MODELING AND ANALYSIS OF
MULTICOMMODITY NETWORK FLOWS
VIA GOAL PROGRAMMING

THESIS

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Abstract
In this work goal programming is used to solve a minimum cost multicommodity network flow problem with multiple goals. A single telecommunication network with multiple commodities (e.g., voice, video, data, etc.) flowing over it is analyzed. This network consists of: linear objective function, linear cost arcs, fixed capacities, specific origin-destination pairs for each commodity. A multicommodity network flow problem with goals can be successfully modeled using linear goal programming techniques. When properly modeled, network flow techniques may be employed to exploit the pure network structure of a multicommodity network flow problem with goals. Lagrangian relaxation captures the essence of the pure network flow problem as a master problem and sub-problems (McGinnis and Rao, 1977). A subgradient algorithm may optimize the Lagrangian function, or the Lagrangian relaxation could be decomposed into subproblems per commodity; each subproblem being a single commodity network flow problem. Parallel to the decomposition of the Lagrangian relaxation, Dantzig-Wolfe decomposition may be implemented to the linear program. Post-optimality analyses provide a variety of options to analyze the robustness of the optimal solution. The options of post-optimality analysis consist of sensitivity analysis and parametric analysis. This mix of modeling options and analyses provide a powerful method to produce insight into the modeling of a multicommodity network flow problem with multiple objectives.

Subject Terms
Multicommodity Network Flow, Goal Programming, Linear Programming, Lagrangian Relaxation, Decomposition

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Matthew A. Scott

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Abstract

Information superiority, along with air superiority, must be achieved and maintained in a theatre of battle in order to increase efficacy and provide protection to our forces, both on the ground and in the air. Information, for the most part, utilizes an underlying network that is identified, and is of vital interest to a commander in a wartime or peacetime scenario. This underlying network must be modeled and analyzed in order to develop target prioritization and to analyze effects of target prioritization implementation.

In this work goal programming is used to solve a minimum cost multicommodity network flow problem with multiple objectives. The networks to be analyzed are telecommunication networks, specifically, a single telecommunication network with multiple commodities (*e.g.*, voice, video, data, etc.) flowing over it. This network consists of: linear objective function, linear cost arcs, fixed capacities, specific origin-destination pairs for each commodity. A minimum cost multicommodity network flow problem with multiple objectives can be successfully modeled using linear goal programming techniques. When properly modeled, network flow techniques may still be employed to exploit the pure network structure of a minimum cost multicommodity network flow problem with multiple objectives. There exist techniques that take advantage of these underlying network properties in a goal programming format; Lagrangian relaxation and Dantzig-Wolfe decomposition. Lagrangian relaxation

captures the essence of the pure network flow problem as a master problem and sub-problems (McGinnis and Rao, 1977). A subgradient algorithm may optimize the Lagrangian function, or the Lagrangian relaxation could be decomposed into subproblems per commodity; each subproblem is a minimum cost single commodity network flow problem. Parallel to the decomposition of the Lagrangian relaxation, Dantzig-Wolfe decomposition may be implemented to the linear program.

All three modeling techniques provide a solution method to effectively analyze the multicommodity network flow problem at hand. Post-optimality analyses provide a variety of options to analyze the robustness of the optimal solution. The options of post-optimality analysis consist of sensitivity analysis and parametric analysis. This mix of modeling options and analyses provide a powerful method to produce insight into the modeling of a multicommodity network flow problem with multiple objectives.

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I. Introduction

Background

During periods of conflict, Information Warfare is the altering, by any means, of the enemy's information and its function, while protecting the United States from information alteration infliction (Defense, 1998:GL-11). Information, according to Air Force Doctrine, encompasses "facts, data, or instructions in any medium or form" (AFDD 2-5, 1998:135). During conflict, information operations are "actions taken to gain, exploit, defend or attack information (IIW) and information warfare (IW)" (AFDD 2-5, 1998:135). Information or knowledge enables a competitive advantage over the enemy.

Information in the battlefield is often encapsulated within networks. According to Joint Pub 6-0, within the past six years there has been an explosion in the number of communication networks within the world (Joint Pub 6-0, 1995:VI-1). Modeling and analysis of communication networks, therefore, would be beneficial to the United States Air Force.

Statement of the Problem

Inherent to the transfer and support of information, networks are formed to send information from point *A* to point *B*. Networks can describe aspects of a country's infrastructure. For instance, networks are embedded in electrical power grids, radio relay facilities, SATCOM links, computer and data processing centers, national C3I centers, telephone exchanges, and logistical mobilization (AFPHM 14-210, 1998:87). A country's infrastructures are often its most vital assets. Specifically, telecommunication networks can be targeted or exploited at the commander's discretion.

Quantitative models are often used to represent networks. A variety of efficient algorithms have been developed for pure network flow problems, such as linear minimum cost single commodity flow problems. Minimum cost multicommodity network flow problems with side constraints are more complex than traditional pure network flow problems, but algorithms also exist for these models.

Telecommunication networks may be modeled as multicommodity network flow problems. Realistically, when dealing with telecommunication networks, there may be multiple, often conflicting, objectives. The analyst of a telecommunication network may want to both minimize the cost of the flow of the commodities on the network, while maximizing the utilization of the network arcs. These conflicting objectives may be formulated and represented as additional side constraints to a network model.

Telecommunication networks are typically vast in size, which causes an increase in modeling complexity and analysis times. The exploitation of the underlying pure network structure of a multicommodity network flow problem with multiple objectives would reduce the computational time. A variety of modeling techniques to exploit the underlying pure network structure of a multicommodity network flow problem with multiple objectives and post optimality analysis are required to extrapolate information of interest in the particular network being analyzed.

Research Approach

In this research, a variety of optimization and modeling techniques (graph theory, network flows, multiple criteria decision making (MCDM), linear goal programming (LGP), Lagrangian relaxation, Dantzig-Wolfe decomposition, LGP sensitivity analysis, and parametric analysis) concepts were implemented to model and analyze minimum cost multicommodity network flow problems with multiple objectives. First, network data must be provided to the level of detail required to conduct the desired analysis. Graph theory and network flow techniques must be employed to produce a graphical representation consisting of commodity, origin-destination pairs, fixed locations, bandwidth, preferences, and distances. Next, MCDM is used to identify multiple objectives (goals) within the network. For example, goals on flow between nodes, goals on flow through a given node, or goals on the cost of flow along any given set of arcs represent preferences that must be defined.

Linear goal programming (LGP) techniques are next implemented to formulate a minimum cost multicommodity network with multiple objectives. If the problem being analyzed is relatively small, with respect to computational time requirements, then the linear problem may be solved. However, if a larger problem is present, then Lagrangian relaxation may be applied to preserve the pure network structure. If the problem consists of a linear objective function, linear arc costs, and fixed capacity, Dantzig-Wolfe decomposition may optionally be applied to solve the minimum cost multicommodity network flow problem with multiple objectives.

Post-optimality analyses, both sensitivity analysis and parametric analysis, are conducted to provide additional insight about the network. GP sensitivity analysis is used to aid in identifying vulnerabilities in the network. Areas of investigation include: (1) changes in the weighting at a priority level; (2) changes in the weighting of deviation variables within a priority level; (3) changes in right-hand-side values; (4) reordering of preemptive priorities. Parametric analysis of the nodes and/or arcs identifies commodity tradeoffs at the source nodes. This provides insight into alternate routes within the network, or cause and effect of reductions in capacity at nodes or arcs of interest.

Scope and Limitations

This effort focuses on modeling and analysis of minimum cost multicommodity network flow problems with multiple objectives. The pedagogical network tested in this thesis is a telecommunications network with commodities voice, video, and data. Additional commodities could be added or replaced to model other multicommodity network flow problems.

Summary

This introduction outlines the foundation of the impetus behind this study's methodology and research approach. Chapter 2 reviews the fundamental elements of the concepts and techniques relevant to this methodology such as graph theory, network flows, multiple criteria decision making, linear goal programming, Lagrangian relaxation, Dantzig-Wolfe decomposition, LGP sensitivity analysis, and parametric analysis.

Chapter 3 describes in detail the methodology employed by this study to obtain and quantify the network problem's multiple objectives, the flow of the commodities, and the utilization of the underlying pure network structure of a minimum cost multicommodity network flow problem with multiple objectives. Chapter 4 presents the implementation and results of the methodology to a notional telecommunications network scenario. Chapter 5 summarizes observations, conclusions of this research, and provides suggested areas for future research.

II. Literature Review

General Network Flows

The field of network flows encompasses "a problem domain that lies at the cusp between several fields of inquiry, including applied mathematics, computer science, engineering, management, and operations research" (Ahuja, Magnanti and Orlin, 1993:1). These network flows exist in everyday activities in modern civilization. All of these underlying networks can be decomposed into an entity (electricity, a consumer product, a person or vehicle, a message, and so forth), which traverses one point to another as efficiently as possible (Jewell, 1966:7; Ahuja, *et al.*, 1993:1). Efficient pure network algorithms may be applied to these decomposed single commodity network flow problems. The modeling and analysis of these underlying networks are categorized *network flow problems.* Network optimization is a link between linear programming and combinatorial optimization; the network structure ensures integer solutions at the extreme points of the feasible polyhedral region (Bertsekas, 1998:ix).

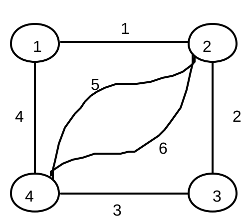
Network Flow and Graph Theory

The field of *Graph Theory* maps ways to understand, classify, and analyze graphs. *Graph* is a theoretical term for a network that includes various nodes connected by links (Pooch, Machuel and McCahn, 1991:204). *Network topology* refers to the connection of links and nodes within a network (Pooch, *et al.*, 1991:204). Formally, Wilson describes a graph to be a collection of *points* (vertices), joined together by *lines* (edges); where each edge links exactly two vertices (Wilson and Watkins, 1990:8). Common terms in network theory like *node*, *link* (or *arc*), and *path* are terms borrowed from graph theory (Pooch, *et al.*, 1991:204).

According to Ahuja, Magnanti, and Orlin, a weighted graph is a "graph whose nodes and/or arcs have associated numerical values (typically, costs, capacities, supplies and demands)" (Ahuja, *et al.*, 1993:24). This graph, or network $G = (N,J)$, includes a set of nodes, $N = \{1,2,\ldots,n\}$ and a set of arcs, $J = \{(i,j), (e,f),\ldots,(l,k)\}$ connecting pairs of vertices (Ahuja, *et al.*, 1993:24; Bazaraa, Jarvis and Sherali, 1997:421; Bertsekas, 1998:3). Each pair of nodes has a specific flow from node i to node j (Ahuja, *et al.*, 1993:24; Bazaraa, *et al.*, 1997:421). An undirected network is a network where there exists no directional orientation of flow.

Node-Arc Incidence Matrix.

Various storage methods can be used to capture the orientation of the network topology. One such storage representation is the node-arc incidence matrix, which captures the network as a $n \times m$ matrix A. This matrix contains one row for each node of the network and one column for each arc number, where n equals the number of nodes and m equals the number of arcs (Ahuja, *et al.*, 1993:33; Bazaraa, *et al.*, 1997:425). This type of representation, shown in Figure 1, is commonly used in network formulations.



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Figure 1. Node-Arc Incidence Matrix

The node-arc incidence matrix form of data representation is used in the formulation presented in this thesis. Often when dealing with communication networks the arcs are undirected, and traditionally are transformed into two directed arcs for every one undirected arc, which contributes to a larger matrix. In the formulation presented in this thesis, the arcs are assigned an arbitrary orientation within the node-arc incidence matrix.

Network Disruption

Network disruption is a topic of interest in the field of network flows. Frank and Frisch state a network is destroyed if at least one of the following criteria exists (Frank and Frisch, 1971:300-301).

1. *G* contains at least two components.
2. There is no directed path *s-t* for a specified set of nodes.
3. The number of vertices in the largest component of *G* is less than some specified number.
4. The shortest *s-t* path is longer (or less reliable) than some specified value.

The same criteria may apply to a *disrupted* network with the exception that the individual components need not be totally severed from one another. Instead, there may be a result of a reduction in capacity or a reduction in commodities. For the purpose of this study, a *disrupted* network is explored.

Minimum Cost Flows

A fundamental problem in network theory is the *minimum cost flow problem*, which means the movement of a commodity from sources that reside at one or more nodes in a network, to meet demands at other nodes, at a cost of shipment (Ahuja, *et al.*, 1993:4; Bazaraa, *et al.*, 1997:420). Examples of such networks include (Ahuja, *et al.*, 1993:4):

- Distribution of a product from manufacturing plants to warehouses
- Work-pieces through the machining stations in a production line
- Routing of calls through a telephone system
- Routing of automobiles through an urban street network

The objective of such a formulation is a solution that provides the least cost to ship commodities through a network subject to supply and demand constraints. Assume there exists a graph $G= (N,A)$ and let graph G be a directed graph with a flow cost c_{ij} and capacity u_{ij} associated with every arc $(i,j) \in A$ (Ahuja, *et al.*, 1993:296; Bazaraa, *et al.*, 1997:420; Bertsekas, 1998:9). The set of all nodes, N, each have an associated supply, demand, or transshipment requirement $b(i)$, depending on whether $b(i) > 0$ (demand), $b(i) < 0$ (supply), or $b(i) = 0$ (transshipment) (Ahuja, *et al.*, 1993:296; Bazaraa, *et al.*, 1997:420; Bertsekas, 1998:6). These may be used to formulate a minimum cost network flow problem:

Minimize:

$$\sum_{(i,j)\in A} c_{ij} x_{ij} \tag{1}$$

Subject to:

$$\sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = r_i \quad \forall i \in N \tag{2}$$

$$0 \le x_{ij} \le b_{ij} \quad \forall (i, j) \in A \tag{3}$$

where,

c_{ij} = The unit cost

x_{ij} = The units of flow from node i to node j

r_i = The number of units of demand, supply, or 0

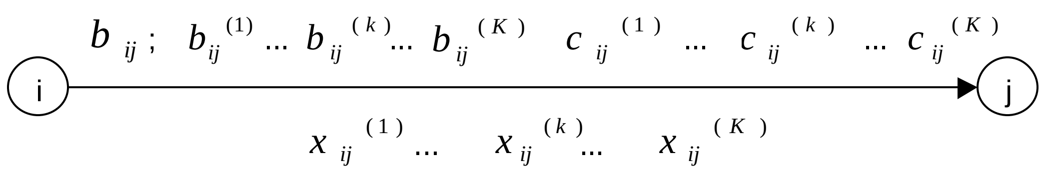
b_{ij} = The upper unit capacity from node i to node j

Unimodularity dictates that the linear pure network flow problem always has an integer solution (assuming that the data are integer) (Ahuja, *et al.*, 1993:543; Bazaraa, *et al.*, 1997:429). A *unimodular matrix* is a matrix that has the property whereby every square submatrix of the matrix has determinant +1, -1, or 0 (Bazaraa, *et al.*, 1997:429). The integrality property does not extend to multicommodity network flow problems, however, due the addition of the mutual capacity flow constraints (Ahuja, *et al.*, 1993:651).

Multicommodity Network Flow

Multicommodity network flow problems are a generalization of pure network flow problems; more than one commodity, distinguished by commodity type or origin-destination pair, share the same network. Multicommodity network problems exist whenever different commodities are shipped simultaneously from their respective origins to their respective destinations throughout the same topological make-up. The multiple commodities compete for the mutual arc capacities in the network. Goldberg, Oldham, Plotkin and Stein describe the minimum cost multicommodity network flow problem as the simultaneous shipping of commodities through a single network, while total flow obeys mutual and individual arc capacities at minimum cost (Goldberg, Oldham, Plotkin and Stein, 1997:1). Figure 2 depicts a typical arc in a minimum cost multicommodity network flow problem (Jewell, 1966:9).

Any arc from a minimum cost multicommodity network flow problem is defined by its own characteristics. In Figure 2, b_{ij} is the mutual flow capacity shared by each commodity. Network feasibility requires that the sum of flow of each commodity be



$i,j = 1,2,\ldots,N$
$k = 1,2,\ldots K$

$x_{ij}^{(k)}$ - non-negative flow of type k in (i,j)

$b_{ij}^{(k)}$ - individual flow capacity of type k

b_{ij} - mutual flow capacity of branch (i,j)

$c_{ij}^{(k)}$ - per-unit cost of type k flow

Figure 2. Representative Arc From a Minimum Cost Multicommodity Network Flow Problem

less than or equal to the capacity of the arc. The multicommodity network flow problem makes it “possible for the per-unit costs of the different kinds of flows to be different, even in a ‘shared’ branch” (Jewell, 1966:7). Equation (6) represents the formulation of the mutual capacity flow constraints in the mathematical formulation of the minimum cost multicommodity network flow problem.

Minimize:

$$\sum_{(i,j)\in A}\sum_{l\le k\le K} c_{ij}^k x_{ij}^k \tag{4}$$

Subject to:

$$\sum_{\{j:(i,j)\in A\}} x_{ij}^{\ k} - \sum_{\{j:(j,i)\in A\}} x_{ij}^{\ k} = r_i^{\ k}\ ,\ \forall i\in N, \forall k\in K \tag{5}$$

$$\sum_{k\in K} x_{ij}^k \le b_{ij}\ ,\ \forall (i,j)\in A \tag{6}$$

$$x_{ij}^k \ge 0\ ,\ \forall (i,j)\in A\ ,\ \forall k\in K \tag{7}$$

where,

N = Set of n nodes

A = Node-arc incidence Matrix

K = Set of k commodities

J = Set of j arcs

R = Set of r constraints

$c_{ij}^{\ k}$ = Unit flow cost of commodity k on arc (i, j)

$x_{ij}^{\ k}$ = Amount of flow of commodity k on arc (i, j)

$r_i^{\ k}$ = Supply/demand of commodity k at node i

b_{ij} = Capacity of arc (i, j)

Generally two assumptions are made with multicommodity flow problems: goods are homogeneous, and there is no congestion (Ahuja, *et al.*, 1993:649). *Homogeneous* *goods* assume that each commodity uses a single unit of capacity of each arc for every unit flow; however, in this thesis this assumption is not made (Assad, 1978:45; Ahuja, *et al.*, 1993:650). *No congestion* assumes that there exits a fixed upper bound on each arc and the cost on each arc is linear in the flow on that arc (Ahuja, *et al.*, 1993:650).

The basic solution methods used to solve minimum cost single commodity network flow problems can be modified to solve the multicommodity case. The bases for multicommodity networks are any basis that contains a spanning tree for each commodity (Detlesfsen and Wallace, 1998:1). If the commodities do not interact in any way, then each single-commodity problem can be solved separately using classic techniques (Ahuja, *et al.*, 1993:649). Linear programming also may be used to solve multicommodity network flow problems, although more practical algorithms may exploit both the block structure of the multicommodity formulation and the structure of each block of flow constraints. This is evident in the Dantzig-Wolfe decomposition solution method (Goldberg, *et al.*, 1997:1; Chardaire and Lisser, 1999:1). Further, the network structure of each block of flow constraints may be manipulated with the relaxation of the coupling constraints (Chardaire and Lisser, 1999:1).

The multicommodity flow problem can be thought of as a capacity allocation problem; commodities are competing for the fixed capacity b_{ij} of every arc(i,j) of the network (Ahuja, *et al.*, 1993:653). Solution approaches begin by allocating portions of the shared capacity to all of the commodities, and then use information gathered from the solution to the single commodity network flow problems to reallocate the portions of the shared capacities in a way to improve the overall system cost and meet mutual capacity constraints (Ahuja, *et al.*, 1993:653).

Price-Directive Decomposition.

In general, a price-directive decomposition method decomposes the problem into a master program and several subprograms, and then coordinates between the two problem goals by changing the objective functions (prices) of the subprograms (Kennington, 1978:220). The coupling constraints of a multicommodity network problem introduce complexity into the solution. If these constraints were ignored, then the multicommodity network problem would decompose into a shortest path solution for each commodity. If these decomposed shortest path problems were solved and none of the coupling constraints were violated, an optimal solution would be present. On the other hand, if the coupling constraints were violated, then modifications on the routing costs would need to be iteratively updated. This is accomplished by placing prices on each use of a link, which correspond to the violated bundle constraints. New shortest path solutions are then iteratively solved, where each new shortest path solution corresponds with the modified routing costs (Chardaire and Lisser, 1999:4). This iterative process alternates between shortest path computation and price computation. At each iteration, the shortest path computed is used in conjunction with previous iterations to determine new prices to find feasible solutions (Chardaire and Lisser, 1999:4).

An example of a price-directive decomposition is Lagrangian relaxation. This method uses the dual function to place Lagrangian multipliers (prices) on the coupling constraints in the objective function. This dual function is concave and continuous but non-differentiable, and places a lower bound on the optimal value of large linear programs (Chardaire and Lisser, 1999:4). A subgradient optimization procedure may be used to maximize the function to find the appropriate prices (Lagrangian multipliers, λ).

The subgradient optimization is an iterative approach that converges to the Lagrangian function's lower bound. The Lagrangian relaxation removes capacity constraints and instead "charges " each commodity for the use of the capacity of each arc (Ahuja, *et al.*, 1993:652). This algorithm develops bounds on the optimal objective function.

Lagrangian relaxation is applied to the following minimum cost single commodity problem.

Minimize:

$$\mathbf{cx} \tag{8}$$

Subject to:

$$A\mathbf{x} = \mathbf{b} \tag{9}$$

The Lagrangian relaxation application of Equations (8) and (9) problem:

$$L(l, x) = \mathbf{cx} + l(A\mathbf{x} - \mathbf{b}) \tag{10}$$

and the dual function is defined by

$$w(l) = \text{Min}_x \{L(l, \mathbf{x})\} \tag{11}$$

where,

c = Cost vector for the arcs of x_{ij}

x = Vector of flow variables for the arcs

A = Node-arc incidence matrix

? = Set of Lagrange multipliers

The Lagrangian function is solved by using subgradient methods to "find good lower bounds on the optimal objective function value" (Kawatra, 1994:296). If Lagrangian relaxation is applied to multicommodity network flow problems then the relaxed problem reduces to solving, independently of one another, the *K*-commodity subproblems. The independent subproblems, without capacity restriction, can therefore be reduced to a shortest path problem with minimum cost.

Another example of a price-directive decomposition is Dantzig-Wolfe decomposition. This decomposition method exploits the network block-diagonal form structure of the flow constraints. Dantzig-Wolfe uses a node-arc formulation of multicommodity flow problems. This method associates a dual variable (price multiplier) with each constraint in the master problems. This procedure breaks up the constraints into a set of easy (network flow) constraints and a set of hard constraints (bundle constraints) (Ahuja, *et al.*, 1993:652).

The master problem is solved as a linear program yielding the set of prices for each subproblem. The subproblems are solved as single commodity, pure network flow problems to produce columns needed for the master problem. There are no flow bounds on the individual commodities other than the bundle constraints. At initialization, a basis matrix and the slack variables are used to produce a set of columns for the first reduced master problem. After each iteration of solving the reduced master problem to optimality, the set of shortest paths of the subproblems are solved to check for optimality, in respect to the full master problem. If the reduced costs of all variables are non-negative, assuming minimization, then an optimal solution is obtained; otherwise, the full master problem is exchanged for new columns of an updated basis, and the procedure repeats. Cremeans, Smith, and Tyndall used linear programming and column generation to solve multicommodity network flow problems that simultaneously considers network chain selection and resource allocation (Cremeans, Smith and Tyndall, 1970:269). The resources of the multicommodity network flow problem include equipment or other mobile assets that are required to accomplish flow on many arcs of the network (Cremeans, *et al.*, 1970:271).

Resource-Directive Decomposition.

Another solution approach to solving multicommodity problems is resource-directive decomposition, which allocates (*right-hand side allocation*) the joint capacity of each link to the commodities, instead of using prices to decompose the problem (Chardaire and Lisser, 1999:12). Resource-directive decomposition methods allocate the available resources from the master problem to the subproblems (Assad, 1978:50).

Partitioning Methods.

Partitioning techniques exploit the block structure of the matrices in the simplex algorithm. This simplex approach is commonly known as the *network simplex algorithm* (Ahuja, *et al.*, 1993:665). These matrices are triangular, enabling the system of linear systems to be solved by forward or backward substitution (Chardaire and Lisser, 1999:15). The network simplex algorithm applied to multicommodity network flow problems requires the maintenance of a partitioning of the basis matrix. Examples of work with partitioning methods include Goffin, Gondzio, Sarkissian and Vial, formulated nonlinear multicommodity flow problems with convex costs. Further examples include, Farvolden, Powell, and Lustig, who used both primal partitioning and decomposition techniques to solve multicommodity network flow problems. Finally, Ford and Fulkerson solved the maximal multicommodity undirected network flow problem with a simplex computation for arc-chain formulation (Ford and Fulkerson, 1958; Farvolden, Powell and Lustig, 1993; Goffin, Gondzio, Sarkissian and Vial, 1995:1).

In 1985, McBride produced EMNET, which can solve generalized network problems with additional constraints and additional (complicating) variables (McBride, 1985:82). EMNET uses a primal partitioning network simplex algorithm to solve large multicommodity flow problems efficiently when coupled with a resource-directive decomposition heuristic (McBride, 1998:947). According to McBride, "the simplex method is the best way to solve large multicommodity flow problems when the network portion is isolated and exploited" (McBride, 1998:954).

A study by Castro and Nabona crosses all three general types of solution methods for multicommodity network flow problems (Castro and Nabona, 1996:37). Castro and Nabona produced PPRN, a code that uses primal partitioning and the Langrangian relaxation method, for solving the multicommodity network flow problem with a linear or nonlinear objective function (Castro and Nabona, 1996:37). In addition, Zenios, Pinar, and Dembo develop a methodology for large scale optimization problems with embedded network structures by using a linear-quadratic penalty (LQP) function as a multiplier to decouple side constraints and produce a sequence of differentiable, but non-separable, linear network problems (Pinar and Zenios, 1992; Zenios, Pinar and Dembo, 1995:220). Examples involving network flows include (Schneur, 1991:47-61; Ahuja, *et al.*, 1993:8):

- The transmission of messages in a communication network between different origin-destination pair
- Transportation of passengers from different origins to different destinations within a city
- Routing of nonhomogeneous tankers
- Optimal location of intermediate distribution facilities between plants and customers
- Railroad traffic scheduling problems
- Multi-vehicle tanker scheduling problem (Bellmore, Bennington and Lubore, 1971)

- The determination of the routing of circuits and construction of additional arc capacities in a communication network so as to satisfy forecasted circuit requirements at minimum cost (McCallum, 1977)

- The formulation of a reliability communication network problem as a multicommodity maximum flow problem (Tsujii, 1979)

To expand upon the first example, a standard telecommunication network houses telephone exchanges and transmission facilities as nodes, copper cables or fiber optic links represent arcs, and transmission of voice, video, and/or data would signify the flow of the commodities.

Communication Systems Fundamentals

Generally, communication systems involve sending information from a *source* node to a *destination* node. Technically, sending information from point to point involves the transmission of electromagnetic waves (Saadawi, Ammar and Hakeem, 1994:50). The transmissions of these electromagnetic waves span a range of frequencies from extremely low to extremely high (Saadawi, *et al.*, 1994:50). These electromagnetic waves are harnessed into an *information signal*. A *communication network* is a collection of *stations* (or devices) to transmit this information signal from source points to destination points via a *transmission medium*. For example, the information flow of telephone calls, characters, video, and so forth, in any communication network, represents a multicommodity flow if the flows share the same underlying network. These specific types of communication networks represent a country's infrastructure, and in turn, represent vital targets to a commander in a theatre. These communication networks could consist of such elements as: a telephone network, wide area data networks (WANs), local area networks (LANs), metropolitan area networks (MANs), radio networks, satellite networks, mobile phone networks, and cable television networks (Saadawi, *et al.*, 1994:21). Due to the vast size of a communication network, the underlying network must be utilized instead of solving the problem as a straight linear program. In order to separate the network properties, communication networks are decomposed into a number of subproblems (Kawatra, 1994:296).

Telecommunications

Telecommunications is "the art and science of communicating at a distance, especially by means of electromagnetic impulses, as in radio, radar, television, telegraphy, telephony, etc" (Pooch, 1991:3). According to the Chairman of the Joint Chiefs of Staff Instruction, telecommunications systems are "interconnected devices used to transmit and/or receive communications or process telecommunication…" (CJCSI 6510.01B, 1997:GL-15). Flood referred to telecommunications networks as large, programmable machines (Flood, 1997:402). Telecommunication networks are growing in importance in all countries: "Just as telecommunications has proved to be the fuel for the engine of growth in the developed worlds it will surely be the same in the twenty-first century for what are presently developing nations" (Winch, 1998:1). As of 1998, the telecommunications market approached US $1 trillion per year (Winch, 1998:1). Winch further states, "The telecommunications objective is to produce high-quality voice, video, and data communication between any pair of desired locations, whether the distance between locations is 1 or 10,000km" (Winch, 1998:1).

The signal (basis of telecommunications) contains the information that is traversed from point *A* to point *B* within a telecommunication system (Pooch, *et al.*, 1991:4). This signal can be transferred by "three basic electronic transmission media: radio (including space based systems), metallic wire, and fiber-optic cable" (Defense, 1995:viii). Likewise, Pooch, Machuel, and McCahn state that transmission media consist of open wire, cable, twisted pair, coaxial cable, optical fiber, microwave, troposcatter (radio HF), and communication satellites (Pooch, *et al.*, 1991:77). The signal transmitted over the media of a telecommunication system consists of voice, telemetry, and complex data messages (Pooch, *et al.*, 1991:61).

The *capacity* is the property that binds all of the commodities together on a network. Capacity in a telecommunications network is the "maximum information rate (called the channel capacity and represented as a rate, in bits per second) that can be transmitted over a given bandwidth [and] depends on the transmitted signal power and the noise characteristics of the channel (as well as the channel bandwidth in hertz)" (Saadawi, *et al.*, 1994:50).

There are data requirements when modeling telecommunication network systems. The following are the requirements for analysis of a communication network (Oettli and Prager, 1971:396-397):

1. Geometrical layout of the proposed network
    a. Nodes (communication centers)
    b. Links (communication channels)
2. Each link cost of a unit capacity (if modeled)
3. Each link capacity
4. Source centers

5. Demand centers

6. Routing congestion for day and night (optional)

The second requirement, if modeling of this cost is desired, represents the cost of installing an arbitrary capacity. Requirement 3 establishes that every link has an associated nonnegative capacity that depicts the allowable amount of flow that may traverse the link (Gomory and Hu, 1964:348). Gomory and Hu point out that, within a communication network, the branch capacities must be large enough to allow all flow of the different commodities to reach their destination simultaneously (Gomory and Hu, 1964:348). Usually in communication networks these message requirements vary with time.

For the purpose of this thesis, a telecommunication network is decomposed into four individual sub-networks:

1. The telephone communication system
2. The cellular phone communication system
3. The microwave communication system
4. The satellite communication system

Telephone System.

The standard telephone system is comprised of two components: the transmission facility and the switching system. The transmission facility is broken up into a local loop, which connects equipment from individual users, and trunk lines, which regulates traffic generated by large users. The customers are part of the local loop, which connects to equipment (*e.g.*, telephone, modem, and office exchange). The media of transfer for the local loop are wire-pair cables and fiber-optic cables. Generally, trunk lines connect two switching systems by a means of wire-pair cables, coaxial cables, microwave radio, satellites, or fiber optic (Saadawi, *et al.*, 1994:22). Switching systems are, generally, central offices and toll offices (Saadawi, *et al.*, 1994:21).

Switching systems connect circuits and route traffic through a network. Switching facilities remove the need for a direct line between each piece of equipment. Local and tandem switching systems are the main groups of a switching system in a telephone network. Local systems (central offices) connect customer loops to other customer loops. Tandem switching systems connect central offices to other central offices (Saadawi, *et al.*, 1994:22). Tandem switching is the long distance connection between local networks. Due to the long distance, the signal has to pass through amplifiers or repeaters.

<u>Cellular Phone Communication System</u>.

The cellular phone network is circuit-switching and uses radio frequency transmission (Saadawi, *et al.*, 1994:26). Circuit-switching serves the function of communication between two customers by capturing channels. A cellular phone network consists of three main parts: the user, the cell site, and the mobile telephone switching office (MTSO). The user is the actual mobile phone. The mobile phone sends a signal to a cell site, a transmission tower connected directly to a MTSO. Next the mobile telephone switching office routes mobile phone signals into the public telephone network (Saadawi, *et al.*, 1994:26-27). This interface system allows mobile phone users to communicate with other telephone users.

Microwave Communication System.

A microwave communication system is essentially composed of at least two microwave antennas and associated transmission facilities. Each microwave antenna is placed 30 to 60 kilometers apart by a line-of-sight transmission (Winch, 1998:139). Each line-of-sight transmission is a single hop. In order to send a radio signal over greater distances, multiple hops are required. Tropospheric scattering may be necessary in conjunction with microwave transmissions of distances over the horizon.

Satellite Communication System.

A satellite communication system consists of ground facilities and a space facility. In essence, a satellite communication system is a microwave communication with the satellite in space acting as the repeater station between the two ground facilities. Each earth-based station uplinks information to the satellite, and the satellite downlinks the information to earth stations (Saadawi, *et al.*, 1994:24-25). The use of the satellite system does not become cost competitive with microwave radio and optical fiber systems until distances greater than 500 km (Winch, 1998:3). Satellite communications and microwave mobile communications are inherently narrowband in nature when compared to optical systems (Winch, 1998:4).

Multiple Criteria Decision Making

Multiple criteria decision making (MCDM) is a means of solving a decision problem with potentially conflicting objectives (Schniederjans, 1995:10). Vincke defines a multicriteria decision problem as:

> A situation in which, having defined a set A of actions and a consistent family F of criteria on A, one wishes

1. to determine a subset of actions considered to be the best with respect to F (choice problem),

2. to divide A into subsets according to some norms (sorting problem), or

3. to rank the actions of A from best to worst (ranking problem). (Vincke, 1992:28)

Real-life problems give rise to a mixture of choice, therefore any decision often relies on one or several criteria and may not yield a single "best" solution as traditional single-objective optimization yields. Rather, real problems often meet a common ground among sorting and ranking of the objectives. MCDM is a methodology to formulate a statement of the problem. According to Korhonen, "Multiple criteria problems are more complex to solve than single ones, because a solution process has also to meet the requirements of behavioral realism" (Korhonen, 1992:550). Behavioral realism refers to focusing on the decision maker's actual behavior. Multicriteria problems are mathematically related to multiple-objective mathematical programs, with both "dealing with the multidimensional nature of control policies and the conflict-laden consideration of real-world decision-making problems" (Haimes and Li, 1988:54). A general model for MCDM is (Hajela and Yoo, 1999:209):

$$\min_{X \in \Omega} \bar{f}(X) \tag{12}$$

$$\bar{f}(x) = \{ f_1(x),\ f_2(x), \ldots,\ f_m(x) \} \tag{13}$$

where,

$$\Omega = \{ X \in R^n \mid g(X) \le 0, h(X) = 0 \}$$

Let $\bar{f}(X)$ and Ω denote the objective function and the feasible set, respectively (Hajela and Yoo, 1999:209). *N* represents the number of decision variables and *m* denotes the number of objective criteria that are to be minimized. The functions $g(X)$ and $h(X)$ define the inequality and equality constraints, respectively" (Hajela and Yoo, 1999:209). If functions $g(X)$ and $h(X)$ depend on x then they are linear. Equations (12) and (13) represent multiple objectives of equal weighting, and can be traded off subjectively. If criteria are going to be traded-off against each other mathematically, then a differential weighted composite objective function must be formulated, represented in Equations (14) and (16) (Hajela and Yoo, 1999:209).

$$f(X) = w_1 f_1 + w_2 f_2 + \ldots + w_m f_m \tag{14}$$

$$\sum_{i=1}^{m} w_i = 1 \tag{15}$$

According to Zionts there exist four sub-areas that make up MCDM, which are presented in Table 1, with their related goal programming equivalents. Linear goal programming is formulated and solved in this thesis.

Table 1. Four Sub Areas of MCDM (Zionts, 1992:567)

MS/OR Sub Areas	Related GP Topics
Multiple Criteria Mathematical Programming	Linear Goal Programming
Multiple Criteria Discrete Alternative	Integer GP and Zero-One GP
Multiattribute Utility Theory	Linear GP, Nonlinear GP, and Fuzzy GP
Negotiation Theory	GP Game Theory Models and Interactive GP

Goal Programming

Goal programming (GP) is an optional framework for conducting MCDM analysis. According to Ignizio, any problem that is a candidate of mathematical programming (optimization) is suitable for GP (Ignizio, 1985:15). Goal programming originated from linear programming. Single objective linear models are restricted to a single objective (*e.g.*, cost for minimization or profit for maximization) (Schniederjans, 1984:3). In a real world environment there is rarely a single objective; most of the time there are multiple, potentially conflicting objective problems (Schniederjans, 1984:3). Deckro and Hebert point out that "The conflicting nature of these objectives results in solutions that involve tradeoffs or compromise" (Deckro and Hebert, 1988:149).

A target level is a suitable level of near-achievement for any attributes warranted by a decision maker, rather than the target level that is satisfied exactly. A formulated objective is a mathematical expression of the attributes. An attribute and a target level make up a goal (Romero, 1991:1). The desires and aspirations of the decision maker make up the right-hand sides (target levels) of the goal; these may or may not be achieved (Romero, 1991:2). In contrast, the right-hand side of rigid constraints must be satisfied to avoid infeasible solutions.

The term "goal programming" first appeared in Charnes and Cooper's 1961 two-volume linear programming textbook, <u>Management Models and Industrial Applications of Linear Programming</u> (Charnes and Cooper, 1961). Charnes and Cooper first suggested the use of goal programming to solve infeasible linear programming (LP) programs (Charnes and Cooper, 1961:215-221).

Since GP is an extension of LP, GP models are based on the canonical form of general LP models given by Equations (16) to (18) (Schniederjans, 1995:2):

Minimize:

$$\sum_{j=1}^{n} c_j x_j \tag{16}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \text{, for } i = 1, \ldots, m \tag{17}$$

$$x_j \geq 0 \text{, for } j = 1, \ldots, n \tag{18}$$

Charnes and Cooper suggested that each constraint be transformed into a functional. These functionals are the transformed constraints, and represent the goals that are attempted to be satisfied. The individual constraints, b_i, are a set of goals that must be achieved in order to have a feasible solution (Schniederjans, 1995:3). Equation (19) is the functional of Equation (17).

$$f_i(x) = \left| \sum_{j=1}^{n} a_{ij} x_{ij} - b_i \right| \text{, for } i = 1, \ldots, m \tag{19}$$

Charnes and Cooper said these goals are to be achieved by minimizing the absolute deviation of the functionals (Charnes and Cooper, 1961). When infeasible solutions occur in a regular linear program, the overachievement and underachievement deviational variables of the target level are inevitable; minimizing the deviation produces the best solution (Schniederjans, 1995:4). Charnes and Cooper stated, “whether goals are attainable or not, an objective may then be stated in which optimization gives a result which comes ‘as close as possible’ to the indicated goals” (Charnes and Cooper, 1961:215).

There are six basic steps in the formulation of a preemptive linear goal-programming model. These steps are similar to that of a regular linear program but with slight additions. The steps are:

1. Define decision variables
2. State Constraints
3. Determine the Preemptive priorities (if applicable)
4. Determine the relative weights (if applicable)
5. State the objective Function
6. State the nonnegativity or given requirements. (Schniederjans, 1995:21; Schniederjans, 1984)

There are three fundamental formulations to consider when addressing goal programming models (Charnes and Cooper, 1977; Schniederjans, 1984; Schniederjans, 1995). The underlying difference between the models is the consideration of non-preemptive versus preemptive goals. This is a subtle difference in the actual notation of the model, but a fundamental difference between models. The basic non-preemptive LGP is:

Minimize:

$$\sum_{i \in m} (d_i^{+} + d_i^{-}) \tag{20}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j - d_i^{+} + d_i^{-} = b_i, \text{ for } i = 1, \ldots, m \tag{21}$$

$$d_i^{+}, d_i^{-}, x_j \geq 0, \text{ for } i = 1, \ldots, m; \text{ for } j = 1, \ldots, n \tag{22}$$

A positive deviation variable, d_i^+, represents an overachievement of a target level in the constraint (or goal) (Schniederjans, 1995:3). A negative deviation variable, d_i^-, represents an underachievement of a target level in the constraint (or goal), the underachievement of a goal (Schniederjans, 1995:3). Notice that the objective function, also called the achievement function, does not contain the traditional cost function (Ignizio, 1985: 25). Instead, linear goal programming is distinguished by placing the deviation variables directly in the achievement function of the model (Schniederjans, 1995:4). The mathematical formulation of the goal program allows for a solution that has an infeasible solution (Schniederjans, 1995:3). The deviation variables allow for the possibility of not obtaining the exact goals but still producing a feasible solution. The following complementary condition must hold: $d_i^+ \times d_i^- = 0$ (Schniederjans, 1995:6). In their basic formulation, the decision maker is indifferent to the achievement of any particular goal over another; non-attainment of the goals is to be equally reduced.

Sometimes there arise situations in which differential weighing of the deviation variables are preferred. For instance, it may be more preferable to underachieve the capacity of one node rather than overachieve the capacity of another node. Therefore, properly elicited differential weights would be assigned to the deviation variables of a given goal to allow for tradeoffs between the different goals.

The second basic model of goal programming includes differential weights to the deviations:

$$Minimize : X = \sum_{i \in m} (w_i^+ d_i^+ + w_i^- d_i^-) \tag{23}$$

This achievement function is subject to equations (21) and (22). The model is a construction of a non-preemptive model with differential weighted deviations from all goals placed in the achievement function. The weighted constants, w_i^+ and w_i^-, represent "the relative weight to be assigned to the respective positive and negative deviation variables" (Schniederjans, 1995:6). According to Schniederjans, differential weights are "Mathematical weights that are expressed as [numbers] (*i.e.*, represented as w_k, where k=1,2,…, k; l = 1,2,…, L) and are used to different the l deviational variables within a single k priority level" (Schniederjans, 1984:68). Within a goal, there exists a preference for obtaining the specific goal. The goals in the model are evenly ranked with regard to each other. However, what if the decision maker prefers particular goals to other goals? Then, as within a goal, there can exist weights between priority goals.

In 1965, Ijiri, introduced preemptive priority factors as a way of ranking goals in the objective function of the linear goal programming model and established the assignment of relative weights to goals in the same priority level (Schniederjans, 1995:6). Deckro and Hebert describe preemptive goal programming as "models to minimize the sum of the weighted deviations from a set of ordered (or prioritized) goals" (Deckro and Hebert, 1988:149). This third model minimizes the sum of the weighted deviations within a set of prioritized goals:

$$Minimize: X = \sum_{i \in m} P_i \sum_{k=1}^{n_i} (w_{ik}^{+} d_i^{+} + w_{ik}^{-} d_i^{-}) \tag{24}$$

The achievement function is subject to Equations (21) and (22). Let there exist w_{ik}^+, w_{ik}^- ≥ 0, which represent the relative weights to be assigned to each $k = 1,\ldots, n_i$ different goals within the ith category to preemptive goal of P_i is assigned. The set of ordered preemptive goals, P_i, serve only as a ranking symbol that can be interpreted to mean that no substitutions across categories of goals will be permitted (Schniederjans, 1984:68; Deckro and Hebert, 1988:149; Schniederjans, 1995:7). This means that the weight associated with the priority level of deviational variables is infinitely greater than the weight of the next lower priority level. The mathematical representation is as follows:

$$P_i(\text{Most Important}) > P_{i+1} > P_{i+2} >>> P_{i+k}(\text{Least Important}) \tag{25}$$

Further, it is assumed that no relative weighting attached to the deviation variable can consist of a combination to produce a substitution across preemptive goals in the process of choosing the x_j (Schniederjans, 1995:7). According to Chandler, "Within a given priority level, discrepancies are weighted according to their importance, relative to other discrepancies at that level" (Chandler, 1982:63). In addition, Chandler points out that "at a given priority level, all discrepancies must have the same unit of measure, but different levels can have different measures" (Chandler, 1982:63).

The selection of a differential weighting scheme for the individual goals is a primary concern with the use of goal programming (Deckro and Hebert, 1988:151). There are two major concerns that must be taken into consideration when modeling. The first concern is naïve relative weights, which occur in models when weights do not accurately reflect the true proportioned weight that is innate in the decision environment. This may be limited by existing weighting methods such as the analytic hierarchy process, conjoint analysis, and even multiple regression analysis (Schniederjans, 1995:28). Decision Analysis offers a number of techniques that may be used to develop weights. The second major concern is the incommensurability of goal constraints. This may occur when different measures of differing goals bias the iterative solution procedure in favor of the parameters that yield the largest reduction in deviation. The bias may be minimized naturally if priority goals are kept the same measures within each priority goal. If priority goals are not used then scaling or normalizing of goal constraint parameters limit the amount of incommensurability of goal constraints (Schniederjans, 1995:37).

Post-Optimal Analyses

Traditionally, sensitivity analysis arises when no alternatives are optimal. The following is a possible list of optimal model analysis to be performed (Ignizio, 1985:63):

1. Changes in the weighting at a priority level
2. Changes in the weighting of deviation variables within a priority level
3. Changes in right-hand-side values
4. Changes in technological coefficients
5. Changes in the number of goals
6. Changes in the number of decision variables
7. Reordering preemptive priorities.

For the purpose of this study, both the reordering of preemptive priorities and changes in selective right-hand-sides are explored. The reordering of preemptive priorities is conducted by obtain solutions for the possible combinations of the ordering of the preemptive goals. The solution sets are then analyzed for comparison and insight.

For large-scale network problems, discrete changes in the right-hand-side become ambiguous; instead, continuous, parametric analysis is usually conducted. This is explained further in Chapter 3.

Past AFIT Academic Research Dealing with Network Flows

An Approach to Disrupting Communication Networks.

Pinkstaff developed a methodology to identify and evaluate candidate target sets for a communications network. Pinkstaff utilizes value focused thinking and multiple objective decision analysis to develop node and arc costs for a minimum cost and a near-minimum cut-set algorithm to produce candidate target sets (Pinkstaff, 2001).

A Network Modeling Tool.

Leinart developed a methodology which quantitatively measures the value of each target set in achieving an objective is needed; assuming a given a network disruption has been identified (Leinart, 1998; Leinart, Deckro, Kloeber Jr. and Jackson, 2002:). Leinart focuses solely on a voice medium in a telecommunication network. Leinart defines two critical parts to a network disruption: "the severance or hindrance of information flow, and the nodes between which this information flow is to be affected" (Leinart, 1998:3). Leinart used a visual Basic/Excel environment to transform an undirected graph into a new graph. This transformed graph is represented as a vertex adjacency list. He used a notional voice telecommunications network, which encompassed ground, cellular, radio, and satellite.

Modeling and Analysis of Social Networks.

Renfro developed a methodology for modeling and analysis of social networks. Renfro uses a flow model with goal programming to capture and represent the multiple objectives inherent in complex social behavior within social networks. Renfro states, "Social networks depict the complex relationships of individuals and groups in multiple overlapping contexts. Influence in a social network impacts behavior and decision making in every setting in which individuals participate" (Renfro, 2001:x).

Other Academic Research Dealing with Network Flows

Scaling Algorithms for Multicommodity Flow Problems and Network Flow Problems with Side Constraints.

Schneur, uses "scaling and e-optimality, together with penalty function methods, to develop algorithms for multicommodity network flow problems and network flow problems with side constraints" (Schneur, 1991:2). Schneur relaxes the coupling constraints and additional side constraints and then places a quadratic penalty term for their violation to the objective function. The scaling algorithm solves the resulting nonlinear function subject to the non-relaxed constraints.

Summary

This chapter discussed the fundamental concepts and literature underlying this thesis work. The literature opened with the discussion of the field of network flows and led into the application of multicommodity network flow models. Modeling techniques and post-optimal analysis options were highlighted to enrich the modeling capability of multicommodity network flows. This chapter built the foundation to develop the application of these concepts into a methodology, discussed in Chapter 3.

III. Methodology

This chapter presents a methodology for modeling and analysis of multicriteria telecommunications networks. This procedure demonstrates a five-step process from model setup to output generation. Figure 3 outlines the flow chart of the methodology.

Research Framework

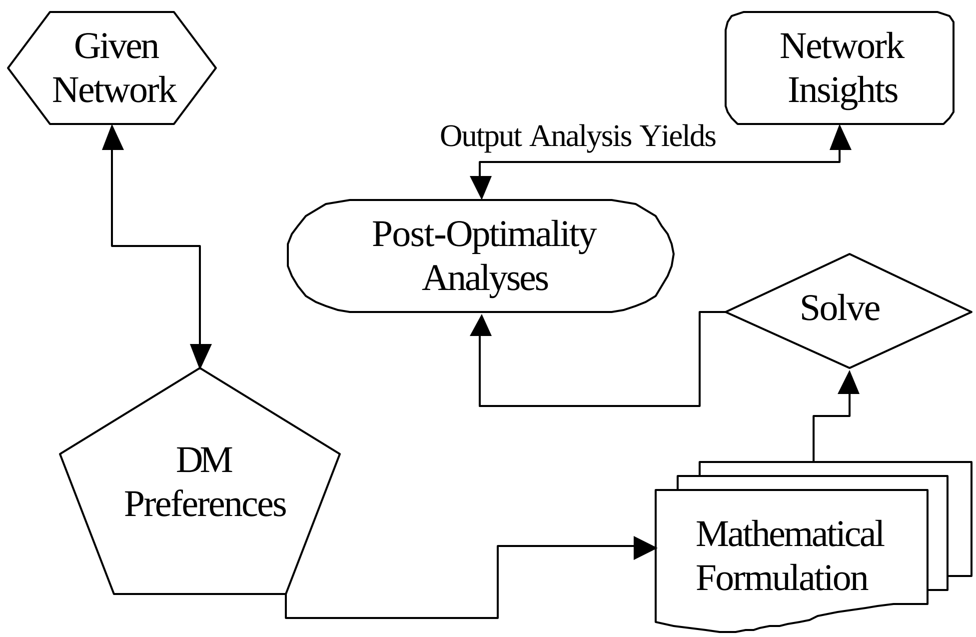


Figure 3. Methodology flow Chart

The first step of the methodology is to obtain the necessary information regarding the existing telecommunications network of interest for analysis. The interested organization must provide the required knowledge and information relating to the topology, attributes, and preferences of the telecommunications network to model and analyze the scenario of concern. Third, the mathematical formulation of the model is constructed, the proper solution approach is selected, and the model is solved. Fourth, post-optimal analyses are conducted on the model output. Sensitivity analysis conducted by a reordering of preemptive goals might be done to analyze changes in the configurations of the model. Parametric analysis of selective right-hand-sides is undertaken to identify possible trade-offs between the commodities of interest and selective right-hand-side values of capacities. The information collected may be depicted numerically or as a graphical representation of the trade-offs of interest. Finally, all analyses are consolidated and pertinent information is extracted. The insight is then provided to the decision maker.

Network Problem

The first step entails eliciting the knowledge of the components of the telecommunications network of interest. Figure 4 identifies the minimum data required to conduct the analysis described in this study.

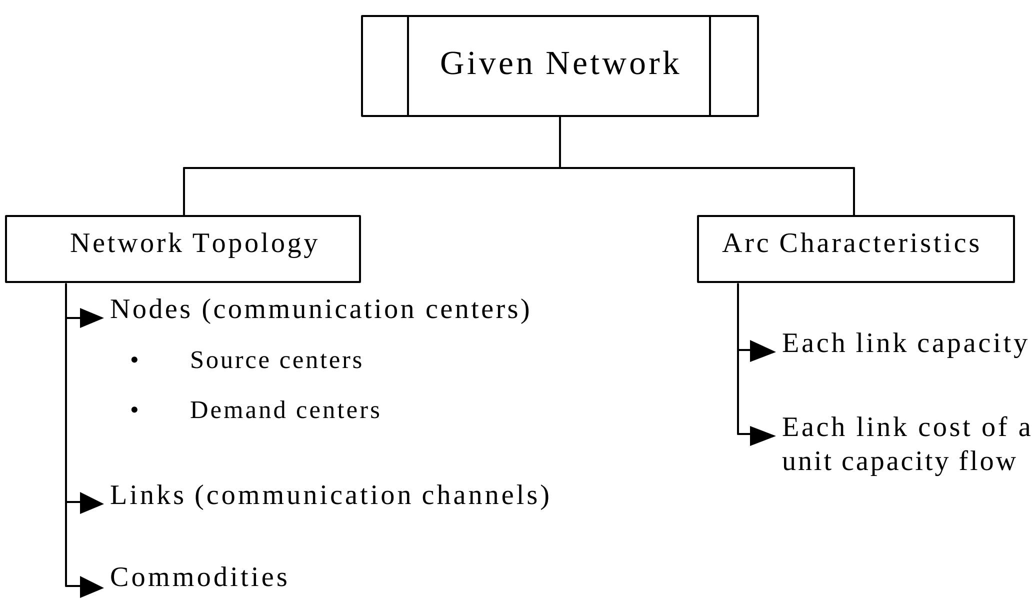


Figure 4. Network Components

The topology of the network must be ascertained; this includes all nodes, representing the equipment, and all links, representing all media types. The possible types of equipment include: transit switches, satellite facilities, a radio facilities, add/drop repeaters, and terminals. The media types include: fiber optic cable, coaxial cable, troposcatter transmission, microwave transmission, and satellite transmission. The given network may be undirected, however it is mathematically described as a directed network in the node-are incidence matrix. The commodities of interest must be identified and described in unit capacity cost of flow (*i.e.*, bits/sec). A voice channel may take up 64 kbits/sec of capacity as opposed to video, 100 kbits/sec. Furthermore, each arc in the network must state an actual or estimated arc maximum capacity (bits/sec). Finally, the source nodes of all commodities and demand nodes of all matching commodities must be identified. Once these basic components are identified, the decision maker constructs the preemptive goals for inspection.

Goal Construction

Goal programming (GP) is a framework for modeling multicriteria decision-making. Explicit weighting of objectives, generalized prioritization of objectives, or no weighting may be used. The decision maker must specify any preemptive goals for investigation. The decision maker may want the goals to disrupt at least 50% of all arcs coming out of command bunkers and to allow only data to be relayed through identified radio towers. Additionally, the goals may be rank-ordered from most preferred to least preferred. The extracting of the proper elements of a model is a key step. Revealing preferences and goals can be an entire project in itself. While not a trivial step, the analyst assumes that the necessary data has been developed. Once this information is provided, a mathematical representation of the network and preferences can be constructed.

Mathematical Formulation

The mutual coupling constraints and/or side constraints destroy the pure network structure of multicommodity network flow problems. Additionally, goal constraints cloud the network structure of a minimum cost multicommodity network flow problem.

McGinnis and Rao point out that the addition of goal programming constraints in network problems "obliterates the problem's natural network structure" (McGinnis and Rao, 1977:243). McGinnis and Rao suggest a way to use the framework of the additional goal programming constraints to isolate the pure network structure. They suggest that by using the partial Langrangian duality approach, the network structure can once again be isolated. McGinnis and Rao proposed this concept in the context of a minimum cost single commodity network flow problem with goal programming.

The same methodology, however, may be extended to goal-based multicommodity network flow problems. Goal constraints may be viewed as additional side constraints. Therefore, a Lagrangian relaxation approach may be applied to both the mutual coupling constraints *and* the goal constraints. This relaxation approach to a goal based multicommodity network flow problem retains and isolates the pure network structure of the underlying network flow model. This framework, while considering goals, facilitates the employment of network algorithms to solve the goal programming multicommodity network flow more efficiently than classic methods such as the *simplex* *goal programming method* (McGinnis and Rao, 1977:243). Figure 5 depicts the flow chart needed for the mathematical formulation.

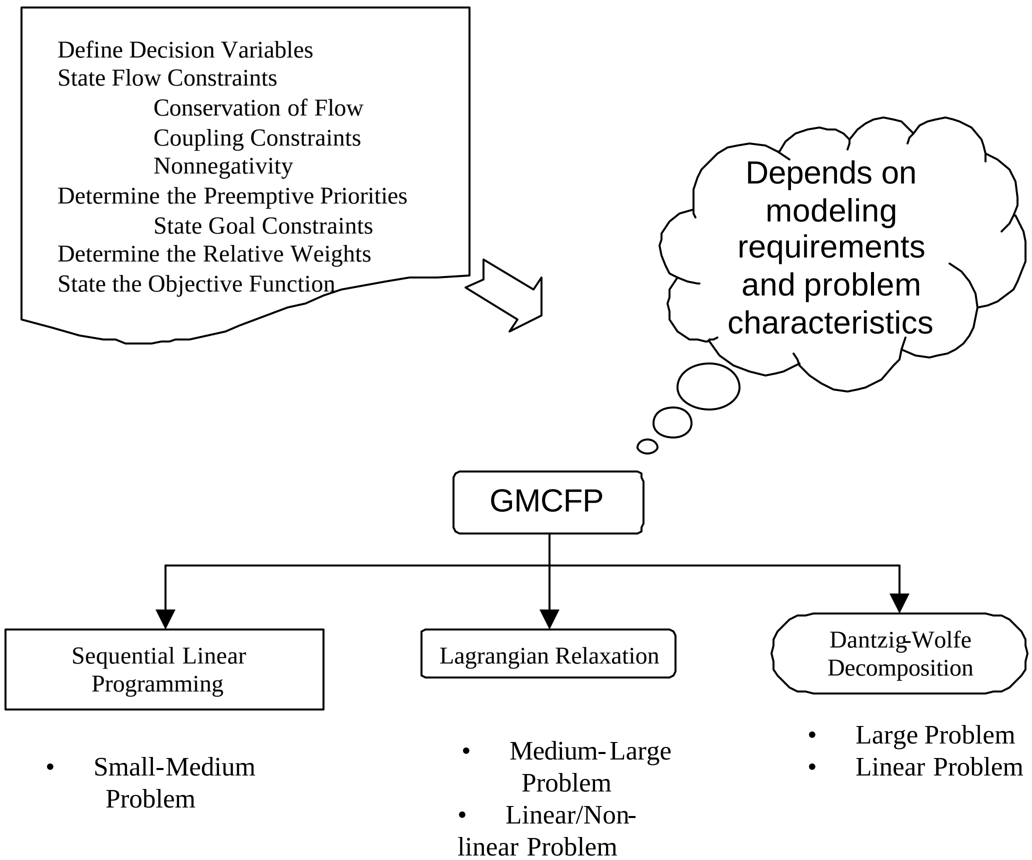


Figure 5. Mathematical Formulation Flow Chart

After the mathematical formulation is established, the appropriate solution technique is implemented depending on modeling requirements and problem characteristics. Linear goal programming may be used on relatively smaller size problems. As the problem of interest grows in size Lagrangian relaxation or Dantzig-Wolfe decomposition is implemented in conjunction with network optimization. If the problem characteristics are linear, then Dantzig-Wolfe decomposition of the linear problem may be preferred; otherwise, Lagrangian relaxation is used. If the *no congestion* assumption does not hold then the problem may be non-linear. Lagrangian relaxation would be the suitable solving option.

For some operational settings, differential weighing of the deviation variables may be preferred. There may be a goal to further restrict a capacity, expressed as an aspiration level in a goal constraint, rather than leave excess capacity at a given node. Therefore, differential weights could be assigned to the deviation variables of a given goal. This depends on the preferences of the decision maker. The overachievement of an aspiration level may be more heavily preferred than the underachievement. There may be other situations where an absolute ranking between the goals is required. In such cases, a model of preemptive goals with differential weighting within each goal would be in order. Once the decision maker defines the preemptive goals, they may be mathematically defined and added to the previous formulation. Equation (4) transforms into Equation (26) and additional goal constraints are added:

$$Minimize: \sum_{1 \le k \le K} \sum_{r \in R} P_r \sum_{q=1}^{Q} w_q d_q^{\pm} \tag{26}$$

Subject to:

$$\sum_{j:(i,j)\in A} x_{ij}^k - \sum_{j:(j,i)\in A} x_{ji}^k = r_i^k, \ \forall i \in N, \forall k \in K \tag{27}$$

$$\sum_{k \in K} x_{ij}^k \le b_{ij}, \ \forall (i,j) \in A \tag{28}$$

$$Gx^k = g^k \tag{29}$$

$$x_{ij}^k \ge 0, \ \forall (i,j) \in A, \ \forall k \in K \tag{30}$$

where,

A = Node-arc incidence Matrix

K = Set of k commodities

J = Set of j arcs

R = Set of r constraints

Q = Set of q goals

P_i = Preemptive priority goal $P_1 >>> P_2 >>> ... >>> P_i$

w_q = Represent the relative weight to be assigned to the respective positive/negative deviations

$d_q^{\pm}$ = Represents the overachievement or underachievement of the q priority

c_{ij}^k = Unit flow cost of commodity k on arc (i, j)

x_{ij}^k = Amount of flow of commodity k on arc (i, j)

r_i^k = Supply/demand of commodity k at node i

b_{ij} = Capacity of arc (i, j)

The additional equations, $Gx^k = g^k$, represent the augmented constraints to construct the goals of the network. A number of options are available to solve the goal multicommodity flow model. This multicommodity problem can be solved with linear programming methods, if the network is relatively small. However, the majority of communication networks are vast in size; it would be advantageous to exploit the embedded network properties of the problem. Due to the nature of goal programming, the goals, Equations (28) and (29), mitigate the pure network properties of the network, preventing the direct use of multicommodity network codes. Most researchers have simply solved single-commodity flow problems as linear programs, foregoing the potential computational advantage of exploiting the underlying network structure. McGinnis and Rao suggested a method to preserve the pure network structure of minimum cost network flow problems with goal programming (McGinnis and Rao, 1977). The methodology in this study extends their work to multicommodity flow problems.

Partial Lagrangian Relaxation.

Lagrangian relaxation, if applied judiciously, maintains the underlying network structure (*i.e.*, unimodularity) of subproblems. The Lagrangian relaxation formulation is given as:

$$\text{Max}_{l}\ L(l, \mathbf{x}) = \text{Min} \sum_{(i,j)\in A} \sum_{1\le k\le K} c_{ij}^{k} x_{ij}^{k} + \sum_{(i,j)\in A} l_{ij} \Big(\sum_{k\in K} x_{ij}^{k} - b_{ij}\Big) + \sum_{1\le k\le K} l_r P_r \sum_{q=1}^{Q} (\mathbf{G}\mathbf{x}^{k} - \mathbf{g}^{k}) + w_q d_q \tag{31}$$

Subject to:

$$\sum_{j:(i,j)\in A} x_{ij}^{k} - \sum_{j:(j,i)\in A} x_{ji}^{k} = r_i^{k}, \quad \forall i \in N, \forall k \in K \tag{32}$$

All variables nonnegative

where,

l = The vector of Lagrange multipliers

A subgradient optimization algorithm may be used to solve this Lagrangian relaxation formulation. A decomposition of this Lagrangian relaxation could also be formed, fully utilizing the underlying pure network structure. The Lagrangian relaxation is usually applied to non-linear functions and could be tedious to implement. If the

Lagrangian-relaxed problem may be solved as a linear program, then the formulation is linear. Performing decomposition of Lagrangian relaxation, while the formulation being linear, is approximately the same as performing Dantzig-Wolfe decomposition of the linear problem (Minoux, 1986:364).

There exist various ways to model a multicommodity network flow problem with multiple objectives that offer the ability to capture different elements. When modeled, the systems have different possible solution approaches. Once an approach has been decided and implement, and an optimal solution is established, sensitivity/parametric analysis may be conducted.

Sensitivity Analysis

Using one of the solution approaches presented in this chapter, a minimum cost multicommodity network flow problem with multiple goals is formulated and solved. However, a solution to a mathematical formulation may not provide all the necessary information to the decision maker. In addition, there may exist concerns regarding data accuracy and the robustness of a solution. Therefore, post-optimal analysis may be conducted on the range of the optimal solution. Sensitivity analysis investigates the effects of discrete parameter changes of a single factor on the optimal solution. The operations research literature is rich with applications of post-optimality analysis of linear programs. Post-optimality analysis allows the analyst to test the robustness of the model, its assumptions, and the values of its parameters. A sensitivity analysis can be tailored to the key aspects of a scenario deemed by a decision maker, or anticipated scenarios of interest. Linear goal programming sensitivity analysis is conducted with more restrictions.

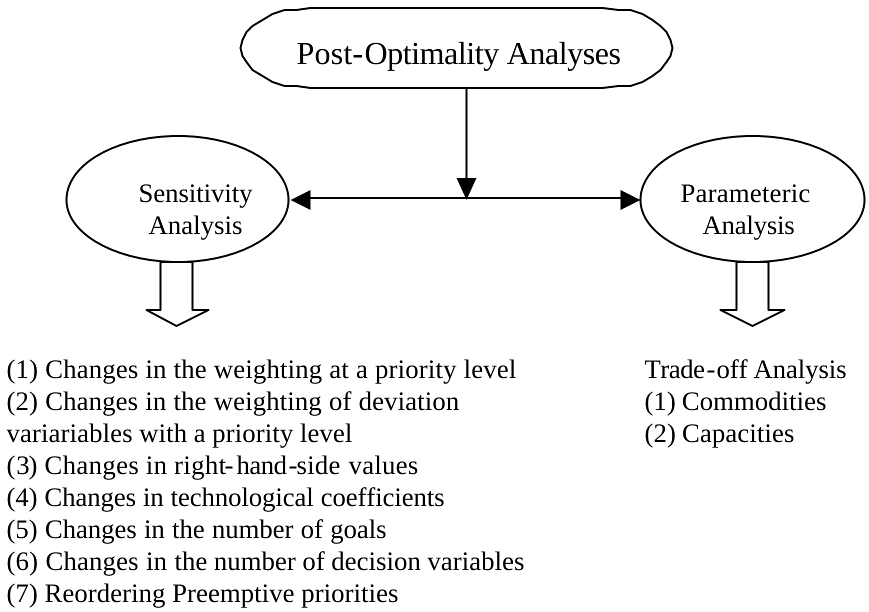


Figure 6. Post-Optimal Analyses Flow Diagram

As outlined in Chapter 2, for this study two particular post-optimality areas are demonstrated:

1. Reordering of preemptive priorities

2. Parametric analysis of selective RHS values of capacities and trade-offs of commodities, in regards to the right-hand sides of the supply-demand constraints

<u>Reordering of Preemptive Priorities</u>.

Sensitivity analysis gives the ability to ask "What if?" questions of changes in the problem statement and to more effectively grasp the notion of both certain and uncertain aspects of the model. Sensitivity analysis of the reordering of preemptive priorities may reveal restrictions a higher preemptive goal places on a lower preemptive goal. The reordering of preemptive priorities may be performed by setting up multiple runs with a different priority scheme of the preemptive goals. The solutions from each preemptive goal scheme will depict how the flow of the commodities attains the goals differently. This will bring insight to the decision maker, particularly in situations where the results of the prioritizations have resulted in unintended consequences.

Changes of Differential Weights Within a Priority Goal.

The differential weighting of goals within a priority goal of a goal program tends to be uncertain, as pointed out in Chapter 2. Therefore, sensitivity analysis of the weighting must be undertaken to identify the appropriate range of differential weighting among within a priority goal. As with the reordering of the preemptive goals, the differential weighting of goals within a priority goal may be merely conducted by setting up runs with different weighting schemes. The resulting solutions provide insight into the range of the optimal solution of the flow of the commodities. For instance, the operating ranges of the number of commodities that maintains a certain attainment level of a goal, or the alternate routes that maintain the same attainment level of a goal provide additional network insight.

Changes to the Right-Hand Sides.

Parametric analysis may be conducted on right-hand sides of large-scale networks to identify ranges of optimality. Parametric analysis studies the effect of predetermined continuous parameter changes on the optimal solution, such as the availability of resources (Taha, 1992:167). There are two cases of parametric analysis: (1) a parameter in the achievement function and (2) a parameter in the right-hand side column. Parameterization of selective RHS capacities of arcs and parameterization of the conservation of flow constraints for supply and demand are the focus of the parametric analysis illustrated in this study.

Changes in Right-Hand-Sides.

Let $\overline{b}$ represent the vector of all right-hand-side resource of interest that are to be parameterized. The right-hand-side resource is parameterized to be **b** + **?ß** , where **ß** is a vector of relative rates of change in b_i as a linear function of **?** (Taha, 1992:171). The new value of the right-hand-side resource, $\tilde{x}_B$, is equal to B^{-1}(**b** + **?ß**). The variable **?** may be added as a variable to the minimum cost multicommodity network flow problem with multiple objectives and solved by the same method of choice. The right-hand-side resource of interest would just need to be augmented with **?** and **ß**.

Network Insight

Solution information from the goal multicommodity network flow problem, coupled with the information from the post optimality analyses, provides an array of insights. For example, information from the parameterized problem may be consolidated and presented as a graph. The information may provide insight as to trade-off in capacity between nodes and/or arcs. In Chapter 4, the methodology is implemented on a notional network of a fictitious scenario involving a rogue nation.

IV. Results and Analysis

Introduction

In this chapter a notional telecommunications network for a fictitious country is modeled and analyzed. This notional network was taken from Pinkstaff's thesis (Pinkstaff, 2001). His scenario and communications network were used as a baseline topology, but were modified to provide illustrations of this approach.

Scenario and Mission Objective

Within this scenario there exists a fictitious rogue nation surrounded by two allied countries: allied country A and allied country B. The United States is in the pre-hostility stages for military action against this foe. The theatre commander has stated a goal of monitoring communications between the enemy's military headquarters and various enemy field command bunkers eluding detection. For the United States to achieve this goal, armed forces should couple with the enemy's telecommunications network infrastructure using a minimum number of assets. Mission requirements dictate specific required minimum disruption of various types of communication facilities and links. A military team is inserted into allied country B and allied country A. Targeted Network.

To model and analyze the rogue nation's telecommunications network, the establishment of a minimum requirement of information must be satisfied. It is assumed that the proper target portfolios have been developed for the network. Figure 7 provides a graphical depiction of an undirected telecommunications network.

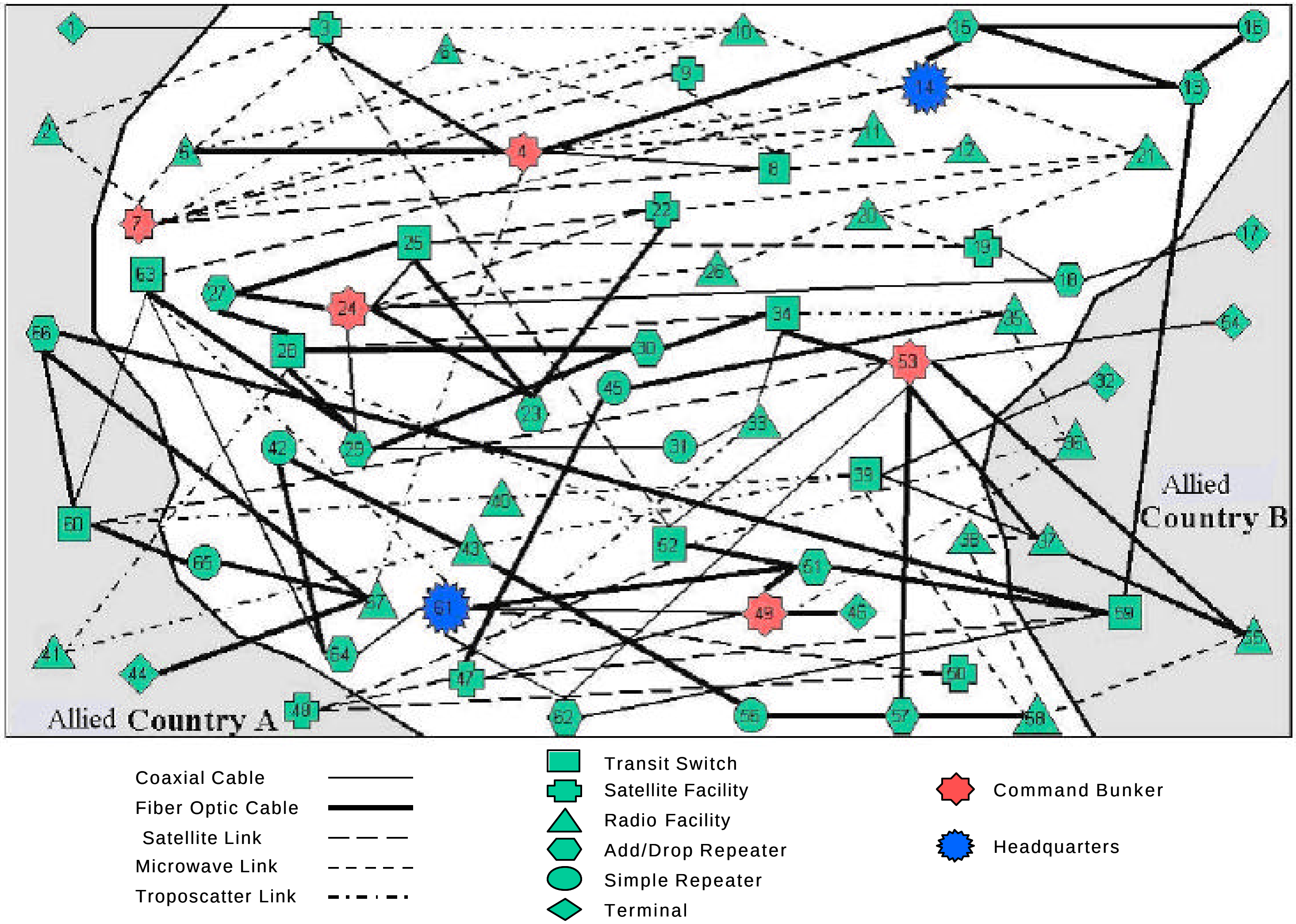


Figure 7. Notional Telecommunications Network

As with almost any nation's infrastructure, portions of the telecommunications network are dedicated to military and government, civilian, and a mixture of both military/government and civilian. The rogue nation's telecommunications network infrastructure consists of 67 nodes; each node contains one or more of the following types of equipment: transit switch, satellite facility, radio facility, add/drop repeater, and terminal. Represented in the set of nodes are also two headquarters and five command bunkers. Additionally, the network contains 119 undirected arcs; each arc represents one of the following media types: fiber optic cable, coaxial cable, troposcatter transmission, microwave transmission, and satellite transmission. All media types in the telecommunications network accommodate three types of commodities, voice, video, and data transmissions, throughout the country.

As mentioned in Chapter 2, the node-arc incidence matrix is arbitrarily assigned 119 directed arcs (located in Appendix A). Table 2 displays the link capacities and message sizes.

Table 2. Link Capacity and Message Size

	Link Capacities					Message Size		
Capacity	Fiber Optic	Coxial	Troposcatter	Microwave	Satellite	Voice	Video	Data
Mbits/sec	51.840	44.736	44.736	3.360	1.544	0.100	0.248	0.025

Intelligence shows that the three commodities that traverse the network are voice, video, and data. The network is modeled as a basic broadcast package. All messages (voice, video, and data) originate from the sole source node at headquarters 14. Command bunkers 4, 7, 24, 49, and 53, and headquarters 61 receive the messages and are represented as the demand nodes in the mathematical formulation. The arc costs are an arbitrary cost to monitor messages per unit flow. They may be any appropriate allied or opposition "cost" factor provided they have been developed in a method appropriate for use as coefficients in linear programs. The necessary requirements have been stated and the goals may now be established.

Decision Maker's Goals.

The theatre commander's goals may be extracted from the mission objectives. The overall objective is to minimize total cost of monitoring message flow. This is equivalent to the minimum amount of risk that the two teams endure while monitoring the network. This notional value is 945.05 if a multicommodity minimum cost flow algorithm is solved. The theatre commander is only willing to allow a maximum risk of 2000 to his two teams, combined.

In addition, the mission objective is to analyze the effects of disrupting the flow of voice, video, and data of the rogue nation's telecommunication network, but not to the point of detection. The team inserted into allied country A is tasked to disrupt at least 60% of capacity of all arcs housed at node 59 without being detected. Node 59 has three fiber optic cables connected to the transit switch, one coaxial cable, and one satellite link.

The third preemptive goal is to disrupt at least 20% of capacity of all arcs at facility 63. Facility 63, a transit switch, has one fiber optic cable, one satellite link, one troposcatter link, and two coaxial cables connected to the transit switch. In addition, the third preemptive goal target is to disrupt at least 40% of arcs housed at facility 62. The terminal at facility 62 has three coaxial cables connected to it. The theatre commander prefers to disrupt facility 63 to facility 62, at a ratio of 3-to-2. Since all goals are now established, the mathematical formulation of the network may be presented.

Mathematical Formulation

The information from the previous two sections may now be used to formulate a mathematical model. The nodes and arcs define the node-arc incidence matrix. All arcs must be converted into directed arcs, so the node-arc incidence matrix be a 67 by 119 matrix. Arcs are identified using a labeling scheme instead of being labeled node *i* to node *j*. The following is a formulation of a minimum cost multicommodity network flow problem (Chardaire and Lisser, 1999):

$$Minimize: \sum_{k=1}^{K} \sum_{j=1}^{J} c_j^k \left| x_j^k \right| \tag{33}$$

Subject to:

$$Ax^k = r^k, \ \forall k \in K \tag{34}$$

$$\sum_{k=1}^{K} \left| x_j^k \right| \leq b_j, \ \forall j \in J \tag{35}$$

where,

N = Set of n nodes

A = Node-Arcincidence Matrix

K = Set of k commodities

J = Set of j arcs

R = Set of r constraints

c_{ij}^k = Unit flow cost of commodity k on arc (i, j)

x_{ij}^k = Amount of flow of commodity k on arc (i, j)

r_i^k = Supply/demand of commodity k at node i

b_j = Capacity of arc j

This formulation is not linear due to the absolute value of the arc variables. However, this formulation can be made linear. The assumption is made that the unit-flow costs must be nonnegative. In addition, introduce nonnegative variables, x_j^{k+} and x_j^{k-}, such that $x_j^k = x_j^{k+} - x_j^{k-}$ and $\left|x_j^k\right| = x_j^{k+} + x_j^{k-}$ to produce the linear equivalent to the formulation represented in Equations (33) through (35) (Minoux, 1986:362). Equations (36) to (39) represent a linear formulation of the previous non-linear formulation (Minoux, 1986:362).

Minimize:

$$\sum_{k=1}^{K} \sum_{j=1}^{J} c_j^k \left(x_j^{k+} + x_j^{k-}\right) \tag{36}$$

Subject to:

$$A(x_j^{k+} - x_j^{k-}) = r^k, \ \forall k \in K \tag{37}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_j^{k+} + x_j^{k-}\right) \le b_j, \ \forall j \in J \tag{38}$$

$$x_j^{k\pm} \ge 0, \ \forall k \in K; \ \forall j \in J \tag{39}$$

The $ratefactor^k$ represents the size of the packet per commodity k. This formulation is solved to find a cost of flow and percentage of arc usage within the network. The results are presented in Table 3 of the solution section of this chapter. Now that the minimum cost multicommodity network flow model has been established and solved, the minimum cost multicommodity network flow with preemptive goals may be formulated.

The first preemptive goal is the maximum allowable cost of risk allowed for the inserted teams to monitor the network. Transforming Equation (36) into the first preemptive goal gives Equation (40):

Minimize:

$$P_1\{d_{1,1}{}^{+}\} \tag{40}$$

Goal Constraint:

$$\sum_{k=1}^{K} \sum_{j=1}^{J} c_j^k \left(x_j^{k+} + x_j^{k-}\right) - d_{1,1}{}^{+} + d_{1,1}{}^{-} = 2000 \tag{41}$$

The formulation minimizes the overachievement of the target value of 2000. The inclusion of $d_{1,1}{}^{+}$ allows a solution above 2000. The minimization of $d_{1,1}{}^{+}$, however, forces a solution within the desired goal level.

The second preemptive goal is to disrupt at least 60% of the capacity of all five arcs connected to facility 59. In order to model this goal, each arc connected to this facility was represented by a goal constraint. For example, the arc connecting facility 13 to facility 59 is illustrated below. All goal constraints pertaining to the second preemptive goal are formulated in Appendix B.

Minimize:

$$P_2\{d_{2,1}{}^{+}\} \tag{42}$$

Goal Constraint:

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(13,59)}^{k+} + x_{(13,59)}^{k-}\right) - d_{2,1}{}^{+} <= 20.736 \text{ (Mbits/sec)} \tag{43}$$

This particular arc illustrated is a fiber optic medium, having maximum capacity of 51.84 (Mbits/sec). Therefore, the target value of the goal constraint is 20.736, 40% of the original maximum capacity. This goal constraint allows for the capacity to be less, meaning greater than 60% of the maximum capacity of the arc was disrupted. The achievement function aims at minimizing the overachievement of the target value.

The third preemptive goal is to disrupt at least 20% of capacity at facility 63 and to disrupt at least 40% of capacity at facility 62. For illustrative purposes, the arc connecting facility 63 to facility 14 and the arc connecting facility 62 to facility 61 are represented in Equations (44) to (46) as goal constraints for the third preemptive goal. All goal constraints pertaining to the third preemptive goal are formulated in Appendix B.

Minimize:

$$P_3\{2 \times d_{3,4}{}^{+} + 3 \times d_{3,6}{}^{+}\} \tag{44}$$

Goal Constraints:

$$\sum_{k=1}^{K} ratefactor^{k} \times \left(x_{(14,63)}^{k+} + x_{(14,63)}^{k-}\right) - d_{3,4}{}^{+} <= 1.2352 \text{ (Mbits/sec)} \tag{45}$$

$$\sum_{k=1}^{K} ratefactor^{k} \times \left(x_{(61,62)}^{k+} + x_{(61,62)}^{k-}\right) - d_{3,6}{}^{+} <= 26.842 \text{ (Mbits/sec)} \tag{46}$$

The particular arc illustrated in Equation (45) is a satellite link with a maximum capacity of 1.544 (Mbits/sec). The target level of 1.2352 represents 80% of the maximum capacity of the satellite link. The particular arc illustrated in Equation (46) is a microwave link with a maximum capacity of 44.736 (Mbits/sec). The target level of 26.842 represents 60% of the maximum capacity of this arc. The achievement function minimizes the overachievement of the targeted value of all goal constraints in this particular preemptive goal. In addition, the decision maker prefers facility 63 3-to-2 over facility 62. In other words, if there is a choice between disrupting facility 63 to disrupting facility 62, disrupting facility 63 is preferred. Therefore, differential weighting must be used within the achievement function of this particular preemptive goal. The overachievement variable pertaining to Equation (45) is differentially weighted by 2 and the overachievement variable pertaining to Equation (46) is differentially weighted by 3.

The solution method will choose to deviate from the overachievement variable pertaining to Equation (45) before deviating from the overachievement variable pertaining to Equation (46). In return, the goal constraints pertaining to facility 63 are attempted to be satisfied before the goal constraints pertaining to facility 62. The full mathematical model is displayed in Appendix B.

Solution Method

Communication network flow problems tend to be very large in terms of the number of nodes and arcs in the complete network. Therefore, solving a communication network flow problem by exploiting the underlying pure network properties potentially saves computation time. The notional telecommunication problem presented in this chapter is not nearly the size that traditional telecommunication network flow problems, therefore, this notional minimum cost multicommodity network flow with goals is solved as a linear program, using ILOG OPL STUDIO 3.51 with a CPLEX solver.

Solution

<u>Model Without Goals</u>.

First, the model without goals was solved to find the minimum amount of risk that the two teams would be exposed to while monitoring the network. All of the arcs were grouped into six categories. Each category represents the range of percentage utilization. Table 3 depicts the six categories and the minimum cost of risk that the two teams are exposed to while monitoring the network.

Table 3. Model Without Goals Utilization

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	18	15.13%
>60-80%	5	4.20%
>40-60%	4	3.36%
>20-40%	4	3.36%
>0-20%	17	14.29%
0%	71	59.66%
Total Cost	945.05	

The two teams monitoring the network are exposed to a minimum risk of 945.05. Only 40% of the 119 available arcs are being used to transmit current communication in this "minimum cost" flow. Of the utilized arcs, 15% are utilized between 80 to 100% of their maximum capacity. The two teams want to monitor the network without being detected. If the flow of supply to demand within the network is disrupted then the two teams are compromised.

Model With First Preemptive Goal.

The first preemptive goal is to keep within a total risk cost factor of 2000. The achievement function of this particular model minimizes the overachievement of the target level of 2000. As the minimum cost multicommodity flow solution had a "cost" of 945.05, it is not surprising that a first goal of holding cost to 2000 or less could be met. Table 4 depicts the six categories of utilization and the risk cost of the two teams.

Table 4. First Preemptive Goal Utilization

BASELINE MODEL		
% Utilized of Max Capacity	**# of Arcs**	**% of all Arcs**
>80-100%	20	16.81%
>60-80%	3	2.52%
>40-60%	3	2.52%
>20-40%	9	7.56%
>0-20%	4	3.36%
0%	80	67.23%
Total Cost	1590.586	

The first preemptive goal is satisfied with a risk cost of 1590.59. The second preemptive goal is implemented. Note that as a goal, the model did not minimize cost once the goal was attained.

Model With First and Second Preemptive Goals.

Solving sequentially by goals, the first preemptive goal constraint was modified to allow zero overachievement of the target level within the model. Table 5 depicts the six categories of utilization and the risk cost of the two teams.

Table 5. First and Second Preemptive Goals Results

BASELINE MODEL		
% Utilized of Max Capacity	**# of Arcs**	**% of all Arcs**
>80-100%	22	18.49%
>60-80%	2	1.68%
>40-60%	11	9.24%
>20-40%	9	7.56%
>0-20%	14	11.76%
0%	61	51.26%
Total Cost	1648.94	

As compared to the model without goals, the unutilized arcs decreased from 60% to 51%. The second preemptive goal is not satisfied. There is one overachievement for one of the arc goal constraints for the second preemptive goal. Table 6 depicts the results of all five-arc target levels of facility 59.

Table 6. Facility 59 Capacity Results

Arc	Target Level	Actual Value	Overachievement
(13,59)	20.736	38.416	17.680
(59,66)	20.736	0.000	-
(51,59)	20.736	20.736	0.000
(59,62)	17.894	17.894	0.000
(47,59)	0.618	0.618	0.000

The only arc to fail to meet its reduced target capacity is arc (13,59). This is a fiber optic link that connects facility 13 to facility 59. It is not possible to reduce its capacity and not interrupt required communication. Figure 8, is a graphical representation of the relative of overachievement of the target level.

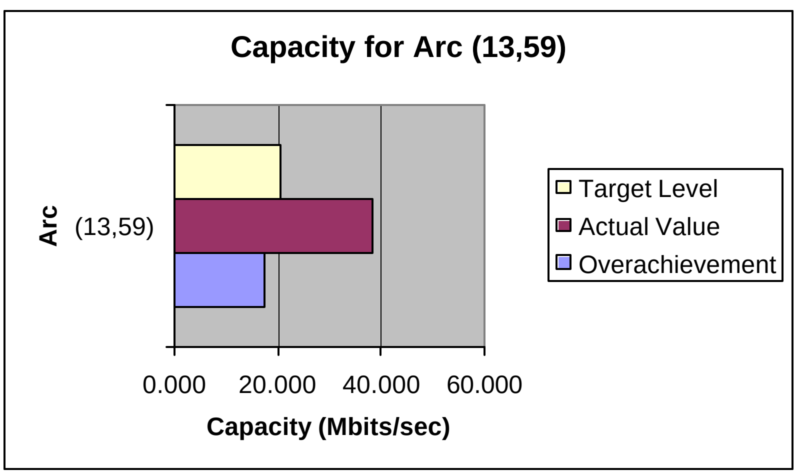


Figure 8. Target Capacity for Arc (13,59)

The second preemptive goal was not satisfied because some of the demand within the network would not be met if all five arcs at facility 59 were disrupted to 60% of their max capacity. If this is the case, the exact commodities causing the overachievement of the target level can be further investigated via sensitivity analysis.

All Preemptive Goals of the Baseline Model.

Now that the first and second preemptive goals have been implemented, the third preemptive goal was sequentially solved, assuming satisfaction of the upper-level goals at the levels already attained. The third preemptive goals consist of disrupting capacity at both facilities 63 and 62; however, facility 63 is preferred 3-to-2 over facility 62. Table 7 depicts the percentage of arc usage from the solution of the third goal.

Table 7. All Preemptive Goals of the Baseline Model Results

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	22	18.49%
>60-80%	8	6.72%
>40-60%	7	5.88%
>20-40%	11	9.24%
>0-20%	8	6.72%
0%	63	52.94%
Total Cost	1791.88	

The third preemptive goal also could not be satisfied. Table 8 depicts the results of the overachievement and underachievement of each goal constraint pertaining to each facility.

Table 8. Facility 63 and 62 Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(29,63)	51.840	41.472	17.062	-	24.410	67.09%
(60,63)	44.736	35.789	15.518	-	20.270	65.31%
(63,64)	44.736	35.789	0.000	-	35.789	100.00%
(61,63)	3.360	2.688	0.000	-	2.688	100.00%
(14,63)	1.544	1.235	1.544	0.309	-	0.00%
(61,62)	44.736	26.842	0.000	-	26.842	100.00%
(53,62)	44.736	26.842	0.000	-	26.842	100.00%
(59,62)	44.736	26.842	0.000	-	26.842	100.00%

The satellite link connecting facility 14 to facility 63 is required to use its full capacity. The third preemptive goal was not satisfied for one of three reasons. The cost goal could not be preventing the satisfaction of the third preemptive goal because the "cost" value is 1791.88; therefore, not binding. If the disruption of the five arcs at facility 63 and the disruption of the three arcs at facility 62 occur to the desired levels, the teams may be detected, as the messages cannot get through. Figure 9 is a bar graph that depicts the amount of overachievement of the target level capacity of the arc (14,63) goal constraint.

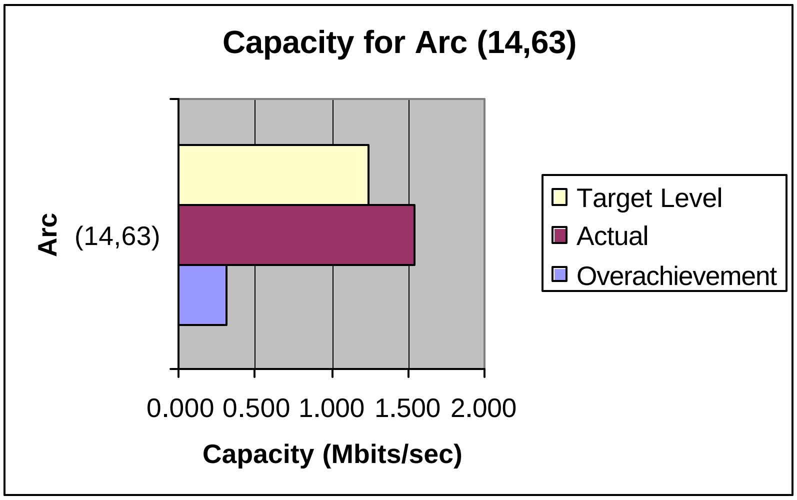


Figure 9. Target Capacity for Arc (14,63)

To conclude, in this notional illustrative example the second and the third preemptive goals had respective goal constraints that overachieved the target level. Post-optimal analysis needs to be conducted to see what is the cause of the dissatisfaction.

Post-Optimal Analysis

In order to investigate the robustness of the solutions of the model, post-optimal analysis needs to be conducted. Post-optimal analysis encompasses two areas that are implemented in this section. First, a sensitivity analysis of the ordering of the preemptive goals was conducted. Table 9 depicts the various scenarios that are analyzed.

Table 9. Preemptive Goals Reordering Scenarios

Scenarios: Preemptive Goals Reordering			
Original	P1	P2	P3
Variation 1	P1	P3	P2
Variation 2	P2	P3	P1
Variation 3	P3	P2	P1
Variation 4 (force partial control to regional HQs)	P2	P3	P1
Variation 5 (force partial control to regional HQs)	P3	P2	P1

The original scenario's results have already been presented. The scenario for variation 1 reorders the second and third preemptive goals. Such a reordering test shows how sensitive the original solution is to the ordering of the priority goals. This could occur if there was some question regarding the analysis. The scenarios for variations 2 and 3 both place the first preemptive goal last in the order of goals. This sheds light as to the degree that the maximum risk of the two teams goal affects the second and third preemptive goals versus the message traffic represented by the rigid constraints (conservation of flow). The scenarios for variations 4 and 5 are the same preemptive goal ordering as variations 2 and 3, respectively, however, the conservation of flow constraints are relaxed. This is analogous to an implementation of a secondary command structure when the main headquarters has been severed from the network. The relaxation of the conservation of flow constraints allows insight into which commodities (messages) are not meeting demand when preemptive goals 2 and 3 are implemented.

The second area of interest in the post-optimal analysis is the parametric analysis of selective right-hand-sides. The theatre commander is considering sending a team to tap into the fiber optic cable connecting facility 14 to facility 15. The theatre commander would like to know how much excess capacity may be reduced without being detected.

Sensitivity Analysis

The complete sensitivity analysis data is located in Appendix C. In this summary section, a goal is stated as "satisfied" or "not satisfied".

Variation 1.

Variation 1 interchanges the second and third preemptive goals. In the baseline model, the third preemptive goal did not satisfy its attainment level. With the interchange, the first and second preemptive goals (P1 and P2) can be satisfied. The third preemptive goal (P3) is not satisfied. Table 10 displays the overachievement and underachievement of the target levels of the third preemptive goal.

Table 10. Variation 1: Second Preemptive Goal (P1, P3, P2)

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(13,59)	51.840	20.736	38.725	17.989	-	25.30%
(59,66)	51.840	20.736	0.000	-	-	100.00%
(51,59)	51.840	20.736	20.736	0.000	0.000	60.00%
(59,62)	44.736	17.894	17.894	0.000	0.000	60.00%
(47,59)	1.544	0.618	0.618	0.000	0.000	60.00%

The same arc as in the baseline model has overachieved its target level. The disruption of facilities 63 and 62 appear to inflict a smaller amount of degradation when compared to the disruption of facility 59. All of the arcs connected to facilities 63 and 62 were disrupted to 100% of their goals of 20% and 40%, respectively. This implies that of the supply of each commodity could be routed without facilities 63 and 62.

From the baseline model and variation 1, the disruption of facilities 63 and 62 have produced minimal degradation of the network, when viewed alone. Therefore, variation 3 and variation 5 have limited impact and will not be discussed further in the chapter (the results may still be referenced in Appendix C). However, there remains the question as to why the disruption of facility 59 is not attaining its attainment levels.

Variation 2.

Variation 2 reorders the second preemptive goal to the highest priority preemptive goal to be satisfied (P2, P3, P1). This relaxes the total cost of flow restriction imposed before the disruption of facility 59 in the baseline model. The second preemptive goal is not satisfied. The third and first preemptive goals are satisfied. Table 11 depicts the resulting attainment levels pertaining to the goal constraints of the second preemptive goal.

Table 11. Variation 2: Second Preemptive Goal (Facility 59) Capacity Results

Arc	Max Capacity	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(13,59)	51.840	20.736	38.42	17.680	-	25.89%
(59,66)	51.840	20.736	0.000	-	20.736	100.00%
(51,59)	51.840	20.736	20.736	0.000	0.000	60.00%
(59,62)	44.736	17.894	17.894	0.000	0.000	60.00%
(47,59)	1.544	0.618	0.618	0.000	0.000	60.00%

Table 11 results are equivalent to the attainment levels from the baseline model. The supply of one or more of the commodities cannot reach their respective demand if the second and third preemptive goals satisfy their target levels. Variation 4 provides insight as to exactly which commodity is not supplying all of its demand.

Variation 4.

Variation 4 pinpoints exactly why the disruption of facility 59 is not being attained. Modifications to the conservation of flow constraints were undertaken to allow for a decrease in supply-demand pairs. In essence, the conservation of flow constraints were transformed into goal constraints. The underachievement variables, associated with each conservation of flow constraint were differentially weighted with the underachievement variables, associated with the overachievement variables associated with each arc at facility 59. The differential weights were chosen to reflect a greater concern to satisfy the attainment level of each goal constraint in the second preemptive goal.

All preemptive goals were satisfied. In order for all preemptive goals to be satisfied, additional supply had to come from various nodes throughout the network. This would be the same as a secondary command structure being implemented when the main headquarters is disconnected from the network. Table 12 identifies the additional supply nodes to satisfy demand within the network.

Table 12. Variation 4: Commodity Supply Augmentation

	Voice	**Video**	**Data**
Node 18	0	6	0
Node 26	0	20	0
Node 48	0	8	0
Node 57	0	39	0

The model chose to provide extra supply of video at facilities 18, 26, 48, and 57, to allow all of the demand destinations to be satisfied. This is as expected since video is the largest bits/rate restriction of all commodities.

Parametric Analysis

The theatre commander is considering inserting a team in country to monitor a fiber optic cable connecting command headquarters located at facility 14 to facility 15. This is a very vital port of information. The monitoring of the line causes a decrease in the capacity of the fiber optic cable. Currently, without any of the previous preemptive goals implemented, the fiber optic cable is 82.92% utilized. The team does not want to be detected, and they can prevent detection by not interrupting the flow of messages. That threshold needs to be identified through parametric analysis.

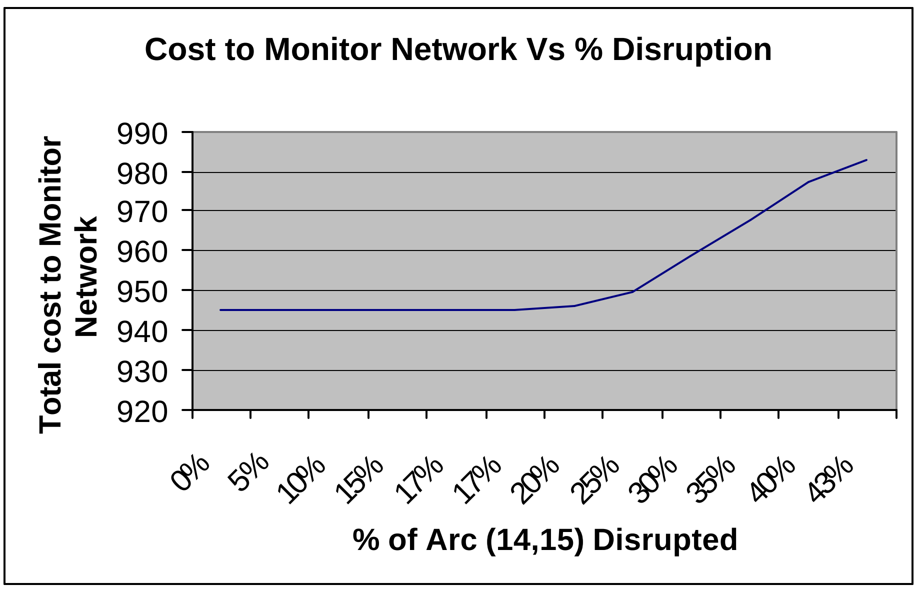


Figure 10. Parametric Analysis of the Capacity of Arc(14,15)

From 0% to 17%, the cost of risk remains at the minimum cost of 945. At 17% there is a break point (a basis change) causing the risk to increase. There are basis changes from 20% to 25%, from 25% to 40%, and from 43%. Above 43% the problem becomes infeasible because not all of the supply can reach demand. This implies that the teams would be detected after degrading the capacity of the link greater than 43%.

Additional parametric analyses of the RHS values of supply constraints were conducted to discover trade-offs between commodities. These results are provided in Appendix D.

Network Insight

From the baseline model alone it is evident that the two preemptive goals pertaining to the disruption of facility 59 and the facilities 63 and 62 were not satisfied. The two teams would have been detected. However, through post-optimal analyses, the second preemptive goal (facility 59) alone decreased the amount of supply that reached demand. It is recommended that the disruption of facility 63 and 62 be carried out over the disruption of the facilities 59 in order to avoid detection.

Summary

The methodology presented in this chapter enhances the formulation and analysis of multicommodity network flow problems. Depending on the network problem size, a variety of modeling formulations are available.

Illustrations of post-optimal analyses were provided to provide additional insight about the multicommodity network flow problem. Examples of sensitivity analysis on the preemptive goals provided insight into the actual effectiveness of each preemptive goal being implemented. In addition, an illustration of the parameterization of the capacity of a vital fiber optic cable in the network demonstrated additional analysis.

V. Conclusions and Recommendations

Overview

This research developed a methodology to provide modeling and analysis of multicommodity network flow problems with multiple objectives via goal programming. A variety of formulations and solution techniques have been provided to exploit, if needed, the pure network structure of the underlying network being analyzed. Additionally, post-optimal analyses options were explored to provide more analysis about the network in question providing a more robust methodology.

Research Results

This thesis showed that the literature of graph theory, network flows, multiple criteria decision making (MCDM), linear goal programming (LGP), Lagrangian relaxation, Dantzig-Wolfe decomposition, LGP sensitivity analysis, and parametric analysis can be interwoven to provide modeling options and post-optimality analysis on multicommodity network flow problems with multiple objectives. In addition, this study builds upon basic concepts of telecommunication networks, and defines a minimum set of criteria of network requirements for analysis.

Depending on the number of nodes, arcs, and goal constraints, the underlying pure network structure of the network problem may be exploited to reduce the number of computations. Lagrangian relaxation may be applied to the initial linear formulation to relax all side constraints. The Lagrangian function may then be solved by a subgradient method or decomposed into subproblems. Furthermore, the initial linear program may be solved by Dantzig-Wolfe decomposition.

Post-optimality analysis options include sensitivity analysis and parametric analysis. Sensitivity was illustrated on the reordering of preemptive goals. Additionally, parametric analysis was used to show the effects of the network when the available capacity of a link is decreased.

The modeling options and post-optimality analyses provide a robust methodology to analyze multicommodity network flow problems with multiple objectives.

Recommendations for Future Research

Multicommodity network flow problems will always have a need for further research due to the growth of larger and more complex problems. Faster and more inclusive models are continually needed for multicommodity network flow problems because of their size and complexity.

Larger networks may be modeled and analyzed using the methodology developed in this thesis. Each solution approached could be applied on the same large network. The computational times could then be compared for efficiency.

Summary

The methodology developed in this thesis enhances the formulation and analysis of multicommodity network flow problems with multiple objectives. The methodology presents a variety of modeling tools such as Lagrangian relaxation and Dantzig-Wolfe decomposition may be exploit the underlying pure network structure of a multicommodity network flow problem with goal programming.

Post-optimal analyses are presented to provide additional insight about the multicommodity network flow problem being investigated. For instance, examples of sensitivity analysis on the preemptive goals provided insight into the actual effectiveness of each preemptive goal being implemented. Both the modeling tools and post-optimality techniques provide for a robust methodology to solve multicommodity network flow problems via goal programming.

Appendix A. Notional Model

A "1" is placed in the column of the type of equipment located at the facility; otherwise, a "0" is placed in the column of the type of equipment. More than 1 piece of equipment may be located at a facility. "Connectivity" identifies the number of arcs connected to each facility.

Table 13. Notional Network Nodes

Quantity and Type of Equipment Located at the Facility								
Facility #	**Terminal**	**Transit Switch**	**Simple Repeater**	**Radio Relay**	**Add/Drop Repeater**	**Satellite Facility**	**Connectivity**	**System Usage**
1	1	0	0	0	0	0	1	Pure Civilian
2	0	0	0	1	0	0	2	Mixed
3	0	0	0	1	1	1	6	Mixed
4	1	1	0	1	1	0	8	Military/Gov't
5	0	0	0	1	1	0	4	Mixed
6	0	0	0	1	0	0	2	Mixed
7	1	1	0	1	1	1	7	Military/Gov't
8	1	1	0	1	1	1	6	Mixed
9	0	0	0	1	0	1	2	Mixed
10	0	0	0	1	1	0	4	Mixed
11	0	0	0	1	0	0	2	Mixed
12	1	0	0	1	0	0	1	Pure Civilian
13	0	0	0	0	1	0	4	Miexed
14	1	1	0	1	1	1	6	Military/Gov't
15	0	0	0	0	1	0	4	Mixed
16	0	0	1	0	0	0	2	Mixed
17	1	0	0	0	0	0	1	Pure Civilian
18	0	0	0	0	1	0	3	Mixed
19	0	0	0	1	1	1	4	Mixed
20	0	0	0	1	1	0	3	Mixed
21	0	0	0	1	1	0	4	Mixed
22	0	0	0	1	1	1	4	Mixed
23	0	0	0	0	1	0	3	Mixed
24	1	1	0	1	1	1	7	Military/Gov't
25	1	1	0	0	1	1	5	Mixed
26	0	0	0	1	0	0	2	Mixed
27	0	0	0	0	1	0	3	Mixed
28	1	1	0	1	1	1	6	Mixed

Quantity and Type of Equipment Located at the Facility								
Facility #	Terminal	Transit Switch	Simple Repeater	Radio Relay	Add/Drop Repeater	Satellite Facility	Connectivity	System Usage
29	0	0	0	0	1	0	5	Mixed
30	0	0	0	0	1	0	3	Mixed
31	0	0	1	0	0	0	2	Mixed
32	1	0	0	0	0	0	1	Pure Civilian
33	0	0	0	1	1	0	3	Mixed
34	1	1	0	1	1	1	5	Mixed
35	0	0	0	1	1	0	3	Mixed
36	0	0	0	1	1	0	3	Mixed
37	0	0	0	1	1	0	4	Mixed
38	0	0	0	1	0	0	2	Mixed
39	1	1	0	1	1	0	5	Mixed
40	0	0	0	1	0	0	2	Mixed
41	0	0	0	1	0	0	2	Mixed
42	0	0	1	0	0	0	2	Mixed
43	0	0	0	1	1	0	4	Mixed
44	1	0	0	0	0	0	1	Pure Civilian
45	0	0	1	0	0	0	2	Mixed
46	1	0	0	0	0	0	1	Military/Gov't
47	0	0	0	1	1	1	5	Mixed
48	0	0	0	0	1	1	3	Mixed
49	1	0	0	1	1	0	5	Military/Gov't
50	0	0	0	0	0	1	2	Mixed
51	0	0	0	0	1	0	4	Mixed
52	1	1	0	1	1	1	6	Mixed
53	1	1	0	0	1	1	8	Military/Gov't
54	1	0	0	0	0	0	1	Military/Gov't
55	0	0	0	1	1	0	3	Mixed
56	0	0	1	0	0	0	2	Mixed
57	0	0	0	0	1	0	3	Mixed
58	0	0	0	1	1	0	4	Mixed
59	1	1	0	0	1	1	5	Mixed
60	1	1	0	1	1	1	5	Mixed
61	1	1	0	1	1	1	6	Military/Gov't
62	0	0	0	0	1	0	3	Mixed
63	1	1	0	1	1	1	5	Mixed
64	0	0	0	0	1	0	3	Mixed
65	0	0	1	0	0	0	2	Mixed
66	0	0	0	0	1	0	3	Mixed
67	0	0	0	1	1	0	4	Mixed

Table 14. Notional Network Links

Medium Type and Max Capacity of Arc #				Arbitrary Arc per Unit Cost		
Arc #	Arbitrary Direction	Media Type	Capacity of Arc (Mbits/sec)	Voice	Video	Data
arc 1	(1,3)	coax	44.736	0.22	0.22	0.22
arc 2	(2,3)	microwave	44.736	0.21	0.21	0.21
arc 3	(2,7)	microwave	44.736	0.14	0.14	0.14
arc 4	(3,4)	fiber	51.840	0.20	0.20	0.20
arc 5	(3,5)	microwave	44.736	0.23	0.23	0.23
arc 6	(3,10)	troposcatter	3.360	0.14	0.14	0.14
arc 7	(3,52)	satellite	1.544	0.19	0.19	0.19
arc 8	(4,5)	fiber	51.840	0.15	0.15	0.15
arc 9	(4,7)	troposcatter	3.360	0.15	0.15	0.15
arc 10	(4,8)	coax	44.736	0.14	0.14	0.14
arc 11	(4,11)	microwave	44.736	0.22	0.22	0.22
arc 12	(4,14)	troposcatter	3.360	0.15	0.15	0.15
arc 13	(4,15)	fiber	51.840	0.23	0.23	0.23
arc 14	(4,67)	troposcatter	3.360	0.15	0.15	0.15
arc 15	(5,7)	microwave	44.736	0.21	0.21	0.21
arc 16	(5,10)	troposcatter	3.360	0.14	0.14	0.14
arc 17	(6,7)	troposcatter	3.360	0.22	0.22	0.22
arc 18	(6,8)	troposcatter	3.360	0.18	0.18	0.18
arc 19	(7,8)	satellite	1.544	0.17	0.17	0.17
arc 20	(7,9)	satellite	1.544	0.22	0.22	0.22
arc 21	(7,10)	troposcatter	3.360	0.22	0.22	0.22
arc 22	(8,9)	microwave	44.736	0.22	0.22	0.22
arc 23	(8,11)	microwave	44.736	0.18	0.18	0.18
arc 24	(8,12)	microwave	44.736	0.13	0.13	0.13
arc 25	(10,14)	troposcatter	3.360	0.22	0.22	0.22
arc 26	(13,14)	fiber	51.840	0.23	0.23	0.23
arc 27	(13,15)	fiber	51.840	0.21	0.21	0.21
arc 28	(13,16)	fiber	51.840	0.20	0.20	0.20
arc 29	(13,59)	fiber	51.840	0.21	0.21	0.21
arc 30	(14,15)	fiber	51.840	0.21	0.21	0.21
arc 31	(14,21)	microwave	44.7360	0.18	0.18	0.18
arc 32	(14,63)	satellite	1.544	0.21	0.21	0.21
arc 33	(15,16)	fiber	51.840	0.17	0.17	0.17
arc 34	(17,18)	coax	44.736	0.14	0.14	0.14
arc 35	(18,19)	coax	44.736	0.15	0.15	0.15
arc 36	(18,24)	coax	44.736	0.23	0.23	0.23
arc 37	(19,20)	microwave	44.736	0.23	0.23	0.23

Medium Type and Max Capacity of Arc #				Arbitrary Arc per Unit Cost		
Arc #	**Arbitrary Direction**	**Media Type**	**Capacity of Arc (Mbits/sec)**	**Voice**	**Video**	**Data**
arc 38	(19,21)	microwave	44.736	0.23	0.23	0.23
arc 39	(19,25)	satellite	1.544	0.13	0.13	0.13
arc 40	(20,21)	microwave	44.736	0.21	0.21	0.21
arc 41	(20,26)	microwave	44.736	0.16	0.16	0.16
arc 42	(21,22)	microwave	44.736	0.23	0.23	0.23
arc 43	(22,23)	fiber	51.840	0.21	0.21	0.21
arc 44	(22,24)	satellite	1.544	0.21	0.21	0.21
arc 45	(22,25)	satellite	1.544	0.23	0.23	0.23
arc 46	(23,24)	fiber	51.840	0.21	0.21	0.21
arc 47	(23,25)	fiber	51.840	0.21	0.21	0.21
arc 48	(24,25)	coax	44.736	0.21	0.21	0.21
arc 49	(24,26)	microwave	44.736	0.22	0.22	0.22
arc 50	(24,27)	fiber	51.840	0.22	0.22	0.22
arc 51	(24,29)	coax	44.736	0.15	0.15	0.15
arc 52	(25,27)	fiber	51.840	0.21	0.21	0.21
arc 53	(27,28)	fiber	51.840	0.23	0.23	0.23
arc 54	(28,29)	fiber	51.840	0.14	0.14	0.14
arc 55	(28,30)	fiber	51.840	0.21	0.21	0.21
arc 56	(28,34)	satellite	1.544	0.14	0.14	0.14
arc 57	(28,41)	troposcatter	3.360	0.16	0.16	0.16
arc 58	(28,52)	troposcatter	3.360	0.23	0.23	0.23
arc 59	(29,30)	fiber	51.840	0.21	0.21	0.21
arc 60	(29,31)	coax	44.736	0.07	0.07	0.07
arc 61	(29,63)	fiber	51.840	0.14	0.14	0.14
arc 62	(30,34)	fiber	51.840	0.24	0.24	0.24
arc 63	(31,33)	coax	44.736	0.21	0.21	0.21
arc 64	(32,39)	coax	44.736	0.06	0.06	0.06
arc 65	(33,34)	coax	44.736	0.15	0.15	0.15
arc 66	(33,47)	troposcatter	3.360	0.23	0.23	0.23
arc 67	(34,35)	troposcatter	3.360	0.23	0.23	0.23
arc 68	(34,53)	fiber	51.840	0.22	0.22	0.22
arc 69	(35,36)	troposcatter	3.360	0.21	0.21	0.21
arc 70	(35,45)	fiber	51.840	0.23	0.23	0.23
arc 71	(36,49)	troposcatter	3.360	0.24	0.24	0.24
arc 72	(36,52)	troposcatter	3.360	0.23	0.23	0.23
arc 73	(37,38)	microwave	44.736	0.22	0.22	0.22
arc 74	(37,39)	coax	44.736	0.21	0.21	0.21
arc 75	(37,53)	fiber	51.840	0.20	0.20	0.20
arc 76	(37,55)	fiber	51.840	0.22	0.22	0.22

Medium Type and Max Capacity of Arc #				Arbitrary Arc per Unit Cost		
Arc #	Arbitrary Direction	Media Type	Capacity of Arc (Mbits/sec)	Voice	Video	Data
arc 77	(38,58)	microwave	44.736	0.14	0.14	0.14
arc 78	(39,40)	troposcatter	3.360	0.22	0.22	0.22
arc 79	(39,43)	troposcatter	3.360	0.14	0.14	0.14
arc 80	(39,58)	microwave	44.736	0.15	0.15	0.15
arc 81	(40,60)	troposcatter	3.360	0.13	0.13	0.13
arc 82	(41,43)	troposcatter	3.360	0.23	0.23	0.23
arc 83	(42,43)	fiber	51.840	0.21	0.21	0.21
arc 84	(42,64)	fiber	51.840	0.20	0.20	0.20
arc 85	(43,56)	fiber	51.840	0.11	0.11	0.11
arc 86	(44,67)	fiber	51.840	0.21	0.21	0.21
arc 87	(45,47)	fiber	51.840	0.23	0.23	0.23
arc 88	(46,49)	fiber	51.840	0.23	0.23	0.23
arc 89	(47,48)	coax	1.544	0.21	0.21	0.21
arc 90	(47,49)	satellite	44.736	0.21	0.21	0.21
arc 91	(47,59)	satellite	1.544	0.22	0.22	0.22
arc 92	(48,50)	satellite	1.544	0.13	0.13	0.13
arc 93	(48,52)	satellite	1.544	0.22	0.22	0.22
arc 94	(49,51)	fiber	51.840	0.23	0.23	0.23
arc 95	(49,61)	coax	44.736	0.21	0.21	0.21
arc 96	(50,61)	satellite	1.544	0.14	0.14	0.14
arc 97	(51,52)	fiber	51.840	0.23	0.23	0.23
arc 98	(51,59)	fiber	51.840	0.24	0.24	0.24
arc 99	(51,61)	fiber	51.8400	0.21	0.21	0.21
arc 100	(52,53)	coax	44.736	0.11	0.11	0.11
arc 101	(53,54)	coax	44.736	0.08	0.08	0.08
arc 102	(53,55)	fiber	51.840	0.23	0.23	0.23
arc 103	(53,57)	fiber	51.840	0.23	0.23	0.23
arc 104	(53,60)	satellite	1.544	0.11	0.11	0.11
arc 105	(53,62)	coax	44.736	0.20	0.20	0.20
arc 106	(55,58)	microwave	44.736	0.20	0.20	0.20
arc 107	(56,57)	fiber	51.840	0.24	0.24	0.24
arc 108	(57,58)	fiber	51.840	0.21	0.21	0.21
arc 109	(59,62)	coax	44.736	0.23	0.23	0.23
arc 110	(59,66)	fiber	51.840	0.15	0.15	0.15
arc 111	(60,63)	coax	44.736	0.13	0.13	0.13
arc 112	(60,65)	fiber	51.840	0.22	0.22	0.22
arc 113	(60,66)	fiber	51.840	0.21	0.21	0.21
arc 114	(61,62)	coax	44.736	0.19	0.19	0.19
arc 115	(61,63)	troposcatter	3.360	0.22	0.22	0.22

Medium Type and Max Capacity of Arc #				Arbitrary Arc per Unit Cost		
Arc #	**Arbitrary Direction**	**Media Type**	**Capacity of Arc (Mbits/sec)**	**Voice**	**Video**	**Data**
arc 116	(61,64)	coax	44.736	0.23	0.23	0.23
arc 117	(63,64)	coax	44.736	0.21	0.21	0.21
arc 118	(65,67)	fiber	51.840	0.23	0.23	0.23
arc 119	(66,67)	fiber	51.840	0.23	0.23	0.23

Appendix B. Full Mathematical Model

Minimize:

$$P_1\{d_{1,1}^{+} + d_{1,1}^{-}\},$$

$$P_2\{d_{2,1}^{+} + d_{2,2}^{+} + d_{2,3}^{+} + d_{2,4}^{+} + d_{2,5}^{+}\},$$

$$P_3\{2d_{3,1}^{+} + 2d_{3,2}^{+} + 2d_{3,3}^{+} + 2d_{3,4}^{+} + 2d_{3,5}^{+} + 3d_{3,6}^{+} + 3d_{3,7}^{+} + 3d_{3,8}^{+}\},$$

Subject to:

$$P_1: \sum_{k=1}^{K} \sum_{j=1}^{J} c_j^k \left(x_j^{k+} + x_j^{k-}\right) - d_{1,1}^{+} + d_{1,1}^{-} \leq 2000$$

$$P_2: \sum_{k=1}^{K} ratefactor^k \times \left(x_{(13,59)}^{k+} + x_{(13,59)}^{k-}\right) - d_{2,1}^{+} <= 20.736 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(59,66)}^{k+} + x_{(59,66)}^{k-}\right) - d_{2,2}^{+} <= 20.736 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(51,59)}^{k+} + x_{(51,59)}^{k-}\right) - d_{2,3}^{+} <= 20.736 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(59,62)}^{k+} + x_{(59,62)}^{k-}\right) - d_{2,4}^{+} <= 17.8944 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(47,59)}^{k+} + x_{(47,59)}^{k-}\right) - d_{2,5}^{+} <= 0.6176 \text{ (Mbits/sec)}$$

$$P_3: \sum_{k=1}^{K} ratefactor^k \times \left(x_{(29,63)}^{k+} + x_{(29,63)}^{k-}\right) - d_{3,1}^{+} <= 41.472 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(60,63)}^{k+} + x_{(60,63)}^{k-}\right) - d_{3,2}^{+} <= 35.7888 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(63,64)}^{k+} + x_{(63,64)}^{k-}\right) - d_{3,3}^{+} <= 35.7888 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(61,63)}^{k+} + x_{(61,63)}^{k-}\right) - d_{3,4}^{+} <= 2.688 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(14,63)}^{k+} + x_{(14,63)}^{k-}\right) - d_{3,5}^{+} <= 1.2352 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(61,62)}^{k+} + x_{(61,62)}^{k-}\right) - d_{3,6}^{+} <= 26.8416 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(53,62)}^{k+} + x_{(53,62)}^{k-}\right) - d_{3,7}^{+} <= 26.8416 \text{ (Mbits/sec)}$$

$$\sum_{k=1}^{K} ratefactor^k \times \left(x_{(59,62)}^{k+} + x_{(59,62)}^{k-}\right) - d_{3,8}^{+} <= 26.8416 \text{ (Mbits/sec)}$$

$$A(x_j^{k+} - x_j^{k-}) = r^k, \ \forall k \in K$$
$$\sum_{k=1}^{K} ratefactor^k \times \left(x_j^{k+} + x_j^{k-}\right) \leq b_j, \ \forall j \in J$$
$$x_j^{k\pm} \geq 0, \ \forall k \in K; \ \forall j \in J$$

where,

N	= Set of n nodes
A	= Node-arc incidence Matrix
K	= Set of k commodities
J	= Set of j arcs
R	= Set of r constraints
P_i	= Preemptive priority goal $P_1 >>> P_2 >>> ... >>> P_i$
$c_{ij}^{\ k}$	= Unit flow cost of commodity k on arc (i, j)
$x_{ij}^{\ k}$	= Amount of flow of commodity k on arc (i, j)
r_i^k	= Supply/demand of commodity k at node i
b_{ij}	= Capacity of arc (i, j)
$d_{i,q}^{\ \pm}$	= The overachievement or underachievement, respectively of the ith priority and q goal constraint of that priority
$ratefactor^k$	= The size of commodity k

Appendix C. Variations Data

Baseline Model

First Preemptive Goal.

Table 15. First Preemptive Goal Results

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	20	16.81%
>60-80%	3	2.52%
>40-60%	3	2.52%
>20-40%	9	7.56%
>0-20%	4	3.36%
0%	80	67.23%
Total Cost	1590.586	

First and Second Preemptive Goals.

Table 16. First and Second Preemptive Goals Results

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	22	18.49%
>60-80%	2	1.68%
>40-60%	11	9.24%
>20-40%	9	7.56%
>0-20%	14	11.76%
0%	61	51.26%
Total Cost	1648.935	

All Preemptive Goals.

Table 17. All Preemptive Goals Results

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	22	18.49%
>60-80%	8	6.72%
>40-60%	7	5.88%
>20-40%	11	9.24%
>0-20%	8	6.72%
0%	63	52.94%
Total Cost	1791.881	

Table 18. Second Preemptive Goal Capacity Results

Arc	Target Level	Actual Value	Overachievement
(13,59)	20.736	38.416	17.680
(59,66)	20.736	0.000	-
(51,59)	20.736	20.736	0.000
(59,62)	17.894	17.894	0.000
(47,59)	0.618	0.618	0.000

Table 19. Third Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachievement	Underachievement	% Disrupted
(29,63)	51.840	41.472	17.062	-	24.410	67.09%
(60,63)	44.736	35.789	15.518	-	20.270	65.31%
(63,64)	44.736	35.789	0.000	-	35.789	100.00%
(61,63)	3.360	2.688	0.000	-	2.688	100.00%
(14,63)	1.544	1.235	1.544	0.309	-	0.00%
(61,62)	44.736	26.842	0.000	-	26.842	100.00%
(53,62)	44.736	26.842	0.000	-	26.842	100.00%
(59,62)	44.736	26.842	0.000	-	26.842	100.00%

Variation 1 (P1, P3, P2)

First Preemptive Goal.

Table 20. First Preemptive Goal Arc Usage Results

% Utilized of Max Capacity	**# of Arcs**	**% of all Arcs**
>80-100%	17	14.29%
>60-80%	3	2.52%
>40-60%	9	7.56%
>20-40%	6	5.04%
>0-20%	8	6.72%
0%	76	63.87%
Total Cost	1520.73	

First and Third Preemptive Goals.

Table 21. First and Second Preemptive Goals Arc Usage Results

% Utilized of Max Capacity	**# of Arcs**	**% of all Arcs**
>80-100%	24	20.17%
>60-80%	3	2.52%
>40-60%	8	6.72%
>20-40%	4	3.36%
>0-20%	11	9.24%
0%	69	57.98%
Total Cost	1646.54	

All Preemptive Goals.

Table 22. All Preemptive Goals Arc Usage Results

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	25	21.01%
>60-80%	2	1.68%
>40-60%	11	9.24%
>20-40%	5	4.20%
>0-20%	15	12.61%
0%	61	51.26%
Total Cost	1628.77	

Table 23. Second Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(13,59)	51.840	20.736	38.725	17.989	-	25.30%
(59,66)	51.840	20.736	0.000	-	-	100.00%
(51,59)	51.840	20.736	20.736	0.000	0.000	60.00%
(59,62)	44.736	17.894	17.894	0.000	0.000	60.00%
(47,59)	1.544	0.618	0.618	0.000	0.000	60.00%

Table 24. Third Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(29,63)	51.840	41.472	0.000	-	41.472	100.00%
(60,63)	44.736	35.789	0.000	-	35.789	100.00%
(63,64)	44.736	35.789	0.000	-	35.789	100.00%
(61,63)	3.360	2.688	0.000	-	2.688	100.00%
(14,63)	1.544	1.235	0.000	-	1.235	100.00%
(61,62)	44.736	26.842	0.000	-	26.842	100.00%
(53,62)	44.736	26.842	0.000	-	26.842	100.00%
(59,62)	44.736	26.842	0.000	-	26.842	100.00%

Variation 2 (P2, P3, P1)

Second Preemptive Goal.

Table 25. Second Preemptive Goal Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	28	23.53%
>60-80%	5	4.20%
>40-60%	8	6.72%
>20-40%	9	7.56%
>0-20%	12	10.08%
0%	57	47.90%
Total Cost	3043.31	

Second and Third Preemptive Goals.

Table 26. Second and Third Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	35	29.41%
>60-80%	5	4.20%
>40-60%	8	6.72%
>20-40%	15	12.61%
>0-20%	12	10.08%
0%	44	36.97%
Total Cost	3526.598	

All Preemptive Goals.

Table 27. All Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	23	19.33%
>60-80%	1	0.84%
>40-60%	7	5.88%
>20-40%	10	8.40%
>0-20%	14	11.76%
0%	64	53.78%
Total Cost	1536.1	

Table 28. Second Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(13,59)	51.840	20.736	38.420	17.680	-	25.89%
(59,66)	51.840	20.736	0.000	-	20.736	100.00%
(51,59)	51.840	20.736	20.736	0.000	0.000	60.00%
(59,62)	44.736	17.894	17.894	0.000	0.000	60.00%
(47,59)	1.5440	0.6180	0.618	0.000	0.000	60.00%

Table 29. Third Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(29,63)	51.840	41.472	1.544	-	39.928	97.02%
(60,63)	44.736	35.789	0.000	-	35.789	100.00%
(63,64)	44.736	35.789	0.000	-	35.789	100.00%
(61,63)	3.360	2.688	0.000	-	2.688	100.00%
(14,63)	1.544	1.235	1.544	0.309	-	0.00%
(61,62)	44.736	26.842	0.000	-	26.842	100.00%
(53,62)	44.736	26.842	15.518	-	11.324	65.31%
(59,62)	44.736	26.842	15.518	-	11.324	65.31%

Variation 3 (P3, P2, P1)

<u>Third Preemptive Goal</u>.

Table 30. Third Preemptive Goal Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	28	23.53%
>60-80%	2	1.68%
>40-60%	7	5.88%
>20-40%	8	6.72%
>0-20%	8	6.72%
0%	66	55.46%
Total Cost	3218.7	

<u>Third and Second Preemptive Goals</u>.

Table 31. Third and Second Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	28	23.53%
>60-80%	5	4.20%
>40-60%	8	6.72%
>20-40%	9	7.56%
>0-20%	12	10.08%
0%	57	47.90%
Total Cost	3055.42	

<u>All Preemptive Goals</u>.

Table 32. All Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	22	18.49%
>60-80%	2	1.68%
>40-60%	11	9.24%
>20-40%	9	7.56%
>0-20%	14	11.76%
0%	61	51.26%
Total Cost	1660.46	

Table 33. Second Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(13,59)	51.840	20.736	38.725	17.989	-	25.30%
(59,66)	51.840	20.736	0.000	-	20.736	100.00%
(51,59)	51.840	20.736	20.736	0.000	0.000	60.00%
(59,62)	44.736	17.894	17.894	0.000	0.000	60.00%
(47,59)	1.544	0.618	0.618	0.000	0.000	60.00%

Table 34. Third Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(29,63)	51.840	41.472	0.000	-	41.472	100.00%
(60,63)	44.736	35.789	0.000	-	35.789	100.00%
(63,64)	44.736	35.789	0.000	-	35.789	100.00%
(61,63)	3.360	2.688	0.000	-	2.688	100.00%
(14,63)	1.544	1.235	0.000	-	1.235	100.00%
(61,62)	44.736	26.842	0.000	-	26.842	100.00%
(53,62)	44.736	26.842	0.000	-	26.842	100.00%
(59,62)	44.736	26.842	0.000	-	26.842	100.00%

Variation 4 (P2, P3, P1) – relaxed conservation of flow constraints

Second Preemptive Goal.

Table 35. Second Preemptive Goal Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	23	19.33%
>60-80%	1	0.84%
>40-60%	6	5.04%
>20-40%	8	6.72%
>0-20%	21	17.65%
0%	60	50.42%
Total Cost	1570.92	

Table 36. Second Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(13,59)	51.840	20.736	20.736	0.000	-	60.00%
(59,66)	51.840	20.736	0.000	-	20.736	100.00%
(51,59)	51.840	20.736	2.224	0.000	18.512	60.00%
(59,62)	44.736	17.894	17.894	0.000	0.000	60.00%
(47,59)	1.544	0.618	0.618	0.000	0.000	60.00%

Table 37. Supplement Supply Nodes – Second Preemptive Goal

	Voice	Video	Data
Node 31	0	21	0
Node 47	0	42	0
Node 56	0	8	0

Second and Third Preemptive Goals.

Table 38. Second and Third Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	25	21.01%
>60-80%	0	0.00%
>40-60%	6	5.04%
>20-40%	10	8.40%
>0-20%	23	19.33%
0%	55	46.22%
Total Cost	1514.06	

Table 39. Third Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(29,63)	51.840	41.472	1.052	-	40.420	97.97%
(60,63)	44.736	35.789	0.401	-	35.388	99.10%
(63,64)	44.736	35.789	0.000	-	35.789	100.00%
(61,63)	3.360	2.688	2.688	0.000	0.000	20.00%
(14,63)	1.544	1.235	1.235	0.000	0.000	20.00%
(61,62)	44.736	26.842	23.758	-	3.084	46.89%
(53,62)	44.736	26.842	5.864	-	20.978	86.89%
(59,62)	44.736	26.842	17.894	-	8.948	60.00%

Table 40. Supplement Supply Nodes – Second and Third Preemptive Goals

	Voice	Video	Data
Node 27	0	17	0
Node 46	0	10	0
Node 52	0	45	0

All Preemptive Goals.

Table 41. All Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	29	24.37%
>60-80%	0	0.00%
>40-60%	5	4.20%
>20-40%	12	10.08%
>0-20%	17	14.29%
0%	56	47.06%
Total Cost	1551.97	

Table 42. Supplement Supply Nodes – All Preemptive Goals

	Voice	Video	Data
Node 18	0	6	0
Node 26	0	20	0
Node 48	0	8	0
Node 57	0	39	0

Variation 5 (P3, P2, P1) – relaxed conservation of flow constraints

Third Preemptive Goal.

Table 43. Third Preemptive Goal Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	23	19.33%
>60-80%	5	4.20%
>40-60%	4	3.36%
>20-40%	12	10.08%
>0-20%	16	13.45%
0%	59	49.58%
Total Cost	1521.77	

Table 44. Third Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(29,63)	51.840	41.472	2.195	-	39.277	95.77%
(60,63)	44.736	35.789	1.544	-	34.245	96.55%
(63,64)	44.736	35.789	0.000	-	35.789	100.00%
(61,63)	3.360	2.688	2.688	0.000	0.000	20.00%
(14,63)	1.544	1.235	1.235	0.000	0.000	20.00%
(61,62)	44.736	26.842	14.159	-	12.683	68.35%
(53,62)	44.736	26.842	14.159	-	12.683	68.35%
(59,62)	44.736	26.842	24.800	-	2.042	44.56%

Supplement Supply Nodes – Third Preemptive Goal

Not Necessary

Third and Second Preemptive Goals.

Table 45. Third and Second Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	24	20.17%
>60-80%	1	0.84%
>40-60%	6	5.04%
>20-40%	8	6.72%
>0-20%	22	18.49%
0%	58	48.74%
Total Cost	1572.69	

Table 46. Second Preemptive Goal Capacity Results

Arc	Max Cap	Target Level	Actual	Overachieve	Underachieve	% Disrupted
(13,59)	51.840	20.736	20.736	0.000	0.000	60.00%
(59,66)	51.840	20.736	0.363	-	20.373	99.30%
(51,59)	51.840	20.736	2.587	-	18.149	95.01%
(59,62)	44.736	17.894	17.894	0.000	0.000	60.00%
(47,59)	1.544	0.618	0.618	0.000	0.000	60.00%

Table 47. Supplement Supply Nodes – Third and Second Preemptive Goals

	Voice	Video	Data
Node 31	0	21	0
Node 47	0	34	0
Node 48	0	6	0
Node 56	0	11	0

All Preemptive Goals.

Table 48. All Preemptive Goals Arc Usage

% Utilized of Max Capacity	# of Arcs	% of all Arcs
>80-100%	29	24.37%
>60-80%	0	0.00%
>40-60%	5	4.20%
>20-40%	12	10.08%
>0-20%	17	14.29%
0%	56	47.06%
Total Cost	1551.97	

Table 49. Supplement Supply Nodes – All Preemptive Goals

	Voice	Video	Data
Node 18	0	6	0
Node 26	0	20	0
Node 48	0	8	0
Node 57	0	39	0

Appendix D. Additional Parametric Analysis

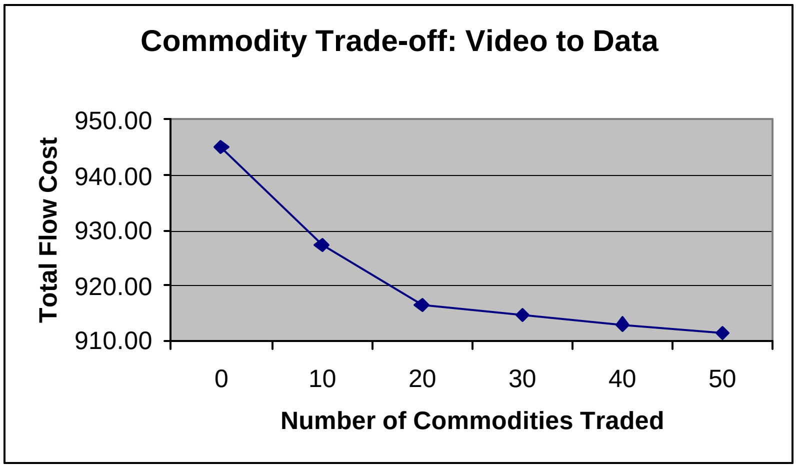
Parametric Analysis

From the sensitivity analysis of the reordering of the goals, all of the sink nodes of video could not be met, in the disrupted network. The parametric analysis section is an illustration to analyze the range of commodity trade-offs, while maintaining the optimal solution.

The first illustration analyzes the commodity trade-off of the model without goals. As a reminder, the minimum amount of risk in the model without goals was 945.05. The commodities video and data were parameterized to find the range of video trade-off to data, maintaining a risk of 945.05. The results showed that all of the fifty messages of video could be traded to data, while maintaining a risk of 945.05. The same results agreed with the trade-off of video to voice. In addition, in the trade-off of voice to video, zero messages could be traded, and the same with the trade-off of data to video.

The second parametric analysis illustration involved using the maximum allowable risk of 2000. First, the commodities video and data were parameterized to allow trade-off of video to data. All of the video messages were traded to data messages, while maintaining a risk of 2000 or less. Figure 11 depicts the risk associated with the number of commodities traded.

Figure 11. Parametric Analysis Trade-off of Video to Data



Starting with a risk of 945.05 and zero video messages traded, the risk decreased to 911.42. As expected, the same results are similar to the trade-off of video to voice. Figure 12 depicts the cost associated with the number of video messages traded to vo ice messages.

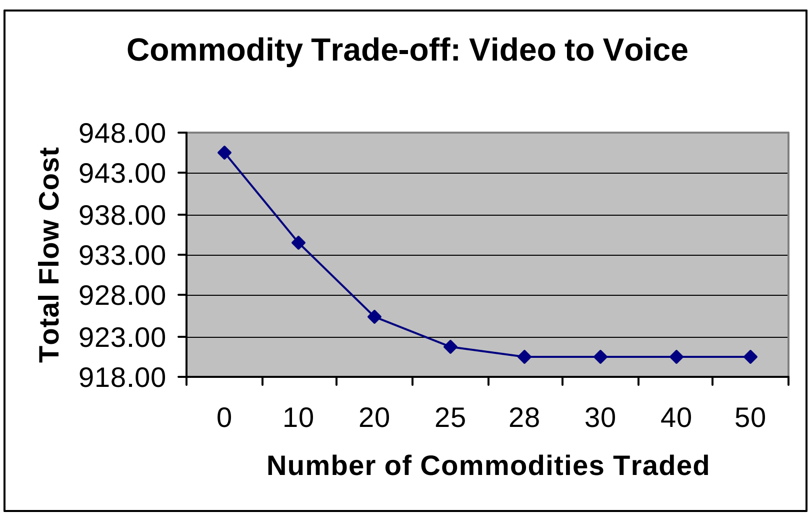
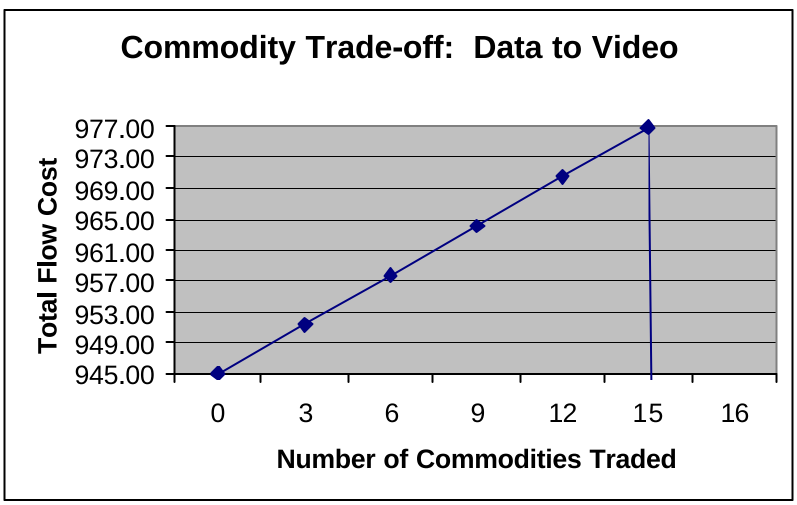


Figure 12. Parametric Analysis Trade-off of Video to Voice

The risk ranges from 945.05 to 920.57, starting with zero video messages to fifty video messages traded. The risk drastically decreased from zero to twenty video messages traded to voice messages, when compared to the slight decrease in risk from twenty-five video messages to fifty video messages; which, ranged from a risk of 921.80 to a risk of 920.57, respectively. Actually, the risk bottomed out at 920.57 at twenty-eight video messages traded.

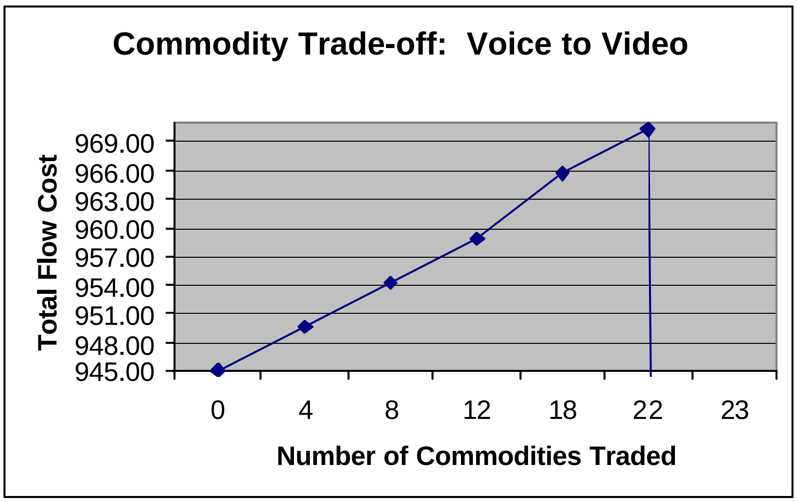
Now that the trade-off of video to another commodity has been parameterized, data and voice are parameterized to investigate a trade-off, individually, to video messages, while maintaining a risk at or below 2000. The trade-off of data to video is parameterized, and results are depicted in Figure 13.

Figure 13. Parametric Analysis Trade-off of Data to Video



As pictured above, the range of the allowable trade of data messages to video messages ranged from zero to fourteen. Fifteen data messages or more traded to video messages would violate the mutual capacity flow constraints of the commodities. Within the allowable range of zero to fourteen of data messages traded, the risk linearly increased in value. The same results are similar to the trade-off analysis of voice messages to video messages as seen in Figure 14.

Figure 14. Parametric Analysis Trade-off of Voice to Video



Opposed to the trade-off of data to video, the range of the number of commodities traded from voice to video is zero to twenty-one voice messages. Twenty-two or greater data messages would cause the mutual capacity flow constraints to violate.

In summary, the number of commodities being traded depends on the packet size of each commodity. Video has the largest packet size causing the greatest strain on the mutual capacity flow constraints. Each will be different as the packet size of each commodity changes in other scenarios.

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14. ABSTRACT

In this work goal programming is used to solve a minimum cost multicommodity network flow problem with goals. A single telecommunication network with multiple commodities (*e.g.*, voice, video, data, etc.) flowing over it is analyzed. This network consists of: linear objective function, linear cost arcs, fixed capacities, specific origin-destination pairs for each commodity. A multicommodity network flow problem with goals can be successfully modeled using linear goal programming techniques. When properly modeled, network flow techniques may be employed to exploit the pure network structure of a multicommodity network flow problem with goals. Lagrangian relaxation captures the essence of the pure network flow problem as a master problem and sub-problems (McGinnis and Rao, 1977). A subgradient algorithm may optimize the Lagrangian function, or the Lagrangian relaxation could be decomposed into subproblems per commodity; each subproblem is a single commodity network flow problem. Parallel to the decomposition of the Lagrangian relaxation, Dantzig-Wolfe decomposition may be implemented to the linear program.

Post-optimality analyses provide a variety of options to analyze the robustness of the optimal solution. The options of post-optimality analysis consist of sensitivity analysis and parametric analysis. This mix of modeling options and analyses provide a powerful method to produce insight into the modeling of a multicommodity network flow problem with multiple objectives.

15. SUBJECT TERMS:
Multicommodity network flow, goal programming, Lagrangian relaxation

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