CASE OF SHORT DATA RECORD FOR BOTH TRAINING AND SIGNAL DETECTION

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1. INTRODUCTION

Detection of a signal embedded in interference is a common problem encountered in radar, sonar and communication systems. In cases where it is known that the interference is low rank (or approximately so) the amount of data required for adaptation can be reduced by using reduced rank estimation methods. Three proposed methods for making the selection of basis vectors are the Cross Spectral Metric (CSM) [1] method, the Principal Components Inverse (PCI) [2] method and Multistage Wiener Filter (MWF) [3]. The examination here is for detection of a signal that may or may not be present within a given set of data. That is, training and signal detection must be performed using the same data set. The case of independent training and test data has been treated in [4, 5].

2. ADAPTIVE CSM, MWF AND PCI

The methods of CSM, PCI and the MWF offer differing ways of providing an adaptive processor in the signal based coordinates,

$$|S^H X - W_{GSLC}^H(\mathbf{B}^H X)| \tag{1}$$

where S is orthogonal to the columns of $\mathbf{B}$. Referring to Figure 1, given a set of K data vector samples, form a $1 \times K$ vector $\mathbf{d} = \begin{bmatrix} d_1 & d_2 & \cdots & d_K \end{bmatrix}$ and a $N-1 \times K$ matrix $\mathbf{Z} = \begin{bmatrix} Z_1 & Z_2 & \cdots & Z_K \end{bmatrix} = \hat{\mathbf{U}}\hat{\mathbf{\Sigma}}\hat{\mathbf{V}}^H$ from the data in the signal and orthogonal space respectively. Subsequently, we will use the ^ symbol to denote estimation from data. For example, $\mathbf{R}_Z$ is the the covariance matrix of the vector Z_k and $\hat{\mathbf{R}}_Z$ is an estimate of $\mathbf{R}_Z$ using a finite number of vector samples.

Adaptive versions of CSM, PCI and the MWF can be constructed by using covariance and cross-covariance estimates.

$$\hat{\mathbf{R}}_Z = \tfrac{1}{K}\mathbf{Z}\mathbf{Z}^H \qquad \hat{r}_{dZ} = \tfrac{1}{K}\mathbf{Z}\mathbf{d}^H \tag{2}$$

For CSM and PCI the weight vector is formed as

$$\hat{W}_{GSLC} = \hat{\mathbf{U}}_p\hat{\mathbf{\Sigma}}_p^{-2}\hat{\mathbf{U}}_p^H\hat{r}_{dZ} \tag{3}$$

using the p singular vectors and values selected by the given method. Adaptive CSM uses the estimated cross spectral metric; whereas, PCI uses only the estimated singular values to determine which singular vectors are kept. The MWF uses the estimated quantities in place of the known quantities in the construction of the multistage decomposition. It should be noted that the philosophy in the development of these methods have differences. CSM and MWF were formulated as a rank reduction for a prescribed rank and general covariance; whereas, PCI was developed with the assumption that the covariance is from a low rank process and the rank is estimated from data over the adaptation interval [6].

3. TRAIN AND TEST ON SAME DATA

Often the scenario is such that the calculation and application of the weight vector is to be performed on the same data set. In this case, with no signal present, CSM has been shown to be the optimal method with respect to mean square error for the selection of singular vectors, and an upper bound to the performance of PCI. Consider, the assumption that the interference has a correct rank such that CSM and PCI should nominally choose the same singular vectors. Suppose the realization of data has a swap [4] in the singular vectors chosen by CSM. Then CSM will choose a set of singular vectors which may not be best for the entire set of all possible realizations, but that doesn't matter since the weight vector will only be used on this realization. PCI on the other hand, chooses singular vectors which should work best on all possible realizations and thus does not perform as well as CSM on this particular realization. That is, the reasoning that allows PCI to outperform CSM in the independent data case is the reason that it is poorer in the same data case with respect to mean square error.

3.1. Toy Example

Let us construct a simple concrete example which can be used to highlight the characteristics of each method. Assume a five element array with four sample snapshots and two jammers. Without loss of generality assume that the beams of the orthogonal space are chosen to be the eigencoordinates. Choose an equal power of 1000 for the jammers and let the signal channel be given as $\mathbf{d}_k = 0.01\mathbf{Z}_{1,k} + 0.1\mathbf{Z}_{2,k} + 0\mathbf{Z}_{3,k} + 0\mathbf{Z}_{4,k}$. Suppose the data with only jamming (no signal or background noise) is

$$\mathbf{d} = \begin{bmatrix} -90 & 90 & 110 & -110 \end{bmatrix} \quad (4)$$

$$\mathbf{Z} = \begin{bmatrix} 1000 & -1000 & 1000 & -1000 \\ -1000 & 1000 & 1000 & -1000 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

The cross covariance of this data is

$$r_{dZ} = \begin{bmatrix} 10000 & 100000 & 0 & 0 \end{bmatrix}^T \quad (6)$$

and the cross correlation is given by

$$\rho_{dZ} = \begin{bmatrix} 0.0995 & 0.9950 & 0 & 0 \end{bmatrix}^T \quad (7)$$

Clearly all three methods will choose identical subspaces and thus have identical performance. Let us now include the effects of background noise, $\mathbf{N}$, in the orthogonal beams such that $\mathbf{Z} + \mathbf{N}$ is now given as

$$\mathbf{Z} + \mathbf{N} = \begin{bmatrix} 1000.3 & -999.2 & 1001 & -1002 \\ -998 & 1001 & 1002 & -1000.2 \\ -1 & -0.5 & 0.5 & 1 \\ 0.5 & 0.2 & 0.1 & -0.5 \end{bmatrix} \quad (8)$$

The cross covariance for this situation has now become

$$r_{dZ} = \begin{bmatrix} 10094 & 100038 & -2.5 & 0.75 \end{bmatrix}^T \quad (9)$$

and the cross correlation has changed to

$$\rho_{dZ} = \begin{bmatrix} 0.1004 & 0.9951 & -0.0315 & 0.0201 \end{bmatrix}^T \quad (10)$$

The addition of the background noise has caused some perturbation but not sufficient to cause PCI and CSM to disagree on the singular vector selection. The first MWF basis vector will also have a negligible change since $10094 >> 2.5$. Let us now introduce a signal level of 100 in the first snapshot such that the signal channel data is given by

$$\mathbf{d} = \begin{bmatrix} 10 & 90 & 110 & -110 \end{bmatrix} \quad (11)$$

The cross covariance has now changed to

$$r_{dZ} = \begin{bmatrix} 35101 & 75088 & -27.5 & 13.25 \end{bmatrix}^T \quad (12)$$

and the cross correlation has changed to

$$\rho_{dZ} = \begin{bmatrix} 0.3898 & 0.8341 & -0.3865 & 0.3970 \end{bmatrix}^T \quad (13)$$

Recall that the PCI choice of subspace is only determined by the power of each eigenchannel and so is unaffected by the presence of signal. Although the values of the cross covariance have changed quite a bit, the first two channels are still the dominant values and the two dimensional subspace chosen by the MWF will only slightly be affected. CSM on the other hand has undergone a change in its choice of subspace due to perturbations in the correlation as a result of the signal presence. CSM will now choose channels 2 and 4 as opposed to 1 and 2. Thus one would now expect decreased jammer suppression as well as increased signal cancellation. The fact that the MWF uses the cross covariance, which is a combination of power and correlation makes it less susceptible to perturbation by introduction of signal in a jamming environment.

Consider now the cases when the estimation of the rank is greater than the true rank. The PCI method will choose basis vectors which contain residual power due to errors in the estimation of the true interference subspace but will be unaffected by the presence of signal. The CSM method will continue to choose singular vectors based on the signal perturbed values of the cross correlation. Once the interference is essentially canceled, CSM is choosing the singular vector for which the noise can best be used to cancel signal and thus will suffer a loss of performance. For the MWF, the subspace selection for ranks at or below the interference rank provides good subspace estimation although some perturbation due to signal presence does exist. However, the strength of the interference in the well estimated subspace will dominate the calculation of the weight for that stage. Once the interference has been suppressed, the MWF will then construct the next basis vector from the residual noise in an attempt to cancel out the signal channel. Unlike CSM which can only choose between singular vectors based upon correlation, the MWF utilizes correlation in the construction of the basis vector. The MWF will therefore suffer significant signal cancellation once the rank is overestimated.

3.2. Single Jammer Simulation

Simulations for the same training and test data were run using a signal plus noise to average noise criteria. The signal used was a single snapshot, random phase signal at broadside to the array. Placing the signal only in a single snapshot is done without loss of generality since it does not statistically change the SVD or cross covariance and cross correlation estimates.

A set of 16 signal free snapshots was created and filtered using the PCI, CSM and MWF methods followed by matched filtering in time with a squaring of the output. The signal was then added to this set of snapshots and the filtering process was repeated on the signal plus noise data. The ratio $\frac{(S+N)_{OUT}}{MEAN(N_{OUT})}$ was computed for each method. The first set of simulations is performed with only one jamming signal present near a null at 22 degrees and 20dB JNR as shown in Figure 2.

The results for PCI and CSM are plotted as scatter plots for varying levels of input SNR in Figure 7. Each dot represents an X-Y plot of the results of two methods to an identical input. The upper left Figure is for the case of no signal. The dots are scattered nearly symmetrically about the diagonal with the two methods rarely producing the same result. There does appear a slight bias of the scattering towards PCI which one would expect since CSM provides the minimum mean squared error. Since the jammer level is 20dB, the PCI choice of basis vector should be nearly constant. Therefore, the disagreement of the methods is a result of the varying CSM choice [4]. Again, from the perspective of mean squared error, the choices are optimum. In the plot at the top right, a signal at 0dB is included in the data. An increase in the shift of the data to the PCI side of the diagonal is evident. Increasing the signal level to $12dB$ in the lower left, the vast majority of the disagreements between PCI and CSM result in a higher output $\frac{(S+N)_{OUT}}{MEAN(N_{OUT})}$ for the PCI method. When the signal level is raised to 24dB, as shown in the lower right plot, essentially all the disagreements of the two methods favors the PCI method.

Scatter plots for PCI and the MWF are plotted in Figure 12. For the cases of no signal and 0dB signal (top left and top right respectively) the two methods produce similar results spread around the diagonal. Recall that since PCI and the MWF construct the basis vectors differently, the agreement along the diagonal would not be exact as in the case of PCI and CSM. When the signal level is increased to 12dB and 24dB (bottom left and bottom right respectively) a slight advantage for the PCI method is created.

In Figure 13 the mean $\left(\frac{S+N}{MEAN(N)}\right)$ is plotted as a function of the signal strength for each method. The three methods are nearly identical at the -6dB signal level but the CSM method begins to show a drop in performance relative to PCI and MWF for signals beginning at 0dB. The difference in the methods holds nearly constant as the signal level is increased past 6dB. The MWF method is slightly below the PCI for larger signal levels although it is difficult to see on the plot.

Let us now examine the performance as a function of rank in Figures 14 and 15. As described earlier, the performance of the MWF drops significantly once the rank is overestimated. The CSM method shows the performance loss for the correct rank and drops faster than the PCI method when the rank used is above the true rank.

3.3. Multiple Jammers

Now consider a five jammer scenario. The pictograph of the scenario is plotted in Figure 16.

The scatter plots for PCI verses CSM are shown in Figure 21. The plots resemble those of the single jammer case

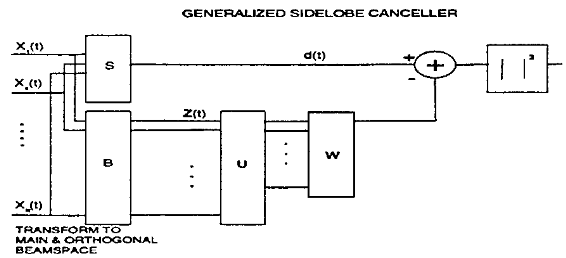

Figure 1: Generalized Sidelobe Canceler Structure

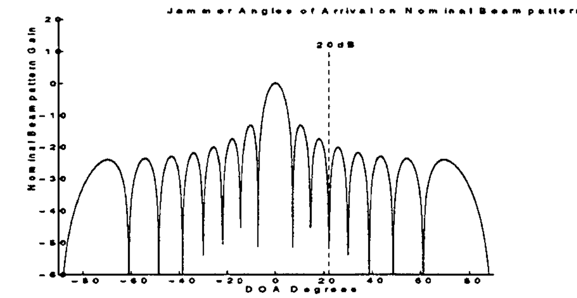

Figure 2: Single Jammer in Null at 20dB JNR

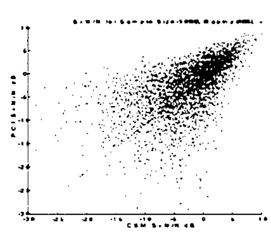

Figure 3: No Signal

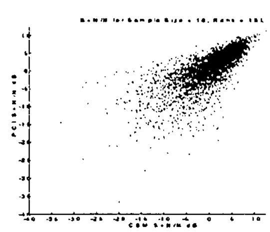

Figure 4: 0dB Signal

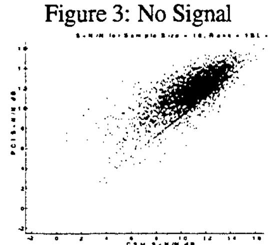

Figure 5: 12dB Signal

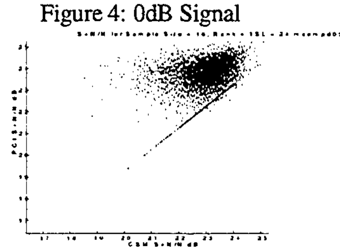

Figure 6: 24dB Signal

Figure 7: PCI Vs CSM $\frac{(S+N)_{out}}{MEAN(N_{out})}$

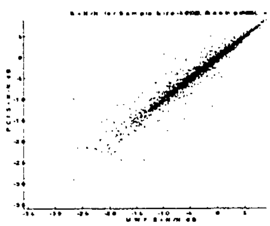

Figure 8: No Signal

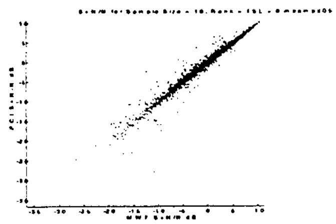

Figure 9: 0dB Signal

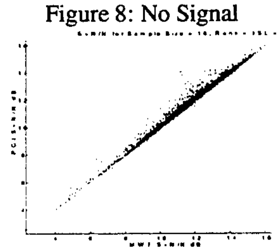

Figure 10: 12dB Signal

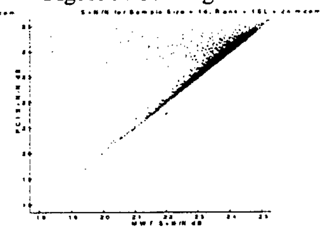

Figure 11: 24dB Signal

Figure 12: PCI Vs MWF $\frac{(S+N)_{out}}{MEAN(N_{out})}$

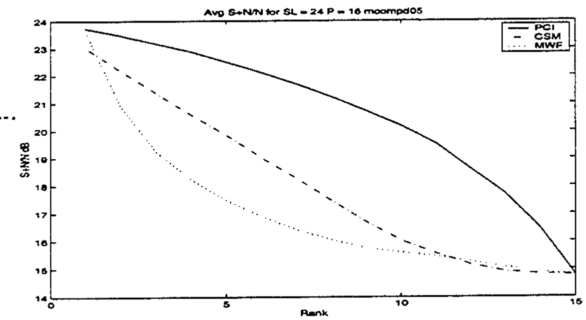

Figure 15: Performance as s Function of Rank for 24dB Signal with Single Jammer

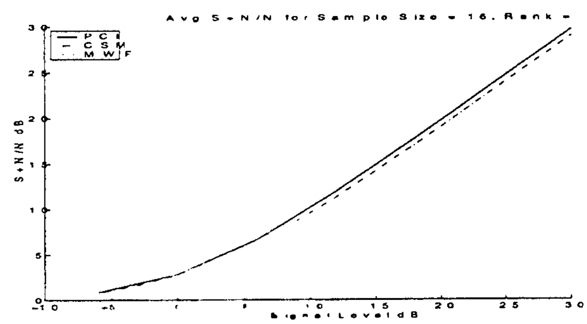

Figure 13: Performance as a Function of Signal Level

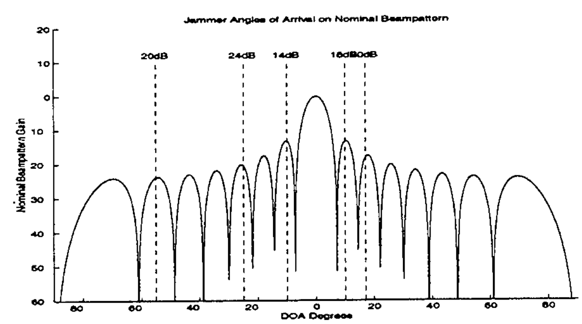

Figure 16: Five Jammer Scenario

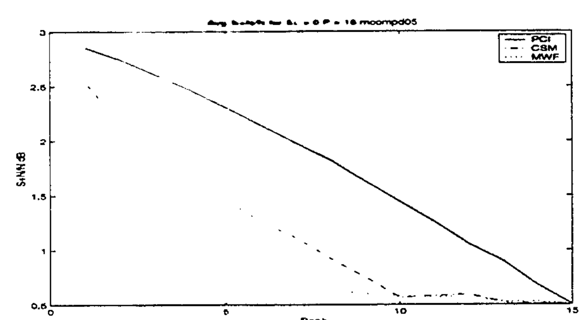

Figure 14: Performance as s Function of Rank for 0dB Signal with Single Jammer

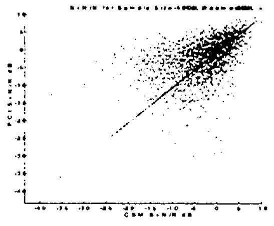

Figure 17: No Signal

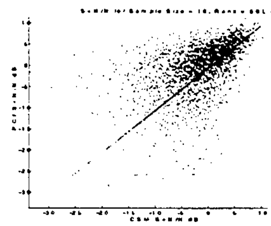

Figure 18: 0dB Signal

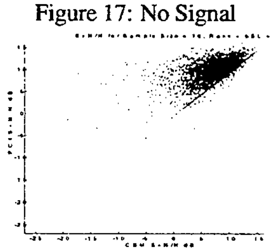

Figure 19: 12dB Signal

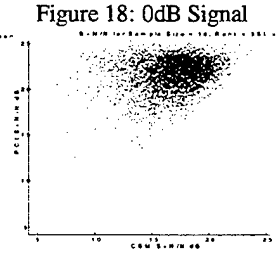

Figure 20: 24dB Signal

Figure 21: PCI Vs CSM $\frac{(S+N)_{out}}{MEAN(N_{out})}$

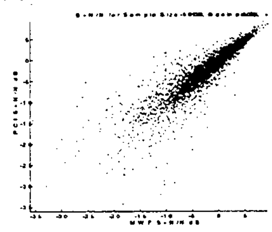

Figure 22: No Signal

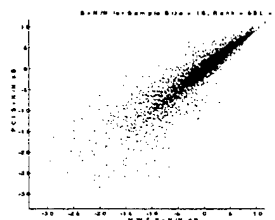

Figure 23: 0dB Signal

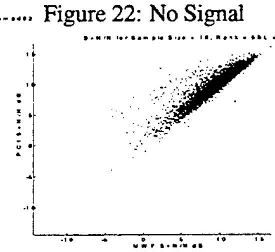

Figure 24: 12dB Signal

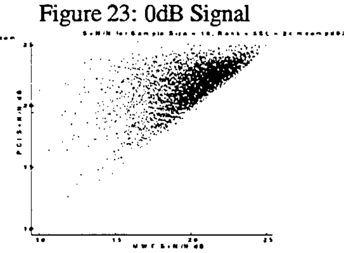

Figure 25: 24dB Signal

Figure 26: PCI Vs MWF $\frac{(S+N)_{out}}{MEAN(N_{out})}$

although the performance difference of the two methods has increased. In Figure 26, the scatter plots are shown for PCI verses the MWF. The performance difference between the two methods is now more noticeable for the higher signal level cases than was apparent with the single jammer. This results from the fact that the power levels in the last two or three basis vectors chosen by the MWF are not nearly as strong as the first two or three.

The view of the performance of the five jammer scenario verses signal level is plotted in Figure 27. As before, the CSM method shows a performance loss even for low signal levels. As the signal level is increased, the performance difference also increases as multiple errors in the choice of basis vectors occur The MWF method agrees well with the PCI method up to a signal level of 6dB at which point the performance of the MWF begins to lag that of PCI. The performance degradation of the MWF grows as the signal level increased.

Turning to the performance verses rank for a 0 dB signal in Figure 28, one first notices that the performance of CSM and the MWF peaks at a rank of 4 as opposed to 5. Estimation of the 5^{th} basis vector is corrupted by signal and better performance results from only using 4 basis vectors. As expected, performance for ranks below the number of jammers is significantly better for the MWF and CSM than the PCI method. However, once the rank is overestimated the performance of the MWF and CSM decrease rapidly for reasons discussed previously. The signal level is increased in Figure 29 to 24dB. Most notable in this plot is the relatively poor performance of the CSM method for all ranks.

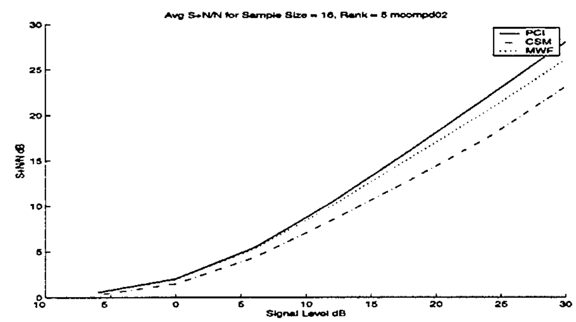

Figure 27: Performance as a Function of Signal Level with 5 Jammers

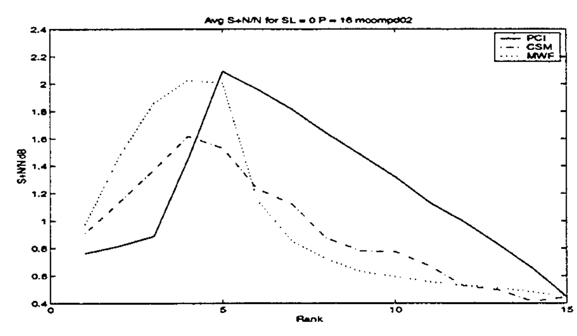

Figure 28: Performance as s Function of Rank for 0dB Signal with 5 Jammers

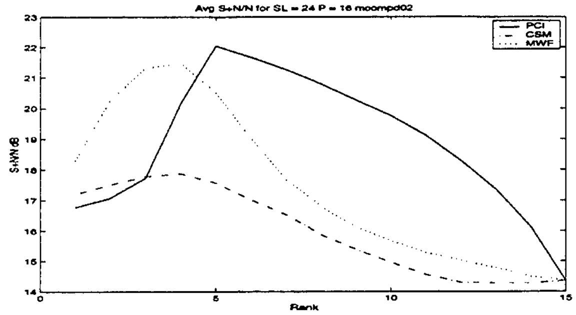

Figure 29: Performance as s Function of Rank for 24dB Signal with 5 Jammers

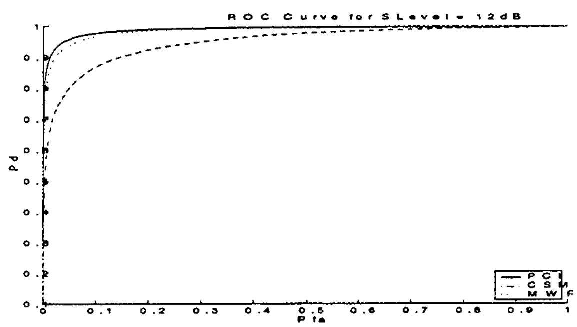

Figure 30: Receiver Operating Characteristic for 12dB Signal

As a final way of looking at the performance of the methods, the Receiver Operating Characteristic (ROC) curves for a 12dB signal level are plotted in Figure 30. The curve was generated using 2^{15} samples. The performance loss of both MWF and CSM is evident.

Overall, the experiments validate the insights that were gained by examining our toy example.

4. CONCLUSIONS

The Cross Spectral Metric (CSM) and the Multistage Wiener Filter (MWF) are two recently introduced alternatives to the Principal Components Inverse (PCI) method of rank reduction for adaptive detection. By gaining insight into the parameters that each method utilizes and the estimation characteristics of those parameters, one can predict how each method will perform under differing scenarios. PCI selects basis vectors by use of an SVD and selects a subspace based upon singular values. The subspace of the SVD is stable under conditions of strong power. CSM selects basis vectors by use of an SVD but then selects a subset based upon correlation with the desired channel. Thus the basis vectors are chosen with respect to power but then a subset is selected by use of the cross spectral metric. Since singular vectors are not necessarily stable, even though a subspace is, CSM has difficulty since the metric depends upon the singular vectors rather than the entire subspace. The MWF forms and chooses basis vectors based upon the cross covariance with the desired channel which is a combination of power and correlation. By creating scenarios where the estimates of these parameters are similar to those obtained by the background white noise, errors in the selection of the basis vectors can be made to occur. These errors are responsible for decreases in the detection performance of the methods.

5. REFERENCES

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