Inductor tool a second time, we rewrote the questions and simplified the expert explanations for the circuit problems. We reduced the total number of questions and redistributed them to create two more evenly balanced and equivalent tests. We changed the order of the procedure for answering questions so that the student was first required to think of and select the primary invariant he or she felt was the key to deriving the solution. After invariant selection, the student chose an answer and provided an explanation, just like the first test. The list of invariants from which to choose was rearranged and reworded to be simpler and less redundant.

In this study, one of our goals was to remove some of the glitches of the first experiment, viz., ambiguous such as the wording of the questions, unwieldy expert explanations, and unbalanced tests. We also directed our study toward determining the influence of invariants on problem solving. To create two conditions, we randomly assigned half of the volunteer participants to a condition in which they did not receive the online instruction on invariants, and they were not prompted to select invariants for each question.

Some of the other issues affecting the prior study remained, however. We had not yet attempted to integrate use of Inductor into an electrical engineering course, and therefore, participation in the study was still voluntary and not directly connected to students' current coursework. Also we had not yet integrated our own resources into Inductor such as simulations and online videos. The resources available in Inductor still consisted of those outside web sites we selected as valuable for learning the concepts presented in our tests.

### Methods

Participants. Twenty undergraduate students in a second year electrical engineering course volunteered to participate online and take the tests.

Procedure and Design. All students completed two 14-item tests with the online Inductor tool. One group of students in this study did not receive instruction about the use of invariants and did not have to select invariants after reading each question. The invariants group did receive instruction about the use of invariants for solving problems before beginning the tests. They also were asked to select the primary invariant property of the circuit for each problem. Besides this, the two groups received identical instruction and feedback. Members of both groups were also pointed to resources in the form of outside web links, and hints on possible conceptual errors they may have made when they missed a question. After answering correctly or revising one's explanation for the answer to a question, the Inductor tool presented an expert explanation for the problem.

The two tests were constructed to be as similar as possible with respect to content and difficulty. Each consisted of four basic DC circuit questions, four basic AC circuit questions, two DC capacitor circuit questions, and four questions concerning AC circuits with capacitors (RC, or filter circuits). In the previous study we have found that students have more difficulty with questions involving capacitors.

### Results of Second Pilot Study

Six of the twenty participants completed both of the tests. We believe the low participation rate was partly due to the fact that use of the tool was not connected to their current class work. In the future, we will be working with professors to make Inductor a supplemental resource for classes. Also, the Inductor tool was described to students only as an online test. In future versions, Inductor will incorporate more instructional resources, and we can better characterize it as an online learning environment rather than an online test system.

Two of the six participants did not receive invariants-based instruction and prompts, and four did receive instruction on the use of invariants.

Test Scores. Overall, the participating students scored an average 61% correct answers on the first test, and 82% correct on the second test, an improvement of 21%. Five of the six participants had improved performance from the first to the second test. There were no significant differences between the scores of the invariants and no-invariants groups, but two of the invariants group members scored perfectly on the second test (see Table 1). One student (ID 6) had a lot of difficulty answering the questions correctly and did not show improvement, but all students were asked to explain their reasoning behind the problems, and thus these explanations reveal more concerning their answers.

| Participant ID | 1st Test | 2nd Test |
|---|---|---|
| 1 (No Invariants) | 57% | 79% |
| 2 (No Invariants) | 71% | 93% |
| 3 (Invariants) | 50% | 86% |
| 4 (Invariants) | 71% | 100% |
| 5 (Invariants) | 71% | 100% |
| 6 (Invariants) | 43% | 36% |

**Table 1: Percentage Correct Answers on Each Test for Each Participant**

As in earlier studies, we found that students had the most difficulty with problems dealing with capacitors. However, in this study, student performance in the second test was higher in all categories, and the largest improvement was in the DC capacitor problems (Fig. 6).

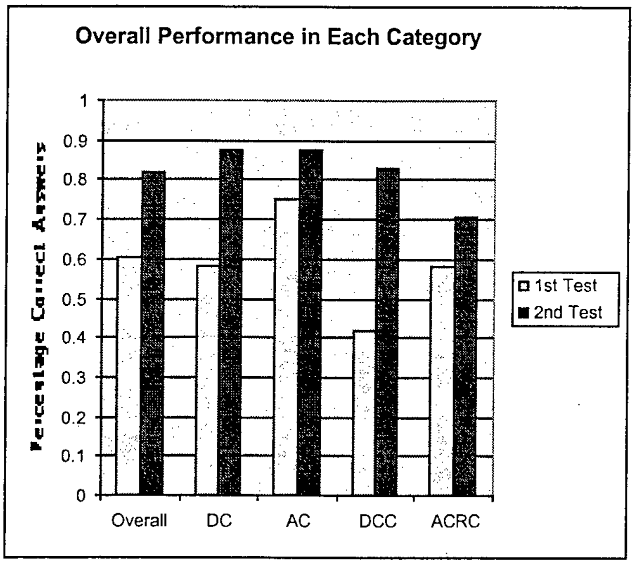


**Figure 6: Percentage Correct Answers in Each Category for First and Second Tests.**

Student Explanations. After reading a question and selecting an answer, students were asked to provide an explanation for their answer. If their answer was incorrect, after receiving hints and links to outside resources, students were asked to revise their explanation for the problem. Finally, regardless of the correctness of their initial answer, the Inductor tool presented an expert explanation for the circuit problem, and students were asked for final comments. Thus students could generate three sets of comments for each question.

The students who did not receive invariant-based instruction and prompts typically wrote shorter and less precise explanations than the group that received invariants instruction. Many of these students seemed to use the outside resources we provided. For example, two students gave explanations for AC capacitor questions resembling the information provided from the resources: *"at high frequencies, it will act like a > short"* and *"at infinity = short."* Both groups of students also initially revealed misconceptions (*"higher resistance means more power is absorbed," "internal resistance rises," "the internal resistance is lower for low frequencies," "John's battery will be required to work harder to push current through the larger resistor"*), errors (*"after long time, all voltage will be across capacitor," "since they are in series the voltage should be the same across all of the bulbs"*), and admitted to making some guesses. However, the responses of students who received invariants prompts and instruction revealed they more often attempted to revise and correct their misconceptions:

- Example 1:
  - 1st explanation: *"Since frequency and current are related linearly, and increase in frequency will increase current."*
  - 2nd explanation: *"Actually, current and frequency have no relation so current does not change with frequency."*

- Example 2:
    - 1st explanation: *"More current flows through the bulbs when they are in series. this makes a brighter light."*
    - 2nd explanation: *"More voltage through the bulbs in parallel will make for brighter bulbs."*
    - 3rd explanation: *"Power is what determines the light intensity. More power makes for more intensity."*

The invariants group also articulated some of the invariant principles they learned:

- *"There is only one path for the current to flow through and both bulbs lie on this path."*
- *"With low frequencies most of the voltage is across the capacitor since its impedance is high when frequency is low."*
- *"Because the lower resistance in Peter's circuit will result in more power consumption. lower resistance = more power"*
- *"Since it is in series connection, the same resistances and current will produce the same power for all the bulb.* $p = i^2 R$*."*
- *"When you go from parallel to series, the intensity will decrease since the voltage is split across both bulb 1 and bulb 2."*
- *"Because the lower resistance will need more power." "the lower resistance will result in more power being dissipated because* $p = \frac{v^2}{R}$*."*

The non-invariants group demonstrated some knowledge of invariants too, however:

- *"The bulbs are in series. Therefore, the current through them must be the same."*
- *"R doesn't depend on frequency."*
- *"The impedance of a capacitor changes inversely with frequency."*
- *"KVL."*

We believe the outside resources may have helped participants in the non-invariants group induce some of the invariant principles. The outside resources were selected because they specifically target concepts related to the invariants. This may also partly explain why the non-invariant group performed as well as the invariants group in correctly answering the questions, and improved as much on the second test as the invariants group. The questions themselves targeted important invariant principles, and were also more clearly written than in the first pilot study (no students mentioned problems with wording in the follow-up survey for the second pilot study).

Student Survey Responses. After completing both tests, students answered questions on a follow-up survey. The students responded that they liked and used the outside resources and the hints we provided after answering incorrectly. One student even suggested, *"I was very impressed with the information provided to learn from mistakes. I* *think that that information should be provided regardless of whether the answer was right or wrong so that if the answer was just a guess I could solidify my understanding."*

Students also mentioned they thought the test questions helped reinforce concepts they had learned, and also helped them to better apply what they had learned. When we asked those participants who received instruction on invariants what they believed invariants are, and how they are used, the students primarily thought of invariants as a method for solving a problem. When asked what is an invariant, one student responded: *"A circuit invariant is a certain method of circuit analysis needed to solve the problems presented.(ex: ohms law, power, etc)."* When asked why it is useful to analyze a circuit by considering the invariants, that student also responded: *"It gives the student an idea of where to start the problem."* Other students also responded: *"It lets you know how to solve the circuit and how to solve like circuits in the future,"* and *"It allows one to find and use the necessary method of solution quicker."*

## 6. SUMMARY AND FUTURE WORK

The complex and invisible nature of the electricity domain makes learning and understanding of electricity concepts difficult. Since the domain is so challenging, and it is not possible to "see" or "feel" current or voltage, it is difficult for students to develop proper conceptions about basic electricity concepts. In the domain of DC, we had discovered a variety of misconceptions about current, voltage, resistance and power (Biswas, et al, 1997). For example, we had found that people have problems differentiating voltage from current, so they talk about voltage "flowing" through the circuit or battery being a source of current. They do not understand that the potential difference or the voltage across the battery causes the current to flow through the wires. Often, they think that current flows sequentially, like water through an empty pipe (so, the first bulb in a row of two bulbs would light first). Or, that the current in a circuit is constant at all times (and that is a property of the battery), that a resistor "slows down" the current so that the flow actually alleviates right "after" the resistor, and so on.

Our more recent studies show that similar misconceptions appear in students' understanding of the basic concepts of the AC domain. Many DC misconceptions seem to carry over to the AC domain. In addition, the time-varying nature of voltage and current, and the fact they go from positive to negative and back confuses students because they think that current cannot change directions and still flow in the circuit. Therefore, they rationalize that the sine wave form represents a spatial rather than temporal property of current, and that current travels as a spatial "wave" through the wires. The key to understanding AC phenomena can be described in two general points: (i) to understand the notions of time varying voltage and current, and (ii) the implications of their time varying nature in analyzing circuit behavior.

We also described our approach to remediating students lack of knowledge and misunderstanding by developing our own dynamic assessment approach that takes into account not only the learning potential of individuals, but the learnability of the domain concepts as well. The results of a preliminary study to test the expectations we had of our approach are promising. We found that some commonly cited misconceptions, (like the difficulty of handling parallel resistors) are not existent by the time students move to more advanced courses in electricity. We found that other misconceptions, like local reasoning about the movement of current from point to point, can be easily remediated and do not have to serve as obstacles to further learning. We also found that some aspects of the domain are difficult to learn even with special attention.

More recently, we have focused on Inductor, a system for self-assessment and learning problem solving methods that are modeled after expert reasoning. We have introduced the notion of invariants to guide the problem solving process, and have developed a web-based multi-choice test that helps students realize their lack of understanding of key domain concepts. The make up of the multiple-choice test was guided by the protocol studies that we had conducted earlier. Preliminary studies demonstrate the effectiveness of the Inductor system in helping student's self-assess their difficulties. Students have also provided positive feedback on the resources we provide, and the general approach to problem solving with invariants.

Our future work will include the further development of our ADL framework by introducing challenge problems that are directly linked to tasks that Naval technicians will be involved in. The challenge is to develop instructional resources that help student technicians better understand basic AC phenomena, and their implications on maintaining and trouble shooting complex equipment. We hope that by focusing on real world devices and applications, students will develop better intuitions about the key characteristics of the phenomena, enhancing their ability to learn and apply the concepts in a variety of real world situations.

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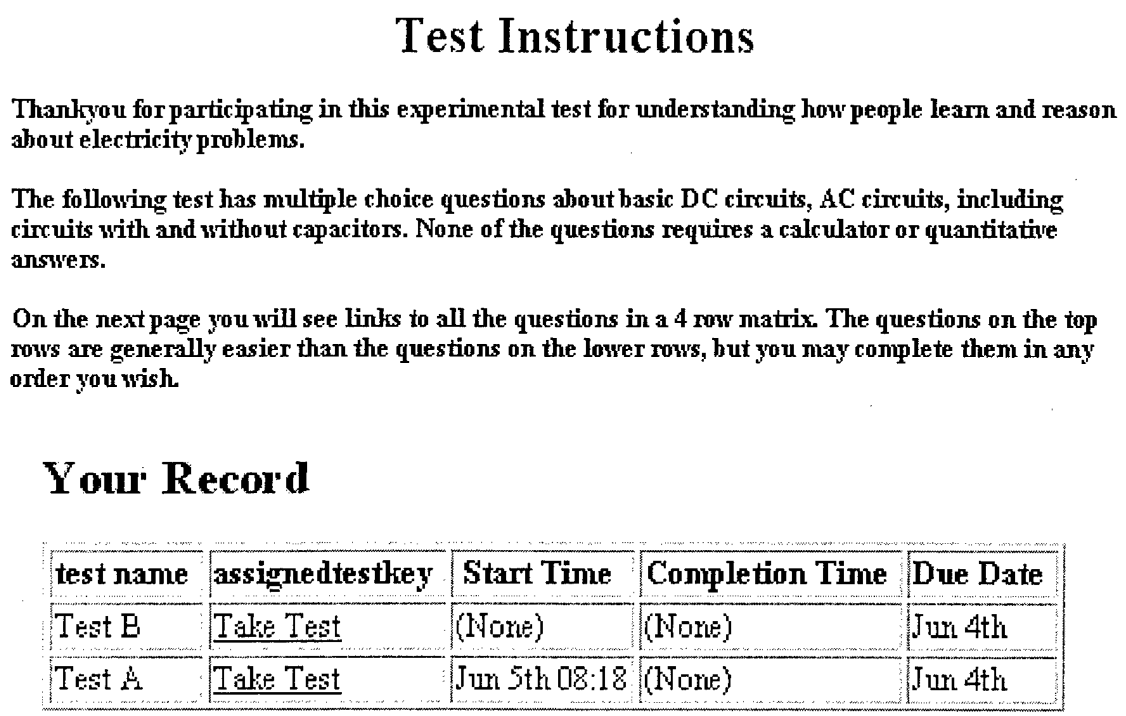
## APPENDIX A

## How a Student Uses Inductor

A student begins by connecting to the Inductor website and selecting a username for him or herself (see: http://relax.ltc.vanderbilt.edu/onr/demo.php ). After typing in a username, the student clicks "Sign Me Up!" and then clicks the login link. The assigned tests are displayed along with progress information and due dates.

Clicking "Take Test" takes the student to a page of instructions. These instructions inform the student about the nature of the test questions, and explain the use of invariants for answering the questions. Recently we have also added a video by Dr. Bhuva explaining what invariants are and how to use them for thinking about a problem.



# Test Instructions

**Thankyou for participating in this experimental test for understanding how people learn and reason about electricity problems.**

**The following test has multiple choice questions about basic DC circuits, AC circuits, including circuits with and without capacitors. None of the questions requires a calculator or quantitative answers.**

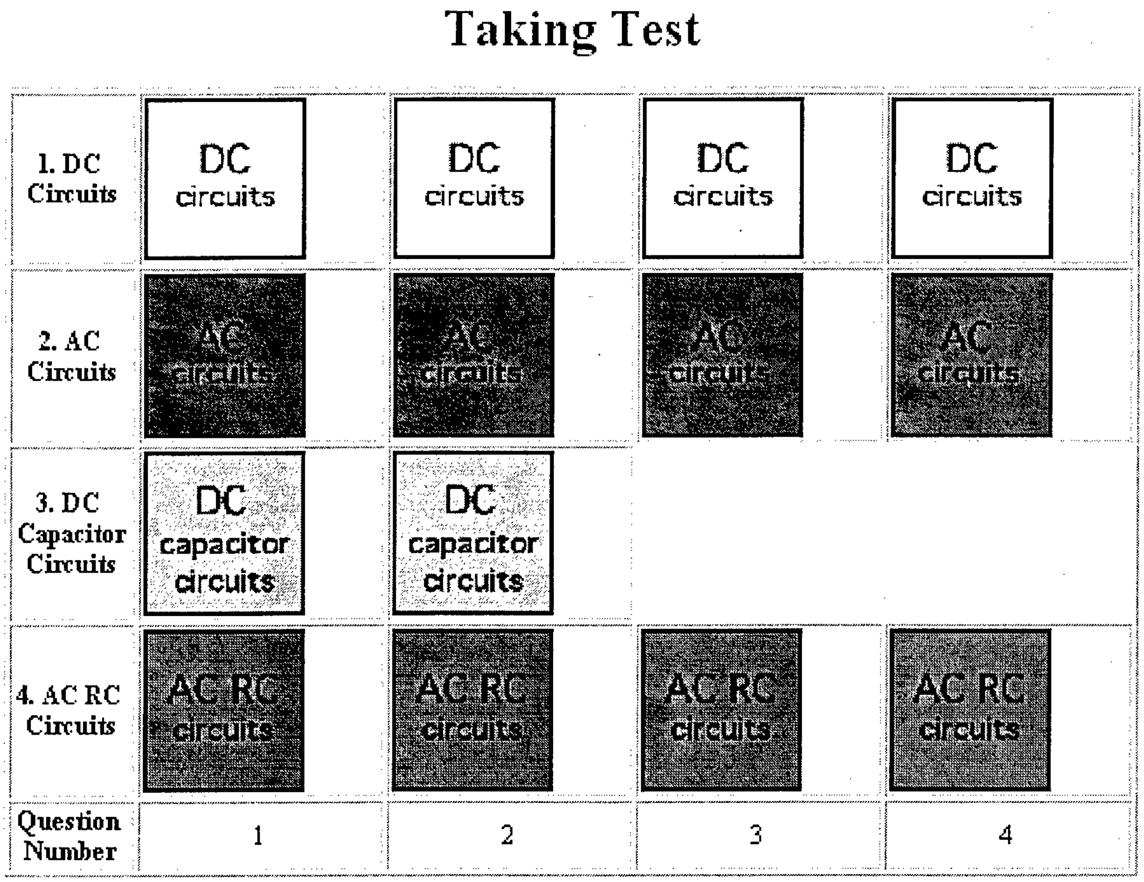
**On the next page you will see links to all the questions in a 4 row matrix. The questions on the top rows are generally easier than the questions on the lower rows, but you may complete them in any order you wish.**

## Your Record

| test name | assignedtestkey | Start Time | Completion Time | Due Date |
|---|---|---|---|---|
| Test B | Take Test | (None) | (None) | Jun 4th |
| Test A | Take Test | Jun 5th 08:18 | (None) | Jun 4th |

## Test Matrix Page

After clicking the "Take Test" link at the bottom of the instructions page, students are taken to the matrix of questions for the test. These are organized by category, and the icons for each question may be used to provide additional information as well. The student may click any of the icons to attempt a question.

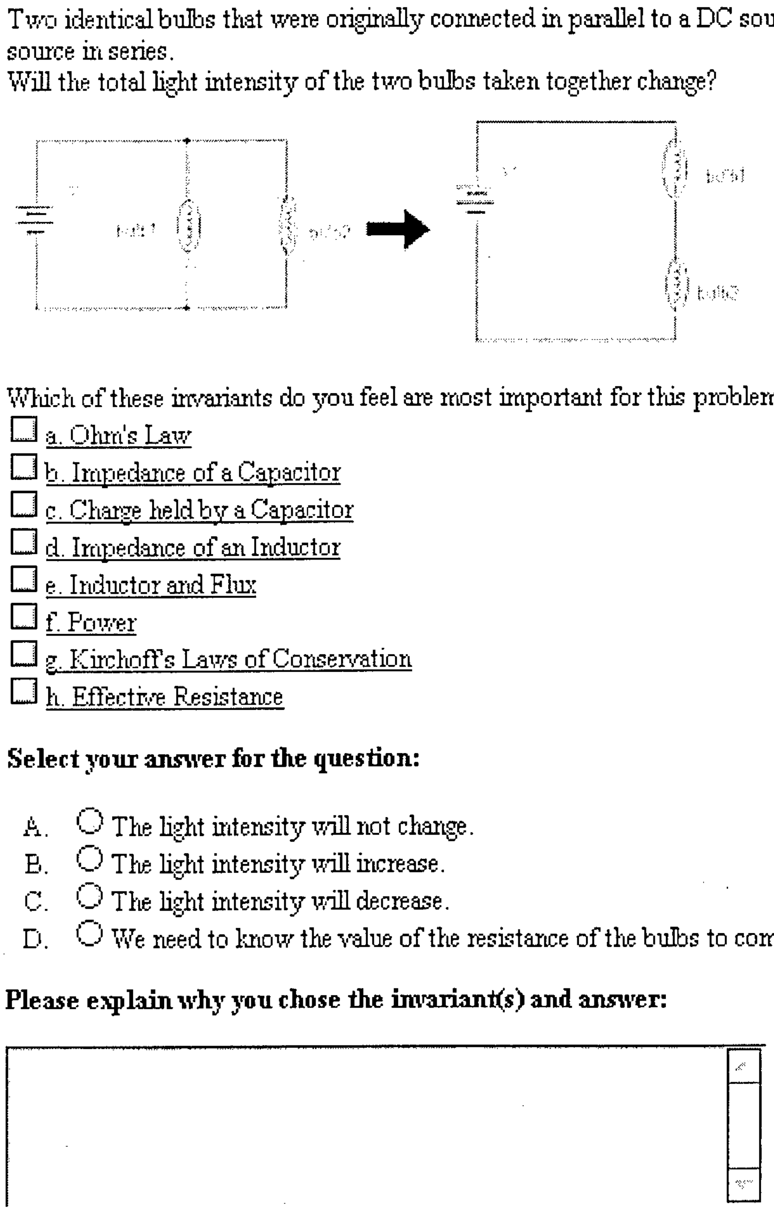


### Taking Test

| | | | | |
|---|---|---|---|---|
| 1. DC Circuits | DC circuits | DC circuits | DC circuits | DC circuits |
| 2. AC Circuits | AC circuits | AC circuits | AC circuits | AC circuits |
| 3. DC Capacitor Circuits | DC capacitor circuits | DC capacitor circuits | | |
| 4. AC RC Circuits | AC RC circuits | AC RC circuits | AC RC circuits | AC RC circuits |
| Question Number | 1 | 2 | 3 | 4 |

## Sample Question Page

Below is a picture of a sample test question. First there is a question along with a picture. Next the students are asked to choose the invariants they feel are important for solving the problem. Each invariant in the list is a link that pops up a new window with more information and resources relevant to the invariants. Next, students select their answer and give an explanation for why they chose the invariants and answer.



Two identical bulbs that were originally connected in parallel to a DC source are now connected to the same source in series.
Will the total light intensity of the two bulbs taken together change?

Which of these invariants do you feel are most important for this problem?

☐ a. Ohm's Law
☐ b. Impedance of a Capacitor
☐ c. Charge held by a Capacitor
☐ d. Impedance of an Inductor
☐ e. Inductor and Flux
☐ f. Power
☐ g. Kirchoff's Laws of Conservation
☐ h. Effective Resistance

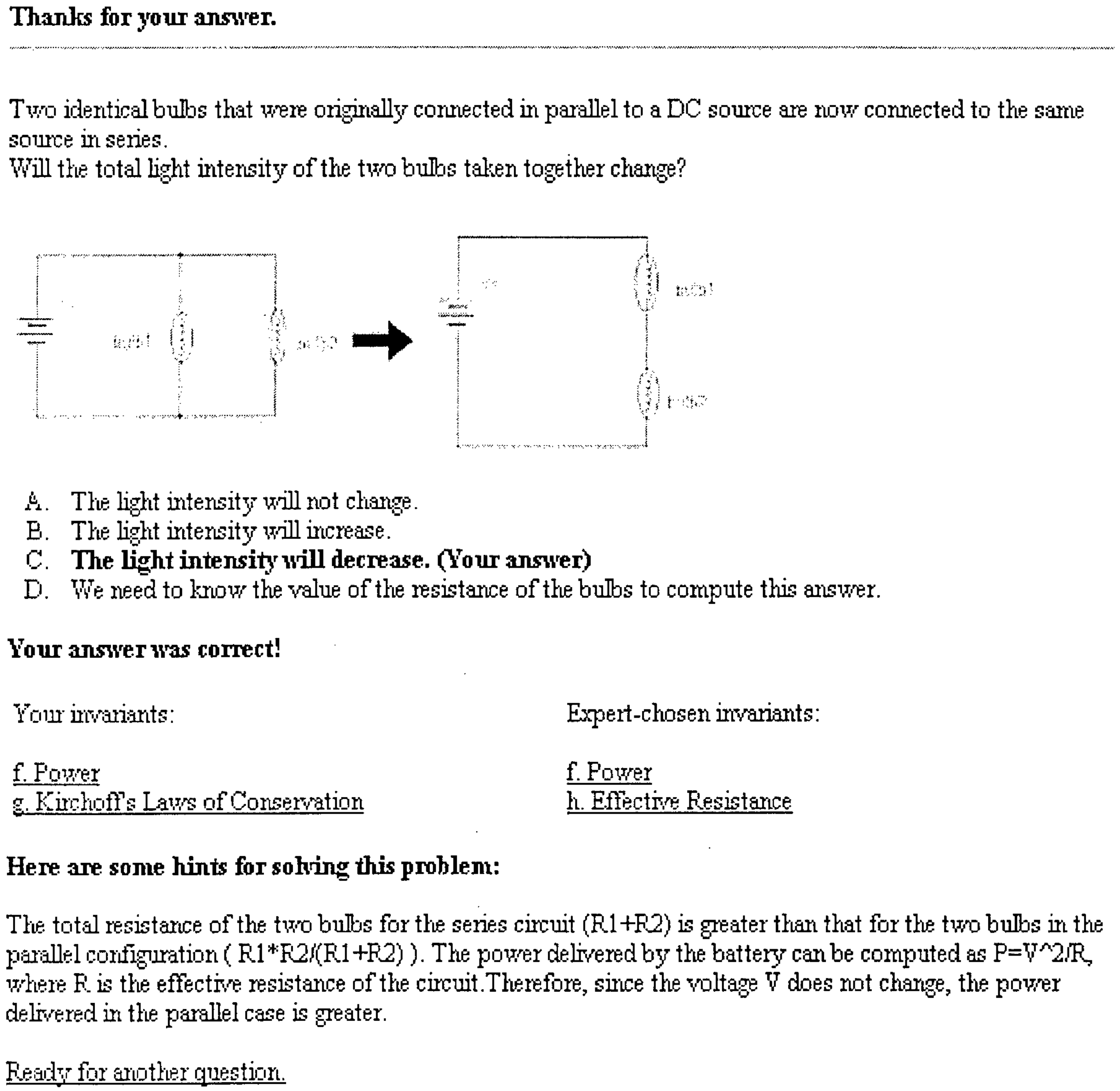
**Select your answer for the question:**

A. ○ The light intensity will not change.
B. ○ The light intensity will increase.
C. ○ The light intensity will decrease.
D. ○ We need to know the value of the resistance of the bulbs to compute this answer.

**Please explain why you chose the invariant(s) and answer:**

Correct Answer Feedback Page

In this example, the answer was correct. The student is still given an expert explanation for the answer.



**Thanks for your answer.**

---

Two identical bulbs that were originally connected in parallel to a DC source are now connected to the same source in series.
Will the total light intensity of the two bulbs taken together change?

A. The light intensity will not change.
B. The light intensity will increase.
C. **The light intensity will decrease. (Your answer)**
D. We need to know the value of the resistance of the bulbs to compute this answer.

**Your answer was correct!**

| Your invariants: | Expert-chosen invariants: |
|---|---|
| f. Power | f. Power |
| g. Kirchoff's Laws of Conservation | h. Effective Resistance |

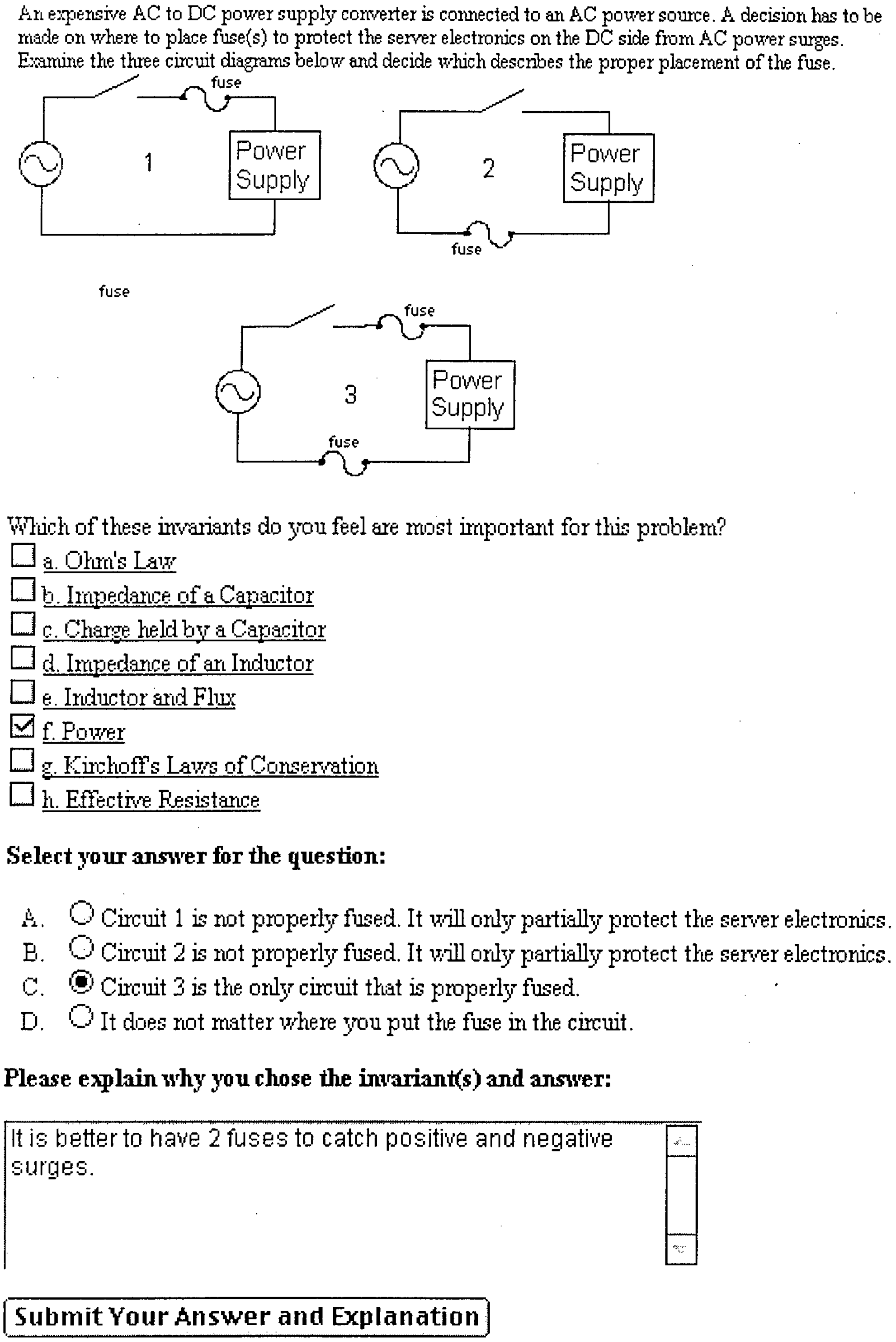
**Here are some hints for solving this problem:**

The total resistance of the two bulbs for the series circuit (R1+R2) is greater than that for the two bulbs in the parallel configuration ( R1*R2/(R1+R2) ). The power delivered by the battery can be computed as P=V^2/R, where R is the effective resistance of the circuit.Therefore, since the voltage V does not change, the power delivered in the parallel case is greater.

Ready for another question.

## Incorrect Answer Example

For this next sample question, an incorrect answer is entered.



An expensive AC to DC power supply converter is connected to an AC power source. A decision has to be made on where to place fuse(s) to protect the server electronics on the DC side from AC power surges. Examine the three circuit diagrams below and decide which describes the proper placement of the fuse.

Which of these invariants do you feel are most important for this problem?

☐ a. Ohm's Law
☐ b. Impedance of a Capacitor
☐ c. Charge held by a Capacitor
☐ d. Impedance of an Inductor
☐ e. Inductor and Flux
☑ f. Power
☐ g. Kirchoff's Laws of Conservation
☐ h. Effective Resistance

**Select your answer for the question:**

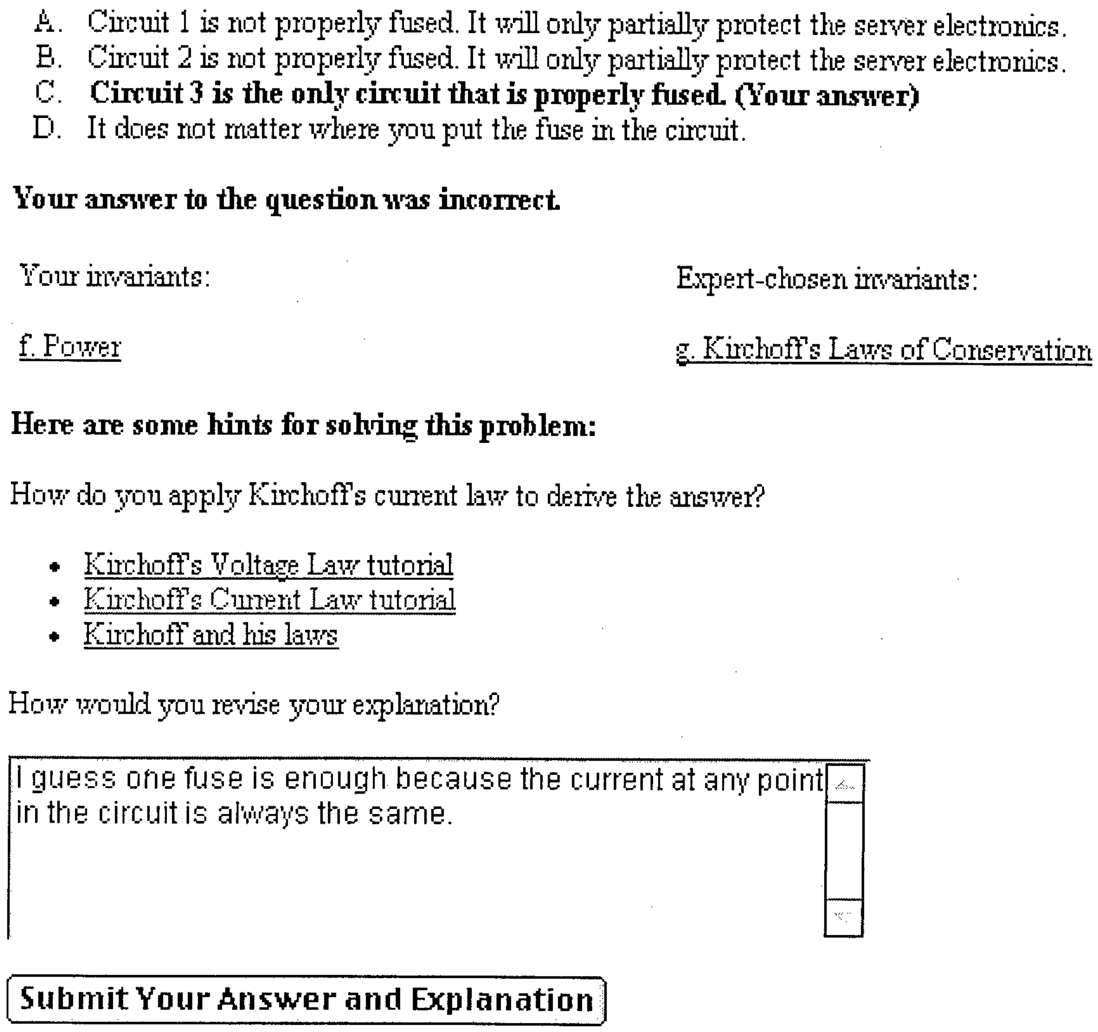
A. ○ Circuit 1 is not properly fused. It will only partially protect the server electronics.
B. ○ Circuit 2 is not properly fused. It will only partially protect the server electronics.
C. ● Circuit 3 is the only circuit that is properly fused.
D. ○ It does not matter where you put the fuse in the circuit.

**Please explain why you chose the invariant(s) and answer:**

It is better to have 2 fuses to catch positive and negative surges.

Incorrect Answer Feedback

The feedback in this example does not tell a student the correct answer yet. First, student is shown the invariants an expert chose for reasoning about this problem. A hint is also given with links to helpful resources, which in the future may also include simulations or videos. The student enters a revised explanation based on this feedback.



A. Circuit 1 is not properly fused. It will only partially protect the server electronics.
B. Circuit 2 is not properly fused. It will only partially protect the server electronics.
C. **Circuit 3 is the only circuit that is properly fused. (Your answer)**
D. It does not matter where you put the fuse in the circuit.

**Your answer to the question was incorrect.**

| Your invariants: | Expert-chosen invariants: |
|---|---|
| f. Power | g. Kirchoff's Laws of Conservation |

**Here are some hints for solving this problem:**

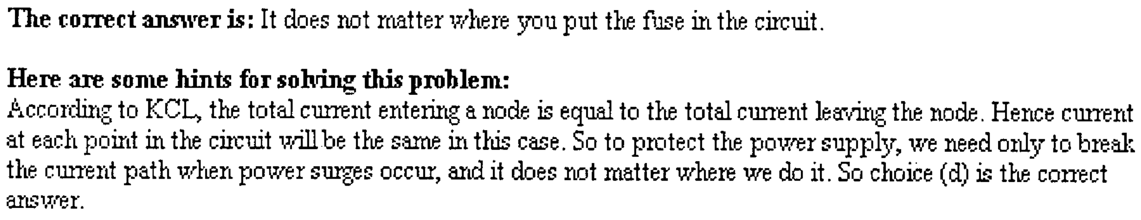
How do you apply Kirchoff's current law to derive the answer?

- Kirchoff's Voltage Law tutorial
- Kirchoff's Current Law tutorial
- Kirchoff and his laws

How would you revise your explanation?

I guess one fuse is enough because the current at any point in the circuit is always the same.

Finally, the student is told the correct answer, and an expert explanation for the problem is given. This expert explanation is given at the end of correctly and incorrectly answered questions.

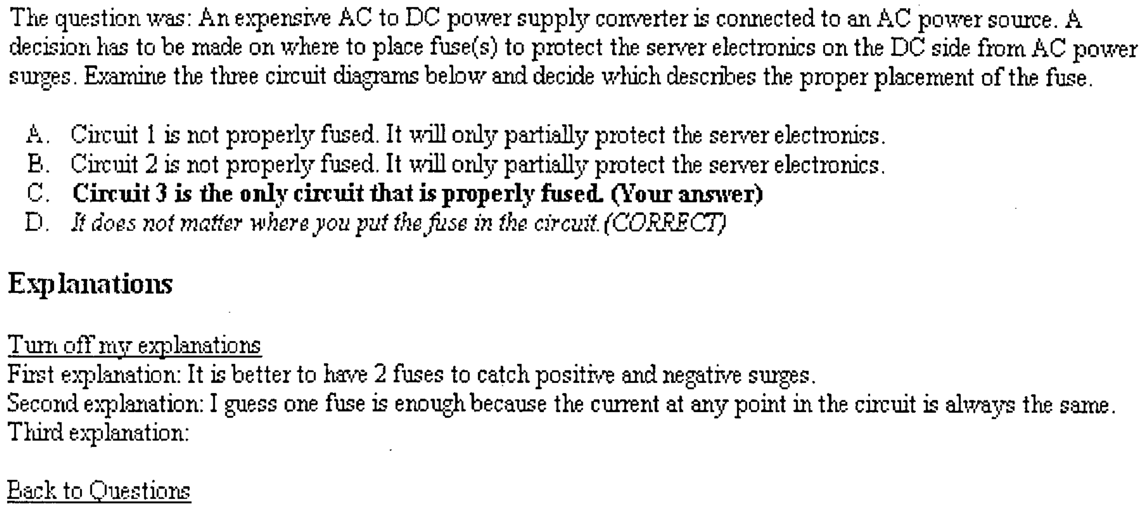


**The correct answer is:** It does not matter where you put the fuse in the circuit.

**Here are some hints for solving this problem:**
According to KCL, the total current entering a node is equal to the total current leaving the node. Hence current at each point in the circuit will be the same in this case. So to protect the power supply, we need only to break the current path when power surges occur, and it does not matter where we do it. So choice (d) is the correct answer.

## Reviewing a Question Previously Answered

After finishing answering a question, the student is taken back to the matrix of questions. Previously answered questions are surrounded with a green (for correct) or red (for incorrect) box and have a link for reviewing the question and one's answers and explanations.



The question was: An expensive AC to DC power supply converter is connected to an AC power source. A decision has to be made on where to place fuse(s) to protect the server electronics on the DC side from AC power surges. Examine the three circuit diagrams below and decide which describes the proper placement of the fuse.

A. Circuit 1 is not properly fused. It will only partially protect the server electronics.
B. Circuit 2 is not properly fused. It will only partially protect the server electronics.
C. **Circuit 3 is the only circuit that is properly fused. (Your answer)**
D. *It does not matter where you put the fuse in the circuit. (CORRECT)*

**Explanations**

Turn off my explanations
First explanation: It is better to have 2 fuses to catch positive and negative surges.
Second explanation: I guess one fuse is enough because the current at any point in the circuit is always the same.
Third explanation:

Back to Questions

# APPENDIX B

## Outside Resources Provided With Inductor

In order to foster further learning during the course of using Inductor, outside resources are provided for students to explore, in addition to the hints and invariant-based explanations given within Inductor. These outside resources consisted of websites with tutorials, explanations, and simulations of DC and AC circuit concepts. Links to these resources are provided in two contexts. On the page listing descriptions of all the invariants, links to outside resources are listed next to the invariant to which they apply (see http://relax.ltc.vanderbilt.edu/onr/invariants/invariants.html ). Resources are also provided for students to explore after first answering a question incorrectly. Inductor gives helpful hints for thinking about the problem, including a list of outside resources targeted towards the invariants involved in the problem. Below is an annotated list of some of the resources.

RLC Filter Circuits:
a) http://users.erols.com/renau/impedance.html (java applet)
b) http://webphysics.ph.msstate.edu/jc/library/21-5/index.html (java applet)
c) http://anthrax.physics.indiana.edu/~dzierba/Honors2/Schedule/Week9/AC/capacitor.html
d) http://anthrax.physics.indiana.edu/~dzierba/Honors2/Schedule/Week9/AC/lowhigh.html

Resources a and b are Java simulations of an RLC AC filter circuit. The simulations are interactive, allowing a student to manipulate the values of the voltage, frequency, capacitance, inductance, and resistance and dynamically update the graphs of the circuit behavior. The circuit behavior is graphed in both the frequency and time domains. The advantage of these features is that students can explore and test the circuit's invariant properties, such as that the impedance of the capacitor (Xc) decreases with frequency. Resources c and d additionally provide the student with two simple fundamental ideas to help their reasoning about these circuits: a capacitor approaches an open circuit at low frequencies and a short circuit at high frequencies, and the difference between a low-pass and high-pass filter.

RC Circuits:
a) http://www.phys.hawaii.edu/~teb/java/ntnujava/rc/rc.html (java applet)
b) http://www.sweethaven.com/acee/forms/frm0802.htm
c) http://www.sweethaven.com/acee/forms/frm1003.htm

Students often have difficulty understanding the behavior of capacitors in DC and AC circuits. Resource a is an illustrative simulation of a DC circuit charging and discharging. It dynamically graphs the output voltage over time, and metaphorically illustrates current flow in the wires in a manner consistent with Kirchoff's current law. Resources b and c also explain the invariant behavior of a capacitor in a DC circuit and link the behaviors shown in the simulation to mathematical formulations.

Kirchoff's Laws of Conservation:

a) http://www.physics.uoguelph.ca/tutorials/ohm/Q.ohm.KVL.html (including java)
b) http://www.physics.uoguelph.ca/tutorials/ohm/Q.ohm.KCL.html
c) http://www.ece.utexas.edu/~aduley/lab/index.html

Resources a and b very simply state the basic rules of Kirchoff's voltage and current laws and how voltage and current are conserved in a circuit. These are two of the most fundamental invariant relationships governing circuit behavior. Resource c additionally provides some history behind these principles and additional circuit examples.

Future Resources. We are currently developing our own resources that more specifically illustrate the invariant relationships in electrical circuits. For example, an interactive graphing tool can illustrate how the impedance of a capacitor changes with the frequency of an AC source, and how that affects output current and voltage. A Java-based circuit simulation tool will show a circuit from different conceptual views. One illustrates the topological relationships in a circuit, providing a diagram of the circuit along with controls for changing various properties such as voltage, frequency, resistance, and capacitance. A second, connected view graphs the properties of the circuit by time or frequency. An expert view will provide text or movies explaining what to notice about the circuit and the invariant properties of the circuit behavior. Additional views might illustrate systems that are analogical to the circuit and its behavior, or describe the mathematical relationships that characterize the circuit.

# APPENDIX C

## Summary of 16 Responses to the Follow-Up Survey Given After the First Test Study (December, 2000)

**1) Do you think having more questions like the ones you answered would make a useful addition to a course? Please give us a few positive reasons and a few negative reasons.**

- 7/16 liked seeing the practical applicability of the concepts
- 4/16 referred to using or reinforcing knowledge they have previously learned
- 2/16 thought it was too time consuming or not time efficient
- 6/16 had problems with the wording and choice of questions themselves

**2) If you can, please write down the different classes of problems that you were asked to solve.**

- 12/16 mentioned AC and DC circuits and components (RLC)
- 7/16 mentioned one or all the categories design, troubleshooting, analysis

**3) Were there features of the on-line assessment system that you found useful or interesting? Please give us a few positive features and a few negative features.**

- 8/16 liked the hints or explanations
- 2/16 mentioned liking typing in their own explanations
- 7/16 mentioned problems with the questions or wording

**4) Did you learn anything about your strengths and weaknesses in the AC domain by completing this assessment?**

- 16/16 learned something about their understanding of AC circuits.
- See the individual responses below, some are very telling.

**5) What is a circuit invariant?**

- 5/16 mentioned components (3 of whom *only* mentioned components)
- 8/16 mentioned rule/law/property
- 6/16 mentioned "unchangeable" or "constant"

**6) Can you think of any reasons that it is useful to begin an analysis of a circuit by considering the invariants?**

- 9/16 mentioned it gives you a foundation/baseline/"place to start with"/outline/picture
- 10/16 mentioned it makes it easier to begin solving the problem, choosing the

equations to use

**7) How useful did you find some of the outside resources in the test, and what kinds of resources would you find most helpful to add?**

- 10/16 found them helpful or useful
- 3/16 thought some were wordy, overwhelming, or too simple
- 5/16 would like more resources/pictures added or improvements to some existing resources

**8) Did the questions, explanations, and resources provided in the first test make it easier for you to derive the answers in the second test you took.**

- 13/16 said yes
- 5/16 said it clued them onto the format of the problems & questions, knew what to expect, questions were similar
- 2/16 said it made them pay more attention, learn from mistakes