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**TURBULENT DRAG REDUCTION USING  
COMPLIANT COATINGS**

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*by*

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30 August 2002

## Summary

An experimental study was carried in a water tunnel, where the drag of several compliant coatings in the turbulent boundary layer was measured. An axi-symmetric test model similar to that used by NUWC was designed and built, and identical compliant coatings were tested as a part of collaboration between UK, US and Russia. While experimental results in the UK show small drag reductions by up to 3% in some of the compliant coatings tested, skin-friction drags of compliant coatings measured at MIT are consistently greater than that of rigid surface. The Russian data are not yet available at the time of writing this report. The aging of the compliant coatings are thought to be the reason for these discrepancies. A better coordination will be required for the rest of the programme to overcome this problem.

### **1. Introduction**

The concept of achieving a reduction in drag force by compliant surfaces originates with Kramer [1] who observed that a dolphin can swim at an exceptionally high speed suggesting that its body may have a very low drag coefficient. Kramer's experiments [2-5] indeed showed a substantial reduction in drag of up to 50% using compliant coatings modelled from a dolphin's skin. Kramer's compliant coatings were closely modelled on his concept of the dolphin's skin, which was believed to reduce drag by bringing about a delay in transition to turbulence. However, all the investigators who tried to repeat Kramer's work have failed to substantiate such a claim despite careful and comprehensive experimental programmes. This led to Benjamin [6] to examine the possibility of obtaining a drag reduction through the beneficial effect of wall compliance on fully turbulent boundary layers. Since then, many experimentalists have tried to find suitable compliant materials to achieve turbulent drag reduction. In particular, a series of wind-tunnel experiments carried out at the University of Oklahoma [7-9] were reported to have shown turbulent drag reduction up to 50%, but here again these results could not be repeated in other tests [10,11].

Later, Chung & Merrill [12] conducted an experiment using a rotating disc with a silicon-polymer coating for which a substantial drag reduction was observed. However, it is not certain whether the flow was laminar or turbulent in this experiment owing to the absence of velocity measurements. Meanwhile, Taylor [13] and Falco & Chu [14] carried out experiments in which they claimed to have obtained drag reduction in turbulent boundary layers. The compliant coatings used in these investigations were very soft, so that deformations of the compliant surfaces could have caused pressure gradients that might have affected the drag measurements.

In 1980s, Semenov's group in Russia (Institute of Thermophysics in Novosibirsk) has conducted a series of field tests of compliant coating, indicating that they have obtained a turbulent drag reduction of up to 20 % [15,16]. In USA, Lee *et al.* [17] conducted an investigation in early 1990s, where a significant reduction in turbulent intensity was observed across the boundary layer over the compliant surface. Unfortunately, no information on the drag reduction is available for this experiment since the skin friction over the compliant surface was not measured.

About the same time, a careful study of turbulent boundary layer over the compliant coating was carried out by Choi *et al.* [18,19] in an effort to *independently* verify the ability of a compliant

surface to reduce the *turbulent* skin-friction drag and surface-flow noise. The experiments were conducted in a water tunnel with test section dimensions 1.28 m x 0.81 m x 3.0 m. A 2.1 m-long slender body of revolution of 0.175 m diameter, equipped with a 0.66 m-long floating cylindrical element to measure the skin-friction drag was used for this study.

The results of floating balance measurement by Choi *et al.* showed that the turbulent skin-friction drag is reduced by up to 7 % by Coating 1. The second compliant coating showed only a marginal drag reduction at the lower end of the velocity range with a slight increase at higher velocities. A typical error in measuring the turbulent skin friction was  $\pm 2$  %. The momentum thickness of the turbulent boundary layer over the test model for a tunnel speed of 4 m/s is approximately 5 % lower over Coating 1 as compared to a rigid surface. This is further evidence that Coating 1 has drag-reducing properties. There were clear reductions in RMS values of skin-friction fluctuations by up to 6 % over the range of test speeds from 1.5 m/s to 4 m/s for Coating 1, which is consistent with the results of skin-friction drag measurement. The wall-pressure intensity reveals reductions for Coating 1 of as much as 19 % compared with that for the rigid surface.

## 2. Required conditions for drag reduction

As one of the conditions for turbulent drag reduction, Semenov [15] suggested that the *dynamic* surface roughness of compliant coatings must be small, below the value to be considered as hydro-dynamically smooth. This is to say that the magnitude of surface deformation of compliant coatings interacting with the turbulent boundary layer should be much less than the viscous sublayer thickness. For compliant coatings used by Russian group, it has been estimated that the non-dimensional amplitude of wall deformation is  $\eta^+ < 1$ . For typical application in turbulent boundary layers, therefore, the dynamic roughness of compliant coatings is not an issue.

Table 1. Boundary layer parameters and the material properties of the compliant coating in the previous studies.

	Kulik <i>et al.</i> [16]	Lee <i>et al.</i> [17]	Choi <i>et al.</i> [19]
$U$ m/s	6.0 ~ 15.0	0.15 ~ 0.51	2.0 ~ 6.0
$\rho$ kg/m <sup>3</sup>	$2.1 \times 10^3$	$1.0 \times 10^3$	$2.1 \times 10^3$
$E$ Pa	$3.7 \times 10^6$	$0.68 \times 10^3$	$2.8 \times 10^6$
$H$ mm	2.5 ~ 7.0	38.0	7.0
$C_T$ m/s	24.0	0.47	20.9
$f_0$ Hz	$1.5 \times 10^3 \sim 4.2 \times 10^3$	5.7	$1.3 \times 10^3$
$t_0^+$	26 ~ 74	5 ~ 63	5 ~ 44
$U/C_T$	0.25 ~ 0.63	0.32 ~ 1.1	0.096 ~ 0.29

Another condition for turbulent drag reduction is that the natural frequency of compliant coatings must be chosen in such a way to give an appropriate response to the fluctuating wall pressure in the turbulent flow [15]. For homogeneous, single-layer material with a modulus of elasticity  $E$ , density  $\rho$  and thickness  $H$ , the fundamental frequency of the coating's longitudinal vibration is given by  $f_0 = \sqrt{E/\rho}/4H$ . Therefore, the corresponding non-dimensional period of the coating's fundamental frequency is given by  $t_0^+ = f_0^{-1} u^*{}^2/\nu$ . The effect of the viscoelasticity on the vibrational characteristics is usually small, which can often be neglected. The values for  $f_0$  and  $t_0^+$  for previous investigations of compliant coatings by Kulik *et al.* [16], Lee *et al.* [17] and Choi *et al.* [19] are summarised in table 1. It can be seen that the non-dimensional period of the first harmonic of the compliant coating falls within  $5 < t_0^+ < 74$  in all of these experiments.

The maximum static deformation of the compliant coating studied by Choi *et al.* [19] is estimated to  $\eta^+ = 0.12$ . Therefore, it is unlikely that the compliant coating is able to interact with the near-wall structure of turbulent boundary layer unless the compliant coating is resonated during the sweep events to give greater wall amplitude. It is known that the period of the pressure pulse during the sweep events is about  $t_0^+ = 20$  [20,21], which is within the range of non-dimensional period of the first harmonic of compliant coatings (see above). This suggests a strong possibility that the pressure pulse is causing a resonance to the coating as it interacts with the surface during the sweep events [22].

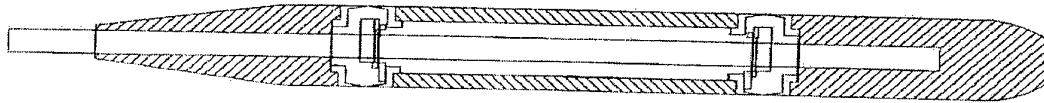


Figure 1. Test model used for the drag reduction study.

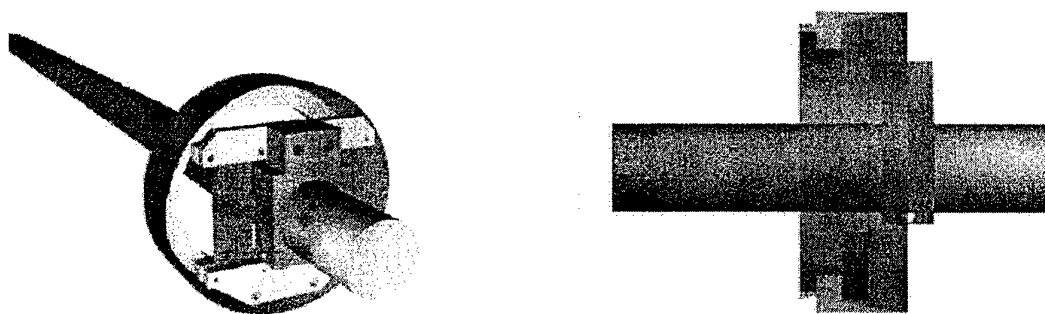


Figure 2. Strain-gauge balance details of the test model.

### 3. Experimental set-up

A water flume at the University of Liverpool, which has been used in our previous investigation [18,19], was used for the present tests. This test facility can be used as an open channel or a closed water tunnel when a cover is fitted over the flume. The basic open channel arrangement of this facility allows easy access to the test section which measures 1.37 m wide x 0.84 m deep x 3.66 m long, thereby minimising the setting-up time of the experiments. The flow velocity of the flume can be controlled from a minimum speed of 0.03 m/s to the maximum of 6.1 m/s.

The test model we originally planned to use was a 2.1 m-long axisymmetric body of 0.175 m diameter, equipped with a 0.66 m-long floating cylindrical element. We have data for the distribution of static pressure over the model. Basic flow characteristics of the boundary layer over the model are available, including the mean velocity profiles, turbulence intensity, skewness and kurtosis. Drag characteristics of the model vs. flow speed are also well documented. It has been agreed before the tests began that the test samples of compliant coatings for this model will be produced in Novosibirsk and shipped to the University of Nottingham, UK for hydrodynamic tests. The Russian team informed us later, however, that compliant coatings to fit to this model could no longer be available due to the damages made to their equipment.

Table 2. Material properties of compliant coatings being investigated. Elasticity and loss tangent are quoted for quasi-equilibrium values at  $f = 1$  Hz.

Material	Thickness mm	UK code (date)	US code (date)	Density $\text{kg/m}^3$	Elasticity MPa	Loss tangent
N3A	5	32 Pink (27 April 01)	Pink 5N (23 April 01)	$2.14 \times 10^3$	1.75 @11days	0.185 @11days
N3A	6	42 Pink (26 April 01)	Pink 6N (03 May 01)	$2.14 \times 10^3$	1.75 @11days	0.185 @11days
N5	3	52 Clear (23 May 01)	Clear 3N (20 April 01)	$1.00 \times 10^3$	0.88 @25days	0.010 @25days
N5	5	51 Clear (17 May 01)	Clear 1N (10 March 01)	$1.00 \times 10^3$	0.88 @25days	0.010 @25days
N5	7	22 Clear (25 May 01)	Clear 7N (25 May 01)	$1.00 \times 10^3$	0.88 @25days	0.010 @25days

After a discussion with Dr Bandyopadhyay of ONR, however, we have decided to produce and use a new test model, which can accommodate small compliant cylinders (76.2mm diameter, 298mm long). Essentially, this is identical to the model used in Russia (Institute of Thermophysics) and in USA (NUWC). The details of our test model are shown in figure 1. The model consists of three parts: a flat nose section (219mm long), a test section (298 mm long) and a tail section (206 mm long). The skin-friction force is measured by a set of strain-gauge balance within the test model housing (see figure 2). Most of the test model parts were made of UPVC to reduce their weight. A thin disk of 0.5 mm protrusion height was sandwiched between the nose and frontal section of the model to tip the boundary layer, thereby fixing the transition point.

First set of compliant coatings was delivered from the Institute of Thermophysics, Russia for drag measurements. The set consisted of five cylinders (see Table 2), of which one cylinder (Coating 42) was badly damaged on transit. The second set of cylinders with different compliant coatings is expected later this summer.

#### 4. Results

Initially we have experienced a problem with the test model. There was some inconsistency in drag measurement with the strain-gauge balance, suggesting that the measured drags were depending on the weight of the cylinder. The new test model we have used in the current tests was much smaller than the previous model, so that the measured skin-friction drag force was quite small (about 20% of that of the previous test model). Accordingly, we had to use a strain gauge that was more sensitive. Perhaps the strain-gauge balance was too sensitive to be affected by the weight of the cylinder through friction force during the translational movement. The drag values given in this report is an average of (more than three) repeated measurements.

The drag force was measured for each compliant coating at flow speeds between 0 to 4.5 m/s. The measurements were then repeated, and the averaged data are shown in figures 3 to 7. The corresponding data for rigid surface test are shown in figure 8. Since there are some offset in each of the drag curve, we have firstly fit a third order polynomials through data points and subtracted the offset from the original data. Figure 9 summarises all the drag data, which are compared with those of based line test using a rigid surface. The percentage drag reductions for each of five coatings are then obtained from these data, which are shown in figure 10.

Contrary to what we believed initially (mainly due to the zero offset present in the measured data), there are some, although small, drag reductions for some of the coatings tested, notably for Coating 51 for up to 3%. Coating 52 also has some extent of drag reductions ( $U = 1$  to 3 m/s). Coating 42 is the damaged one, but it also shows some drag reductions at low flow speeds up to 3 m/s. Drag of Coating 42 increases somewhat at higher flow speed, however, probably because of roughness effect due to the damaged surface.

The preliminary MIT data obtained by C. W. Hensch (NUWC, Newport) are shown in figure 11 for comparison. Initial observation of the figure indicated that MIT data are somewhat scattered, while drag curves obtained in the UK are very smooth. It was also noted that the drag curve for MIT data seems to behave quite differently from that of UK data, particularly at low flow speeds, having a kink at around the flow speed of 2 m/s. It will be shown later that this is caused by the lack of transition trip on the test model used.

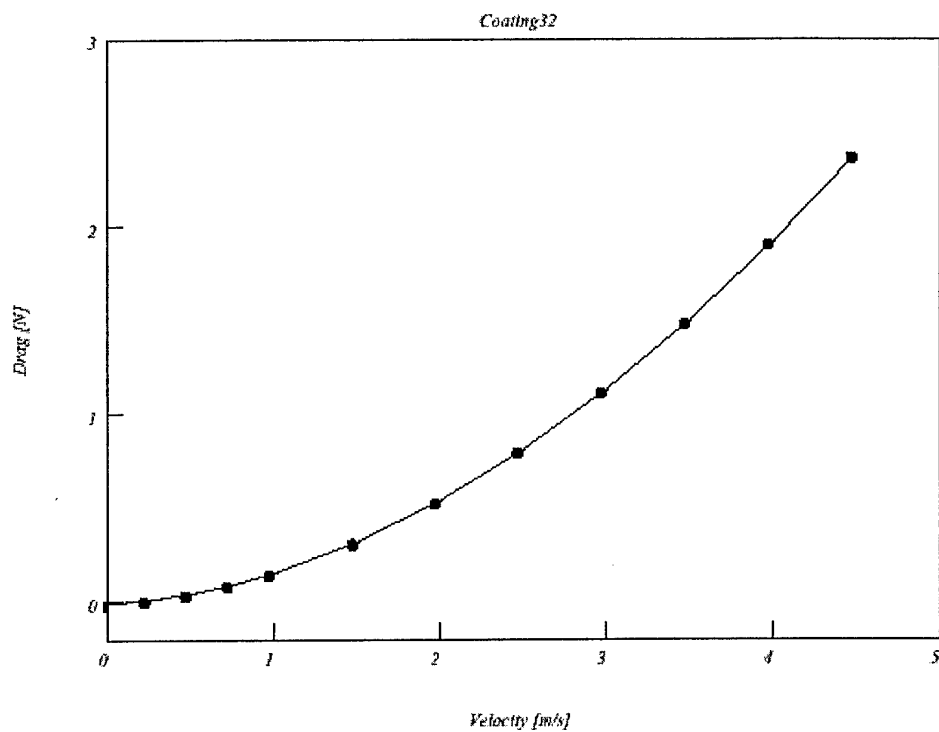


Figure 3. Skin-friction drag of Coating 32.

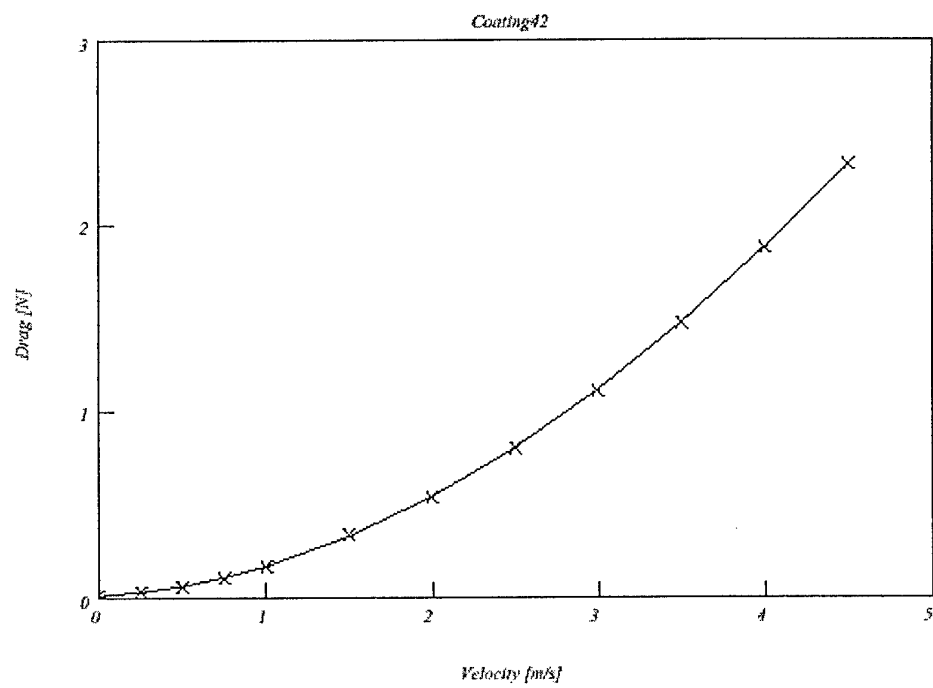


Figure 4. Skin-friction drag of Coating 42.

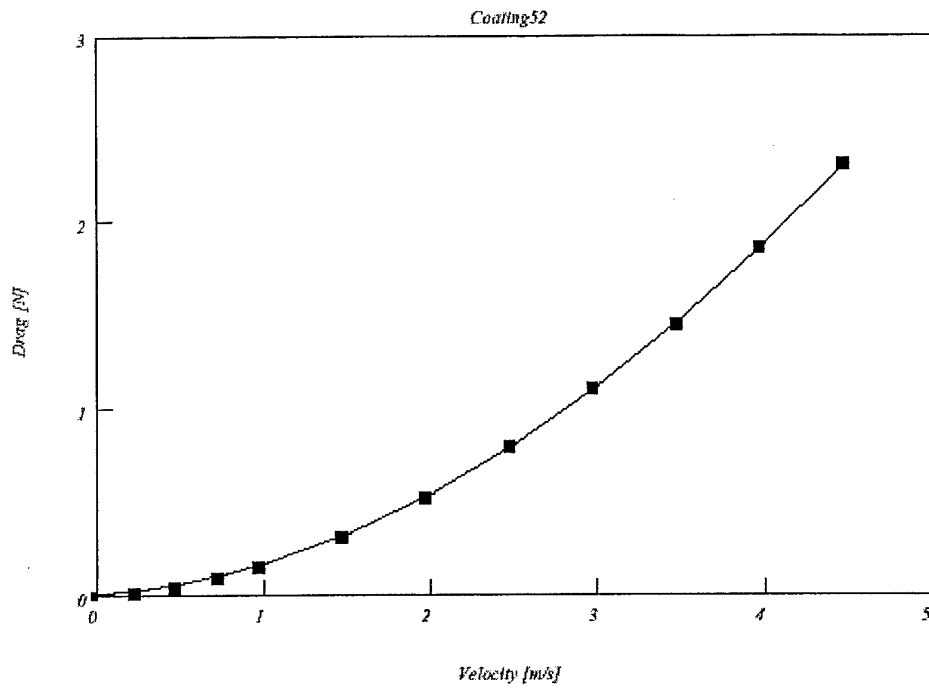


Figure 5. Skin-friction drag of Coating 52.

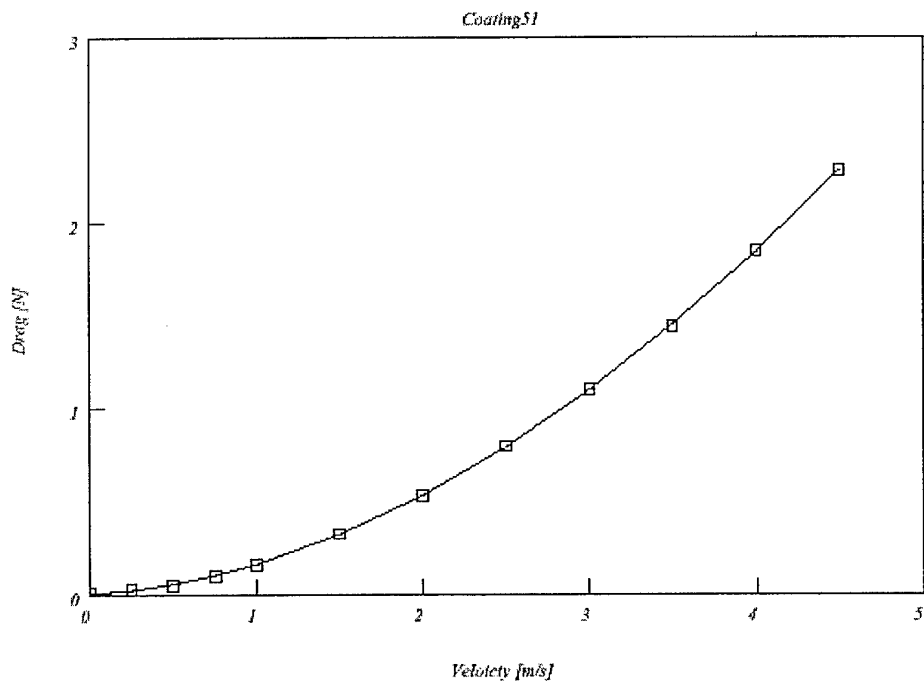


Figure 6. Skin-friction drag of Coating 51.

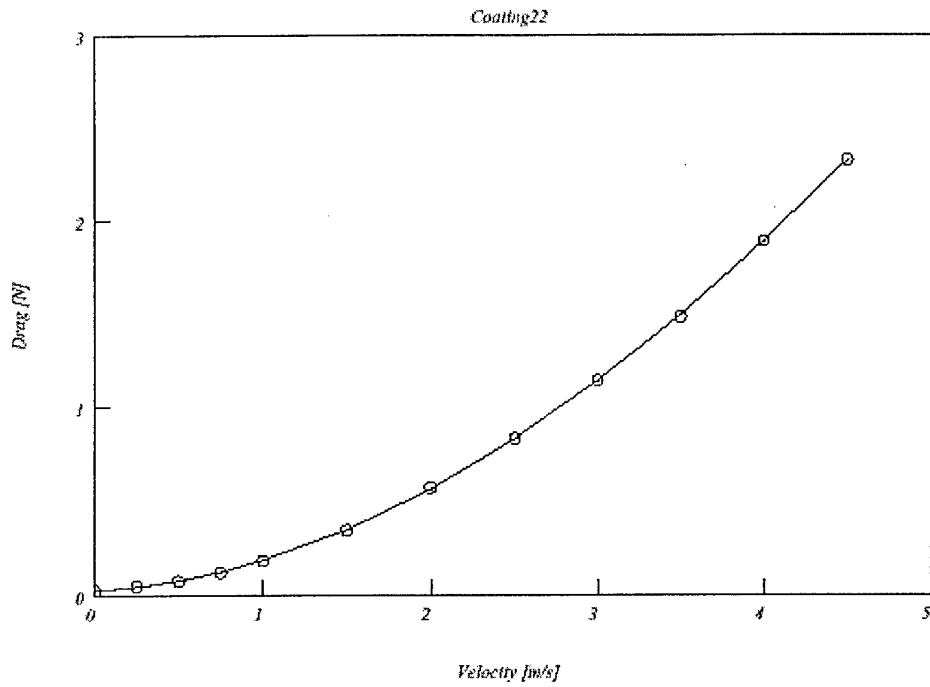


Figure 7. Skin-friction drag of Coating 22.

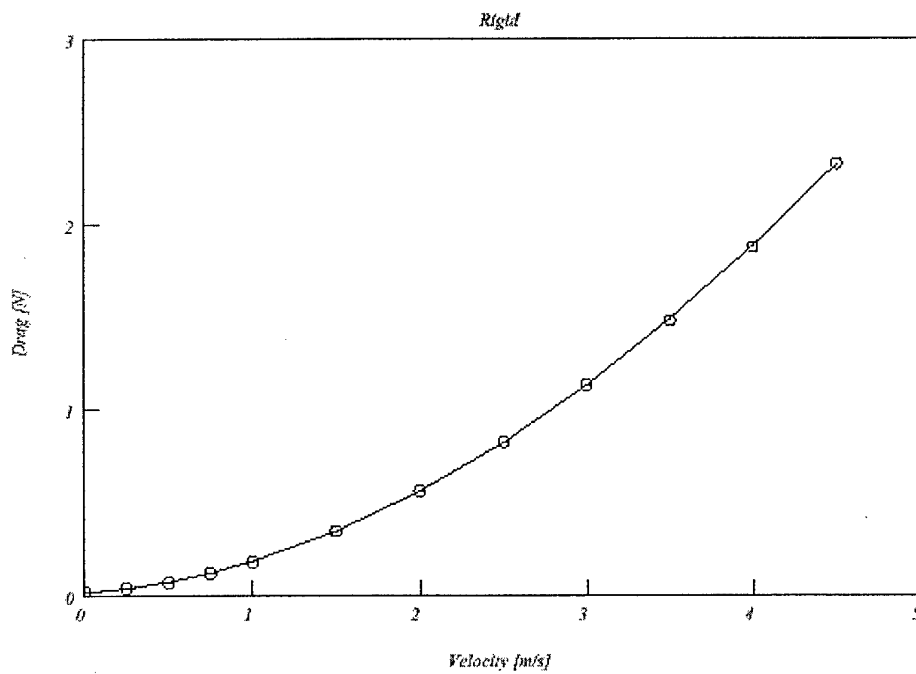


Figure 8. Skin-friction drag of the rigid surface.

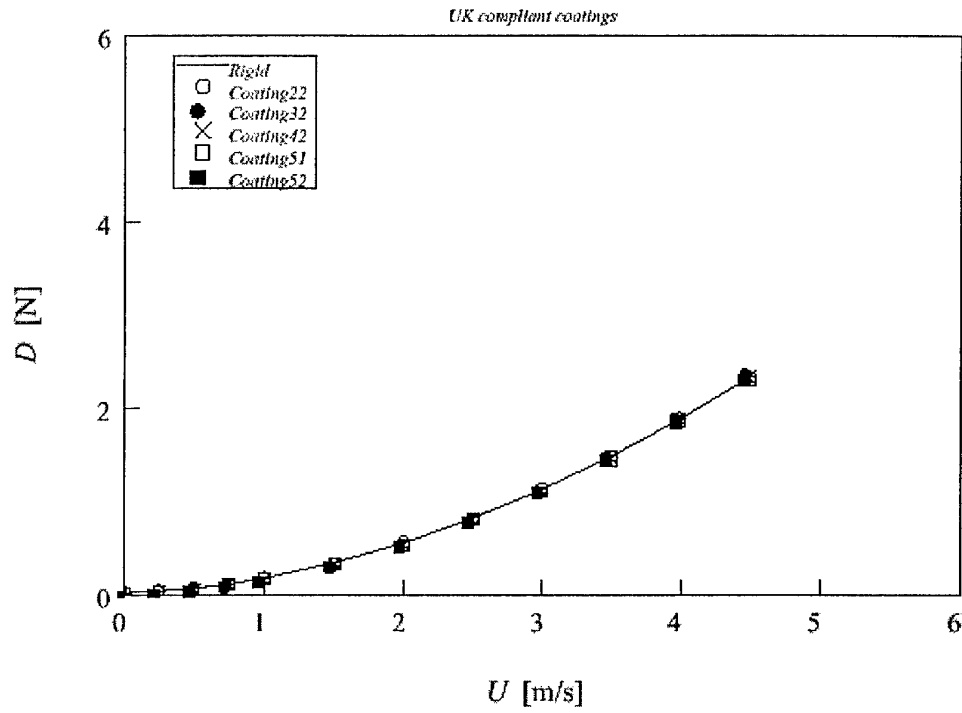


Figure 9. Skin-friction drag of compliant coatings.

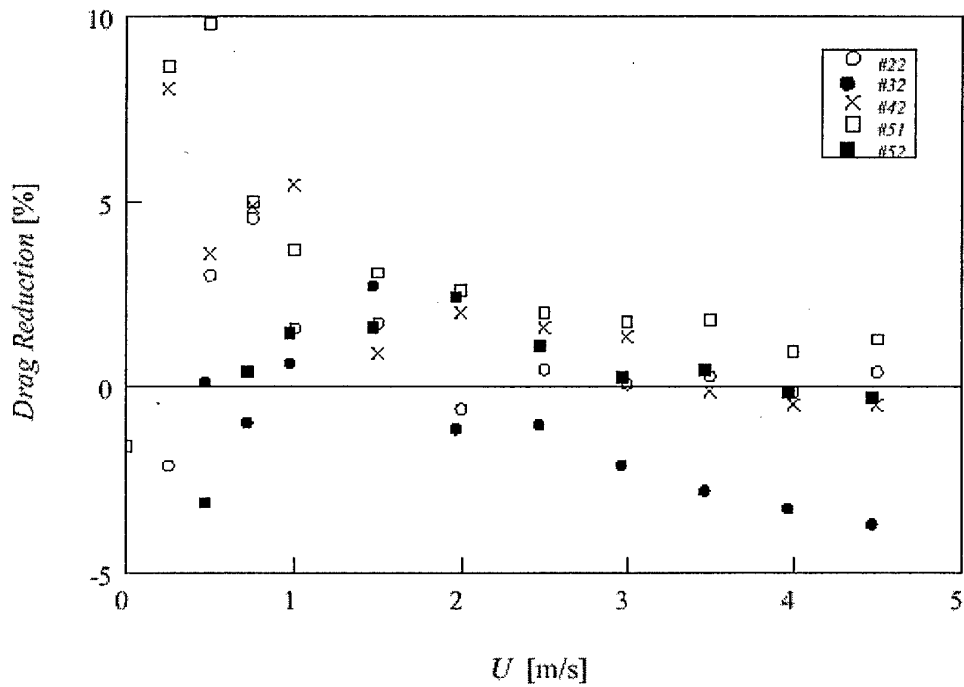


Figure 10. Drag reduction of compliant coatings.

As an effort to reduce the amount of scatter in the experimental data, we have applied a similar data reduction technique (described above) to MIT data. This time, a third-order polynomial was fit through only those data points where the boundary layer over the test model is thought to be turbulent. Figure 12 shows all the baseline data for rigid surface test with a curve drawn through data points for flow speeds between  $U = 2.5$  m/s and 5.5 m/s. Here, the second data set (Base2N) was not used to obtain the least-squared fit curve, since it was quite different from the other data set. Inclusion of this data set would increase the amount of drag reduction by compliant coatings, however. Figure 13 compares the skin-friction drag of each of compliant coating with the baseline drag curve given in figure 12.

While experimental results in the UK show small drag reductions (by up to 3%, see figure 10) in some of the compliant coatings tested, skin-friction drags of compliant coatings measured at MIT are consistently greater (see figure 14) than that of rigid surface. Nevertheless, there is some consistency between the UK and US results in the relative magnitude of skin-friction drag for each compliant coating. In other words, the skin-friction drag of Coating 22 (Clear 1) was always least among the compliant coatings tested. On the other hand, the skin-friction drag of Coating 32 (Pink 5) was always greatest. It is also noted that the maximum drag reduction in the UK test was found at flow speeds between 1.5 m/s and 3.5 m/s, while the least drag increase in MIT tests was found at much higher speeds. Figure 14 even suggests that there might be a drag reduction for some of the compliant coatings at much higher flow speeds, say  $U > 7$  m/s.

Figure 15 shows the coefficient of friction drag  $C_D$  vs. the flow speed  $U$ , confirming that the boundary layer over the compliant coating surface was fully turbulent for the UK measurements, since  $C_D$  value decreases with an increase in flow speed as predicted. Indeed, the skin-friction drag over the rigid surface is very similar to that of empirical data [23] for  $U > 1$  m/s. Slightly smaller values in  $C_D$  could be due to the trip mounted at the nose section of the test model, which may have increased the virtual origin of the boundary layer development through an increase in the boundary layer thickness.

On the contrary, the MIT data given in figure 16 shows that  $C_D$  value increases initially at low flow speeds ( $U < 2.5$  m/s), and then decreases with an increase in flow speed. This suggests that the boundary layer over the test model at MIT was partially laminar for flow speeds up to 3 m/s. It may be speculated further that the transition to turbulence may have been affected by the surface finish of the coatings, which could explain large differences in drag between compliant coatings and rigid surface (baseline) in MIT tests.

The measured drag force seems to be similar between the UK baseline data (see figure 8) and the MIT baseline data (figure 12) for flow speeds up to 2m/s. For flow speed above 2m/s, however, MIT data indicate much greater drag (by as much as 40%) as compared to those measured in the UK. This suggests that the drag measurements at MIT may have been affected by the lack of transition trip, as suggested above.

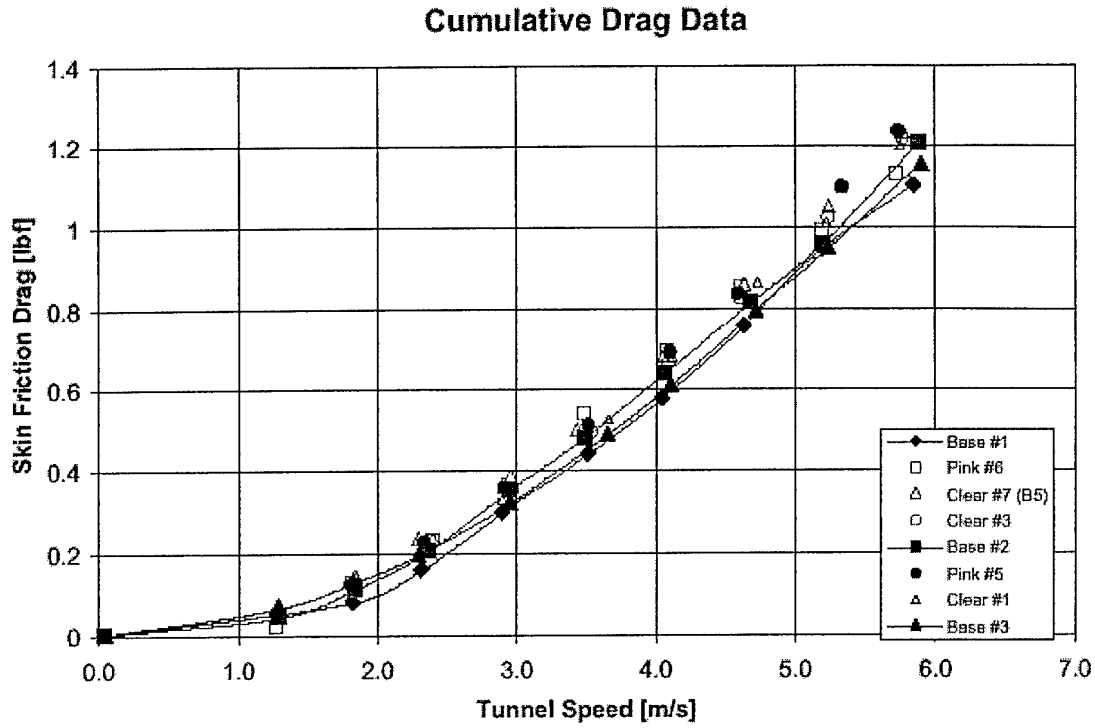


Figure 11. Skin-friction drag of compliant coatings in MIT tests (original data supplied by C.W. Henoch).

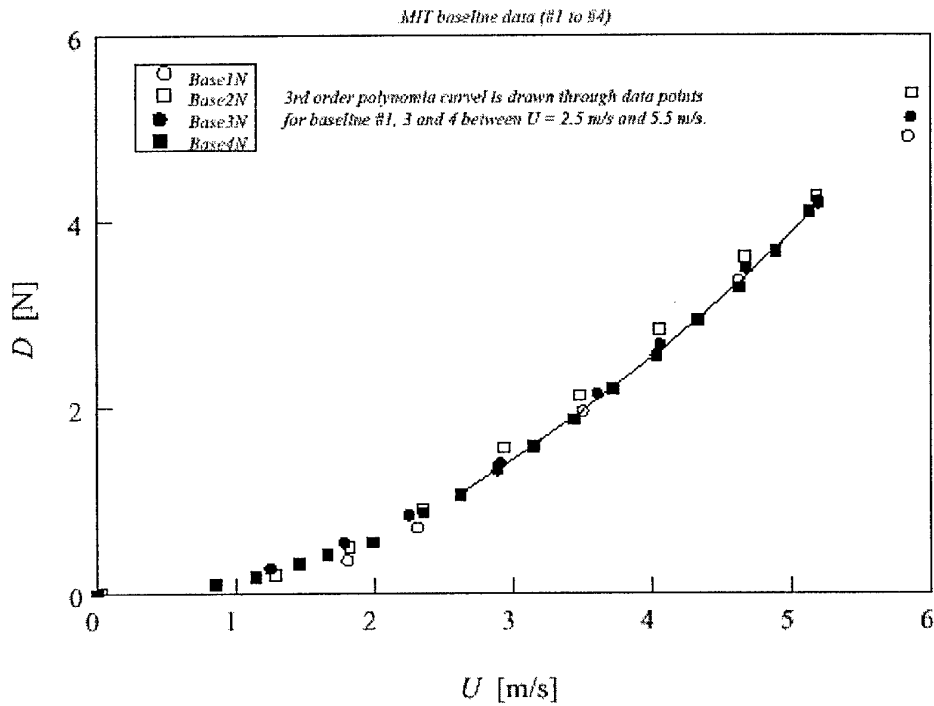


Figure 12. Skin-friction drag of rigid surface in MIT test.

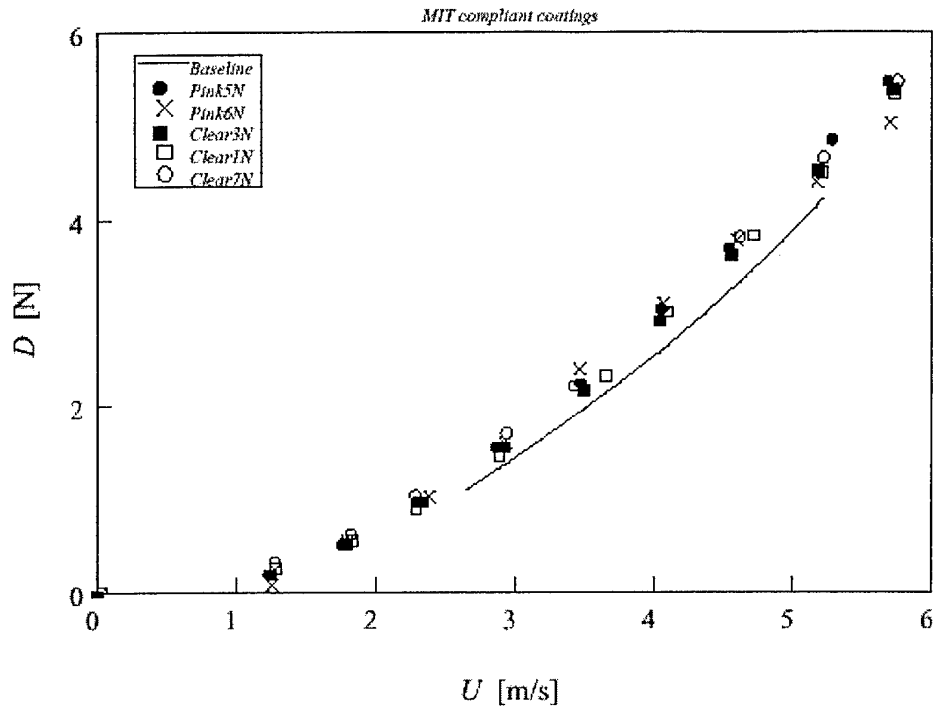


Figure 13. Skin-friction drag of compliant coatings in MIT tests.

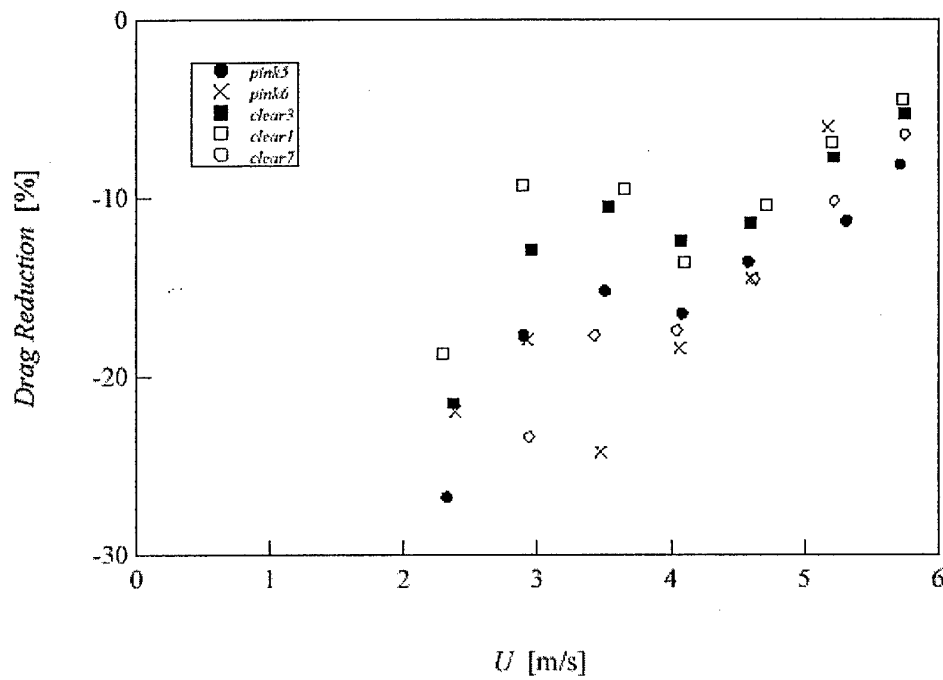


Figure 14. Drag reduction of compliant coatings in MIT tests.

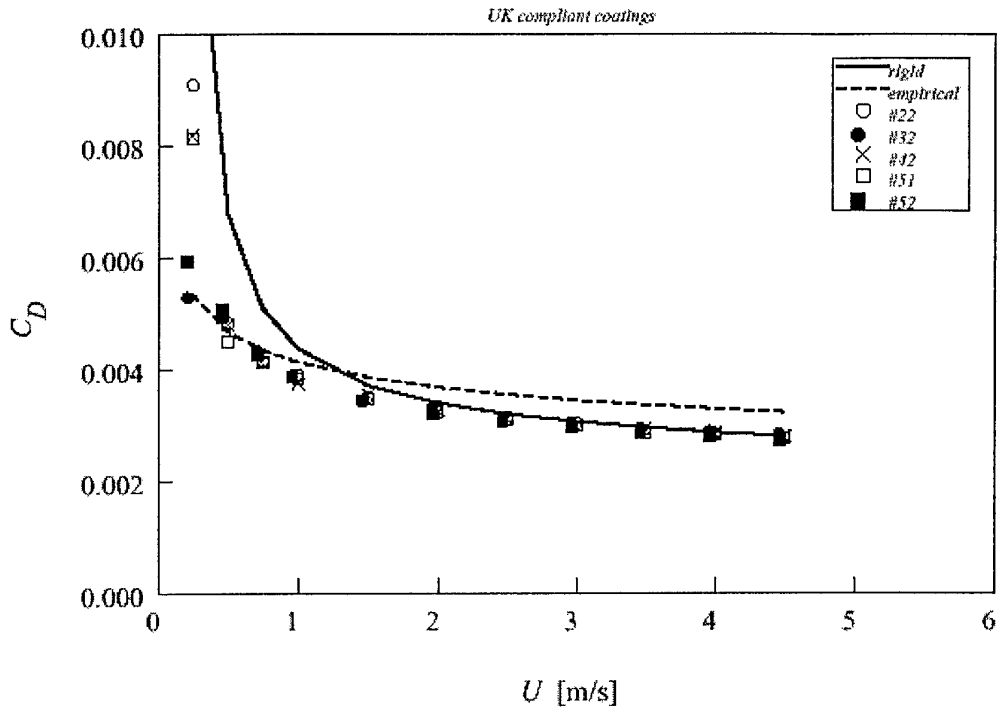


Figure 15. Skin-friction coefficient of compliant coatings in UK tests.

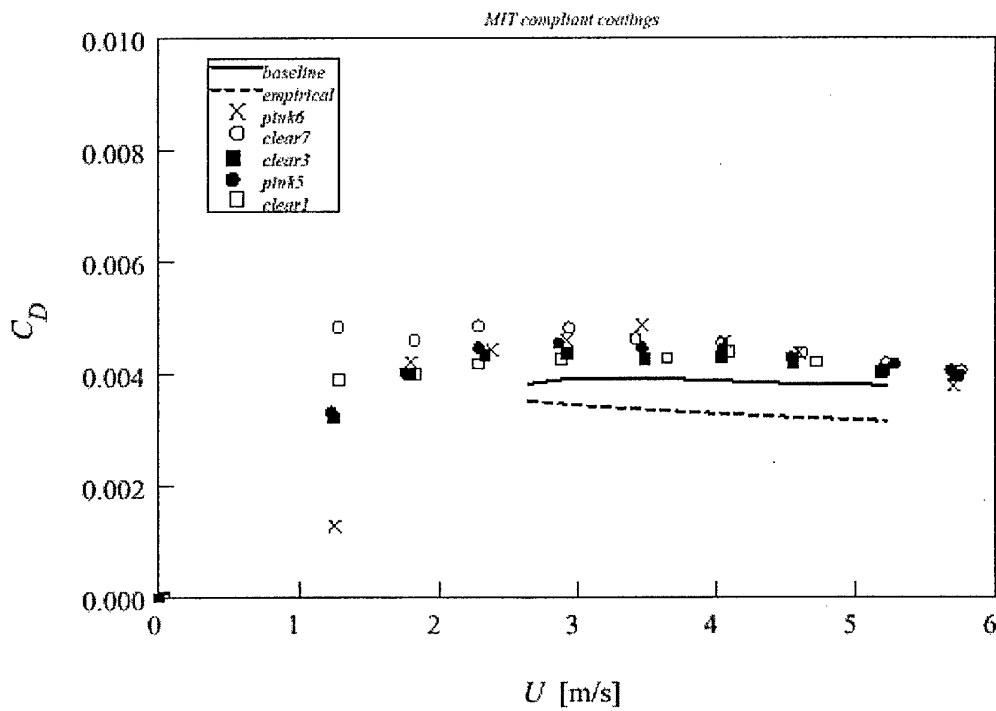


Figure 16. Skin-friction coefficient of compliant coatings in MIT tests.

## 5. Concluding remarks

The skin-friction drag of compliant coatings was measured using an axi-symmetric test model in a water tunnel for flow speeds up to 4.5 m/s. The test model had a transition trip at the end of the flat nose section, ensuring that the boundary layer over the cylindrical test section is fully turbulent. The results indicated that there are some, although small, drag reductions for some of the compliant coatings tested by up to 3%. However, the skin-friction drags of compliant coatings measured at MIT (carried out by NUWC, Newport) are consistently greater than the rigid surface drag. Nevertheless, there is some consistency between the UK and US results in the relative magnitude of skin-friction drag for each compliant coating. It is also noted that the maximum drag reduction in the UK test was found at lower range of flow speed, while the least drag increase in MIT tests was found at much higher flow speeds.

Figures 17 and 18 show the aging profile of one of the coatings (Coating 52; Clear 3) tested, which were obtained from Dr Kulik of the Institute of Thermophysics, Russia. The figures show the elasticity  $E$  (figure 17) and the loss tangent  $\eta$  (figure 18) against frequency  $f$ , with time (given in days) since manufacturing the coating as a parameter. In only 3 months the coating hardened (in terms of the modulus of elasticity) by twice, while the material damping (measured by the loss coefficient) reduced to one third. Since the fundamental frequency of the coating's longitudinal vibration is proportional to the square root of the modulus of elasticity ( $f_0 \sim \sqrt{E}$ ), the effect of material aging will push the effective frequency of near-wall turbulence. This means that the optimum flow speed for drag reduction will increase, although the effect of reduction in loss tangent to drag reduction is not clear.

Drag measurements in the UK were made between July and September 2001, while tests at MIT was carried in March and April 2002. Since all the coatings were made in April and May 2001, the coatings have aged by approximately 3 months by the time the UK tests were carried out. It is also clear that 11 months have already passed for the MIT tests since manufacturing of the compliant coatings. This would explain the reasons why the measured drags were different between the two; the optimum velocity for drag reduction have shifted to higher flow speeds in the MIT tests.

When we make drag measurements on the next set of compliant coatings, we must make sure that we will carry out the tests as soon as we receive the coatings. If this is difficult for practical reasons, we should try to make measurements at the same time to ensure that we will test compliant coatings with identical material properties.

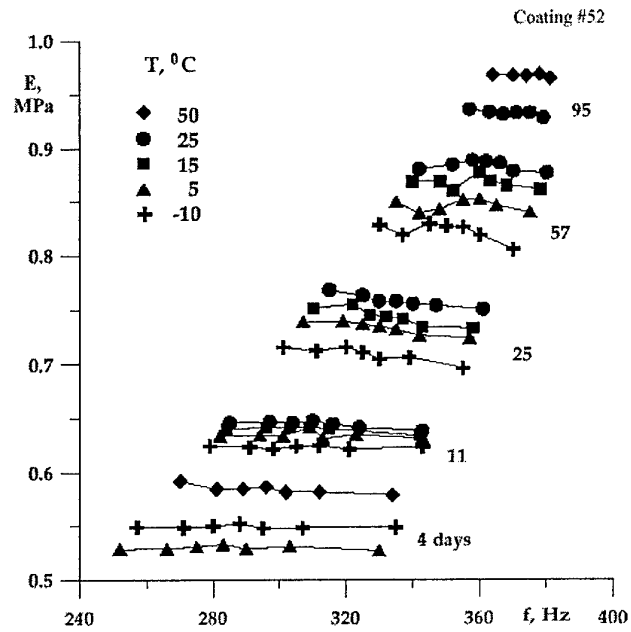


Figure 17. Change in the modulus of elasticity due to aging of Coating 52.

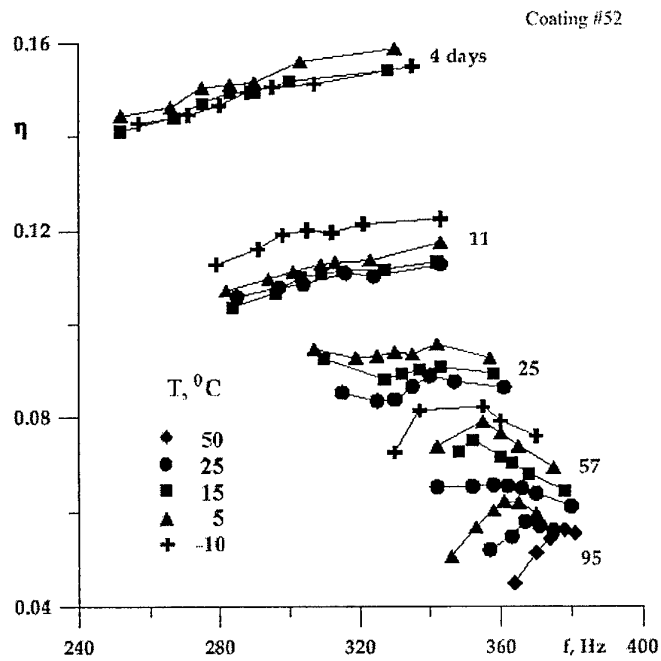


Figure 18. Change in the loss tangent due to aging of Coating 52.

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